Prudence versus Predation and the Gains from Trade[†]

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Abstract. We analyze a dynamic, two-country model that highlights the various trade-offs each country faces between current consumption and competing investments in its future productive and military capacities as it prepares for a possible future conflict. Our focus is on the circumstances under which the effects of current trade between the two countries on the future balance of power render trade unappealing to one. We find that a strictly positive probability of future conflict induces the country with less resource wealth to "prey" on the relatively more "prudent" behavior of its richer rival, and more so as conflict becomes more likely. While a shift from autarky to trade always raises the current incomes of both countries, the poorer country realizes the relatively larger income gain from trade and also devotes a relatively larger share of its income gain towards arming. Consequently, the richer country rationally forecloses on trade today when the difference in *ex ante* resource wealth is sufficiently large and is more likely to prefer autarky when the probability of future conflict is higher. An empirical analysis of the period surrounding the end of the Cold War provides suggestive evidence in support of the theory.

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1 Introduction

Despite a resurgence in global tensions in recent years, we live in an era of unprecedented peace between nations. Seventy years have gone by without a repeat of the major conflicts of the early twentieth century; and, since the end of the Cold War, the number of inter-state conflicts has steadily declined to practically zero. The expansion of international trade since World War II, in particular, is credited with ensuring a more secure global order by raising the economic costs of war (Polachek, 1980; Martin et al., 2008; Glick and Taylor, 2010). Nonetheless, global expenditure on defense in 2017 was roughly \$1.7 trillion, a 70% increase in real terms from the post-Cold War lows of the early-1990s. Aside from the West's recent interventions in the Middle East, a major driver of this trend is the rapid expansion of defense spending by emerging economies that have become more integrated into the world trading system in the past few decades.[1]

Economists have recently begun to expand the scope of international trade theory to explore the importance of arming and conflict.[2] These theories, however, are silent on why we continue to observe increases in arming in a seemingly peaceful and continually globalizing world. Furthermore, while "realist" security scholars have long expressed the view that the standard "gains from trade" could be outweighed by the negative consequences for a country's future security if these gains are asymmetrically distributed, trade theory has not examined this argument specifically.[3] This is a regrettable oversight that diminishes our understanding of why and when nations expand trade with one another. Even in the present day, policymakers openly regard trade policy as an instrument for achieving security-related goals.[4]

In this paper, with an aim to address these and related issues, we present a simple dynamic theory of trade, investment, and arming, focusing on two countries. Central to both our motivation and our analysis is the conceit that, while peace prevails in the present, the two countries make these decisions in the shadow of a future conflict that emerges with some

[1] Data from SIPRI (2018), available at www.sipri.org, show China (182%), Russia (128%), Vietnam (171%), and the former communist nations of Eastern Europe (collectively, 122%) have each increased their defense spending tremendously since 2005 alone, continuing long term trends that soon followed the liberalization of their economies during the 1990s. Recent upward trends are also common across Africa, Asia, and Central and South America.

[2] For example, see Skaperdas and Syropoulos (2001) and Garfinkel et al. (2015). We discuss these papers in more detail along with other related ones below.

[3] As Morelli and Sonno (2017) discuss, "realist" scholars generally treat dependence on trade with other nations as a source of diminished security (c.f., Waltz, 1979; Grieco, 1990), whereas the opposing, "liberal" view argues that the efficiency gains from trade should raise the opportunity cost of war. A third view presented in Copeland (2015) combines elements of both views, positing that wars may arise because of uncertainty over future trade (see also Bonfatti and O'Rourke, 2018).

[4] For example, former U.S. Secretary of Defense Ash Carter stated, "you may not expect to hear this from a Secretary of Defense, but in terms of a rebalance in the broadest sense, passing TPP [the Trans-Pacific Partnership] is as important to me as another aircraft carrier." See "Remarks on the Next Phase of the U.S. Rebalance to the Asia-Pacific" delivered at the McCain Institute (Arizona State University, April 6, 2015) and available at www.defense.gov.

known, positive probability. This setting naturally delivers motivations for costly arming in the midst of an ongoing peace—namely, as necessary (and/or opportunistic) preparations made for an uncertain future. More importantly, the nature of the interaction between the two countries, with each having to balance how its decisions today will affect outcomes under both peace and conflict tomorrow, allows us to examine how differences in the initial distribution of resources translate into differences in military power and, ultimately, in preferences toward trade. In doing so, the analysis yields a clear prediction regarding the effectiveness and credibility of security-based arguments for trade restrictions: a sufficiently high threat of conflict can eliminate a larger country's incentive to trade with a smaller rival, but only if the difference in *ex ante* economic size between the two countries is sufficiently large; otherwise, trade in the present remains the best policy for both countries, despite the possibility of future conflict.

Our theory highlights two types of trade-offs that countries must weigh in order to optimize their expected payoffs in the midst of an ongoing rivalry. First, both countries must invest some of their current resources to provide for future consumption—a standard "*inter*-temporal" trade-off. Second, they face an "*intra*-temporal" trade-off between two types of investment that support future consumption in distinct ways: "saving", which yields resources for future consumption, and "arming", which determines how these resources will be divided in the event of a future war. Naturally, countries with larger initial resource endowments are in a better position to satisfy their current consumption needs. As expected, then, a relatively larger country chooses higher levels of arming and saving in equilibrium than its smaller rival. But, the degree to which it enjoys an arming advantage does not depend simply on the difference in endowment sizes (as might be presumed), but rather on how such differences and the likelihood of conflict jointly shape each country's strategic incentives for both arming and saving. In equilibrium, the *ex ante* smaller country allocates a relatively smaller share of its income to saving and a relatively larger share to arming compared with its larger counterpart. Thus, a strictly positive probability of future conflict enables the smaller country to "prey" on the more "prudent" behavior of its counterpart, thereby making it disproportionately powerful as compared with its initial size.

The relevance of trade in this setting derives from its effects on the current resources of the two countries. As is true for most static trade models without security concerns—and as we will illustrate using a simple "Ricardian" example—trade does not reduce the productive efficiency of either country and usually generates real income gains in absolute terms for both. At the same time, we find the intuitive result that the smaller country always enjoys the relatively larger gain from trade.[5] Though unequal distributions of the gains from trade would not ordinarily prevent trade from taking place, our dynamic setting where future security concerns enter highlights a clear set of circumstances under which a larger

[5]By "gains from trade", we refer to the ratio of a country's real income under trade to its real income under autarky.

country will refuse to trade in the present. More precisely, as the initial size difference between the two countries increases, the larger country's gains from trade become smaller, the smaller country's gains from trade become larger, and—because of how the possibility of conflict affects relative arming choices versus relative saving choices—the smaller country increasingly allocates a larger portion of its income gain towards arming versus saving. To be sure, the presence of dual strategic interactions in both arming and saving makes characterizing the effects of trade in this setting quite complex (and potentially ambiguous). Nonetheless, it is always possible to identify a sufficiently unequal distribution of initial endowments beyond which the larger country prefers autarky to trade.[6] Furthermore, the larger country's preference for autarky tends to emerge for a wider range of relative endowment sizes when the probability of conflict is higher.[7]

That a higher risk of conflict should weigh more heavily on incentives to trade when countries are dissimilar in size draws our interest empirically. Obviously, assessing the "likelihood of conflict" from the data requires some creativity on our part, since it cannot be observed directly. The most related empirical study to ours in this regard is Long (2008), who uses data on bilateral "strategic rivalries" (originally from Thompson, 2001) to examine the effects of such rivalries on trade during the widespread strategic re-alignment that occurred in the years surrounding the end of the Cold War. We similarly view this period as especially appealing to study, as the heightened tensions which preceded the Cold War's conclusion provide plausibly large sources of variation in the perceived risk of conflict both across pairs of countries as well as over time. A panel "gravity" framework applied to this unique era of history finds support for the prediction that the effect of a strategic rivalry on trade is only present for countries that initially differ in economic size.[8]

Both history and current events offer numerous episodes to which the insights from our analysis can be applied, including the accession of China to the WTO, the U.S.'s carefully calculated trade policy towards the Soviet Union during the Cold War, and the implementation of similar embargo and sanction policies dating from the Age of Discovery up to present day.[9] Take China's accession to the WTO as an example. As Mann (1999) summarizes,

[6]Remarkably, this finding holds not only in our simple Ricardian setting, but also in many other, more complex trade environments that feature trade costs, incomplete specialization, intra-industry trade, increasing returns and/or heterogeneous firms. As we show in the Appendix, the robustness of this result arises as the gains from trade in all of these very different models behave very similarly in the limit as one country becomes infinitesimal (a novel result in its own right).

[7]Our finding that one country might choose not to trade in the present resembles Gonzalez (2005)'s finding that agents might adopt inefficient technologies to discourage future aggression by rivals. Though Gonzalez (2005) also relies on a dynamic model, his work differs from ours in that disincentives for technology adoption arise from how technology adoption shapes future (contested) output, whereas disincentives for trade in our analysis arise from how trade shapes current (secure) output.

[8]Our empirical framework differs from Long (2008) in that we use Poisson Pseudo-maximum Likelihood and a rich set of fixed effects, as is now standard in the international trade literature.

[9]Numerous anecdotes illustrate the strategic and historic relevance of trade embargoes. The rise of the Dutch Republic as a major naval power in the 17th century is closely linked with the end of intermittent Spanish embargoes imposed during the Eighty Years' War (Israel, 1989). Napolean's invasions of both Spain

until the 1990s, U.S. commercial policies towards China had long been decided mainly by the U.S.'s national security apparatus, with a view towards "containing" the rise of China as a potential future rival. But, starting with the Clinton administration this strategy gave way to "engagement", whereby the economic benefits of trade for both countries would facilitate cooperation in other arenas as well. One interpretation of this dynamic in terms of our model is simply that, between its initial, failed application to the GATT in 1987 and its eventual accession in 2003, China had grown sufficiently large on its own such that the commercial benefits to the U.S. of expanding trade with China began to outweigh any extenuating security concerns. Yet, more recent policy overtures—such as the prominent strategic undercurrent in the Obama administration's vision for a Trans-Pacific trade bloc that would have excluded China—suggest that the U.S.'s perception of China as a military threat has risen.

Our choice to abstract from the possibility that either country can take actions to try to influence the likelihood of conflict is driven by our perspective in relation to the literature. Theories underlining the "liberal peace" argument that trade may serve as a deterrent to war (e.g., Martin et al., 2008) typically examine how the choice go to war is affected by exogenous changes in the trade regime.[10] Our approach pursues a kind of converse: even acknowledging that trade may be useful for ensuring peace, it is also worth investigating why and when peace may be necessary for ensuring trade. As an illustration, our empirical section touches on the case of Jordan and Israel, which first signed a peace agreement in 1994 before negotiating a series of trade and investment agreements shortly thereafter.

In shedding light on these issues, our analysis builds on and synthesizes several disparate strands of the relevant literature. The first of these is the "relative gains" argument for restricting economic cooperation articulated by the "neorealist" school of international relations. In this tradition, as in our model, "cooperation that creates and distributes wealth affects security as well as welfare" (Liberman, 1996); thus, countries must be strategic in choosing with whom they cooperate and when. However, formal presentations of this argument—e.g., Powell (1991); Gowa (1995)—have not considered explicitly the means by which relative wealth translates into relative power and the conditions under which this relationship could be stronger or weaker.

Second, we share with Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011),

and Russia were partly motivated by a desire to make his blockade against British trade more effective (Woolf, 2002). Contemporary sanctioning episodes involving Russia, Iran, Cuba, and North Korea also feature a notable strategic dimension.

[10]The "liberal peace" hypothesis finds some support in the empirical literature—see the recent survey by Morelli and Sonno (2017). However, this literature has also highlighted some interesting exceptions relevant to our analysis. Most notably, Hegre (2004) and Morelli and Sonno (2017) respectively find that the peace-promoting effects of trade could be conditional on asymmetries in trade dependence and on asymmetries in resource wealth. In other related work, Seitz et al. (2015) show that if trade lowers the probability of conflict, the resulting reduction in the need for defense spending may generate substantial economic benefits on top of the usual gains from trade.

Garfinkel et al. (2008, 2015), and Garfinkel et al. (2018) an interest in how trade affects the relative appeal of arming versus other, more productive activities. The first four papers focus on a factor-price channel that can render trade by countries involved in conflict unappealing even when neither adversarial country gains relatively more from trade. Garfinkel et al. (2018) find, as we do, that trade between adversaries can be relatively unappealing to one country in the presence of sharp resource asymmetries, but there the mechanism is a terms-of-trade channel.[11] In any case, with an emphasis on how the anticipation of trade influences arming incentives, neither of these two approaches captures the possibility that increases in national income due to trade have direct implications for military spending, as is apparent from the arming statistics cited above. Our analysis, by contrast, centers on an income channel that is more directly relevant for understanding why relative gains from trade might matter.

Third, our model gives rise to a variant of Hirshleifer (1991)'s concept of the "paradox of power", which states that players with fewer resources initially devote disproportionately more resources to appropriative activities because they have less to lose from distributional conflicts than their larger rivals. Our setting's version of this paradox increases in importance when the likelihood of conflict increases, capturing the idea that the threat of conflict itself can be a meaningful source of power for a smaller player.

The rest of the paper is organized as follows. The next section describes the basic elements of our model. Section 3 then characterizes how equilibrium arming, savings, and payoffs respond to (exogenous) changes in first-period incomes. In Section 4, we close the model by allowing first-period incomes to be determined by the trade regime and examine each country's preferences towards trade. Section 5 explores empirically the theory's predictions for trade. Section 6 concludes. All technical details, including proofs and a discussion of the implications of alternative models of trade, are provided in an online appendix.

2 A Dynamic Model of Prudence Versus Predation

We consider a two-period model of the world economy populated by two countries that are identified by a superscript $i = 1, 2$. The key feature of our setting is that, while peace always prevails in the first period ($t = 1$), war emerges in the second period ($t = 2$) with a strictly positive probability. In the case of peace in period $t = 2$, each country's output, produced using the resources generated from their savings/investments in period $t = 1$, is secure.

[11] Skaperdas and Syropoulos (2001), Dal Bó and Dal Bó (2011), and Garfinkel et al. (2008, 2015) each study extended Hecksher-Ohlin settings with small countries that trade with the rest of the world where changes in relative world prices induce changes in relative factor prices of capital and labor that in turn influence the relative costs of arming versus producing useful output. Garfinkel et al. (2018) analyze a modified Ricardian model of trade between large adversarial countries, intentionally abstracting from both factor-price and income channels to isolate the importance of a terms-of-trade channel that generally induces countries to internalize the negative externality of their arming and thus more often than not generates gains from trade that are greater than those implied by conventional analyses that abstract from conflict altogether.

But, if war erupts, then some fraction of each country's output is subject to expropriation by the other.[12]

Our principal aim in this setting is to explore how the trading relationship in period $t = 1$ influences each country's saving and arming decisions in that same period and how that matters for relative power, growth and, ultimately, national welfare over the two periods. In our initial exposition, we strive to motivate our results for trade in a simple yet general way, deferring the formal details regarding trade until Section 4. Accordingly, for now, we briefly characterize each country by its possession of an initial resource endowment R^i and a native autarky technology level A^i. A^i can be thought of as aggregating a country's ability, under autarky, to produce multiple intermediate inputs used for production of final output (or income) Y^i: $Y^i = A^i R^i$. But, under trade, at least one country—and possibly both—realizes efficiency gains from specializing in the input in which it has comparative advantage. That is, output under trade can be written instead as $Y^i = T^i R^i$, where $T^i \equiv T^i(R^i, R^j)$ depends on initial resource endowments of both countries as well as technologies and where $T^i \geq A^i$ holds as a strict inequality for at least one country, reflecting the possible gains from trade.

The most salient feature of trade for our current purposes, however, is how these income gains are distributed across countries. Specifically, as we demonstrate below using a simple "Ricardian" example and as we can show in other trade models,[13] the smaller of the two countries under autarky can expect to enjoy relatively larger income gains from trade than its larger trading partner:

$$\textit{if} \;\; A^j R^j > A^i R^i, \quad \textit{then} \;\; \frac{T^i R^i}{A^i R^i} > \frac{T^j R^j}{A^j R^j}.$$

This forthcoming result should be kept in mind as we develop the intuition behind our results for trade by first focusing on exogenous changes in relative output.

Setting aside (for now) the decision to trade, a central component of our analysis is how each country subsequently decides to allocate its first-period income Y^i. Specifically, each country divides its output between current consumption C^i and two distinct types of activities that can augment future consumption $\widetilde{C}^i$: "arming" (which we denote by G^i) and "saving" (which we denote by Z^i). (Throughout, we use a tilde ($\sim$) over a variable

[12] Our assumption that war emerges in the future with some probability represents an important departure from much of the conflict literature that assumes conflict emerges with certainty. Even in analyses that study the choice between war and peace (e.g., Jackson and Morelli, 2007), war emerges with either probability 1 or probability 0. We view our approach as appealing since it allows us to study not only these special cases, but also intermediate cases where arming is *ex ante* prudent but not necessarily *ex post*. Nevertheless, we recognize the value of exploring the endogenous determination of the probability of future conflict, especially in relation to the gains from trade in the future, as taken up in Section 4.3.

[13] Indeed, our analysis generalizes under a variety of trade models, including the standard neoclassical ("Heckscher-Ohlin" and "Ricardo-Viner") frameworks (assuming fixed relative factor endowments) as well as the more recent paradigms described in Armington (1969), Krugman (1980), Eaton and Kortum (2002), and Melitz (2003) (see Appendix C).

to indicate its value in the second period, $t = 2$.) This choice must satisfy the following resource constraint:

$$C^i + G^i + Z^i \leq Y^i, \text{ for } i = 1, 2. \tag{1}$$

In period $t = 2$, each country i's first-period saving yields $\widetilde{R}^i = Z^i$ units of the productive resource. Assuming technologies available to each country do not change over time, but trade is not possible in period $t = 2$, that resource in turn is transformed into $\widetilde{Y}^i = A^i Z^i$ units of output.[14]

The output held securely by each country in period $t = 2$ and thus the return from such saving, however, are subject to uncertainty due to the possibility of a future war. The weight of this uncertainty is governed by two parameters, the probability of conflict (denoted by $q \in (0, 1]$) and the degree of output insecurity under conflict (denoted by $\kappa \in (0, 1]$). More precisely, in the event that peace prevails, which occurs with probability $1 - q$, country i enjoys its entire output: $\widetilde{C}^i = A^i Z^i$, $i = 1, 2$. But, if war erupts, which occurs with probability q, only a fraction $1 - \kappa$ of a country i's output is secure; the remaining fraction goes into a contested pool, $\kappa \left(A^i Z^i + A^j Z^j\right)$, $i \neq j = 1, 2$.

When war breaks out, country i's share ϕ^i of the contested pool in period $t = 2$ depends on arming by both countries (G^i, G^j) chosen in period $t = 1$.[15] This share takes the standard ratio form:

$$\phi^i(G^i, G^j) \equiv \frac{(G^i)^m}{(G^i)^m + (G^j)^m}, \quad \text{if } G^i + G^j > 0 \text{ for } i \neq j = 1, 2, \tag{2}$$

where $m \in (0, 1]$ reflects the effectiveness of arming; if $G^i + G^j = 0$ then $\phi^i = \phi^j = \frac{1}{2}$.[16] This specification implies a country's share is increasing in its own arming (i.e., $\phi^i_{G^i} \equiv \partial\phi^i/\partial G^i = m\phi^i\phi^j/G^i > 0$) and decreasing in the opponent's arming (i.e., $\phi^i_{G^j} \equiv \partial\phi^i/\partial G^j = -m\phi^i\phi^j/G^j < 0$). Furthermore, this conflict technology is symmetric (i.e., whenever $G^i = G^j$, $\phi^i = \phi^j = \frac{1}{2}$ holds).

Each country i chooses its allocation of current income Y^i to arming G^i and saving Z^i to maximize expected lifetime utility: $U^i = u(C^i) + \delta E\{u(\widetilde{C}^i)\}$, where $\delta \in (0, 1]$ represents the common discount factor and $u(\cdot)$ has the usual properties: $u' > 0$, $u'' < 0$, and $\lim_{C \to 0} u'(C) = \infty$.[17] To keep the analysis as simple as possible, we assume logarithmic

[14] Although a central objective in this paper is to explore the influence that the trade regime in place in $t = 1$ has on current equilibrium allocations to saving and arming when war in the next period possibly materializes, our analysis could be extended to consider also the possibility of trade in $t = 2$ if peace prevails. Briefly discussed in Section 4.3, such an extension would leave our central results unchanged. Building on that possibility, we also briefly consider how future trade could influence the probability of a future war and thereby affect the preference for trade in the first period.

[15] As discussed in Section 4.3, modeling conflict instead as a "winner-take-all contest", with ϕ^i representing country i's probability of winning, complicates the analysis but does not alter our central results.

[16] Tullock (1980) and Skaperdas (1996) provide discussion and analyses of this form. The restriction that $m \leq 1$ is often imposed in the contest literature for technical reasons.

[17] These properties prove helpful in establishing quasi-concavity of payoff functions and ensure strictly positive allocations to both saving and arming.

preferences:[18]

$$U^i = \ln C^i + \delta E\left\{\ln \widetilde{C}^i\right\} \text{ for } i = 1, 2. \tag{3}$$

This maximization problem for each country i, which takes rival country j's choices as given, is subject to (1) as a strict equality, (2) and, for $i \neq j = 1, 2$,

$$\widetilde{C}^i = \begin{cases} A^i Z^i & \text{with probability } 1 - q \\ \phi^i \kappa (A^i Z^i + A^j Z^j) + (1 - \kappa) A^i Z^i & \text{with probability } q. \end{cases} \tag{4}$$

As (4) shows, a country's arming matters for future consumption only in the event of war and, even then, only to the extent that property is insecure ($\kappa > 0$).[19]

The timing of the extended policy game is as follows. First, at the beginning of period $t = 1$, the two countries' policymakers simultaneously and noncooperatively choose their individually preferred trade regimes. If both countries announce "trade" (T), then the two countries exchange their intermediate goods, and each country i's output level is $Y^i = T^i(R^i, R^j)R^i$ for $i \neq j = 1, 2$; if, however, at least one country announces "autarky" (A), then no trade takes place, and each country i's output level is $Y^i = A^i R^i$. Second, once first-period output levels are determined, each country i chooses G^i and Z^i noncooperatively and simultaneously and consumes the remaining income C^i. In period $t = 2$, each country uses its available resource $\widetilde{R}^i = Z^i$ to produce the intermediate goods and then output $\widetilde{Y}^i = A^i Z^i$. The amount consumed that period by each country depends on whether or not war breaks out and of course on both countries' first-period choices as shown in (4).[20]

3 Equilibrium Arming, Saving and Payoffs Given Income

Given the dynamic structure of the model, we find the subgame perfect equilibrium by solving the model backwards. Specifically, in this section, we characterize the Nash equilibrium of the simultaneous-move subgame in arming and saving and the associated discounted payoffs given Y^i for $i = 1, 2$, deferring until the next section our discussion of trade. Using

[18] Even though the logarithmic form restricts the intertemporal elasticity of substitution to equal 1, our key findings remain intact when $u(C) = C^{1-\rho}/(1-\rho)$ for $0 < \rho \neq 1$, where ρ is the coefficient of relative risk aversion and $1/\rho$ is the elasticity of intertemporal substitution.

[19] When either $q = 0$ or $\kappa = 0$, the model simplifies to a standard consumption/investment savings model (i.e., with $G^i = G^j = 0$), a useful benchmark for highlighting the importance of insecurity and uncertainty for such dynamic problems. We could also modify the model so that conflict results in costly destruction (where, for example, only a fraction of the contested pool remains intact). We do not consider this possibility here, because it does not substantively alter our conclusions, as discussed in Section 4.3.

[20] This interaction differs substantively from standard games of distributive conflict, where each contender typically controls only one instrument (arming) that in turn determines the size of the contested prize. By contrast, in our model, each contender chooses its savings in addition to arming, and these two choices jointly determine the size of the contested prize in period $t = 2$ in the event of war. As will become evident below, our analysis of this more complex problem makes a methodological contribution to the literature as well, even before considering trade.

equations (1) and (4), let us rewrite country i's expected, two-period payoff (3) as follows:

$$U^i = \ln(Y^i - G^i - Z^i) \\ + \delta\left[q \ln\left[\phi^i \kappa \left(A^i Z^i + A^j Z^j\right) + (1-\kappa) A^i Z^i\right] + (1-q)\ln\left(A^i Z^i\right)\right], \tag{5}$$

for $i \neq j = 1, 2$, where $\phi^i = \phi^i(G^i, G^j)$ is shown in (2) and where $Y^i = A^i R^i$ under autarky and $Y^i = T^i(R^i, R^j)R^i$ under free trade. Country i's choices of arming G^i and saving Z^i, then, satisfy the following first-order conditions (FOCs):

$$U^i_{G^i} = \delta q \frac{\phi^i_{G^i}\kappa\left(A^i Z^i + A^j Z^j\right)}{\phi^i\kappa\left(A^i Z^i + A^j Z^j\right) + (1-\kappa)A^i Z^i} - \frac{1}{Y^i - G^i - Z^i} \leq 0 \tag{6a}$$

$$U^i_{Z^i} = \delta\left[\frac{q\left(\phi^i\kappa + 1 - \kappa\right)A^i}{\phi^i\kappa\left(A^i Z^i + A^j Z^j\right) + (1-\kappa)A^i Z^i} + \frac{1-q}{Z^i}\right] - \frac{1}{Y^i - G^i - Z^i} \leq 0, \tag{6b}$$

for $i \neq j = 1, 2$, which is a system of four equations in four unknowns.

The second terms in the right-hand sides (RHS) of (6a) and (6b) respectively represent the marginal costs to country i of arming (MC^i_G) and saving (MC^i_Z) that arise as such activities reduce current consumption, $C^i = Y^i - G^i - Z^i > 0$. Because G^i and Z^i constitute competing uses of $t = 1$ output and they displace the same quantity of current consumption, their marginal costs are identical (i.e., $MC^i_G = MC^i_Z$) and always reflect the *inter*-temporal trade-off between present and future consumption. In addition, both MC^i_G and MC^i_Z are increasing and convex in G^i and Z^i (respectively) and are decreasing in country i's $t = 1$ output Y^i; that is, $\lim_{G^i + Z^i \to Y^i} MC^i_J = \infty$ and $\partial MC^i_J / \partial Y^i < 0$ for $J = G, Z$.

The first term in the RHS of (6a) represents country i's expected, discounted marginal benefit of producing an additional gun (MB^i_G). This benefit derives from the effect of increased arming to expand country i's share of the contested output and thereby augment its future consumption $\widetilde{C}^i$ in the event of war. Observe that MB^i_G depends positively on both the probability of war q and the degree of resource insecurity κ, as well as the discount factor δ. The properties of the conflict technology in (2) imply further that MB^i_G is decreasing in country i's own arming G^i. Since MC^i_G is increasing in G^i (as noted above), country i's payoff is strictly concave in G^i (i.e., $U^i_{G^i G^i} < 0$). By contrast, the qualitative influence of country j's arming on MB^i_G (or the sign of $U^i_{G^i G^j}$) depends on both countries' arming and saving decisions, as well as the degree of output insecurity κ. More precisely, one can show that when $\kappa = 1$ (i.e., property is totally insecure), $U^i_{G^i G^j} > 0$ holds for all feasible (G^i, G^j).[21] However, when $\kappa < 1$, $U^i_{G^i G^j} > 0$ holds for any given $G^i > 0$ only if the levels of G^i and G^j are sufficiently similar.[22]

[21] To see this, note generally, from (2), that $\phi^i_{G^i} = m\phi^i\phi^j/G^i$. Thus, when $\kappa = 1$, country i's marginal benefit of arming shown as the first term in (6a) simplifies as $MB^i_G = \delta q \phi^i_{G^i}/\phi^i = \delta q m \phi^j / G^i$, which is clearly increasing in G^j.

[22] When property is totally insecure ($\kappa = 1$), each country's marginal benefit of arming is independent of both Z^i and Z^j and, thus, becomes symmetric across the two countries such that $MB^i_G = MB^j_G$ whenever

Country i's expected marginal benefit of saving (MB_Z^i) is captured by the first term in the RHS of (6b). Unlike MB_G^i, this expected benefit derives from two distinct sources, one which is operative in the event of conflict and one which takes precedence only in the event of peace. If conflict arises, increases in savings affect both the total pie of insecure future output to be contested and one's own secure supply of resources. If instead peace prevails, each country's savings then convert entirely to future consumption. As might be expected, MB_Z^i falls with increases in the likelihood of conflict q and the degree of insecurity κ, and rises with increases in the discount factor δ. Further inspection also reveals MB_Z^i is decreasing in the country's own saving Z^i, and, when $q < 1$, $\lim_{Z^i \to 0} MB_Z^i = \infty$. Thus, provided the probability of future peace is strictly positive ($q < 1$), both countries choose strictly positive savings: $Z^i > 0$ for $i = 1, 2$. Since MC_Z^i is increasing in Z^i, these properties imply country i's payoff is strictly concave in Z^i (i.e., $U_{Z^i Z^i}^i < 0$). Furthermore, since MB_Z^i is decreasing in Z^j, the countries' savings choices are strategic substitutes (i.e., $U_{Z^i Z^j}^i < 0$).[23]

3.1 Equilibrium in Shares

With the relationships outlined above, we now turn to defining and characterizing the equilibrium implied by (6). In view of the complexity of this strategic environment, we first reduce the dimensionality of the problem in order to obtain what we call an "equilibrium in shares" representation. This approach enables us to illuminate how country "size" translates into "power" as well as how this relationship is moderated by changes in the probability of conflict q and/or the degree of insecurity κ, thereby paving the way for our upcoming analysis of how the trade regime matters for equilibrium outcomes and payoffs.

To proceed, define the share that country i contributes to the (potentially) contested pool of future output as

$$\theta^i(Z^i, Z^j) \equiv \frac{A^i Z^i}{A^i Z^i + A^j Z^j} \text{ if } A^i Z^i + A^j Z^j > 0 \text{ for } i \neq j = 1, 2, \tag{7}$$

where $\theta^j = 1 - \theta^i$. One can easily verify that $q < 1$ ensures $\theta^i, \theta^j > 0$, $\theta_{Z^i}^i = \theta^i \theta^j / Z^i > 0$, and $\theta_{Z^j}^i = -\theta^i \theta^j / Z^j < 0$. The value of the definition in (7) is that it allows us to characterize relative saving choices across countries in terms of a single endogenous parameter: $Z^i/Z^j = A^j \theta^i / A^i(\theta^j) = A^j \theta^i / A^i(1 - \theta^i)$ for $i \neq j = 1, 2$. Similarly using the conflict technology in (2), we characterize relative arming choices in terms of another single endogenous parameter: $G^i/G^j = (\phi^i/\phi^j)^{1/m} = (\phi^i/(1-\phi^i))^{1/m}$ for $i \neq j = 1, 2$. Using these two relationships, we then transform (6) (a system of four equations in four unknowns) into a system of two

$G^i = G^j$. However, when property is partially secure ($\kappa < 1$), MB_G^i is increasing in Z^j and decreasing in Z^i, implying that country i tends to be more aggressive in its security policy if its rival j ($\neq i$) saves more, but arms by less as its own saving increases.

[23] Each country's marginal benefit of saving can also depend on both countries' arming, but only when property is partially secure (i.e., when $\kappa < 1$). Specifically, in this case, MB_Z^i is increasing in G^j and decreasing in G^i.

equations in just two unknowns—specifically, the appropriative and contributive shares, ϕ^i and θ^i.

To derive the first of these equations, we proceed in two steps. First, observe how the *relative* marginal benefits of arming and saving, given respectively by

$$\frac{MB_G^j}{MB_G^i} = \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}} \left[\frac{\kappa\phi^i + (1-\kappa)\theta^i}{\kappa\phi^j + (1-\kappa)\theta^j}\right] \quad \text{and} \quad \frac{MB_Z^j}{MB_Z^i} = \frac{A^j\theta^i}{A^i\theta^j}\left[\frac{q\frac{\kappa\phi^j\theta^j+(1-\kappa)\theta^j}{\kappa\phi^j+(1-\kappa)\theta^j}+1-q}{q\frac{\kappa\phi^i\theta^i+(1-\kappa)\theta^i}{\kappa\phi^i+(1-\kappa)\theta^i}+1-q}\right],$$

can be written so that they each depend only on ϕ^i, θ^i, q, and κ. Second, we exploit the fact that, in any equilibrium, the marginal benefits of arming and saving must equalize for each country (since $MC_G^i = MC_Z^i$ implies $MB_G^i = MB_Z^i$ for $i \neq j = 1, 2$). Thus, the following equalities must also hold in equilibrium:

$$\frac{MB_G^j/MB_Z^j}{MB_G^i/MB_Z^i} = \frac{A^i}{A^j}\left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}} \left[\frac{1-\kappa+\kappa\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)}{1-\kappa+\kappa\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}\right] = \frac{MC_G^j/MC_Z^j}{MC_G^i/MC_Z^i} = 1. \tag{8}$$

To limit the range of possibilities, we adopt the following assumption:

Assumption A $A^i = A^j = A$ *for* $i \neq j$.[24]

With this assumption and (8), we obtain the first of two conditions that define an equilibrium:

$$S^i\left(\phi^i, \theta^i; q, \kappa\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{\frac{1}{m}} \left[\frac{1-\kappa+\kappa\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)}{1-\kappa+\kappa\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}\right] - 1 = 0. \tag{9}$$

The first term on the RHS represents the ratio of the relative marginal benefits of arming and saving across countries, whereas the second term reflects the ratio of the relative marginal costs of arming and saving across countries that must equal 1. Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the condition in (9) implicitly defines an equilibrium relationship between θ^i and ϕ^i that we henceforth refer to as the "S^i-curve" or, alternatively, as "schedule S^i". The lemma below describes the key properties of this schedule, named for its "S" shape as depicted in Fig. 1:[25]

Lemma 1 *Under Assumption A, there exist pairs* $\left(\phi^i, \theta^i\right) \in (0,1) \times (0,1)$ *that solve (9), thus defining implicitly the* S^i*-curve. Such pairs have the following properties:*

(a) $\theta^i \lesseqqgtr \phi^i \lesseqqgtr \frac{1}{2} \lesseqqgtr \phi^j \lesseqqgtr \theta^j$ *along the* S^i*-curve for* $i \neq j$;

(b) $\lim_{\phi^i \to 0} \theta^i/\phi^i\big|_{S^i=0} = 0$ *and* $\lim_{\phi^i \to 1} \theta^i/\phi^i\big|_{S^i=0} = 1$;

(c) $S_{\phi^i}^i > 0$, $S_{\theta^i}^i < 0$ *and* $d\theta^i/d\phi^i\big|_{S^i=0} = -S_{\phi^i}^i/S_{\theta^i}^i > 0$;

(d) $\lim_{\phi^i \to 0} d\theta^i/d\phi^i\big|_{S^i=0} = \lim_{\phi^i \to 1} d\theta^i/d\phi^i\big|_{S^i=0} = 0$ *and* $\lim_{\phi^i \to \frac{1}{2}} d\theta^i/d\phi^i\big|_{S^i=0} > 1$.

