Comparing solutions to the linking problem using an integrated quantitative framework of language acquisition

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Abstract

To successfully learn language – and more specifically how to use verbs correctly – children must solve the linking problem: they must learn the mapping between the thematic roles specified by a verb’s lexical semantics and the syntactic argument positions specified by a verb’s syntactic frame. One central debate in the linguistics literature is whether this mapping is innately specified (innate-mapping) or learned (derived-mapping) during language acquisition. We use an empirically-grounded and integrated quantitative framework involving corpus analysis, experimental meta-analysis, and computational modeling to implement minimally distinct versions of innate-mapping and derived-mapping approaches and two different types of thematic role systems (categorical vs. relative). Using successful verb class learning as an evaluation metric, we embed each approach within a concrete model of the acquisition process and see which learning assumptions are able to match children’s verb learning behavior at three, four, and five years old. Our results support a trajectory where children (i) do not have built-in expectations about linking patterns to begin with, instead developing them over time, and (ii) begin with a relative thematic system, progressing towards optionality between a relative and a categorical system. We discuss implications of our results for both theories of syntactic representation and theories of how those representations are acquired.

1 Introduction

To successfully learn how to use a verb, children must learn (at least) three pieces of information: (i) the syntactic properties of the verb, such as the syntactic frames that it can appear in, (ii) the lexical semantics of the verb, including the thematic roles assigned by the verb, and (iii) a mapping between the thematic roles specified by the verb’s lexical semantics and the syntactic argument positions specified by the verb’s syntactic frame(s). The learning of this third component is often called the linking problem.

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At the level of individual verbs and individual syntactic frames, the linking problem doesn’t appear to be much of a problem. We might imagine that children simply learn the mapping between thematic roles and syntactic positions for each combination of a verb and syntactic frame one at a time. However, this doesn’t account for children’s ability to generalize their knowledge to new verbs. That is, if the linking between thematic roles and syntactic positions is only ever learned on a verb-by-verb basis, how could children use a new verb appropriately without hearing all its possible uses? Because children are capable of generalizing linking patterns from one verb to another (sometimes incorrectly during the course of development) (Gropen, Pinker, Hollander, Goldberg, & Wilson, 1989; L. Naigles, 1990; L. G. Naigles & Kako, 1993; Gelman & Koenig, 2001; Bunger & Lidz, 2004; Huttenlocher, Vasilyeva, & Shimpi, 2004; Kidd, Lieven, & Tomasello, 2006; Conwell & Demuth, 2007; Papafragou, Cassidy, & Gleitman, 2007; Bunger & Lidz, 2008; Thothathiri & Snedeker, 2008; Scott & Fisher, 2009; Yuan & Fisher, 2009; Kidd, Lieven, & Tomasello, 2010; Becker, 2014; Hartshorne, Pogue, & Snedeker, 2015), it seems they must be learning linking patterns at a more abstract level.

The additional complexity of the linking problem becomes apparent when we consider the broader linking patterns that we see cross-linguistically. Two core linking patterns emerge (we will be using SMALL CAPITALS to indicate thematic roles and italics to indicate syntactic positions):

(i) For the vast majority of verbs in accusative languages, AGENT-like thematic roles tend to appear in syntactic subject position, PATIENT-like thematic roles tend to appear in syntactic object position, and INSTRUMENT/SOURCE/GOAL-like roles tend to appear in oblique syntactic positions such as indirect object or object of PP.

(ii) Exceptions to this pattern tend to be contained within very specific semantic classes of verbs (see section 2 for examples).

How and why does this regularity in linking patterns emerge? There are currently two general approaches. The first is that the linking patterns could result from children possessing relatively direct innate knowledge of the linking patterns themselves, such that the linking pattern does not need to be learned during development. We will call these innate-mapping approaches. The second possibility is that the linking patterns could derive from the interplay between the input that children receive and the learning mechanisms underlying verb learning. We will call these derived-mapping approaches. Our primary goal in this paper is to compare innate-mapping and derived-mapping approaches to the acquisition of linking patterns.

To empirically compare these two approaches, we must create a framework that meets two criteria: it must be possible to (i) systematically manipulate the presence or absence of innate knowledge of linking patterns, and (ii) evaluate both approaches on a neutral metric of success. Note that achieving knowledge of the linking pattern itself can’t be the metric of success because the innate-mapping approach builds that pattern into the learner directly, and thus would automatically “win” under such a metric. With this in mind, we propose to measure success by assess-

\footnote{Here and throughout we set aside the specific challenges posed by “deep” ergative languages to the characterization of the linking pattern. This is so that we can abstract away from the debate between monostratal and multistratal approaches to syntax when we establish the first set of results from the framework proposed here. We hope to integrate these higher-order questions into future work, as any general solution to the linking problem and verb class learning must also account for “deep” ergative languages.}
ing one prominent type of acquired knowledge that relies on learning linking patterns: whether developmentally-attested verb classes can be learned from the data children encounter, given a computationally modeled child implementing either an innate-mapping or derived-mapping approach to the linking problem. In other words, we will use an argument from acquisition to evaluate theories of knowledge representation (Pearl, Ho, & Detrano, 2016; Pearl, 2017) for linking patterns.

Though the verb class learning literature and the linking pattern literature do not always intersect (presumably because the verb class learning literature focuses on development, and the linking pattern literature focuses on adult end states), we believe that verb class learning is a useful common denominator for evaluating the two major approaches for several reasons. First, the linking pattern is defined over verb classes (see section 2). Therefore, under the assumption that constraints on the adult end state are also operative during language learning, it is plausible to expect modeled learners that incorporate those end state constraints to better match the observed developmental trajectory of human children. Exactly how well innate knowledge aids a modeled learner in achieving children’s observed verb class learning behavior is an empirical question – one that we investigate here using our integrated quantitative framework of the acquisition process.

For this study, we explore two of the most prominent innate-mapping approaches in the literature that are built on cognitively plausible assumptions about the thematic role systems available to children during development: the Uniformity of Theta Assignment Hypothesis (UTAH: Baker 1988, building on Perlmutter and Postal 1984), and the relativized Uniformity of Theta Assignment Hypothesis (rUTAH: Larson 1990, Speas 1990, Grimshaw 1990). We contrast these with derived-mapping versions that use the same thematic systems, but which don’t build in knowledge of how to map to syntactic positions. In this way, these two derived-mapping approaches are minimally different from UTAH and rUTAH, leveraging the cognitively plausible approaches to thematic systems but without the added assumption of innate linking patterns. Our results therefore contribute to two sets of debates: the debate between innate-mapping and derived-mapping approaches, and the debate about the details of the thematic role system.

Within our integrated quantitative framework, we create computationally modeled learners that rely on different combinations of assumptions (e.g., innate-mapping vs. derived-mapping, UTAH vs. rUTAH). The framework uses existing corpus and behavioral data to quantitatively specify the input available to children at different ages as well as their developing knowledge of verb classes at ages three, four, and five. These empirical data therefore determine the input for each modeled learner and the target output knowledge. The modeled learners use hierarchical Bayesian inference to infer verb classes from realistic input distributions, and these inferred verb classes are compared against the target verb classes at different ages.

The rest of this paper is organized as follows. We first discuss the linking problem in more detail, along with the theoretically-motivated solutions mentioned above: the innate-mapping UTAH and rUTAH, and their derived-mapping equivalents. We then discuss our use of verb class learning as a neutral evaluation for comparing innate-mapping and derived-mapping approaches. We also present the verb classes that children have acquired by ages 3, 4, and 5, as derived from a review of 37 studies from the experimental acquisition literature; we additionally review the verb behaviors examined in those studies, where verb behavior refers to which syntactic frames a verb
can appear in, and the thematic role information of its arguments within each frame. We subsequently introduce our acquisition modeling framework, highlighting (i) the components necessary to implement a modeled learner who attempts to learn verb classes, and (ii) how different learning assumptions impact a modeled learner. This includes discussion of how a modeled learner interprets the syntactic and conceptual information available in the input, as well as the empirical data from the CHILDES Treebank (Pearl & Sprouse, 2013a) that the modeled learner’s input is based on. We also discuss the hierarchical Bayesian inference process that allows the modeled learner to use the available input to infer verb classes.

Our first key finding is that all modeled learners perform relatively well, which affirms that verb classes can be probabilistically learned from relatively sparse linguistic and conceptual information, as opposed to requiring richer information. Our second key finding is that the modeled learner (and therefore, the specific learning assumption combination) that best matches children’s verb class knowledge changes over time. Here, we assume that the progression of learning assumptions that best matches children’s verb class knowledge is a reasonable reflection of children’s true learning assumptions. With this as a working hypothesis about children’s underlying knowledge, our results support a developmental trajectory that begins at three years old with a rUTAH-like thematic system and no built-in mapping; it progresses towards optionality between UTAH- and rUTAH-like thematic systems with an expectation of a mapping that looks like the general linking pattern at five years old. We discuss the implications of our results for syntactic theory, acquisition theory, and future experimental and computational studies of verb learning.

2 The linking problem and its potential solutions

2.1 A brief introduction to the linking problem

As mentioned above, the linking problem is predicated upon two observations. First, there seems to be a primary pattern robustly observed cross-linguistically (see Baker 1997 for a review), as shown in the English examples in (1). This pattern has AGENT-like roles in the syntactic subject position, PATIENT-like roles in syntactic object position, and INSTRUMENT/SOURCE/OBJECT-like roles in oblique syntactic positions.

(1) The primary pattern

a. Jack cut the pie with a knife.
   \(subject = AGENT, object = PATIENT, object of PP = INSTRUMENT\)

b. Jack stole the jewels from the store.
   \(subject = AGENT, object = PATIENT, object of PP = SOURCE\)

c. Lily sent the letter to her parents.
   \(subject = AGENT, object = PATIENT, object of PP = GOAL\)

Second, verbs that are exceptions to this primary pattern tend to form well-defined semantic classes (again, see Baker 1997 for a brief review). For instance, in English, one example is the semantic class known as psych-verbs, which involve one of the verb arguments experiencing a psychological or mental state (see Postal [1971], Belletti & Rizzi [1988] and Dowty [1991] among many
The psych-verb pair in (2) involves two verbs, *fear* and *frighten*, that have very similar lexical semantics but nonetheless yield two distinct linking patterns: the EXPERIENCER of the psychological state and the apparent CAUSER of the psychological state alternate syntactic positions. Interestingly, we don’t tend to find this sort of alternation for verbs from other semantic classes.

(2) Psych-verb examples
a. Lily fears spiders.
   \( \text{(subject} = \text{EXPERIENCER, object} = \text{CAUSER}) \)

b. Spiders frighten Lily.
   \( \text{(subject} = \text{CAUSER, object} = \text{EXPERIENCER}) \)

A second example of exceptions connected to semantically-defined verb classes involves *split-intransitivity*, where intransitive verbs can be subdivided into two or more subclasses (sometimes called *unergative* and *unaccusative*) that are derived from the lexical semantics of the verbs (Perlmutter 1978; Burzio 1986; Levin & Rappaport Hovav 1995; Sorace 2000). In English, we can see this in the examples in (3): unergative *sneeze* maps an AGENT to the subject position while unaccusative *arrive* maps a PATIENT to the subject position.

(3) Split-intransitivity examples
a. Jack sneezed during the meeting.
   \( \text{(subject} = \text{AGENT}) \)

b. The package arrived during the meeting.
   \( \text{(subject} = \text{PATIENT}) \)

The regularity of the primary pattern cross-linguistically and the semantic coherence of the exceptions to it have spurred theories of representation that compactly encode this regularity. From a representational standpoint, this compact representation would allow easier storage and use of the relevant knowledge linking thematic roles to syntactic positions. From a developmental standpoint, this compact representation would helpfully constrain children’s hypotheses and so enable them to solve the linking problem more quickly (Pearl et al. 2016; Pearl 2017).

As mentioned above, developmental approaches diverge on whether this linking pattern representation is available innately (innate-mapping) or is instead derived from language experience (derived-mapping). For innate-mapping approaches, the primary pattern comes for free and children learn exceptions (such as certain psych-verbs and split-intransitivity verbs) through language experience, drawing on learned knowledge of lexical semantics and specific grammatical mechanisms (e.g., the movement operation in Government-and-Binding (GB) theory/Minimalism). In contrast, for derived-mapping approaches, all linking patterns (both the primary one and any exceptions) are inferred from experience with particular verbs. General mechanisms of abstraction allow children to generalize across verbs and learn any linking patterns that exist, based on the input available.

To be clear, there can be significant variability among the theories within each type: innate-mapping theories can vary substantially in how they capture the exceptions to the mapping (Fillmore 1968; Perlmutter & Postal 1984; Jackendoff 1987; Larson 1988; Grimshaw 1990; Larson 1990;
Speas, 1990; Dowty, 1991; Baker, 1997), and derived-mapping theories can vary substantially in how they capture regularities (Bowerman, 1988; Tomasello, 1992; Braine & Brooks, 1995; Goldberg, 1995; Tomasello, 2003; Goldberg, 2006; Boyd & Goldberg, 2011; Goldberg, 2013). Given this, we intend to compare modeled learners instantiating (i) innate-mapping and derived-mapping approaches, and (ii) more fine-grained differences within each approach. To that end, we have focused on two prominent innate-mapping solutions, UTAH and rUTAH, and their derived-mapping counterparts; these approaches rest on cognitively plausible assumptions about the complexity of the thematic system available during development (discussed in section 4.2.2) but differ in the thematic system details.

2.2 The Uniformity of Theta Assignment Hypothesis (UTAH)

UTAH (Fillmore, 1968; Perlmutter & Postal, 1984; Jackendoff, 1987; Baker, 1988; Grimshaw, 1990; Speas, 1990; Dowty, 1991; Baker, 1997) has two components: (i) an inventory of thematic roles that will be used for the calculation of syntactic position, and (ii) an expected mapping between each of the thematic roles and syntactic positions. Here, we assume the implementation from Baker (1997), which posits an inventory of three thematic (macro)roles: proto-AGENT, proto-PATIENT, and OTHER. This implementation is agnostic about the existence of finer-grained thematic roles at a semantic level. All it requires is that any finer-grained typology of thematic roles map to the three macroroles that are necessary for the syntactic calculation. In this way, UTAH represents a categorical approach to the thematic system, where each macrorole is a thematic category. Under this implementation, thematic roles that tend to involve internal causation (Levin & Rappaport Hovav, 1995) map to proto-AGENT, roles that tend to involve external causation (Levin & Rappaport Hovav, 1995) map to proto-PATIENT, and all other roles map to OTHER. Example (4) lists 13 common finer-grained thematic roles from the literature, and how they would map to the three macroroles in this implementation.

