Strategic Discipline in Monetary Policy with Private Information: Optimal Targeting Horizons

By Michelle R. Garfinkel and Seonghwan Oh*

This paper analyzes a multiperiod monetary targeting procedure as a possible resolution to the credibility problem in policy when the monetary authority has some private information. By limiting the degree of flexibility permitted in policy, this procedure mitigates the credibility problem. As the length of the targeting horizon decreases, the severity of the credibility problem falls, but at the expense of weakening the monetary authority's ability to pursue its stabilization goals. Based on model simulations, the analysis studies the determinants of the optimal targeting horizon that balances the benefits of flexibility and discipline in policy. (JEL E50, F61)

How much flexibility or discretion should be given to the monetary authority? The answer to this time-honored question, in the monetarist spirit, is essentially none. Under the presumption that the monetary authority lacks an ability to forecast accurately or is able to forecast, at best, as well as economic agents, discretionary policy only creates an additional element of uncertainty that unnecessarily complicates economic agents' decision problems. Hence, the monetary authority should follow a strict rule, for example, a constant-money-growth-rate rule. Against the monetarist prescription, proponents of an activist rule for the monetary authority argue that the inflexibility of a strict rule precludes an optimal response to unanticipated disturbances.

To this already complicated question, recent developments in the literature have added more complexity, highlighting the strategic elements of monetary policy. When the market-determined output or employment level is deemed suboptimal, a benevolent monetary authority might try to raise employment or output by surprising agents with high inflation if temporary nominal rigidities are present. Rational, forward-looking agents, recognizing this incentive, set high rates of wage inflation to discourage the monetary authority from trying to reduce their real wage below their target level. Hence, even if the monetary authority is perfectly benevolent in that it maximizes

*Garfinkel: Department of Economics, University of California-Irvine, Irvine, CA 92717; Oh: Department of Economics, University of California-Los Angeles, Los Angeles, CA 90024, and Department of Economics, Seoul National University, Seoul, 151-742, Korea. The first draft of this paper was completed while the first author was an economist and the second author was a visiting scholar at the Federal Reserve Bank of St. Louis. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis or the Federal Reserve System. We thank, without implicating, Gary Hansen, Gregory Hess, Axel Leijonhufvud, participants in the UCLA Money Workshop, and especially Guido Tabellini and two anonymous referees for their invaluable comments on earlier drafts of this paper. Scott Leitz provided useful technical assistance.

1See Allan H. Meltzer (1990), whose skepticism about the Fed's forecasting ability leads him to advocate a constant-money-growth-rate rule. Axel Leijonhufvud (1984) provides a detailed discussion about the uncertainty generated by discretionary policy under different monetary regimes.
social welfare, its policy might be inefficient; the equilibrium could be characterized by excessively high average inflation—an inflationary bias. As recognized since Finn Kydland and Edward Prescott’s (1977) seminal paper, the efficient policy that avoids this bias might not be dynamically consistent or, more precisely, incentive-compatible.

Generally, when there is complete information, in the sense that individuals know the monetary authority’s preferences and observe the realizations of the stochastic variables that constrain its choices, the reputational mechanism can serve to eliminate or, at least, to diminish the extent of the suboptimality arising from this dynamic consistency problem (see Robert J. Barro and David Gordon, 1983; Kenneth Rogoff, 1987). If, however, the monetary authority has some private information, the efficacy of the reputational mechanism is called into question. Specifically, as shown by Matthew B. Canzoneri (1985), when the information structure is incomplete so that individuals cannot verify that the monetary authority has not intentionally invalidated their expectations, the efficacy of reputational considerations to resolve the credibility problem is weakened. Moreover, private information precludes the effectiveness of any commitment technology to force the monetary authority to adhere to the optimal rule unless there is a separate mechanism to force the monetary authority to reveal its private information truthfully.

Given these difficulties, Canzoneri (1985) suggests taking a legislative approach in order to resolve the credibility problem when the policymaker has private information. This approach assumes an incomplete commitment technology that cannot force the monetary authority to reveal its private information truthfully. Nevertheless, since a legislated procedure can be specified in terms of variables that are observed by all market participants (e.g., the growth rates of the monetary aggregates), this approach is potentially attractive. One example, studied by Canzoneri, is a two-period targeting procedure requiring that the average money growth rate per two periods equal the socially desired rate. More generally, provided that the procedure does not depend on the monetary authority’s private information and the monetary authority can control perfectly the targeted variable, the procedure is operational.

This paper extends Canzoneri’s (1985) analysis of a two-period targeting procedure in a multiperiod setting to investigate the scope of flexibility, in terms of the number of periods in the targeting horizon, that should be given to the monetary authority facing a trade-off between its output and inflation objectives. Legislation passed by Congress, for example, could mandate that the monetary authority announce a targeting horizon, over which time the growth of the money stock must average the rate corresponding to the (given) socially optimal inflation rate. Such an extension permits the analysis to identify the key factors that influence the optimal choice of the targeting horizon which balances credibility and flex-

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2 For a general discussion about dynamic inconsistency, see Brian Hillier and James Malcomson (1984). A necessary (but not sufficient) condition for dynamic inconsistency is that the government have fewer instruments than objectives.

3 Following Edward J. Green and Robert H. Porter (1984), however, Canzoneri (1985) shows how reputational considerations can mitigate the credibility problem partially, even in the presence of private information. The solution rests on a trigger strategy that does not require explicit verification of the monetary authority’s private information. However, this resolution is not complete. While the monetary authority adheres to the “reputational policy,” excessively high rates of inflation would emerge on occasion. This solution, then, generates an expected rate of inflation in excess of the socially desired rate.

4 For complementary analyses of monetary policy with private information, see Alex Cukierman and Meltzer (1986) and Jeremy C. Stein (1989). In contrast to these analyses, which mainly focus on maintained secrecy in monetary policy arising from private information, the present paper tries to specify an operational rule that improves upon the deadlock situation arising from the credibility problem.
ibility in monetary policy when there is private information. 

Under this procedure, the longer the targeting horizon, the greater is the degree of flexibility permitted in policy. However, as the targeting horizon increases, the greater is the monetary authority's willingness to act on its incentive to create surprise inflation in each period, and thus the greater is the (gross) inflationary bias in each period. In addition, the overall targeting constraint requires that these biases be reversed in subsequent periods. As the targeting horizon increases, then, the magnitude of those required reversals also increases. Thus, as shown below, the contribution of the remaining credibility problem to the variance in inflation around the socially optimal rate increases as the targeting horizon increases. The optimal length of the targeting horizon embodies the optimal trade-off between the cost of the remaining credibility problem and the cost of limiting flexibility to accommodate the predicted component of money demand shocks (i.e., the monetary authority's private information) relative to the policy that is fully discretionary.