[24]As will become clear later on when we introduce trade, this assumption is satisfied for countries that have "mirror-image" technologies.

[25]Ignore the other curves for now.

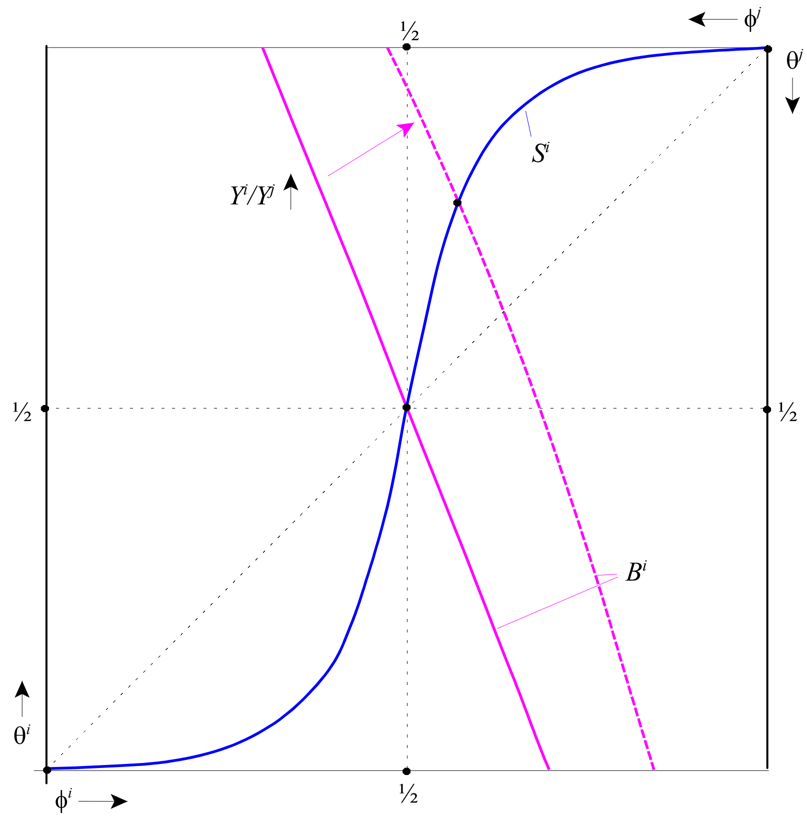

Figure 1: The Determination of Countries' Equilibrium Shares in Appropriative and Productive Investments

Part (a) points out that the less powerful country's contributive share to the potentially contested pool of future income is not only less than that of its relatively more powerful rival but also less than its own appropriative share. The rest of Lemma 1 spells out the remaining details needed to characterize the shape of the S^i-curve. Part (b) combined with part (a) establishes that the S^i-curve starts at $(\phi^i, \theta^i) = (0, 0)$, crosses the midpoint $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$ and ends up at $(\phi^i, \theta^i) = (1, 1)$. Part (c) establishes that the curve is increasing over the entire range of values, with points to the right (left) and below (above) the curve implying $S^i > 0$ ($S^i < 0$). However, from part (d), it is "flat" at the endpoints and steeper than 1 at the midpoint $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$.

For some intuition regarding the ϕ^i/θ^i relationship along the S^i-curve, recall that it represents a balance between the countries' marginal benefits of arming relative to the marginal benefits of saving and their respective relative marginal costs. As shown in (8), because the ratio of the two countries' marginal costs is fixed at 1, adjustments in ϕ^i/θ^i as we move along the entire length of the curve are due solely to changes in the ratio of countries' relative marginal benefits. To dig a little deeper, consider the midpoint of the S^i-curve, at $(\phi^i, \theta^i) = (\frac{1}{2}, \frac{1}{2})$, where $\phi^i/\theta^i = \phi^j/\theta^j = 1$ for $i \neq j$. Now consider how the ratio of relative marginal benefits would change if ϕ^i and θ^i increased proportionately (i.e., if we moved NE along the 45° line in the figure). Since ϕ^i and θ^i increase (and thus ϕ^j and θ^j decrease), while $\phi^i/\theta^i = \phi^j/\theta^j$ remain unchanged at 1, the ratio of marginal benefits in (8) rises above 1; thus, $S^i > 0$, which implies that we travel below schedule S^i. For any given increase in a country's appropriative share ϕ^i above $\frac{1}{2}$, its contributive share θ^i must rise by more (such that $\theta^i > \phi^i$) to keep the value of the ratio of marginal benefits equal to 1 and thus remain on the S^i-curve, as emphasized in Lemma 1(a). However, as part (d) establishes, this tendency becomes less pronounced for country i as $\phi^i \to 1$.

As shown in the definition in (9) and as we discuss in detail below, the shape of the S^i-curve also depends on the probability of conflict q and the degree of insecurity κ. It does not, however, depend on income levels Y^i and Y^j or on the discount factor δ; thus, while any equilibrium in (ϕ^i, θ^i) must lie somewhere on the S^i-curve, determining its exact location requires a second condition capturing the influence of these other variables on relative arming and saving decisions.

To derive this second condition, we solve for each country i's arming and saving decisions, G^i and Z^i, from the FOCs in (6), in order to obtain:

$$G^i = \frac{\gamma^i}{1+\gamma^i+\zeta^i} Y^i \quad \text{and} \quad Z^i = \frac{\zeta^i}{1+\gamma^i+\zeta^i} Y^i, \tag{10}$$

where

$$\gamma^i = \gamma^i\left(\phi^i, \theta^i\right) \equiv \delta q m \left(1-\phi^i\right) \frac{\kappa\phi^i}{\kappa\phi^i + (1-\kappa)\,\theta^i} > 0 \tag{11a}$$

$$\zeta^i = \zeta^i\left(\phi^i, \theta^i\right) \equiv \delta\left[q\frac{\kappa\phi^i\theta^i + (1-\kappa)\theta^i}{\kappa\phi^i + (1-\kappa)\,\theta^i} + 1 - q\right] > 0, \tag{11b}$$

represent weights that govern spending on arming and saving respectively per unit of income spent on current consumption.[26] Clearly, the shares of income that country i channels into arming and saving, shown in (10), depend on ϕ^i and θ^i. To proceed, observe from (10) that the ratio G^i/G^j can be written as a function of the two countries' expenditure shares and recall that the specification of ϕ^i in (2) implies $G^i/G^j = (\phi^i/\phi^j)^{1/m}$ for $i \neq j = 1, 2$. Together, these implications give us our second equilibrium condition:

$$B^i\left(\phi^i, \theta^i; Y^i/Y^j, q, \kappa\right) \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} - \frac{\gamma^i\left(1+\gamma^j+\zeta^j\right)}{\gamma^j\left(1+\gamma^i+\zeta^i\right)}\left(\frac{Y^i}{Y^j}\right) = 0. \tag{12}$$

Since $\phi^i + \phi^j = 1$ and $\theta^i + \theta^j = 1$, the above equation with the definitions of γ^i and ζ^i for $i \neq j = 1, 2$ given in (11) implicitly define another relationship between ϕ^i and θ^i, which we call the "B^i-curve" or "schedule B^i".

The next lemma establishes its existence and characterizes its shape.

[26] Note the share of income spent on first-period consumption is always given by $1/(1+\gamma^i+\zeta^i)$.

Lemma 2 *There exist pairs* $(\phi^i, \theta^i) \in (0,1) \times (0,1)$ *that solve* (12), *thus implicitly defining the* B^i*-curve. Since* $B^i_{\phi^i} > 0$ *and* $B^i_{\theta^i} > 0$, $d\theta^i/d\phi^i|_{B^i=0} = -B^i_{\phi^i}/B^i_{\theta^i} < 0$.

Schedule B^i is the negatively sloped curve in Fig. 1 that, drawn for $Y^i = Y^j$, goes through the midpoint where $\phi^i = \theta^i = \frac{1}{2}$.[27] Points to the right (left) and above (below) the curve imply $B^i > 0$ ($B^i < 0$).

Observe that, like the definition of schedule S^i, the definition of schedule B^i uses both countries' FOCs. However, its derivation relies more directly on the two countries' arming decisions, which in turn explains why the ratio of incomes $y^i \equiv Y^i/Y^j$ and (through the parameters γ^i and ζ^i) the discount factor δ appear in the RHS of (12).[28] Observe, in addition as revealed by inspection of (12) with (11), the shape and location of the B^i-curve also depend on the probability of conflict q and the degree of insecurity κ as well as relative first-period incomes, y^i.

Using Lemmas 1 and 2, we now turn to the determination of equilibrium shares, ϕ^i and θ^i, and their properties.

Proposition 1 (Equilibrium in Shares.) *Suppose* $q \in (0,1)$, $\kappa \in (0,1]$, $\delta \in (0,1]$, $y^i \in (0,\infty)$, *and Assumption A is satisfied. Then, a unique equilibrium in pure strategies* $(\phi^{i*}, \theta^{i*}) \in (0,1) \times (0,1)$ *exists in which both countries allocate positive quantities of their incomes to arming and saving. If* $y^i \gtreqless 1$, *then the equilibrium appropriative and contributive shares satisfy the following for* $i \neq j = 1,2$:

(a) $\theta^{i*} \gtreqless \phi^{i*} \gtreqless \frac{1}{2} \gtreqless \phi^{j*} \gtreqless \theta^{j*}$, *with* $\lim_{y^i \to 0} \phi^i = \lim_{y^i \to 0} \theta^i = 0$, $\lim_{y^i \to 0} \theta^i/\phi^i = 0$, *and* $\lim_{y^i \to 0} \phi^j = \lim_{y^i \to 0} \theta^j = 1$ *and* $\lim_{y^i \to 0} \theta^j/\phi^j = 1$.

(b) $\partial\phi^{i*}/\partial\kappa \gtreqless 0$ *while* $\partial\phi^{i*}/\partial q \lesseqgtr 0$, *with* $\lim_{\kappa \to 0} \phi^{i*} = \lim_{q \to 1} \phi^{i*} = \frac{1}{2}$ *for any* $y^i \in (0,\infty)$.

(c) $\partial\phi^{i*}/\partial\delta \lesseqgtr 0$ *and* $\partial\theta^{i*}/\partial\delta \lesseqgtr 0$.

Fig. 1 illustrates the equilibrium shares in appropriative and productive investments, depicted by the intersection of the (i) the B^i-curve associated with the FOCs for arming and (ii) the S^i-curve associated with the FOCs for both arming and saving. As illustrated in the figure, when $Y^i = Y^j$ (or $y^i = 1$), the intersection of the two schedules occurs at the midpoint where the two countries are equally powerful as well as equal contributors to future income ($\phi^{i*} = \theta^{i*} = \frac{1}{2}$). An increase in Y^i given Y^j (equivalently, an increase in y^i) relaxes country i's *inter*-temporal trade-off between current and future consumption, thereby reducing its marginal costs of arming and saving and causing the B^i-curve to shift rightward. The equilibria induced by such changes in Y^i then trace out the S^i-curve, reflecting changes in the relative marginal benefits of arming and saving as country i changes in size. Thus, as pointed out in part (a), when country i is initially larger (i.e., $Y^i > Y^j$), the B^i-curve

[27] That $\phi^i = \theta^i = \frac{1}{2}$ is a point on schedule B^i when $Y^i = Y^j$ can be confirmed using equation (11) with the definition of $B^i = 0$ in (12).

[28] An alternative but equivalent approach here would be to consider the saving decisions by using the definition of θ^i in (7) and the expression for Z^i in (10).

intersects the S^i-curve to the right and above the midpoint, implying it is more powerful than country j and an even bigger relative contributor to future income (i.e., $\theta^i > \phi^i > \frac{1}{2}$).

Part (b) reveals the differential influence of q and κ on each country's *intra*-temporal trade-off between arming and saving that, in turn, weighs on the balance of power. Specifically, it establishes that a deterioration of the security of property ($\kappa \uparrow$) tends to amplify relative differences in power, whereas a deterioration of international relations ($q \uparrow$) tends to diminish such differences. Furthermore, as property becomes perfectly secure ($\kappa \to 0$) or the likelihood of a future war becomes a certainty ($q \to 1$), differences in power disappear regardless of differences in the countries' income levels. Since, as mentioned earlier, the marginal benefit of arming MB^i_G shown as the first term in (6a) is increasing in q and κ while the marginal benefit of saving MB^i_Z shown as the first term in (6b) is decreasing in q and κ, an increase in either q or k increases MB^i_G/MB^i_Z or equivalently decreases the opportunity cost of arming for each country i.

To tease out some intuition here, let us start with q; and, to isolate its effect, consider the case of totally insecure property ($\kappa = 1$), such that MB^i_G depends symmetrically on country i's own arms G^i and those of the rival G^j, $i \neq j = 1, 2$. Under Assumption A, the marginal benefit of saving MB^i_Z is nearly symmetric across countries i. The sole difference appears in the second term, $(1-q)/Z^i$. Underscoring the importance of saving for the possibility of peace, this term governs the relationship between differences in country size (which do not otherwise enter when $\kappa = 1$) and the *intra*-temporal trade-off. To fix ideas, suppose $Y^i > Y^j$, which implies by part (a) of the proposition that $\theta^i > \frac{1}{2}$ and thus $Z^i > Z^j$. Accordingly, all else the same, this second term is smaller for the larger country (i), which means its opportunity cost of shifting resources from saving to arming is smaller, thereby giving it a military advantage. Now, observe that as the probability of conflict rises ($q \uparrow$), the importance of the term $(1-q)/Z^i$ falls, thereby weakening the larger country's military advantage and eventually eliminating it in any interior solution when $q = 1$.[29] Partially differentiating the expression in (8) with respect to q shows more generally that, for any $\kappa \in (0, 1]$, an increase in q reduces the opportunity cost of arming by more for the smaller country (j) than the larger one (i). This finding suggests that the mere possibility of a future conflict ($q > 0$) effectively enables the smaller country to "prey" on its larger rival's savings via aggressive arming, and more so as q increases. An increase in the security of income ($\kappa \downarrow$) works analogously. Specifically, a decrease in κ reduces the marginal benefit of arming relative to saving for both countries and to a greater extent for the larger country that tends to save more, and thereby reduces the larger country's relative military strength.[30]

[29] At this limit, both MB^i_G and MB^i_Z are symmetric across i, such that the FOCs in (6) can be satisfied for each country i only where $G^i = G^j$. In this special case, the larger country necessarily saves more, such that $C^i = C^j$ despite differences in income. Furthermore, since $q = \kappa = 1$ by assumption, $\widetilde{C}^i = \widetilde{C}^j$ holds, such that the two countries enjoy identical payoffs in any interior solution. However, corner solutions where the smaller country does not save at all also become possible when $q = \kappa = 1$.

[30] The effects of either an increase in q or a decrease in κ on the *intra*-temporal trade-off can be visualized

Finally, part (c) of the proposition shows that an increase in the discount factor δ tends to reduce differences in power ϕ^{i*} and in contributive shares θ^{i*} across countries. As discussed earlier, an increase in δ magnifies the marginal benefits of both arming and saving for each country. This magnification effect is larger for the smaller country, however, causing it to become more aggressive and, at the same time, more prudent relative to its larger rival.[31]

In sum, Proposition 1 tells us how the countries' appropriative and contributive shares (or relative arming and saving) are related to relative incomes, as well as how the distribution of power adjusts to changes in the security of property, the probability of a future war, and time preferences. But, it leaves unanswered the question of how such changes influence the arming and saving decisions of each country in levels. For example, while we know that an increase in country i's relative income makes that country more powerful, it is unclear whether each country devotes more or less resources to arming. Similarly, while we know that country i's saving rises relative to that of its rival, we do not know yet whether they save more or less.

Nonetheless, an appealing feature of our "equilibrium in shares" approach is that the share variables ϕ^{i*} and θ^{i*} pin down the fractions of current income allocated to arming and saving via (11) with (10), allowing us to recover equilibrium spending choices by each country i on G^{i*} and Z^{i*} as functions of ϕ^{i*} and θ^{i*}. Then, having identified the effects of changes in relative income Y^i/Y^j on ϕ^{i*} and θ^{i*}, we can characterize their effects on G^{i*} and Z^{i*}. This characterization not only allows us to flesh out further the implications of Proposition 1, but also leads directly into our study of the effects of trade on equilibrium arming, saving, and payoffs.

3.2 Income Changes and Equilibrium Arming, Saving, and Payoffs

We now turn to examine the implications of changes in one country's income for both countries' equilibrium choices and payoffs. Letting a hat ($\wedge$) over variables denote percent changes (e.g., $\widehat{x} \equiv dx/x$), the following proposition characterizes these effects.

Proposition 2 (Equilibrium Arming and Saving.) *Under the assumptions of Proposition*

in the setting of Fig. 1, as a counterclockwise rotation of the S^i-curve around the midpoint $(\frac{1}{2}, \frac{1}{2})$, with the endpoints unchanged, thereby making the relationship between saving (and thus country size) and power less linear. At the same time, the B^i-curve also rotates in a counterclockwise direction around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $Y^i < Y^j$ ($Y^i > Y^j$) and $m < 1$. When $Y^i = Y^j$, the curves rotate as just described, but around their intersection at the midpoint, such that ϕ^{i*} is not affected. Part (a) of Proposition A.1, presented in Appendix A, states further that θ^{i*} is similarly independent of κ and q when $Y^i = Y^j$. Although we cannot pin down the influences of these parameters on θ^{i*} for all Y^i and Y^j, part (b) of Proposition A.1 shows that, in the case of an extreme asymmetry as $Y^i \to 0$ for given $Y^j \in (0, \infty)$, θ^i is independent of κ and is decreasing in q.

[31] The effect of an increase in δ can be visualized as a counterclockwise rotation of the B^i-curve around the point where it intersects the 45° line when $m = 1$ or above (below) that intersection when $m < 1$ and $Y^i < Y^j$ ($Y^i > Y^j$).

1, an increase in country i's income (i.e., $\widehat{Y}^i > 0$) affects equilibrium arming and saving in country $i \neq j = 1, 2$ as follows:

(a) $0 < \widehat{G}^{i*} < \widehat{Y}^i < \widehat{Z}^{i*}$.

(b) $\widehat{Z}^{j*} < 0 < \widehat{G}^{j*} < \widehat{G}^{i*}$.

Generally speaking, an increase in country i's income (given Y^j) generates a positive, direct effect on country i's own arming and saving primarily by reducing the marginal cost of both activities and thereby directly relaxing the country's *inter*-temporal trade-off between present and future consumption. These changes, however, induce the rival country (j) to make adjustments in its arming and saving, and these adjustments naturally feed back into country i's choices.

Sorting through the various direct and indirect effects, part (a) of Proposition 2 establishes that a given increase in country i's first-period income induces an increase in both saving and arming, but a larger (percentage) change in saving.[32] The reasoning here builds on the set of strategic interactions we discussed earlier in connection with the FOCs (6) and relate to the results in part (b). Specifically, the countries' saving choices are always strategic substitutes. Furthermore, their arming choices are strategic complements either when $\kappa = 1$ or when $\kappa < 1$ and their arming choices are sufficiently even.[33] Part (b) of the proposition shows for all $\kappa \in (0, 1]$, country j responds to increases in G^{i*} and Z^{i*} induced by an increase in Y^i by shifting resources from saving to arming.[34] The reduction in country j's saving induces i to increase its own saving, further clarifying the intuition for why Z^{i*} expands more quickly than G^{i*} in part (a).[35]

Characterizing analytically the effects of the probability of future conflict (q) and the degree of output insecurity (κ) on spending levels here proves to be challenging, because we cannot sign the effects of changes in these parameters on θ^i and thus cannot identify their effects on the two countries' arming and saving decisions for all $Y^i > 0$ and $Y^j > 0$. However, numerical analysis shows that an increase in either of these parameters induces each country to substitute out of saving into arming.[36] In turn, Proposition 1(b) indicates

[32] Proposition A.2(a) presented in Appendix A shows country i's current consumption also rises.

[33] If country j is larger than country i, then its arming choice always depends positively on G^i. In addition, when $\kappa < 1$, country j's arming also depends positively on country i's saving choice (see footnote 22 above.)

[34] Proposition A.2(b) indicates the effect of an increase in the opponent's income (Y^i) on country j's current consumption (C^j) is non-monotonic. In particular, as $Y^i \to Y^j$, an increase in Y^i implies C^j rises. However, for extreme differences in initial income where either $Y^i \to 0$ or $Y^i \to \infty$ given $Y^j \in (0, \infty)$ initially, C^j falls with increases in Y^i.

[35] The observation from Fig. 1 that θ^i/ϕ^i increases as we move NE along the S^i-curve for sufficiently low Y^i ($< Y^j$) can also be understood along these lines. Specifically, an increase in Y^i implies increases in both θ^i and ϕ^i as Z^i and G^i rise. Since such spending changes induce Z^j to fall and G^j to rise (with $\widehat{G}^j < \widehat{G}^i$), θ^i necessarily rises more quickly than ϕ^i. However, as we move NE along the curve beyond the midpoint (i.e., where $Y^i > Y^j$), the now-smaller country's increasing relative specialization in arming is decreasingly reflected in the ratio θ^i/ϕ^i, causing the S^i-curve to flatten for sufficiently large Y^i.

[36] Also see Proposition A.3 presented in Appendix A that characterizes these effects on arming, saving, and first-period consumption when incomes across countries become either very similar (i.e., as $Y^i \to Y^j$)

that the effect of an increase in q on the smaller country's arming is proportionately greater to augment that country's relative power, while the effect of increase in κ disproportionately increases the larger country's arming such that it gains in terms of relative power. Although we cannot pin down, in general, the effects on their relative contributive shares to world savings and thus the pool of contestable output, numerical analysis shows further that an increase in either q or κ has a disproportionately negative effect on the smaller country's savings, implying an increase in the larger country's relative contribution. Still, these tendencies are consistent with the intuitive idea that increased international tensions or increased insecurity can have adverse consequences for growth.

In any case, Proposition 2 clarifies several ambiguities left over from our representation of the problem in terms of shares. Having fully characterized how the two countries' choices (G^{i*} and Z^{i*}) in both levels and shares depend on income levels, we now turn our attention to the more intricate problem of identifying how exogenous income changes affect each country's equilibrium payoff:

Proposition 3 (Income and Equilibrium Payoffs.) *If the assumptions of Proposition 1 are satisfied, then for any given Y^j, a change in country i's income Y^i affects the two countries' equilibrium payoffs, U^{i*} and U^{j*} $(i \neq j = 1, 2)$, as follows:*

(a) $dU^{i*}/dY^i > 0$.

(b) *There exist threshold income levels $\underline{Y}^i$ and $\overline{Y}^i$ satisfying $\underline{Y}^i \leq \overline{Y}^i (< Y^j)$ such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$ whereas $dU^{j*}/dY^i > 0$ for all $Y^i \geq \overline{Y}^i$.*

As suggested by Proposition 2, an increase in country i's first-period income generates both positive and negative welfare effects for both countries. For country i, the increase in Y^i has the direct, positive effect of increasing country i's first-period consumption.[37] At the same time, the other country's ($j \neq i$) responses in terms of increased arming and decreased saving generate indirect, negative effects on country i's payoff. Part (a) establishes the direct, positive effect dominates, such that an increase in country i's first-period income always increases on net its own payoff.

The more interesting set of effects is for the rival country $j \neq i$. On the one hand, the increase in arming by country i implies a negative security externality for country j. On the other hand, the increase in saving by country i implies a larger pool of future output to be contested, thereby creating a positive externality. Which effect dominates depends on country i's first-period income relative to that of country j. In particular, by Propositions 1 and 2, when the countries are more similar in size initially, the smaller country's (i) saving rises faster than its arming for a given increase in its income, such that its contributive share of future output increases faster than its share of the balance of power. Thus, as shown in Proposition 3(b), growth in country i can affect its potential rival's payoff adversely *only*

or extremely different (i.e., $Y^i \to 0$, while $Y^j \in (0, \infty)$).

[37] Again, see Proposition A.2 in Appendix A.

if country i is sufficiently smaller than country j to start with. Importantly, this finding arises regardless of the possible absence or presence of trade and is unrelated to price (or terms-of-trade) effects. Nevertheless, the result is crucial in our analysis below finding that trade could be unappealing to *ex ante* "larger" countries.

4 Equilibria under Autarky and Trade

Given our characterization of how equilibrium outcomes depend on initial income levels (Y^i, Y^j), the implications of "trade" follow naturally. Our formal trade setting involves a Ricardian model with potential specialization in tradable intermediate goods.[38] In what follows, we first show how this trade setting implies larger relative income gains for countries initially having smaller resource endowments (or *ex ante* "smaller" countries). We then consider whether, as a result, situations can arise where *ex ante* larger countries will choose not to trade with their smaller rivals.[39]

4.1 Resource Endowments and the "Gains from Trade"

Recall that, at the beginning of period $t = 1$, each country i is endowed with R^i $(i = 1, 2)$ units of a productive resource (e.g., capital). This resource is employed to produce two intermediate goods that serve as inputs in the production of the final consumption good Y^i. Denote the quantity of country i's production of intermediate good j by X^i_j, $j = 1, 2$. The technology for X^i_j requires α^i_j units of country i's resource to produce one unit of intermediate good j. Therefore, the resource constraint for intermediate inputs in country i $(= 1, 2)$ is

$$\alpha^i_1 X^i_1 + \alpha^i_2 X^i_2 \leq R^i, \text{ where } X^i_j \geq 0, \;\; j = 1, 2. \tag{13}$$

As in a standard Ricardian setting, differences in technologies across countries give rise to comparative advantage. To fix ideas, we assume that country i has a comparative advantage in the production of good i—that is, $\alpha^i_j/\alpha^i_i \geq \alpha^j_j/\alpha^j_i$, where α^i_j/α^i_i is country i's opportunity cost of producing good j measured in units of good i $(\neq j = 1, 2)$. Depending on the trade regime that prevails in period $t = 1$—either autarky or trade—each country i might produce both intermediate goods domestically (as would be the case under autarky) or it might specialize on the basis of its comparative advantage, producing only good i and exchanging some of that for good j (as could be the case under trade).

Let D^i_j denote the quantity of intermediate good j $(= 1, 2)$ that becomes available

[38] Since our central result holds for a variety of trade models as mentioned above (and discussed in Appendix C), we chose the relatively simplest one. The Ricardian model also has the advantage of allowing for the possibility that the elasticity of substitution between intermediate inputs (σ) is less than 1. As will become evident below, when $\sigma < 1$, trade can actually induce a reversal in power as compared with autarky, but this reversal alone is not what drives our main result.

[39] As discussed in Section 4.3, our results would remain valid if we were to consider a free trade agreement versus noncooperatively determined trade polices instead of trade versus autarky.

domestically (under autarky or through trade) to producers of the final good in country i $(= 1, 2)$. The technology for production of the final good is described by the following symmetric CES production function:

$$Y^i = \left[\sum_{k=i,j} \left(D_k^i \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{14}$$

where $\sigma > \underline{\sigma} \in [0, 1)$ represents the (constant) elasticity of substitution between intermediate inputs.[40]

Whether the two countries trade intermediate goods or not, the maximal quantity of output that country i can produce is $Y^i = r^i R^i / P^i$, where r^i is the price of the domestic resource, $P^i \equiv \left[\sum_{k=i,j} (p_k^i)^{1-\sigma} \right]^{1/(1-\sigma)}$ is the price index, and p_k^i is the price of intermediate good k. Since country i has a comparative advantage in producing intermediate good i, it always produces this good. Perfect competition in all markets implies $r^i = p_i^i / \alpha_i^i$ for $i = 1, 2$. Now let $p^i \equiv p_j^i / p_i^i$ be the domestic relative price of intermediate good j $(\neq i)$ in which country i has a comparative disadvantage. Then, one can verify $Y^i = \left[1 + (p^i)^{1-\sigma} \right]^{-\frac{1}{1-\sigma}} R^i / \alpha_i^i$ under either trade or autarky.[41]

Final output Y^i differs under autarky and trade through the determination of the relative price p^i. Let p_A^i and p_T^i denote the market-clearing (relative) prices under autarky and under trade, respectively. Under autarky, country i produces both intermediate goods domestically, implying p_A^i equals the country's domestic opportunity cost: $p_A^i = \alpha_j^i / \alpha_i^i$ for $j \neq i = 1, 2$. Thus, as one can easily verify, final output under autarky is given by $Y_A^i = A^i R^i$, where $A^i \equiv [1 + (\alpha_j^i / \alpha_i^i)^{1-\sigma}]^{-\frac{1}{1-\sigma}} / \alpha_i^i$ $(i \neq j = 1, 2)$ is a constant that depends only on country i's technology for producing intermediate goods. Assumption A in the previous section (that $A^i = A^j$) amounts to assuming that countries have symmetric (i.e., mirror-image) comparative and absolute advantage—that is, $\alpha_j^i = \alpha_i^j$ and $\alpha_i^i = \alpha_j^j$, which imply $p_A^i = p_A^j$ for $i \neq j = 1, 2$.[42] To simplify notation further, without loss of generality, let $\alpha_i^i = \alpha_j^j = 1$ and $\alpha_j^i = \alpha_i^j = \alpha > 1$, implying that $p_A^i = p_A^j = \alpha$ and

$$A^i = A = \left[1 + \alpha^{1-\sigma} \right]^{-\frac{1}{1-\sigma}}, \; i = 1, 2. \tag{15}$$

Note, absent trade in period $t = 2$, country i's income in that period, regardless of whether or not war breaks out, is given by $\widetilde{Y}^i = \widetilde{Y}_A^i = AZ^i$ for $i = 1, 2$.

Under trade in period $t = 1$, the market-clearing relative price p_T^i is the price that

[40] The lower bound on σ generally ensures stability and uniqueness of competitive equilibria under trade. However, absent trade costs, this limit is simply given by $\underline{\sigma} = 0$.

[41] In particular, the maximization of Y^i subject to (13) and (14) and given prices p_j^i $(j = 1, 2)$ implies the following input demand functions: $D_j^i = [p_j^i / P^i]^{1-\sigma} r^i R^i / p_j^i$ for $j = 1, 2$. Recalling that $r^i = p_i^i / \alpha_i^i$, substitution of these demand functions with $p^i = p_j^i / p_i^i$ into (14) gives the expression shown in the text.

[42] As an alternative approach for analyzing the effects of asymmetries in this context, one could suppose that initial endowments are identical, but allow for differences in absolute advantage.

balances world trade. To keep matters simple, we abstract here from trade costs and thus focus on free trade, such that world and domestic prices coincide (and also ensuring $\underline{\sigma} = 0$).[43] Now, define $\pi^i \equiv (R^i/R^j)^{1/\sigma}$ for $i \neq j = 1, 2$. Then, the equilibrium relative world price of good i satisfies the following:

$$p_T^i = \begin{cases} 1/p_A^j & \text{if } \pi^i \leq 1/\alpha \\ \pi^i & \text{if } 1/\alpha < \pi^i < \alpha \\ p_A^i & \text{if } \pi^i \geq \alpha, \end{cases} \tag{16}$$

for $i \neq j = 1, 2$. When $p_T^i = \pi^i \in (1/p_A^j, p_A^i)$ (as shown in the second line), each country specializes completely in the production of the good in which it has a comparative advantage.[44] However, depending on the distribution of initial resource endowments and technologies (specifically, σ and α), the value of π^i could be larger than $p_A^i = \alpha$ or smaller than $1/p_A^j = 1/\alpha$. Since country i would not be willing to pay more than its own relative autarkic price (α) for good j and, similarly, country j would not be willing to accept a price less than its own autarkic relative price of good j ($1/\alpha$), one country might diversify in production of the two intermediate goods. In this case, p_T^i is determined by the larger country's autarkic relative price (as shown in lines 1 and 3 above). In any case, under trade with p_T^i given by (16), country equilibrium i's output in period $t = 1$ equals $Y_T^i = T^i(R^i, R^j)R^i$, where

$$T^i(R^i, R^j) \equiv \left[1 + (p_T^i)^{1-\sigma}\right]^{-\frac{1}{1-\sigma}}, \quad i \neq j = 1, 2. \tag{17}$$

Since $p_T^i \leq p_A^i = \alpha$, $T^i \geq A$, as originally posited in Section 2.

Building on the analysis above, we now consider how the introduction of trade influences the equilibrium determination of first-period income in levels and relative terms (Y^i, Y^j and Y^i/Y^j) given the distribution of resource endowments R^i and R^j. As a starting point, consider the following result, which is standard and well-understood in the trade literature (assuming perfect security or no conflict) and, as such, serves as a useful benchmark for our analysis.

Benchmark Result: (Income Levels.) *Trade in period $t = 1$ reduces neither country's income that period. Indeed, the country with the relatively smaller resource endowment always enjoys an increase in income relative to that under autarky. The larger country, too, realizes a higher income under trade, provided the international distribution of resource endowments is sufficiently even to induce complete specialization by that country as well. Otherwise, the relatively larger country's income under trade will be identical to that under*

[43] Introducing trade costs (at non-prohibitive levels) would not substantively change our welfare results to follow. For some details, see Appendix C.

[44] Let π_j denote the world price of good $j = 1, 2$. Since world and domestic prices coincide under free trade, $\pi_j = p_j^i = p_j^j$ holds for $i \neq j = 1, 2$. Then, under complete specialization, the relative world price $\pi^i \equiv \pi_j/\pi_i$ shown in the text satisfies $\pi_j D_j^i = \pi_i D_i^j$ (where D_j^i is given in footnote 41), such that the value of country i's imports equals the value of this country's exports to country j.

autarky.