(4) Example UTAH mapping with three fixed macroroles

a. proto-AGENT: AGENT, CAUSER, EXPERIENCER (when internally-caused), POSSESSOR
b. proto-PATIENT: PATIENT, THEME, EXPERIENCER (when externally-caused), SUBJECT MATTER
c. OTHER: LOCATION, SOURCE, GOAL, BENEFICARO, INSTRUMENT

Baker’s (1997) implementation assumes that the proto-AGENT role maps to the syntactic subject position, the proto-PATIENT role maps to the syntactic object position, and that the OTHER role maps to oblique object positions (such as object of PP).

To see this UTAH implementation in action, we can apply it to examples of primary and exceptional patterns, as shown in (5a)-(5e).

(5) Example UTAH utterance mappings

a. Available roles in Jack cut the pie with a knife = AGENT, PATIENT, INSTRUMENT
AGENT = proto-AGENT, PATIENT = proto-PATIENT, INSTRUMENT = OTHER

b. Available roles in Lily fears spiders
   = EXPERIENCER, SUBJECT MATTER
   EXPERIENCER = proto-AGENT, SUBJECT MATTER = proto-PATIENT

c. Available roles in Spiders frighten Lily
   = CAUSER, EXPERIENCER
   CAUSER = proto-AGENT, EXPERIENCER = proto-PATIENT

d. Available roles in Jack sneezed during the meeting
   = AGENT
   AGENT = proto-AGENT

e. Available roles in The package arrived during the meeting
   = PATIENT
   PATIENT = proto-PATIENT

For primary pattern sentences like (5a), the subject of each sentence is a proto-AGENT, the direct object of each sentence is a proto-PATIENT, and the oblique object of each sentence is OTHER. For the psych-verbs in (5b)-(5c) this implementation of UTAH leverages the internal-vs-external causation distinction: in Lily fears spiders, Lily is causing her own mental state, and is thus a proto-AGENT; in Spiders frighten Lily, spiders are causing Lily’s mental state, and thus Lily is the proto-PATIENT. For the unergative sneezed in (5d), Jack is the proto-AGENT, and mapped to the subject. For the unaccusative arrived in (5e) this implementation would claim that the package enters the syntactic derivation as the object of arrive, thus respecting UTAH. The package would then be moved to the subject position by an additional mechanism (such as the movement operation in GB/Minimalism).

2.3 The relativized Uniformity of Theta Assignment Hypothesis (rUTAH)

rUTAH (Larson, 1988, 1990; Grimshaw, 1990; Speas, 1990) also has two components: (i) a hierarchy of thematic roles that will be used for the calculation of syntactic position, and (ii) an expected mapping between the relative position of thematic roles on the hierarchy and syntactic positions. The basic idea is that for any given utterance, the rUTAH calculation requires the learner to first determine an ordering relation among the utterance’s thematic roles, based on a previously-established thematic role hierarchy. This hierarchy is presumably based on some sort of relative salience of the different thematic roles, possibly even outside of the domain of language itself (though most rUTAH-based analyses leave open the etiology of the thematic role hierarchy). The learner can then use that ordering relation of the utterance’s roles to map each role to a syntactic position: the thematic role that is highest in the hierarchy will map to the (structurally) highest syntactic position, the next highest thematic role will map to the next highest syntactic position, and so on. Here, we created a thematic role hierarchy based on the hierarchies developed in Larson (1988, 1990) using the 13 common thematic roles from the literature mentioned above. This hierarchy is given in (6a) and example utterance mappings are in (6b)-(6f). Note that some roles may not be strictly ordered with respect to each other in the hierarchy. For instance, LOCATION and SOURCE are equally salient in the hierarchy in (6a).
For this implementation, we assume that syntactic *subjects* are structurally higher than syntactic *objects*, which in turn are higher than oblique objects.

(6)  
a. Hierarchy:
   AGENT > CAUSER > EXPERIENCER > POSSESSOR >
   SUBJECT MATTER > CAUSEE > THEME > PATIENT >
   (LOCATION, SOURCE, GOAL, BENEFAC'TOR, INSTRUMENT)

b. Available roles in *Jack cut the pie with a knife*
   = AGENT, PATIENT, INSTRUMENT
   AGENT > PATIENT > INSTRUMENT
   AGENT = HIGHEST, PATIENT = 2ND-HIGHEST, INSTRUMENT = 3RD-HIGHEST

b. Available roles in *Lily fears spiders*
   = EXPERIENCER, SUBJECT MATTER
   EXPERIENCER > SUBJECT MATTER
   EXPERIENCER = HIGHEST, SUBJECT MATTER = 2ND-HIGHEST

c. Available roles in *Spiders frighten Lily*
   = CAUSER, EXPERIENCER
   CAUSER > EXPERIENCER
   CAUSER = HIGHEST, EXPERIENCER = 2ND-HIGHEST

d. Available roles in *Jack sneezed during the meeting*
   = AGENT
   AGENT = HIGHEST

e. Available roles in *The package arrived during the meeting*
   = PATIENT
   PATIENT = HIGHEST

For primary pattern sentences like [(6b)], there are three thematic roles: AGENT, PATIENT, and INSTRUMENT. The thematic hierarchy places them in that order (AGENT > PATIENT > INSTRUMENT), so they map to *subject*, *object*, and oblique *object* positions respectively. For psych-verbs like *fear* in [(6c)], rUTAH would posit that *Lily* is an EXPERIENCER, while *spiders* is a SUBJECT MATTER. As such, *Lily* will map to the *subject* position, and *spiders* will map to the *object* position. In contrast, for psych-verbs like *frighten* in [(6d)], rUTAH would posit that *spiders* is now a CAUSER, though *Lily* is still an EXPERIENCER. Because CAUSER > EXPERIENCER, *spiders* will map to the *subject* position, and *Lily* will map to the *object* position. Finally, for the intransitive verbs *sneezed* and *arrived* in [(6e)-(6f)], both verbs only have one syntactic position and one thematic role, and so the argument appears in *subject* position regardless of its thematic role.

2.4 UTAH vs. rUTAH

To be clear, the implementations of UTAH and rUTAH that we adopt here are just two of many possible implementations of these theories. We don’t believe that there’s anything special about the specific implementations that we chose (and future studies should investigate other implementations). What’s critical for our purposes is that UTAH and rUTAH represent two distinct types of
theories that are cognitively plausible given what we know about language development. That is, to map thematic roles onto syntactic positions, children are likely to either (i) make a small number of coarse intermediate categories of thematic roles corresponding to proto-roles, or (ii) view some roles as more salient than others, and order roles accordingly. In each case, the critical step is limiting the number of thematic roles that children must attend to and track statistically, either in absolute terms, or in relative terms. That said, we do believe that the implementations of UTAH and rUTAH that we have chosen for our models are relatively representative of the theory types as a whole, at least as far as the two theories are represented in the theoretical literature.

2.5 Derived-mapping equivalents of UTAH and rUTAH

Derived-mapping approaches don’t postulate any expected mapping between thematic roles and syntactic positions at the beginning of acquisition. Instead, some verbs and their linking patterns are first learned in isolation; then, over time, if enough verbs are learned with the same properties, a class is formed via general-purpose learning mechanisms that allows these linking patterns (and other verb behaviors) to generalize. In other words, over time, children will build verb classes that can be used to make predictions about novel verbs. In this way, derived-mapping approaches can capture both the regularities and the exceptions that we observe within and across languages. Both result from different verb classes derived in a bottom-up way from experience. More specifically, children learn patterns associated with individual verbs, create verb classes based off of those verbs, and then generalize to more abstract patterns (e.g., an expected linking pattern within a given verb class). So, children derive an expectation for linking pattern mappings over time, rather than being innately equipped with this expectation.

2.6 Evaluating the expectation for a mapping: innate vs. derived

To evaluate the role of expected mappings in acquisition, we begin with the thematic role systems from either UTAH (an absolute set of 3 roles) or rUTAH (a relative hierarchy), and manipulate the presence (innate-mapping) or absence (derived-mapping) of an expectation between thematic roles and syntactic positions. A key motivation for investigating UTAH and rUTAH is that they are prominent innate-mapping approaches, and can easily generate minimally different derived-mapping versions. It is also the case that recent work in the theoretical syntax literature has uncovered potential empirical challenges to UTAH and rUTAH among accusative languages (Wood, 2015; Kastner, 2016; Myler, 2016), prompting interesting re-evaluations of solutions to the linking problem. We can contribute to these re-evaluations by examining acquisition-based evidence for UTAH and rUTAH.
3 Verb classes as an evaluation metric

3.1 Verb classes defined by verb behaviors

To compare innate-mapping and derived-mapping approaches to each other as well as compare competing thematic role systems, we evaluate the different approaches on a shared goal: the acquisition of developmentally observed verb classes. The predominant approach to defining verb classes in the literature (e.g., Levin 1993) is by verb behavior: which syntactic frames a verb can appear in, as well as the thematic role information of its arguments within each frame. For example, both want and seem can appear in the syntactic frame NP V IP_{finite} (e.g., Jack wants/seems to laugh). However, want gives the subject NP Jack an EXPERIENCER role while seem gives the subject NP no role (instead, that NP’s role comes only from the embedded verb). We additionally include animacy information of a verb’s arguments (e.g., Jack is +animate) as part of a verb’s behavior. (See section 4.2 for the developmental motivation to include animacy information). A verb class can then be defined as a distribution over verb behavior, i.e., the combination of syntactic frames, positional thematic roles, and animacy of arguments a verb appears with. For example, one verb class during the course of development may consist of the verbs that are known only to be passivizable by a certain age (+passive): they appear with the syntactic frame NP be/get V_{participle} (e.g., The cookie got eaten), and in that frame, the subject NP is the PATIENT though the NP can be either +/-animate. Another verb class may consist of verbs known to both passivizable and able to take a non-finite to sentential complement (+passive, +non-finite to): they exhibit the passive behavior noted above, and in addition allow the syntactic frame NP V IP_{finite}.

3.2 Target states: Verb classes known by children at different ages

To evaluate the performance of our modeled learners, we need to establish a target knowledge state for them to reach. We are also interested in the developmental trajectory of verb class knowledge, and so wanted to assess a modeled learner’s ability to capture child knowledge at different ages. Importantly, English child verb classes may well differ from English adult verb classes and so we use the experimental acquisition literature on children’s comprehension and production of verbs as evidence of children’s knowledge of verb classes.

To derive those verb classes, we first did a meta-analysis of 37 articles from the experimental acquisition literature (see Appendix A). Based on this, we extracted (i) the set of verbs that children comprehend and/or produce at different ages, and (ii) the set of verb behaviors that are associated with these verbs at those ages. This meta-analysis yielded 12 verb behaviors (see Table 1) for 86 verbs that can be used to define child verb classes in English. The full results of our meta-analysis of the experimental literature can be found in Appendix A.

Because the input data available to our modeled learners from the CHILDES Treebank (Pearl & Sprouse 2013b) range up to five years old, we focused on the verb classes children seem to know by age three, four, and five. (These corpus data are discussed in more detail in section 4.2.) We additionally restricted these classes to verbs appearing five or more times in the age-appropriate input sets for three-, four-, and five-year-olds, with the idea that a modeled learner could infer something from the distribution of verbs appearing at least this often. This resulted in the verbs and
Table 1: Verb behaviors associated with specific verbs from child behavioral study meta-analysis.

<table>
<thead>
<tr>
<th>Verb behavior</th>
<th>Example</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaccusative</td>
<td>The ice melted.</td>
<td>Intransitive (NP V) frame where subject is PATIENT.</td>
</tr>
<tr>
<td>Ditransitive</td>
<td>Jack sent Lily the apple.</td>
<td>Verb allows double object construction (NP V NP NP).</td>
</tr>
<tr>
<td>Passivizable</td>
<td>Jack was tricked/laughed at.</td>
<td>Verb allows passive frame (NP be/get (V_{\text{participle}})) where subject is PATIENT of verb or verbal complex.</td>
</tr>
<tr>
<td>Control object</td>
<td>Lily asked him to escape.</td>
<td>Embedded subject is GOAL of matrix verb and AGENT of embedded verb in NP V NP IP(_{-\text{finite}}) frame.</td>
</tr>
<tr>
<td>Raising object</td>
<td>Lily wanted him to escape.</td>
<td>Embedded subject is AGENT of embedded verb only in NP V NP IP(_{-\text{finite}}) frame.</td>
</tr>
<tr>
<td>Control subject</td>
<td>Jack tried to escape.</td>
<td>subject is AGENT of matrix verb and embedded verb in NP V IP(_{-\text{finite}}) frame.</td>
</tr>
<tr>
<td>Raising subject</td>
<td>Jack happened to escape.</td>
<td>subject is AGENT of embedded verb only in NP V IP(_{-\text{finite}}) frame.</td>
</tr>
<tr>
<td>Psych: Subject experiencer</td>
<td>Jack loved Lily.</td>
<td>subject is EXPERIENER of verb in NP V NP frame.</td>
</tr>
<tr>
<td>Psych: Object experiencer</td>
<td>The giant frightened Jack.</td>
<td>object is EXPERIENER of verb in NP V NP frame.</td>
</tr>
<tr>
<td>Non-finite to complement</td>
<td>I want (him) to go.</td>
<td>Verb allows a non-finite to complement, with or without an embedded subject (NP V (NP) IP(_{-\text{finite}})).</td>
</tr>
<tr>
<td>that-complement</td>
<td>Lily hoped that Jack escaped.</td>
<td>Verb allows finite complement headed by that (NP V CP(_{\text{that}})).</td>
</tr>
<tr>
<td>whether/if -complement</td>
<td>Lily wondered whether Jack escaped.</td>
<td>Verb allows finite complement headed by whether or if (NP V CP(_{\text{whether/if}})).</td>
</tr>
</tbody>
</table>

derived verb classes characterized by different verb behaviors that are summarized in Table 2 for a total of 15-25 verb classes comprising 60-84 verbs from ages three to five. The full description of these child verb classes are in Appendix B (Tables 7, 8, and 9).

