Appendix

Proof. Proposition 1. According to steady-state demand condition,

$$D = (PA)^{\frac{1}{1-\alpha}} \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} f^{ss}(\theta_u, a; D)\, \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} f^{ss}(\theta_g, a; D)\, \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} \end{array} \right\}. \tag{A1}$$

, where $f^{ss}(\theta^e, a; D)$ is the steady-state measure of plants with age a and the expected idiosyncratic productivity θ^e. More specifically,

$$\begin{aligned} f^{ss}(\theta_u, a; D) &= f^{ss}(0, D)(1-p)^a \\ f^{ss}(\theta_g, a; D) &= f^{ss}(0, D)\, \varphi \left[1 - (1-p)^a\right] \end{aligned} \tag{A2}$$

By definition, a steady state features time-invariant distribution of plants across a and θ^e. This implies that PA has to be time-invariant for (A1) to hold.

In addition to (A1), $f^{ss}(0, D)$, $\overline{a}_u^{ss}(D)$ and $\overline{a}_g^{ss}(D)$ have to satisfy the following conditions. The exit condition for a good plant is:

$$\left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_g}{(1+\gamma)^{\overline{a}_g^{ss}(D)}}\right]^{\frac{1}{1-\alpha}} - \Psi = 0 \tag{A3}$$

The exit condition for an unsure plant is

$$\begin{aligned} 0 \;=\; & \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_u}{(1+\gamma)^{\overline{a}_u^{ss}(D)}}\right]^{\frac{1}{1-\alpha}} - \Psi + \\ & p\varphi \sum_{a=\overline{a}_u^{ss}(D)+1}^{\overline{a}_g^{ss}(D)} \beta^{a-\overline{a}_u^{ss}(D)} \left\{ \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\}; \end{aligned} \tag{A4}$$

The free entry condition is:

$$\begin{aligned} c_0 + c_1 f^{ss}(0, D) \;=\; & \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \left\{ \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\} + \\ & \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \left\{ \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \left[\frac{PA\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi \right\}. \end{aligned} \tag{A5}$$

Furthermore, (A3) suggests:

$$(PA)^{\frac{1}{1-\alpha}} = \frac{\Psi}{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \tag{A6}$$

Plugging (A6) and (A2) into (A1) gives

$$D = \frac{\Psi}{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} f^{ss}(0,D) \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} (1-p)^a \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \varphi \left[1 - (1-p)^a \right] \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} \end{array} \right\}. \tag{A7}$$

Plugging (A6) into (A4) gives

$$\begin{aligned} \frac{1-\beta^{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)+1}}{1-\beta} &= \left(\frac{\theta_u}{\theta_g} \right)^{\frac{1}{1-\alpha}} (1+\gamma)^{\frac{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)}{1-\alpha}} + \\ p\varphi \frac{\frac{\beta}{(1+\gamma)^{\frac{1}{1-\alpha}}}}{1-\frac{\beta}{(1+\gamma)^{\frac{1}{1-\alpha}}}} &\left[(1+\gamma)^{\frac{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)}{1-\alpha}} - \beta^{\overline{a}_g^{ss}(D)-\overline{a}_u^{ss}(D)} \right] \end{aligned} \tag{A8}$$

Notice that D does not enter (A8), so that, as long as (A8) determines an unique value for $\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)$, (A7) and (A5) (with (A6) plugged in) would jointly determine $\overline{a}_g^{ss}(D)$ and $f^{ss}(0,D)$ with $\overline{a}_u^{ss}(D) = \overline{a}_g^{ss}(D) - \left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$. It turns out that, for (A8) to reveal a unique solution for $\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)$, it requires that $\theta_u < \theta_g$, which holds by definition. This proves Proposition 1. ■

Proof. Proposition 4. Plugging (13) into (12) gives

$$\begin{aligned} c_0 + c_1 f^{ss}(0,D) &= \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} + \\ &\sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a \right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\}, \end{aligned} \tag{A9}$$

which suggests

$$f^{ss}(0,D) = \left(\begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a \right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g} \right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a} \right]^{\frac{1}{1-\alpha}} - 1 \right\} - c_0 \end{array} \right) / c_1. \tag{A10}$$

