Optimal Time-Consistent Monetary Policy in the New Keynesian Model with Repeated Simultaneous Play

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Summary

- There are two definitions of “discretion” in the literature
- These definitions differ in terms of within-period timing of play
- Within-period timing makes a huge difference
- In the New Keynesian model with repeated Stackelberg play, there are multiple equilibria (King-Wolman, 2004)
- In the New Keynesian model with repeated simultaneous play, there is a unique equilibrium (this paper)
- Empirical relevance: Will the 1970s repeat itself?
Background and Motivation

Time-consistent (discretionary) policy: Kydland and Prescott (1977)

There are multiple equilibria under discretion:
- Barro and Gordon (1983)
- Chari, Christiano, Eichenbaum (1998)

Critiques of the Barro-Gordon/CEE result:
- enormous number, range of equilibria make theory impossible to test or reject
- equilibria require fantastic sophistication, coordination across continuum of atomistic agents
Background and Motivation

Literature has thus changed focus to *Markov perfect equilibria*:

- King and Wolman (2004)

King and Wolman (2004):

- standard New Keynesian model
- assume repeated Stackelberg within-period play
- there are two Markov perfect equilibria

But recall LQ literature:

- Pearlman (1994)
- assume repeated simultaneous within-period play
Comparison: Fiscal Policy

- two definitions of discretion in the tax literature
- Klein, Krusell, Rios-Rull (2004): repeated Stackelberg
- different timing assumption lead to different equilibria, welfare

In this paper:
- defining repeated simultaneous play is more subtle: Walras
- timing assumption changes not just payoffs, welfare, but多licity of equilibria
The Game $\Gamma_0$

Discretion is a game between private sector and central bank.

For clarity, begin definition of game without central bank:
- Assume interest rate process $\{r_t\}$ is i.i.d.
- Call this game $\Gamma_0$

Game $\Gamma_0$:
- Players
- Payoffs
- Information sets
- Action spaces
Game $\Gamma_0$: Players and Payoffs

1. Firms indexed by $i \in [0, 1]$:
   produce differentiated products; face Dixit-Stiglitz demand curves; have production function $y_t(i) = l_t(i)$; hire labor at wage rate $w_t$; payoff each period is profit:
   $$\Pi_t(i) = p_t(i)y_t(i) - w_t l_t(i)$$

2. Households indexed by $j \in [0, 1]$:
   supply labor $L_t(j)$; consume final good $C_t(j)$; borrow or lend a one-period nominal bond $B_t(j)$; payoff each period is utility flow:
   $$u(C_s(j), L_s(j)) = \frac{C_s(j)^{1-\varphi} - 1}{1 - \varphi} - \chi_0 \frac{L_s(j)^{1+\chi}}{1 + \chi}$$

Note: there is a final good aggregator that is not a player of $\Gamma_0$
Game $\Gamma_0$: Information Sets

Individual households and firms are anonymous:
- only aggregate variables and aggregate outcomes are publicly observed

Information set of each firm $i$ at time $t$ is thus:
- history of aggregate outcomes: $\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}$, $s < t$
- history of firm $i$’s own actions

Information set of each household $j$ at time $t$ is thus:
- history of aggregate outcomes: $\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}$, $s < t$
- history of household $j$’s own actions
Aggregate Resource Constraints

In games of industry competition:
- Bertrand
- Cournot
- Stackelberg

Action spaces are just real numbers: e.g., price, quantity

In a macroeconomic game, there are aggregate resource constraints that must be respected, e.g.:
- total labor supplied by households must equal total labor demanded by firms
- total output supplied by firms must equal total consumption demanded by households
- money supplied by central bank must equal total money demanded by households (in game $\Gamma_1$)
To ensure that aggregate resource constraints are respected, we introduce a Walrasian auctioneer

- Instead of playing a price $p_t$, firms now play a price schedule $p_t(X_t)$, where $X_t$ denotes aggregate variables realized at $t$
- this is just the usual NK assumption that firms take wages, interest rate, aggregates at time $t$ as given
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- Instead of playing a consumption-labor pair $(C_t, L_t)$, households play a joint schedule $(C_t(X_t), L_t(X_t))$
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Walrasian Auctioneer

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Walrasian auctioneer then determines the equilibrium $X_t$ that satisfies aggregate resource constraints
Game $\Gamma_0$: Action Spaces

