The Role of the Military Draft in Optimal Fiscal Policy*

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I. Introduction

Recent contributions to the theory of optimal taxation have simultaneously contributed to the theory of optimal public debt. In particular, Barro [2], who effectively interprets the government’s optimal taxation problem as a dynamic control problem, demonstrates that optimal debt creation serves to smooth out the distortions associated with income taxes over time. In his analysis, the value of outstanding debt is determined uniquely by the theory of optimal taxation, which roughly dictates constant tax rates over time.¹

This paper introduces a military draft into the theory of optimal fiscal policy. In addition to taxing labor income and borrowing, the government can conscript labor to mobilize the economy. Studying the optimality of the draft in an intertemporal, general equilibrium framework, under the assumption that government commitments to future debt servicing and draft policies are irrevocable, permits the analysis to illustrate how the presence of the draft can influence the cross-sectional and time-series properties optimal fiscal policy, including income taxation and debt creation.

The analysis formalizes the point made by Friedman [5], among others, that although conscription of labor generates a deadweight loss, if government spending is extremely and temporarily high,—i.e., during large scale wars—then optimal fiscal policy can involve the military draft. The basic idea here is that the government trades off the distortions arising from the draft against distortions arising from the income tax.² Optimality implies that the two instruments of fiscal policy are negatively related.

*The author is grateful to Peter M. Garber and anonymous referees for helpful comments on earlier versions of this paper. The opinions expressed in this paper are those of the author and do not necessarily represent the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

1. Kydland and Prescott’s seminal paper [8], however, suggests that the approach to the study of the macroeconomic control problem adopted by Barro, among others, can be misleading. Specifically, future income tax policies considered to be optimal, if the government had complete control over individuals’ expectations of future policies, generally fail to be considered optimal in the future. As a result, that sequence of future policies fails to be time consistent. As emphasized by Kydland and Prescott [9], the feasibility of the optimal policy crucially hinges on the government’s ability to adhere to the rule of constant tax rates. Lucas and Stokey [10], who study optimal fiscal policy in a more articulated general equilibrium model assuming that a government commitment to future debt servicing is irrevocable, find that debt with a sufficiently rich maturity structure provides the mechanism by which the government can make the optimal fiscal policy time consistent. In their analysis, which supports the general implication of Barro’s study [2], the optimal, time consistent fiscal policy implies a unique maturity structure of outstanding debt.

2. Recently, in a complementary analysis of military manpower procurement, in which the military draft is precluded, Harford and Marcus [7] demonstrate that if the risk of danger to civilians and soldiers is decreasing in the number
To show how peacetime fiscal policy depends on future (potential) involvement in a war, when it might be optimal to impose the military draft, the model is simulated. Numerical examples illustrate that when it is optimal to impose a military draft, although consumption will be smoother than predicted by a model in which the draft is precluded, explicit (labor) tax rates can more volatile and the deficit can be smoother. Nevertheless, the analysis is consistent with Barro’s finding [2], that debt creation smooths out the distortions associated with labor tax rates over time. But to the extent that the draft enables the government to reduce the explicit (labor tax) burden associated with a given amount of government spending, the draft lowers the amount of distortions that debt creation otherwise would smooth out over time, and thereby serves as a partial substitute for debt creation as well as income taxation in optimal fiscal policy.

In what follows, the next section develops the analytical framework, which is a modest extension of the Lucas and Stokey [10] general equilibrium model of optimal fiscal policy—henceforth, LS. Here, the necessary conditions for optimal government policy are derived. Section III focuses on an example to provide some intuition for the nature of the solution to the government’s policy, given that one exists, and how that solution might differ from that in the LS model where the draft is not available as an instrument for fiscal policy. Section IV presents some preliminary evidence supporting the theory. Finally, section V offers some concluding remarks.

II. A Model of Optimal Fiscal Policy

To examine the possibility of the emergence of the draft as a part of optimal war finance, this section considers a simple barter economy where a single, non-storable good is produced. The only source of uncertainty in the economy is the government consumption of this good, the amount of which is given exogenously by a stochastic process. Using LS notation, $F$ denotes the joint distribution of the realizations of this consumption, $g \equiv (g_0, \ldots, g_e)$ where $g$ is expressed in per capita terms; the marginal distribution of $g' \equiv (g_0, \ldots, g_e)$, the vector of the current and future government consumption until time $t$, is given by $F_t'$; and the distribution of $g_s' \equiv (g_s, \ldots, g_t)$, which is the vector of government consumption from time $s$ to time $t$, conditioned on $g^{s-1}$, is given by $F_{s}$. It is assumed that these distributions are known by all agents in the economy.