The key idea here is that a country's income gains from trade depend on whether trade causes world prices to change (and if so, by how much). If world prices change, then trade improves welfare and more so with larger changes; otherwise, trade does not affect welfare, *ceteris parabus*. Of course, these claims abstract from security considerations (essentially assuming in our model that either $q = 0$ or $\kappa = 0$.) As we will argue, if conflict arises with a strictly positive probability ($q > 0$) in the future in which case some positive fraction of second period output ($\kappa > 0$) is contested, then one country might find trade in the current period unappealing.

But, first, the next proposition offers a more detailed view of how the distribution of endowments translates into the distribution of first-period incomes under trade:

Proposition 4 (Relative Incomes.) *If relative resource endowments and technologies are such that trade in period $t = 1$ induces each country to specialize in the production of the intermediate good in which it has a comparative advantage, then their income levels compare as follows:*

(a) *for $\sigma > 1$, $Y_T^i/Y_T^j \gtreqless 1$ if $R^i/R^j \gtreqless 1$;*

(b) *for $\sigma = 1$, $Y_T^i/Y_T^j = 1$;*

(c) *for $\sigma \in (0,1)$, $Y_T^i/Y_T^j \gtreqless 1$ if $R^i/R^j \lesseqgtr 1$.*

Additionally, under the maintained assumption that both countries specialize, Y_T^i/Y_A^i is decreasing in R^i/R^j. However, regardless of whether one or both countries specialize, $(Y_T^i/Y_A^i)/(Y_T^j/Y_A^j) \gtreqless 1$ if $R^i/R^j \lesseqgtr 1$ for any $\sigma > 0$.

Figs. 2(a)–(c) respectively illustrate parts (a)–(c) of the proposition along with the Benchmark Result. In the intermediate regions of each figure, both countries completely specialize in production, implying that both of their income levels under trade, given by the dashed lines, exceed their incomes under autarky, given by the solid lines. In other words, both countries gain from trade' in income terms. Furthermore, if both countries specialize, country i's relative income gains Y_T^i/Y_A^i are inversely related to its relative resource endowment R^i/R^j. However, if either country is sufficiently large relative to the potential rival, as occurs towards either side of these figures, the world price converges to that country's autarky price (by (16)). As a result, that country diversifies in production and its income gains from trade are zero; the smaller country, meanwhile, still enjoys a large relative income benefit.

Interestingly, when both countries specialize, the country with the greater resource endowment (which also has the higher autarky income) need not be the one that enjoys the higher level of income under trade. Specifically, such a ranking follows only if traded inputs are gross substitutes in production (as in Fig. 2(a)). If instead they are gross complements, the country with the smaller resource endowment enjoys greater income under trade (as in

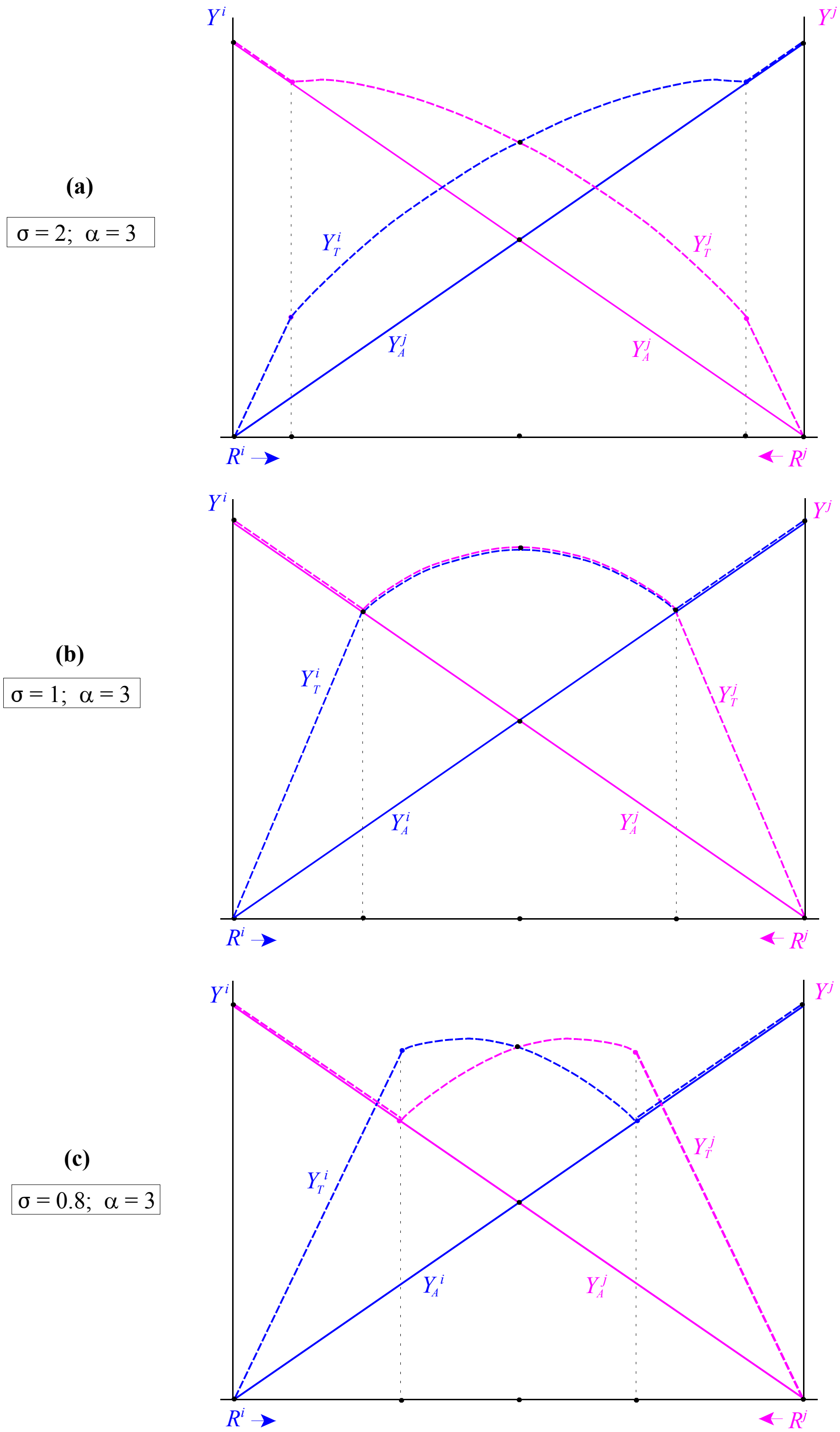

Figure 2: First-period Income Levels as functions of Trade Regimes and the Distribution of Resource Endowments

Fig. 2(c)).[45] In the intermediate case of a Cobb-Douglas production technology ($\sigma = 1$, as in Fig. 2(b)), income levels under trade are identical across countries when both specialize. In any case, the last part of the proposition, which holds whether one or both countries specialize in production and for all $\sigma > 0$, shows that the *ex ante* smaller country (i.e., with $R^i < R^j$) always enjoys a relatively larger income gain from trade.[46]

4.2 Trade, Power, and Welfare

We move now to the main objective of our analysis: to see how and when the security considerations brought on by the possibility of future conflict can limit—or even overwhelm completely—the standard gains from trade for either country. To begin, we characterize the security externalities associated with trade, synthesizing our key results thus far for the effects of trade on the balance of power. Let ϕ_A^i $(i = 1, 2)$ denote country i's equilibrium power under autarky. Proposition 1(a) implies that if country i is the *ex ante* larger country (i.e., with $R^i > R^j$), then under autarky it always appropriates a larger share of the contested output in the event of a future war: $\phi_A^i > \phi_A^j$. Letting ϕ_T^i denote country i's $(i = 1, 2)$ power under trade, we have:

Proposition 5 (Trade and Power.) *If the relative endowments and technologies are such that trade in period $t = 1$ induces complete specialization in the production of intermediate goods in both countries, then the balance of power under trade depends on σ as follows:*

(a) *for $\sigma > 1$, $\phi_T^i/\phi_T^j \gtreqless 1$, if $R^i/R^j \gtreqless 1$;*

(b) *for $\sigma = 1$, $\phi_T^i/\phi_T^j = 1$;*

(c) *for $\sigma \in (0, 1)$, $\phi_T^i/\phi_T^j \gtreqless 1$, if $R^i/R^j \lesseqgtr 1$,*

for $i \neq j = 1, 2$. Furthermore, whether only one or both countries completely specialize in the production of intermediate goods, $\phi_T^i/\phi_A^i \gtreqless 1$ also holds for any $\sigma > 0$, if $R^i/R^j \lesseqgtr 1$.

The implications of Proposition 5 are shown in Fig. 3(a) under alternative values of σ: $\sigma > 1$, $\sigma = 1$ and $\sigma < 1$.[47] Because trade raises the first-period income of one and possibly both countries relative to their respective autarky incomes, we know, by Proposition 2, that trade necessarily induces both countries to produce more guns. However, Proposition 4 also establishes that the *ex ante* smaller country always realizes a relatively larger income gain from trade. Thus, by Proposition 1, the introduction of trade reduces the *ex ante* larger country's military advantage as compared with autarky, thereby leading to a more equitable

[45] This reversal in the ranking of income levels, which is consistent with the Benchmark Result, arises because relative demands are skewed towards the relatively more expensive commodity, enhancing the terms of trade of the (*ex ante* smaller) country that specializes in the scarcer input.

[46] As established in the proof, when $p_T^i < \alpha$ holds for both countries $i = 1, 2$, the divergence in gains is decreasing in σ.

[47] In that figure and in the ones to follow, $q = 0.9$, $\kappa = 1$ and $\delta = .95$.

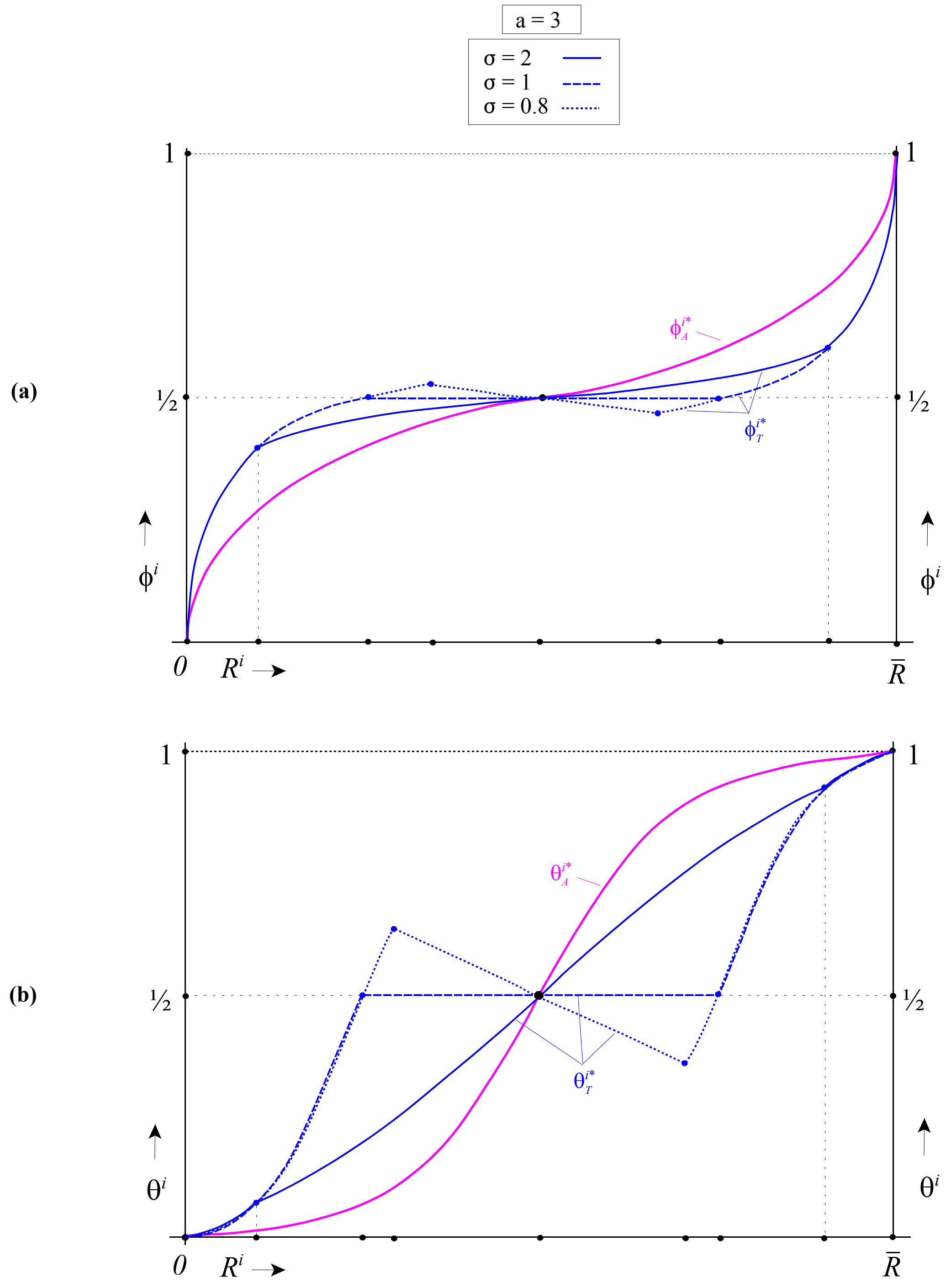

Figure 3: Appropriative and Contributive Shares under Autarky and Trade for Various Distributions of Resource Endowments

division of the contested output in the event of war.[48] Indeed, depending on the value of σ which determines the effect of trade on relative incomes by Proposition 4, trade could eliminate any differences in power ($\sigma = 1$) or even reverse the balance of power ($\sigma < 1$).

Trade also impacts savings decisions, of course. Notably, as illustrated in Fig. 3(b), for each value of σ considered, trade induces the *ex ante* smaller (larger) country to become a relatively larger (smaller) contributor to future income as compared with what happens under autarky. Thus, trade generates both security and savings externalities. Indeed, in the case that $\sigma \leq 1$ and both countries specialize completely in the production of intermediate goods, the smaller country contributes half or more to future output.

Building on our insights above, we state our results for welfare as follows:

Proposition 6 (Payoffs.) *If the international distribution of resource endowments is sufficiently even, introducing trade in period* 1 *improves both countries' equilibrium discounted payoffs. However, if this distribution is sufficiently uneven, then the ex ante larger country will find trade unappealing as compared with autarky.*

The main ideas behind Proposition 6 follow readily from Propositions 3 and 4. In particular, these propositions taken together imply that the country with the relatively smaller initial resource base unambiguously realizes income gains from trade in period $t = 1$. At the same time, by Proposition 3(b), the savings and security externalities associated with the change in relative incomes have a net positive and thus reinforcing effect on the smaller country's expected two-period payoff.

The relatively larger country can also realize income gains from trade, but such gains are necessarily smaller in comparison; and, in extreme cases, they are nonexistent. If the adverse security externality from the *ex ante* smaller country's increased relative power induced by its increased income under trade exceeds both the positive externality from the smaller country's increased contributive share and the *ex ante* larger country's income gains from trade, the larger country will find trade unappealing relative to autarky. Proposition 3(b) shows that the adverse security externality is more likely to swamp the positive savings externality for the *ex ante* larger country when the initial difference in country size is more pronounced; and, by Proposition 4, the larger country's own income gains from trade also fall with increased differences in initial size. Thus, the distribution of relative income gains from trade negatively affects the larger country's payoff whenever the initial distribution of resource wealth is sufficiently uneven.

Figs. 4(a)–(c) illustrate these results. Again, we draw distinctions depending on whether $\sigma \lesseqgtr 1$, which affects the ranking of income gains under trade. In all cases, each country's

[48] The result can also be seen in terms of Fig. 1. Suppose that country i has a larger endowment of the resource, implying an equilibrium under autarky above the midpoint of the S^i-curve. Then, the effect of trade to reduce Y_T^i/Y_T^j (relative to Y_A^i/Y_A^j) is to shift the B^i-curve down, implying a reduction in ϕ^i as well as in θ^i.

payoff under trade (given by the dashed lines) falls below its payoff under autarky when it becomes sufficiently larger than its trading partner. This reversal tends to happen near or in the region where the larger country no longer completely specializes in production, because in our Ricardian setting its income gains from trade vanish in this region. However, the reversal in payoffs also holds true in other prominent trade models that do not feature complete specialization.[49]

The implications for the extended policy game should be clear: whenever one country is sufficiently larger than its rival, it rationally chooses to foreclose on trade altogether, because it relinquishes power without gaining much back in return from trade. However, based on our numerical analysis given relative incomes described in Section 3, one would expect the negative strategic payoff effects for the larger country to be smaller and the positive indirect (as well as the direct) payoff effects it enjoys to be larger with decreases in the probability of conflict q and/or the degree of insecurity κ. Indeed, additional numerical analysis of payoffs under autarky and trade (assuming $\sigma > 1$) shows the range of relative endowment sizes for which the larger country prefers trade over autarky tends to expand as either q or κ decreases.[50] Numerical analysis suggests further that decreases in σ (implying greater gains from trade), m (implying less effective arming) and/or δ (implying greater discounting of the future) similarly expand the range of relative endowment sizes for which the larger country prefers trade. Our results for q in particular motivate the empirical analysis that soon follows, in which we consider the relationship between strategic rivalries and trade during the period surrounding the end of the Cold War.

4.3 Possible Extensions and Generalizations

Before turning to our empirical analysis, let us consider briefly a few extensions that relax some of the simplifying assumptions we have imposed to make the analysis as transparent as possible. For example, we could change the nature of conflict to a "winner-take-all" contest where, in the event of war, ϕ^i represents country i's winning probability instead of its share of contested output. Due to risk aversion, each country's expected payoff (for given arming choices) would be lower in this case than when ϕ^i is a share. Nonetheless, the negative strategic payoff effect of trade in the current period for the larger country would still overwhelm the positive effects of its own increased income and increased saving by the smaller country, particularly if the initial international distribution of resources is sufficiently uneven. By the same token, allowing for the destructive effects of war (another possible extension) would lower expected payoffs for both countries, but, again, the result that the larger country may find trade unappealing would remain intact.

[49] See Appendix C for some details.

[50] These results always hold in the Armington (1969) ($\alpha = \infty$) and Krugman (1980) models and, provided that σ and α are sufficiently large, in the Ricardian model. But, when α and σ are relatively small, the range of relative endowments can expand with an increase in q or the relationship can be non-monotonic.

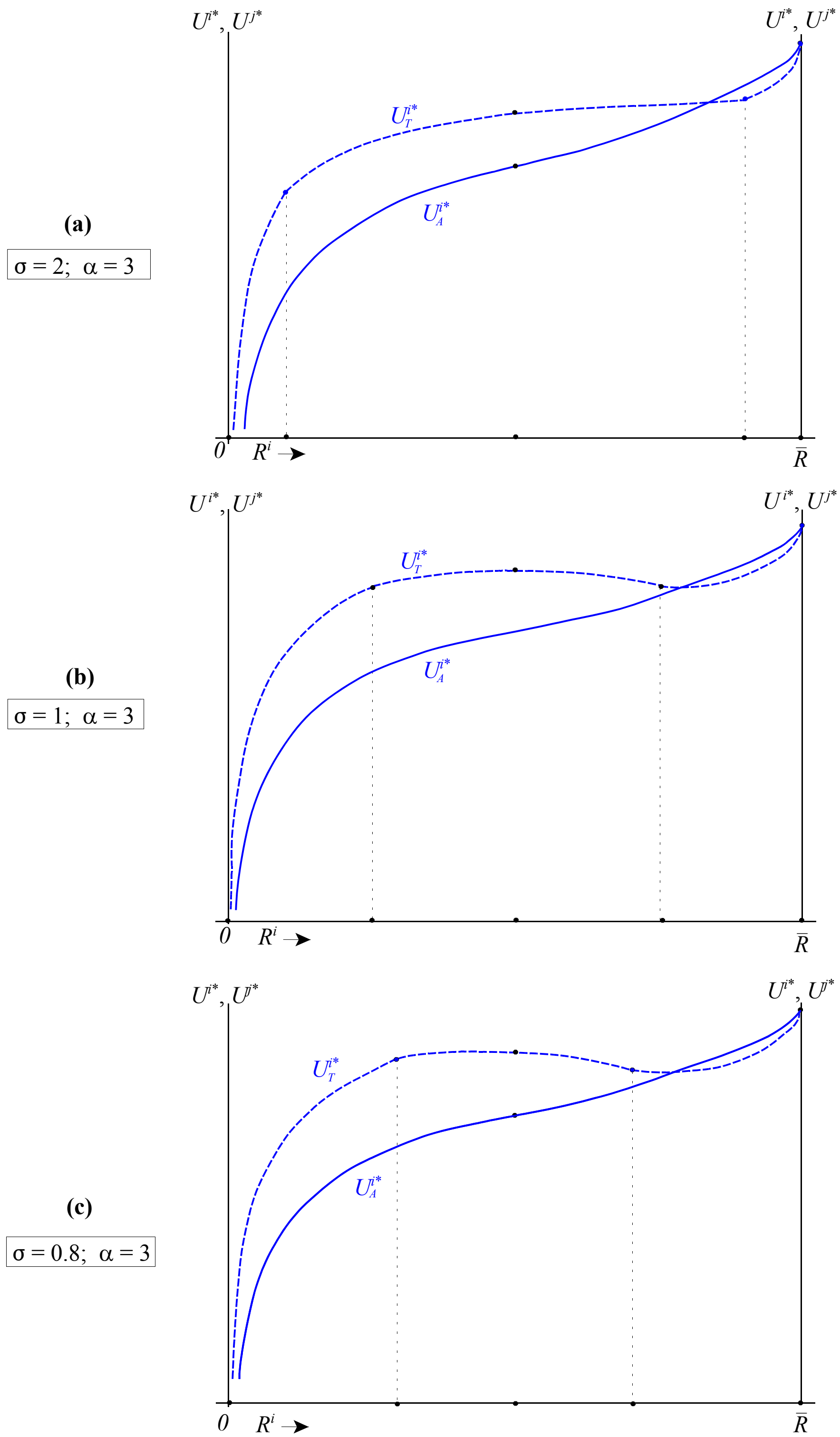

Figure 4: Discounted Payoffs under Autarky and Trade for Various Distributions of Resource Endowments

We can also consider several other revealing extensions related to the modeling of trade. As we mention above (and as we demonstrate in Appendix C), our central result that the larger country could find trade in the first period unappealing turns out to be generally robust across a variety of trade models, including those with trade costs, incomplete specialization, intra-industry trade, heterogeneous firms, and/or increasing returns. What's more, this result would remain valid across any of these models if we were to compare payoffs under non-cooperative trade policies and payoffs under a free-trade agreement instead of autarky versus free trade; all that would be required is that the *ex ante* smaller country realize part of the surplus created by the trade agreement.

More fundamentally, we could alter the analysis to allow the two countries to trade in the second period if peace prevails then, as opposed to only allowing trade in the first period. Importantly, while this is a natural generalization of our setting (and while it also lends itself to further extensions of interest), it does not materially alter the rationale underlying the larger country's preference not to trade in the first period, which arises strictly from how trade affects first-period incomes. It does, however, introduce an additional channel of influence—namely, a terms-of-trade channel—with further implications regarding the countries' preferences over *future* trade. Specifically, because increases in a country's future output via savings would worsen its terms of trade in the future, this type of trade would lower the marginal benefit of saving for both countries and, as a consequence, induce them to arm more. Thus, the possibility of trade in the second period could lower the expected two-period payoffs of one or both countries, but for reasons that are altogether different than why the larger country could prefer not to trade in the first period.

Allowing for trade in the second period also allows us to address one other possible critique of our analysis. As the "liberal peace" argument mentioned in our Introduction would suggest, if countries can trade in the future, the probability of conflict should not be treated as fixed, but rather should depend on how countries value the potential future trade that would be lost in the event of war. Incorporating this possibility requires a more fully articulated model that captures the countries' choices between peaceful settlement and war in the second period. To fix ideas, suppose that a dispute arises between the two countries in the second period with some exogenous probability, but the countries have an opportunity in this case to settle their differences peacefully. If a settlement is reached, the two sides are able to trade in the second period; if not, war breaks out, destroying the possibility of second-period trade. Clearly, such an extension is quite complicated and would take us well beyond the scope of the present analysis.[51] Nonetheless, we can examine the

[51] To be sure while it seems highly unlikely, given the countries' first-period arming choices and assuming complete information, that a dispute would escalate into war, the payoffs under a negotiated settlement would still depend on the countries threat-point payoffs (depending on their arming choices). As such, arming decisions would matter. We cannot be sure of the implications for first-period arming incentives, but we conjecture that greater gains from trade in the second period would increase the payoffs of current trade to both countries.

potential feedback from the possibility of future trade to countries' preferences over current trade by assuming that the probability of war (as perceived by the two countries) depends negatively on the total surplus. Specifically, let this surplus, denoted by Π, be equal to the additional world output in the second period made possible with trade. Based on the Armington (1969) and Krugman (1980) models of trade, Π is an inverse U-shaped function of the savings decisions and thus is a concave function of R^i/R^j, reaching a maximum at $R^1 = R^2$ and approaching 0 as either R^1 or R^2 approaches 0.[52] Now, let the probability of future conflict be determined as $q = 1/(1 + b + n\Pi)$, where $b > 0$ and $n > 0$.[53] To simplify further, suppose that the two countries ignore the influence of their first-period choices on the size of the surplus and thus on q.[54] Numerical analysis confirms our intuition that there exists a set of circumstances under which a sufficiently uneven distribution of endowments in the first period renders trade in the first period unappealing to the larger country. The range of relative endowments for which this preference holds vanishes when the elasticity of substitution σ is sufficiently small to imply larger gains from trade (and thus a larger surplus) and/or when q's sensitivity to the surplus n is sufficiently large. Although more work needs to be done here, this extended analysis does suggest that, through the endogeneity of the probability of future conflict, the possibility of future trade can amplify the relative appeal of trade today.

5 Empirical Evidence: Rivalries and Trade

Motivated by the insights from Section 4.2, in this section, we study empirically the relationship between conflict expectations and trade. To operationalize our simple two-country setting in terms of real-world data, we make the following adjustments. First, to take full advantage of the rich variation present in the trade data, we loosely interpret the determination of the *decision to trade* from our theory as being similarly salient for the *level of trade* in our empirics. Second, like Long (2008), we exploit Thompson (2001)'s historical database on "strategic rivalries" between pairs of countries as a rough measure of the probability of conflict (i.e., "q" in our analysis).[55]

[52] In the Armington (1969) model where $\alpha = \infty$, world output under free trade would be determined by a CES production function depending on Z^i and Z^j as the relevant inputs, whereas total output under conflict would equal $Z^i + Z^j$. Total surplus would, then, be given by $\Pi = [(Z^i)^c + (Z^j)^c]^{1/c} - [Z^i + Z^j]$, where $c = (\sigma-1)/\sigma$. In the Krugman (1980) model, the surplus would equal $\Pi = h[Z^i+Z^j]^{1/c} - h[(Z^i)^{1/c}+(Z^j)^{1/c}]$, where $h > 0$ is a common constant that depends on σ and variable and fixed costs.

[53] When $n = 0$, we are back to the setting of our main analysis, with $q = 1/(1+b)$ being entirely exogenous.

[54] Although Π does not directly depend on first-period arming choices, it does depend on first-period savings and thus indirectly on arming choices. These influences, which also affect q, make the first-period expected payoff-maximizing allocation problem intractable. Even if we simplify by assuming that countries do not account for the effect of their saving decisions on q, this extension results in a complex system of nonlinear FOCs that cannot be solved analytically. Still, they can be studied numerically.

[55] Note that, due to the absence of a similar proxy for the degree of insecurity (κ), we implicitly assume κ is uncorrelated with the presence of a rivalry. To the extent that insecurity and the risk of conflict are positively correlated (as one might expect), our theoretical results regarding κ suggest that the former could be contributing to our results in a way that would still be consistent with our theory.

In contrast to measures of conflict based on the presence of an actual war or recent history of war (as in, e.g., Martin et al., 2008), this variable records the ongoing threat of war based on observed diplomatic tensions; thus, war need not take place for a strategic rivalry to occur. Furthermore, to capture plausibly concrete variations in such tensions, we focus on a particular historical episode—the end of the Cold War—which featured both a major realignment in strategic rivalries as well as a general reduction in the likelihood of conflict. Third, we follow the latest developments in the empirical trade literature in implementing a gravity model with a rich set of exporter- and importer-specific fixed effects. As is widely understood, using such fixed effects that absorb all country-specific and multilateral influences on trade allows the estimation to focus strictly on standard bilateral determinants of trade (such as geographic distance, the signing of a regional trade agreement, and so on) as well as, in our case, on the presence of bilateral rivalries.[56]

Adhering to the "gravity with fixed effects" approach, and given that we observe trade over a number of years, we can express an estimating equation for trade flows from country i to country j in period t (X_{ijt}), as

$$\begin{aligned} X_{ijt} = \exp\Big[& F_{it} + F_{jt} + b_1 \ln \mathit{DIST}_{ij} + b_2 \mathit{CONTIG}_{ij} + b_3 \mathit{COLONY}_{ij} + b_4 \mathit{COMLEG}_{ij} \\ & + b_5 \mathit{COMLANG}_{ij} + b_6 \mathit{FTA}_{ijt} + b_7 \mathit{CONFLICT}_{ijt} \\ & + b_8 \mathit{RECENT\_CONFLICT}_{ijt} + b_9 \mathit{RIVAL}_{ijt} \\ & + b_{10} \mathit{RIVAL}_{ijt} \times |\Delta \mathit{SIZE}|_{ijt} + b_{11} |\Delta \mathit{SIZE}|_{ijt} \Big] + \varepsilon_{ijt}, \end{aligned} \tag{18}$$

where F_{it} and F_{jt} are the time-varying exporter and importer fixed effects discussed above. Our bilateral variables include the standard set of controls for distance (*DIST*), common borders (*CONTIG*), past colonial relationships (*COLONY*), common legal systems (*COMLEG*), common languages (*COMLANG*), and free trade agreements (*FTA*). To these, we add indicators for an ongoing armed conflict (*CONFLICT*) as well as for a "recent" conflict (i.e., within ten years, *RECENT_CONFLICT*), following Martin et al. (2008)'s finding that the trade-disrupting effects of conflict persist for up to ten years afterwards.

Our main variable of interest, however, is *RIVAL*, an indicator for the presence of an ongoing strategic rivalry. One might naturally expect the influence of *RIVAL* on trade to be negative—i.e., that $b_9 < 0$. However, the predictions of our theory suggest that this relationship should be conditioned on the *difference in size* between the countries. To be precise, if we let $|\Delta \mathit{SIZE}|$ denote the absolute difference in log GDP, then the coefficient on the interaction between *RIVAL* and absolute size differences, b_{10}, should be *negative*. In terms of our model, and loosely interpreting *RIVAL* as a proxy for q, the formation of a rivalry should only affect decisions to trade if countries are sufficiently different in size. The

[56] Anderson and van Wincoop (2003) famously show that all third-country effects on bilateral trade can be usefully aggregated into country-specific "multilateral resistance" indices, which here will be absorbed by exporter- and importer-specific fixed effects.

converse should also hold for the dissolution of a rivalry.

Two remaining details with respect to the estimating equation then remain to be specified. First, note that (18) is written in nonlinear form. This is because, following the recommendations of Santos Silva and Tenreyro (2006), we will use Poisson Pseudo-maximum Likelihood (PPML) to estimate (18). As Santos Silva and Tenreyro (2006) discuss, log-transforming the dependent variable in gravity estimation leads to biased, inconsistent estimates when the error term is heteroscedastic. Using PPML both avoids this problem and generally performs well under different patterns of heteroscedasticity. Second, one shortcoming of the proposed approach is that it does not fully exploit the time-variation in our main covariates of interest. Thus, we will also consider alternative specifications with pair-specific fixed effects and/or time trends, as discussed further below.[57]

The starting point for our data is the data set assembled by Long (2008), which conveniently combines data on international trade, GDPs, and international conflicts (all from the Correlates of War database) with data on strategic rivalries from Thompson (2001). It also centers on the period around surrounding the end of the Cold War, which we find desirable. To this existing data set, we add a standard set of controls from the gravity literature; these data are taken from standard sources.[58] We also add data on prior conflicts pre-dating the sample from Correlates of War. Altogether, the data set covers 128 countries over the 14 year period 1984–1997. The number of strategic rivalries is 100 at the beginning of the period, peaks at 104 over 1985-1986, and falls to 62 by the end of the period, as shown in Fig. 5.

One particularly interesting episode in our data that exemplifies some of the main ideas evoked by our theory is Israel's rivalry with Jordan, which ran from the Arab-Israeli War in 1948 until the the signing of the Washington Declaration and a subsequent peace treaty in 1994. Notably, despite the fact that Jordan and Israel share a common border, we do not observe any trade between them in our data before the two countries signed a trade agreement in 1995, the year after their peace agreement. The timing and motivation behind these events is very relevant for our analysis: Jordan recognized that a peace agreement with Israel (the economically larger country in this example) was essential for obtaining what Gal and Rock (2018) call the "fruit of peace"—a series of subsequent trade and investment agreements with both Israel and the U.S. that led to a decade-long economic boom in

[57] Our use of a rich set of fixed effects and a nonlinear estimator mark important methodological differences from Long (2008), who also includes many monadic variables associated with conflict expectations, such as country-specific political risk. We adopt PPML with fixed effects to follow as closely as possible the best practices recommended in the trade literature and to focus specifically on the bilateral dimension of the data, as we have noted. Another well-known advantage of PPML is that it allows for zero trade flows. Our results are similar with or without zeroes, however.

[58] The set of gravity controls are from CEPII (Mayer and Zignango, 2011), except for free trade agreements, which we take from Baier and Bergstrand (2015)'s highly detailed database on bilateral trade agreements. Specifically, we construct our "FTA" variable from their data on free trade agreements and deeper economic unions.