As the monetary authority attaches greater importance to its objective for inflation relative to its objective for output, the optimal targeting horizon becomes longer. The basic intuition for this result is that the monetary authority's incentive to create surprises decreases, thereby decreasing the severity of the credibility problem and, hence, increasing the possible benefits of flexibility afforded by the $N$-period targeting procedure to permit the monetary authority to react to its private information. For the extreme case in which the inflationary bias that emerges in the full-discretionary regime is so small that the constraint imposed by the targeting procedure becomes unnecessary, the optimal number of periods in the targeting horizon is infinite. For the other extreme case, in which the credibility problem is sufficiently severe to make the optimal targeting horizon equal to one period, the procedure simplifies to a constant-money-growth rule. Such a rule eliminates the equilibrium inflationary bias associated with the credibility problem, but at the cost of removing all flexibility in policy. Nevertheless, as the weight that the monetary authority attaches to its inflation objective relative to its output objective approaches zero, the constant-money-growth rule is more likely to emerge as the optimal targeting policy.

In what follows, the next section briefly presents a model of monetary policy with private information in a multiperiod setting and analyzes the monetary authority's optimal, time-consistent monetary policy subject to the $N$-period average targeting constraint. Section II presents simulations of the model to illustrate how the parameters of the model influence the optimal choice of
the targeting horizon. These simulations suggest that, for many parameter values, the cost of limiting flexibility to reduce the severity of the credibility problem will be too large to render a finite-period targeting procedure desirable; but, for many other parameter values, some discipline in policy will be warranted, even if the monetary authority's forecasting ability is fairly accurate. Indeed, for a large part of the parameter space, the constant-money-growth rule emerges as the optimal policy. The present analysis suggests, then, that the credibility problem, as well as the monetary authority's forecasting ability, plays an important role in the optimal design of feasible monetary rules. Finally, Section III offers some concluding remarks, including possible extensions of the analysis.

I. A Model of Monetary Policy with Multiperiod Targeting

This section presents a simple economic model to investigate the efficacy of multiperiod average targeting procedures to support a better outcome than that achieved with full discretion when the monetary authority has private information. Following conventional practice, the analysis builds on a standard rational-expectations supply function,

\[ y_t = y^n + \theta(\pi_t - \pi^e_t) \quad \theta > 0 \]

where \( y_t \) denotes the logarithm of output in period \( t \), \( y^n \) denotes the logarithm of the natural level of output, \( \pi_t \) is the inflation rate in period \( t \), and \( \pi^e_t \) is the wage setters' expectation of inflation conditional on information available at the end of period \( t - 1 \).\(^6\)

The following money demand equation determines the equilibrium price level:

\[ m_t - p_t = y^n - v_t \]

where \( m_t \) and \( p_t \) equal, respectively, the logarithms of the money stock and the price level in period \( t \) and \( v_t \) is a money demand disturbance realized at the end of period \( t \). The disturbance is assumed to follow a random walk. The equilibrium inflation rate is obtained by taking the first difference of (2),

\[ g_t - \pi_t = \delta_t \]

where \( \delta_t = v_{t-1} - v_t \) and \( g_t \) is the growth rate of money, the monetary authority's instrument;\(^3\) \( \delta_t \) is an independently and identically distributed random variable with a zero mean and a finite variance, \( \sigma^2_\delta \). When wages are set, \( \delta_t \) is not known (i.e., \( \delta^e_t = 0 \)). The expression in (3), then, implies that wage setters' expectation of inflation depends solely on the monetary authority's policy, \( g_t \). This expectation, \( g^e_t \), ultimately depends on the monetary authority's strategy.

\(^6\)The emergence of the inflationary bias, discussed later in the paper, rests on the assumption embedded in this equation that workers supply labor along the demand schedule. However, as Cukierman (1992 Ch. 3) argues, it is highly unlikely that employment is determined along the labor demand curve when labor demand is in excess of supply. Cukierman demonstrates that, alternatively, when employment is viewed as being determined by the "short side" of the market, the monetary authority's incentive to create surprise inflation disappears and so does the inflationary bias. Nevertheless, some degree of unionization that keeps real wages above the real market-clearing wage can give the monetary authority an incentive to create surprise inflation to reduce (unexpectedly) the real wage, because the union's best strategy is to encourage its members to supply labor ex post along their individual labor supply curves. See Cukierman (1992) for the conditions under which the presence of unions gives rise to an inflationary bias.

\(^3\)As indicated earlier, for the targeting procedure to work, the analysis must assume that the monetary authority can control the growth rate of money perfectly if that is its target. That the Federal Reserve System chooses to target the more broadly defined monetary aggregates (i.e., M2 and M3) rather than those over which it has more control (i.e., high-powered money and M1) could be related to the secrecy problem. See, for example, Cukierman and Meltzer (1986) who show that the monetary authority might choose a less precise procedure for policy implementation so as to maintain some degree of ambiguity, whereby the secrecy of its information could be maintained to some extent. The present analysis, however, does not permit meaningful distinctions among the various monetary aggregates.
To study alternative strategies for the monetary authority, the analysis to follow assumes that the monetary authority chooses its policy to maximize its expected $N$-period average utility:

$$U' = E_0 \left\{ \frac{1}{N} \sum_{t=1}^{N} u_t \right\} \quad 1 \leq N < \infty$$

where one-period utility in period $t$ is given by

$$u_t = -(y_t - ky^n)^2 - s(\pi_t - \pi^*)^2 \quad k > 1.$$  

$E_0(\cdot)$ is an expectations operator conditional on period $t = \tau$ information, and $N$ denotes the number of periods in the targeting horizon. For $N = \infty$, the expected average utility is given by $E_0(u_t)$. The logarithm of the socially desirable output level is represented as $ky^n$, and $\pi^*$ is the socially determined optimal rate of inflation; $s$ is the weight the monetary authority attaches to its goal of inflation stability relative to its goal of output stability.

Letting $U \equiv U'/\theta^2$ and $u_t \equiv u_t'/\theta^2$ and using (1) and (3) with the assumption that $\delta_t^0 = 0$, the monetary authority's expected average $N$-period utility can be rewritten as

$$U = E_0 \left\{ \frac{1}{N} \sum_{t=1}^{N} u_t \right\}$$

where

$$u_t = - (g_t - g_t^e - \delta_t - y^*)^2 - f(g_t - \delta_t - \pi^*)^2$$

and $f \equiv (s/\theta^2)$ and $y^* \equiv (k - 1)y^n/\theta$.

After period-$t$ wages are set, the monetary authority chooses its policy, $g_t$. In contrast to wage-setters, the monetary authority receives some information about the disturbance to money demand ($\delta_t$) after wages are set but before policy actions are taken. Specifically, it has a private forecast of this disturbance, $d_t = E_t[\delta_t]$, that satisfies

$$\delta_t = d_t + \epsilon_t$$

where $\epsilon_t$ is an independently and identically distributed disturbance realized after policy is implemented. This forecast error has a zero mean, a finite variance, $\sigma^2$, and no correlation with $d_t$. Similarly, $d_t$ is independently and identically distributed with a zero mean and a finite variance, $\sigma^2_d$. Thus, $\sigma^2 = \sigma^2 + \sigma^2_d$. Section II discusses the case when $d_t$ is serially correlated. Although wage-setters observe $\delta_t$ and $\pi_t$ after $g_t$ is set, they cannot distinguish the monetary authority's forecast, $d_t$, from the forecast error, $\epsilon_t$.