In each period, individuals are endowed with one unit of labor. The technology is partially defined by the following equation:

$$c_t + x_t + d_t + a_t \leq 1 \quad (1a)$$

where,

- $c_t = \text{private consumption of goods produced in } t$.
- $x_t = \text{consumption of leisure in } t$.
- $d_t = \text{drafted labor to produce } g \text{ in } t$.
- $a_t = \text{labor hired from the market to produce } g \text{ in } t$.

of soldiers employed and the marginal rate of decrease is greater for soldiers than for civilians, the supply curve for volunteer soldiers can be downward sloping. In this case, an exogenous increase in the threat to civilians or to soldiers alone (for example, during a large scale war) will call for an increase in the number of volunteer soldiers and a decrease in the military wage. Nevertheless, the tax rate will rise. Given that the higher tax rate generates greater deadweight losses, the military draft might arise as an optimal policy in their general equilibrium framework.

3. Fisher [4], for example, estimates that eliminating the draft in the U.S. would have increased the payroll cost of maintaining a given amount of military manpower by about 5 to 7.5 billion dollars.
The technology is characterized by constant returns to scale in that one unit labor can produce one unit of the good for private consumption or one unit of leisure consumption, or it can be allocated towards the production of $g_t$. The government has the option to draft the labor necessary to produce a portion or the full amount of a given level of $g_t$ or simply to allow the private market to produce it. In the case that $d = 0, a = g$ and the model specializes to LS. When $d_t$ is positive, all individuals must allocate $d_t$ of the labor endowment to the production of $g_t$. In other words, the draft is a universal service.\(^4\)

From the perspective of the individual, in the context of the present analysis, the universal draft system is a lump sum tax. Given the amount of labor drafted, individuals are free to choose the allocation of the remainder of their labor endowment among the production of goods in the private market and leisure. Because the amount of the individual’s endowment that is drafted does not depend explicitly on this allocation, the draft does not distort relative (contemporaneous) prices of private consumption goods and leisure in the competitive equilibrium.

In order to capture the negative effects of drafting labor to produce $g_t$, i.e., those costs not included in the notion of the draft as a lump sum tax—it is assumed that there is a productivity loss when labor is drafted. In particular, the technology for the production of the goods to be consumed by the government is given by

$$g_t = d_t(1 - \varepsilon) + a_t,$$  \(^{(1b)}\)

where $\varepsilon$ denotes the loss in productivity. According to (1b), if $g_t$ is interpreted as wartime expenditures, then there is no peacetime draft since the draft variable can only be positive when $g_t$ is positive. Alternatively, if $g_t$ is interpreted as defense expenditures, including preparation for war, then a temporarily high level of $g_t$ would represent wartime expenditures. A peacetime draft would be consistent with this more general interpretation of $g_t$. In any case, given $g_t$, the government chooses $d_t$, and so implicitly chooses $a_t$.

The presence of $\varepsilon$ reflects the notion that drafted labor is less productive than the labor supplied by an individual at his own choice. One possible interpretation of $\varepsilon$ is that the draft does not result in the minimization of the opportunity costs of allocating labor resources among the military and civilian sectors. For example, a large value of $\varepsilon$ might correspond to a lottery system, in which little or no effort is made by the government to identify and draft labor that is more productive in defense activities than in private market activities. And, a small value of $\varepsilon$ might be associated with a selective service, in which a set of criteria are established in an attempt to draft those individuals whose opportunity costs of serving in the military are smallest. The main idea is that because of the productivity loss, the draft reduces the sum of goods available for private consumption and the amount of leisure consumption that is feasible for a given level of goods to be consumed by the government.\(^5\) The total deadweight loss of the draft in time $t$ is $d_t \varepsilon$.\(^6\)

\(^4\) By modelling the draft as a universal system, the analysis abstracts from some important distortionary effects of the draft. Specifically, the formulation does not capture the distortionary behavior of individuals induced by a system of determents and exemptions, i.e., a selective service—and by the additional element of uncertainty created by a lottery system. (See Hansen and Weisbrod [6] for a detailed discussion of the different economic effects of the military draft in its various forms. For a survey of the existing theoretical and empirical studies of the military draft, see Amacher et al. [1].)

\(^5\) It should be noted that although the draft has a constant marginal deadweight loss in labor units, the marginal value in terms of utility units is increasing in the size of the draft since utility is concave.

\(^6\) On the basis of his projections of the composition of a hypothetical volunteer force, Oi [11] estimates that the draft reduced the payroll cost of securing an active duty force of 2.65 million for the fiscal year 1965 by approximately
Following the strategy of LS, assume that the infinitely-lived individuals in the economy have identical preferences and wish to maximize expected utility. Preferences of any individual are embodied in the following von Neumann-Morgenstern utility function:

$$E_0(U_0) = \sum_{t=0}^{\infty} B^t \int u(c_t, x_t) dF_t, \quad 0 < \beta < 1,$$

where $E_0(\cdot)$ denotes the expectations operator conditioned on information available in time period $s$, and $\beta$ is the constant discount factor. $u$ is assumed to be increasing in leisure and the private consumption good, both of which are non-inferior, and is assumed to be strictly concave.