Figure 5: Changes in the number of strategic rivalries over time

Jordan. For our purposes, the key observation is that Israel and the U.S. did not view trade with Jordan by itself as sufficient for ensuring future peace; rather, the implications for future security had to be settled first.[59]

Turning our attention to the data in full, column 1 of Table 1 shows results from a baseline specification based on (18) but excluding the interaction between $RIVAL$ and $|\Delta SIZE|$. All years are included. Interestingly, the presence of a strategic rivalry initially does not enter significantly. Our other controls mostly enter as expected. Both distance and actual conflicts—both current and recent—relate negatively to trade, while contiguity, common languages, common legal systems, and regional trade agreements each affect trade positively. The one exception is the sharing of a past colonial relationship, which enters negatively.[60]

Column 2 shifts to a test of our main hypothesis laid out in (18). As expected, the interaction between the presence of a rivalry and the absolute difference in size is indeed negative and highly significant. Furthermore, the main effect of $RIVAL$ remains insignificant on its own, consistent with the observation from our theory that trade between similarly-sized rivals need not be affected by the possibility of a future conflict. All other covariates enter similarly as before.

Columns 3 and 4 narrow the focus, showing year-specific results for the years 1986 and 1996, which we take as representative based on the peak and trough in the number

[59]Other, similar rivalries between nearby countries of differing sizes (and with surprisingly little trade taking place given their proximity) include Argentina and Chile, Egypt and Syria, and South Africa and Zambia. A complete list of the rivalries in our data can be found in Table 1 of Thompson (2001).

[60]We are agnostic on why the sign for colonial relationships should be negative. However, Head et al. (2010) have previously shown that the importance of past colonial relationships for trade has fallen dramatically over time.

Table 1: Gravity Results - Poisson PML

	Dependent variable: Bilateral Trade Flows					
	All years		*1986*	*1996*	*All years*	
	(1)	(2)	(3)	(4)	(5)	(6)
ln DIST	-0.730***	-0.735***	-0.774***	-0.657***		
	(0.031)	(0.031)	(0.040)	(0.034)		
CONTIG	0.423***	0.421***	0.307***	0.541***		
	(0.068)	(0.067)	(0.082)	(0.074)		
COLONY	-0.162*	-0.167*	-0.223*	-0.102		
	(0.098)	(0.096)	(0.115)	(0.098)		
COMLANG	0.160**	0.149*	0.260**	0.056		
	(0.080)	(0.077)	(0.106)	(0.070)		
COMLEG	0.250***	0.254***	0.330***	0.199***		
	(0.046)	(0.045)	(0.064)	(0.048)		
CONFLICT	-1.782***	-1.649***	-3.152***	-1.355**	-0.697**	-0.541
	(0.393)	(0.363)	(0.878)	(0.592)	(0.313)	(0.339)
RECENT_CONFLICT	-1.243***	-1.123***	-2.188***	-1.179***	-0.466**	-0.522
	(0.294)	(0.350)	(0.415)	(0.369)	(0.212)	(0.353)
FTA	0.533***	0.525***	0.467***	0.787***	0.386***	0.072***
	(0.066)	(0.068)	(0.113)	(0.072)	(0.034)	(0.024)
RIVAL	0.115	0.612	0.168	0.711*	-0.515**	0.243
	(0.414)	(0.454)	(0.581)	(0.401)	(0.231)	(0.148)
RIVAL $\times$ $\lvert\Delta$SIZE$\rvert$		-0.420***	-0.320*	-0.227**		
		(0.119)	(0.173)	(0.112)		
RIVAL $\times$ $\lvert\Delta$SIZE$\rvert_{1984}$					0.008	-0.209**
					(0.162)	(0.098)
$\lvert\Delta$SIZE$\rvert$		0.005	0.010	-0.003		
		(0.026)	(0.035)	(0.022)		
Exporter-time FEs	x	x	x	x	x	x
Importer-time FEs	x	x	x	x	x	x
Pair FEs					x	x
Pair trends						x
Observations	215614	215614	15500	15750	186895	186895
R^2	0.927	0.926	0.893	0.950	0.994	0.998

Gravity estimates are obtained using Poisson Pseudo-maximum Likelihood. The sample covers trade between 128 countries over the period 1984-1997. Standard errors, which appear in parentheses, are clustered by country-pair. Zeroes are included. Variation in N between columns 1-2 and 5-6 occurs because pairs who never trade are dropped when pair fixed effects are used. Columns 5 and 6 do not include pairs involving the former Soviet Union. Full annual results for the $RIVAL \times |\Delta SIZE|$ term are shown in an accompanying figure.
* $p < 0.10$, ** $p < .05$, *** $p < .01$.

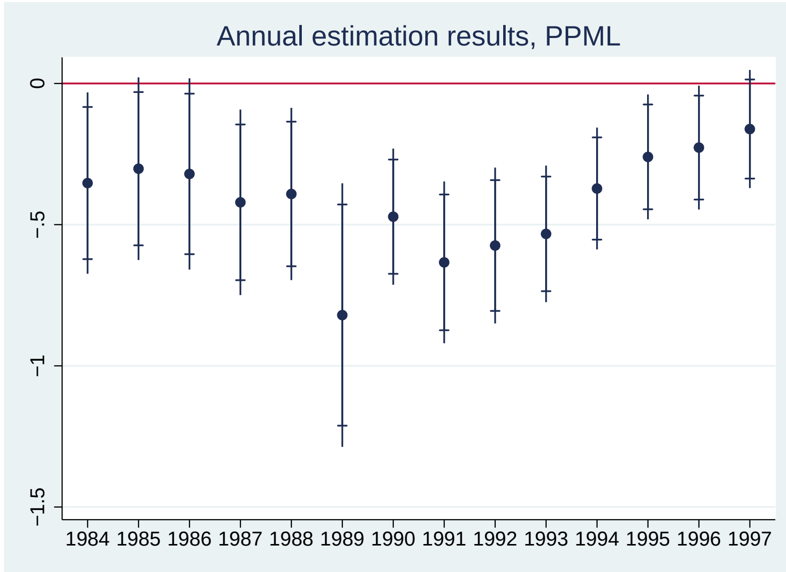

Figure 6: Estimated coefficients for $RIVAL \times |\Delta SIZE|$ from annual regressions (with 95% and 90% C.I.s)

of rivalries shown in Fig. 5. A full set of year-specific results for the *RIVAL* $\times$ $|\Delta$*SIZE*$|$ interaction term is also summarized in Fig. 6. All in all, the results are consistent across the different time periods, with the interaction term being at least weakly significant (i.e., at the $p < .1$ level) in every year except 1997 and strongly significant ($p < .05$) in 11 out of 14 years.

These results suggest that, throughout the sample, rival pairs of different size trade less with one another as compared to non-rival pairs and rivals of similar size. But what about when rivalries end—as many did around the middle of the period? Does the end of a rivalry have similarly differential effects on subsequent trade growth depending on the difference in country size? To examine this latter question, we follow the empirical literature on trade agreements (c.f., Baier and Bergstrand, 2007; Piermartini and Yotov, 2016) in introducing additional, *pair*-specific fixed effects (in columns 5 and 6) as well as pair-specific time trends (in column 6 only).[61] As in standard panel analysis, the added pair fixed effects ensure that our estimates can only be identified off of within-pair changes in trade following a change in rivalry status or in the other time-varying regressors. The trend terms added in column 6 then further require that estimates are identified relative to existing trends in bilateral trade. Furthermore, carefully noting that our theory specifically concerns *ex ante* size differences, we now fix $|\Delta$*SIZE*$|$ at 1984 levels such that the interaction between *RIVAL*

[61] Note that we cannot compute "within" changes in trade for trade with the Soviet Union, since it does not exist continuously throughout the period (an obvious consideration in this context). We therefore drop the Soviet Union whenever pair fixed effects are used, such that the breakup of the Soviet Union does not actually contribute to these results. Our prior results are also robust to dropping the Soviet Union.

and $|\Delta SIZE|$ may be identified solely through on time-variation in *RIVAL*.[62] The results in column 5 with added pair-specific fixed effects seem inconsistent with our earlier results, as the main effect for *RIVAL* is now significant but the interaction $RIVAL \times |\Delta SIZE|$ is not. However, as we show in column 6—where we also add pair time trends—this finding is due to differences in pre-existing trends in trade across different pairs. When these trends are taken into account, we again confirm our main results from before. In descriptive terms, trade appears to be trending downwards between rivals of differing size, then takes a large, discrete jump as many such rivalries came to an end at the close of the Cold War.

In sum, our gravity results show, in a variety of different ways, that having a higher probability of conflict is more negatively associated with trade when countries differ in size. Nonetheless, while the Cold War is a useful period to study for our purposes, we are reluctant to present these estimates as reflecting a natural experiment. Rivalries do not begin or end exogenously and there are likely other explanations behind some of the data points we observe. Indeed, as our motivation suggests, causality could flow either way here: either increased trade could be stimulating countries to end their rivalries (a variation of the "Liberal Peace" argument) or a lower probability of conflict could be creating more favorable conditions for trade (as we ourselves have argued). Our last specification that specifically accounts for pre-existing trends in trade goes a significant distance towards validating the latter as a reasonable interpretation.

6 Concluding Remarks

Trade and security are inseparable pillars of international policy. Yet the study of international trade largely abstracts from how the vast sums that are spent on national defense are affected by international commerce. Similarly, the conflict literature lacks discursive frameworks formally relating the distribution of economic activity to the determination of power, especially in a mostly peaceful world where the objectives underlying continued military spending are clearly forward-looking. In this paper, we analyze a dynamic, two-country model of trade and arming interactions, where arming arises naturally as preparation for an uncertain future. Notably, we show how arming decisions reflect not only the economic capabilities of each country, but also how the marginal benefit of more productive investments (i.e., saving) varies with the degree of uncertainty and the degree of resource insecurity.

A key implication of the theory, which we test and find suggestive support for, is that larger countries will find it unappealing to trade in the shadow of a possible future conflict with smaller rivals when the difference in *ex ante* size is sufficiently large. This prediction derives generally from the nonlinearity that occurs in the relationship between "size" and "power" when the probability of conflict is high. In other words, we show that the threat of conflict could itself be a source of "power" for an *ex ante* disadvantaged country.

[62] Our earlier cross-sectional results are also similar if we fix $|\Delta SIZE|$ at 1984 levels.

This last observation could be particularly salient for informing conflict management policies in situations where disproportionate compensation to seemingly weaker rivals would be necessary to entice them to improve diplomatic relations. Our framework could be extended to allow countries to make investments in institutions that manage conflict and reduce q (e.g., institutions that keep lines of communication open), before they make their arming and saving decisions, to improve expected payoffs.[63] Of course, such investments are costly. Nonetheless, more affluent countries would benefit directly and indirectly from a reduction in q under either autarky or trade; and, insofar as the benefits are larger under trade, such investments could induce the larger country to trade. Similarly, though sufficiently less affluent countries do not necessarily benefit from a reduction in q, the opportunity to realize relatively large gains from trade could entice them to make significant contributions of their own to these types of institutions.

Another interesting and important extension, already touched upon in Section 4.3, would be to allow for trade to take place in the future in the event of peace. As we have discussed, what is intriguing about this modified setting is how each country's incentives for productive investments in the present are dampened by the adverse consequences for its future terms of trade, thereby making more resources available for arming. A more careful investigation of these possibilities could illuminate the importance of a country's expected future terms of trade for its current security policies and preferences towards present trade. Importantly, this mechanism could be woven into an infinite period model where countries repeatedly make decisions about trade and arming under continually evolving beliefs about what the future may hold.

[63] See Meirowitz et al. (2019) for a recent analysis of "peace talks" between adversaries, in a setting where each country has private information regarding its own strength. An interesting and important finding that could inform this proposed extension is that such talks, if not mediated by a third party, could yield the perverse effect of inducing greater military spending and consequently a greater likelihood of destructive warfare; by contrast, talks mediated by third parties would reduce military spending and that likelihood.

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Online Appendix (not for publication)

This Appendix consists of three parts. In the first, we provide proofs of the lemmas and propositions presented in the paper and supplementary lemmas and propositions (along with their proofs). In section B, we provide additional details to help the reader work through some of the more tedious calculations. Finally, in section C, we provide brief notes in support of our claim that the possibility the larger country could prefer autarky over trade extends to other standard models of trade.

A Proofs and Additional Lemmas and Propositions

Proof of Lemma 1. For the analysis to follow, we rewrite $S^i = 0$ in (9) as:

$$S^i(\cdot) \equiv E^i - H^i = 0, \tag{A.1a}$$

where

$$E^i \equiv \left(\frac{\phi^i}{\phi^j}\right)^{1/m} \tag{A.1b}$$

$$H^i \equiv \left(\frac{\theta^i}{\theta^j}\right)\left(\frac{\Lambda^j}{\Lambda^i}\right) \tag{A.1c}$$

$$\Lambda^r \equiv (1-\kappa)\,\theta^r + \kappa\phi^r\,(1-q+q\theta^r) \in (0,1)\,, \quad r = i \neq j. \tag{A.1d}$$

Part (a): By the definition of E^i, we have $E^i \gtreqless 1 \iff \phi^i/\phi^j \gtreqless 1$. The requirement that $S^i = 0$ impies $E^i = H^i$, and thus $E^i \gtreqless 1 \iff H^i \gtreqless 1$ or, equivalently, after combining (A.1c) and (A.1d),

$$E^i = \frac{\phi^i}{\phi^j} \gtreqless 1 \iff H^i = \frac{1-\kappa+\kappa\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)}{1-\kappa+\kappa\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)} \gtreqless 1 \tag{A.2}$$

along the S^i-curve. The second set of inequalities in the line above can be rewritten as follows:

$$(1-q)\left(\theta^i-\phi^i\right) \gtreqless q\theta^i\theta^j\left(\phi^i-\phi^j\right). \tag{A.3}$$

Now, suppose that $\phi^i = \phi^j = \frac{1}{2}$, implying that $E^i = 1$ and thus (A.3) must hold as an equality. Accordingly, $\theta^i = \phi^i = \frac{1}{2}$ holds. Next, suppose that $\phi^i > \phi^j$, implying $E^i > 1$. Since $H^i > 1$ must hold, (A.3) requires that $\theta^i > \phi^i$ hold. Conversely, if $\phi^i < \phi^j$, we have $E^i < 1$, requiring that $H^i < 1$, and thus from (A.3) $\theta^i < \phi^i$.

Part (b): Recall that $\phi^j = 1 - \phi^i$, $\theta^j = 1 - \theta^i$ and $m \in (0, 1]$. Now suppose $\phi^i \to 0$ which implies $\phi^j \to 1$. Since $\theta^i < \phi^i$, from part (a), it must the case that θ^i is arbitrarily close to 0 as well; that is, $\theta^i \to 0$ which implies $\theta^j \to 1$. Thus, $\theta^j/\phi^j \to 1$ as $\phi^i \to 0$. Next, observe from (A.1b) that $\phi^i \to 0$ also implies $E^i \to 0$. But, the definition of $S^i = 0$ in (A.1a) implies that H^i must also converge to 0. Since $\phi^j/\theta^j \to 1$, the numerator of H^i in (A.2) is finite. Thus, $H^i \to 0$ only if the denominator of H^i becomes infinitely large as $\phi^i \to 0$. Inspection of (A.2) readily reveals that $\lim_{\phi^i \to 0} H^i = 0$ only if $\phi^i/\theta^i \to \infty$ or, equivalently, if $\theta^i/\phi^i \to 0$. This suggests that the S^i-curve becomes flat as $\phi^i \to 0$. The second portion of part (b) should be obvious.

Part (c): Partially differentiating the E^i and H^i components of $S^i(\cdot) = 0$ in (A.1), while keeping in mind that $\phi^j = 1 - \phi^i$ and $\theta^j = 1 - \theta^i$, yields

$$\frac{E^i_{\phi^i}}{E^i} = \sum_{r=i,j} \left[\frac{1}{m\phi^r}\right] = \frac{1}{m\phi^i\phi^j} > 0 \tag{A.4a}$$

$$E^i_{\theta^i} = 0 \tag{A.4b}$$

$$\frac{H^i_{\phi^i}}{H^i} = \frac{\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\Lambda^i_{\phi^i}}{\Lambda^i} = -\sum_{r=i,j} \left[\frac{\kappa\left(1-q+q\theta^r\right)}{\Lambda^r}\right] < 0 \tag{A.4c}$$

$$\frac{H^i_{\theta^i}}{H^i} = \frac{1}{\theta^i\theta^j} + \frac{\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\Lambda^i_{\theta^i}}{\Lambda^i} = \sum_{r=i,j} \left[\frac{\kappa\left(1-q\right)\phi^r/\theta^r}{\Lambda^r}\right] > 0, \tag{A.4d}$$

for $\phi^i \in (0,1)$. From (A.1a), we have $S^i_\eta = E^i_\eta - H^i_\eta$ for $\eta = \phi^i, \theta^i$. Then, the signs of the above expressions imply that $S^i_{\phi^i} > 0$, $S^i_{\theta^i} < 0$. By the implicit function theorem and the requirement that $E^i = H^i$ along the S^i-curve, we have

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{S^i=0} = -\frac{S^i_{\phi^i}}{S^i_{\theta^i}} = \frac{\frac{1}{m\phi^i\phi^j} + \sum_{r=i,j}\left[\frac{\kappa(1-q+q\theta^r)}{\Lambda^r}\right]}{\sum_{r=i,j}\left[\frac{\kappa(1-q)\phi^r/\theta^r}{\Lambda^r}\right]} > 0. \tag{A.5}$$

Thus, schedule S^i is increasing in $\phi^i \in (0,1)$, as stated in part (c) and depicted in Fig. 1.

Part (d): By substituting the expressions for Λ^r shown in (A.1d) into (A.5) and rearranging some, one can verify that the following holds along the S^i-curve:

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{S^i=0} = \left[\frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right]\frac{\theta^i}{\phi^i}. \tag{A.6}$$

The first point of part (d) can be established by evaluating the expression in the square brackets above as $\phi^i \to 0$, which implies $\phi^j \to 1$ and, from part (b), $\theta^i/\phi^i \to 0$ and $\theta^j/\phi^j \to 1$ along the S^i-curve, to find it simplifies to a finite term, $1+\frac{1}{m}$; since $\theta^i/\phi^i \to 0$ as $\phi^i \to 0$, the entire expression goes to 0. One can similarly show that $\lim_{\phi^i \to 1} d\theta^i/d\phi^i = 0$. The last point in part (d) follows by evaluating the expression in (A.6) at $\phi^i = \theta^i = \frac{1}{2}$. After rearranging, the expression becomes

$$\lim_{\phi^i \to \frac{1}{2}}\left(\left.\frac{d\theta^i}{d\phi^i}\right|_{S^i=0}\right) = 1 + \frac{1}{m} + \frac{1-\kappa+\kappa q\left(1+m\right)/2}{\kappa m\left(1-q\right)} > 1.$$

One should now be able to see that schedule S^i has an inflection point at $\phi^i = 1/2$.

The above relationship reveals that schedule S^i becomes perfectly inelastic at $\phi^i \to 1/2$—such that the flats at either endpoint of S^i become elongated—under two distinct circumstances: (i) when the probability of conflict is very high ($q \to 1$); and, (ii) when property is very secure ($\kappa \to 0$). These findings suggest that, under such circumstances, adjustments along schedule S^i for moderate values of ϕ^i (perhaps due to changes in income levels as analyzed below) will involve primarily changes in investment shares θ^i, with power (captured by ϕ^i) being relatively unresponsive. $\quad ||$

The next lemma builds on Lemma 1 to provide additional implications for ϕ^i and θ^i along the S^i-curve that are useful for proving some of the propositions to follow. Recall that a hat over a variable indicates percent change in that variable (i.e., $\widehat{x} = dx/x$).

Lemma A.1 *Percent changes in shares, $\widehat{\theta}^i$ and $\widehat{\phi}^i$, along the S^i-curve have the following properties:*

(a) (i) $\lim_{\phi^i \to 0} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} = 1 + \frac{1}{m}$

(ii) $\lim_{\phi^i \to \frac{1}{2}} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} = 1 + \frac{1}{m} + \frac{1-\kappa+\kappa q(1+m)/2}{m\kappa(1-q)}$

(iii) $\lim_{\phi^i \to 1} \left(\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0}\right) = 0.$

(b) $\lambda \times \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} > 1 + \frac{1}{m}$ *where* $\lambda = 1$ *for* $\phi^i \in \left(0, \frac{1}{2}\right]$ *and* $\lambda = \frac{\theta^i\phi^j}{\theta^j\phi^i}$ *for* $\phi^i \in \left[\frac{1}{2}, 1\right)$.

(c) $\arg\max_{\phi^i} \left(\widehat{\theta}^i/\widehat{\phi}^i\right)|_{S^i=0} \in \left(0, \frac{1}{2}\right)$.

Proof: Noting $\widehat{\theta}^i/\widehat{\phi}^i = (d\theta^i/d\phi^i)(\phi^i/\theta^i)$, the proof is based on the expression for $d\theta^i/d\phi^i|_{S^i=0}$ shown in (A.6) pre-multiplied by ϕ^i/θ^i:

$$\left.\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\right|_{S^i=0} = \frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}. \tag{A.7}$$

Part (a): The three items of this part are established by taking the appropriate limits of the expression above. (See the proof of Lemma 1(d) for more details.)

Part (b): This part of the lemma divides the parameter space for ϕ^i as follows: (i) $\phi^i \in (0, \frac{1}{2}]$, where we set $\lambda = 1$; and, (ii) $\phi^i \in [\frac{1}{2}, 1)$, where we set $\lambda = \theta^i\phi^j/\theta^j\phi^i$:

(i) For $\phi^i \in (0, \frac{1}{2}]$ (or $\lambda = 1$), Lemma 1(a) implies $\phi^i/\phi^j > \theta^i/\theta^j$. Then, substituting ϕ^i/ϕ^j for θ^i/θ^j in the denominator of (A.7) gives

$$\begin{aligned}\left.\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\right|_{S^i=0} &> \frac{\frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(1+\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}} \\ &> 1 + \frac{1}{m}\left[\frac{1+\left(\frac{\phi^i}{\phi^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \left(\frac{\phi^i}{\phi^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right] \\ &> 1 + \frac{1}{m}.\end{aligned}$$

The inequality on the first line immediately follows, since $\theta^i, \theta^j > 0$ and $\phi^i/\phi^j > \theta^i/\theta^j$. The inequality in the second line can be confirmed by comparing the various components that appear in the numerator and the denominator in the RHS of the first inequality. The inequality in the third line is obtained by applying the same logic to the expressions inside the square brackets of the second inequality.

(ii) For $\phi^i \in [\frac{1}{2}, 1)$ (or $\lambda = \theta^i\phi^j/\theta^j\phi^i$), Lemma 1(a) implies $\lambda > 1$. Pre-multiply (A.7) by λ. After some simplifying, the resulting expression shows

$$\begin{aligned}\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} &> 1+\frac{1}{m}\left[\frac{1+\left(\frac{\theta^i}{\theta^j}\right)}{\frac{\kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)}+\left(\frac{\theta^i}{\theta^j}\right)\frac{\kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}\right] \\ &> 1+\frac{1}{m}.\end{aligned}$$

The inequality on the first line can be confirmed by using the facts that $\theta^i/\theta^j > \phi^i/\phi^j$ in this case and $\theta^i, \theta^j > 0$ and comparing the various components of the numerator and denominator of the resulting expression (not shown). The inequality on the second line can similarly be confirmed by comparing the components of the expressions that appear in the numerator and the denominator of the RHS of the first inequality.

Part (c): Parts (a) and (b) suggest that, even though both ϕ^i and θ^i rise along the S^i-curve, the increase in θ^i tapers off after some value of ϕ^i and vanishes as $\phi^i \to \frac{1}{2}$. Part (c) asserts that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0}$ attains a maximum prior to arriving at $\phi^i = \frac{1}{2}$. To see this point, note that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0} = \left(\frac{d\theta^i/d\phi^i}{\theta^i/\phi^i}\right)\Big|_{S^i=0}$ and that its rate of change along the S^i-curve is

$$\widehat{\left(\widehat{\theta}^i/\widehat{\phi}^i\right)} = (\widehat{d\theta^i/d\phi^i}) - (\widehat{\theta^i/\phi^i}).$$

When evaluated at $\phi^i = \frac{1}{2}$, the first term in the RHS equals 0 because, by symmetry, the S^i-curve has an inflection point $\phi^i = \frac{1}{2}$. Furthermore, at $\phi^i = \frac{1}{2}$, $(\widehat{\theta^i/\phi^i}) > 0$ by part (a), suggesting that $\widehat{\theta}^i/\widehat{\phi}^i|_{S^i=0}$ is decreasing as $\phi^i \to \frac{1}{2}$, thus establishing part (c). ||

Proof of Lemma 2. It convenient to rewrite (12) as

$$B^i(\cdot) \equiv E^i - F^i = 0 \tag{A.8a}$$

where E^i was defined in (A.1b) and

$$\begin{aligned} F^i &\equiv \left(\frac{\Omega^j}{\Omega^i}\right)\left(\frac{Y^i}{Y^j}\right) && \text{(A.8b)} \\ \Omega^r &\equiv (1+\delta)(1-\kappa)\theta^r + \kappa\phi^r[1+\delta(1-q)+q\delta(\theta^r+m(1-\phi^r)] > 0, && \text{(A.8c)}\end{aligned}$$

for $r = i \neq j = 1, 2$. The expression for F^i in (A.8b) is obtained by substituting the values of γ^i and ζ^i described in (11) into the second expression of (12). Partial differentiation of

F^i gives

$$\frac{F^i_{\phi^i}}{F^i} = \frac{\Omega^j_{\phi^i}}{\Omega^j} - \frac{\Omega^i_{\phi^i}}{\Omega^i} = -\kappa \sum_{r=i,j} \left[\frac{1+\delta(1-q)+q\delta\left(\theta^r+m-2m\phi^r\right)}{\Omega^r} \right] < 0 \quad \text{(A.9a)}$$

$$\frac{F^i_{\theta^i}}{F^i} = \frac{\Omega^j_{\theta^i}}{\Omega^j} - \frac{\Omega^i_{\theta^i}}{\Omega^i} = -\sum_{r=i,j} \left[\frac{(1-\kappa)(1+\delta)+\kappa q\delta\phi^r}{\Omega^r} \right] < 0. \quad \text{(A.9b)}$$

By definition (A.8a), $B^i_\eta = E^i_\eta - F^i_\eta$ for $\eta = \phi^i, \theta^i$. Since $E^i_{\phi^i} > 0$ and $F^i_{\phi^i} < 0$, we have $B^i_{\phi^i} > 0$. Similarly, since $E^i_{\theta^i} = 0$ and $F^i_{\theta^i} < 0$, $B^i_{\theta^i} > 0$ holds. Applying the implicit function theorem to (A.8a) while keeping in mind that $E^i = F^i$ along that curve, thus, yields

$$\left.\frac{d\theta^i}{d\phi^i}\right|_{B^i=0} = -\frac{B^i_{\phi^i}}{B^i_{\theta^i}} = -\frac{\frac{1}{m\phi^i\phi^j} + \sum_{r=i,j}\left[\frac{\kappa[1+\delta(1-q)+q\delta(\theta^r+m-2m\phi^r)]}{\Omega^r}\right]}{\sum_{r=i,j}\left[\frac{(1-\kappa)(1+\delta)+\kappa q\delta\phi^r}{\Omega^r}\right]} < 0,$$

which implies θ^i and ϕ^i are negatively related along schedule B^i.

Next, consider the values of ϕ^i along B^i in the extremes where $\theta^i = 0$ and $\theta^i = 1$. One can verify, from (A.8) and (A.1b), that, for $\theta^i = 0$, we have $\lim_{\phi^i \to 0} B^i < 0$ and $\lim_{\phi^i \to 1} B^i > 0$. Since $B^i_{\phi^i} > 0$, there exists a unique value of $\phi^i \in (0,1)$ such that $B^i(\cdot) = 0$. One can also verify that the same argument applies for $\theta^i = 1$. We can thus conclude that, at $\theta^i = 0$ and $\theta^i = 1$, schedule B^i cuts the horizontal axes, as illustrated in Fig. 1. ||

Proof of Proposition 1. Totally differentiating the S^i- and B^i-curves in (A.1) and (A.8) respectively, while focusing on percent changes, allows us to rewrite the system of equations involving changes in these curves in a more convenient way for the purpose of proving this proposition and others to follow. In particular, letting $y^i \equiv Y^i/Y^j$, we have

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \widehat{\phi}^{i*} \\ \widehat{\theta}^{i*} \end{pmatrix} + \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} \widehat{y}^i + \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix} \widehat{\kappa} + \begin{pmatrix} b_{13} \\ b_{23} \end{pmatrix} \widehat{q} + \begin{pmatrix} b_{14} \\ b_{24} \end{pmatrix} \widehat{\delta} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad \text{(A.10)}$$

From (A.1) and (A.8) with (A.4) and (A.9), the a-coefficients can be written as[1]

$$a_{11} = \frac{\phi^i E^i_{\phi^i}}{E^i} - \frac{\phi^i H^i_{\phi^i}}{H^i} = \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j}$$

$$= \frac{1}{m\phi^j} + \kappa\phi^i \left[\sum_{r=i,j} \frac{1-q+q\theta^r}{\Lambda^r} \right] > 0 \quad \text{(A.11a)}$$

$$a_{12} = \frac{\theta^i E^i_{\theta^i}}{E^i} - \frac{\theta^i H^i_{\phi^i}}{H^i} = -\frac{1}{\theta^j} + \frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j}$$

$$= -\frac{\kappa(1-q)}{\theta^j} \left[\sum_{r=i,j} \frac{\phi^r(1-\theta^r)}{\Lambda^r} \right] < 0 \quad \text{(A.11b)}$$

[1]For more details on deriving the individual components, see equations (B.2) and (B.3) presented in Appendix B.

$$
\begin{aligned}
a_{21} &= \frac{\phi^i E^i_{\phi^i}}{E^i} - \frac{\phi^i F^i_{\phi^i}}{F^i} = \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} \\
&= \frac{1}{m\phi^j} + \kappa\phi^i \left[\sum_{r=i,j} \frac{1+(1-q)\delta + q\delta[\theta^r + m(1-2\phi^r)]}{\Omega^r} \right] > 0 \qquad \text{(A.11c)} \\
a_{22} &= \frac{\theta^i E^i_{\theta^i}}{E^i} - \frac{\theta^i F^i_{\phi^i}}{F^i} = \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} \\
&= \theta^i \left[\sum_{r=i,j} \frac{(1-\kappa)(1-q) + \kappa q \delta \phi^r}{\Lambda^r} \right] > 0. \qquad \text{(A.11d)}
\end{aligned}
$$

We derive the b-coefficients below.

The proof of existence and uniqueness of equilibrium follows readily from Lemmas 1 and 2, which described the properties of the schedules S^i and B^i, respectively. In particular, the shapes of these schedules imply that they will intersect once and only once since

$$
\mathcal{D} \equiv \det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \overset{(+)}{a_{11}}\overset{(+)}{a_{22}} - \overset{(-)}{a_{12}}\overset{(+)}{a_{21}} > 0. \qquad \text{(A.12)}
$$

The emergence of a unique point $(\phi^{i*}, \theta^{i*}) \in (0,1) \times (0,1)$, together with (10) and (11), enable us to recover G^{i*} and Z^{i*} from (ϕ^{i*}, θ^{i*}), thereby completing this part of the proof. Henceforth, we drop "$*$" from equilibrium values to avoid cluttering.

Part (a): As discussed in the text, the S^i-curve is independent of the two countries' first-period relative incomes $y^i \equiv Y^i/Y^j$, always going through the midpoint where $\phi^i = \theta^i = \frac{1}{2}$ for $i \neq j = 1, 2$. By contrast, the B^i-curve goes through that midpoint only when $y^i = 1$ and shifts out (in) as y^i increases above (decreases below) 1. More formally, this part of the proposition can be established, by setting $\widehat{\kappa} = \widehat{q} = \widehat{\delta} = 0$, and solving (A.10) as follows:

$$
\begin{pmatrix} \widehat{\phi}^i \\ \widehat{\theta}^i \end{pmatrix} = \frac{1}{\mathcal{D}} \begin{pmatrix} -a_{22}b_{11} + a_{12}b_{21} \\ a_{21}b_{11} - a_{11}b_{21} \end{pmatrix} \widehat{y}^i, \qquad \text{(A.13)}
$$

where, from (A.1) and (A.8), the coefficients on $\widehat{y}^i$ in (A.10) are given by

$$
\begin{aligned}
b_{11} &= \frac{y^i E^i_{y^i}}{E^i} - \frac{y^i H^i_{y^i}}{H^i} = 0 \qquad \text{(A.14a)} \\
b_{21} &= \frac{y^i E^i_{y^i}}{E^i} - \frac{y^i F^i_{y^i}}{F^i} = -1. \qquad \text{(A.14b)}
\end{aligned}
$$

Then, substituting these expressions along with those for a_{11} and a_{12} shown respectively in (A.11a) and (A.11b) into (A.13) gives

$$
\widehat{\phi}^i = -\frac{a_{12}}{\mathcal{D}} \widehat{y}^i = \frac{1}{\mathcal{D}} \left[\frac{\kappa(1-q)}{\theta^j} \left(\sum_{r=i,j} \frac{\phi^r(1-\theta^r)}{\Lambda^r} \right) \right] \widehat{y}^i > 0 \qquad \text{(A.15a)}
$$

$$\widehat{\theta}^i = \frac{a_{11}}{\mathcal{D}}\widehat{y}^i = \frac{1}{\mathcal{D}}\left[\frac{1}{m\phi^j} + \kappa\phi^i\left(\sum_{r=i,j}\frac{1-q+q\theta^r}{\Lambda^r}\right)\right]\widehat{y}^i > 0. \tag{A.15b}$$

Thus, both ϕ^i and θ^i rise with increases in y^i and fall with decreases in y^i. The remaining points in part (a) can be seen, from parts (a) and (b) of Lemma 1, by noting that upward shifts in schedule B^i in Fig. 1 (induced by increases in y^i) trace out the points along the S^i-curve. Interestingly, the shape of this curve implies the ratio θ^i/ϕ^i rises initially with increases in y^i; however, eventually, when y^i crosses a certain threshold, the relative appeal of saving to arming θ^i/ϕ^i falls.