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8$N$ can be thought of as the time horizon of an individual policymaker. However, given the stationary nature of the model, the solution would not change if, instead, the policymaker were infinitely-lived since the infinite-horizon problem can be decomposed into independent $N$-period-horizon problems. The implicit assumption here, that the monetary authority does not discount the future, is not strong. A discount factor less than 1 only complicates the analysis without providing much additional insight. (A copy of the solution in the case of discounting is available from the authors upon request, and the effects of discounting on the optimal targeting horizon are discussed briefly at the end of Section II.)

9See Canzoneri (1985 pp. 1058–9) for a detailed discussion of the possible interpretations of the assumption that $k > 1$, an assumption that is important for explaining the emergence of a positive inflationary bias. These interpretations build on distortions (presumed to exist in the economy) that depress average output, or the natural level of output, below the level considered to be optimal from a social-welfare perspective (but see footnote 6 and Cukierman [1992 Ch. 3], who provides a useful critique of this social-welfare approach). As an alternative, Cukierman (1992) adopts a political approach to study central-bank behavior. This approach interprets the monetary authority's output objective with $k > 1$ as representing those interest groups advocating economic stimulation. Under this interpretation, the parameter $s$ reflects "a compromise reached through the central bank between the advocates of economic stimulation and the advocates of price stability" (Cukierman, 1992 p. 43). This approach would seem inappropriate for the present representative-agent model, however.

10Given the sequence of actions by wage-setters and the monetary authority relative to the timing of the realization of the monetary authority's private informa-
Before the optimal, time-consistent monetary policy with multiperiod targeting is derived, two benchmark solutions are presented. The first solution, the efficient solution, is that which would obtain if there were a full commitment technology permitting the monetary authority to adhere to a contingent rule, while truthfully revealing its private information. The second solution, the full-discretionary solution, assumes that the monetary authority takes wage-setters’ expectations as given. It is identical to the solution that would emerge under an infinite targeting procedure, since \( N = \infty \) is equivalent to no constraints on policy.

**A. The Efficient Solution**

Assuming a full commitment technology to force the monetary authority to reveal its private information truthfully and to adhere to a contingent rule so that it can influence wage-setters’ expectations effectively in a way that is consistent with its policy, the optimal monetary policy, \( \hat{\sigma} \), is given by

\[
\hat{\sigma}_t = \pi^* + d_t
\]

for \( t = 1, 2, \ldots \), which yields the following expected average utility for the monetary authority:

\[
\hat{U} = -(1 + f) \sigma^2 - (y^*)^2.
\]

Note that the money growth rule in (7) completely accommodates money demand shocks to stabilize inflation around the socially desired rate.

**B. The Full-Discretionary Solution**

Under an alternative assumption that the monetary authority behaves as if it can have no impact on wage-setters’ expectations, it maximizes the expectation of (5) conditional on its forecast of the disturbance to money demand, subject to (6) and taking \( g_i^c \) as given. The wage-setters’ expectation of the associated first-order condition implies \( g_i^c = \pi^* + y^*/f \). By substituting \( g_i^c \) back into the first-order condition, one can verify that the money growth rate under this regime, \( \bar{\sigma} \), is given by

\[
\bar{\sigma}_t = \pi^* + d_t + \frac{y^*}{f}.
\]

Note that the monetary authority’s private information is fully revealed in the outcome. Although the monetary authority can do no better than to set its policy according to (9) given wage-setters’ expectations, the monetary authority’s incentive to create surprise inflation generates an inflationary bias, \( y^*/f \). Even if the monetary authority had no private information, this bias would emerge, provided that it took expectations as given.

The inefficiency of the full-discretionary solution is revealed by comparing the expected average utility of the monetary authority in this regime, given by

\[
\bar{U} = -(1 + f) \sigma^2 - \left(1 + \frac{1}{f}\right)(y^*)^2
\]

to that obtained in the efficient regime, given by (8). The difference between (8) and (10) in the static framework, this solution is equivalent to the (one-shot) Nash solution. As already noted and as discussed in part C of this section, however, this solution more generally is interpreted as an infinite-period, average targeting procedure.
(10), \((y^*)^2/f\), captures the disutility of the inflationary bias.

C. The N-Period Targeting Solution

As is widely recognized, while the full-discretionary policy is not first-best, the efficient solution is not necessarily a feasible outcome in the absence of a full commitment technology. That is, the monetary policy in (7) is not incentive-compatible. Once wages are set, the monetary authority’s optimal policy no longer corresponds to (7). Rather, the monetary authority would like to set \(g_t = \pi^* + d_t + y^*/(1+f)\). In addition, the monetary authority’s private information obscures the role of reputational considerations to diminish the inflationary bias. Specifically, without a complete information structure, wage-setters cannot verify that the monetary authority has followed the efficient policy.\(^{12}\) Moreover, the monetary authority’s private information precludes the effectiveness of legislation requiring the monetary authority to implement policy according to (7). Even if the legislation were binding, the monetary authority would have an incentive to lie, claiming that its forecast of \(\delta_e\) equaled \(d_t + y^*/(1+f)\), whereby it could disguise the optimal cheating policy (given \(g_e^* = \pi^*\)) as the efficient one. Without any additional constraints on policy, the presence of nonverifiable private information means that the monetary authority cannot influence wage-setters’ expectations effectively, and the full-discretionary policy emerges in the equilibrium outcome.

As a possible solution to the inefficiency of the inflationary bias when the monetary authority cannot credibly reveal its private information, Canzoneri (1985) studies a two-period average targeting procedure, requiring \(g_1 + g_2 = 2\pi^*\). This procedure is attractive not only in its simplicity, but in its independence from the monetary authority’s private forecast. Such independence makes it operational, for enforcement of the (precommitted) constraint does not require verification of \(d_t\).\(^{13}\) As a generalization of Canzoneri’s analysis, consider an extended average targeting procedure that requires

\[
\sum_{t-1}^{N} g_t = \pi^* N
\]

where \(1 \leq N < \infty\). For \(N = \infty\), there is no relevant constraint on the conduct of policy.