In the presence of lump sum taxes, optimal allocations would satisfy (1) and the following marginal condition:

$$u^i_t = u^j_t,$$

where $u^i_t$ denotes the marginal utility with respect to $i$, evaluated at the consumption plan in time $t$. If lump sum income taxes were possible, the optimal tax would be set equal to the realization of $g_t$. In this case, $a_t = g_t$. Without lump sum taxes, the government can finance its consumption through a tax, $\tau_t$, proportional to labor income, $(1 - x_t - d_t)$, through debt creation, $i \cdot b$ and/or through the draft. In the special case in which $\varepsilon = 0$, only conscription would be optimal. In this case, $d_t = g_t, \tau_t = i \cdot b = 0$, and the optimal allocation of resources would satisfy (1) and (3).

In the more general case, in which $\varepsilon > 0$, the government might find it desirable to resort to levying an income tax, in addition to, or rather than implementing the draft. Furthermore, the government might choose to issue debt: $i \cdot b = \{i \cdot b_t\}_{t=1}^{\infty}$, where each element, $i \cdot b_t$, is a claim (possibly negative) held by individuals at the end of $t$, to consumption in time $s$ contingent on $g_t$, given $g_t^{t-1}$ having occurred.

Before characterizing the optimal fiscal policy, it is first necessary to study the optimization problem of the representative consumer, who takes prices, the income tax rate, the draft and government consumption as given. In each period $t$, $g_t$ and the marginal (conditional) distribution function are known to all agents. Individuals trade in the market for current consumption goods and labor, and they trade in a complete market for contingent claims for future consumption. Given consumer behavior, which takes the government’s behavior as given, it is possible to determine the competitive equilibrium. Then, given the competitive equilibrium condition, it is possible to study the optimal behavior of the government, who acts to promote the interests of the “representative” consumer. Taking the impact of its policy actions on consumer behavior into consideration, the government chooses how to finance its current expenditures, including its current consumption and its current debt obligations.

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$4$ billion. He also estimates that the full economic cost of the draft, or equivalently, the implicit taxes levied on military personnel was about $5.4$ billion. The divergence between the full economic cost of the draft and the financial benefit of the draft, as measured by the reduction in resources required to maintain a military force of a given size, is due to the differences of the compositions of a hypothetical volunteer force and the actual mixed force. Because, with the draft, the government is not employing labor most productive for defense purposes, the draft generates a deadweight loss, which Oi estimates to be about $1.4$ billion.

7. Because the government acts to promote the interests of the “representative” individual and the draft involves a universal military service, the present analysis abstracts from the important distributional effects of the draft. As such, the analysis provides an explanation of the emergence of the draft complementary to those existing, positive analyses that build primarily on distributional considerations—for example, Tollison [13] and Wagner [16].
The Consumer’s Optimization Problem

The representative consumer takes the sequence of contingent taxes, \( \tau = \{\tau t\}_{t=0}^{\infty} \) and \( d = \{d t\}_{t=0}^{\infty} \), and prices, \( p = \{p t\}_{t=0}^{\infty} \) as given. The price of a claim to a unit of current consumption is \( p t(g t) \). Similarly, for a given \( g' \), \( p s(g s, g t+1) \) denotes the price of a claim to a unit of consumption in period \( s \) contingent on \( g t+1 \). In any time \( t \), the individual trades current consumption, \( (c t, x t) \), and holdings of contingent claims, \( t+1 b \), and chooses or plans an optimal sequence of future consumption, \( (c, x) = \{c s, x s\}_{s=t+1}^{\infty} \). The individual’s choice is the solution to the maximization of (2) subject to the following constraint:

\[
p o(c 0 - (1 - \tau 0)(1 - x 0 - d 0) - 0 b) + \sum_{t=1}^{\infty} \int p t(c t - (1 - \tau t)(1 - x t - d t) - 0 b t) d F t' \leq 0,
\]

(4)

where \( \tau, p, d, F \) and \( g 0 \) are known and taken as given, and \( 0 b \) is given historically. As is obvious from the budget constraint, drafted labor is not compensated. Accordingly, the representative individual views the draft as a pure lump sum tax in kind. \(^8\)

Provided that the solution to the consumer’s problem is interior, as assumed, equations (1) and (4) as equalities and the following first order conditions characterize that solution:

\[
\frac{u t'}{u t} = 1 - \tau t
\]

(5a)

\[
\beta'(u t'/u t) = p t
\]

(5b)

for \( t = 0, 1, \ldots \), all \( g' \), and where \( p 0 = 1 \). The assumption that \( u(c t, x t) \) is strictly concave implies that the solution \( (c t, x t) \) is unique.