Part (b): To prove part (b), we set $\widehat{y}^i = \widehat{\delta} = 0$ and solve (A.10) for $\widehat{\phi}^i$ to obtain:

$$\widehat{\phi}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{22}b_{12} + a_{12}b_{22})\,\widehat{\kappa} + (-a_{22}b_{13} + a_{12}b_{23})\,\widehat{q}\,\right], \tag{A.16}$$

where, since E^i does not depend on κ, (A.1) and (A.8) imply $b_{12} \equiv -\kappa H^i_\kappa/H^i$ and $b_{22} \equiv -\kappa F^i_\kappa/F^i$ or equivalently[2]

$$\begin{aligned}
b_{12} &= \frac{\kappa\Lambda^i_\kappa}{\Lambda^i} - \frac{\kappa\Lambda^j_\kappa}{\Lambda^j} = \frac{\kappa\phi^i\left(1-q+q\theta^i\right)-\kappa\theta^i}{\kappa\phi^i\left(1-q+q\theta^i\right)+(1-\kappa)\,\theta^i} \\
&\qquad - \frac{\kappa\phi^j\left(1-q+q\theta^j\right)-\kappa\theta^j}{\kappa\phi^j\left(1-q+q\theta^j\right)+(1-\kappa)\,\theta^j} \\
b_{22} &= \frac{\kappa\Omega^i_\kappa}{\Omega^i} - \frac{\kappa\Omega^j_\kappa}{\Omega^j} = \frac{\kappa\phi^i\left[1+\delta\left(1-q\right)+\delta q\left(\theta^i+m\phi^j\right)\right]-\kappa\left(1+\delta\right)\theta^i}{\kappa\phi^i\left[1+\delta\left(1-q\right)+\delta q\left(\theta^i+m\phi^j\right)\right]+\left(1-\kappa\right)\left(1+\delta\right)\theta^i} \\
&\qquad - \frac{\kappa\phi^j\left[1+\delta\left(1-q\right)+\delta q\left(\theta^j+m\phi^i\right)\right]-\kappa\left(1+\delta\right)\theta^j}{\kappa\phi^j\left[1+\delta\left(1-q\right)+\delta q\left(\theta^j+m\phi^i\right)\right]+\left(1-\kappa\right)\left(1+\delta\right)\theta^j};
\end{aligned} \tag{A.17a}$$

and, similarly, since E^i does not depend on q, (A.1) and (A.8) imply $b_{13} \equiv -qH^i_q/H^i$ and $b_{23} \equiv -qF^i_q/F^i$ or[3]

$$\begin{aligned}
b_{13} &= \frac{q\Lambda^i_q}{\Lambda^i} - \frac{q\Lambda^j_q}{\Lambda^j} = \frac{\kappa q\theta^i\phi^j}{\kappa\phi^j\left(1-q+q\theta^j\right)+(1-\kappa)\,\theta^j} \\
&\qquad - \frac{\kappa q\theta^j\phi^i}{\kappa\phi^i\left(1-q+q\theta^i\right)+(1-\kappa)\,\theta^i} \\
b_{23} &= \frac{q\Omega^i_q}{\Omega^i} - \frac{q\Omega^j_q}{\Omega^j} = \frac{\kappa q\delta\left(m\phi^j-\theta^j\right)\phi^i}{\kappa\phi^i\left[1+\delta\left(1-q\right)+\delta q\left(\theta^i+m\phi^j\right)\right]+\left(1-\kappa\right)\left(1+\delta\right)\theta^i} \\
&\qquad - \frac{\kappa q\delta\left(m\phi^i-\theta^i\right)\phi^j}{\kappa\phi^j\left[1+\delta\left(1-q\right)+\delta q\left(\theta^j+m\phi^i\right)\right]+\left(1-\kappa\right)\left(1+\delta\right)\theta^j}
\end{aligned} \tag{A.17b}$$

We have already shown that $a_{11} > 0$, $a_{12} < 0$, $a_{21} > 0$, $a_{22} > 0$ and $\mathcal{D} > 0$. Furthermore, from part (a), we have $\phi^i = \phi^j$ and $\theta^i = \theta^j$ when $Y^i = Y^j$. Thus, inspection of the expressions in (A.17a) and (A.17b) reveals that $b_{12} = b_{22} = b_{13} = b_{23} = 0$ when $Y^i = Y^j$,

[2] For more details on deriving the individual components, see equation (B.4) presented in Appendix B.

[3] Again, see equation (B.4) presented in Appendix B.

so that ϕ^{i*} and θ^{i*} are not affected by changes in κ or q affect ϕ^{i*} in this symmetric case.

To prove the remaining components of part (b), we focus on the case where $Y^i < Y^j$, and establish that $b_{12} > 0$ and $b_{22} > 0$ in (A.17a), while $b_{13} < 0$ and $b_{23} < 0$ in (A.17b).[4] We start with b_{12}, which can be simplified, by adding and subtracting $(1-\kappa)\theta^i$ from the numerator of the first term and then adding and subtracting $(1-\kappa)\theta^j$ from the numerator of the second term on the far RHS of the first expression in (A.17a), as follows:

$$b_{12} = 1 - \frac{\theta^i}{\Lambda^i} - 1 + \frac{\theta^j}{\Lambda^j} = \frac{\theta^j}{\Lambda^j} - \frac{\theta^i}{\Lambda^i}. \tag{A.18}$$

To evaluate the sign of this expression, recall from (A.1) that $E^i = H^i$ or equivalently $\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j\Lambda^i}{\theta^i\Lambda^j}\right) = 1$ along the S^i-curve. Furthermore, we have

$$\frac{\theta^j}{\Lambda^j} > \frac{\theta^i}{\Lambda^i} \iff 1 > \frac{\theta^i\Lambda^j}{\theta^j\Lambda^i}.$$

Now multiply both sides of the second inequality above by $\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j\Lambda^i}{\theta^i\Lambda^j}\right) = 1$ to find

$$1 > \frac{\theta^i\Lambda^j}{\theta^j\Lambda^i} \times \left[\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j\Lambda^i}{\theta^i\Lambda^j}\right)\right] = \left(\phi^i/\phi^j\right)^{\frac{1}{m}}.$$

Our assumption that $Y^i < Y^j$ implies, from part (a), $\phi^i < \phi^j$. Thus, the latter inequality holds, and $b_{12} > 0$.

Turning to b_{22}, we first simplify the expression, with a similar procedure used above for b_{12}, adding and subtracting $(1-\kappa)(1+\delta)\theta^i$ from the numerator of the first term and then adding and subtracting $(1-\kappa)(1+\delta)\theta^j$ from the numerator of the second term on the far RHS of the second expression in (A.17a) to find

$$b_{22} = 1 - \frac{(1+\delta)\,\theta^i}{\Omega^i} - 1 + \frac{(1+\delta)\,\theta^j}{\Omega^j} = (1+\delta)\left[\frac{\theta^j}{\Omega^j} - \frac{\theta^i}{\Omega^i}\right]. \tag{A.19}$$

To evaluate the sign of the far RHS expression, first note that

$$\frac{\theta^j}{\Omega^j} > \frac{\theta^i}{\Omega^i} \iff 1 > \frac{\theta^i\Omega^j}{\theta^j\Omega^i}.$$

Following our strategy above to evaluate the sign of b_{12}, we multiply both sides of the second inequality above by $\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \times \left(\frac{\theta^j\Lambda^i}{\theta^i\Lambda^j}\right) = 1$ (as required along the S^i-curve) to rewrite it as

$$1 > \frac{\theta^i\Omega^j}{\theta^j\Omega^i} \times \left[\left(\phi^i/\phi^j\right)^{\frac{1}{m}} \left(\frac{\theta^j\Lambda^i}{\theta^i\Lambda^j}\right)\right] = \left(\phi^i/\phi^j\right)^{\frac{1}{m}} \frac{\Omega^j\Lambda^i}{\Omega^i\Lambda^j}.$$

The definitions of the relevant variables imply that the expression on the far RHS of the

[4] The proof for $Y^i > Y^j$ can be established along similar lines, showing $b_{12} < 0$ and $b_{22} < 0$, while $b_{13} > 0$ and $b_{23} > 0$ in this case.

inequality is a function of (ϕ^i, θ^i). We now argue that the last inequality holds true for values of $\phi^i \in \left(0, \frac{1}{2}\right)$ and $\theta^i \in \left(0, \frac{1}{2}\right)$ and not just the combinations of ϕ^i and θ^i along the S^i-curve. To see this, consider any $\phi^i \in \left(0, \frac{1}{2}\right)$ and suppose that $\theta^i = \phi^i$ initially. It is straightforward for one to verify (after some tedious but straightforward calculations) that

$$\left.\frac{\Omega^j \Lambda^i}{\Omega^i \Lambda^j}\right|_{\theta^i=\phi^i \in \left(0,\frac{1}{2}\right)} = 1 + \frac{\kappa q \left(1+\delta m\right) \phi^i \phi^j \left(\phi^i - \phi^j\right)}{\Omega^i \Lambda^j} < 1,$$

thereby suggesting that the inequality of interest is satisfied at any $\phi^i \in \left(0, \frac{1}{2}\right)$ when $\theta^i = \phi^i$. We now show that, for any given $\phi^i \in \left(0, \frac{1}{2}\right)$, $\frac{\Omega^j \Lambda^i}{\Omega^i \Lambda^j}$ is increasing in θ^i:

$$\begin{aligned}
\left(\widehat{\frac{\Omega^j \Lambda^i}{\Omega^i \Lambda^j}}\right) / \widehat{\theta^i} &= \frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} - \frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} + \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} \\
&= \frac{\theta^i \left(1-\kappa+\kappa q \phi^i\right)}{\Lambda^i} - \frac{\theta^i \left[\left(1-\kappa\right)\left(1+\delta\right)+\kappa q \delta \phi^i\right]}{\Omega^i} \\
&\quad + \frac{\theta^i \left(1-\kappa+\kappa q \phi^j\right)}{\Lambda^j} - \frac{\theta^i \left[\left(1-\kappa\right)\left(1+\delta\right)+\kappa q \delta \phi^j\right]}{\Omega^j} \\
&= \frac{\kappa q \theta^i \phi^i \left[\left(1-\kappa\right)\left(1+\delta m \phi^j\right)+\kappa \phi^i \left(1+q\delta m \phi^j\right)\right]}{\Lambda^i \Omega^i} \\
&\quad + \frac{\kappa q \theta^i \phi^j \left[\left(1-\kappa\right)\left(1+\delta m \phi^i\right)+\kappa \phi^j \left(1+q\delta m \phi^i\right)\right]}{\Lambda^j \Omega^j} > 0.
\end{aligned}$$

Since the above expression is positive, a decrease in θ^i implies that the inequality will hold true for all $\theta^i \leq \phi^i \in \left(0, \frac{1}{2}\right)$, including the points along the S^i-curve. In turn, we have that $b_{22} > 0$ holds.

Our findings that $b_{12} > 0$ and $b_{22} > 0$ in the context of (A.16) imply that $d\phi^i/d\kappa < 0$ (when $Y^i < Y^j$).[5] An increase in the degree of insecurity κ causes the S^i-curve to rotate clockwise around the point where $\phi^i = \theta^i = \frac{1}{2}$ with the endpoints $(0,0)$ and $(1,1)$ unchanged; the B^i-curve also rotates clockwise with a pivot point located at its intersection with the 45° line when $m = 1$ and below that intersection for $m < 1$.[6]

Turning to the effect of changes in the probability of conflict q, one can easily confirm from the first expression in (A.17b) that b_{13} can be written as

$$b_{13} = \kappa q \left[\frac{\theta^i}{\Lambda^j/\phi^j} - \frac{\theta^j}{\Lambda^i/\phi^i}\right]. \tag{A.20}$$

[5] Applying analogous reasoning, one can show that, when $Y^i > Y^j$, $b_{12} < 0$ and $b_{22} < 0$, and thus $d\phi^i/d\kappa > 0$.

[6] Recall that for points to the right and below the S^i-curve $S^i > 0$, while points to the right and above the B^i-curve imply $B^i > 0$. To verify the pivot point for the S^i-curve, evaluate the expression for b_{12} in (A.18) at the midpoint, to find that it equals zero. Similarly, to verify the pivot point of the B^i-curve, evaluate the expression in b_{22} in (A.19) at any $\phi^i = \theta^i < \frac{1}{2}$ (i.e., along the lower segment of the 45° line). Some straightforward algebra reveals that the sign of this expression equals $\text{sign}\{(\phi^i - \phi^j)(1-m)\}$. Thus, the pivot point for the B^i-curve lies on the 45° line when $m = 1$ and (since by assumption $Y^i < Y^j$ and thus $\phi^i < \phi^j$) lies below it when $m < 1$.

To sign this expression, first note that, from part (a) of this proposition, our assumption that $Y^i < Y^j$ implies $\theta^i < \theta^j$. Hence, a sufficient condition for $b_{13} < 0$ is that $\Lambda^i/\phi^i < \Lambda^j/\phi^j$. To proceed, observe that

$$\Lambda^i/\phi^i = (1-\kappa)\left(\theta^i/\phi^i\right) + \kappa\left[1 - q + q\theta^i\right],$$

which enables us to form the difference

$$\Lambda^i/\phi^i - \Lambda^j/\phi^j = (1-\kappa)\left[\frac{\theta^i}{\phi^i} - \frac{\theta^j}{\phi^j}\right] + \kappa q\left[\theta^i - \theta^j\right] < 0.$$

The negative sign of the above expressions follows again, from part (a) of the proposition, that $Y^i < Y^j$ implies $\theta^i/\phi^i - \theta^j/\phi^j < 0$ and $\theta^i - \theta^j < 0$. (Note that the inequality above also implies that $\Lambda^i - \Lambda^j < 0$ for $Y^i < Y^j$.) Hence, $b_{13} < 0$.

Turning to the last coefficient of interest, note that b_{23} in the second expression of (A.17b) can be written as

$$b_{23} = \kappa q\delta\left[\frac{\theta^i - m\phi^i}{\Omega^j/\phi^j} - \frac{\theta^j - m\phi^j}{\Omega^i/\phi^i}\right]. \tag{A.21}$$

To confirm that the sign of the expression above is negative, first note that $\theta^i - m\phi^i - (\theta^j - m\phi^j) = \phi^i\left(\frac{\theta^i}{\phi^i} - m\right) - \phi^j\left(\frac{\theta^j}{\phi^j} - m\right)$, which is negative, since $Y^i < Y^j$ implies $\phi^i < \phi^j$ and $\theta^i/\phi^i < \theta^j/\phi^j$ from part (a) of the proposition. Thus, $\theta^i - m\phi^i < \theta^j - m\phi^j$, and we need only to establish that $\left(\Omega^i/\phi^i\right) < \left(\Omega^j/\phi^j\right)$. Noting that

$$\Omega^i/\phi^i = (1-\kappa)(1+\delta)\left(\theta^i/\phi^i\right) + \kappa\left[1 + (1-q)\delta + q\delta\left(\theta^i + m\phi^j\right)\right]$$

for $i \neq j = 1, 2$, we find

$$\Omega^i/\phi^i - \Omega^j/\phi^j = (1-\kappa)(1+\delta)\left[\frac{\theta^i}{\phi^i} - \frac{\theta^j}{\phi^j}\right] + \kappa q\delta\left[\theta^i + m\phi^j - \theta^j - m\phi^i\right] < 0.$$

The above inequality is obtained from our finding immediately above that $\theta^i - m\phi^i < \theta^j - m\phi^j$ and the assumption that $Y^i < Y^j$ which implies (again, by part (a) of the proposition) that $\theta^i/\phi^i - \theta^j/\phi^j < 0$. (The inequality above also implies that $\Omega^i - \Omega^j < 0$ for $Y^i < Y^j$.) Thus, $b_{23} < 0$.

Applying the observations that $b_{13} < 0$ and $b_{23} < 0$ to (A.16) gives $d\phi^i/dq > 0$ (when $Y^i < Y^j$).[7] The effect of an increase in q can be seen graphically as a counter-clockwise rotation of the S^i-curve around the $\phi^i = \theta^i = \frac{1}{2}$ pivot point; at the same time, the B^i-curve rotates in a counterclockwise direction around a pivot point that lies on its intersection with

[7] Taking a similar approach, one can establish that $b_{13} > 0$ and $b_{23} > 0$ and thus $d\phi^i/dq < 0$ when $Y^i > Y^j$.

the 45° line when $m = 1$ and above (below) it when $m < 1$ and $Y^i < Y^j$ ($Y^i > Y^j$).[8]

To establish the limits stated part (b) of the proposition, consider first what happens to the S^i-curve as $\kappa \to 0$ such that property becomes very secure. From the expression for H^i shown in (A.2), we have $\lim_{\kappa\to 0} H^i = 1$ for all $\theta^i \in (0, 1)$ or equivalently when $y^i \in (0, \infty)$. Since $E^i = (\phi^i/\phi^j)^{1/m}$, the equilibrium condition $S^i = E^i - H^i = 0$ can be satisfied only for $\phi^i = \phi^j = \frac{1}{2}$. As property becomes perfectly secure, the S^i-curve becomes vertical for all $\theta^i \in (0, 1)$ at $\phi^i = \frac{1}{2}$, such that the positioning of the B^i-curve alone determines the equilibrium value of θ^i; specifically, $\theta^i \gtreqqless \frac{1}{2}$ when $Y^i \gtreqqless Y^j$. Next consider what happens as $q \to 1$. From the expression for H^i again shown in (A.2), we have $\lim_{q\to 1} H^i = (1 - \kappa\phi^i)/(1 - \kappa\phi^j)$ for all $\theta^i \in (0, 1)$. As before, the equilibrium condition $S^i = E^i - H^i = 0$ can be satisfied only at $\phi^i = \phi^j = \frac{1}{2}$, and the positioning of the B^i-curve in turn pins down the equilibrium value for θ^i, again with $\theta^i \gtreqqless \frac{1}{2}$ when $Y^i \gtreqqless Y^j$.

Part (c): To prove this part of the proof, we set $\widehat{y}^i = \widehat{q} = \widehat{\kappa} = 0$ and solve (A.10) for $\widehat{\phi}^i$ to obtain:

$$\begin{pmatrix} \widehat{\phi}^i \\ \widehat{\theta}^i \end{pmatrix} = \frac{1}{\mathcal{D}} \begin{pmatrix} -a_{22}b_{14} + a_{12}b_{24} \\ a_{21}b_{14} - a_{11}b_{24} \end{pmatrix} \widehat{\delta}, \tag{A.22}$$

where the a-coefficients are given in equation (A.11). Since neither E^i nor H^i depends on δ, $b_{14} = 0$. Thus, we need only to sign the coefficient b_{24}.

Let us define $\widehat{\delta} \equiv \frac{d\delta}{1+\delta}$, which implies $b_{24} = -(1 + \delta)F^i_\delta/F^i$ or equivalently

$$\begin{aligned} b_{24} &= -\frac{(1+\delta)\,\Omega^j_\delta}{\Omega^j} + \frac{(1+\delta)\,\Omega^i_\delta}{\Omega^i} = \left[\frac{(1+\delta)\,\Omega^j_\delta}{\Omega^j} - 1\right] + \left[\frac{(1+\delta)\,\Omega^i_\delta}{\Omega^i} - 1\right] \\ &= \left[\frac{\kappa q\phi^j\,(\theta^i - m\phi^i)}{\Omega^j}\right] - \left[\frac{\kappa q\phi^i\,(\theta^j - m\phi^j)}{\Omega^i}\right] \\ &= \kappa q\phi^i\phi^j \left[\frac{\theta^i/\phi^i - m}{\Omega^j} - \frac{\theta^j/\phi^j - m}{\Omega^i}\right]. \end{aligned} \tag{A.23}$$

Recall that $a_{12} < 0$ and $a_{11} > 0$. Therefore, to establish this part of the proof with our maintained focus on the case where $Y^i < Y^j$, it suffices to show that $b_{24} < 0$. From our analysis in the proof of part (b), we know that $Y^i < Y^j$ implies $\Omega^i < \Omega^j$ (which, in turn, implies $1/\Omega^j < 1/\Omega^i$). Furthermore, from part (a), $Y^i < Y^j$ implies $\theta^i/\phi^i < \theta^j/\phi^j$. These two results taken together imply $b_{24} < 0$ and thus $d\phi^i/d\delta > 0$ and $d\theta^i/d\delta > 0$ when $Y^i < Y^j$.[9] The effect in an increase in δ can be illustrated as a counterclockwise rotation of the B^i curve around a pivot point that lies on the 45° line when $m = 1$ and above (below) it when $Y^i < Y^j$ ($Y^i > Y^j$).[10] ||

[8] Along the same logic spelled out in footnote 6, one can verify these pivot points using the expressions for b_{13} and b_{23} shown respectively in (A.20) and (A.21).

[9] One can similarly establish that when $Y^i > Y^j$, $b_{24} > 0$, so that $d\phi^i/d\delta < 0$ and $d\theta^i/d\delta < 0$ in this case.

[10] Once again, the reader can confirm these pivot points applying the logic spelled out in footnote 6 with (A.23).

Proposition A.1 *Suppose that condition A satisfied. Then, the influences of changes in insecurity of property κ and the probability of a future war q on equilibrium shares in the cases of perfect symmetry and extreme asymmetry are as follows:*

(a) *When $Y^i = Y^j = Y \in (0,\infty)$, the equilibrium shares are $\phi^{i*} = \theta^{i*} = \frac{1}{2}$ for all $\kappa \in (0,1]$ and $q \in (0,1)$.*

(b) *For given $Y^j \in (0,\infty)$, $\lim_{Y^i\to 0} \widehat{\phi}^{i*}/\widehat{\kappa} < 0$, $\lim_{Y^i\to 0} \widehat{\phi}^{i*}/\widehat{q} > 0$, $\lim_{Y^i\to 0} \widehat{\theta}^{i*}/\widehat{\kappa} = 0$, and $\lim_{Y^i\to 0} \widehat{\theta}^{i*}/\widehat{q} < 0$.*

Proof: The system of equations in (A.10) implies

$$\widehat{\phi}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{22}b_{12} + a_{12}b_{22})\,\widehat{\kappa} + (-a_{22}b_{13} + a_{12}b_{23})\,\widehat{q}\,\right] \tag{A.24a}$$

$$\widehat{\theta}^i = \frac{1}{\mathcal{D}}\left[\,(-a_{21}b_{12} + a_{11}b_{22})\,\widehat{\kappa} + (-a_{21}b_{13} + a_{11}b_{23})\,\widehat{q}\,\right], \tag{A.24b}$$

where $\mathcal{D} = a_{11}a_{22} - a_{12}a_{21} > 0$ and the a- and b-coefficients are shown respectively in (A.11) and (A.17).

Part (a): In the proof of Proposition 1(b) we have already established that, when $Y^i = Y^j > 0$, $\phi^i = \theta^i = \frac{1}{2}$, such that from (A.17) $b_{12} = b_{22} = b_{13} = b_{23} = 0$. Thus, evaluating the expressions in (A.24) where $Y^i = Y^j$ shows that changes in in κ and q have no effects on the equilibrium values of ϕ^i and θ^i.

Part (b): For the case of extreme asymmetry, we evaluate the expressions in (A.24) in the limit as $Y^i \to 0$ with $Y^j \in (0,\infty)$. First, using the calculations shown in Appendix B, one can find the appropriate limits of the a-coefficients in (A.11):[11]

$$\lim_{Y^i\to 0} a_{11} = 1 + \frac{1}{m}, \;\; \lim_{Y^i\to 0} a_{12} = -1, \;\; \lim_{Y^i\to 0} a_{21} = 1 + \frac{1}{m}, \;\text{and}\; \lim_{Y^i\to 0} a_{22} = 0, \tag{A.25}$$

and

$$\lim_{Y^i\to 0} \mathcal{D} = \lim_{Y^i\to 0} \left[a_{11}a_{22} - a_{12}a_{21}\right] = 1 + \frac{1}{m}. \tag{A.26}$$

Similarly, using (A.17), one can establish the following for the b-coefficients:[12]

$$\lim_{Y^i\to 0} b_{12} = \lim_{Y^i\to 0} b_{22} = 1, \;\; \lim_{Y^i\to 0} b_{13} = -\frac{q}{1-q}, \;\text{and}\; \lim_{Y^i\to 0} b_{23} = \frac{q\delta(m-1)}{1+\delta+q\delta(m-1)}$$

With these results and (A.25) and (A.26), one can take the limit of each expression in (A.24) to find

$$\lim_{Y^i\to 0} \widehat{\theta}^{i*} = -\frac{q}{1-q}\left[\frac{1+\delta m}{1+\delta-q\delta\,(1-m)}\right]\widehat{q} \tag{A.27a}$$

$$\lim_{Y^i\to 0} \widehat{\phi}^{i*} = \frac{m}{1+m}\left[-\widehat{\kappa} + \frac{q\delta\,(1-m)}{1+\delta-q\delta\,(1-m)}\widehat{q}\right], \tag{A.27b}$$

[11] The derivation, based on equations (B.6) and (B.7), is shown in (B.8).

[12] For details, see the derivation of (B.10) using (B.9), as presented in Appendix B.

thereby confirming the signs of the limits stated in the proposition. (Note that these findings for $\widehat{\phi}^{i*}$ are consistent with the results stated in Proposition 1(b) in the case that $0 < Y^i < Y^j < \infty$.) $\quad ||$

Proof of Proposition 2. In this proof, we study the response of equilibrium arming and saving to a change in country i's income Y^i. In doing so, we keep Y^j fixed, so that any changes in relative income y^i will be solely due to changes in Y^i. Using (11) with (A.1d) and (A.8c), we rewrite the expressions for G^i and Z^i shown in (10):

$$G^i = \kappa m q \left(\frac{\phi^i \phi^j}{\Omega^i}\right) \delta Y^i \tag{A.28a}$$

$$Z^i = \left(\frac{\Lambda^i}{\Omega^i}\right) \delta Y^i \tag{A.28b}$$

for $i = 1, 2$, where Ω^i was defined in (A.8c) and Λ^i was defined in (A.1d). Logarithmic differentiation of the above quantities gives

$$\widehat{G}^i = \widehat{Y}^i - \left(\overset{(+)}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}}\right)\widehat{\theta}^i - \left(\frac{\phi^i}{\phi^j} - 1 + \overset{(+)}{\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}\right)\widehat{\phi}^i \tag{A.29a}$$

$$\widehat{Z}^i = \widehat{Y}^i + \left(\overset{(+)}{\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i}} - \overset{(+)}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}}\right)\widehat{\theta}^i + \left(\overset{(+)}{\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i}} - \overset{(+)}{\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}\right)\widehat{\phi}^i \tag{A.29b}$$

and

$$\widehat{G}^j = -\left(\overset{(-)}{\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}}\right)\widehat{\theta}^i + \left(1 - \frac{\phi^i}{\phi^j} - \overset{(-)}{\frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}}\right)\widehat{\phi}^i \tag{A.30a}$$

$$\widehat{Z}^j = \left(\overset{(-)}{\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j}} - \overset{(-)}{\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}}\right)\widehat{\theta}^i + \left(\overset{(-)}{\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j}} - \overset{(-)}{\frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}}\right)\widehat{\phi}^i \tag{A.30b}$$

To prove each part, we must distinguish between two cases: (i) $Y^i \leq Y^j$ and (ii) $Y^i > Y^j$. Furthermore, we start by analyzing the response of country j's choices to an increase in Y^i (given Y^j) to prove part (b). We, then, return to prove part (a).[13]

Part (b): The following fill out the details for the coefficients on $\widehat{\theta}^i$ and $\widehat{\phi}^i$ in the expressions

[13] More details regarding the individual terms in the coefficients on $\widehat{\phi}^i$ and $\widehat{\theta}^i$ in the expressions above in (A.29) and (A.30), including their signs, can be found in (B.2) and (B.3) presented in Appendix B.

for $\widehat{G}^j$ and $\widehat{Z}^j$ in (A.30):

$$
\begin{aligned}
\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j} &= -\frac{\theta^i\left[(1-\kappa)(1+\delta)+\kappa q\delta\phi^j\right]}{\Omega^j} \\
1-\frac{\phi^i}{\phi^j}-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j} &= \frac{(1-\kappa)(1+\delta)\,\theta^j\left(\phi^j-\phi^i\right)+\kappa\left[1+\delta\left(1-q+q\theta^j\right)\right]\left(\phi^j\right)^2}{\phi^j\Omega^j} \\
\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}-\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j} &= -\frac{\kappa q\theta^i\phi^j\left[(1-\kappa)\left(1+m\delta\phi^i\right)+\kappa\phi^j\left(1+mq\delta\phi^i\right)\right]}{\Lambda^j\Omega^j}<0 \\
\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j} &= -\frac{\kappa q\phi^i\left[(1-\kappa)\,\theta^j\left[m\delta\left(\phi^j-\phi^i\right)-\theta^i\right]+\kappa m\delta\left(1-q+q\theta^j\right)\left(\phi^j\right)^2\right]}{\Lambda^j\Omega^j}.
\end{aligned}
$$

With the help of the above expressions, one can verify from (A.30) that

$$
\operatorname{sign}\left\{\widehat{G}^j\right\} = \operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i+\Xi^j_G\right\} \tag{A.31a}
$$

$$
\operatorname{sign}\left\{\widehat{Z}^j\right\} = -\operatorname{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i+\Xi^j_Z\right\} \tag{A.31b}
$$

where

$$
\begin{aligned}
\Xi^j_G &\equiv \left(1-\frac{\phi^i}{\phi^j}-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\Big/\left(\frac{\theta^i\Omega^j_{\theta}}{\Omega^j}\right) \\
&= \frac{(1-\kappa)(1+\delta)\,\theta^j\left(\phi^j-\phi^i\right)+\kappa\left[1+\delta\left(1-q+q\theta^j\right)\right]\left(\phi^j\right)^2}{\theta^i\phi^j\left[(1-\kappa)(1+\delta)+\kappa q\delta\phi^j\right]}
\end{aligned} \tag{A.32a}
$$

and

$$
\begin{aligned}
\Xi^j_Z &\equiv \left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\Big/\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}-\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) \\
&= \left(\frac{\phi^i}{\theta^i\phi^j}\right)\left\{\frac{(1-\kappa)\,\theta^j\left[-\theta^i+m\delta\left(\phi^j-\phi^i\right)\right]+\kappa m\delta\left(1-q+q\theta^j\right)\left(\phi^j\right)^2}{(1-\kappa)\left(1+m\delta\phi^i\right)+\kappa\phi^j\left(1+mq\delta\phi^i\right)}\right\}.
\end{aligned} \tag{A.32b}
$$

(i) $Y^i/Y^j \leq 1$: In this case, Proposition 1(a) implies $\phi^j-\phi^i \geq 0$, which from (A.32a) implies $\Xi^j_G>0$. By Lemma A.1(a), we also have $\widehat{\theta}^i/\widehat{\phi}^i > 1+\frac{1}{m}$ (for movements along the S^i curve) as $\widehat{Y}^i>0$. It thus follows from (A.31a) that $\widehat{G}^j>0$ for increases in Y^i. In addition, because $\widehat{\phi}^i>0$ as $\widehat{Y}^i>0$ from (A.15a), it must also be the case that $\widehat{G}^j<\widehat{G}^i$.

Turning to $\widehat{Z}^j$, the sign of Ξ^j_Z in (A.32b) can be negative. Nonetheless, because $\phi^j\geq\phi^i$ in this case, we can infer from inspection of the components inside the curly brackets in (A.32b) that

$$
\Xi^j_Z > -\left(\frac{\phi^i\theta^j}{\phi^j}\right)\left\{\frac{(1-\kappa)}{(1-\kappa)\left(1+m\delta\phi^i\right)+\kappa\phi^j\left(1+mq\delta\phi^i\right)}\right\} > -\left(\frac{\phi^i}{\phi^j}\right)\theta^j > -1.
$$

Since $\widehat{\theta^i}/\widehat{\phi^i} > 1 + \frac{1}{m}$ by part (a) of Lemma A.1, we must have $\widehat{\theta^i}/\widehat{\phi^i} + \Xi_Z^j > 0$; therefore, from (A.31b), $\widehat{Z^j} < 0$ holds.

(ii) $Y^i/Y^j > 1$: Keep in mind that, in this case from Proposition 1(a), we have $\phi^j - \phi^i < 0$ and $\frac{\theta^i\phi^j}{\theta^j\phi^i} > 1$. Turning back to the sign of $\widehat{G^j}$, we can adjust the RHS of (A.32a) to show that

$$\Xi_G^j > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right)\left\{\frac{(1-\kappa)(1+\delta)}{(1-\kappa)(1+\delta)+\kappa q\delta\phi^j}\right\} > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right).$$

Multiplying $\widehat{\theta^i}/\widehat{\phi^i} + \Xi_G^j$ inside the curly brackets in (A.31a) by $\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)$ and using the above inequality gives

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta^i}/\widehat{\phi^i}\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_G^j > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta^i}/\widehat{\phi^i}\right) - 1 > 0.$$

The last inequality in the above expression follows from part (b) of Lemma A.1 for $\lambda = \frac{\theta^i\phi^j}{\theta^j\phi^i} > 1$. Thus, $\widehat{G^j} > 0$ holds, so an increase in Y^i increases G^j for all $Y^i/Y^j > 1$.

To identify the sign of $\widehat{Z^j}$ for $Y^i/Y^j > 1$, note from (A.32b) that

$$\Xi_Z^j > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right)\left\{\frac{(1-\kappa)\left[\theta^i + m\delta\phi^i\right]}{(1-\kappa)(1+m\delta\phi^i)+\kappa\phi^j(1+mq\delta\phi^i)}\right\} > -\left(\frac{\phi^i\theta^j}{\theta^i\phi^j}\right).$$

Following our strategy above for $\widehat{G^j}$, we multiply $\widehat{\theta^i}/\widehat{\phi^i} + \Xi_Z^j$ in the curly brackets in (A.31b) by $\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)$ to obtain

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta^i}/\widehat{\phi^i}\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^j > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta^i}/\widehat{\phi^i}\right) - 1 > 0.$$

Again, the last inequality follows from part (b) in Lemma A.1. Thus, $\widehat{Z^j} < 0$ in this case, thereby completing the proof of part (b): $\widehat{Z^j} < 0 < \widehat{G^j}$ for all Y^i/Y^j.

Part (a): It is straightforward to establish $\widehat{G^i} > 0$ for all Y^i/Y^j. Specifically, recall from the definition of ϕ^i in (2) that $G^i/G^j (= E^i) = (\phi^i/\phi^j)^{\frac{1}{m}}$, which implies

$$\widehat{G^i} - \widehat{G^j} = \frac{1}{m}\left(\widehat{\phi^i} - \widehat{\phi^j}\right) = \frac{1}{m}\left(\widehat{\phi^i} + \frac{\phi^i}{\phi^j}\widehat{\phi^i}\right) = \frac{1}{m\phi^j}\widehat{\phi^i}.$$

Since $\widehat{\phi^i} > 0$ and (as already established in part (b)) $\widehat{G^j} > 0$ for $\widehat{Y^i} > 0$, $\widehat{G^i} = \frac{1}{m\phi^j}\widehat{\phi^i} + \widehat{G^j} > 0$.