One important property of the child verb classes serving as the modeled learner target state is that a specific verb can change its verb class over time (based on child behavior with that verb); this therefore means the content of verb classes can change over time. For example, the class where verbs are known only to be passivizable ([+passive]) at age three contains 20 verbs, while the same class at age four contains 26 verbs (it adds 6 verbs over time). As another example, see belongs to the passivizable ([+passive]) class at age three, the passivizable class that also allows that complements ([+passive, +that-complement]) at age four, and the passivizable class that allows both that and whether/if complements ([+passive, +that-complement, +whether/if-complement]) at age five. We note that because these verb classes are derived from existing behavioral data, the changes to a verb’s class represent either (i) development of verb class knowledge, or (ii) a (current) lack of empirical data about knowledge of verb behavior at younger ages. Under the working assumption that these are developmental changes to verb class knowledge over time, we test our modeled learners at three ages, determining which modeled learners (representing different learning assumption
Table 2: Summary of verb classes derived from child behavioral data for three, four, and five-year-olds. This includes the number of derived verb classes, the number of verbs appearing five or more times in the dataset captured by those classes, and the verb behaviors that comprise the derived verb classes.

<table>
<thead>
<tr>
<th>age</th>
<th># classes</th>
<th># verbs</th>
<th>verb behaviors</th>
</tr>
</thead>
<tbody>
<tr>
<td>3yrs</td>
<td>15</td>
<td>60</td>
<td>unaccusative, ditransitive, non-finite to complement, passivizable, that complement</td>
</tr>
<tr>
<td>4yrs</td>
<td>23</td>
<td>76</td>
<td>unaccusative, ditransitive, non-finite to complement, passivizable, that complement, control object, control subject, psych object experiencer, psych subject experiencer, raising object, raising subject</td>
</tr>
<tr>
<td>5yrs</td>
<td>25</td>
<td>84</td>
<td>unaccusative, ditransitive, non-finite to complement, passivizable, that complement, control object, control subject, psych object experiencer, psych subject experiencer, raising object, raising subject, whether/if complement</td>
</tr>
</tbody>
</table>

combinations) can best match children’s verb class knowledge development.

### 3.3 Metrics for assessing verb class learning

#### 3.3.1 The RI Score

Each modeled learner outputs a clustering of verbs into classes, and we want to assess how well these inferred verb classes match the true verb classes derived from observed child behavior. There are several metrics available for analyzing clustering. One measure we use is the Rand Index (RI; Rand, 1971) because it’s a common measure in the clustering literature and it has an intuitive absolute interpretation.

The RI is a pairwise measure derived from signal detection theory. When considering a pair of verbs, there are two possible true states: the two verbs are clustered together into a single class, or the two verbs are separated into two distinct classes. Similarly, there are two possible modeled learner output states: the two verbs are clustered into a single class, or the two verbs are separated into two distinct classes. Crossing the true and modeled learner output states leads to four possible combinations, as shown in Table 3.

<table>
<thead>
<tr>
<th>True state: Together</th>
<th>True state: Separate</th>
</tr>
</thead>
<tbody>
<tr>
<td>True positive (TP)</td>
<td>True negative (TN)</td>
</tr>
<tr>
<td>False positive (FP)</td>
<td>False negative (FN)</td>
</tr>
</tbody>
</table>

When two verbs are the same kind in the true state, they should be clustered together in the modeled learner output. A true positive (TP) occurs when the modeled learner clusters these verbs together, while a false negative (FN) occurs when the modeled learner separates these verbs. When two verbs are not the same kind in the true state, they should be separated by the modeled learner. A true negative (TN) occurs when the modeled learner does separate them, while a false positive (FP) occurs when the modeled learner clusters them together. The RI is the ratio of correct classifications (true positives and true negatives) to the total number of classifications made (true positives, true negatives, false positives, and false negatives): $\frac{TP+TN}{TP+TN+FP+FN}$. The intuitive appeal of this ratio is that credit is
given both for correctly putting verbs together into the same class and for correctly keeping them separate. The RI ranges between 0 (no classifications are correct) and 1 (all classifications are correct): $0 \leq RI \leq 1$. The interpretation of the RI is intuitive in an absolute sense: an RI of .5 means that half of the classifications were correct; or equivalently, for any randomly chosen verb pair, there is a probability of .5 that the modeled learner’s output will agree with the true state.

Table 3: Signal detection theory distinctions relevant for the Rand index (RI) when applied to a verb pair.

<table>
<thead>
<tr>
<th>True state</th>
<th>Modeled learner output state</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Together</td>
</tr>
<tr>
<td>Together</td>
<td>True Positive</td>
</tr>
<tr>
<td>Separate</td>
<td>False Positive</td>
</tr>
</tbody>
</table>

### 3.3.2 Evaluating an RI score relative to chance

One limitation of the RI is that the distribution of RI scores for any given number of classes is not known; therefore we cannot determine if the RI score we obtain is particularly good for the number of classes there are, or particularly bad. We might therefore want to perform some sort of test that compares the observed RI score to the distribution of RI scores expected by a null hypothesis (either chance, or some other null expectation). One solution to this problem is to use a permutation test: we keep the number and sizes of the classes fixed at the number and sizes in the modeled learner’s output, and randomly assign the verbs to each class. We can then calculate an RI score for this permuted set of classes, which is equivalent to an RI under the null hypothesis that the verbs assigned to each class are exchangeable (while the number and sizes of the classes are fixed). We can repeat this process some large number of times (e.g., 10,000) to estimate a distribution of RI scores under this null hypothesis. We can then calculate the probability of obtaining our observed RI score (or one more extreme) under the null hypothesis using this distribution. Here, we will report the estimated $p$-values, and qualitatively interpret $p$-values less than .01 as significant.

Another common method for evaluating RI scores relative to chance is to convert RI scores into a new measure known as the adjusted Rand Index (ARI) (Hubert & Arabie, 1985). The ARI uses a specific randomness model (the generalized hypergeometric distribution) to calculate an expected value for the specific number and size of classes returned by the modeled learner. The RI is then scaled relative to this expected value such that the expected value is 0, an RI of 1 is still 1, and an RI of 0 becomes -1. The result is the ARI, which ranges between -1 and 1 ($-1 \leq ARI \leq 1$), with scores less than 0 indicating worse than chance performance relative to the randomness model, 0 indicating chance performance relative to the randomness model, and scores greater than 0 indicating better than chance performance. Note that 1 is still perfect performance, as in the original RI. We implemented the ARI calculation using the python function `sklearn.metrics.adjusted_rand_score`.
Both the permutation test approach and the ARI approach come with well-known limitations. The permutation test inherits all the limitations of null hypothesis testing (e.g., less-than-intuitive logic, the winner’s curse, arbitrary selection of critical \(p\)-values, etc). The ARI does not specify how far above 0 counts as a significant departure from the value expected by chance; it also assumes a specific model of randomness that may or may not hold for a given empirical domain. To minimize these individual limitations, we will evaluate both the RI permutation tests and ARI scores independently, and jointly in the discussion that follows.

4 Computationally modeling the acquisition of verb classes

4.1 The acquisition framework

We follow the view that language acquisition is an information-processing task, where children use their available input to build an internal system of linguistic knowledge whose behavioral output we can observe \cite{LidzGagliardi2015, OmakiLidz2015, Pearl2015}. The framework of Pearl (under review), building on that of Lidz and Gagliardi \cite{LidzGagliardi2015} and Omaki and Lidz \cite{OmakiLidz2015}, articulates several crucial components of this task, underscoring how theories of representation and theories of the learning process work together to create a complete theory of acquisition.

For our purposes, there are three crucial pathways. First, there is the input-intake pathway, where the external signal, the input, is encoded by the child into an internal mental representation we will call the linguistic intake\(^2\) The parts of the linguistic intake that are identified by the acquisition system as relevant for acquisition are called the acquisitional intake. For example, an input utterance of *What’s she climbing over?* might be encoded by the child as containing certain syntactic and conceptual information – this is the linguistic intake, which serves as the child’s representation of that utterance at this stage of development. This encoding process will depend on the child’s ability to deploy her existing linguistic and extralinguistic knowledge in real time, given her developing cognitive abilities. The acquisitional intake is the portion of that representation relevant for the acquisition task at hand – for example, perhaps only syntactic structure may be relevant for learning about certain constraints on \(wh\)-dependencies (as in \cite{PearlSprouse2013a}), but perhaps conceptual information may be relevant for learning about the verb argument structure of *climb*. The acquisitional intake is determined by the child’s learning biases about what information is relevant in the linguistic intake. For verb class learning, this pathway will determine how the age-appropriate child-directed speech samples serving as input are transformed into different acquisitional intakes, depending on the modeled learner’s learning assumptions.

The second pathway is the intake-inference pathway, which takes the acquisitional intake and does inference on that intake to generate the most up-to-date hypotheses about the linguistic system encoded by the developing grammar. The exact update procedures used will depend on the child’s current learning biases. For example, a child might use purely statistical inference within a hypothesis space defined in terms of clusters of salient features, or a hypothesis-testing approach

\(^2\)What we call the linguistic intake has been referred to in the framework mentioned above as “perceptual intake” because it is what the child is capable of perceiving from the available input at that point in development; we choose “linguistic” to highlight that this representation includes more than just perceptual information.
within a hypothesis space defined in terms of linguistic parameters. For verb class learning, this pathway will involve hierarchical Bayesian learning that generates the verb classes in the modeled learner’s developing grammar, based on the syntactic, conceptual, and linking information in the acquisitional intake.

The third pathway is the **grammar-behavior** pathway. This pathway describes how the child’s internal representations (encoded by the linguistic intake of the moment and the developing grammar) are transformed into various types of external behavior that we can observe, such as utterance generation, truth-value judgments, or looking times. This depends both on the state of the child’s internal representations and the production systems that operate on those representations to produce observable behavior. For example, an internal representation of *What’s she climbing over?* that involves both syntactic and conceptual information might cause a child to generate the utterance *What’s she dancing on?* using her developing grammar, because the new utterance has similar syntactic and conceptual properties to the utterance in the linguistic intake. For verb class learning, this pathway will involve how the verb classes in the modeled learner’s developing grammar compare to the verb classes derived from observed child behavior at ages three, four, and five.

By using this framework – and, more specifically, these three pathways – we can make theories of acquisition (which involve both theories of representation and theories of the learning process) explicit and testable against available empirical data (Pearl, 2014; Pearl & Sprouse, 2015; Pearl, 2017, under review). Here, this means that we can evaluate different theories of how to solve the linking problem by how well they enable a modeled learner to learn verb classes the way children seem to. More specifically, each modeled learner implements a combination of learning assumptions that corresponds to different theoretical claims (e.g., UTAH vs. rUTAH, innate-mapping vs. derived-mapping). By seeing if a given modeled learner can learn the verb classes children do at different ages, we can evaluate the utility of these assumptions for acquisition.

### 4.2 The input-intake pathway

#### 4.2.1 Input

Children’s input signal can include both linguistic information (e.g., spoken or signed productions) and non-linguistic information (e.g., contextual information about intended meaning). We take realistic samples of this input signal from the CHILDES Treebank (Pearl & Sprouse, 2013a), which contains speech directed at children between one and five years old, annotated with linguistic and non-linguistic information. In particular, around 180,000 child-directed speech utterances from the BrownEve, BrownAdam, and Valian corpora (Brown, 1973; Valian, 1991) have been annotated with syntactic, conceptual, and thematic information. First, these utterances have syntactic phrase structure, based on an adapted version of the Penn Treebank annotation system. This annotation was done using a combination of automated and hand annotation (see Pearl and Sprouse (2013a) and the included readme file at [http://www.socsci.uci.edu/~lpearl/CoLaLab/CHILDESTreebank/childestreebank.html](http://www.socsci.uci.edu/~lpearl/CoLaLab/CHILDESTreebank/childestreebank.html) for details). Second, animacy for each NP argument was annotated by hand. We included animacy because a number of acquisition studies have demonstrated that animacy is a useful cue for learning verb classes (Scott & Fisher, 2009; Becker, 2009; Kirby, 2009a, 2010; Becker & Estigarribia, 2013; Becker, 2014, 2015; Hartshorne et al., 2015). Third, thematic roles...
for the arguments of each verb (except the copula be) were annotated by hand using 13 thematic role labels that are common in the literature (again, see the readme file mentioned above for details).

We divided these utterances into age ranges based on the age of the child the speech was directed at: less than 3 years of age, less than 4 years of age, and less than 5 years of age. We then constructed datasets representing the input to a child of a particular age. We note that the datasets used as input for models of older children (e.g., <4yrs, representing a four-year-old child) include the data directed at younger children (e.g., <3yrs + data directed at children between the ages of three and four). This is because we assume older children would learn from all the data they’ve heard up until that point. Table 4 provides a detailed summary of the statistics for each input dataset.

Table 4: Child-directed speech data used as input to modeled three-year-old, four-year-old, and five-year-old children. This includes the sources of these data in the CHILDES Treebank, the number of children the speech was directed at, the age range of the children the speech was directed at, the total number of utterances and words, the total number of verb types, and the number of verb types appearing 5 or more times in the dataset.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Sources</th>
<th># children</th>
<th>ages</th>
<th># utt</th>
<th># words</th>
<th># vbs</th>
<th># vbs &gt;5</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3yrs</td>
<td>BrownEve, Valian</td>
<td>22</td>
<td>1;6-2;8</td>
<td>≈39.8K</td>
<td>≈197K</td>
<td>555</td>
<td>239</td>
</tr>
<tr>
<td>&lt;4yrs</td>
<td>BrownEve, Valian, BrownAdam3to4</td>
<td>23</td>
<td>1;6-4;0</td>
<td>≈50.7K</td>
<td>≈254K</td>
<td>617</td>
<td>267</td>
</tr>
<tr>
<td>&lt;5yrs</td>
<td>BrownEve, Valian, BrownAdam3to4, BrownAdam4up</td>
<td>23</td>
<td>1;6-4;10</td>
<td>≈56.5K</td>
<td>≈285K</td>
<td>651</td>
<td>285</td>
</tr>
</tbody>
</table>

4.2.2 Linguistic intake

From the input signal, children extract their linguistic intake. The information they extract depends on what information is salient to them and what they can plausibly extract from the input in real time. We consider several types of information children could plausibly extract for learning about verb classes: one syntactic, one conceptual, and one linking conceptual and syntactic information.