To ensure that the consumer’s plan of future consumption \( (ct+1, xt+1) \) is feasible for any realization of \( gt+1 \), given \( g' \), bond holdings in the beginning of the next period, \( t+1 b \), must satisfy the following condition:

\[
p t+1 t+1 b t+1 + \sum_{s=t+2}^{\infty} \int p t+1 t+2 b s d F t+2 s = p t+1(c t+1 - (1 - \tau t+1)(1 - x t+1 - d t+1)) + \sum_{s=t+2}^{\infty} \int p s(c s - (1 - \tau s)(1 - x s - d s)) d F t+2 s,
\]

(6)

for \( t = 0, 1, \ldots, \infty \) and all \( gt+1 \), given \( g' \). LS demonstrate that the required \( t+1 b \) is always in the consumer’s budget set.

The Competitive Equilibrium

Having identified the necessary conditions for an optimal solution to the consumer’s problem, it is possible to determine the resource allocation in a competitive equilibrium. For any given \( \tau, d, \) and \( F \), equations (1) and (5) uniquely determine this allocation. The prices that support this allocation are determined from (5b). Using (4) as an equality and (5) yields the following compact condition that must hold in a competitive equilibrium:

\(^8\) The introduction of a wage for drafted labor, presumably less than one, makes the solution more complicated without providing any additional insight.
\[
\sum_{t=0}^{\infty} \beta^t \int (((c_t - b_t)u_t^c - (1 - x_t - d_t)u_t^x)dF^t = 0. 
\]

Equations (1) and (7) now completely describe the competitive equilibrium given \(F, \tau,\) and \(o b\).

The Government’s Optimization Problem

Above, the government’s finance policy was taken as given. This section examines the optimal behavior of the government who takes the behavior of consumers into consideration. Given \(F\) and \(o b\), the government can induce the resource allocation defined by (1) and (7), by choosing the sequences \(\tau\) and \(d\) that satisfy (5a). Then equation (5b) determines the prices supporting this allocation. To make this allocation feasible, the government might have to sell or purchase bonds at market prices. In doing so, the end of period debt, \(i b\), must satisfy the expression in (6).

In order to study the optimal behavior of the government, it is assumed that the government is able to bind itself to any chosen sequence of taxes, \(\tau\) and \(d\), and that it is able to commit itself not to repudiate its debt.\(^9\) Furthermore, the analysis assumes that the government’s objective is to maximize the welfare of the representative consumer. Accordingly, in time period \(t = 0\), the government maximizes (2) subject to (1) and (7), and chooses the optimal contingent allocation of resources and the corresponding sequences of \(\tau\), \(d\), \(i b\) which are necessary to induce that allocation, given \(o b\), \(g_0\) and \(F\).

Letting \(\mu_{o} \geq 0\) and \(\nu_{o} \leq 0\) be the multipliers associated with (1a) and (1b) respectively, and letting \(\lambda_0\) be the multiplier associated with the competitive equilibrium constraint, (7), the first order conditions to the government’s problem are given by the following:

\[
(1 + \lambda_0)u'^c_t + \lambda_0((c_t - b_t)u'^c_{c_t} - (1 - x_t - d_t)u'^c_{x_t}) - \mu_{o, t} = 0 
\]

\[\text{(8a)}\]

\[
(1 + \lambda_0)u'^x_t + \lambda_0((c_t - b_t)u'^x_{c_t} - (1 - x_t - d_t)u'^x_{x_t}) - \mu_{o, t} = 0 
\]

\[\text{(8b)}\]

\[
\lambda_0 u'^t_t - \mu_{o, t} - \nu_{o, t}(1 - \varepsilon) = 0 \quad \text{or,} \quad d_t = 0 
\]

\[\text{(8c)}\]

\[
- \mu_{o, t} - \nu_{o, t} = 0 \quad \text{or,} \quad a_t = 0. 
\]

\[\text{(8d)}\]

Maintaining the assumption that the solution \((c_t, x_t)\) is interior, the conditions given by (8) must hold for all \(g^t\), \(t = 0, 1, \ldots, \infty\). In the special case in which \(\varepsilon\) is equal to one, from (1), \(a_t = g_t\) and \(d_t = 0\) for all \(t\). In this case, the model specializes to LS. In another special case in which \(\varepsilon = 0\), as mentioned above, \(d_t = g_t\) and \(a_t = 0\). More generally, where \(0 < \varepsilon < 1\), it is not clear from these conditions alone whether \(d_t = g_t/(1 - \varepsilon)\) and \(a_t = 0\), or \(d_t = 0\) and \(a_t = g_t\), or there is some interior solution, where both \(a_t\) and \(d_t\) are strictly positive. The answer to this question depends not only on the preferences of the representative agent, but on the value of \(\varepsilon\) and the particular stochastic process for government consumption.