Having shown $\widehat{G^i} > 0$, we now go on one step further to show that $\widehat{G^i} < \widehat{Y^i}$. With the help of (A.29a) we can subtract $\widehat{G}^i$ from $\widehat{Y}^i$ to find

$$\widehat{Y}^i - \widehat{G}^i = \left(\overset{(+)}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}}\right)\widehat{\theta}^i + \left(\frac{\phi^i}{\phi^j} - 1 + \overset{(+)}{\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}\right)\widehat{\phi}^i.$$

Inspection of the RHS of the above equation reveals that it is always positive for $\phi^i/\phi^j > 1$ which, of course, is true when $Y^i/Y^j > 1$. However, since $\phi^i/\phi^j < 1$ for $Y^i/Y^j < 1$, the sign of the RHS is not immediately obvious in this case. Nonetheless, this sign is the same as

$$\text{sign}\left\{\widehat{Y}^i - \widehat{G}^i\right\} = \text{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi^i_G\right\},$$

where

$$\begin{aligned}
\Xi^i_G &\equiv \frac{\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}} = \frac{\theta^i (1-\kappa)(1+\delta)\left(\phi^i - \phi^j\right) + \kappa\left[1 + \delta\left(1 - q + q\theta^i\right)\right]}{\theta^i \phi^j\left[(1-\kappa)(1+\delta) + \kappa\delta q \phi^i\right]} \Rightarrow \\
\Xi^i_G &> -\frac{(1-\kappa)(1+\delta)}{(1-\kappa)(1+\delta) + \kappa\delta q \phi^i} > -1.
\end{aligned}$$

Since Lemma A.1(b) shows $\widehat{\theta}^i/\widehat{\phi}^i > 1 + \frac{1}{m}$ for $\phi^i \in (0, \frac{1}{2}]$ or equivalently by Proposition 1(a) $Y^i/Y^j \leq 1$, the above inequality implies $\widehat{\theta}^i/\widehat{\phi}^i + \Xi^i_G > \frac{1}{m}$. As such, $0 < \widehat{G}^i < \widehat{Y}^i$ holds for all possible Y^i/Y^j.

Turning to $\widehat{Z}^i$, observe from (A.29b), that

$$\widehat{Z}^i - \widehat{Y}^i = \left(\overset{(+)}{\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i}} - \overset{(+)}{\frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}}\right)\widehat{\theta}^i + \left(\overset{(+)}{\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i}} - \overset{(+)}{\frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}\right)\widehat{\phi}^i, \tag{A.33}$$

where

$$\begin{aligned}
\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} &= \frac{\kappa q \phi^i \theta^i}{\Lambda^i \Omega^i}\left[(1-\kappa)\left(1 + m\delta\phi^j\right) + \kappa\phi^i\left(1 + mq\delta\phi^j\right)\right] > 0 \\
\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} &= \frac{\kappa q \phi^i \theta^i}{\Lambda^i \Omega^i}\left[(1-\kappa)\left[m\delta\left(\phi^i - \phi^j\right) - \theta^j\right] + \kappa m\delta\left(1 - q + q\theta^i\right)\phi^i\left(\phi^i/\theta^i\right)\right].
\end{aligned}$$

The expressions directly above along with (A.33) imply that

$$\text{sign}\left\{\widehat{Z}^i - \widehat{Y}^i\right\} = \text{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i + \Xi^i_Z\right\}, \tag{A.34}$$

where

$$\Xi^i_Z \equiv \frac{\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}}{\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}} = \frac{\left[\theta^i (1-\kappa)\left[-\theta^j + m\delta\left(\phi^i - \phi^j\right)\right] + \kappa m\delta\left(1 - q + q\theta^i\right)\left(\phi^i\right)^2\right]}{\theta^i\left[(1-\kappa)\left(1 + m\delta\phi^j\right) + \kappa\phi^i\left(1 + mq\delta\phi^j\right)\right]}.$$

In view of the ambiguity in the sign of Ξ_Z^i, we consider the two cases: (i) $Y^i/Y^j \leq 1$ and (ii) $Y^i/Y^j > 1$.

(i) $Y^i/Y^j \leq 1$: From the definition of Ξ_Z^i and the fact that $\phi^i - \phi^j < 0$ in this case,

$$\Xi_Z^i > -\frac{(1-\kappa)\left(\theta^j + m\delta\phi^j\right)}{(1-\kappa)\left(1+m\delta\phi^j\right) + \kappa\phi^i\left(1+mq\delta\phi^j\right)} > -1.$$

Thus $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i > \widehat{\theta}^i/\widehat{\phi}^i - 1 > \frac{1}{m}$ from part (b) of Lemma A.1. Therefore, from (A.34), $\widehat{Z}^i - \widehat{Y}^i > 0$ for $Y^i/Y^j \leq 1$.

(ii) $Y^i/Y^j > 1$: Since $\phi^i - \phi^j > 0$ ($\Rightarrow \phi^j/\phi^i < 1$) in this case, we have

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^i > -\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left[\frac{(1-\kappa)\,\theta^j}{(1-\kappa)\left(1+m\delta\phi^j\right)+\kappa\phi^i\left(1+mq\delta\phi^j\right)}\right] > -\left(\frac{\phi^j}{\phi^i}\right)\theta^i > -1.$$

Multiplying $\widehat{\theta}^i/\widehat{\phi}^i + \Xi_Z^i$ inside the curly brackets in (A.34) by $\frac{\theta^i\phi^j}{\theta^j\phi^i}$ gives

$$\left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) + \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\Xi_Z^i > \left(\frac{\theta^i\phi^j}{\theta^j\phi^i}\right)\left(\widehat{\theta}^i/\widehat{\phi}^i\right) - 1 > 0,$$

where, again, the last inequality follows from part (b) of Lemma A.1. Thus, (A.34) implies $\widehat{Z}^i - \widehat{Y}^i > 0$ in this case as well.

The above analysis implies $0 < \widehat{G}^i < \widehat{Y}^i < \widehat{Z}^i$, thereby completing the proof to part (a), and thus the proposition. ||

Proposition A.2 *If the assumptions of Proposition 1 are satisfied, then an increase in country i's income (i.e., $\widehat{Y}^i > 0$) affects equilibrium first-period consumption in countries $i \neq j = 1, 2$ as follows:*

(a) $0 < \widehat{C}^{i*}$.

(b) $\lim_{Y^i \to 0} \widehat{C}^{j*} < 0$, $\lim_{Y^i \to Y^j} \widehat{C}^{j*} > 0$ *and* $\lim_{Y^i \to \infty} \widehat{C}^{j*} < 0$.

Proof: Here we follow the strategy adopted in the proof of Proposition 2, to study how current-period consumption for each country responds to a change in Y^i, treating Y^j as fixed. Using (A.28a) and (A.28b) in $C^i = Y^i - G^i - Z^i$ yields

$$C^i = \left(\frac{\kappa\phi^i + (1-\kappa)\,\theta^i}{\Omega^i}\right)Y^i, \tag{A.35}$$

Part (a): Logarithmic differentiation of (A.35) gives

$$\widehat{C}^i = \widehat{Y}^i + \left(\overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} - \overset{(+)}{\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}}\right)\widehat{\theta}^i + \left(\overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i + \kappa\phi^i}} - \overset{(+)}{\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}}\right)\widehat{\phi}^i \tag{A.36}$$

Substituting the values of $\widehat{\phi}^i$ and $\widehat{\theta}^i$ from (A.15) in (A.36) gives after some algebra

$$\begin{aligned}
\widehat{C}^i &= \widehat{Y}^i + \left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right]\frac{a_{11}}{\mathcal{D}}\widehat{Y}^i + \left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right]\frac{(-a_{12})}{\mathcal{D}}\widehat{Y}^i \\
&= \frac{\widehat{Y}^i}{\mathcal{D}}\left\{[a_{11}a_{22}-a_{12}a_{21}] + a_{11}\left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right]\right. \\
&\qquad \left. -a_{12}\left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right]\right\} \\
&= \frac{\widehat{Y}^i}{\mathcal{D}}\left\{a_{11}\left[a_{22} + \frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right] - a_{12}\left[a_{21} + \frac{\kappa\phi^i}{(1-\kappa)\,\theta^i+\kappa\phi^i} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right]\right\}
\end{aligned}$$

The second line was obtained by substituting in for the value of $D = a_{11}a_{22} - a_{12}a_{21}$ (>0), and the expression in the third line by taking a_{11} and a_{12} as common factors. Substituting the expressions for a_{22} and a_{21} from (A.11), the above equation can be written as

$$\widehat{C}^i = \frac{\widehat{Y}^i}{\mathcal{D}}\left\{\overset{(+)}{a_{11}}\left[-\overset{(-)}{\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}} + \overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^i+\kappa\phi^i}}\right] - \overset{(-)}{a_{12}}\left[\overset{(+)}{\frac{\phi^i E^i_{\phi^i}}{E^i}} - \overset{(-)}{\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}} + \overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^i+\kappa\phi^i}}\right]\right\},$$

which is positive. Thus, an increase in Y^i raises country i's period $t=1$ consumption.

Part (b): Logarithmic differentiation of (A.35) for country j keeping Y^j fixed yields

$$\widehat{C}^j = -\left(\overset{(+)}{\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^j+\kappa\phi^j}} + \overset{(-)}{\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}}\right)\widehat{\theta}^i - \left(\overset{(+)}{\frac{\kappa\phi^i}{(1-\kappa)\,\theta^j+\kappa\phi^j}} + \overset{(-)}{\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}}\right)\widehat{\phi}^i. \tag{A.37}$$

By rearranging (A.37) one can show that

$$\text{sign}\left\{\widehat{C}^j\right\} = \text{sign}\left\{\widehat{\theta}^i/\widehat{\phi}^i - \left(\frac{\theta^j/\phi^j}{\theta^i/\phi^i}\right)\Xi^j_C\right\} \tag{A.38a}$$

where

$$\Xi^j_C \;\equiv\; \frac{(1-\kappa)\left(\theta^i + m\phi^j - m\phi^i\right) + \kappa m\phi^j\left(\phi^j/\theta^j\right)}{(1-\kappa)\left(1-m\phi^i\right)+\kappa\phi^j}. \tag{A.38b}$$

Let us first consider the case where countries have identical income levels $Y^i = Y^j$, which implies $\phi^i = \phi^j = \theta^i = \theta^j = \frac{1}{2}$. It is easy to verify that, in this case, we have $\frac{\theta^j/\phi^j}{\theta^i/\phi^i} = 1$ and $\Xi^j_C = \frac{1-\kappa+\kappa m}{1-\kappa+\kappa m+1-m} \leq 1$. But from Lemma A.1(a), we know that $\lim_{Y^i\to Y^j}(\widehat{\theta}^i/\widehat{\phi}^i) > 1+\frac{1}{m}$. It thus becomes clear from (A.38a) that $\lim_{Y^i\to Y^j}\left(\widehat{C}^j\right) > 0$.

We now consider the extremes cases of (i) $Y^i \to 0$ and (ii) $Y^i \to \infty$, for any given finite $Y^j > 0$.

(i) $Y^i \to 0$: From Proposition 1(a), we have $\phi^i \to 0$, $\theta^i \to 0$ and $\theta^i/\phi^i \to 0$, while $\phi^j \to 1$, $\theta^j \to 0$ and $\theta^j/\phi^j \to 1$ in this case. Thus, $\lim_{Y^i \to 0}\left(\frac{\theta^j/\phi^j}{\theta^i/\phi^i}\right) = \infty$ and $\lim_{Y^i \to 0} \Xi_C^j = m$. Since by Lemma A.1(a) $\lim_{Y^i \to 0}(\widehat{\theta}^i/\widehat{\phi}^i) = 1 + \frac{1}{m}$, (A.38a) implies $\widehat{C}^j < 0$ for Y^i sufficiently close to 0.

(ii) $Y^i \to \infty$: Multiplying the expression inside (A.38a) by $\frac{\theta^i \phi^j}{\theta^j \phi^i}$ does not change the sign of that expression, but allows us to rewrite it as

$$\text{sign}\left\{\lim_{Y^i \to \infty} \widehat{C}^j\right\} = \text{sign}\left\{\lim_{Y^i \to \infty}\left[\frac{\theta^i \phi^j}{\theta^j \phi^i}\left(\widehat{\theta}^i/\widehat{\phi}^i\right)\right] - \lim_{Y^i \to \infty} \Xi_C^j\right\}.$$

Let us now study the two components inside the curly brackets, starting with the first. Multiplying both sides of (A.7) by $(\theta^i \phi^j)/(\theta^j \phi^i)$ yields

$$\left(\frac{\theta^i \phi^j}{\theta^j \phi^i}\right) \frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} = \frac{\left(\frac{\theta^i \phi^j}{\phi^i}\right) \frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \frac{\theta^i \kappa\left(1-q+q\theta^j\right)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)} + \frac{1}{m}\left(\frac{\theta^i}{\phi^i}\right)}{\frac{\theta^j \kappa(1-q)}{(1-\kappa)(\theta^i/\phi^i)+\kappa(1-q+q\theta^i)} + \frac{\theta^i \kappa(1-q)}{(1-\kappa)(\theta^j/\phi^j)+\kappa(1-q+q\theta^j)}}.$$

Next, we take the limit of the expression above as $Y^i \to \infty$. To do so, recall that $\lim_{Y^i \to \infty} \theta^i = \lim_{Y^i \to \infty} \phi^i = 1$, whereas $\lim_{Y^i \to \infty} \theta^j = \lim_{Y^i \to \infty} \phi^j = 0$ and $\lim_{Y^i \to \infty} \theta^j/\phi^j = 0$. One can then verify the following:

$$\begin{aligned}
\lim_{Y^i \to \infty}\left[\left(\frac{\theta^i \phi^j}{\phi^i}\right) \frac{\kappa\left(1-q+q\theta^i\right)}{(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)}\right] &= 0 \\
\lim_{Y^i \to \infty}\left[\frac{\theta^i \kappa\left(1-q+q\theta^j\right)}{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)}\right] &= 1 \\
\lim_{Y^i \to \infty}\left[\frac{1}{m}\left(\frac{\theta^i}{\phi^i}\right)\right] &= \frac{1}{m} \\
\lim_{Y^i \to \infty}\left[\frac{\theta^j \kappa(1-q)}{(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)}\right] &= 0 \\
\lim_{Y^i \to \infty}\left[\frac{\theta^i \kappa(1-q)}{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)}\right] &= 1
\end{aligned}$$

Using the expressions above gives

$$\lim_{Y^i \to \infty}\left(\frac{\theta^i \phi^j}{\theta^j \phi^i}\right) \frac{\widehat{\theta}^i}{\widehat{\phi}^i}\bigg|_{S^i=0} = 1 + \frac{1}{m}.$$

Thus, to complete the proof, it suffices to show that $\lim_{Y^i \to \infty} \Xi_C^j > 1 + \frac{1}{m}$. Take that limit to find

$$\lim_{Y^i \to \infty} \Xi_C^j = 1 + \frac{\kappa m}{(1-\kappa)(1-m)} \lim_{Y^i \to \infty}\left[\frac{\phi^j}{\theta^j/\phi^j}\right]. \tag{A.39}$$

Since any change in relative incomes always moves us along the S^i-curve, we use the definition of this curve to establish the following:

$$\frac{\phi^j}{\theta^j/\phi^j} = \left(\phi^j\right)^{1-\frac{1}{m}} \left\{ \frac{\left(\phi^i\right)^{\frac{1}{m}} \left[\left(1-\kappa\right)\left(\theta^i/\phi^i\right) + \kappa\left(1-q+q\theta^i\right)\right]}{\left(\theta^i/\phi^i\right)\left[\left(1-\kappa\right)\left(\theta^j/\phi^j\right) + \kappa\left(1-q+q\theta^j\right)\right]} \right\}.$$

Taking limits gives

$$\lim_{Y^i\to\infty} \left[\frac{\phi^j}{\theta^j/\phi^j}\right] = \lim_{Y^i\to\infty} \left[\left(\phi^j\right)^{1-\frac{1}{m}}\right] \left\{\frac{1}{\kappa\left(1-q\right)}\right\}.$$

Now substitute the above expression in (A.39):

$$\lim_{Y^i\to\infty} \Xi_C^j = 1 + \frac{m}{\left(1-\kappa\right)\left(1-m\right)\left(1-q\right)} \lim_{Y^i\to\infty} \left[\left(\phi^j\right)^{1-\frac{1}{m}}\right].$$

Since $\lim_{Y^i\to\infty} \phi^j = 0$ and $m \in (0,1]$, we have $\lim_{Y^i\to\infty} \Xi_C^j > 1 + \frac{1}{m}$ for all parameter values. $||$

Proposition A.3 *Assuming condition A satisfied, changes in the degree of property security $\kappa \in (0,1]$ and the probability of a future war $q \in (0,1)$ influence each countries' allocation of income to arming, savings and first-period consumption as follows in the two benchmark cases of perfect symmetry and extreme asymmetry in country sizes:*

(a) *If $Y^i = Y^j \in (0,\infty)$, then*
  (i) $dG^{i*}/d\kappa = dG^{j*}/d\kappa > 0$, $dZ^{i*}/d\kappa = dZ^{j*}/d\kappa < 0$, *and* $dC^{i*}/d\kappa = dC^{j*}/d\kappa > 0$.
  (ii) $dG^{i*}/dq = dG^{j*}/dq > 0$, $dZ^{i*}/dq = dZ^{j*}/dq < 0$, *and* $dC^{i*}/dq = dC^{j*}/dq > 0$.

(b) *For given $Y^j \in (0,\infty)$, we have*
  (i) $\lim_{Y^i\to 0} \widehat{G}^{i*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{Z}^{i*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{C}^{i*}/\widehat{\kappa} = 0$, $\lim_{Y^i\to 0} \widehat{G}^{i*}/\widehat{q} > 0$, $\lim_{Y^i\to 0} \widehat{Z}^{i*}/\widehat{q} < 0$, *and* $\lim_{Y^i\to 0} \widehat{C}^{i*}/\widehat{q} > 0$.
  (ii) $\lim_{Y^i\to 0} \widehat{G}^{j*}/\widehat{\kappa} > 0$, $\lim_{Y^i\to 0} \widehat{G}^{j*}/\widehat{q} > 0$, *while* $\lim_{Y^i\to 0} \widehat{Z}^{j*}/\widehat{\kappa} = \lim_{Y^i\to 0} \widehat{Z}^{j*}/\widehat{q} = \lim_{Y^i\to 0} \widehat{C}^{j*}/\widehat{\kappa} = \widehat{C}^{j*}/\widehat{q} = 0$.

Proof: The effects of changes in κ and q on first-period allocations consist of both direct and indirect effects through the implied changes in θ^{i*} and ϕ^{i*}, and they can be found by appropriately differentiating (A.28) and (A.35) to find

$$\begin{aligned} \widehat{G}^{i*} &= \left(1 - \frac{\kappa\Omega^i_\kappa}{\Omega^i}\right)\widehat{\kappa} + \left(1 - \frac{q\Omega^i_q}{\Omega^i}\right)\widehat{q} \\ &\quad - \left(\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right)\widehat{\theta}^{i*} - \left(\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right)\widehat{\phi}^{i*} \qquad \text{(A.40a)} \\ \widehat{G}^{j*} &= \left(1 - \frac{\kappa\Omega^j_\kappa}{\Omega^j}\right)\widehat{\kappa} + \left(1 - \frac{q\Omega^j_q}{\Omega^j}\right)\widehat{q} \end{aligned}$$

$$-\left(\frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}\right)\widehat{\theta}^{i*} - \left(\frac{\phi^i}{\phi^j} - 1 + \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}\right)\widehat{\phi}^{i*}, \tag{A.40b}$$

$$\widehat{Z}^{i*} = \left(\frac{\kappa \Lambda^i_\kappa}{\Lambda^i} - \frac{\kappa \Omega^i_\kappa}{\Omega^i}\right)\widehat{\kappa} + \left(\frac{q\Lambda^i_q}{\Lambda^i} - \frac{q\Omega^i_q}{\Omega^i}\right)\widehat{q}$$
$$+ \left(\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i}\right)\widehat{\theta}^{i*} + \left(\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i}\right)\widehat{\phi}^{i*} \tag{A.41a}$$

$$\widehat{Z}^{j*} = \left(\frac{\kappa \Lambda^j_\kappa}{\Lambda^j} - \frac{\kappa \Omega^j_\kappa}{\Omega^j}\right)\widehat{\kappa} + \left(\frac{q\Lambda^j_q}{\Lambda^j} - \frac{q\Omega^j_q}{\Omega^j}\right)\widehat{q}$$
$$+ \left(\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j}\right)\widehat{\theta}^{i*} + \left(\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j}\right)\widehat{\phi}^{i*} \tag{A.41b}$$

and

$$\widehat{C}^{i*} = \left[\frac{\kappa\left(1-\theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\kappa\Omega^i_\kappa}{\Omega^i}\right]\widehat{\kappa} - \frac{q\Omega^i_q}{\Omega^i}\widehat{q}$$
$$+ \left[\frac{(1-\kappa)\left(\theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right]\widehat{\theta}^{i*} + \left[\frac{\kappa}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)} - \frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right]\widehat{\phi}^{i*} \tag{A.42a}$$

$$\widehat{C}^{j*} = \left[\frac{\kappa\left(1-\theta^j/\phi^j\right)}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)} - \frac{\kappa\Omega^j_\kappa}{\Omega^j}\right]\widehat{\kappa} - \frac{q\Omega^j_q}{\Omega^j}\widehat{q}$$
$$- \left[\frac{(1-\kappa)\,\theta^i}{(1-\kappa)\,\theta^j + \kappa\phi^j} + \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right]\widehat{\theta}^{i*} - \left[\frac{\kappa\phi^i}{(1-\kappa)\,\theta^j + \kappa\phi^j} + \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right]\widehat{\phi}^{i*}. \tag{A.42b}$$

Part (a): As established in Proposition A.1(a), when $Y^i = Y^j > 0$, $\phi^{i*} = \theta^{i*} = \frac{1}{2}$ regardless of the values of $\kappa \in (0,1]$ and $q \in (0,1)$. Therefore, only the direct effects of changes in these two parameters matter for first-period allocations of income. To evaluate those effects, we differentiate Λ^r and Ω^r (for $r = i,j$) with respect to κ and q,[14] and then evaluate those expressions at $\phi^i = \theta^i = \frac{1}{2}$:

$$\left.\frac{\kappa\Lambda^i_\kappa}{\Lambda^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = \left.\frac{q\Lambda^i_q}{\Lambda^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = -\frac{\frac{1}{2}\kappa q}{1-\frac{1}{2}\kappa q} < 0$$
$$\left.\frac{\kappa\Omega^i_\kappa}{\Omega^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = \left.\frac{q\Omega^i_q}{\Omega^i}\right|_{\phi^i=\theta^i=\frac{1}{2}} = -\frac{\frac{1}{2}\kappa q\delta\,(1-m)}{1+\delta-\frac{1}{2}\kappa q\delta\,(1-m)} < 0.$$

By substituting these expressions into (A.40)–(A.42) evaluated at $\phi^i = \theta^i = \frac{1}{2}$, one can

[14] The resulting partial derivatives are shown in (A.17). Also see (B.4) presented in Appendix B.

then confirm the following:

$$\widehat{G}^{i*} = \widehat{G}^{j*} = \frac{1+\delta}{1+\delta-\frac{1}{2}\kappa q\delta\left(1-m\right)}\left(\widehat{\kappa}+\widehat{q}\right) > 0 \tag{A.43a}$$

$$\widehat{Z}^{i*} = \widehat{Z}^{j*} = -\frac{\frac{1}{2}\kappa q\left(1+m\delta\right)}{\left(1-\frac{1}{2}\kappa q\right)\left[1+\delta-\frac{1}{2}\kappa q\delta\left(1-m\right)\right]}\left(\widehat{\kappa}+\widehat{q}\right) < 0 \tag{A.43b}$$

$$\widehat{C}^{i*} = \widehat{C}^{j*} = \frac{\frac{1}{2}\kappa q\delta\left(1-m\right)}{1+\delta-\frac{1}{2}\kappa q\delta\left(1-m\right)}\left(\widehat{\kappa}+\widehat{q}\right) > 0, \tag{A.43c}$$

which completes the proof of part (a).

Part (b): For country i, combining the limit results shown in (B.6), (B.7) and (B.9) presented in Appendix B and (A.40)–(A.42) shown above, one can verify the following:

$$\begin{aligned}
\lim_{Y^i\to 0}\widehat{G}^{i*} &= (1-1)\times\widehat{\kappa}+\left(1+\frac{q\delta\left(1-m\right)}{1+\delta-q\delta\left(1-m\right)}\right)\times\widehat{q}-0\times\widehat{\theta}^{i}-(0-1+1)\times\widehat{\phi}^{i}\\
&= \frac{1+\delta}{1+\delta-q\delta\left(1-m\right)}\widehat{q} \qquad\qquad (A.44a)\\
\lim_{Y^i\to 0}\widehat{Z}^{i*} &= (1-1)\times\widehat{\kappa}-\left[\frac{q}{1-q}+\frac{q\delta\left(1-m\right)}{1+\delta-q\delta\left(1-m\right)}\right]\times\widehat{q}\\
&\quad +(0-0)\times\widehat{\theta}^{i*}+(1-1)\times\widehat{\phi}^{i*}\\
&= -\frac{q\left(1+\delta m\right)}{\left(1-q\right)\left[1+\delta-q\delta\left(1-m\right)\right]}\widehat{q} \qquad\qquad (A.44b)\\
\lim_{Y^i\to 0}\widehat{C}^{i*} &= [1-1]\times\widehat{\kappa}+\left[\frac{q\delta\left(1-m\right)}{1+\delta-q\delta\left(1-m\right)}\right]\times\widehat{q}+[0-0]\times\widehat{\theta}^{i*}+[1-1]\times\widehat{\phi}^{i*}\\
&= \frac{q\delta\left(1-m\right)}{1+\delta-q\delta\left(1-m\right)}\widehat{q}. \qquad\qquad (A.44c)
\end{aligned}$$

These results confirm the findings stated in part (b-i). Similarly, one can find the following for country j:

$$\begin{aligned}
\lim_{Y^i\to 0}\widehat{G}^{j*} &= \widehat{\kappa}+\widehat{q}+\frac{m}{1+m}\left[-\widehat{\kappa}+\frac{q\delta\left(1-m\right)}{1+\delta-q\delta\left(1-m\right)}\widehat{q}\right]\\
&= \frac{1}{1+m}\widehat{\kappa}+\left[1+\frac{mq\delta\left(1-m\right)}{\left(1+m\right)\left[1+\delta-q\delta\left(1-m\right)\right]}\right]\widehat{q} \qquad\qquad (A.45a)\\
\lim_{Y^i\to 0}\widehat{Z}^{j*} &= (0-0)\times\widehat{\kappa}+(0-0)\times\widehat{q}+(0-0)\times\widehat{\theta}^{i*}+(0-0)\times\widehat{\phi}^{i*}=0 \qquad\qquad (A.45b)\\
\lim_{Y^i\to 0}\widehat{C}^{j*} &= [0-0]\times\widehat{\kappa}-0\times\widehat{q}-[0+0]\times\widehat{\theta}^{i*}-[0+0]\times\widehat{\phi}^{i*}=0, \qquad\qquad (A.45c)
\end{aligned}$$

thereby completing the proof. ||

Proof of Proposition 3. *Part (a):* To identify the effect of a change in Y^i on U^i, we differentiate U^i and invoke the envelope theorem to obtain

$$dU^{i*} = \Xi_U^i \left[\widehat{Y}^i - \left(\frac{G^i}{Y^i}\right)\widehat{G}^{j*} + \left(\frac{G^i}{Y^i}\right)\left(\frac{\theta^j}{m\phi^j}\right)\widehat{Z}^{j*}\right]$$

where

$$\Xi_U^i \equiv \frac{\Omega^i}{(1-\kappa)\,\theta^i + \kappa\phi^i} > 0.$$

Using the expression for G^i from (A.28a), the expressions for $\widehat{G}^{j*}$ and $\widehat{Z}^{j*}$ from (A.29a) and (A.29b), respectively, and the expressions for $\widehat{\theta}^i$ and $\widehat{\phi}^i$ from (A.15) gives

$$\begin{aligned}
\frac{dU^{i*}}{dY^i} &= \frac{\Xi_U^i}{Y^i}\left\{1 - \left(\frac{G^i}{Y^i}\right)\left[-\left(\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right)\frac{\widehat{\theta}^i}{\widehat{Y}^i} + \left(1 - \frac{\phi^i}{\phi^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\frac{\widehat{\phi}^i}{\widehat{Y}^i}\right]\right. \\
&\quad \left. + \left(\frac{G^i}{Y^i}\right)\left(\frac{\theta^j}{m\phi^j}\right)\left[\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right)\frac{\widehat{\theta}^i}{\widehat{Y}^i} + \left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\frac{\widehat{\phi}^i}{\widehat{Y}^i}\right]\right\} \\
&= \frac{\Xi_U^i}{Y^i}\left\{1 - \frac{\kappa m q\delta\phi^i\phi^j}{\Omega^i}\left[-\left(\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right)\left(\frac{a_{11}}{\mathcal{D}}\right) + \left(1 - \frac{\phi^i}{\phi^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\left(\frac{-a_{12}}{\mathcal{D}}\right)\right]\right. \\
&\quad \left. + \frac{\kappa q\delta\phi^i\theta^j}{\Omega^i}\left[\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right)\left(\frac{a_{11}}{\mathcal{D}}\right) + \left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right)\left(\frac{-a_{12}}{\mathcal{D}}\right)\right]\right\}.
\end{aligned}$$

Multiply the second equation by $\Omega^i\mathcal{D} > 0$, while recalling that $\mathcal{D} \equiv a_{11}a_{22} - a_{12}a_{21}$. Then, one can rearrange terms by pulling a_{11} (> 0) and $-a_{12}$ (> 0) as common factors to establish

$$\text{sign}\left\{\frac{dU^{i*}}{dY^i}\right\} = \text{sign}\left\{\Gamma_1 a_{11} + \Gamma_2(-a_{12})\right\}$$

where

$$\begin{aligned}
\Gamma_1 &\equiv a_{22}\Omega^i + \kappa m q\delta\phi^i\phi^j\left(\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) + \kappa q\delta\phi^i\theta^j\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j} - \frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) \\
\Gamma_2 &\equiv a_{21}\Omega^i + \kappa m q\delta\phi^i\phi^j\left(-1 + \frac{\phi^i}{\phi^j} + \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right) + \kappa q\delta\phi^i\theta^j\left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j} - \frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right).
\end{aligned}$$

Since $a_{11} > 0$ and $a_{12} < 0$, we need only to demonstrate that $\Gamma_1 > 0$ and $\Gamma_2 > 0$ hold to establish this part of the proposition.

To proceed, we substitute the values of $a_{22} > 0$ and $a_{21} > 0$ from (A.11) and the definition of Ω^i from (A.8c) into the expressions immediately above. Upon simplifying, we can rewrite Γ_1 and Γ_2 as follows:

$$\Gamma_1 = (1+\delta)\left[(1-\kappa)\,\theta^i + \kappa\phi^i\right]\left(-\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) + M_1$$

$$\Gamma_2 = (1+\delta)\left[(1-\kappa)\theta^i + \kappa\phi^i\right]\left(-\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right) + M_2$$

where

$$\begin{aligned} M_1 &\equiv \theta^i\Omega^i_{\theta^i} - \kappa q\delta\phi^i\theta^j\left(-\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}\right) \\ M_2 &\equiv \kappa\phi^i\left[1+\delta\left(1-q+q\theta^i\right)\right] + \frac{\Omega^i}{m\phi^j} - \kappa q\delta\phi^i\theta^j\left(-\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}\right) \end{aligned}$$

Since $\Omega^j_{\theta^i} < 0$ and $\Omega^j_{\phi^i} < 0$, the first term in Γ_1 and that in Γ_2 are positive. Hence, to complete the proof, it suffices to show that $M_1 > 0$ and $M_2 > 0$. Expanding the terms in M_1 allows us to rewrite it as

$$\begin{aligned} M_1 &\equiv \theta^i\left\{(1-\kappa)(1+\delta) + \kappa q\delta\phi^i - \kappa q\delta\phi^i\left[\frac{(1-\kappa)\theta^j + \kappa q\theta^j\phi^j}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j}\right]\right\} \\ &= \theta^i\left\{(1-\kappa)(1+\delta) + \kappa q\delta\phi^i\left[\frac{\kappa(1-q)\phi^j}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j}\right]\right\} > 0. \end{aligned}$$

Hence, $\Gamma_1 > 0$. Turning to M_2, its first term is positive, and the algebraic sum of its last two terms can be written as

$$\begin{aligned} &\frac{(1-\kappa)(1+\delta)\theta^i + \kappa\phi^i[1+(1-q\ )\delta + q\delta\left(\theta^i + m\phi^j\right)]}{m\phi^j} \\ &\quad -\kappa q\delta\phi^i\theta^j\left[\frac{\kappa(1-q+q\theta^j)\phi^i}{(1-\kappa)\theta^j + \kappa(1-q+q\theta^j)\phi^j}\right] \\ &> \frac{\kappa\phi^i\left[1+(1-q)\delta + q\delta\left(\theta^i + m\phi^j\right)\right]}{m\phi^j} - \kappa q\delta\phi^i\theta^j\left[\frac{\phi^i}{\phi^j}\right] \\ &= \frac{\kappa\phi^i}{m\phi^j}\left[1+(1-q)\delta + q\delta\left(\theta^i - m\phi^i\theta^j + m\phi^j\right)\right] > 0. \end{aligned}$$

This last inequality implies $M_2 > 0$ and thus $\Gamma_2 > 0$, thereby completing the proof of part (a).

Part (b): By the envelope theorem, an increase in the income of country j's rival (Y^i) influences its payoff only through the effects on country i's arming and saving. Differentiating U^{j*} appropriately gives

$$dU^{j*} = \frac{\delta q\kappa\phi^i\phi^j}{(1-\kappa)\theta^j + \kappa\phi^j}\left[\left(\theta^i/\phi^i\right)\widehat{Z}^{i*} - m\widehat{G}^{i*}\right],$$

which implies

$$\operatorname{sign}\left\{dU^{j*}/dY^i\right\} = \operatorname{sign}\left\{\left(\theta^i/\phi^i\right)\widehat{Z}^{i*} - m\widehat{G}^{i*}\right\} \tag{A.46}$$

for $\widehat{Y}^i > 0$.