Syntactic information seems plausible, as children are known to be adept at syntactic bootstrapping – that is, using the syntactic context – when learning about verbs (Landau & Gleitman, 1985; Gleitman, 1990; Gillette, Gleitman, Gleitman, & Lederer, 1999; Fisher, Gertner, Scott, & Yuan, 2010; Gutman, Dautriche, Crabbé, & Christophe, 2015; Harrigan, Hacquard, & Lidz, 2016). One way to implement syntactic information is via phrase structure, with verb argument positions like subject labeled, as shown in (7a).
Another plausible information source is the concept of animacy (e.g., a penguin is animate, while an ice cube isn’t). Animacy is something young children are known to both be sensitive to as a general property and also use as a cue in experimental studies to predict how verbs will behave (Scott & Fisher, 2009; Becker, 2009; Kirby, 2009a, 2010; Becker, 2014, 2015; Hartshorne et al., 2015). Moreover, if children are able to harness animacy effectively in their input, it’s possible to use the animacy of a verb’s arguments (in particular, whether the argument is inanimate) to distinguish verb behaviors such as those associated with subject-raising, subject-control, object-raising, and object-control (Kirby, 2009a, 2010; Becker & Estigarribia, 2013; Becker, 2014). One way to implement this conceptual information is for the verb’s NP arguments to be labeled as +/-animate, as in (7b).

A third source of information corresponds directly to UTAH and rUTAH, linking conceptual information like thematic roles to syntactic position. More specifically, infants under a year old are sensitive to the presence of thematic roles (<10 months: Gordon, 2003; <6 months: Hamlin, Wynn, & Bloom, 2007; Hamlin, Wynn, Bloom, & Mahajan, 2011), making thematic roles a plausible information source for learning verb classes. Innate-mapping versions of UTAH and rUTAH assume a built-in mapping from the intermediate thematic representation (whether fixed macro-roles like UTAH or an ordered hierarchy like rUTAH) to syntactic positions like subject; derived-mapping versions don’t assume this mapping is present initially. Importantly, all versions require the child to extract the syntactic positions of the verb’s arguments, and be aware of their thematic role, as shown in (7c).

Here, we make the simplifying assumption that the perceptual encoding process creating the linguistic intake is perfectly reliable (an assumption that can be relaxed in future work). Implementationally speaking, this means we assume that when given an input utterance like it’s falling off from the BrownEve corpus in the CHILDES Treebank (Pearl & Sprouse, 2013a), we assume a linguistic intake that encodes syntactic and conceptual information, such as (7).

(7) Example linguistic intake for it’s falling off:

a. Syntactic information for falling:

\[
\text{IP} \to \text{NP}_{\text{subject}} \to \text{PRON} \to \text{AUX} \to \text{VP} \to \text{V}_{\text{prog}} \to \text{PRT} \to \text{falling} \to \text{off}
\]

b. Animacy information:

\[\text{it (subject}_{\text{falling}}) = \text{-animate}\]

c. Thematic information:

\[\text{it (subject}_{\text{falling}}) = \text{THEME}_{\text{falling}}\]
4.2.3 The acquisitional intake

From this linguistic intake, the modeled learners extract their acquisitional intake. The exact acquisitional intake depends on the learning assumptions the learner is using.

For the syntactic information, syntactic frames encoding surface argument structure can be derived from the phrase structure of the verb usage. For example, the utterance *it’s falling off* might yield a frame for *fall* involving the NP *subject* and the particle, either with or without the progressive morphology that surfaces on the verb itself (*+/−surface-morphology*), as in (8). Whether children heed the verbal surface morphology when encoding syntactic frames for their acquisitional intake is an additional point of variation. So, our modeled learners will also therefore vary on whether they encode the verb’s surface morphology in their syntactic frames.

(8) *fall* syntactic frame options for *it’s falling off*

a. *+surface-morphology*: NP \( V_{+prog} \) PRT
b. *−surface-morphology*: NP \( V \) PRT

Another key point of variation is whether the mapping from the intermediate thematic representation is present (as it would be initially in innate-mapping approaches) or not (as it would be initially in derived-mapping approaches). This affects whether the modeled learner expects a mapping *a priori* (*expect-mapping*).

If the modeled learner expects a mapping (+expect-mapping), then it will be sensitive to violations of that expectation. Our modeled learners interpret these violations as instances of movement. That is, the learner will abstract away from the specific roles and positions, and instead take in the fact that movement occurred, as shown in (9b). If instead the modeled learner doesn’t yet expect a mapping (−expect-mapping), the learner will track the distribution of the intermediate thematic representation. That is, the learner will take in the details of which role occurred in which position, as shown in (9c). In this way, the expectation of a mapping that distinguishes innate-mapping vs. derived-mapping approaches directly impacts the learner’s acquisitional intake.

(9) Acquisitional intake for *The ice was melted by the girl*, using *+/−expect-mapping*

a. *The ice* = PATIENT = *subject*
   *the girl* = AGENT = *object of PP*

b. +expect-mapping
   (i) UTAH: proto-PATIENT = *subject*, proto-AGENT = *object of PP*
       Unexpected. Indicates +movement.
       **Acquisitional intake**: 2 movement
   (ii) rUTAH: 2ND-HIGHEST = *subject*, HIGHEST = *object of PP*
       Unexpected. Indicates +movement.
       **Acquisitional intake**: 2 movement

c. −expect-mapping
   (i) UTAH: proto-PATIENT = *subject*, proto-AGENT = *object of PP*
       **Acquisitional intake**: 1 proto-PATIENT as *subject*, 1 proto-AGENT as *object of PP*
   (ii) rUTAH: 2ND-HIGHEST = *Subject*, HIGHEST = *object of PP*
Acquisitional intake:
1 2ND-HIGHEST as subject, 1 HIGHEST as object of PP

The different learning assumptions affecting the learner’s acquisitional intake and their different combinations are shown in Table 5. Given the 3 binary choices (+/-surface-morphology, UTAH/rUTAH thematic system, and +/-expect-mapping), we implement 8 modeled learners. Note that all modeled learners use the animacy of a verb’s arguments, in addition to syntactic frame information and thematic role information. Where they differ is how exactly they use the syntactic frame and thematic role information.

Table 5: The eight potential learning assumption combinations for verb classes, based on the available sources of information and a child’s example linguistic intake from (7) of the utterance it’s falling off. Each combination has different effects on the learner’s acquisitional intake.

<table>
<thead>
<tr>
<th>Learning assumption combinations</th>
<th>Acquisitional intake</th>
</tr>
</thead>
<tbody>
<tr>
<td>All learners use verb argument animacy</td>
<td>subject−anim = 1</td>
</tr>
<tr>
<td>surface morphology</td>
<td>thematic system</td>
</tr>
<tr>
<td>-</td>
<td>UTAH</td>
</tr>
<tr>
<td>-</td>
<td>UTAH</td>
</tr>
<tr>
<td>-</td>
<td>rUTAH</td>
</tr>
<tr>
<td>-</td>
<td>rUTAH</td>
</tr>
<tr>
<td>+</td>
<td>UTAH</td>
</tr>
<tr>
<td>+</td>
<td>UTAH</td>
</tr>
<tr>
<td>+</td>
<td>rUTAH</td>
</tr>
<tr>
<td>+</td>
<td>rUTAH</td>
</tr>
</tbody>
</table>

As an example, let’s consider the different acquisitional intakes for the utterance it’s falling off, whose linguistic intake was shown in (7). All learners encode one instance of an inanimate argument in subject position (subject−anim). So, there is no difference in the acquisitional intake with respect to animacy. For those learners ignoring surface morphology on the verb, only the core verb frame would be extracted: NP V PRT. For learners heeding surface morphology, the fact that the verb is in the progressive would additionally be included: NP V<sub>prog</sub> PRT.

The exact thematic information extracted depends on the thematic system (UTAH/rUTAH): with a UTAH thematic system, the thematic role of the subject (THEME) is mapped to proto-PATIENT; with a rUTAH thematic system, the learner uses the thematic role hierarchy to map the thematic role of the subject (THEME) to the HIGHEST role because it’s the only thematic role present. If there is no expectation of mapping, the learner encodes the distribution of thematic representations (here, proto-PATIENT or HIGHEST in subject position). If there is in fact an expectation of mapping, the learner encodes whether the observed mapping obeys or violates that expectation. For the UTAH representation, a proto-PATIENT in subject position violates the expected mapping and so is interpreted as movement; in contrast, for the rUTAH representation, the HIGHEST role in subject position obeys the expected mapping and so is interpreted as no movement.
4.3 The intake-inference pathway

Each modeled learner uses the acquisitional intake defined by its respective learning assumption combination to update its hypotheses about verb classes; a successful assumption combination will allow the learner to match children’s observable behavior for verb classes. We implement this update process using hierarchical Bayesian inference, where the learner assumes the generative process depicted in Figure 1 (the generative process is represented with standard plate diagram notation for hierarchical Bayesian modeling). The observable verb data V in the acquisitional intake are generated by combining the available syntactic, animacy, and thematic information in the acquisitional intake, mediated by the latent representation of verb classes C.

![Plate diagram for a generative model of verb classes](image)

Figure 1: Plate diagram for a generative model of verb classes, based on syntactic, animacy, and thematic information from individual verbs in the input. Observable verb data V (and specifically verb usage instances $F_{ji}$) are generated based on the underlying verb class information $C$, which involves different characteristics $M$ and $B$ tracked by modeled learners (specifically, multinomial characteristics $\psi$ like syntactic frame information and binomial characteristics $\phi$ like argument animacy information).

Observable data are available for each verb $v_j \in V$, in the form of the frames $F$ that verb is used in, which include the syntactic structure, the animacy of the arguments, and the thematic roles present. For example, the verb *fall* may appear multiple times, in instances such as *it’s falling off*, *she fell down*, *don’t fall!*, and *London Bridge is falling down*. Each frame instance $F_j$ for a verb appears with some frequency $F_{ji}$ – for example, *it’s falling off* might occur three times.

The objective of the modeled learner is to infer the set of verb classes $C$ that generate the observable verb data. Each verb $v_j$ belongs to its verb class $c_j$. The learner doesn’t know beforehand how many verb classes there are, or which verb belongs to which. However, via the verb class hy-
perparameters \( \theta_c \) and \( \gamma_c \), the learner has a bias for fewer classes (see Appendix C for more details on the inference implementation).

Each verb class \( c_j \) has certain binomial characteristics \( B \) and multinomial characteristics \( M \) associated with it. Binary characteristics \( \phi \in B \) include whether the subject, object, and oblique object are animate (+/-animate). If the modeled learner involves an expected mapping, then whether the mapping was violated and so interpreted as movement is also a binary characteristic. Each class will have some probability of preferring each option \( \pi_{\phi c} \). For example, a class \( c_j \) might prefer inanimate to animate subjects, with \( \pi_{-\text{animate, subject}} = 0.70 \) and \( \pi_{+\text{animate, subject}} = 0.30 \). During the course of learning, the learner infers these probabilities for each verb class. The hyperparameters \((\beta_{\phi n}, \beta_{\phi o})\) implement an initial uniform probability over the possible binary options, thereby implementing no bias \textit{a priori}.

Multinomial characteristics \( \psi \in M \) include which syntactic frame a verb appears in (e.g., NP V PRT for \textit{it’s falling down}). If the modeled learner doesn’t assume a mapping between thematic roles and syntactic positions, then the syntactic position is also a multinomial property (e.g., if the proto-agent appears in subject, object, or oblique object position). Each class will have some probability of preferring each option \( \theta_{\psi c} \). For example, a class \( c_j \) might primarily prefer the NP V PRT and NP V syntactic frames, giving them higher probabilities, and disprefer the frame NP V IP \((\theta_{\text{NP V PRT}} = 0.50, \theta_{\text{NP V}} = 0.40, ..., \theta_{\text{NP V IP}} \approx 0.00)\). During the course of learning, the learner infers these probabilities for each verb class. The hyperparameter \( \alpha_{\psi} \) implements an initial uniform probability over the possible multinomial options, thereby implementing no bias \textit{a priori}.

Importantly, the learner infers different verb classes precisely because the characteristics of verb classes differ sufficiently. In particular, given the observed instances of verb usage, the learner uses Bayesian inference to infer (i) how many verb classes there are, (ii) what the characteristics of each verb class are, and (iii) which class each verb belongs to. The best hypothesis is the one that maximizes the probability of the observed data, balanced against the prior preference for fewer verb classes.

This inference is accomplished via Gibbs sampling operating over the data as a single batch (see Appendix C), which is guaranteed to converge on the optimal answer if given sufficient time to search the hypothesis space (i.e., Gibbs sampling is an optimal inference process). This is part of what makes the modeled learners \textit{ideal learners} – the inference computation is implemented by an optimal inference process that is \textit{not} intended to be realistically constrained. Instead, humans likely approximate this inference process to accomplish the same computation and execute inference incrementally as data are encountered.

A reasonable question is why we should use an ideal inference process rather than a realistically constrained process to model language acquisition. Typically, acquisition modelers will start with an optimal inference process in order to know if the mental computation specified by the model is a potential match to human behavior (here, child language acquisition behavior) \textit{(Pearl under review)}. If not, this is a signal that the learning assumptions encoded in the model are unlikely to be right. That is, if a modeled learner can’t get close to human behavior even when the mental computation is performed as perfectly as possible, then that computation is probably not the right one. This would mean the learning assumptions that circumscribe that mental computation (here: using syntactic, animacy, and thematic information in particular ways) are not useful.
In contrast, if a modeled learner using optimal inference can match human behavior, this suggests the learning assumptions are plausible. Subsequent work could then explore how acquisition unfolds when inference is non-optimal (e.g., subject to the cognitive constraints children have and the incremental nature of learning). In the meantime, the ideal learning model using optimal inference serves as a useful proof-of-concept in the search for learning assumptions that can potentially solve the acquisition problem under investigation. More generally, it’s important to determine that learning assumptions are potentially useful to children before investigating if they’re usable by children. This is the approach we pursue here.

4.4 The grammar-behavior pathway

This pathway determines how a modeled learner’s output will be evaluated when the target is observed behavior. In section 3.2, we described the verb classes derived from observed child behavior. It’s reasonable to believe that such verb classes are a legitimate target state reflecting children’s underlying knowledge because of how we think of the grammar-behavior pathway. In particular, we assume here that if children’s comprehension and/or production indicate that they treat two verbs similarly with respect to some verb behavior (e.g., being passivizable), this transparently reflects children’s developing grammars – that is, the two verbs in question are clustered together with respect to that verb behavior. If children’s verb comprehension and/or production indicate two verbs are clustered together for all currently tested verb behaviors for those verbs, then we assume that the verbs are in the same class in children’s developing grammars. This is how the verb classes in Tables 7-9 in Appendix B were derived.