In the case where \(a_t\) and \(d_t\) are optimally chosen to be strictly positive, (8c) and (8d) are combined to eliminate \(\nu_{o, t}\), thereby giving the following condition:

\[
\lambda_0 u'^t_t - \mu_{o, t} \varepsilon = 0 
\]

\[\text{(8d)}\]

\(^9\) In contrast to LS, even when debt repudiation is precluded, the optimal fiscal policy \((\tau_t)_{t=0}^{\infty}\) and \((d_t)_{t=0}^{\infty}\) is not time consistent. Specifically, the inclusion of the draft as a possible instrument for finance drives a wedge between the optimal policy when precommitment is possible and the time consistent policy because time-consistency generally requires a commitment instrument for each distortionary (and discretionary) instrument. The present model effectively precludes time consistency since it introduces a distortionary instrument without also providing an additional commitment instrument.
or, equivalently

$$\lambda_0 u_i^t (1 - \tau_i) - \lambda_0 \varepsilon = 0,$$  

(9)

by (5a). The above expressions represent the marginal utility from decreasing the amount of expenditures to be financed through labor taxation or debt creation by increasing the amount of drafted labor. The first term, $\lambda_0 u_i^t$, or $\lambda_0 u_i^t (1 - \tau_i)$ is the marginal effect of the decreased proportional taxation net of the effect of the decreased tax base. As shown by Persson and Svensson [12], $\lambda_0 u_i^t$ indicates the change in utility from a hypothetical switch from a distortory income tax to a lump sum tax, for a given level of government consumption. The term $-\lambda_0 u_i^t \tau_i$ is the effect on utility from the decrease in the tax base associated with an increase in drafted labor. The second term in (9), $-\lambda_0 \varepsilon$, is the marginal, pure resource effect arising from the productivity loss. The conditions in (9) state that these marginal effects must sum to zero.

The fundamental idea here is that the government trades off the losses arising from the distortory effects of the draft against the losses arising from the distortions associated with the income tax. It is important to note that the predicted relation between the two fiscal instruments is negative, since the marginal “benefit” of the draft is negatively related to the labor tax rate. For a given $g_t$, an increase in $d_t$ implies a decrease in $\tau_t$. This implication is tested in section IV.

In any case, the conditions given by (8) generally characterize the optimal solution to the government’s optimization problem by characterizing the optimal trade-off of the distortory effects. As in LS, because the second order conditions involve third derivatives, the questions of existence and uniqueness of a solution are not easily resolved. The appendix derives the necessary conditions for existence and analytical solutions for the case of quadratic utility. Provided that a solution does exist, it must satisfy (1), (7), and (8). Specifically, one could, in principle, derive explicit solutions. Equations (1) and (8) would yield solutions for $c$, $x$, and $d$ in terms of $0 \bar{b}$, $g$, $\varepsilon$, and $\lambda_0$. Equation (7) could then be used to solve for $\lambda_0$ in terms of the exogenous variables. The expression in (5a) would determine the sequences $d$ and $\tau$ necessary to induce this allocation, and the expression in (5b) would determine the supporting prices. To ensure that the government can meet its current expenditures, given the optimal choices of $d$ and $\tau$, it might have to restructure the debt, making sure that (6) is satisfied. The requirement imposed by (6) only places a restriction on the total value of outstanding debt, since the commitment to future tax policies implies that the relative intertemporal (contingent) prices satisfying (5b) are determined for all time. 10

10. By assuming that, in addition to no debt repudiation, the government can commit to the optimal sequence for the draft, $(d_t)_{t=1}^\infty$, then, as in LS, it is possible to verify that the government can make the optimal tax policy $(\tau_t)_{t=1}^\infty$ time consistent by manipulating the maturity structure of outstanding debt to satisfy the following,

$$\lambda_1 b_t = \lambda_0 b_t + (1 - \lambda_0) b_t$$

for $t = 1, 2, \ldots, \infty$ and all $g^t$, where

$$h_t = ((u_{r_t} - u_{r_{t-1}}) + (u_{c_t} - u_{c_{t-1}})c_t + (u_{t} - u_{t-1})(1 - x_t - d_t))(u_{c_t} - u_{c_{t-1}})^{-1},$$

and $\lambda$ equals the multiplier on the competitive equilibrium constraint faced by the government in $t = 1$. By restructing debt such that the above condition is satisfied, the current administration can induce the next administration to carry out the optimal tax policy originally announced in $t = 0$, $(\tau_t)_{t=1}^\infty$, assuming that it can bind future administrations to follow the optimal draft sequence chosen in $t = 0$, $(d_t)_{t=1}^\infty$. Similarly, the future administrations can restructure the debt according to a condition analogous to the expression above and ensure that their successors will adhere to that same optimally chosen
III. The Role of the Draft in Optimal Policy: Some Numerical Examples

Given the complexity of the model, it is virtually impossible to examine the impact of the availability of the draft on the optimal fiscal policy by directly using the equations that characterize the solution to the government’s problem. This section simulates the model for examples in which the parameters for the utility function, the amount of historically given debt and the level of government consumption are specified. The examples, which differ with respect to the specification of the process of government consumption, illustrate how the cross-sectional and time-series properties of optimal fiscal policy, including income taxation and debt creation depend on the presence of the draft and the magnitude of its distortionary effect.