To derive the optimal, dynamically consistent monetary policy under this (precommitted) targeting regime, a backward solution concept is appropriate. Specifically, in period \(t = N - 1\), the monetary authority maximizes the sum of the expected value of (5) in \(t = N\) and \(t = N - 1\) with respect to \(g_{N-1}\) subject to the forecast \(d_{N-1}\) and (11). In solving this optimization problem, the monetary authority takes \(g_{N}^e\) and \(g_{N-1}^e\) as given while recognizing the impact that \(g_{N-1}^e\) will have on future expectations, \(\tilde{g}_N^e\), through the constraint in (11). Repeating this exercise for \(t = N - 2, N - 3, \ldots\), one can find that

\[
\tilde{g}_t = \tilde{g}_t^e + \frac{(N-t)(1+f)}{N-t+(N-t+1)f} d_t
\]

for \(t = 1, 2, \ldots, N\), where \(\tilde{g}\) denotes the

\(^{12}\)But see footnote 3.

\(^{13}\)To be sure, the analysis to follow implicitly assumes that a precommitment to adhere to the targeting constraint in (11) is credible. Even if the monetary authority’s expected average utility under this procedure (for a given \(N\)) exceeds that which would be obtained under the full-discretionary solution (\(N = \infty\)), the constraint need not be incentive-compatible in any period during the targeting horizon. In other words, in any period \(t \leq N < \infty\), given expectations, the monetary authority has an incentive to violate the constraint. This incentive undermines the credibility of the precommitment. The present analysis implicitly assumes, then, that there exists an external enforcement mechanism (e.g., legislation effectively requiring that the monetary authority be severely punished if it fails to hit that average period target over the prespecified horizon, \(N\)) to support the procedure provided that it yields a better outcome than that obtained under full discretion, a condition that is examined in Section II. Note that, when this condition is satisfied, reputational considerations might be able to substitute for the assumed commitment technology to support the targeting rule in equilibrium.
money growth policy under the $N$-period targeting procedure and

$$
\bar{g}_t^e = \frac{N\pi^*}{N - t + 1} + \frac{(N - t)y^*}{(N - t + 1)f} - \frac{1}{N} \sum_{\tau = 1}^{t-1} \frac{\bar{g}_\tau}{N - t + 1}
$$

for $t = 1, 2, \ldots, N$. Note that, in the expression above, the summation is taken only for $t > 1$. (In what follows, $\Sigma_t^s$ is taken to be equal to zero whenever $s > t$.) The Appendix illustrates the solution method used here more explicitly.

To eliminate past $\bar{g}$ so as to express the optimal policy only in terms of current and past $d$ as well as the parameters of the model, (12) is used to find $\bar{g}_1$ and sequentially used to find $\bar{g}_t$, $t = 2, 3, \ldots, N$, by substituting in past $\bar{g}_t$. That is,

$$
(13) \quad \bar{g}_t = \Gamma(t) - \frac{1}{N} \sum_{\tau = 1}^{t-1} \frac{\Gamma(\tau)}{N - \tau}
$$

where

$$
\Gamma(t) = \frac{N\pi^*}{N - t + 1} + \frac{(N - t)y^*}{(N - t + 1)f} + \frac{(N - t)(1 + f)}{N - t + (N - t + 1)f} d_t
$$

for $t = 1, \ldots, N$. By rearranging (13) and simplifying, one can verify that the optimal monetary policy subject to the $N$-period targeting constraint is given by the following:

$$
(14) \quad \bar{g}_t = \pi^* + \frac{(N - t)y^*}{(N - t + 1)f} + \frac{(N - t)(1 + f)}{N - t + (N - t + 1)f} d_t - \frac{1}{N} \sum_{\tau = 1}^{t-1} \left[ \frac{y^*}{(N - \tau + 1)f} + \frac{1 + f}{N - \tau + (N - \tau + 1)f} d_\tau \right]
$$

for $t = 1, 2, \ldots, N$.

Expected average utility under this regime is given by

$$
(15) \quad \bar{U} = -(y^*)^2 - (1 + f)\sigma_d^2
$$

for $t > 1$. (In what follows, $\Sigma_t^s$ is taken to be equal to zero whenever $s > t$.) The Appendix illustrates the solution method used here more explicitly.

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$$

where

$$
\Gamma(t) = \frac{N\pi^*}{N - t + 1} + \frac{(N - t)y^*}{(N - t + 1)f} + \frac{(N - t)(1 + f)}{N - t + (N - t + 1)f} d_t
$$

for $t = 1, \ldots, N$. By rearranging (13) and simplifying, one can verify that the optimal monetary policy subject to the $N$-period targeting constraint is given by the following:

$$
(14) \quad \bar{g}_t = \pi^* + \frac{(N - t)y^*}{(N - t + 1)f} + \frac{(N - t)(1 + f)}{N - t + (N - t + 1)f} d_t - \frac{1}{N} \sum_{\tau = 1}^{t-1} \left[ \frac{y^*}{(N - \tau + 1)f} + \frac{1 + f}{N - \tau + (N - \tau + 1)f} d_\tau \right]
$$

for $t = 1, 2, \ldots, N$.

Note that the sum of the first two terms in the above expression equals the expected average utility that would be realized if the efficient outcome could be supported in equilibrium, (8). Hence, the last three terms reflect the inefficiency of the targeting procedure.\footnote{For $N = 1$, equation (15) reduces to the average expected utility under a constant-money-growth rule, given by $-(y^*)^2 - (1 + f)\sigma_d^2$. In this case, the last three terms reduce to $(1 + f)\sigma_d^2$, the cost arising from removing all flexibility in policy. As $N$ approaches infinity, (15) reduces to (10), the average expected utility under the full-discretionary regime. To verify this, note that the third term approaches $(y^*)^2/fN$ as $N$ approaches infinity, while the last two terms converge to zero. The convergence of the last term to zero can be verified easily by observing that the double summation of that term can be written as}

$$
\frac{1}{N} \sum_{t = 1}^{N} \sum_{\tau = 1}^{t-1} \frac{t - 1}{(t - 1 + tf)^2}
$$

Since the limit of $(t - 1)/(t - 1 + tf)^2$ converges to zero as $t$ approaches infinity, the arithmetic mean of such a sequence will also be zero (see Walter Rudin, 1976 p. 80). Thus, when $N = \infty$, the last three terms in (15) boil down to $(y^*)^2/f$, the disutility of the inflation that would emerge in the full-discretionary solution.
The optimal targeting horizon, \( N^* \), maximizes (15). Assuming that \( N^* \) is finite, the average inefficiency is increasing in \( N \) for \( N > N^* \), as shown with an example in Figure 1. The horizontal line in the figure measures the normalized inefficiency of the full-disciplinary solution when \( f = 1.0 \) and \( y^*/\sigma_d = 1.35 \) [i.e., \((y^*)^2/\sigma_d^2 = 1.82\)]. The inverted V-shaped curve measures the normalized, average inefficiency of a multi-period targeting over various \( N \), showing that \( N^* = 6 \) for the chosen parameter values.\(^{16}\)