We take these verb classes, derived for children ages three, four, and five, as representative of the developing grammars of children of these ages. So, modeled learner output is compared against them using the measures discussed in section 3.3.

5 Modeling results

Recall that each of the eight modeled learners uses a different combination of learning assumptions, based on how linguistic and non-linguistic information are used (Table 5). For each learner, we ran an ideal learner implementation 10 times over each age-based dataset (<3yrs, <4yrs, or <5yrs). The resulting ten sets of inferred verb clusterings were aggregated into a single set of verb classes, using a simple threshold: any verb pair together in more than 75% of the runs (i.e., >7 of 10) was put together in the aggregate verb clustering; similarly, any verb that was in a class of its own (a singleton) for more than 75% of the run was put as a singleton in the aggregate verb clustering for that modeled learner. Each modeled learner’s aggregate verb clusterings for each dataset appear in Appendix D. Figure 2 shows RI and ARI scores when compared to the behaviorally derived verb classes relevant for different ages of acquisition (i.e., verb classes learned by age three for the <3yrs dataset; verb classes learned by age four for the <4yrs dataset; verb classes learned by age five for the <5yrs dataset). For the RI permutation tests, we use a threshold of p<.01 for significance (two-tailed). We have indicated the threshold for each model with a horizontal line, and added an asterisk (*) for each model that surpasses this threshold.
We turn first to the results using the RI and permutation tests. For the three-year-old modeled learners (<3 years), the ones ignoring surface morphology (-surface-morphology) and using the rUTAH thematic system pass the p<.01 threshold. It doesn’t appear to matter whether the learner expects a mapping between the relative thematic roles and syntactic positions (+expect-mapping) or not (-expect-mapping), so either is possible. For the four-year-old modeled learners (<4 years), the only one to pass the threshold heeds surface morphology (+surface-morphology), uses the UTAH thematic system, and does not expect a mapping between thematic roles and syntactic positions (-expect-mapping). For the five-year-old modeled learners (<5 years), four pass the threshold: two that heed surface morphology and two that ignore it, and two each of UTAH and rUTAH thematic systems. If we assume continuity from four to five years old with respect to heeding surface morphology (+surface-morphology), learners using either thematic system (UTAH or rUTAH) pass the threshold as long as they expect a mapping between thematic roles and syntactic positions (+expect-mapping).

We turn next to the results using the ARI. Because the ARI builds in a randomness model, such that an ARI of 0 represents chance performance, we didn’t perform any statistical tests on the ARI results. Instead, we can compare the relative magnitude of any ARI results that are above 0. For
the three-year-old modeled learners (<3years), all four that ignore surface morphology (-surface-morphology) perform better than the four that heed surface morphology. This accords well with the RI results. The only difference is that with ARI, the best performing modeled learner uses the UTAH thematic system without an expectation of a mapping. For the four-year-old modeled learners (<4years), the best performing one is the same as the best performing one for the RI results: heeding surface morphology, using the UTAH thematic system, and not expecting a mapping. For the five-year-old modeled learners (<5years), the two that appear to substantially outperform the others heed surface morphology (+surface-morphology), use either the UTAH or rUTAH thematic system, and expect a mapping (+expect-mapping). Again, this accords nicely with the RI results. The fact that the RI and ARI results accord well suggests that the choice of randomness model has relatively little impact on the evaluation of the models.

If we take the best performing modeled learner as a working hypothesis about children’s learning assumptions at each age, then these results suggest there are distinct developmental stages for children’s use of these information types. This trajectory is summarized in Table 6. First, there appear to be two stages with respect to verbal surface morphology: children ignore it before age three and use it thereafter. Second, for thematic roles systems, there appears to be a slight advantage for the rUTAH thematic system by age 3 (both systems perform roughly equally by ARI, but the rUTAH thematic system performs better by RI). This switches to an advantage for the UTAH thematic system by age 4, and equivalence between the two thematic systems by age 5. Finally, for the expectation of a mapping between thematic roles and syntactic positions, there appears to be an equivalence by age 3, an advantage for no expectation by age 4, and an advantage for expecting a mapping by age 5.

Table 6: Developmental trajectory of learning assumptions by age based on agreement between RI and ARI scores: whether surface morphology on verbs is heeded for verb syntactic frames (surface-morphology), which thematic system is used (UTAH/rUTAH), and whether a mapping from thematic roles to syntactic positions is expected (expect-mapping).

<table>
<thead>
<tr>
<th>Age</th>
<th>surface-morphology</th>
<th>(r)UTAH</th>
<th>expect-mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;3yrs</td>
<td>-</td>
<td>rUTAH</td>
<td>+/-</td>
</tr>
<tr>
<td>&lt;4yrs</td>
<td>+</td>
<td>UTAH</td>
<td>-</td>
</tr>
<tr>
<td>&lt;5yrs</td>
<td>+</td>
<td>UTAH/rUTAH</td>
<td>+</td>
</tr>
</tbody>
</table>

This developmental trajectory has some intuitive appeal and could also be cognitively plausible. For verbal surface morphology, it may be that children’s processing limitations prevent them from reliably extracting (or noticing) verbal morphology at the level of syntactic frames. So, younger children would ignore verbal surface morphology while older children would heed it. For the thematic role system, it could be that younger children haven’t yet categorized the myriad thematic roles into categories like proto-AGENT and proto-PATIENT, but instead simply rely on a relative sense of how salient a given thematic role is in a given meaning/context. Older children, however, could have formed these broad conceptual categories. The fact that UTAH and rUTAH thematic systems (with an expected mapping) are equivalent for the oldest children accords well with the
literature on adult syntax, as both are considered equal competitors for the explanation of adult knowledge of the linking problem. For the expectation of a mapping, children could begin with no expectation and learn an expectation over time from their input. Once children have a sense of the mapping tendencies, they subsequently use this to guide their expectations of where verb arguments with specific thematic roles ought to appear.

Taken together, we interpret these results as support for the derived-mapping approach to the linking problem: children build up verb classes without an expected mapping (though with some simplifying theories of thematic roles), and only later begin to expect a mapping. This interpretation is of course based on several assumptions, such as a strict mapping between best performance and children’s learning assumptions, and an ontological preference for no expected mapping when the empirical results between no expected mapping and expected mapping are equivalent. In addition, these results support both UTAH and rUTAH thematic systems at different ages, with both as viable options for five-year-olds. We explore this interpretation in more detail in the discussion section.

6 Discussion

6.1 Implications for theories of representation and development

Given the complexity of the learning problem of creating dozens of verb classes, and given that the learning assumptions tested here only involve a subset of all the possible information children could be using to solve that problem, our first noteworthy result is that any of the learning assumption combinations are successful. Though we started this project from the assumption that syntactic frames, thematic information, and animacy information from child-directed input would be sufficient to learn verb frames, there is no a priori reason to believe that this information would be sufficient. So, these results empirically support the common assumption in the literature that these specific pieces of information are sufficient to learn verb classes the way children seem to.

From the perspective of theories of linguistic representation, these results have two implications. First, both UTAH and rUTAH in their classic (+expect-mapping) forms are reasonably accurate at capturing children’s representations at some point in development. This provides developmental support that they are plausible representational theories. Moreover, because they are compatible with the oldest children’s verb behavior (at five years old), they are also plausible representational theories for adults.

However, one particularly notable finding is that not all the options capture younger children’s behavior equally (at three and four years old). The results here suggest three-year-olds are more likely to rely on a rUTAH-like thematic system, and four-year-olds are more likely to use a UTAH-like thematic system and not yet expect a mapping between their thematic roles and syntactic positions. This has real implications for what needs to be built in to yield the linguistic development we observe in children. Here, it seems that the conceptual categories corresponding to protoroles are not required (as in the UTAH thematic system), nor is an expectation of a mapping between thematic roles and syntactic positions (as in the innate-mapping approaches). Instead, both of these could potentially develop later. This means that they could be derived from language
experience, rather than being innately specified. This is, of course, a promissory note – it remains to be seen exactly how the conceptual categories of UTAH and the expectation of a mapping would be derived from children’s input. Future computational modeling may be able to contribute to this investigation.

Of course, the fact that our modeling results suggest that a derived-mapping approach performs better than an innate-mapping approach on verb class learning does not in and of itself disprove innate-mapping theories of why the linking patterns are the way they are cross-linguistically. What it does mean is that innate mapping is neither necessary, nor particularly helpful, for verb class learning and the emergence of the linking pattern knowledge across verbs in children. This in itself is surprising, as it’s logically possible that innate knowledge of linking patterns could be helpful in learning verb classes. Moreover, we might expect that the robustness of the pattern in adult representations would be aided by innate knowledge guiding development of that knowledge. Instead, our results suggest that verb class learning can’t be used as an argument in favor of innate-mapping approaches, and instead is an argument in favor of derived-mapping approaches. What this means in practice is that the next step in the debate between innate-mapping and derived-mapping approaches should focus on the precise mechanisms that derived-mapping approaches can use to explain the distribution of linking patterns. In particular, derived-mapping approaches would need to demonstrate how the linking patterns observed cross-linguistically can be derived from children’s input in each language.

6.2 Open questions

There are a number of open questions that the current results highlight. Here we classify these questions into three types: experimental, computational, and theoretical avenues of inquiry.

6.2.1 Experimental avenues

One avenue for future experimental work is to increase the number of verbs and verb classes that are used in early acquisition studies. Though our corpus analysis yielded up to 285 verbs appearing five or more times (<3yrs: 239, <4yrs: 267, <5yrs: 285) in the CHILDES Treebank, the available experimental data about specific verb behaviors yielded far fewer verbs on which to evaluate our simulations (<3yrs: 60, <4yrs: 76, <5yrs: 84). This means there are nearly 200 verbs for each age group that we have model predictions for, but no behavioral data about (and therefore were not reported here). With more targeted child language experiments, we will have a broader empirical basis on which to evaluate our acquisition theories. For example, at three years old, the rUTAH-based learner which doesn’t expect a mapping puts together keep and stop in one class and verbs like miss and say in a separate class, while the rUTAH-based learner which does expect a mapping groups all these verbs together into the same class. Do three-year-olds expect different behaviors for these four verbs, or the same behaviors? Once we know, we can better choose between the two learning assumption combinations that currently best fit three-year-old behavior.

This lack of behavioral data also applies to the verb behaviors we know about – here, there were 12 attested verb behaviors, but there are many more where we need knowledge of how specific verbs behave (e.g., intransitivity, monotransitivity, unergativity, verbs taking non-finite com-
plements with -ing, verbs taking small clause complements, wager-class verbs). Again, with a broader child behavioral foundation, we will be better able to choose among the modeled learner options and the learning assumptions they encode.

6.2.2 Computational avenues

One avenue for computational work is complementary to the future experimental work with children. Each modeled learner here has generated a set of verb classes which is that learner’s internal representation of which verbs behave like other verbs. Each verb class has a set of characteristics (involving syntactic and conceptual preferences) that can be used to generate precise predictions for any experimental setup. For example, we can calculate the probability distribution over verbs that a modeled child will prefer to use with a particular utterance that has certain syntactic and conceptual characteristics (e.g., $She \_to\_laugh = subject_{+anim}, NP\ V\ IP_{-finite}, subject_{EXPERIENCER}$). This corresponds to what might be observed in child productions. We can also calculate the probability distribution over utterances that a modeled child will prefer to use for a particular verb (e.g., want might have a high probability for $She \_to\_laugh$ while make has a low probability). This corresponds to both the productions a child might generate, and also the ease with which a child would comprehend a verb used in a particular utterance. Both of these are examples of the modeled learner generating concrete behavioral predictions that can be experimentally evaluated.

When the predictions diverge and only one matches children’s behavior, we then have additional empirical support for whichever modeled child (and therefore, whichever specific set of learning assumptions) was successful.

We can also make more sophisticated computational models that capture both the incremental nature of children’s learning and children’s cognitive constraints. Recall that the ideal learner model implementations here operate by learning from all the data at once that children of a certain age will have seen, and optimally executing the inference over those data. As mentioned, this is a first step in understanding the mental computations that occur during acquisition. Future work can relax some of the idealized assumptions present in the ideal modeled learners used here. For example, one option is to make more realistic learners that: (i) learn from data as they are encountered one utterance at a time (rather than as a batch), and (ii) use an inference approximation, rather than Gibbs sampling, to converge on the final set of verb classes (e.g., see the learning approaches of Fazly, Alishahi, & Stevenson, 2010, Barak, Fazly, & Stevenson, 2014b, and Barak, Fazly, & Stevenson, 2014a). Unlike the ideal learner model implementations, these more realistic modeled learners would be executing a potential inference algorithm that children could be capable of – this makes these future models algorithmic-level (rather than computational-level) in the sense of Marr (1982).

The utility of algorithmic-level implementations is to see if the learning assumptions that were useful for a computational-level learner are still useful when incremental learning and cognitive constraints are present (Pearl, 2014, Phillips & Pearl, 2015, Pearl, under review). That is, algorithmic-level implementations can tell us if the learning assumptions that seem to be useful for optimal learning models are actually usable by real children, who have various constraints on their acquisition computation. This isn’t always the case – it could turn out that certain learning assumptions are less helpful to a cognitively constrained learner while other assumptions are more
helpful (Phillips & Pearl, 2015; Pearl & Phillips, 2018).

Assuming that the developmental trajectory suggested by these results holds under future experimental and (incremental) modeling work, another open question is how the primary linking pattern and the secondary exception patterns arise under a derived-mapping approach. That is, how does the expectation for the “right” mapping between thematic representations and syntactic positions develop between age four and age five? Without a built-in expectation of specific mappings, these patterns are dependent on the content of the input. If there are a sufficient number of primary pattern verbs in the input (and/or verb classes) learned at early stages, then this will lead to the development of the primary pattern expectation. Mathematical analyses of children’s input that predict when children will make a generalization vs. not, such as the Tolerance Principle (Yang, 2005; Legate & Yang, 2013; Schuler, Yang, & Newport, 2016; Yang & Montrul, 2017), can provide an answer. Such analyses can either support the ability of realistic input to help children derive the primary mapping or demonstrate the obstacles to be surmounted under the derived-mapping approach.