Tables I and II report the solutions to the government’s problem in a three period example, where \( u(c, x) = c + x + (1/2)c x - (1/2)(c^2 + x^2) \), \( \beta = 1 \), \( gb = (0, 0, 0) \), and \( g = (G^p, G^p + G^T, G^p) \) for different values of \( G^p \), \( G^T \) and \( \epsilon \). The tables also report the solutions to the LS model—i.e., assuming that \( d_t = 0 \).

Provided that the productivity loss is sufficiently low, the draft provides the means by which the burden of the war (\( G^T \)) felt in all \( t \), through the distortionary tax, is alleviated. (See Table I.) In particular, a comparison of the solutions for the present model to the solutions for the LS model reveals that the draft enables the government to approach the first best solution, \( \bar{c} = (1/2)(1 - G^p) \), during “peacetime” periods and that the use of the draft dampens the percentage fluctuations in consumption over time, relative to LS. (See Table II.) The tax rate will be lower and the absolute value of outstanding debt in the war and post-war periods will be lower than in the nation where the draft is not permitted. One striking result is that percentage changes in the tax rate are larger than predicted by the LS model.

Notice that \( d_t \) and \( \tau_t \) are negatively related as predicted above. Furthermore, it is important
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a. Since $c_0 = c_2$, $x_0 = x_2$, and $\tau_0 = \tau_2$ only $c_0$, $x_0$, and $\tau_0$ are reported. The reported claims, $t_{b_1}$, $t_{b_2}$, and $z_{b_2}$, are determined by the condition for the time consistency of the optimal labor tax. See footnote 10.
to see that the draft serves not only as a partial substitute for income taxes, but for debt creation. Specifically, the magnitude of the change in the value of outstanding debt in \( t = 0 \) and \( t = 1 \), given respectively by \( p_1 b_1 + b_2 \) and \( b_2 - (p_1 b_1 + b_2) \), is smaller than predicted by the LS model.

A close examination of Table II reveals that, for \( G^P = 0 \), the LS model and the present model have very different implications for the cross-sectional relations between the amount of transitory government consumption, \( G^T \), and private consumption, leisure, taxes, taxes and outstanding debt. Specifically, the LS model predicts that larger values of \( G^T \) will be associated with lower levels of private consumption of the produced good and greater consumption of leisure. In contrast, the present model predicts, that provided \( d_1 > 0 \) when \( G^P = 0 \), a hypothetical increase in \( G^T \) implies higher levels of private consumption of the produced good in peacetime periods and lower levels of consumption of leisure in all periods. Accordingly, the two models have opposite predictions about the cross-sectional relationship between income taxes and the amount \( G^T \). The LS model implies that income tax rates in all periods will be larger, the larger is \( G^T \), while the present model predicts that income tax rates can be smaller, provided \( d_1 > 0 \), the larger is \( G^T \). Furthermore, the LS model, as well as Barro [2], predicts a positive cross-sectional relationship between the absolute value of outstanding public debt and \( G^T \), while the present model predicts a negative relation provided that \( d_1 > 0 \).

To see how the time series relation between \( d_t \) and \( g_t \) depends on the productivity loss, \( \varepsilon \), look at the examples where \( G^P > 0 \). Not surprisingly, when \( \varepsilon \) is very small, the draft is positive in those periods when government consumption is positive. In fact, for \( \varepsilon = .05 \) and larger \( G^P \), the draft is used to the fullest extent in period \( t = 1 \). In these cases, the change in \( d_t \) from period \( t = 0 \) to period \( t = 1 \) is greater than the change in \( g_t \). For larger productivity losses associated with the draft, the draft generally is not used, except when temporary government consumption is very high. Specifically, if \( \varepsilon = .15 \), then the draft is not implemented except for \( G^T = .10 \), when the draft is imposed to finance just about that amount of transitory government consumption.

Inspection of Table II for \( G^P > 0 \) also indicates that the claims held by individuals against the government, that would make the optimal tax policy time consistent (see footnote 10), qualitatively depend on the value of \( \varepsilon \). In contrast to LS, for some cases when \( \varepsilon \) is small, the government issues debt in \( t = 0 \), pays its debt obligations, \( \beta_1 \), buys its outstanding claims for the next period, and buys bonds issued by individuals in \( t = 1 \). In these cases, the draft is used to such a large extent in \( t = 1 \) that the government’s deficit need not be positively related to temporary government consumption.

It should be noted that these examples are not meant to yield any general implications of the model of optimal fiscal policy presented in this paper. Rather, these examples serve to illustrate that when it is optimal to choose \( d_t > 0 \), the implications of the present model can depart significantly from the implications of the LS model. Although the analysis is not inconsistent with Barro’s tax-smoothing model [2], the military draft can serve as a partial substitute for debt creation in optimal fiscal policy.