Combining (A7) and (A10) gives

$$D = \frac{\Psi}{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})} \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g}\right]^{\frac{1}{1-\alpha}} \left\{ \begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} (1-p)^a \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \varphi \left[1-(1-p)^a\right] \alpha^{\frac{\alpha}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} \end{array} \right\} \cdot \tag{A11}$$
$$\left(\begin{array}{c} \sum_{a=0}^{\overline{a}_u^{ss}(D)} \beta^a (1-p)^a \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g}\right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_u}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - 1 \right\} + \\ \sum_{a=0}^{\overline{a}_g^{ss}(D)} \beta^a \varphi \left[1-(1-p)^a\right] \Psi \left\{ \left[\frac{(1+\gamma)^{\overline{a}_g^{ss}(D)}}{\theta_g}\right]^{\frac{1}{1-\alpha}} \left[\frac{\theta_g}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - 1 \right\} - c_0 \end{array} \right) / c_1.$$

where $\overline{a}_u^{ss}(D) = \overline{a}_g^{ss}(D) - \left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$ with $\left(\overline{a}_g^{ss}(D) - \overline{a}_u^{ss}(D)\right)$ determined by (A8) independently. Apparently, the right-hand side of (A11) increases monotonically in $\overline{a}_g^{ss}(D)$. This implies that higher D leads to higher $\overline{a}_g^{ss}(D)$ and $\overline{a}_u^{ss}(D)$. Moreover, the right-hand side of (A10) also increases monotonically in $\overline{a}_g^{ss}(D)$, which suggests that, by causing higher $\overline{a}_g^{ss}(D)$, higher D will also give higher $f^{ss}(0, D)$. This proves Proposition 4. ■

0.1 Approximating Value Functions with Krusell & Smith (1998) Approach

The key computational task is to map F, the plant distribution across ages and idiosyncratic productivity, given demand level D, into a set of value functions $V(\theta^e, a; F, D)$. To make the state space tractable, we define a variable X such that:

$$X(F) = \sum_a \sum_{\theta^e} f(\theta^e, a)\, q(\theta^e, a) \tag{A12}$$

where $f(\theta^e, a)$, as a component of F, measures the mass of plants with expected idiosyncratic productivity θ^e and age a. Apparently,

$$P(F, D)\, A = \frac{D}{X(F')} = \frac{D}{X(H(F, D))}. \tag{A13}$$

F' is the updated plant distribution after entry and exit and $F' = H(F, D)$; $P(F, D)$ is the equilibrium price in a period with initial aggregate state (F, D). Plugging (A13) into (4) gives

$$\pi(a, \theta; F, D) = (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}) \left[\frac{D}{X(H(F, D))}\right]^{\frac{1}{1-\alpha}} \left[\frac{\theta^e}{(1+\gamma)^a}\right]^{\frac{1}{1-\alpha}} - \Psi. \tag{A14}$$

Thus, the aggregate state (F, D) and its law of motion help plants to predict future profitability by suggesting sequences of X's from today onward under different paths of demand realizations. The question then is: what is the plant's critical level of knowledge of F that allows it to predict the sequence of $X's$ over time? Although plants would ideally have full information about F, this is not computationally feasible.

Ω	$\{X\}$
H_Ω	$H_x(X, D_h)$: $\log X' = 0.0321 + 0.9837 \log X$ $H_x(X, D_l): \log X' = 0.0568 + 0.9354 \log X$
R^2	for D_h: 0.9999 for D_l: 0.9924
standard forecast error	for D_h: $5.4 \times 10^{-7}\%$ for D_l : $7.7 \times 10^{-7}\%$
maximum forecast error	for D_h: $3.24 \times 10^{-6}\%$ for D_l: $4.38 \times 10^{-6}\%$
Den Haan & Marcet test statistic (χ^2_7)	0.4336

Table 1: The Estimated Laws of Motion and Measures of Fit

Therefore we need to find an information set Ω that delivers a good approximation of plants' equilibrium behavior, yet is small enough to reduce the computational difficulty.