Given the \( N \)-period targeting constraint, the monetary policy in (14) is dynamically consistent, like the full-disciplinary solution without any constraints. Note, however, that for finite \( N \) the gross inflationary bias [i.e., the second term in (14)] is lower than that in the full-disciplinary solution. From (13) or (14) in \( t = 1 \), for example, one can verify that the bias is \((N-1)y^*/Nf\), which is reversed in each of the subsequent \( N-1 \) periods, in increments of \( y^*/Nf \). The increments are of the same magnitude since the monetary authority does not discount the future. Similarly, the bias in \( t = 2 \) is \((N-2)y^*/(N-1)f\), which is reversed in the subsequent \( N-2 \) periods in increments of \( y^*/(N-1)f \). As \( t \) approaches \( N \), the monetary authority's willingness to act on its incentive to create surprise inflation falls, and so does the gross inflationary bias. At the same time, there is an increasing accumulation of past biases to be reversed. Thus, as \( t \) approaches \( N \), the period-\( t \) inflationary bias net of the sum of current reversals of previous periods' biases (referred to here as the remaining or net inflationary bias) falls and eventually becomes negative to make

\(^{15}\)Although writing down a closed-form solution for \( N^* \) is extremely difficult, it is possible to verify that \( N^* \) can approach infinity for a certain set of parameter specifications (see Section II).

\(^{16}\)It should be noted that, for \( \sigma_d < 1.5 \), Canzoneri's (1985) reputational solution would not be feasible since it requires that \( y^*/f > 2 \) (p. 1065). For this numerical example, then, the reputational solution is clearly dominated by the institutional solution studied in this paper, when \( \sigma_d < 1.5 \).
the average remaining inflationary bias over the targeting horizon zero.

The movement in the remaining inflationary bias (i.e., the unconditional expectation of money growth in excess of the socially determined optimal inflation rate) over \( t \) is given in Table 1 for \( 1 \leq N \leq 10 \). The table is constructed on the basis of the previous example with the additional assumptions that \( \pi^* = 0.02 \) and \( y^* = 0.78 \) (implying that \( \sigma_d^2 = 1/3 \)). For this example, the inflationary bias that would emerge in the full-discretionary regime is 0.78 for all \( t \). As revealed by the table, although the remaining inflationary bias averages zero over the horizon, this remaining bias in absolute terms can exceed the bias that would emerge in the full-discretionary solution. In addition, the table indicates that increases in \( N \) imply a greater average of the absolute value of the remaining inflationary biases that emerge over the targeting horizon due to the constraint (11). The basic idea here is similar to that behind the downward movement of the remaining inflationary bias over the horizon. As \( N \) increases, the targeting constraint (11) appears less binding to the monetary authority at any given period \( t \). Accordingly, the monetary authority is more inclined to act on its incentive to create surprise inflation, which is recognized by wage-setters. Of course, the future reversals must be greater, in magnitude, if the targeting constraint (11) is to be satisfied. Thus, as \( N \) increases, the variance of the remaining bias around zero increases. However, because the remaining biases are perfectly anticipated by wage-setters, the increased variance of the remaining bias has no implications for the variation of output.

The disutility of the remaining inflationary biases is captured by the third term in (15). For \( N = 1 \), this term equals zero. Because any period’s (gross) inflationary bias (which must eventually be reversed over the remainder of the targeting horizon) increases as \( N \) increases, the loss in utility due to the effects of the remaining inflationary bias on the variability of inflation around \( \pi^* \) increases as \( N \) becomes large. As \( N \) goes to infinity, this loss converges to \( (y^*)^2/f \); in the example above, this loss, reported in the last column of Table 1, converges to 0.608.

In contrast to the full-discretionary and the efficient solutions, the monetary policy in (14) involves only a partial accommodation of the current shock. The targeting procedure limits the monetary authority’s flexibility to react to its forecast. However, as \( N \) approaches infinity, the accommodation is full. The effect of increasing the monetary authority’s flexibility to react to its private information is partly captured by the fourth term in (15), which equals \( (1 + f)\sigma_d^2 \) for \( N = 1 \) and is decreasing in \( N \), approaching zero as \( N \) approaches infinity. That is, by relaxing the monetary authority’s con-
straint, increasing the length of the targeting horizon provides more leeway for the monetary authority to achieve its output and inflation-stabilization goals.

The constraint in (11), however, requires that the accommodation in every period be reversed in subsequent periods. The effect of this requirement can be observed easily in (13) or (14). In particular, as the constraint becomes more binding (i.e., \( t \) approaches \( N \)), the monetary authority’s reaction to the current shock approaches zero. The reaction to the shock in time \( t, d_t \), is reversed in increments of

\[
\frac{1 + f}{N - t + (N - t + 1)f} d_t
\]

in each of the remaining \( N - t \) periods. Again, that the increments of the reversal of a given reaction to a time-\( t \) shock are of equal magnitude is driven by the assumption that the monetary authority does not discount the future. Note that these reversals have no implications on the variability of output. In this model, the optimal policy, given by (12), implies that past money disturbances, \( d_t \), are fully revealed to wage-setters upon their observing \( g_t \). Thus, wage-setters fully incorporate these reversals into expectations. However, the reversals do influence the variability of inflation around \( \pi^* \).

The fifth term in (15) reflects the cost of the increased variability of inflation due to the requirement of having to reverse earlier reactions to shocks. As \( N \) increases from 1, that loss in expected utility initially increases from zero. That loss, however, decreases as \( N \) becomes sufficiently large. At this point, increasing flexibility reduces the expected cost of engineering those reversals, so that the loss captured by the fifth term falls and finally converges to zero. Combining the fourth and fifth terms of (15) yields the net effect of limiting the scope of flexibility as permitted by the targeting procedure.

That the sum of the last terms in (15) is increasing in magnitude in \( N \) for \( N > N^* \) implies that the targeting procedure becomes more inefficient as \( N \) increases above the optimal targeting horizon. Nevertheless, note that a sufficient condition for \( N^* \) to be finite is that \((1 + f)\sigma_t^2 \leq (y^*)^2 / f \) (i.e., that a strict one-period targeting rule dominates the full-discretionary solution) as can be verified by comparing the expected average utility under the targeting procedure, given by (15) with \( N = 1 \), to the expected average utility in the full-discretionary regime, given by (10) or by (15) with \( N = \infty \). This condition is not necessary, however. More generally, as revealed by a comparison of (10) and (15), the targeting procedure will dominate the full-discretionary solution (\( N = \infty \)) provided that the magnitude of the sum of those terms, for any finite \( N \), is less than \((y^*)^2 / f \), the disutility of the inflationary bias that would emerge in the full-discretionary equilibrium.