6.2.3 Theoretical avenues

From a theoretical perspective, there may be other solutions to the linking problem that we wish to investigate using this integrated quantitative framework. Here, we focused on two prominent options discussed in the theoretical literature (UTAH and rUTAH) that (i) take thematic roles as their basis, and (ii) involve either a categorical (UTAH) or relative (rUTAH) perception of these thematic roles. While these both seem cognitively plausible, other options are certainly available. For example, perhaps children abstract across thematic roles in different categorical or relative ways than the implementations explored here (more than 3 protoroles, different definitions of protoroles, different orderings in the role hierarchy, etc). Relatedly, there could also be different thematic role distinctions at the basic conceptual level – the 13 roles here were chosen to make the CHILDES Treebank as useful as possible to the widest range of users (Pearl & Sprouse, 2013a). That said, there are a number of specific proposals for thematic role systems in the literature; and the diversity of theories only increases when one considers that children’s thematic distinctions might differ from adults’ in complex ways (especially very young children’s). It could also be that children begin by not abstracting over thematic roles at all. Instead, they might track mappings from the individual thematic roles directly to syntactic positions. Finally, as briefly mentioned in section 2.6, it is also possible that the source of the linking patterns we see lies outside of syntax (so, not in principles like UTAH or rUTAH), and is instead a consequence of a constraint on the types of semantic representations that language allows (Wood, 2015; Kastner, 2016; Myler, 2016). This is still a type of innate knowledge, therefore the quantitative framework developed here could be modified to compare modeled learners with knowledge of that constraint versus modeled learners without knowledge of that constraint.

Related to the idea of different underlying thematic systems and how they might change during development, there may also be a change to the information children are sensitive to in the input. For example, while younger children may rely on syntactic frames, older children may rely on additional and/or more abstract syntactic information. For example, when encountering the utterance She seemed to laugh, a younger child might extract the syntactic frame NP _ IP_finite for seem. In
contrast, an older child might also perceive the raising dependency, and so encode *seem*’s syntax as \( \text{NP}_1 \rightarrow \{ \text{IP}_t \ \text{VP}_{\text{finite}} \} \). Knowing exactly what information children of different ages are able to both extract from their input and use for learning depends on having precise theories of acquisition that combine developing representations with developing abilities to use those representations in real time.

### 7 Conclusion

To successfully learn language – and more specifically how to use verbs correctly – children must solve the linking problem: they must learn the mapping between the thematic roles specified by a verb’s lexical semantics and the syntactic argument positions specified by a verb’s syntactic frame. Here we have constructed an argument from acquisition for different theoretical approaches to solving the linking problem. In particular, we have used acquisition of verb classes as an evaluation metric for theories of solving the linking problem, with the idea that a good theory will be able to account for children’s developing knowledge of verb classes over time. We made different theoretical options concrete within an integrated quantitative framework of the acquisition process that relies on corpus analysis, experimental meta-analysis, and computational modeling. More specifically, we compared innate-mapping vs. derived-mapping approaches to the linking problem, as well as different underlying thematic representations to be linked to syntactic positions.

Our results allowed us to specify for the first time a developmental trajectory of mental representations and learning assumptions children may have when learning verb classes. Importantly, this specification supports a derived-mapping approach to the linking problem, where children are not innately equipped with knowledge of how thematic roles link to syntactic positions. So, the only advantage innate-mapping approaches retain over derived-mapping approaches is the ease of explaining the cross-linguistic regularity in linking patterns. But, they must then explain why this innate knowledge of how verb thematic roles are linked to verb syntactic arguments is not apparently used during verb knowledge development before age five. One fruitful avenue of future work for derived-mapping approaches is thus to understand how children derive the regularity we see in linking patterns from their input. Beyond this, our results support both categorical (rUTAH) and relative (rUTAH) thematic representations at different ages, with both potentially available for five-year-olds. More generally, our quantitative approach to language acquisition allows us to connect together theories of linguistic representation and theories of the learning process, and so better understand both as part of an integrated theory of language.

### References


Pearl, L., & Sprouse, J. (2015). Computational modeling for language acquisition: A tutorial with


A Survey of child behavioral results

A.1 Child behavioral data

This is a synthesis of 37 behavioral acquisition studies relating to verb behaviors known by children by five years old. The specific verbs attested are used to identify which particular verbs ought (or ought not) to cluster together at different ages.

A.1.1 Passivizable, intransitive, & monotransitive.

A verb that’s passivizable is often one that can be used transitively (i.e., it allows an object). For example, eat is both passivizable (It was/got eaten) and (optionally) transitive (I ate it). However, verbs can also be used in the passive form even if they’re intransitive, because they can take an indirect object. We can see this with sneeze: It was/got sneezed at and I sneezed at it).

In terms of acquisition evidence, we should be able to look at studies that investigate comprehension of transitive verbs and transitive cues, as well as studies that investigate children’s comprehension of passives. Moreover, for passives, comprehension of a “short” passive (e.g., It was/got eaten) should be sufficient, rather than requiring comprehension of a “long” passive (e.g., It was/got eaten by the cat). If children comprehend the long passive, they should be able to comprehend the short passive for that verb.

The ability to comprehend the passive usage of a verb correctly seems to come significantly after the cues to transitivity are recognized. For example, by two years old, English children recognize that the frame She’s Xing the man indicates X is a transitive verb and so expect a transitive meaning where one agent affects another; they also recognize the frame She’s Xing indicates X is an intransitive verb, and so expect an intransitive (e.g., unaccusative or unergative) meaning with only a single agent (L. Naigles, 1990; L. G. Naigles & Kako, 1993; Yuan & Fisher, 2009). At 28 months, they also recognize cues involving multiple frames to identify verbs as optionally transitive vs. unaccusative (Scott & Fisher, 2009).

Maratsos 1974. M. P. Maratsos (1974) finds that children can pass an act-out task with full passives using the verbs bump and push by age four and a half.

Maratsos et al. 1985. M. Maratsos, Fox, Becker, and Chalkley (1985) found that children can comprehend long passives for these verbs by age 4: find, hold, kick, kiss, push, wash. They comprehend long passives for these verbs by age 5: like, love. They comprehend long passives for these verbs by age 9: hate, remember, see.

Gordon & Chafetz 1990. Gordon and Chafetz (1990) found that children can comprehend both short and long passives for these verbs by age 3: drop, eat, carry, hold, hug, kick, kiss, shake, wash, watch. However, they fail to comprehend either long or short passives for these by age 3 to 4: believe, forget, hate, hear, know, like, remember, see.
O’Brien et al. 2006 & Nguyen et al. 2016 O’Brien, Grolla, and Lillo-Martin (2006) found that children can comprehend long passives for these verbs by age 3 (and 4) when the pragmatic context makes the use of the passive more felicitous: hug, chase, like, see. However, Nguyen, Lillo-Martin, and Snyder (2016) found that three- and four-year-olds only seem to comprehend long passives for hug and chase, though they comprehend short passives for all four verbs.

Crain et al. 2009. Crain, Thornton, and Murasugi (2009) report samples of elicited long passives from 9 three-year-olds, 22 four-year-olds, and one five-year-old. We can assume that if these children produce a passive structure, they have classified the verbs as passivizable. Based on the samples from the three-year-olds (3;4-3;11), by age three children should have grouped these verbs together: bleed, crash, eat, hit, hurt, lick, knock, marry, punch, pick up, push, scratch, trip up, turn (over). Based on the samples from the four- and five-year-olds (4;1-5;0), by age four children should have additionally grouped these verbs together: bite, hug, jump, kill, kick, kiss, knock, lick, ride, shoot.

Messenger et al. 2009. Messenger, Branigan, McLean, and Sorace (2009) found that children can comprehend long passives for these verbs by age 3-4: frighten, shock, annoy, upset, surprise, scare, pat, bite, pull, hit, carry, squash. However, they were unable to comprehend long passives at age 3-4 for these verbs: see, hear, love, ignore, remember, hate.

A.1.2 Ditransitive.

Corpus analysis and experimental work discussed below suggests that some verbs that allow (or require) two objects are identified by age three (Gropen et al. 1989; Snyder & Stromswold 1997; Campbell & Tomasello 2001; Conwell & Demuth 2007; Thothathiri & Snedeker 2008), though there may be some overgeneralizations where verbs are assumed to have the ditransitive behavior for awhile that shouldn’t (ex: say).

Gropen et al. 1989. Gropen et al. (1989) conducted a corpus analysis of five children’s productions (including Brown-Eve) and found that the double-object construction was often first produced between ages 1;8 and 3;3. Their corpus analysis showed that at least two of the five children had used the following verbs in both dative constructions, with the earliest age of use in parenthesis: give (1;9), get (2;0), read (2;0), bring (2;3), buy (2;11), show (3;0), make (3;4), tell (3;4). This same analysis showed that at least two of the five children had used the following verbs in the double-object dative construction, with the earliest age of use in parenthesis: read (1;8), give (1;9), show (1;9), bring (1;10), get (2;0), buy (2;11), pour (2;11), tell (3;0), draw (3;4), make (3;4), teach (3;6), ask (4;7).

This suggests that these verbs may have the ditransitive behavior by these ages: two = read, give, show, bring, get; three = buy, pour, tell; four = draw, make, teach; five = ask.

In addition, Gropen et al. (1989) also observed overproductive uses of the double-object construction at these ages for these verbs: write (2;3), say (2;8), eat (3;3), keep (3;8), spend (4;0), put on (4;1), finish (4;11), fix (5;2). So, the above clusterings may also include these errors (i.e.,
including these verbs at these ages). So, the groupings may look more like this for the ditransitive behavior: two = read, give, show, bring, get; three = buy, pour, tell, write, say; four = draw, make, teach, eat, keep, spend; five = ask, put on, finish; six = fix.

Snedeker & Huang 2015, Campbell & Tomasello 2001. Snedeker and Huang (2015), citing Campbell and Tomasello (2001), note that both constructions associated with the dative (i.e., subject verb object$_{indir}$ object$_{dir}$ [double-object]=She gave him a penguin and subject verb object$_{dir}$ preposition object$_{indir}$ [prepositional]=She gave a penguin to him) are acquired before age three. More specifically, Campbell and Tomasello (2001) conducted a corpus analysis of seven children in the CHILDES database and found that both dative constructions are first produced by age three (at the latest), and often before age two and a half (five of seven children). The specific verbs where the majority of children produced multiple dative construction types by age three are bring, get, give, make, read, and show. So, it is likely children should have grouped these verbs together by age three. Moreover, if we focus on the verbs that were used in the double-object construction specifically, these are grouped together: bring, buy, get, give, make, read, show, tell.

Huttenlocher et al. 2004. Huttenlocher et al. (2004) found that four- and five-year-olds had structural priming for both dative constructions across these verbs: bake, bring, buy, deliver, feed, give, serve, show, teach, throw. This suggests these verbs have been clustered together by age four into a class that allows the double-object dative and the prepositional dative.

Conwell & Demuth 2007. Conwell and Demuth (2007) find that three-year-olds demonstrate abstract structural knowledge of both the double-object and prepositional dative constructions, using elicited repetition with novel verbs. In particular, three-year-olds will use the prepositional construction for a novel verb when it’s been modeled in the double-object construction, suggesting they understand these are related. So, it is likely three-year-olds have a verb class that includes the double-object construction (i.e., the ditransitive behavior).

Thothathiri & Snedeker 2008. Thothathiri and Snedeker (2008) note that naturalistic production isn’t definitive about categorization – in particular, children’s productions could be simple imitations of their input rather than generalizations formed via a verb class. Thothathiri and Snedeker (2008) conduct priming studies with three- and four-year-olds children to determine whether class-level knowledge exists for dative constructions (double-object and prepositional). This presumably also indicates which verbs belong to that class, as children who can generalize with a construction to a new usage are doing so because the verb belongs to the class. In the first set of experiments, four-year-olds show both within-verb priming (with give) and across-verb priming (show priming give for the double-object construction, bring priming give for the prepositional dative). These results suggest that four-year-olds cluster together show and give for the double-object construction, and potentially also bring for datives in general. In the second set of experiments, three-year-olds also show both within-verb and across-verb priming with these verbs: bring, hand, pass, send, show, throw, toss. The priming effect was stronger for the double-object construction, which suggests this class has definitely been formed.
So, by three, children have likely clustered together these verbs (along with give) into a class that allows the ditransitive construction.

A.1.3 Unaccusatives and unergatives.

Both experimental evidence (Bunger & Lidz, 2004) and analysis of naturalistic productions (Déprez & Pierce, 1993; Snyder & Stromswold, 1997) suggest that English children have begun forming a class of unaccusative verbs by age two. Experimental evidence (Bunger & Lidz, 2008) additionally suggests that English children have begun forming a class of unergative verbs by age two.

Déprez & Pierce 1993. Déprez and Pierce (1993) investigated three English children’s naturally produced speech between the ages of one and a half and two years old. They found that post-verbal subjects (i.e., VS order, like going it) occurred only with be and with unaccusatives such as break, go, come, and fall. This suggests that English children have grouped these unaccusatives together by age two (presumably noting the PATIENT-like role of the subject).

Snyder & Stromswold 1997. Snyder and Stromswold (1997) conducted a corpus analysis in the naturalistic productions of 12 English children to determine the age at which the verbs break, come, fall, go, grow, and leave were first produced in an “unaccusative context”. This age range was found to be between 1;6 and 2;7. So, we might interpret this as children recognizing that these six verbs can be used unaccusatively by age two and clustering them together.

Gelman & Koenig 2001. Gelman and Koenig (2001) find that five-year-olds and adults use the animacy of a subject to determine how to interpret the verb move in intransitive uses such as Was this X moving?. In particular, +animate subjects yield an unergative reading while -animate subjects yield an unaccusative reading. So, by five, children seemed to have created a distinction between the unergative and unaccusative classes that depends on the animacy of the subject. The three-year-olds in the study were trending towards this behavior, but their behavioral results weren’t statistically significant.

Bunger & Lidz 2004, 2008. Bunger and Lidz (2004) demonstrate experimentally that 2-year-old English children are able to use syntactic distributional cues to determine that a verb is unaccusative. For example, when presented with “The ball is pimming” (sometimes accompanied with “The girl is pimming the ball”), 2-year-olds infer that pim refers to the “results-focused” action (like the ball bouncing) that unaccusatives have. This suggests that children have begun forming an unaccusative verb class by age two, and it has both syntactic and semantic cues associated with it.