We look for an Ω through the following procedure. In step 1, we choose a candidate Ω. In step 2, we postulate perceived laws of motion for all members of Ω, denoted H_Ω, such that $\Omega' = H_\Omega(\Omega, D)$. In step 3, given H_Ω, we calculate plants' value functions on a grid of points in the state space of Ω applying value function iteration, and obtain the corresponding industry-level decision rules − entry sizes and exit ages across aggregate states. In step 4, given such decision rules and an initial plant distribution. We simulate the behavior of a continuum of plants along a random path of demand realizations, and derive the implied aggregate behavior — a time series of Ω. In step 5, we use the stationary region of the simulated series to estimate the *implied* laws of motion and compare them with the *perceived* H_Ω; if different, we update H_Ω, return to step 3 and continue until convergence. In step 6, once H_Ω converges, we evaluate the fit of H_Ω in terms of tracking the aggregate behavior. If the fit is satisfactory, we stop; if not, we return to step 1, make plants more knowledgeable by expanding Ω, and repeat the procedure.

We start with $\Omega = \{X\}$ — plants observe X instead of F. We further assume that plants perceive the sequence of future coming $X's$ as depending on nothing more than the current observed X and the state of demand. The perceived law of motion for X is denoted H_x so that $X' = H_x(X, D)$. We then apply the procedure described above and simulate the behavior of a continuum of plants over 10000 periods. The results are presented in Table 5.

As shown in Table 5, the estimated H_x is log-linear. The fit of H_x is quite good, as suggested by the high R^2, the low standard forecast error, and the low maximum forecast error. The good fit when $\Omega = \{X\}$ implies that plants perceiving these simple laws of motion make only small mistakes in forecasting future prices. To explore the extent to which the forecast error can be explained by variables other than X, we implement the Den Haan and Marcet (1994) test using instruments $[1, X, \mu_a, \sigma_a, \gamma_a, \kappa_a, r_u]$, where μ_a, σ_a, γ_a, κ_a,r_u are the mean, standard deviation, skewness, and kurtosis of the age distribution of plants, and

Ω	$\{X, \sigma_a\}$
H_Ω	booms ($\log X$): $\log X' = -1.0406 + 0.9954 \log X + 0.1262\sigma_a$ booms(σ_a): $\sigma_a' = 0.2785 - 0.0068 \log X + 0.9754\sigma_a$ recessions($\log X$): $\log X' = -1.0371 + 0.9963 \log X + 0.8988\sigma_a$ recessions(σ_a): $\sigma_a' = 0.2775 - 0.0065 \log X + 0.9751\sigma_a$
R^2	booms ($\log X$): 0.9999 recessions($\log X$): 0.99999 booms (σ_a): 0.9989 recessions(σ_a): 0.9990
standard forecast error	booms ($\log X$): $1.1 \times 10^{-8}\%$ recessions($\log X$): $1.2 \times 10^{-8}\%$ booms (σ_a): $6.4 \times 10^{-9}\%$ recessions(σ_a):$6.25 \times 10^{-9}\%$
maximum forecast error	booms ($\log X$): $4.87 \times 10^{-8}\%$ recessions($\log X$):$5.05 \times 10^{-8}\%$ booms (σ_a):$1.48 \times 10^{-8}\%$ recessions(σ_a):$1.51 \times 10^{-8}\%$
Den Haan & Marcet test statistic $\left(\chi_7^2\right)$	0.4375

Table 2: The Estimated Laws of Motion with two moments and Measures of Fit

the fraction of unsure plants, respectively. The test statistic is 0.4336, well below the critical value at the 1% level. This suggests that given the estimated laws of motion, we do not find much additional forecasting power contained in other variables. Nevertheless, we expand Ω further to include σ_a, the standard deviation of the age distribution of firms. The results when $\Omega = \{X, \sigma_a\}$ are shown in Table 1.