The basic idea here is as follows: as \( N \) goes from 1 to infinity, the variance of output falls, approaching that which would obtain in the full-discretionary regime. At the same time, the variance of inflation around \( \pi^* \) due to the remaining credibility problem increases, approaching that which would obtain in the full-discretionary regime. Although increasing \( N \) also implies more flexibility to insulate inflation and output from money demand shocks, the required reversals of reactions to past \( d_t \), adds to the variability of inflation around \( \pi^* \), at least initially as \( N \) increases from 1. The variability due to such reversals over the targeting horizon falls sometime thereafter, however, approaching zero as \( N \) goes to infinity. Thus, even if the strict one-period targeting procedure does not dominate the full-discretionary policy, some limits on flexibility might be desirable. For \( N^* \) to be finite under conditions for which the full-discretionary policy dominates the constant-money-growth rule, the disutility of the increased variability of inflation around \( \pi^* \) due to the reversals of earlier reactions to \( d_t \) must fall soon after \( N \) increases from 1 and approach zero at a rapid rate as \( N \) continues to increase. Otherwise, the reduction in the variability of inflation around the socially determined rate below that which obtains in the full-discretionary regime will
not be sufficient to compensate for the increased variability in output induced by the targeting constraint; in this case, no constraints on policy through the targeting procedure would be desirable.

II. Optimal Targeting Horizons: Simulations of the Model

The optimal degree of flexibility depends on the accuracy of the monetary authority's forecasts of money demand disturbances, as measured by \( \sigma_d^2 \) for a given \( \sigma_e^2 \) or a given \( \sigma_e^2 \), and the severity of the credibility problem, as measured by \( y^*/f \). As in much of this literature, however, analytically deriving the optimal targeting horizon, \( N^* \), that maximizes expected average utility, as given by (15), is not possible despite the simplicity of the model. This section summarizes the results of simulations that specify the two key parameters of the model: \( y^*/\sigma_d \) (the weighted difference between the player's output goals relative to the magnitude of the predictable part of the shock) and \( f \) (the ratio of the weight attached to inflation stability relative to output stability in the monetary authority's preferences, \( s \), to the squared elasticity of output with respect to unanticipated inflation, \( \theta \)). For various parameter specifications, \( N^* \) is reported in Table 2. Discrete jumps in \( N^* \), as the parameter values vary, are due to the fact that changes in the parameter values are not sufficiently small.

A close inspection of the table reveals two important implications of the simulation exercise. First, for a given \( y^*/\sigma_d \), as \( f \) increases, the optimal targeting horizon gets longer. That is, as the weight of importance that the monetary authority attaches to its inflation objective relative to that for its output objective increases, the optimal length of the targeting horizon increases.\(^{17}\)

\(^{17}\)Note that an analogous comparative-static exercise involving a hypothetical decrease in the elasticity of output with respect to unanticipated inflation, \( \theta \), would not necessarily produce the same result. A smaller \( \theta \) implies not only a larger \( f \), but a larger \( y^* \). As we will discuss, a larger \( y^* \), for a given \( f \), implies a smaller \( N^* \). Thus, hypothetical changes in \( f \) should be interpreted as changes in \( s \) only. Similarly, hypothetical changes in \( y^* \) should be interpreted as changes in either \( k \) or \( y^* \) only.
The basic idea behind this implication is rather simple. As \( f \) increases, the monetary authority's incentive to push output beyond its natural level falls so that the inflationary bias that would emerge in the full-discretionary equilibrium falls. Accordingly, the optimal scope of flexibility under the targeting procedure increases. For large values of \( f \), the optimal targeting solution boils down to the full-discretionary solution, in which there is complete flexibility. In this case, the inflationary bias that emerges under full discretion becomes so trivial that the limits on flexibility imposed by the targeting procedure are unnecessary.

This implication is similar to that of Rogoff's (1985) analysis. In contrast to the present analysis, however, Rogoff distinguishes the monetary authority's preferences from that of society and thereby gives an institutional interpretation to the monetary authority's preferences. The difference emerges in the weight of importance attached to the inflation goal relative to the output goal; for convenience, the difference is denoted here by \( \mu = f_m - f > 0 \), where \( f \) reflects society's preferences and \( f_m \) reflects the (perverse) monetary authority's preferences. Nevertheless, as in Rogoff's analysis, the inflationary bias that emerges in the full-discretionary outcome becomes less severe, as the monetary authority's dislike for deviations of actual inflation away from the socially determined optimal rate increases. The important distinction between the present analysis and that of Rogoff lies in his interpretation of flexibility. Specifically, in a one-shot-game setting, he roughly interprets increasing the monetary authority's relative concern for inflation stability around \( \pi^* \) (i.e., increasing \( f_m \) for a given \( f \)) as limiting flexibility, for it implies less complete and, for a given \( f \), less efficient accommodations of supply shocks by the monetary authority. In contrast to Rogoff (1985), the present multiperiod analysis interprets increases in flexibility in terms of increasing \( N \), finding that the more perverse is the monetary authority (i.e., the larger is \( f \) the greater are the advantages of full flexibility in monetary policy.

The second implication of the simulation exercise is that, for a given \( f \), the optimal targeting horizon becomes shorter as \( y^* / \sigma_d \) increases. There are two potential forces at work here. As \( y^* \) increases for a given \( \sigma_d \), the difference between the socially optimal output target and the natural level of output becomes larger; as a consequence, the inflationary bias that would emerge in the full-discretionary solution becomes larger, thereby detracting from the benefits of flexibility afforded by the targeting procedure. Furthermore, as \( \sigma_d \) falls for a given \( y^* \), the expected value of reacting to current shocks (the monetary authority's private information) to stabilize inflation and output falls. Since \( \sigma_d^2 = \sigma^2 - \sigma^2_e \), a decrease in \( \sigma_d^2 \) can be thought of as a decrease in \( \sigma^2_e \) or as an increase in \( \sigma^2_e \). In either case, the reduction in \( \sigma_d^2 \) implies a reduction in the degree of accuracy of the monetary authority's forecast, \( \sigma_d^2 / \sigma^2 \). Not surprisingly, then, the simulation exercise suggests that, with a

\[\text{by the policymakers with preferences corresponding exactly to those of society. In her interesting analysis of what incentive structure should be imposed on the monetary authority, Lohmann (1992) adds another dimension to this interpretation of flexibility with a notion of central-bank independence—in particular, independence of the monetary authority from the policymakers having preferences identical to those of society. Building on Flood and Isard (1989), her notion of independence revolves around the possibility that the policymakers can, at a cost, dismiss the monetary authority and implement the full-discretionary policy. In equilibrium, the monetary authority follows a policy involving a flexible escape clause, whereby it can avoid dismissal. The smaller the cost of dismissal, the smaller is the degree of independence and, for a given \( f_m \), the closer is the monetary authority's reaction to the shock to what the policymaker's reaction would be if it were to formulate and otherwise implement policy directly. Thus, for a given \( f_m \), a decrease in independence could be interpreted as an increase in flexibility. (Note that Rogoff's [1985] analysis implicitly assumes that the cost of dismissal is infinite, implying no independence.)}\]

\[\text{18} \text{Also see Lohmann (1992). For a general survey on this and related issues, see Torsten Persson and Guido Tabellini (1990).}\]

\[\text{19} \text{Under this interpretation, then, the fully flexible policy is that which would otherwise be implemented}\]
reduction in the monetary authority’s forecasting ability, the optimal limits on flexibility are greater (i.e., $N^*$ is smaller). The strict one-period targeting rule is more likely to be optimal as $y^* / \sigma_d$ rises.