Bunger and Lidz (2008) demonstrate experimentally that 2-year-old English children also are able to use both syntactic distributional cues and semantic role information to determine that a verb is unergative. For example, when presented with “The boy is blicking”, 2-year-olds infer that blick refers to the “means-focused” action (like pumping a bicycle pump to spin an attached flower) that
unergatives have. This suggests that children have begun forming an unergative verb class by age two, and it has both syntactic and semantic cues associated with it.

A.1.4 Control object and raising object.

Kirby 2009, 2009, 2010 & Becker 2014. Becker (2014) discusses a set of experiments by Kirby (2009b) in which the raising object verbs want and need were investigated along with the control object verbs ask and tell. Kirby (2009b, 2011) found that four- and five-year-olds were able to correctly interpret raising object verb and control objects verb utterances when the embedded clause is active (e.g., “He wanted Winnie the Pooh to kiss Patrick”, “He asked the farmer to comb the horse”). Both ages of children are also able to interpret raising object utterances when the embedded non-finite clause is a passive (“He wanted Tigger to be called by Elmo.”). However, only five-year-olds were above chance on interpreting control objects with embedded non-finite passives (“She told the policeman to be sniffed by the dog.”)

Additionally, as reported in Kirby (2009a, 2010), both four- and five-year-olds are sensitive to the animacy restrictions for control object verbs (and reject utterances like “Elmo told the toys to be smaller” as “weird” more often than chance). This behavior is different for raising object verbs (with utterances like “The boy wanted the cake to be chocolate”), where both ages of children were at chance for accepting the utterances as “okay”. Similarly, as also reported in Kirby (2009a, 2010), four- and five-year-olds treat control object and raising object verbs differently when judging the acceptability of expletive subjects in the embedded clause (e.g., control object: *The girl told it to be warm, raising subject: The girl wanted there to be cookies in the bag.). Adult-like judgments do vary by age: four-year-olds only have adult judgments for control object verbs and reject expletive subjects while five-year-olds only have adult judgments for raising object verbs and accept expletive subjects. Still, the main point is that they recognize that these types of verbs behave differently from each other but similarly to ones of the same kind (i.e., want patterns with need and ask patterns with tell).

This suggests five-year-olds have distinguished raising object from control object verbs, and four-year-olds may have as well. More specifically, we might expect four- and five-year-olds to group together want and need in one class and ask and tell in a separate class.

A.1.5 Control subject and raising subject.

Becker 2006. The experiment with children in Becker (2006) suggests that five-year-olds have adult-like judgments for verbs allowing inanimate subjects. In particular, they accept inanimate subjects only for subject raising verbs like seem and appear (e.g., The flower seems to be pink), and not for control subject verbs like want and try (e.g. The flower wants to be pink). In contrast, three- and four-year-olds allow all verbs to have inanimate subjects (and so have not differentiated want and try from seem and appear in this respect). For purposes of verb classification, this suggests that children place want and try together, separately from seem and appear by five years old.
Becker 2007 and Becker 2009. The experiments in Becker (2007, 2009) suggest that three- and four-year-olds accept expletive subjects for seem and appear (e.g., It seems to be windy, It appears to be warm) either 83% (three-year-olds) or 91.7% (four-year-olds) of the time. This may indicate that they recognize that seem and appear are the same kind of predicate. However, these children also accepted expletive subjects for the control subject verbs want and try either 66.7% (three-year-olds) or 68.1% (four-year-olds) of the time. Becker (2009) suggests this is because children of both ages recognize the expletive subject as a cue for raising subject verbs – and so try to cast control subject verbs as raising subject verbs when they’re used with expletive subjects (It wants to be raining = modal interpretation ≈ It's going to rain).

For the purposes of verb classification, only four-year-olds – but not three-year-olds – accepted raising subject verbs with expletive subjects more often than control subject verbs with expletive subjects. Becker (2009) suggests that three-year-olds are still determining the class membership for raising vs. control subject verbs. In particular, only four-year-olds recognize that want and try should be in a different class than seem and appear (the class that naturally allows expletive subjects).

Becker 2014. Becker (2014) discusses a pilot study (pp.206-207) with 5 five-year-olds, and finds that being used with an inanimate subject is a strong signal to expect that verb to also be used with the there-expletive (Did there meb to be a banana in the soup?). So, five-year-olds have a strong sense of cues to raising verb analyses.

A.1.6 Subject-experiencer & object-experiencer verbs.

Experimental studies suggest that children are still in the process of learning how to interpret subject-experiencer verbs like like, love, and hate in all contexts, even though they use these words frequently in their naturalistic output and hear them often in their input (Hartshorne et al., 2015). Five-year-olds, however, can interpret subject-experiencer verbs correctly above chance. In contrast, four-year-olds are better at sorting out object-experiencer verbs like surprise and frighten and interpreting them correctly above chance (Hartshorne et al., 2015) when both the subject and object are animate. This suggests that there may be an object-experiencer verb class by age 4, as well as an emerging subject-experiencer verb class.

Hartshorne et al. 2015. Hartshorne et al. (2015) conducted experimental studies testing children’s comprehension of several subject-experiencer verbs (admire, fear, hate, like, love, trust) and object-experiencer verbs (amaze, bore, confuse, frighten, scare, surprise). They found that four-year-olds correctly interpreted three higher-frequency object-experiencer verbs above chance (surprise, frighten, scare) and no subject-experiencer verbs above chance when both subject and object were animate. In fact, two low frequency subject-experiencer verbs were interpreted as object-experiencer verbs above chance (fear, trust). This may suggest that four-year-olds have grouped these five verbs together: fear, frighten, scare, surprise, trust. When the subject was animate while the object was inanimate, four-year-olds are above chance at interpreting the three most frequent subject-experiencer verbs correctly: like, love, hate. This suggests that four-year-
olds have grouped these verbs together, though they may not always interpret them correctly if the object is animate.

### A.1.7 Complements: -ing, to, that, whether/if.

**Bloom et al. 1984, 1989.** Bloom, Tackeff, and Lahey (1984) conducted an analysis of child-produced speech from four children between the ages of two and three years old. They found suggestive evidence that children have productive use of non-finite to in “V-to-V” constructions, such as I want to see Mommy (though not in “V-NP-to-V” constructions like I want Mommy to get balloon). This suggests that by three years old, children have begun forming a class of verbs that take non-finite to as a complement. The specific verbs that involved complement verb contexts in child productions were these: ask, forget, get, go, have, know, like, need, show, start, suppose, teach, tell, try, use, want. A subset of these were used specifically with “how to” instructive contexts: know, show, teach. Bloom et al. (1984) note that no -ing complements were produced by any of the children they looked at. This suggests acquisition of -ing complements occurs after three years of age.

Bloom, Rispoli, Gartner, and Hafitz (1989) examined the naturalistic productions from these same children for evidence of productive use of (1) “S-complements” (including both non-finite -ing clauses such as I see Mommy wash(ing) her hands and finite -that clauses such as I see that Mommy is washing her hands, and (2) wh-complements, such as I know what the little bear’s eating. They found that think and see were produced with S-complements while know and see were produced with wh-complements. However, their evidence did not indicate that these children realized there was a general rule or class of such verbs. Instead, it seemed “children learned this for each matrix verb separately”. This suggests that verb classes involving S-complements and wh-complements (i.e., interrogative clauses, which would include whether/if clauses) aren’t formed till after three years of age.

**Diessel & Tomasello 2001.** Diessel and Tomasello (2001) conducted a corpus analysis of seven English-speaking children’s spontaneous speech between the ages of 1;2 and 5;2. They identified seven verbs taking an if-complement: see (69/98), tell (14/98), wonder, ask, care, know, happen (15/98). This suggests that by five, children have clustered together see and tell, and may have also clustered together ask, care, happen, know, and wonder.

**Papafragou et al. 2007.** Papafragou et al. (2007) investigated the cues children (and adults) use to identify the meaning of a verb, including the syntactic cue of taking a tensed clausal complement introduced by complementizer that (e.g., Matt grops that his grandmother is under the covers.). When eliciting guesses about verb meaning from 34 children ages 3;7-5;9, they found that this syntactic frame increased the chances of children guessing belief verbs. In particular, children’s guesses included these verbs: be surprised, dream, fall for, guess, know, lie, pretend, think, trick. This suggests that by four to five, children are forming classes of verbs that allow a tensed sentential complement (specifically here, the belief verb class).
Kidd et al. 2006, 2010. Kidd et al. (2006) conducted a sentence recall/lexical priming study on three-, four-, and five-year-olds involving both high and low frequency verbs that take finite complements (e.g., \textit{think:} “I think she is riding away on the horse.”). Interestingly, children often made substitutions with different verbs, instead of repeating the verbs previously presented to them. Three-year-olds substituted in these verbs more than once: \textit{think} (179/266), \textit{bet} (34/266), \textit{hope} (24/266), \textit{see} (11/266), \textit{know} (9/266), \textit{say} (4/266), and \textit{hear} (4/266). Four- and five-year-olds substituted in the same verbs, though with higher incidences for some lower frequency verbs (\textit{know, say}) than the three-year-olds. This suggests that by age three, children may have clustered together these \textit{that}-complement taking verbs: \textit{bet, hear, hope, know, say, see, think}.

Kidd et al. (2010) conducted a similar sentence recall/lexical priming study on four- and six-year-olds involving both high and low frequency verbs that take finite complements (e.g., \textit{say:} ‘Mickey says that Minnie is wearing a lovely dress.”). Again, children often made substitutions with different verbs, instead of repeating the verbs previously presented to them. Four-year-olds substituted \textit{think} nearly 88\% of the time (126/143), and used \textit{get} (1/143), \textit{see} (3/143), \textit{tell} (3/143), and \textit{wish} (6/143) for the remaining 12\%. Six-year-olds substituted \textit{think} nearly 75\% of the time (102/135), and used \textit{find} (2/135), \textit{laugh} (1/135), \textit{like} (2/135), \textit{persuade} (1/135), \textit{reckon} (13/135), \textit{see} (5/135), \textit{tell} (1/135), and \textit{wish} (1/135) for the remaining 25\%. If we assume that any verb substituted in more than once is reliably affiliated with the class of complementizer-\textit{that} verbs, we have the following clusters: at four, children have clustered together \textit{see, tell, think, and wish}; at six, they have additionally clustered in \textit{find, like, and reckon}.

A.2 Verb classes derived from behavioral data

This is a synthesis of the 32 behavioral studies describing the behavior of specific verbs at specific ages.

A.2.1 Passivizable.

By age three, children should cluster together these: \textit{bleed, carry, chase, crash, drop, eat, hit, hold, hug, hurt, kick, kiss, knock, lick, like, marry, punch, pick up, push, scratch, see, shake, trip up, turn (over), wash, watch} (Gordon & Chafetz, 1990) short and long passives, O’Brien et al. 2006 pragmatically felicitous long passives, Nguyen et al. 2016 short passives; Crain et al. 2009 long passives). Moreover, they should keep them separate from these: \textit{believe, forget, hate, hear, ignore, know, love, remember} (Gordon & Chafetz, 1990) short and long passives; Messenger et al. 2009 long passives).


By age five, children should additionally include these: \textit{love} (M. Maratsos et al. 1985 long passives).

Note that the primes didn’t involve complementizer \textit{that} even though the verbs could all allow it. That is, all the verbs belonged to the class that optionally allows complementizer \textit{that}.
By age nine, children should additionally include these: hate, remember (M. Maratsos et al., 1985: long passives).

### A.2.2 Unaccusative.

By two, children have recognized that these verbs can be used unaccusatively and so should be clustered together: break, come, fall, go, grow, leave (Déprez & Pierce, 1993; Snyder & Stromswold, 1997: naturalistic production).

### A.2.3 Control object and raising object.

The study by Kirby (2009b) suggests that children should group want and need together (raising object), and cluster together ask and tell (control object) separately by age four.

### A.2.4 Control subject and raising subject.

Studies by Becker suggest that children should group want and try (control subject) together, and separate them from seem and appear (raising subject), by either four years old (Becker, 2007, 2009) or five years old (Becker, 2006).

### A.2.5 Subject-experiencer and object-experiencer.

By age four, children have grouped these high-frequency subject-experiencer verbs together: like, love, hate (Hartshorne et al., 2015: truth-value judgment task). They have also grouped together several object-experiencer verbs (frighten, scare, surprise) with lower-frequency subject-experiencer verbs (fear, trust) (Hartshorne et al., 2015: truth-value judgment task).

### A.2.6 Complements: to, that, whether.

**to-complement.** By age three, children have likely clustered together these verbs that can take a non-finite to complement: ask, forget, get, go, have, know, like, need, show, start, suppose, teach, tell, try, use, want. A subset of these may be in a separable “how to” instructive context class: know, show, teach (Bloom et al., 1984: corpus analysis of naturalistic productions).

**that-complement.** By age three, children have likely clustered together these verbs that optionally take a that-complement: bet, hear, hope, know, say, see, think (Kidd et al., 2006: sentence recall/lexical priming substitution). By age four, children have likely clustered together these verbs as well: tell, wish (Kidd et al., 2010: sentence recall/lexical priming substitution). By age five, children have likely grouped in these belief verbs together that take a sentential complement introduced by complementizer that: be surprised, dream, fall for, guess, lie, pretend, trick (Papafragou et al., 2007: elicited verbs). By age six, they have additionally clustered in find, like, and reckon (Kidd et al., 2010: sentence recall/lexical priming substitution).
whether/if-complement. By age five, children have clustered together these verbs that allow if-complements: see and tell. They may have also clustered together ask, care, happen, know, and wonder (Diessel & Tomasello, 2001: spontaneous speech productions).

B Verb classes derived from child behavioral data

Table 7: Attested verb behaviors and verb classes derived from child behavioral data for verbs in the <3yrs dataset appearing 5 or more times.