To start, we establish that there exists a threshold level of income for country i, $\overline{Y}^i \leq Y^j$, such that for $Y^i \geq \overline{Y}^i$, $dU^{j*}/dY^i \geq 0$. Recall, from Proposition 2(a), that $\widehat{Z}^{i*} > \widehat{Y}^i > \widehat{G}^{i*} > 0$ holds. Thus, while both arming and saving rise in the country that grows, $\widehat{Y}^i$ constitutes an upper (respectively, lower) bound to $\widehat{G}^{i*}$ (respectively, $\widehat{Z}^{i*}$). Applying these bounds to the RHS of the expression above gives

$$\frac{\theta^i}{\phi^i}\widehat{Z}^{i*} - m\widehat{G}^{i*} > \frac{\theta^i}{\phi^i}\widehat{Y}^i - m\widehat{Y}^i = \left(\frac{\theta^i}{\phi^i} - m\right)\widehat{Y}^i,$$

for $\widehat{Y}^i > 0$ at any $Y^i > 0$. Since θ^i/ϕ^i varies continuously with Y^i along the S^i-curve such that $\theta^i/\phi^i \in (0, 1]$ for $Y^i \leq Y^j$ while $\theta^i/\phi^i > 1$ for $Y^i > Y^j$, there must exist a threshold level $\overline{Y}^i \leq Y^j$, whose value depends on m, that causes the RHS of the above expression to be non-negative. It then follows that the LHS will be non-negative for all $Y^i \geq \overline{Y}^i$, which implies $dU^{j*}/dY^i > 0$ in this case.

Of course, since $\frac{\theta^i}{\phi^i}\widehat{Z}^{i*} - m\widehat{G}^{i*} > (\frac{\theta^i}{\phi^i} - m)\widehat{Y}^i$, it is possible for an increase in Y^i to be welfare-enhancing for country j even when $\frac{\theta^i}{\phi^i} < m$. However, we now argue that there exists another threshold level $\underline{Y}^i$ ($\leq \overline{Y}^i$) such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$. We do this by studying the behavior of U^{j*} as $Y^i \to 0$. To proceed, recall from the expressions of $\widehat{G}^{i*}$ and $\widehat{Z}^{i*}$ shown respectively in (A.29a) and (A.29b), that arming and saving depend on $\widehat{Y}^i$ directly and indirectly through its effect on shares θ^i and ϕ^i. Using the calculations shown in Appendix B, one can find the direct effects as follows:[15]

$$\begin{aligned}
&\lim_{Y^i \to 0}\left(\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i}\right) = \lim_{Y^i \to 0}\left(\frac{\theta^i\Lambda^i_{\theta^i}}{\Lambda^i}\right) = 0, \ \lim_{Y^i \to 0}\left(\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i}\right) = \lim_{Y^i \to 0}\left(\frac{\phi^i\Lambda^i_{\phi^i}}{\Lambda^i}\right) = 1, \text{ and} \\
&\lim_{Y^i \to 0}\left(\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j}\right) = \lim_{Y^i \to 0}\left(\frac{\theta^i\Lambda^j_{\theta^i}}{\Lambda^j}\right) = \lim_{Y^i \to 0}\left(\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j}\right) = \lim_{Y^i \to 0}\left(\frac{\phi^i\Lambda^j_{\phi^i}}{\Lambda^j}\right) = 0.
\end{aligned} \tag{A.47}$$

As for the indirect effects through $\hat{\theta}^i$ and $\hat{\phi}^i$, we apply our earlier findings for the limits of the a-coefficients and $\mathcal{D}$ as $Y^i \to 0$ shown respectively in (A.25) and (A.26) in (A.15) to establish the following:[16]

$$\begin{aligned}
\lim_{Y^i \to 0}\left(\widehat{\theta}^i/\widehat{Y}^i\right) &= \lim_{Y^i \to 0}\left(\frac{a_{11}}{\mathcal{D}}\right) = 1 \\
\lim_{Y^i \to 0}\left(\widehat{\phi}^i/\widehat{Y}^i\right) &= \lim_{Y^i \to 0}\left(-\frac{a_{12}}{\mathcal{D}}\right) = \frac{1}{1 + 1/m}.
\end{aligned}$$

Now, going back to the expressions for $\widehat{G}^{i*}$ and $\widehat{Z}^{i*}$ in (A.29a) and (A.29b), the results

[15] See specifically the derivations of (B.6) and (B.7).

[16] Notice that the expressions below are consistent with our finding in Lemma A.1(a) that $\lim_{\phi^i \to 0}(\widehat{\theta}^i/\widehat{\phi}^i) = 1 + \frac{1}{m}$.

above with the finding in Proposition 1(a) that $\lim_{Y^i \to 0}(\theta^i/\phi^i) = 0$ give us the following:[17]

$$\lim_{Y^i \to 0} \left(\widehat{Z}^{i*}/\widehat{Y}^i\right) = \lim_{Y^i \to 0} \left(\widehat{G}^{i*}/\widehat{Y}^i\right) = 1.$$

On the basis of the above, we thus have

$$\lim_{Y^i \to 0} \left[\frac{\theta^i}{\phi^i}\left(\widehat{Z}^{i*}/\widehat{Y}^i\right) - m\left(\widehat{G}^{i*}/\widehat{Y}^i\right)\right] = \lim_{Y^i \to 0} \left(\frac{\theta^i}{\phi^i} - m\right) = -m < 0,$$

which implies, by (A.46), that $\lim_{Y^i \to 0} dU^{j*}/dY^i < 0$. Since U^{j*} is continuous in Y^i and $dU^{j*}/dY^i > 0$ at a sufficiently high Y^i level, there exists (at least one) threshold level, $\underline{Y}^i \leq \overline{Y}^i$, such that $dU^{j*}/dY^i < 0$ for all $Y^i < \underline{Y}^i$. ||

Proposition A.4 *Suppose that assumption A is satisfied. Then, the welfare implications of changes in the degree of insecurity of future income $\kappa \in (0,1]$ and the probability of a future war $q \in (0,1)$ depend on the relative sizes of the two countries. For the two benchmark cases considered earlier, we have*

(a) $\lim_{Y^i \to Y^j} dU^{i*}/d\kappa = \lim_{Y^i \to Y^j} dU^{j*}/d\kappa < 0$ *and* $\lim_{Y^i \to Y^j} dU^{i*}/dq = \lim_{Y^i \to Y^j} dU^{j*}/dq < 0$*; and*

(b) *given* $Y^j \in (0,\infty)$, $\lim_{Y^i \to 0} dU^{i*}/d\kappa > 0$ *and* $\lim_{Y^i \to 0} dU^{i*}/dq > 0$, *while* $\lim_{Y^i \to 0} dU^{j*}/d\kappa = \lim_{Y^i \to 0} dU^{j*}/dq = 0$.

Proof: The effects of an increase in κ and q on the two countries' payoffs generally consist of both direct effects and indirect effects through their influence on the opponent's choices of arming and savings:

$$\begin{aligned} dU^{i*} = q\delta \Bigg\{ &\left[\frac{\kappa\left(1-\theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)}\right]\widehat{\kappa} - \widehat{q}\ln\left[\frac{\left(\theta^i/\phi^i\right)}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)}\right] \\ &- \left[\frac{\kappa m \phi^j}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)}\right]\widehat{G}^{j*} + \left[\frac{\kappa \theta^j}{\kappa + (1-\kappa)\left(\theta^i/\phi^i\right)}\right]\widehat{Z}^{j*}\Bigg\} \end{aligned} \tag{A.48a}$$

$$\begin{aligned} dU^{j*} = q\delta \Bigg\{ &\left[\frac{\kappa\left(1-\theta^j/\phi^j\right)}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)}\right]\widehat{\kappa} - \widehat{q}\ln\left[\frac{\left(\theta^j/\phi^j\right)}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)}\right] \\ &- \left[\frac{\kappa m \phi^i}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)}\right]\widehat{G}^{i*} + \left[\frac{\kappa \theta^i}{\kappa + (1-\kappa)\left(\theta^j/\phi^j\right)}\right]\widehat{Z}^{i*}\Bigg\}. \end{aligned} \tag{A.48b}$$

Part (a): The first lines of the two expressions above show that, when $Y^i = Y^j$ which implies $\phi^i = \theta^i = \frac{1}{2}$, the direct effects of changes in either κ or q equal zero. Turning our attention to the indirect effects, recall from the proof of Proposition A.3(a) that, when $Y^i = Y^j$, the effects of an increase in κ on $G^i = G^j$ and $Z^i = Z^j$ are identical to the respective effects of an increase in q (see equation (A.43)). Denote the corresponding percentage-changes

[17] Noting that $\lim_{Y^i \to 0}((1-\kappa)\theta^i/[(1-\kappa)\theta^i + \kappa\phi^i]) = 0$ and $\lim_{Y^i \to 0}(\kappa\phi^i/[(1-\kappa)\theta^i + \kappa\phi^i]) = 1$, one can also confirm, from (A.36), that $\lim_{Y^i \to 0}(\widehat{C}^{i*}/\widehat{Y}^i) = 1$.

respectively by $\widehat{G}^*$ and $\widehat{Z}^*$. Then, the welfare effects of changes in κ and q in this case can be written as follows:

$$dU^{i*}|_{Y^i=Y^j} = dU^{j*}|_{Y^i=Y^j} = \frac{1}{2}\kappa q\delta\left[-m\widehat{G}^* + \widehat{Z}^*\right](\widehat{\kappa}+\widehat{q}).$$

From Proposition A.3(a), we know $\widehat{G}^* > 0$ and $\widehat{Z}^* < 0$, implying that an increase in either κ or q reduces payoffs for both countries.

Part (b): When $Y^i \to 0$ given $Y^j \in (0,\infty)$, the welfare effects of an increase in κ and q can include both direct and indirect effects, and generally the combined effects on payoffs are unequal across the two countries. For the calculations to follow, keep in mind that, that as $Y^i \to 0$ for a positive and finite value of Y^j, $\phi^i \to 0$, $\theta^i \to 0$, $\theta^i/\phi^i \to 0$ while $\theta^j/\phi^j \to 1$ (see Proposition 1(a)). We start with the smaller country i, taking the limit of dU^{i*} in (A.48a) as $Y^i \to 0$ using (A.45):

$$\begin{aligned}\lim_{Y^i\to 0} dU^{i*} &= q\delta\Big\{1\times\widehat{\kappa} + \widehat{q}\times\infty \\ &\qquad - m\times\left[\frac{1}{1+m}\widehat{\kappa} + \left(1+\frac{mq\delta(1-m)}{(1+m)[1+\delta-q\delta(1-m)]}\right)\times\widehat{q}\right] + 1\times 0\Big\} \\ &= q\delta\left\{\frac{1}{1+m}\widehat{\kappa} + \left[\infty - m\left(1+\frac{mq\delta(1-m)}{(1+m)[1+\delta-q\delta(1-m)]}\right)\right]\widehat{q}\right\}.\end{aligned}$$

Hence, an increase in either κ or q is welfare-enhancing for the smaller country. Turning to the larger country (j), we similarly take the limit of dU^{j*} in (A.48b) as $Y^i \to 0$ to find

$$\lim_{Y^i\to 0} dU^{j*} = q\delta\left\{0\times\widehat{\kappa} - 0\times\widehat{q} - 0\times\widehat{G}^{i*} + 0\times\widehat{Z}^{i*}\right\} = 0.$$

Thus, the larger country's payoff is independent of κ and q. ||

Proof of Proposition 4. *Parts (a)–(c):* With both countries specializing in production under trade, we have $p_T^i = (R^i/R^j)^{1/\sigma} \in (1/\alpha,\alpha)$. Then, since $Y_T^i = T^i(R^i,R^j)R^i$, we can write country i's income under trade using (17) as follows:

$$Y_T^i = \left[1+\left(R^i/R^j\right)^{\frac{1-\sigma}{\sigma}}\right]^{-\frac{1}{1-\sigma}} R^i,\ i \neq j = 1,2.$$

From this expression, one can easily verify $Y_T^i/Y_T^j = [R^i/R^j]^{(\sigma-1)/\sigma}$, which can be evaluated for $\sigma > 1$, $\sigma = 1$ and $\sigma < 1$ to establish the rankings stated respectively in parts (a)–(c).

For the second main component of the proof, recall that $Y_A^i = AR^i$ where A is shown in (15), implying country i's income under autarky can be written as

$$Y_A^i = [1+\alpha^{1-\sigma}]^{-\frac{1}{1-\sigma}} R^i, \quad i = 1,2.$$

Thus, when $(R^i/R^j)^{1/\sigma} \in (1/\alpha,\alpha)$ such that both countries specialize in production, the (income) gains from trade for country i can be written as

$$\frac{Y_T^i}{Y_A^i} = \left[\frac{1+\alpha^{1-\sigma}}{1+(R^i/R^j)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{1}{1-\sigma}} > 1,\ i \neq j = 1, 2,$$

which is decreasing in R^i/R^j for all $\sigma > 0$. Using this expression for both countries, after some rearranging, gives the gains from trade for country i relative to those for country j:

$$\frac{Y_T^i}{Y_A^i}\bigg/\frac{Y_T^j}{Y_A^j} = \left[\frac{R^j}{R^i}\right]^{1/\sigma}.$$

Thus, provided $(R^i/R^j)^{1/\sigma} \in (1/\alpha, \alpha)$, the relative gains from trade are larger for the *ex ante* smaller country for any $\sigma > 0$, with the relative difference depending negatively on σ.

In the case that $(R^i/R^j)^{1/\sigma} \geq \alpha$ for one country i such that $p_T^i = p_A^i = \alpha$ and it diversifies in production of the intermediate goods, its relative income gains from trade equal zero: $Y_T^i = Y_A^i$. At the same time, the relative income gains from trade for the *ex ante* smaller country ($j \neq i$) that continues to specialize in production with $p_T^j = 1/\alpha$, are

$$\frac{Y_T^j}{Y_A^j} = \left[\frac{1+\alpha^{1-\sigma}}{1+(1/\alpha)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} = \alpha > 1,$$

thereby completing the proof. ||

Proof of Proposition 5. To start, assume that $(R^i/R^j)^{1/\sigma} \in (1/\alpha, \alpha)$, which implies that both countries specialize in the production of the intermediate good for which they have a comparative advantage. Parts (a), (b) and (c) follow respectively from parts (a), (b) and (c) of Proposition 4, which show how different rankings in the countries' initial resource endowments (or R^i/R^j) imply different rankings in their first-period incomes under trade (or Y_T^i/Y_T^j) depending on the value of σ, combined with part (a) of Proposition 1, which shows how different rankings of their incomes translate into the balance of power between them under trade (or ϕ_T^i/ϕ_T^j).

Going beyond the parameter space that ensures specialization in production by both countries, let us fix ideas by supposing that $(R^i/R^j)^{1/\sigma} \geq \alpha$ (> 1) or equivalently that $R^i/R^j \geq \alpha^\sigma$ (> 1). Since, as can be confirmed in this case, country j alone specializes in production under trade with $p_T^j = 1/\alpha$, we have

$$\frac{Y_T^i}{Y_T^j} = \frac{R^i}{R^j}\left[\frac{1+1/\alpha^{1-\sigma}}{1+\alpha^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} = \frac{R^i}{\alpha R^j} \geq \alpha^{\sigma-1}.$$

Assuming diversification in production by country i, this expression shows that, for $\sigma \geq 1$, $R^i > R^j$ implies $Y_T^i > Y_T^j$. Therefore, by Proposition 1, country i continues to enjoy a military advantage: $\phi_T^i > \phi_T^j$. Turning to the case where $\sigma < 1$, suppose that $R^i/R^j = \alpha^\sigma$.

At that point, from the expression above, $Y_T^i/Y_T^j = \alpha^{\sigma-1} < 1$, implying that $\phi_T^i < \phi_T^j$ even though $R^i > R^j$. Nonetheless, since Y_T^i/Y_T^j is increasing in R^i/R^j and approaches ∞ as $R^i/R^j \to \infty$, there exists a threshold value $\overline{R^i/R^j} > \alpha^\sigma$ such that, for $R^i/R^j = \overline{R^i/R^j}$, $Y_T^i/Y_T^j = 1$ and thus $\phi_T^i = \phi_T^j$; and, for values of $R^i/R^j > \overline{R^i/R^j}$, $\phi_T^i > \phi_T^j$ holds. This threshold is illustrated in Fig. 3(a), where the dotted curve crosses the $\phi^i = \frac{1}{2}$ line.

To prove the second main component of the proposition, we appeal to the second main component of Proposition 4, which establishes that for all $\sigma > 0$ and all possible values of R^i/R^j, the *ex ante* smaller country enjoys larger relative income gains from trade and thus

$$\text{if} \quad \frac{R^i}{R^j} \gtreqless 1, \quad \text{then} \quad \frac{Y_T^i}{Y_T^j} \lesseqgtr \frac{Y_A^i}{Y_A^j}.$$

Applying Proposition 1(a) to the second set of inequalities gives the desired result. ||

Although Proposition 6 is valid for a variety of canonical models of trade in the presence of trade costs, our proof to follow focuses on the Ricardian model with no trade costs as presented in the text. See Proposition C.1 presented in Appendix C for a more general statement.

Proof of Proposition 6. We know from Proposition 3 that the smaller country always benefits from an increase in its own income as well as from an increase in the larger country's income; thus, a smaller country will always prefer trade to autarky. However, we also know from Proposition 3(b) that, while the larger country likewise always benefits from an increase in its own income, it does not benefit from an increase in the smaller country's income for sufficiently uneven initial distributions of resources (and when the probability of a future war is positive). Furthermore, Proposition 4 shows that the larger country's relative income gain is decreasing in its relative initial size. Hence, we focus here on establishing the possibility that the larger country could prefer autarky.[18] To do so, suppose $(R^i/R^j)^{1/\sigma} < 1/\alpha$, which implies that country i's *ex ante* relative size is sufficiently small so that it specializes completely in production, while country j's ($\neq i$) production is diversified. In this case, we have $p_T^j = p_A^j = \alpha = 1/p_T^i$ and $Y_T^j = Y_A^j = AR^j$, whereas $Y_T^i = \alpha Y_A^i = \alpha AR^i$. Additionally, since $y^i \equiv Y^i/Y^j$, we also have $y_T^i/y_A^i = \alpha$, $\widehat{y}_T^i = \widehat{y}_A^i = \widehat{R^i/R^j}$ and $\lim_{R^i \to 0} y_T^i = \lim_{R^i \to 0} y_A^i = 0$. Without loss of generality, we can attribute any changes in y^i solely to changes in R^i.

[18]Since the larger country's income gain goes to zero in the Ricardian model when endowments are sufficiently uneven, readers may wonder why this result does not simply follow from Proposition 3(b), which describes a similar result for small changes in the smaller country's income. The key distinction is that trade induces a *discrete* change in incomes; we therefore must rule out the possibility that the smaller country's gain from trade is large enough that the (positive) savings externality dominates the (negative) security externality. As we go on to discuss in Appendix C, the security externality can dominate the positive benefits of trade for the larger country for a wide variety of trade models.

Next we rearrange the various terms of the B^i-curve in (A.8) to rewrite it as

$$\left(\frac{\phi^i}{\phi^j}\right)^{\frac{1+m}{m}}/y^i = \frac{\left[(1-\kappa)(1+\delta)\left(\theta^j/\phi^j\right)+\kappa\left(1+(1-q)\delta+q\delta\left(\theta^j+m(1-\phi^j)\right)\right)\right]}{\left[(1-\kappa)(1+\delta)\left(\theta^i/\phi^i\right)+\kappa\left(1+(1-q)\delta+q\delta\left(\theta^i+m\left(1-\phi^i\right)\right)\right)\right]}$$

both under trade and under autarky. Recalling from Proposition 1(a) that $\lim_{y^i\to 0}\phi^i = \lim_{y^i\to 0}\theta^i = 0$, $\lim_{y^i\to 0}\phi^j = \lim_{y^i\to 0}\theta^j = 1$ and $\lim_{y^i\to 0}\left(\theta^i/\phi^i\right) = 0$ gives

$$\lim_{R^i\to 0}\left[\left(\phi^i\right)^{\frac{1+m}{m}}/y^i\right] = \frac{1+\delta}{\kappa\left[1+\delta\left(1-q+qm\right)\right]},$$

again both under trade and under autarky. It thus follows that

$$\lim_{R^i\to 0}\left(\frac{\phi^i_T}{\phi^i_A}\right) = \lim_{R^i\to 0}\left(\frac{y^i_T}{y^i_A}\right)^{\frac{m}{1+m}} = \alpha^{\frac{m}{1+m}} > 1. \tag{A.49}$$

Let us now consider country j's payoff, U^j. As shown in the proof to Proposition 2, we can write the equilibrium values of arming and saving for country j (G^j and Z^j), as functions of j's income (Y^j) and the equilibrium appropriative and contributive shares ($\phi^i(y^i)$ and $\theta^i(y^i)$) that simultaneously satisfy the S^i and B^i conditions shown in (A.1) and (A.8) given relative the countries' relative income ($y^i \equiv Y^i/Y^j$). Then, using (A.28) with those equilibrium conditions in (5), we can write country j's payoff as $V^j = U^j\left(\phi^i\left(y^i\right), \theta^i\left(y^i\right), Y^j\right)$:

$$\begin{aligned} V^j \;=\;& \delta\ln\left[\delta A\right] + (1+\delta)\ln\left[Y^j\right] - q\delta\ln\left[\theta^j\right] + (1+q\delta)\ln\left[\kappa\phi^j + (1-\kappa)\theta^j\right] + \\ & \delta\ln\left[\kappa\phi^j(1-q+q\theta^j) + (1-\kappa)\theta^j\right] - \\ & (1+\delta)\ln\left[\kappa\phi^j\left(1+\delta(1-q)+\delta q(\theta^j+m\phi^i)\right) + (1-\kappa)\,\theta^j\left(1+\delta\right)\right]. \end{aligned}$$

Because $Y^j_T = Y^j_A = AR^j$ holds for all allocations that satisfy $\left(R^i/R^j\right)^{1/\sigma} < 1/\alpha$ (including the limiting case where $R^i \to 0$), a small change in R^i can affect V^j only through the changes it induces in θ^i and ϕ^i. Differentiation of V^j gives $dV^j = V^j_{\theta^i}d\theta^i + V^j_{\phi^i}d\phi^i$ which can be rewritten as

$$\frac{dV^j/\widehat{y}^i}{\phi^i} = \left[\theta^i/\phi^i\right][V^j_{\theta^i}][\widehat{\theta}^i/\widehat{y}^i] + [V^j_{\phi^i}][\widehat{\phi}^i/\widehat{y}^i], \tag{A.50}$$

where (since $d\theta^j = -d\theta^i$ and $d\phi^j = -d\phi^i$),

$$\begin{aligned} V^j_{\theta^i} =& \frac{q\delta}{\theta^j} - \frac{(1-\kappa)(1+q\delta)}{(1-\kappa)\,\theta^j + \kappa\phi^j} - \frac{\delta\left(1-\kappa+q\kappa\phi^j\right)}{(1-\kappa)\,\theta^j + \kappa\phi^j\left(1-q+q\theta^j\right)} \\ &+ \frac{(1+\delta)\left[(1-\kappa)(1+\delta) + \kappa q\delta\phi^j\right]}{(1-\kappa)(1+\delta)\,\theta^j + \kappa\phi^j\left[1+(1-q)\,\delta + q\delta\left(\theta^j + m\phi^i\right)\right]} \\ V^j_{\phi^i} =& -\frac{\kappa\left(1+q\delta\right)}{(1-\kappa)\,\theta^j + \kappa\phi^j} - \frac{\kappa\delta\left(1-q+q\theta^j\right)}{(1-\kappa)\,\theta^j + \kappa\phi^j\left(1-q+q\theta^j\right)} \end{aligned}$$

$$+\frac{\kappa(1+\delta)\left[1+(1-q)\delta+q\delta\left(\theta^j+m\phi^i-m\phi^j\right)\right]}{(1-\kappa)(1+\delta)\theta^j+\kappa\phi^j\left[1+(1-q)\delta+q\delta\left(\theta^j+m\phi^i\right)\right]},$$

and $\widehat{y}^i = \widehat{R}^i$. Again, keeping in mind that $R^i \to 0$ implies $Y^i \to 0$ as well as $y^i \to 0$ and making use of the limit results from Proposition 1(a) in the expressions above, one can confirm that $\lim_{R^i\to 0}[V^j_{\theta^i}] = q\kappa\delta$ and $\lim_{R^i\to 0}[V^j_{\phi^i}] = -q\kappa\delta(1+m)$. Furthermore, $\lim_{R^i\to 0}\left[\theta^i/\phi^i\right] = 0$. Finally, observe that, from (A.15), (A.25) and (A.26), we have $\lim_{R^i\to 0}[\widehat{\theta}^i/\widehat{y}^i] = 1$ and $\lim_{R^i\to 0}[\widehat{\phi}^i/\widehat{y}^i] = \frac{m}{1+m}$. With an application of these limits that hold true under both trade and autarky in (A.50) noting that $\widehat{y}^i_T = \widehat{y}^i_A$, one can verify the following:

$$\frac{\lim_{R^i\to 0} dV^j_T}{\lim_{R^i\to 0} dV^j_A} = \lim_{R^i\to 0}\left[\frac{-q\kappa\delta m\left(\phi^i_T\widehat{y}^i_T\right)}{-q\kappa\delta m\left(\phi^i_A\widehat{y}^i_A\right)}\right] = \lim_{R^i\to 0}\left(\frac{\phi^i_T}{\phi^i_A}\right) = \alpha^{\frac{m}{1+m}} > 1,$$

where the last equality follows from (A.49). Since $\lim_{R^i\to 0} V^j_T = \lim_{R^i\to 0} V^j_A$ initially and $dV^j < 0$ for sufficiently small Y^i under either trade regime as shown in the proof to Proposition 3(b), it follows that a marginal increase in R^i will reduce j's payoff under trade by more than it reduces j's payoff under autarky. Thus, at least for sufficiently small R^i, we must have $V^j_T < V^j_A$, thereby completing the proof. $\|$

B More Details

In this part of the Appendix, we provide more details regarding some of the calculations used in the Appendix A. Recall that

$$E^i \equiv \left(\phi^i/\phi^i\right)^{1/m} > 0 \tag{B.1a}$$

$$\Lambda^i \equiv (1-\kappa)\theta^i + \kappa\left(1-q+q\theta^i\right)\phi^i > 0 \tag{B.1b}$$

$$\Omega^i \equiv (1-\kappa)(1+\delta)\theta^i + \kappa\phi^i\left[1+(1-q)\delta+q\delta\left(\theta^i+m\phi^j\right)\right] > 0. \tag{B.1c}$$

Using these definitions and keeping in mind that $\phi^j = 1-\phi^i$ and $\theta^j = 1-\theta^i$ (where $i \neq j$), we take and sign the following derivatives:

$$\frac{\phi^i E^i_{\phi^i}}{E^i} = \frac{1}{m\phi^j} > 0 \tag{B.2a}$$

$$\frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} = \frac{\kappa\phi^i\left[1-q+q\theta^i\right]}{\Lambda^i} > 0 \tag{B.2b}$$

$$\frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} = -\frac{\kappa\phi^i\left[1-q+q\theta^j\right]}{\Lambda^j} < 0 \tag{B.2c}$$

$$\frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} = \frac{\theta^i\left[1-\kappa+\kappa q\phi^i\right]}{\Lambda^i} > 0 \tag{B.2d}$$

$$\frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} = -\frac{\theta^i\left[1-\kappa+\kappa q\phi^j\right]}{\Lambda^j} < 0. \tag{B.2e}$$

and

$$\frac{\phi^i\Omega^i_{\phi^i}}{\Omega^i} = \frac{\kappa\phi^i\left[1+(1-q)\,\delta+q\delta\left(\theta^i-m\phi^i+m\phi^j\right)\right]}{\Omega^i} > 0 \tag{B.3a}$$

$$\frac{\phi^i\Omega^j_{\phi^i}}{\Omega^j} = -\frac{\kappa\phi^i\left[1+(1-q)\,\delta+q\delta\left(\theta^j-m\phi^j+m\phi^i\right)\right]}{\Omega^j} < 0 \tag{B.3b}$$

$$\frac{\theta^i\Omega^i_{\theta^i}}{\Omega^i} = \frac{\theta^i\left[(1-\kappa)\,(1+\delta)+\kappa q\delta\phi^i\right]}{\Omega^i} > 0 \tag{B.3c}$$

$$\frac{\theta^i\Omega^j_{\theta^i}}{\Omega^j} = -\frac{\theta^i\left[(1-\kappa)\,(1+\delta)+\kappa q\delta\phi^j\right]}{\Omega^j} < 0. \tag{B.3d}$$

For our calculations regarding the effects of changes in κ and q, observe from the definitions of Λ^r and Ω^r respectively shown in (A.1d) and (A.8c) that

$$\frac{\kappa\Lambda^i_\kappa}{\Lambda^i} = \frac{\kappa\phi^i\left(1-q+q\theta^i\right)-\kappa\theta^i}{\Lambda^i} \tag{B.4a}$$

$$\frac{\kappa\Omega^i_\kappa}{\Omega^i} = \frac{\kappa\phi^i\left[1+\delta+q\delta\phi^j\left(m-\theta^j/\phi^j\right)\right]-\kappa\theta^i\,(1+\delta)}{\Omega^i} \tag{B.4b}$$

$$\frac{q\Lambda^i_q}{\Lambda^i} = -\frac{\kappa q\phi^i\theta^j}{\Lambda^i} \tag{B.4c}$$

$$\frac{q\Omega^i_q}{\Omega^i} = \frac{\kappa q\delta\phi^i\phi^j\left(m-\theta^j/\phi^j\right)}{\Omega^i}. \tag{B.4d}$$

The following are useful for calculating the limits of the a- and b-coefficients in (A.10), as $Y^i \to 0$ for finite $Y^j > 0$, which, by Proposition 1(a), implies $\phi^i \to 0$ while $\phi^j \to 1$, $\theta^i \to 0$ while $\theta^j \to 1$, and $\theta^i/\phi^i \to 0$ while $\theta^j/\phi^j \to 1$:

$$\lim_{Y^i\to 0}\left(\Lambda^i/\phi^i\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)\left(\theta^i/\phi^i\right)+\kappa\left(1-q+q\theta^i\right)\right\} = \kappa\,(1-q) \tag{B.5a}$$

$$\lim_{Y^i\to 0}\left(\Lambda^j/\phi^j\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)\left(\theta^j/\phi^j\right)+\kappa\left(1-q+q\theta^j\right)\right\} = 1 \tag{B.5b}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^i/\phi^i\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)\,(1+\delta)\left(\theta^i/\phi^i\right)+\kappa\left[1+(1-q)\,\delta+q\delta\left(\theta^i+m\phi^j\right)\right]\right\}\\ &= \kappa\left[1+\delta+q\delta\,(m-1)\right]\end{aligned} \tag{B.5c}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^j/\phi^j\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)\,(1+\delta)\left(\theta^j/\phi^j\right)+\kappa\left[1+(1-q)\,\delta+q\delta\left(\theta^j+m\phi^i\right)\right]\right\}\\ &= 1+\delta\end{aligned} \tag{B.5d}$$

$$\lim_{Y^i\to 0}\left(\Lambda^i/\theta^i\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)+\kappa\left(1-q+q\theta^i\right)\left(\phi^i/\theta^i\right)\right\} = \infty \tag{B.5e}$$

$$\lim_{Y^i\to 0}\left(\Lambda^j/\theta^j\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)+\kappa\left(1-q+q\theta^j\right)\left(\phi^j/\theta^j\right)\right\} = 1 \tag{B.5f}$$

$$\begin{aligned}\lim_{Y^i\to 0}\left(\Omega^i/\theta^i\right) &= \lim_{Y^i\to 0}\left\{(1-\kappa)\,(1+\delta)+\kappa\left[1+(1-q)\,\delta+q\delta\left(\theta^i+m\phi^j\right)\right]\left(\phi^i/\theta^i\right)\right\}\\ &= \infty\end{aligned} \tag{B.5g}$$

$$\lim_{Y^i\to 0}\left(\Omega^j/\theta^j\right) = \lim_{Y^i\to 0}\left\{(1-\kappa)\,(1+\delta)+\kappa\left[1+(1-q)\,\delta+q\delta\left(\theta^j+m\phi^i\right)\right]\left(\phi^j/\theta^j\right)\right\}$$

$$= 1 + \delta. \tag{B.5h}$$

Next, we evaluate the limits as $Y^i \to 0$ of the expressions in (B.2) and (B.3), relying on the above:

$$\lim_{Y^i \to 0} \frac{\phi^i E^i_{\phi^i}}{E^i} = \lim_{Y^i \to 0} \frac{1}{m\phi^j} = \frac{1}{m} \tag{B.6a}$$

$$\lim_{Y^i \to 0} \frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} = \lim_{Y^i \to 0} \frac{\kappa \left[1 - q + q\theta^i\right]}{\Lambda^i/\phi^i} = 1 \tag{B.6b}$$

$$\lim_{Y^i \to 0} \frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} = -\lim_{Y^i \to 0} \frac{\kappa \frac{\phi^i}{\phi^j} \left[1 - q + q\theta^j\right]}{\Lambda^j/\phi^j} = 0 \tag{B.6c}$$

$$\lim_{Y^i \to 0} \frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} = \lim_{Y^i \to 0} \frac{\left[1 - \kappa + \kappa q\phi^i\right]}{\Lambda^i/\theta^i} = 0 \tag{B.6d}$$

$$\lim_{Y^i \to 0} \frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} = -\lim_{Y^i \to 0} \frac{\frac{\theta^i}{\theta^j} \left[1 - \kappa + \kappa q\phi^j\right]}{\Lambda^j/\theta^j} = 0, \tag{B.6e}$$

and

$$\lim_{Y^i \to 0} \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} = \lim_{Y^i \to 0} \frac{\kappa \left[1 + (1-q)\,\delta + q\delta \left(\theta^i - m\phi^i + m\phi^j\right)\right]}{\Omega^i/\phi^i} = 1 \tag{B.7a}$$

$$\lim_{Y^i \to 0} \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} = -\lim_{Y^i \to 0} \frac{\kappa \frac{\phi^i}{\phi^j} \left[1 + (1-q)\,\delta + q\delta \left(\theta^j - m\phi^j + m\phi^i\right)\right]}{\Omega^j/\phi^j} = 0 \tag{B.7b}$$

$$\lim_{Y^i \to 0} \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} = \lim_{Y^i \to 0} \frac{\left[(1-\kappa)(1+\delta) + \kappa q\delta\phi^i\right]}{\Omega^i/\theta^i} = 0 \tag{B.7c}$$

$$\lim_{Y^i \to 0} \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} = -\lim_{Y^i \to 0} \frac{\frac{\theta^i}{\theta^j} \left[(1-\kappa)(1+\delta) + \kappa q\delta\phi^j\right]}{\Omega^j/\theta^j} = 0. \tag{B.7d}$$