This implication further distinguishes the results of the present analysis from those of Rogoff (1985). Specifically, Rogoff finds that some flexibility in monetary policy is invariably better than none (i.e., a constant-money-growth rule or, in terms of their analyses, an extremely perverse monetary authority with $\mu = \infty$). In contrast, the present analysis finds that a constant-growth rule ($N^* = 1$) can emerge as the optimal policy for a wide range of parameter values. There are two reasons for this distinction. First, the present analysis explicitly considers the monetary authority’s forecasting ability or lack thereof. Reducing the monetary authority’s ability to predict money demand shocks, which should be accommodated to stabilize inflation and output, weakens the case for maintained flexibility in policy. Second, the present analysis allows for the possibility of extending the one-period game setting to a multiperiod game setting where, through the targeting constraint, the monetary authority can partially influence future expectations through its current policy. Even if there were supply shocks, as in Rogoff’s model, this distinction would emerge. Nevertheless, for those parameter values under which the present analysis finds that $N^* = \infty$, there could be room for improvement by limiting flexibility through the alternative institutional arrangements studied by Rogoff (1985) and Lohmann (1992): specifically, through the appointment of a perverse central banker with some degree of independence. More generally, as noted in Section III, it might be possible to combine these institutional arrangements with the multiperiod targeting procedure to effect a more efficient balance between credibility and flexibility.

It is difficult to interpret the simulation results directly in terms of actual data because the difference in output goals is measured in (log) levels, and the scale that would be appropriate is not clear. Interpreting the simulation results is more difficult without knowledge of the value of the parameter $f$. Nevertheless, Table 2 indicates that the full-discretionary solution dominates any finite targeting procedure for a large part of the parameter space. However, for those parameter values in which there is room for improvement by imposing a binding constraint on monetary policy, the table illustrates that, often, $N^* = 1$. This result is rather interesting, considering the fact that, in many nations where central banks have adopted some form of monetary targeting procedure, the targeting horizon coincides with the wage contract period, typically one year.

The result is also illustrated by Figure 2, which depicts regions for $N^* = 1, 2, 3,$ and $4$ and for $N^* \geq 5$ in terms of combinations of $f$ and $y^* / \sigma_d$. Even with this limited opportunity set, the figure shows that the regions for which $N^* = 1$ and $N^* \geq 5$ are the largest areas among those under consideration. More generally, by sequentially increasing the opportunity set and deriving explicit regions for $N^* \geq 5$ (e.g., $N^* = 5, N^* = 6$, etc.), it is possible to see that regions for $N^*$ become progressively narrower as $N^* < \infty$ increases. Hence, assuming that $N^*$ is finite, the optimal degree of flexibility is likely to be extremely limited if not entirely eliminated. Of course, this claim implicitly assumes a uniform distribution for the actual

\[20\text{Similarly, Lohmann (1992) finds that complete independence is not optimal (see footnote 19). It should be noted, however, that both Rogoff (1985) and Lohmann (1992) also find that some limits on flexibility are always better than none (i.e., $\mu = 0$ is never optimal). (Lohmann also finds that some independence is always better than none.) This distinction emerges since, in their analyses, limited flexibility ($\mu > 0$) does not constrain reactions to money demand disturbances.}

\[21\text{Additional simulations confirm this interpretation of the table. Moreover, they show that, for $f \geq y^* / \sigma_d$, the full-discretionary solution dominates all finite targeting procedures with $N \leq 1,000$.}
values of the parameters. Without knowledge of the empirical distribution of the key parameters, the figure shows only the conditions (in terms of combinations of parameter values) under which certain regimes would be predicted to emerge by the present analysis. In any case, the simulation exercise provides additional support to the notion that it is important to take into account the credibility problem, in addition to the monetary authority's forecasting ability, when designing feasible optimal rules for monetary policy.

One important caveat to the above analysis should be noted. In particular, the assumption that $d_\epsilon$ is not serially correlated is a strong assumption. Indeed, the extreme results above (i.e., given $N^*$ is finite, extremely limited flexibility is often desirable) might be driven partly by the transitory nature of the monetary authority's private information. While the effect of persistence of shocks to money demand on the optimal targeting horizon is not obvious in the context of this model, the analysis by Cukierman and Meltzer (1986) suggests that the introduction of such persistence is likely to increase the monetary authority's desire for maintained flexibility. In particular, their analysis shows how persistence in the monetary authority's private information, which is never directly observed by the public, provides a channel through which the monetary authority can influence (indirectly and imperfectly) future expectations, without imposing any explicit limits on flexibility. Persistence, in the context of this model, then might imply a smaller role for a restrictive targeting procedure as an imperfect mechanism by which the monetary authority can influence future expectations indirectly.

However, the qualitative results obtained above are not sensitive to the assumption that the monetary authority does not discount the future. That is, $N^*$ is increasing in $f$ and decreasing in $y^*/\sigma_d$, even when the monetary authority discounts the future. In the case of discounting, under a given $N$-period targeting constraint, the monetary authority's willingness to act on its incentive to create surprise inflation would be greater, and its optimal policy would involve more
complete accommodations of the predicted part of the money demand disturbances, since future reversals are perceived as being less of a burden. As such, when the monetary authority discounts the future, a given $N$-period targeting solution retains more flexibility but does not gain as much credibility over the full-discretionary solution, relative to when the monetary authority does not discount the future. An examination of the utilities obtained under various $N$-period targeting constraints when the monetary authority discounts the future suggests that, as the discount rate increases, the optimal targeting horizon, $N^\ast$, falls for relatively higher values of $y^\ast / \sigma_d$ and smaller values of $f$; but, for relatively smaller values of $y^\ast / \sigma_d$ and larger values of $f$, an increase in the discount rate will imply an increase in $N^\ast$.

III. Concluding Remarks

This paper has investigated the efficacy of average monetary targeting to reduce the inflationary bias that otherwise emerges as a result of the monetary authority’s incentive to surprise agents in an effort to increase output beyond its natural level. Reputational considerations and the imposition of a contingent rule, as reasonable methods of eliminating the bias while retaining flexibility for output and inflation stabilization, are called into question when the monetary authority has some private information and there is no mechanism to enforce a truthful dissemination of that information. Provided that the inflationary bias is sufficiently large, an average monetary targeting procedure to eliminate the bias will be appealing. As the targeting horizon gets longer, the amount of flexibility permitted by the procedure increases, but at the cost of a higher variance of the unconditionally expected inflation rate around its socially determined optimal rate. Hence, the optimal targeting horizon defines the optimal trade-off between the cost of limited flexibility and the cost of the remaining inefficiencies of the credibility problem.