IV. Evidence

The theory of optimal fiscal policy prescribes how the distortions arising from the income tax and draft should be traded off intertemporally. This section presents a simple test to determine if the theory is in some sense descriptive. Specifically, the positive implication of the model, that
the two tools of government finance are substitutes and, controlling for government expenditures, negatively related, is tested for the United States.

Annual data on average (weighted arithmetically) marginal tax rates, MTAX, are taken from Barro and Sahasakul [3, 434–35], and data on the number of drafted individuals and federal government expenditures are taken from U.S. Bureau of the Census Historical Statistics of the United States: Colonial Times to 1970 [14] and various editions of the Statistical Abstract [15]. To calculate the percentage of drafted individuals, DR, the number of inductees are divided by U.S. adult population (age 14 and older before 1947, and age 16 and older thereafter). Federal government expenditures divided by GNP defines the variable FEXP. Table III presents the annual data from 1941–1973.\textsuperscript{11}

Regressing the number of drafted as a fraction of the adult population (DR) on the marginal tax rate (MTAX) and on federal government expenditures a fraction of GNP (FEXP) from 1941 to 1973 when the draft was implemented yields

\[
DR = .009 - .105MTAX + .093FEXP \\
( .005 ) ( .022 ) ( .008 )
\]

\[ N = 33 \quad \bar{R}^2 = .803 \quad D.W. = 1.55 \quad s.e.e. = .004, \]

where standard errors are in parentheses. The Durbin-Watson statistic does not indicate statistically significant serial autocorrelation of the residual. As one would expect, government expenditures are significantly and positively related to the draft variable. Moreover, as predicted by the theory, the coefficient on MTAX is significantly negative.

Alternative specifications, not presented here, yield similar results. For example, a time trend does not significantly enter into the equation and leaves the coefficients on the other regressors virtually unchanged. Using average taxes, measured by federal government receipts divided by GNP, instead of the marginal tax rate, yields virtually the same results, as would be expected since the correlation coefficient between the two tax measures over the sample period is .75. Furthermore, breaking the sample into two subperiods to check for coefficient stability, does not change the signs of the coefficients; however, the coefficients on MTAX and FEXP become insignificant in the 1957–1973 period. In any case, the data are not inconsistent with the predicted negative relation between labor taxes and the draft.\textsuperscript{12}

V. Concluding Remarks

The theory of optimal fiscal policy shows that the draft, examined in the context of a general equilibrium framework, is not necessarily suboptimal. Whether or not the draft option is actually exercised depends, of course, on the stochastic process of government consumption, the

\textsuperscript{11} It should be noted that the data available for the empirical analysis do not exactly correspond to the theoretic constructs. First, the DR variable measures the number of new inductees each year, whereas the appropriate measure would be the number of those serving in the Armed Forces as a result of the draft. Second, the FEXP variable does not capture the full product of the Armed Forces. Hence, the evidence presented here is intended to be only suggestive.

\textsuperscript{12} Note that because the observations with \( DR = 0 \) are omitted from the sample, the estimated coefficients are biased. However, that the observed distribution is truncated at zero implies that the coefficients are biased toward zero and accordingly does not weaken the evidence substantially. A more sophisticated methodology involving censored regression techniques, left for future research, should reveal a similar negative relation.
Table III.

<table>
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<tr>
<th>Year</th>
<th>Drafted individuals as a percentage of the adult population (DR)</th>
<th>Average marginal tax rates (MTAX)</th>
<th>Federal government expenditures as a fraction of GNP (FEXP)</th>
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Productivity loss associated with the draft, and preferences. Only when the productivity loss is identically zero, will the draft unambiguously emerge as a part of optimal fiscal policy. 13 Numerical examples illustrate that the conditions when the draft is actually implemented are large wars and low productivity losses, such that the deadweight loss associated with the draft is sufficiently small relative to the distortions which would arise if the draft were not imposed. Under these conditions, taxes are lower than predicted by the LS model, and debt becomes less important in smoothing out distortions, because, in reducing the burden of a war of a given size, the draft

13. The appendix shows that when a solution does not exist in the LS model, the draft must be a part of any solution that does exist in this slightly more general model under identical circumstances. In terms of the numerical simulations, for example, when $G^p = 0$ and $G^T > .6$, there is no solution in the LS model, but there exists a solution in the present model with $d_t > 0$. 
reduces the need for taxation during the war, and hence reduces the distortions that have to be smoothed out in wartime and peacetime years. As such, the draft serves not only as a partial substitute for income taxation, but debt creation. Furthermore, the theory provides one possible explanation for the observation that, controlling for government expenditures, labor tax rates and the draft are negatively correlated.