1. Firms
   - set prices for two periods in Taylor contracts; must supply whatever output is demanded at posted price
   - firms in $[0, 1/2)$:
     - for $t$ odd, action space is set of measurable functions $p_t(X_t)$
     - for $t$ even, action space is trivial
   - firms in $[1/2, 1)$:
     - for $t$ even, action space is set of measurable functions $p_t(X_t)$
     - for $t$ odd, action space is trivial

2. Households
   - in each period, action space is set of measurable functions $(C_t(X_t), L_t(X_t))$
Note:
- all firms $i$ and households $j$ play simultaneously in each period $t$
- Walrasian auctioneer clears markets, aggregate resource constraints

Also, do not confuse action spaces here with strategies:
- a strategy is a mapping from history $h^t$ to the action space
- here, action spaces are functions of aggregate variables realized at $t$
- but strategies are unrestricted, may depend on arbitrary history of aggregate variables (until we impose Markovian restriction)
The Game \( \Gamma_1 \)

Now, extend the game \( \Gamma_0 \) to include an optimizing central bank:

- interest rate \( r_t \) is set by central bank each period
- call this game \( \Gamma_1 \)

First two sets of players (firms and households) are defined exactly as in \( \Gamma_0 \)
Game $\Gamma_1$: Central Bank

3. Central bank:

sets one-period nominal interest rate $r_t$; payoff each period is given by average household welfare:

$$\int \frac{C_s(j)^{1-\varphi} - 1}{1 - \varphi} - \chi_0 \frac{L_s(j)^{1+\chi}}{1 + \chi} \, dj$$

Central bank’s information set is the history of aggregate outcomes:

$\{C_s, L_s, P_s, r_s, w_s, \Pi_s\}, \ s < t$

Note:

- central bank has no ability to commit to future actions (discretion)
- central bank is *monolithic*, while private sector is *atomistic*
Within-Period Timing of Play

Repeated Stackelberg play:
- each period divided into two halves
- first, central bank precommits to a value for $r_t$ (or $m_t$)
- second, firms and households play simultaneously
- Walrasian auctioneer determines equilibrium
  note: one can drop the Walrasian auctioneer here if willing to ignore out-of-equilibrium play by positive $\mu$ of firms, households

Repeated simultaneous play:
- firms, households, and central bank all play simultaneously
- Walrasian auctioneer determines equilibrium
  note: Walrasian auctioneer is crucial, cannot be dropped (central bank is nonatomistic)
Game $\Gamma_1$: Action Spaces

In defining the game $\Gamma_1$, we assume repeated simultaneous play:

- firms $i$, households $j$, and central bank all play simultaneously in each period $t$
- action spaces of firms, households are same as in $\Gamma_0$
- for central bank, action space each period is set of measurable functions $r_t(X_t)$ (simultaneous play)
- Walrasian auctioneer clears markets, aggregate resource constraints

Again, do not confuse action spaces with strategies:

- strategies are unrestricted, may depend on arbitrary history of aggregate variables (until we impose Markovian restriction)
Why Assume Simultaneous Play?

Practical considerations/realism:
- Makes no difference whether monetary instrument is $r_t$ or $m_t$
- Central banks monitor economic conditions continuously, adjust policy as needed

Theoretical considerations:
- Why treat central bank, private sector so asymmetrically?
- LQ literature (Svensson-Woodford 2003, 2004, Woodford 2003, Pearlman 1994, etc.) assumes simultaneous play
- Investigate sensitivity of multiple equilibria to within-period timing
Solving for Markov Perfect Equilibria

- State Variables of the Game $\Gamma_1$
- Policymaker Bellman Equation
- Markov Perfect Equilibria of the Game $\Gamma_1$
State Variables of the Game $\Gamma_1$

There are two sets of state variables for the game $\Gamma_1$ (and also $\Gamma_0$):
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- distribution of household bond holdings, $B_{t-1}(j), j \in [0, 1]$
- two measures of the distribution of inherited prices:

\[
\int p_{t-1}(i)^{-1/\theta} \, di
\]

and

\[
\int p_{t-1}(i)^{-\frac{(1+\theta)}{\theta}} \, di
\]
State Variables of the Game $\Gamma_1$

However, starting from symmetric initial conditions in period $t - 1$:

Proposition 1:
- household optimality conditions imply all households play identically in period $t$ in any subgame perfect equilibrium of $\Gamma_1$
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That is, starting from symmetric initial conditions in period $t_0$, we show these state variables are degenerate in any subgame perfect equilibrium of $\Gamma_1$ for all times $t \geq t_0$. 
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We henceforth restrict definition of game $\Gamma_1$ to case of symmetric initial conditions in period $t_0$. 
Policymaker Bellman Equation

\[ V_t = \max_{\{r_t\}} \left\{ \int \frac{Y_t(j)^{1-\varphi}}{1 - \varphi} - \chi_0 \frac{L_t(j)^{1+\chi}}{1 + \chi} \, dj + \beta E_t V_{t+1} \right\} \]
Policymaker Bellman Equation

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subject to:

\[ \frac{L_t}{Y_t} = 2^\theta \frac{1 + x_t^{(1+\theta)/\theta}}{(1 + x_t^{1/\theta})^{1+\theta}}, \]

\[ Y_t^{-\varphi}(1 + x_t^{1/\theta}) = \beta(1 + r_t)h_{1t}, \]

\[ 2^{-\theta}(1+x_t^{1/\theta})^\theta \left[ Y_t^{1-\varphi} + \beta(1 + x_t^{1/\theta})h_{2t} \right] = (1+\theta)\chi_0 \left[ Y_t L_t^x + \beta(1+x_t^{1/\theta})^{1+\theta} h_{3t} \right]. \]
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where expectations of next period variables are given functions of this period’s economic state: \( h_{1t}, h_{2t}, h_{3t} \) (discretion)
Markov Perfect Equilibria of the Game $\Gamma_1$

In any Markov Perfect Equilibrium of $\Gamma_1$, state variables are degenerate (only operative off of the equilibrium path)
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As a result, along the equilibrium path:

\[
\begin{align*}
    h_{1t} &= E_t Y_{t+1}^{-\varphi}(1 + x_{t+1}^{-1/\theta}) = h_1 \\
    h_{2t} &= E_t \frac{Y_{t+1}^{1-\varphi}}{1 + x_{t+1}^{-1/\theta}} = h_2 \\
    h_{3t} &= E_t \frac{Y_{t+1} L_{t+1}^x}{(1 + x_{t+1}^{-1/\theta})^{1+\theta}} = h_3
\end{align*}
\]
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$$h_{3t} = E_t \frac{Y_{t+1} L_{t+1}^x}{(1 + x_{t+1}^{-1/\theta})^{1+\theta}} = h_3$$

Note: we will not write out how play evolves off of the equilibrium path, but simply assert that it agents will continue to play optimally (Phelan-Stachetti, 2001)
Solving for Markov Perfect Equilibria

Solve: \[ V_t = \max_{\{r_t\}} \left\{ \frac{Y_t^{1-\varphi}}{1-\varphi} - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1} \right\} \]
subject to:

\[ \frac{L_t}{Y_t} = 2^\theta \frac{1 + x_t^{(1+\theta)/\theta}}{(1 + x_t^{1/\theta})^{1+\theta}}, \]

\[ Y_t^{-\varphi}(1 + x_t^{1/\theta}) = \beta(1 + r_t)h_1, \]

\[ 2^{-\theta}(1+x_t^{1/\theta})^\theta \left[ Y_t^{1-\varphi} + \beta(1 + x_t^{1/\theta})h_2 \right] = (1+\theta)\chi_0 \left[ Y_t L_t^\chi + \beta(1+x_t^{1/\theta})^{1+\theta} h_3 \right]. \]

where \( h_1, h_2, h_3 \) are exogenous constants.
Solving for Markov Perfect Equilibria

Solve: \[ V_t = \max_{\{r_t\}} \left\{ \frac{Y_t^{1-\varphi}}{1-\varphi} - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1} \right\} \]
subject to:
\[ \frac{L_t}{Y_t} = 2^{\theta} \frac{1 + x_t^{(1+\theta)/\theta}}{(1 + x_t^{1/\theta})^{1+\theta}}, \]
\[ Y_t^{-\varphi} (1 + x_t^{1/\theta}) = \beta (1 + r_t) h_1, \]
\[ 2^{-\theta} (1 + x_t^{1/\theta})^\theta \left[ Y_t^{1-\varphi} + \beta (1 + x_t^{1/\theta}) h_2 \right] = (1+\theta) \chi_0 \left[ Y_t L_t^\chi + \beta (1 + x_t^{1/\theta})^{1+\theta} h_3 \right]. \]

where \( h_1, h_2, h_3 \) are exogenous constants.