As indicated earlier, that the optimal targeting horizon is often very short might be accounted for, in part, by the transitory nature of the monetary authority’s private information. Thus an interesting extension of the analysis, left for future research, involves giving that information some persistence and examining the degree to which such persistence will increase the monetary authority’s desire for maintaining some degree of flexibility. Also, given this persistence, the analysis could consider the possibility that the monetary authority might employ Crawford and Sobel’s (1982) concept of cheap talk to reveal its private information, at least partially. Specifically, the monetary authority could make an imprecise announcement (in terms of a range) that partially reveals the persistent (permanent) component of its private information. Accordingly, the monetary authority could keep the average equilibrium inflationary bias low in absolute terms throughout the horizon and, at the same time, enhance the scope of flexibility permitted by the targeting procedure. While the optimal targeting horizon, $N^\ast$, would be determined from the stationary part of the model, the degree of precision of the announcements (in terms of the number and width of the ranges) would be determined on the basis of the persistent component of the monetary authority’s private information.

Another interesting extension left for future research involves the introduction of (verifiable) supply shocks as in Rogoff (1985), Flood and Isard (1989), and Lohmann (1992) into the model. The richer model specification in a multiperiod setting would permit a fruitful investigation of the merits of the multiperiod targeting procedure relative to alternative institutional solutions—in particular, Rogoff’s perverse policymaker solution, Flood and Isard’s mixed-strategy solution involving a simple constant-money-growth rule with an escape clause to permit temporary (i.e., one-period) reversions to the full-discretionary policy upon the realization of a seldom experienced shock, and a combination of those two solutions as suggested by Lohmann. Moreover, the analysis could determine the extent to which these alternative institutional arrangements could be supplemented by the
multiperiod targeting procedure to achieve a more efficient balance between flexibility and credibility.

**Appendix**

This appendix sketches a few steps of the method used to solve the monetary authority's optimization problem subject to the $N$-period average target constraint. (For simplicity, the tilde ("~") notation is suppressed here.) In period $t = N$, the monetary authority has no choice. Its policy is determined by (11) in the main text, given the past sequence $(g_{t-1})_{t=1}^{N-1}$. In period $t = N - 1$, it solves the following problem, recognizing the impact its policy in the current period $t = N - 1$ has on future expectations, and subject to (11) in the main text:

$$
A(1) \max_{g_{N-1}} E_{N-1} \left\{ \sum_{t = N-1}^{N} \left[ -f(g_t - \delta_t - \pi^*)^2 \right. \right. \\
\left. \left. - (g_t - g_t^e - \delta_t - y^*)^2 \right] \right\}.
$$

In light of the constraint on its policy in the next period ($t = N$), the monetary authority does not expect that it can create surprise inflation in the last period. Since (11) implies $g_N^e = g_N$, the last term for $t = N$ in (A1) is irrelevant in the optimization problem. The expectation of the first-order condition, conditional on $d_{N-1}$ is given by

$$
A(2) -2f(g_{N-1} - d_{N-1} - \pi^*)
$$

$$
-2(g_{N-1} - g_{N-1}^e - d_{N-1} - y^*)
$$

$$
+ 2f \left( N\pi^* - \sum_{t=1}^{N-1} g_t - \pi^* \right) = 0
$$

where (11) has been used. Taking the wage-setters' expectation of (A2) and rearranging yields

$$
A(3) \quad g_{N-1}^e = \left( \frac{y^*}{f} + N\pi^* - \sum_{t=1}^{N-2} g_t \right) / 2.
$$

The solution to the monetary authority’s problem in period $t = N - 1$ is obtained by substituting (A3) back into (A2):

$$
A(4) \quad g_{N-1} = g_{N-1}^e + \frac{1 + f}{1 + 2f} d_{N-1}
$$

where $g_{N-1}^e$ is given by (A3).

In period $t = N - 2$, the monetary authority solves a similar problem:

$$
\max_{g_{N-2}} E_{N-2} \left\{ \sum_{t = N-2}^{N} \left[ -f(g_t - \delta_t - \pi^*)^2 \right. \right. \\
\left. \left. - (g_t - g_t^e - \delta_t - y^*)^2 \right] \right\}
$$

subject to (11). Under the assumption that the monetary authority recognizes the impact that its current policy has on future expectations, the second terms for $t = N$ and $t = N - 1$ in the optimization problem for period $t = N - 2$ are irrelevant. Because the monetary authority expects to implement the policy in (A4) in the next period ($t = N - 1$) and believes that the wage-setters’ expectations of its future ($t = N - 1$) policy are consistent with (A4), it recognizes that it cannot create inflation surprises then or any time in the future with its current policy. Using (A3), (A4), and (11) and rearranging, the problem becomes equation (A5), below. Since $E_{N-2}[d_s] = 0$ and

$$
A(5) \max_{g_{N-2}} E_{N-2} \left\{ -f(g_{N-2} - \delta_{N-2} - \pi^*)^2 - (g_{N-2} - g_{N-2}^e - \delta_{N-2} - y^*)^2 \right\}
$$

$$
- f \left( \left( \frac{y^*}{f} + (N - 2)\pi^* + \sum_{t=1}^{N-2} g_t \right) / 2 \right)^2 + \frac{1 + f}{1 + 2f} d_{N-1} - \delta_{N-1} \right)^2
$$

$$
- f \left( - \left( \frac{y^*}{f} - (N - 2)\pi^* - \sum_{t=1}^{N-2} g_t \right) / 2 \right)^2 - \frac{1 + f}{1 + 2f} d_{N-1} - \delta_{N-1} \right)^2
$$

. \right\}.
\[ E_{N-2}(\delta_s) = 0 \text{ for } s > N-2, \text{ the first-order condition to this problem implies} \]

\( g_{N-2}^c = \left( N\pi^* + 2y^* / f - \sum_{t=1}^{N-3} g_t \right) / 3 \)

and

\( g_{N-2} = g_{N-2}^e + \frac{2(1+f)}{2+3f} d_{N-2}. \)

Similarly, the optimization problem for period \( t = N-3 \) has the following solution:

\( g_{N-3}^c = \left( N\pi^* + 3y^* / f - \sum_{t=1}^{N-4} g_t \right) / 4 \)

\( g_{N-3} = g_{N-3}^e + \frac{3(1+f)}{3+4f} d_{N-3}. \)

Repeating this exercise for periods \( t = N-\rho, \) one can verify that the general solution to this problem is given by

\( g_{N-\rho}^c = \left( N\pi^* + \rho y^* / f - \sum_{t=1}^{N-\rho-1} g_t \right) / (\rho+1) \)

and

\( g_{N-\rho} = g_{N-\rho}^e + \frac{\rho(1+f)}{\rho + (\rho+1)f} d_{N-\rho} \)

for \( \rho = 1, 2, 3, \ldots, N-1. \) The solution given in the text, equation (12), follows from (A10) and (A11), noting that \( \rho = N-t. \) [A proof via mathematic induction that (12) is the solution is available from the authors upon request.]

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