Appendix

This appendix derives the analytical solutions to the government’s problem for the special case of quadratic utility. Following LS in the appendix to their paper, $(\bar{c}, \bar{x})$ is defined to be the solution to the maximization of $u(c, x)$ subject to $c + x \leq 1$. Let $\delta$ denote the value of marginal utility evaluated at that optimal consumption bundle—i.e., $\delta = u_c(\bar{c}, \bar{x}) = u_x(\bar{c}, \bar{x})$. Then a Taylor series expansion around $(\bar{c}, \bar{x})$, using (1), gives the following expressions for marginal utility:

\begin{align}
  u_c(c, x) & = \delta + (u_{cc} - u_{cx})(c - \bar{c}) - u_{cx}(g + d\epsilon) \quad (A1) \\
  u_x(c, x) & = \delta + (u_{cx} - u_{xx})(c - \bar{c}) - u_{xx}(g + d\epsilon). \quad (A2)
\end{align}

As utility is quadratic, $u_{cc}$, $u_{xx}$, and $u_{cx}$ are constants, and the expressions in (A1) and (A2) are exact representations.

By using the first order conditions (1) and (8), from the main text, along with the expressions for marginal utility above, it becomes apparent that the solutions for $c$ and $x$ are given by the following:

\begin{align}
  c_t & = (1 - \phi_0)\bar{c} - v g_t + \phi_0 (1 - v) b_t + v d_t (\phi_0 - \epsilon) \quad (A3) \\
  x_t & = 1 - (1 - \phi_0)\bar{x} - (1 - v)(g_t + \phi_0 b_t) - d_t((1 - \epsilon)\bar{c} + v d_0) \quad (A4)
\end{align}

where $\phi_0 = \lambda_0 / (1 + 2\lambda_0)$, $v = -\Delta^{-1}(u_{xx} - u_{cx})$, and $\Delta = -(u_{xx} - 2u_{cx} + u_{cc})$. Since utility is concave, $\Delta > 0$. $v$ can be interpreted as the derivative of demand for leisure with respect to the labor endowment. Similarly, $1 - v$ is the derivative of demand for the private consumption good with respect to the labor endowment. As both goods are non-inferior, $0 < v < 1$.

By using (1), (A1), and the first order conditions given by (8), one can find an explicit solution for the draft variable in terms of $g_t$, $\phi_0$, and $\phi_0$, assuming that $g_t > 0$ and that $a_t$ and $d_t$ are both strictly positive:

\begin{align}
  d_t & = \{g_t \alpha (e - \phi_0) + b_t \phi_0 (e - \Delta v(1 - v)\phi_0) + \delta((1 - \phi_0)\epsilon - \phi_0) + \phi_0^2 \delta v \bar{c}^2 \} \\
  & /\{\Delta \phi_0^2 v^2 + \alpha \epsilon (2\phi_0 - \epsilon)\} \quad (A5)
\end{align}

where $\alpha = \Delta^{-1}(u_{cc}u_{xx} - u_{cx}^2)$. Provided that consumers are risk averse, $\alpha > 0$. Note, that a sufficient condition for the draft to be positively related to $g_t$ is that $\phi_0 < \epsilon < 2\phi_0$.

Using the condition for competitive equilibrium (7), and the resource constraints given by (1), the solution for $\phi_0$ in the case of quadratic utility is given by the following:

\begin{align}
  \phi_0(1 - \phi_0) & = \{\sum_{i=0}^{\infty} \beta^i \left( (g_t + \phi_0(b_t - d_t(1 - \epsilon)))(\delta + \alpha(d_t e + g_t))dF^t \right) \\
  & /\{\Delta \sum_{i=0}^{\infty} \beta^i (\bar{c} - v d_t - (1 - \epsilon)b_t)^2 dF^t\}. \quad (A6)
\end{align}

Unlike (A5), (A6) is a necessary condition. In the case that $\phi b \geq 0$, it is possible to show that $\phi_0 \geq 0$ when government consumption is positive for some $t$. This sort of reasoning provides a lower bound for $\phi_0$. To ensure that $\phi_0$ is a real number requires that $\phi_0(1 - \phi_0) \leq 1/4$. Assuming that this condition is met, there are two possible solutions for $\phi_0$. The smaller of the two maximizes welfare, and consequently, the upper bound for $\phi_0$ is 1/2.
Comparing (A6) to the analogous equation in LS,

$$
\phi_0^{LS} (1 - \phi_0^{LS}) = \left\{ \sum_{i=0}^{\infty} \beta^i \left[ (\phi_i + g_i)(\delta + \alpha g_i) dF_i \right] / \left\{ \Delta \sum_{i=0}^{\infty} \beta^i \left[ (c - (1 - \nu)\phi_i)^2 dF_i \right] \right\}
$$

reveals that to rule out a priori that the draft will not be used requires that a real solution does not exist in the simple LS model. When the government consumption is extremely high, so that there is no solution for optimal fiscal policy in the LS model, if, in fact, there is a solution in present model, then the draft must be a part of the optimal fiscal policy. Intuitively, during periods of large scale wartime activity, the draft becomes a necessary tool to mobilize the labor force without placing an enormous burden on the entire economy. Hence, this model might predict the observation that the draft is used primarily by nations who are fighting large wars.

References