Finally, impose equilibrium conditions: \( h_1 = E_t Y_{t+1}^{-\varphi} (1 + x_{t+1}^{-1/\theta}), \)
\( h_2 = E_t \frac{Y_{t+1}^{1-\varphi}}{1 + x_{t+1}^{1/\theta}}, \) \( h_3 = E_t \frac{Y_{t+1} L_{t+1}^\chi}{(1 + x_{t+1}^{-1/\theta})^{1+\theta}}. \)
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Solve: \[ V_t = \max_{\{r_t\}} \left\{ \frac{Y_t^{1-\varphi}}{1-\varphi} - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1} \right\} \]

subject to:

\[ L_t = \frac{2^\theta \left(1 + x_t^{(1+\theta)/\theta}\right)}{(1 + x_t^{1/\theta})^{1+\theta}}, \]

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\[ 2^{-\theta}(1+x_t^{1/\theta})^{\theta} [Y_t^{1-\varphi} + \beta(1 + x_t^{1/\theta})h_2] = (1+\theta)\chi_0 [Y_t L_t^{\chi} + \beta(1+x_t^{1/\theta})^{1+\theta} h_3]. \]

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\( h_2 = E_t \frac{Y_{t+1}^{1-\varphi}}{1+x_{t+1}^{-1/\theta}}, h_3 = E_t \frac{Y_{t+1} L_{t+1}^{\chi}}{(1+x_{t+1}^{-1/\theta})^{1+\theta}}. \)

Note: there can still be multiplicity here, e.g. if \( h_1, h_2, h_3 \) are “bad”
Proposition 6: The inflation rate $\pi$ in any Markov Perfect Equilibrium of the game $\Gamma_1$ must satisfy the condition:

$$\frac{1 + \beta \pi^{(1+\theta)/\theta}}{1 + \beta \pi^{1/\theta}} \times \frac{1 + \pi^{1/\theta}}{1 + \pi^{(1+\theta)/\theta}} \times \left\{ \begin{array}{l}
1 - \frac{(\pi - 1) \left[ 1 + \chi - (1 - \varphi) \frac{1 + \beta \pi^{(1+\theta)/\theta}}{1 + \beta \pi^{1/\theta}} \right]}{(\pi - 1) \left[ 1 - (1 - \varphi) \frac{1 + \beta \pi^{(1+\theta)/\theta}}{1 + \beta \pi^{1/\theta}} \right] + (1 + \pi^{(1+\theta)/\theta}) \left[ 1 - \frac{1}{1 + \theta} \frac{1 + \beta \pi^{(1+\theta)/\theta}}{1 + \beta \pi^{1/\theta}} \right]} \end{array} \right\} = \frac{1}{1 + \theta} \quad (\star)$$
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\]

Proposition 7: Let $\varphi = 1$, $\chi = 0$, and $\beta > \max\{1/2, 1/(1 + 2\theta)\}$. Then there is precisely one value of $\pi$ that satisfies equation (\ast).
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\frac{1 + \beta \pi^{(1+\theta)/\theta}}{1 + \beta \pi^{1/\theta}} \times \frac{1 + \pi^{1/\theta}}{1 + \pi^{(1+\theta)/\theta}} \times \frac{(\pi - 1)}{(\pi - 1)^2 + (1 - \varphi) \frac{1+\beta \pi^{(1+\theta)/\theta}}{1+\beta \pi^{1/\theta}}} + (1 + \pi^{(1+\theta)/\theta}) \left[ 1 - \frac{1}{1+\theta} \frac{1+\beta \pi^{(1+\theta)/\theta}}{1+\beta \pi^{1/\theta}} \right] = \frac{1}{1+\theta}
\]

Proposition 7: Let $\varphi = 1$, $\chi = 0$, and $\beta > \max\{1/2, 1/(1 + 2\theta)\}$.

*Then there is precisely one value of $\pi$ that satisfies equation (*)*. 

Note:

- $\varphi = 1$, $\chi = 0$ are not special, but simplify algebra in proofs
- there is a unique equilibrium for wide range of parameters
- confirmed by extensive numerical simulation in Matlab
Conclusions

- There are two definitions of “discretion” in the literature.
- These definitions differ in terms of within-period timing of play.
- Within-period timing makes a huge difference.
- In the New Keynesian model with repeated Stackelberg play, there are multiple equilibria (King-Wolman, 2004).
- In the New Keynesian model with repeated simultaneous play, there is a unique equilibrium (this paper).
- Open questions: other NK models, models with a (nondegenerate) state variable.