With these expressions, we can evaluate the limits of the a-coefficients shown in (A.11) as $Y^i \to 0$:

$$\lim_{Y^i \to 0} a_{11} = \lim_{Y^i \to 0} \left\{ \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Lambda^i_{\phi^i}}{\Lambda^i} - \frac{\phi^i \Lambda^j_{\phi^i}}{\Lambda^j} \right\} = \frac{1}{m} + 1 \tag{B.8a}$$

$$\lim_{Y^i \to 0} a_{12} = \lim_{Y^i \to 0} \left\{ -\frac{1}{\theta^j} + \frac{\theta^i \Lambda^i_{\theta^i}}{\Lambda^i} - \frac{\theta^i \Lambda^j_{\theta^i}}{\Lambda^j} \right\} = -1 \tag{B.8b}$$

$$\lim_{Y^i \to 0} a_{21} = \lim_{Y^i \to 0} \left\{ \frac{\phi^i E^i_{\phi^i}}{E^i} + \frac{\phi^i \Omega^i_{\phi^i}}{\Omega^i} - \frac{\phi^i \Omega^j_{\phi^i}}{\Omega^j} \right\} = 1 + \frac{1}{m} \tag{B.8c}$$

$$\lim_{Y^i \to 0} a_{22} = \lim_{Y^i \to 0} \left\{ \frac{\theta^i \Omega^i_{\theta^i}}{\Omega^i} - \frac{\theta^i \Omega^j_{\theta^i}}{\Omega^j} \right\} = 0. \tag{B.8d}$$

Next, we turn to the b-coefficients shown in (A.17). First, we evaluate the limits of the expressions in (B.4) as $Y^i \to 0$, again using (B.5):

$$\lim_{Y^i\to 0}\frac{\kappa\Lambda^i_\kappa}{\Lambda^i} = \lim_{Y^i\to 0}\frac{\kappa\left(1-q+q\theta^i\right)-\kappa(\theta^i/\phi^i)}{\Lambda^i/\phi^i} = 1 \tag{B.9a}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Lambda^j_\kappa}{\Lambda^j} = \lim_{Y^i\to 0}\frac{\kappa\left(1-q+q\theta^j\right)-\kappa(\theta^j/\phi^j)}{\Lambda^j/\phi^j} = 0 \tag{B.9b}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Omega^i_\kappa}{\Omega^i} = \lim_{Y^i\to 0}\frac{\kappa\left[1+\delta+q\delta\phi^j\left(m-\theta^j/\phi^j\right)\right]-\kappa\left(1+\delta\right)\left(\theta^i/\phi^i\right)}{\Omega^i/\phi^i} = 1 \tag{B.9c}$$

$$\lim_{Y^i\to 0}\frac{\kappa\Omega^j_\kappa}{\Omega^j} = \lim_{Y^i\to 0}\frac{\kappa\left[1+\delta+q\delta\phi^i\left(m-\theta^j/\phi^i\right)\right]-\kappa\left(1+\delta\right)\left(\theta^j/\phi^j\right)}{\Omega^j/\phi^j} = 0 \tag{B.9d}$$

$$\lim_{Y^i\to 0}\frac{q\Lambda^i_q}{\Lambda^i} = -\lim_{Y^i\to 0}\frac{\kappa q\theta^j}{\Lambda^i/\phi^i} = -\frac{q}{1-q} \tag{B.9e}$$

$$\lim_{Y^i\to 0}\frac{q\Lambda^j_q}{\Lambda^j} = -\lim_{Y^i\to 0}\frac{\kappa q\theta^i}{\Lambda^j/\phi^j} = 0 \tag{B.9f}$$

$$\lim_{Y^i\to 0}\frac{q\Omega^i_q}{\Omega^i} = \lim_{Y^i\to 0}\frac{\kappa q\delta\phi^j\left(m-\theta^j/\phi^j\right)}{\Omega^i/\phi^i} = \frac{q\delta(m-1)}{1+\delta+q\delta(m-1)} \tag{B.9g}$$

$$\lim_{Y^i\to 0}\frac{q\Omega^j_q}{\Omega^j} = \lim_{Y^i\to 0}\frac{\kappa q\delta\phi^i\left(m-\theta^i/\phi^i\right)}{\Omega^j/\phi^j} = 0. \tag{B.9h}$$

Then, using (A.17), one can confirm the following:

$$\lim_{Y^i\to 0} b_{12} = \lim_{Y^i\to 0}\left\{\frac{\kappa\Lambda^i_\kappa}{\Lambda^i}-\frac{\kappa\Lambda^j_\kappa}{\Lambda^j}\right\} = 1 \tag{B.10a}$$

$$\lim_{Y^i\to 0} b_{22} = \lim_{Y^i\to 0}\left\{\frac{\kappa\Omega^i_\kappa}{\Omega^i}-\frac{\kappa\Omega^j_\kappa}{\Omega^j}\right\} = 1 \tag{B.10b}$$

$$\lim_{Y^i\to 0} b_{13} = \lim_{Y^i\to 0}\left\{\frac{q\Lambda^i_q}{\Lambda^i}-\frac{q\Lambda^j_q}{\Lambda^j}\right\} = -\frac{q}{1-q} \tag{B.10c}$$

$$\lim_{Y^i\to 0} b_{23} = \lim_{Y^i\to 0}\left\{\frac{q\Omega^i_q}{\Omega^i}-\frac{q\Omega^j_q}{\Omega^j}\right\} = \frac{q\delta(m-1)}{1+\delta+q\delta(m-1)}. \tag{B.10d}$$

C Alternative Models of Trade

One might naturally wonder if the possibility that the larger country prefers autarky over trade extends to other trade models. In these brief notes, we explain that the answer is yes. Note first that $\lim_{R^i\to 0} U^{j*}_T = \lim_{R^i\to 0} U^{j*}_A$ (an infinitely large country does not gain from trade and is not threatened by appropriation). Letting j always be the larger country in what follows, our general strategy therefore will be to show that for a marginal increase in the smaller country's "scale" R^i, we must have that $\lim_{R^i\to 0} dU^{j*}_T < \lim_{R^i\to 0} dU^{j*}_A$. In other words, starting from a situation where j's trade and autarky payoffs are known to be equal (the limit where $R^i \to 0$), we will verify that there exist sufficiently small values of R^i for which j prefers autarky to trade.[19]

We perform this comparison in the context of four trade models: (1) Neoclassical trade models; (2) Armington (1969); (3) Krugman (1980) and, (4) Melitz (2003) – Chaney (2008).[20] In a departure from our current setup, we also allow for the possibility that trade could be subject to (possibly asymmetric) "iceberg" type trade costs. For models (2)-(4), we focus on the one-sector versions of these models. In model (1), we consider a general neoclassical model with multiple resources and internationally diversified production. For each of these models, the crucial step is to prove the following proposition:

Proposition C.1 *Consider any of the trade models* (1) – (4). *A marginal increase in the scale of country i (i.e., $R^i \uparrow$) reduces the payoff of its rival (j) under trade by more than under autarky if country i is sufficiently small:* $\lim_{R^i \to 0} \left(dU_T^{j*}/dU_A^{j*} \right) > 1$ $(j \neq i = 1, 2)$.

The result that a country chooses to foreclose on trade if the rival country is sufficiently small then follows from the fact that $\lim_{R^i \to 0} U_T^{j*} = \lim_{R^i \to 0} U_A^{j*}$ and the fact that $\lim_{R^i \to 0} dU_A^{j*}$ and $\lim_{R^i \to 0} dU_T^{j*}$ are both negative (by Proposition 3(b)).

Proof: First, recall from our proof of Proposition 3(b) that

$$dU^{j*} = \frac{\delta q \kappa \phi^i}{(1-\kappa)\left(\theta^j/\phi^j\right) + \kappa} \left[\left(\theta^i/\phi^i\right) \left(\widehat{Z}^{i*}/\widehat{Y}^i\right) - m \left(\widehat{G}^{i*}/\widehat{Y}^i\right) \right] \widehat{Y}^i \tag{C.1}$$

gives the change in j's payoff as a function of the (log) change in country i's income $\widehat{Y}^i$ due to a change in its scale R^i. Since (C.1) holds under both trade and autarky, we can next write that

$$\begin{aligned} \lim_{R^i \to 0} \left(\frac{dU_T^{j*}}{dU_A^{j*}} \right) &= \lim_{R^i \to 0} \left(\frac{\phi_T^i}{\phi_A^i} \right) \times \frac{\lim_{R^i \to 0} \left[(1-\kappa) \left(\theta_A^j/\phi_A^j \right) + \kappa \right]}{\lim_{R^i \to 0} \left[(1-\kappa) \left(\theta_T^j/\phi_T^j \right) + \kappa \right]} \times \\ &\quad \frac{\lim_{R^i \to 0} \left[\left(\theta_T^i/\phi_T^i\right) \left(\widehat{Z}^{i*}/\widehat{Y}^i\right) - m \left(\widehat{G}^{i*}/\widehat{Y}^i\right) \right]}{\lim_{R^i \to 0} \left[\left(\theta_A^i/\phi_A^i\right) \left(\widehat{Z}^{i*}/\widehat{Y}^i\right) - m \left(\widehat{G}^{i*}/\widehat{Y}^i\right) \right]} \times \lim_{R^i \to 0} \left(\frac{\widehat{Y}_T^i/\widehat{R}^i}{\widehat{Y}_A^i/\widehat{R}^i} \right) \Rightarrow \\ \lim_{R^i \to 0} \left(\frac{dU_T^{j*}}{dU_A^{j*}} \right) &= \lim_{R^i \to 0} \left(\frac{\phi_T^i}{\phi_A^i} \right) \times \lim_{R^i \to 0} \left(\frac{\widehat{Y}_T^i}{\widehat{Y}_A^i} \right), \quad \text{for } j \neq i = 1, 2. \end{aligned} \tag{C.2}$$

The last expression in the RHS of (C.2) is obtained for the following reasons. The second

[19] The reason why these results do not simply follow from Proposition 3 is because, even though trade generally has the effect of increasing a small country's income Y^i, the increase in Y^i is not necessarily small. Thus, a detailed proof is required to establish when the increase in Y^i is sufficiently small so that, from the larger country's perspective, the (positive) savings externality generated by trade does not overwhelm the (negative) security externality.

[20] The first category includes the Heckscher-Ohlin and Specific Factors models. Note further, because the equilibrium conditions of the Armington model and the many-good Ricardian model of Eaton and Kortum (2002) are largely isomorphic to one another, our results extend to this latter model as well. The main difference is that the trade elasticity in Eaton and Kortum (2002) is not given by $\sigma - 1$, but rather by θ, a shape parameter from the Fréchet distribution that governs the dispersion of productivities across goods.

multiplicative term in the RHS of the first equation equals 1 because $\lim_{R^i \to 0} Y^i = 0$ under all trade regimes in all the relevant trade models, which in turn implies, by Proposition 1(a), that $\lim_{Y^i \to 0} \left(\theta^j/\phi^j\right) = 1$.[21] The third term of this equation also equals 1 because $\lim_{R^i \to 0} \left(\theta^i/\phi^i\right) = 0$ and, as shown in the proof to Proposition 3(b), $\lim_{R^i \to 0} \left(\widehat{Z}^{i*}/\widehat{Y}^i\right) = \lim_{R^i \to 0} \left(\widehat{G}^{i*}/\widehat{Y}^i\right) = 1$ under all trade regimes and all trade models considered.

But, since $\lim_{R^i \to 0} \phi^i_T = \lim_{R^i \to 0} \phi^i_A = 0$, we need to verify that $\lim_{R^i \to 0} \left(\phi^i_T/\phi^i_A\right)$ in (C.2) exists.[22] Using the definition of the B^i-curve in (A.8), we can multiply E^i and F^i by ϕ^i/ϕ^j in order to obtain the following relationship:

$$\left(\frac{\phi^i}{\phi^j}\right)^{\frac{1+m}{m}} = \left(\frac{Y^i}{Y^j}\right) \frac{\left[(1-\kappa)(1+\delta)\left(\theta^j/\phi^j\right) + \kappa\left(1 + (1-q)\delta + q\delta\left(\theta^j + m\left(1-\phi^j\right)\right)\right)\right]}{\left[(1-\kappa)(1+\delta)\left(\theta^i/\phi^i\right) + \kappa\left(1 + (1-q)\delta + q\delta\left(\theta^i + m\left(1-\phi^i\right)\right)\right)\right]},$$

which holds true both under trade and under autarky. Evaluating the above expression under trade and autarky, and taking ratios appropriately, we next obtain

$$\frac{\phi^i_T}{\phi^i_A} = \left(\frac{\phi^j_T}{\phi^j_A}\right)\left(\frac{Y^i_T Y^j_A}{Y^i_A Y^j_T}\right)^{\frac{m}{1+m}} \left(K^j/K^i\right)^{\frac{m}{1+m}},$$

where

$$K^j \equiv \frac{\left[(1-\kappa)(1+\delta)\left(\theta^j_T/\phi^j_T\right) + \kappa\left(1 + (1-q)\delta + q\delta\left(\theta^j_T + m(1-\phi^j_T)\right)\right)\right]}{\left[(1-\kappa)(1+\delta)\left(\theta^j_A/\phi^j_A\right) + \kappa\left(1 + (1-q)\delta + q\delta\left(\theta^j_T + m(1-\phi^j_T)\right)\right)\right]}.$$

From Proposition 1, $\lim_{R^i \to 0} Y^i = 0$ implies $\lim_{R^i \to 0} \left(\theta^j/\phi^j\right) = 1$, $\lim_{R^i \to 0} \left(\theta^i/\phi^i\right) = 0$, $\lim_{R^i \to 0} \theta^i = \lim_{R^i \to 0} \phi^i = 0$, and $\lim_{R^i \to 0} \theta^j = \lim_{R^i \to 0} \phi^j = 1$ under either trade regime. Then, from the expression above, we have $\lim_{R^i \to 0} K^j = \lim_{R^i \to 0} K^i = 1$ and $\lim_{R^i \to 0}(\phi^j_T/\phi^j_A) = 1$, which in turn give

$$\lim_{R^i \to 0}\left(\frac{\phi^i_T}{\phi^i_A}\right) = \lim_{R^i \to 0}\left(\frac{Y^i_T Y^j_A}{Y^i_A Y^j_T}\right)^{\frac{m}{1+m}}. \tag{C.3}$$

Finally, we have $\lim_{R^i \to 0}\left(Y^j_T/Y^j_A\right) = 1$, because infinitely large countries do not gain from trade. Thus, the above relationship suggests that the ratio of an infinitesimal economy's appropriative shares under trade and autarky (i.e., $\lim_{R^i \to 0} \phi^i_T/\phi^i_A$) is increasing (at a decreasing rate) in the limit of the ratio of its income under trade over its income under autarky (i.e., $\lim_{R^i \to 0} Y^i_T/Y^i_A$). With this in mind, we can rewrite (C.2) as follows:

$$\lim_{R^i \to 0}\left(\frac{dU^{j*}_T}{dU^{j*}_A}\right) = \lim_{R^i \to 0}\left(\frac{Y^i_T}{Y^i_A}\right)^{\frac{m}{1+m}} \times \lim_{R^i \to 0}\left(\frac{\widehat{Y}^i_T}{\widehat{Y}^i_A}\right), \quad \text{for } j \neq i = 1, 2. \tag{C.4}$$

[21] The reason that $\lim_{R^i \to 0} Y^i = 0$ will become clear in due course.

[22] Moreover, it is not yet clear whether $\lim_{R^i \to 0}(\widehat{Y}^i_T/\widehat{Y}^i_A)$ in the same equation exists. As one might expect, the answer depends on the particular trade model one considers.

For each of the models under consideration, we will show that the product of the two terms on the RHS of (C.4) is greater than 1.

To fill in these remaining blanks, we start by describing general results for the consequences of trade that hold across a wide variety of commonly used trade models. Details showing how these results map onto each specific model are then shown in Table C.1. To set up the general framework, let $p^i \equiv p^i_j/p^i_i$ be the internal relative price of i's importable, measured in units of its exportable. This price can differ from the corresponding world price (p^j_j/p^i_i) because of trade costs. Also let $I^i(p^i_i, p^i_j, V^i)$ be the maximized value of i's production of intermediate goods (i.e., the "revenue function"), given its technology and the vector of its factor endowments V^i. To aim for generality, we will assume that, even if the endowment vector V^i has multiple elements (as in neoclassical trade models), increases in a country's "scale" R^i raise all elements of V^i with the same "scale elasticity" $s_m \geq 1$.[23] Furthermore, to also admit models with monopolistic competition, such as the Krugman or Melitz models, we may treat the price terms p^i_i and p^i_j as generally representing the *price indices* of domestically-produced and imported intermediate bundles, potentially reflecting both the number of available varieties within each bundle as well as the prices of each underlying variety.

We assume the production function for final goods is CRS; thus, under both autarky and trade, national income can be written as

$$Y^i = \eta^i(p^i_i, p^i_j)I^i \tag{C.5}$$

where the marginal utility of income $\eta(p^i_i, p^i_j)$ acts as an (inverse) price index for final output and is decreasing in each of its arguments. Also note that $I^i = I^i(p^i_i, p^i_j, V^i)$ has the standard properties that it is linearly homogeneous, increasing and convex in prices (p^i_i, p^i_j). By the envelope theorem, country i's production of any good can be derived as $Q^i_j = I^i_{p^i_j}$. Also let $D^i_j(p^i_i, p^i_j, I^i)$ denote the Marshallian demand function for good j and let $X^i_j = D^i_j - Q^i_j$ denote excess demand. D^i_j and Q^i_j increase proportionally with I^i; therefore, an increase in R^i at fixed prices results in equi-proportional increases in Q^i_j, D^i_j, and X^i_j through its effects on I^i.

Differentiating (C.5), we obtain

$$\begin{aligned}
dY^i &= Y^i_{p^i_i}dp^i_i + Y^i_{p^i_j}dp^i_j + Y^i_{I^i}\left(I^i_{p^i_i}dp^i_i + I^i_{p^i_j}dp^i_j + \sum\nolimits_k I^i_{V^i_k}dV^i_k\right) \\
&= Y^i_{I^i}\left[Y^i_{p^i_i}/Y^i_{I^i}dp^i_i + Y^i_{p^i_j}/Y^i_{I^i}dp^i_j + I^i_{p^i_i}dp^i_i + I^i_{p^i_j}dp^i_j + \sum\nolimits_k V^i_k I^i_{V^i_k}\widehat{V}^i_k\right] \\
&= Y^i_{I^i}\left[\left(-D^i_i + Q^i_i\right)dp^i_i + \left(-D^i_j + Q^i_j\right)dp^i_j + s_m I^i \widehat{R}^i\right],
\end{aligned}$$

[23]To fix ideas, $s_m = 1$ in any model with constant returns. In the CES-variants of the Krugman and Melitz models we consider, $s_m = \sigma/(\sigma-1) > 1$, where σ is the elasticity of substitution between varieties. See Table C.1 for further details clarifying how revenue functions differ across models.

where it is useful to note that $Y^i_{p^i_j}/Y^i_{I^i} = -D^i_I$ (by Roy's identity) and that $\sum_k V^i_k I^i_{V^i_k} = s_m I^i$, with $s_m \geq 1$ a model-specific constant reflecting returns to scale. To simplify further, note that balanced trade requires that $p^i_i(D^i_i - Q^i_i) = p^i_j(D^i_j - Q^i_j) = p^i_j X^i_j$ and that $Y^i_{I^i} \cdot I^i = Y^i$. Thus, percentage changes in Y^i are always given by

$$\widehat{Y}^i \quad = \quad s_m \widehat{R}^i - \mu^i_j \widehat{p}^i, \tag{C.6}$$

where $\mu^i_j \equiv p^i_j X^i_j / I^i$ will henceforth denote country i's expenditure share on imported goods (likewise, let $\mu^i_i \equiv 1 - \mu^i_j$ denote the expenditure share on domestically produced goods, a useful object to work with in several of these models.) Under autarky, clearly $\mu^i_j = 0$, so that $\widehat{Y}^i_A = s_m \widehat{R}^i$. Since the log-change in a country's "gains from trade" can be written as $\widehat{Y}^i_T - \widehat{Y}^i_A$, it follows that a country's relative gains from trade are inversely related to its relative size if $\widehat{p}^i/\widehat{R}^i > 0$ and $\widehat{p}^i/\widehat{R}^j < 0$, which Table C.1 shows to be true across all the models we consider. It also follows that

$$\lim_{R^i \to 0} \frac{\widehat{Y}^i_T}{\widehat{Y}^i_A} = 1 - \lim_{R^i \to 0} \frac{\mu^i_j \widehat{p}^i}{s_m \widehat{R}^i}, \tag{C.7}$$

which will be > 0 so long as $\lim_{R^i \to 0} \mu^i_j \widehat{p}^i / s_m \widehat{R}^i < 1$.

To say more, we need to characterize more fully the manner in which $\widehat{p}^i$ is determined and its limit as $R^i \to 0$. For each model, let τ^{ji} be the amount of country i's importable that must be shipped from country j for 1 unit to arrive in country i—i.e., inclusive of trade costs. For the Neoclassical, Armington, and Krugman models, this means we always have that $p^i_j = p^j_j \tau^{ji}$ and, furthermore, the balanced trade condition is always given by

$$p^i_i \tau^{ij} X^j_i = p^i_j X^i_j \Rightarrow p^i = \frac{\tau^{ij} X^i_j}{X^j_i} \tag{C.8}$$

The Melitz model also features other types of trade costs, however, which are described further in Table C.1. Regardless of the assumed model, however, we can obtain $\lim_{R^i \to 0} \widehat{p}^i / \widehat{R}^i$ by differentiating the relevant balanced trade condition.[24] Furthermore, under each model, it is possible to show the product of $\lim_{R^i \to 0} Y^i_T / Y^i_A$ and $\lim_{R^i \to 0} \widehat{Y}^i_T / \widehat{Y}_A$ is greater than 1, such that (C.4) ensures that $\lim_{R^i \to 0} dU^{j*}_T / dU^{j*}_A > 1$. The key steps needed to obtain these results are shown in Table C.1. Full details are available on request.

[24]In general, $\lim_{R^i \to 0} \widehat{p}^i / \widehat{R}^i > 0$. In the Krugman and Melitz models, this result occurs through the effect of an increase in R^i on the number of varieties produced in country i, which lowers the overall price index for these varieties p^i_i, even though the price charged for each variety actually increases through the home market effects inherent to these models.

Table C.1: Key results for different trade models

Model	Price indices (p_i^i,p_j^i)	Revenue function (I^i)	Scale elasticity (s_m)	Balanced trade condition	Relative price change ($\widehat{p}^i$)	$\lim_{R^i \to 0} Y_T^i/Y_A^i$	$\lim_{R^i \to 0} \widehat{Y}_T/\widehat{Y}_A$	Notes
Ricardian	$p_i^i = \alpha_i^i r^i$; $p_j^i = p_j^j \tau^{ji}$	$I^i = r^i R^i = (\alpha_i^i)^{-1} p_i^i R^i$ (if completely specialized) $= (\alpha_i^i)^{-1} p_i^i R_i^i + (\alpha_j^i)^{-1} p_j^i R_j^i$(if not)	1	$p^i = \frac{\tau^{ij} X_j^i}{X_i^j}$	$\widehat{p}^i = \frac{1}{\Delta_r}\left(\widehat{R}^i - \widehat{R}^j\right)$	> 1 if $p_T^i \neq p_A^i$	1	R_j^i denotes the amount of country i's endowment employed in the production of good j. We assume $p_A^i < 1/p_A^j$ such that good i is country i's comparative advantage sector. This also ensures $\lim_{R^i \to 0} Y_T^i/Y_A^i > 0$. $\Delta_r \equiv \varepsilon^i + \varepsilon^j - 1$, where $\varepsilon^i \equiv -\frac{p_j^i \partial X_j^i}{X_j^i \partial p_j^i}$ is the price elasticity of country i's import demand. Note $\lim_{R^i \to 0} p_T^j = p_A^j$ implies that $\lim_{R^i \to 0} \varepsilon^j = \infty$ and, in turn, that $\lim_{R^i \to 0} \widehat{p}^i/\widehat{R}^i = 0$.
Neoclassical	$p_i^i = \sum_V \alpha_{Vi}^i r_V^i$; $p_j^i = \sum_V \alpha_{Vj}^i r_V^i = p_j^j \tau^{ji}$	$I^i = \sum_V r_v^v V^i$	1	$p^i = \frac{\tau^{ij} X_j^i}{X_i^j}$	$\widehat{p}^i = \frac{1}{\Delta_n}\left(\widehat{R}^i - \widehat{R}^j\right)$	> 1 if $p_T^i \neq p_A^i$	1	We consider an increase in R^i as increasing all endowments proportionately: $\widehat{V}^i/\widehat{R}^i = 1$ for all V^i (as in Syropoulos, 2002). This is what ensures $s_m = \widehat{I}^i/\widehat{R}^i = 1$. r_V^i is the factor payment received by owners of factor V^i in country i and α_{Vj}^i is the amount of factor V^i required to produce a unit of good j in country i. $\Delta_r \equiv \varepsilon^i + \varepsilon^j - 1$, where $\varepsilon^i \equiv -\frac{p_j^i \partial X_j^i}{X_j^i \partial p_j^i}$ is the price elasticity of country i's import demand. Note $\lim_{R^i \to 0} p_T^j = p_A^j$ implies that $\lim_{R^i \to 0} \varepsilon^j = \infty$ and, in turn, that $\lim_{R^i \to 0} \widehat{p}^i/\widehat{R}^i = 0$.
Armington	$p_i^i = r^i$; $p_j^i = p_j^j \tau^{ji}$.	$I^i = r^i R^i$; $p_i^i = r^i \implies Q_i^i = R^i$	1	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $p^i = \left[\frac{1+\left(p^i/\tau^{ji}\tau^{ij}\right)^{1-\sigma}}{1+(1/p^i)^{1-\sigma}}\right]\left[\frac{\tau^{ji} R^i}{R^j}\right]$	$\widehat{p}^i = \frac{1}{\Delta_a}\left(\widehat{R}^i - \widehat{R}^j\right)$	$\lim_{R^i \to 0} (\mu_i^i)^{\frac{1}{1-\sigma}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}^i}{\widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a}$ $= \frac{\sigma-1}{\sigma} > 0$	$\sigma > 1$ is the elasticity of substitution. The overall price index is given by $P^i \equiv [(p_i^i)^{1-\sigma} + (p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{1-\sigma}$; $\mu_j^i = (p_j^i/P^i)^{1-\sigma}$. $\Delta_a \equiv 1 + (\sigma-1)\left(\mu_i^i + \mu_j^j\right) > 0$. Also note that equilibria are possible where $\sigma \leq 1$. In these cases, $Y_A^i = [(Q_i^i)^{\frac{\sigma-1}{\sigma}} + 0^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} = 0$ and neither country would ever foreclose on trade.
Krugman	$p_i^i \equiv \left[\int_{N^i} p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} =$ $(N^i)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} r^i = \left(\frac{R^i}{\sigma f}\right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} r^i$ $p_j^i \equiv p_j^j \tau^{ji}$ (Note free entry implies that $N^i = \frac{R^i}{\sigma f}$.)	$I^i = r^i R^i = \left(\frac{\sigma-1}{\sigma}\right)(N^i)^{\frac{1}{\sigma-1}} p_i^i R^i$ $\implies I^i = (\sigma f)^{\frac{1}{1-\sigma}}\left(\frac{\sigma-1}{\sigma}\right) p_i^i (R^i)^{\frac{\sigma}{\sigma-1}}$	$\frac{\sigma}{\sigma-1} > 1$	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $p^i = \left[\frac{1+\left(p^i/\tau^{ji}\tau^{ij}\right)^{1-\sigma}}{1+(1/p^i)^{1-\sigma}}\right] \tau^{ji}\left[\frac{R^i}{R^j}\right]^{\frac{\sigma}{\sigma-1}}$	$\widehat{p}^i = \frac{1}{\Delta_k}\left(\widehat{R}^i - \widehat{R}^j\right)$	$\lim_{R^i \to 0} (\mu_i^i)^{\frac{1}{1-\sigma}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}^i}{s_m \widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{s_m \Delta_k}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a}$ $= \frac{\sigma-1}{\sigma} > 0$	$\sigma > 1$ is the elasticity of substitution. The overall price index is given by $P^i \equiv [(p_i^i)^{1-\sigma} + (p_j^i)^{1-\sigma}]^{1/(1-\sigma)}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{1-\sigma}$; $\mu_j^i = (p_j^i/P^i)^{1-\sigma}$. N^i is the mass of firms in i. f is the fixed cost of production (paid in units of R^i). $\Delta_k \equiv [(\sigma-1)/\sigma]\Delta_a \in (0, \Delta_a)$.
Melitz	$p_i^i \equiv \left[\Lambda_1 \left(\frac{I^i}{f_d}\right)^{\xi} N^i \underline{\varphi}_i^{\theta} (r^i)^{-\theta-\xi}\right]^{-\frac{1}{\theta}}$ where $\xi \equiv \frac{\theta-(\sigma-1)}{\sigma-1}$ and $\Lambda_1 \equiv (\frac{\sigma-1}{\sigma})^{-\theta}\sigma^{-\xi}\frac{\theta}{\sigma-1-\theta}$. $p_j^i \equiv p_j^j \left(\frac{I^i}{I^j}\right)^{-\frac{\xi}{\theta}}\left(\frac{f_x^{ji}}{f_d}\right)^{-\frac{\xi}{\theta}}(\tau^{ji})$. (Note the overall price index is given by $P^i = [(p_i^i)^{-\theta} + (p_j^i)^{-\theta}]^{-\frac{1}{\theta}}$.)	$I^i = r^i R^i =$ $\left[\Lambda_1 \left(\frac{R^i}{f_d}\right)^{\xi} N^i \underline{\varphi}_i^{\theta}\right]^{\frac{1}{\theta}} p_i^i R^i =$ $\Lambda_2^{\frac{1}{\theta}} p_i^i (R^i)^{\frac{\sigma}{\sigma-1}}$ where $\Lambda_2 = \frac{\sigma-1}{\theta\sigma f_e}\underline{\varphi}_i f_d^{-\xi}\Lambda_1 > 0$. (Note free entry delivers $N^i = \frac{R^i(\sigma-1)}{f_e\theta\sigma}$.)	$\frac{\sigma}{\sigma-1} > 1$	$\mu_i^j r^j R^j = \mu_j^i r^i R^i \implies$ $\frac{r^j}{r^i} = \frac{\mu_j^i R^i}{\mu_i^j R^j} =$ $\left[\frac{\overline{\tau}^{ji}+\left(R^j/R^i\right)\left(r^j/r^i\right)^{-\theta-\xi}}{\overline{\tau}^{ij}+(R^i/R^j)(r^i/r^j)^{-\theta-\xi}}\right]\left[\frac{\overline{\tau}^{ij} R^i}{\overline{\tau}^{ji} R^j}\right]$ where $\overline{\tau}^{ij} \equiv (\tau^{ij})^{-\theta}(f_x^{ij}/f_d)^{-\xi}(\underline{\varphi}_i/\underline{\varphi}_j)^{\theta}$	$\widehat{p}^i = \frac{1}{\Delta_m}\left(\widehat{R}^i - \widehat{R}^j\right)$	$\lim_{R^i \to 0} (\mu_i^i)^{-\frac{1}{\theta}} = \infty$	$\lim_{R^i \to 0} 1 - \mu_j^i \frac{\widehat{p}^i}{s_m \widehat{R}^i}$ $= \lim_{R^i \to 0} 1 - \frac{1}{s_m \Delta_m}$ $=$ $\lim_{R^i \to 0} 1 - \frac{1}{s_m \Delta_k + s_m[\theta-(\sigma-1)]}$ $= \lim_{R^i \to 0} 1 - \frac{1}{\Delta_a + \frac{\theta\sigma}{\sigma-1} - \sigma} =$ $1 - \frac{\sigma-1}{\theta\sigma} \in (0,1)$	$\sigma > 1$ is the elasticity of substitution. $\theta > \sigma - 1$ is the shape parameter of the Pareto firm productivity distribution, with pdf $\psi^i(\varphi) = \theta\varphi^{\theta-1}/\underline{\varphi}_i^{\theta}$. Expenditure shares are given by: $\mu_i^i = (p_i^i/P^i)^{-\theta}$; $\mu_j^i = (p_j^i/P^i)^{-\theta}$. φ_d^i is the minimum productivity needed for firms in i to produce for the domestic market. φ_x^i is the minimum productivity needed to export to j. f_e, f_d, and f_x respectively are the fixed costs of entry, production, and exporting. $\Delta_m \equiv \Delta_k + [\theta - (\sigma-1)](\mu_i^i + \mu_j^j) > \Delta_k > 0$.

For completeness, we also include results for the Ricardian model. The key result is whether $\lim_{R^i \to 0} Y_T^i/Y_A^i \times \lim_{R^i \to 0} \widehat{Y}_T/\widehat{Y}_A > 0$, which is satisfied for all models shown. Detailed derivations of this result for each of these models are available by request. The "scale elasticity" (s_m) can be obtained in each model as $s_m \equiv \partial \ln I^i / \partial \ln R^i$, after first writing the revenue function in the form $I^i \equiv I(p_i^i, p_j^i, R^i)$ and applying the relevant definitions of p_i^i and p_j^i for each model. Some additional useful references include Arkolakis (2012) and Demidova and Rodríguez-Clare (2013).

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