<table>
<thead>
<tr>
<th>Verb behavior</th>
<th>Attested verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaccusative</td>
<td>6: break, come, fall, go, grow, leave</td>
</tr>
<tr>
<td>Ditransitive</td>
<td>14: bring, buy, give, make, pass, pour, read, say, send, show, tell, throw, write</td>
</tr>
<tr>
<td>Non-finite to complement</td>
<td>16: ask, forget, get, go, have, know, like, need, show, start, suppose, teach, tell, try, use, want</td>
</tr>
<tr>
<td>Passivizable</td>
<td>27: carry, chase, crash, drop, eat, hit, hold, hurt, jump, kick, kiss, knock, lick, like, punch, push, scratch, see, shake, turn, wash, watch</td>
</tr>
<tr>
<td>that-complement</td>
<td>+: believe, forget, hear, know, remember</td>
</tr>
<tr>
<td></td>
<td>-: bet, hear, hope, know, say, see, tell, think, wish</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Verb classes</th>
<th>15 classes, 60 verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>[+unaccusative]</td>
<td>5: break, come, fall, grow, leave</td>
</tr>
<tr>
<td>[+ditrans]</td>
<td>10: bring, buy, give, make, pass, pour, read, send, throw, write</td>
</tr>
<tr>
<td>[+non-finite to]</td>
<td>9: ask, have, need, start, suppose, teach, try, use, want</td>
</tr>
<tr>
<td>[+passive]</td>
<td>20: carry, chase, crash, drop, eat, hit, hold, hurt, jump, kick, kiss, knock, lick, like, punch, push, scratch, see, shake, turn, wash, watch</td>
</tr>
<tr>
<td>[-passive]</td>
<td>2: believe, remember</td>
</tr>
<tr>
<td>[+that-comp]</td>
<td>4: bet, hope, think, wish</td>
</tr>
<tr>
<td>[+unaccusative, +non-finite to]</td>
<td>1: go</td>
</tr>
<tr>
<td>[+ditrans, +non-finite to]</td>
<td>2: get, show</td>
</tr>
<tr>
<td>[+passive, +that-comp]</td>
<td>1: say</td>
</tr>
<tr>
<td>[+passive, +non-finite to]</td>
<td>1: like</td>
</tr>
<tr>
<td>[-passive, +that-comp]</td>
<td>1: see</td>
</tr>
<tr>
<td>[-passive, +non-finite to]</td>
<td>1: forget</td>
</tr>
<tr>
<td>[-passive, +that-comp]</td>
<td>1: hear</td>
</tr>
<tr>
<td>[-passive, +that-comp, +non-finite to]</td>
<td>1: know</td>
</tr>
</tbody>
</table>
Table 8: Attested verb behaviors and verb classes derived from child behavioral data for verbs in the <4yrs dataset appearing 5 or more times.

<table>
<thead>
<tr>
<th>Verb behavior</th>
<th>Attested verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaccusative</td>
<td>6: break, come, fall, go, grow, leave</td>
</tr>
<tr>
<td>Ditransitive</td>
<td>21: bake, bring, buy, draw, eat, feed, give, keep, make, pass, pour, read, say, send</td>
</tr>
<tr>
<td>Non-finite to complement</td>
<td>16: ask, forget, get, go, have, know, like, need, show, start, suppose, teach, tell, try, use, want</td>
</tr>
<tr>
<td>Passivizable</td>
<td>38: bite, bump, carry, chase, crash, drop, eat, find, frighten, hit, hold, hurt, jump, kick, kill, kiss, knock, lick, like, pull, punch, push, ride, scare, scratch, see, shake, shoot, surprise, turn, wash, watch</td>
</tr>
<tr>
<td>that-complement</td>
<td>9: believe, forget, hear, know, love, remember</td>
</tr>
<tr>
<td>Control object</td>
<td>2: ask, tell</td>
</tr>
<tr>
<td>Control subject</td>
<td>2: try, want</td>
</tr>
<tr>
<td>Psych: Obj Exp</td>
<td>3: frighten, scare, surprise</td>
</tr>
<tr>
<td>Psych: Subj Exp</td>
<td>2: frighten, like, love</td>
</tr>
<tr>
<td>Raising object</td>
<td>2: need, want</td>
</tr>
<tr>
<td>Raising subject</td>
<td>1: seem</td>
</tr>
<tr>
<td>Verb classes</td>
<td>23 classes, 76 verbs</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[+unaccusative]</td>
<td>5: break, come, fall, grow, leave</td>
</tr>
<tr>
<td>[+ditrans]</td>
<td>15: bake, bring, buy, draw, eat, feed, give, keep, make, pass, pour, read, send, serve, throw, write</td>
</tr>
<tr>
<td>[+non-finite to]</td>
<td>4: have, start, suppose, use</td>
</tr>
<tr>
<td>[+passive]</td>
<td>26: bite, bump, carry, chase, crash, drop, eat, find, hit, hold, hurt, jump, kick, kill, kiss, knock, lick, pull, punch, push, ride, scare, scratch, see, shake, shoot, surprise, turn, wash, watch</td>
</tr>
<tr>
<td>[-passive]</td>
<td>1: seem</td>
</tr>
<tr>
<td>[+that-comp]</td>
<td>1: seem</td>
</tr>
<tr>
<td>[+unaccusative, +non-finite to]</td>
<td>1: go</td>
</tr>
<tr>
<td>[+ditrans, +non-finite to]</td>
<td>3: get, show, teach</td>
</tr>
<tr>
<td>[+ditrans, +passive]</td>
<td>1: eat</td>
</tr>
<tr>
<td>[+ditrans, +that-comp]</td>
<td>1: say</td>
</tr>
<tr>
<td>[+ditrans, +that-comp, +non-finite to, +control-obj]</td>
<td>1: tell</td>
</tr>
<tr>
<td>[+non-finite to, +control-obj]</td>
<td>1: ask</td>
</tr>
<tr>
<td>[+non-finite to, +control-subj]</td>
<td>1: try</td>
</tr>
<tr>
<td>[+non-finite to, +raising-obj]</td>
<td>1: need</td>
</tr>
<tr>
<td>[+non-finite to, +raising-obj, +control-subj]</td>
<td>1: want</td>
</tr>
<tr>
<td>[+passive, +non-finite to, +psych-subj]</td>
<td>1: like</td>
</tr>
<tr>
<td>[+passive, +that-comp]</td>
<td>1: see</td>
</tr>
<tr>
<td>[+passive, +psych-obj]</td>
<td>2: scare, surprise</td>
</tr>
<tr>
<td>[-passive, +non-finite to]</td>
<td>1: forget</td>
</tr>
<tr>
<td>[-passive, +that-comp]</td>
<td>1: hear</td>
</tr>
<tr>
<td>[-passive, +psych-subj]</td>
<td>1: know</td>
</tr>
<tr>
<td>[-passive, +psych-subj]</td>
<td>1: love</td>
</tr>
</tbody>
</table>
Table 9: Attested verb behaviors and verb classes derived from child behavioral data for verbs in the <5yrs dataset appearing 5 or more times.

<table>
<thead>
<tr>
<th>Verb behavior</th>
<th>Attested verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unaccusative</td>
<td>6: break, come, fall, go, grow, leave</td>
</tr>
<tr>
<td>Ditransitive</td>
<td>23: ask, bake, bring, buy, draw, eat, feed, finish, get, give, keep, make, pass, pour, read, say, send, serve, show, teach, tell, throw, write</td>
</tr>
<tr>
<td>Non-finite to complement</td>
<td>16: ask, forget, get, go, have, know, like, need, show, start, suppose, teach, tell, try, use, want</td>
</tr>
<tr>
<td>Passivizable</td>
<td>38.</td>
</tr>
<tr>
<td>that-complement</td>
<td>1: bite, bump, carry, chase, crash, drop, eat, find, frighten, hit, hold, hurt, jump, kick, kill, kiss, knock, lick, like, love, pull, punch, push, ride, scare, scratch, see, shake, shoot, surprise, turn, wash, watch</td>
</tr>
<tr>
<td>Control object</td>
<td>2: ask, tell</td>
</tr>
<tr>
<td>Control subject</td>
<td>2: try, want</td>
</tr>
<tr>
<td>Psych: Obj Exp</td>
<td>3: frighten, scare, surprise</td>
</tr>
<tr>
<td>Psych: Subj Exp</td>
<td>2: frighten, like, love</td>
</tr>
<tr>
<td>Raising object</td>
<td>2: need, want</td>
</tr>
<tr>
<td>Raising subject</td>
<td>1: seem</td>
</tr>
<tr>
<td>whether/if-complement</td>
<td>7: ask, care, happen, know, see, tell, think, wish</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Verb classes</th>
<th>25 classes, 84 verbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>[+unaccusative]</td>
<td>8: break, come, fall, grow, leave</td>
</tr>
<tr>
<td>[+ditrans]</td>
<td>16: bake, bring, buy, draw, eat, feed, finish, give, keep, make, pass, pour, read, say, send, serve, show, teach, tell, throw, write</td>
</tr>
<tr>
<td>[+non-finite to]</td>
<td>4: have, start, suppose, use</td>
</tr>
<tr>
<td>[+passive]</td>
<td>25: bite, bump, carry, chase, crash, drop, eat, find, frighten, hit, hold, hurt, jump, kick, kill, kiss, knock, lick pull, push, ride, scratch, shake, shoot, turn, wash, watch</td>
</tr>
<tr>
<td>[-passive]</td>
<td>2: believe, remember</td>
</tr>
<tr>
<td>[+that-comp]</td>
<td>8: bet, dream, guess, hope, lie, pretend, think, wish</td>
</tr>
<tr>
<td>[+raising-subj]</td>
<td>1: seem</td>
</tr>
<tr>
<td>[+unaccusative, +non-finite to]</td>
<td>1: go</td>
</tr>
<tr>
<td>[+ditrans, +non-finite to]</td>
<td>3: care, happen, wonder</td>
</tr>
<tr>
<td>[+ditrans, +passive]</td>
<td>1: eat</td>
</tr>
<tr>
<td>[+ditrans, +that-comp]</td>
<td>1: say</td>
</tr>
<tr>
<td>[+ditrans, +that-comp, +non-finite to, +control-obj, +whether/if-comp]</td>
<td>1: tell</td>
</tr>
<tr>
<td>[+ditrans, +non-finite to, +control-obj, +whether/if-comp]</td>
<td>1: ask</td>
</tr>
<tr>
<td>[+non-finite to, +control-subj]</td>
<td>1: try</td>
</tr>
<tr>
<td>[+non-finite to, +raising-obj]</td>
<td>1: need</td>
</tr>
<tr>
<td>[+non-finite to, +raising-obj, +control-subj]</td>
<td>1: want</td>
</tr>
<tr>
<td>[+passive, +non-finite to]</td>
<td>1: like</td>
</tr>
<tr>
<td>[+passive, +that-comp, +whether/if-comp]</td>
<td>1: see</td>
</tr>
<tr>
<td>[+passive, +psych-subj]</td>
<td>2: scare, surprise</td>
</tr>
<tr>
<td>[+passive, +psych-subj]</td>
<td>1: love</td>
</tr>
<tr>
<td>[-passive, +that-comp]</td>
<td>1: forget</td>
</tr>
<tr>
<td>[-passive, +non-finite to, +that-comp, whether/if-comp]</td>
<td>1: know</td>
</tr>
<tr>
<td>[-passive, +that-comp]</td>
<td>1: hear</td>
</tr>
<tr>
<td>[-passive, +psych-subj]</td>
<td>1: love</td>
</tr>
</tbody>
</table>
C  Gibbs sampling in the generative model

The plate diagram for the generative model is presented here again for ease of reference, and Table [10] describes the variables in the plate diagram. Gibbs sampling is done following the process laid out in the plate diagram and described in more detail in the rest of this appendix. Depending on the modeled learner, between 2000 and 4000 iterations of Gibbs sampling were run, with each iteration involving the sampling of all classes, class properties, and hyperparameters, as described below.

![Plate Diagram](image)

Figure 3: Plate diagram for the generative model of verb classes.

C.1  How Gibbs sampling works

The general form of the sampler for a situation where outcome \( x_i \) takes value \( k \), given previous outcomes \( x_{-i} = (x_1, \ldots, x_{i-1}) \), and hyperparameter/pseudocount \( \beta \) for all \( K \) possible outcomes and multinomial distribution \( \theta \) is:

\[
P(k|x_{-i}, \beta) = \int P(k|\theta)P(\theta|x_{-i}, \beta)d\theta = \frac{n_k + \beta}{i - 1 + K\beta}
\]

according to the derivation in Goldwater and Griffiths (2007), who refer to the derivation in MacKay and Peto (1995), where \( n_k \) is the number of times \( k \) occurred in \( x_{-i} \).

This is the basis that we can use to do the sampling for all categorical and binary variables that correspond to a single frame instance (ex: verb classes, +animate-subject, etc). The basic sampling process involves removing the information of the current item being considered, and then calculating the sampling probabilities as directed. Once sampling is complete and the item’s
Table 10: Variables used in the plate diagram for the generative model and in the details of the Gibbs sampling process.

Information is updated, we add the new information back in. See Resnik and Hardisty (2010) for an excellent tutorial about Gibbs sampling.

C.2 Probability of the category label for a given verb

For all currently existing categories \( c_j \) in \( C \) and the possibility \( \gamma_c \) of creating a new category, calculate

\[
p_{\text{cat}_j} = P(c_j|c_{-j}, \gamma_c) = \frac{n_{c_j} + \gamma_c}{n_{all} + C \gamma_c}
\]

(2)

where \( n_{c_j} \) is the number of verbs with category \( c_j \) (excluding the verb whose class is currently being sampled \( c_{-j} \)), \( n_{all} \) is the total number of verbs excluding the one being sampled, and \( C \) is the total number of classes currently. So, the equivalent for the possibility of creating a new category would be

\[
P(c_{\text{new}}|c_{-j}, \gamma_c) = \frac{\gamma_c}{n_{all} + C \gamma_c}
\]

(3)

These should then all sum to 1.
C.3 Sampling binary properties

These will include +anim-subj, +anim-obj, and +anim-iobj for all model types. For the model assuming UTAH is already known, +mvmt would indicate whether the observed syntactic positions obey r/UTAH transparently or have moved. All of these apply to the probability of an individual frame \( f_{ji} \) appearing with the property within a given verb class \( c_j \) for a verb \( v_j \), with \( F_{c,j} \) referring to all the frames in class \( c_j \) except for the ones from \( v_j \). These properties are represented by \( \pi_{\phi_c} \) in the plate diagram.

\[
p_{\phi_{i,cj}} = P(f_{ji}|c_j, c_{-j}, F_{c_{-j}}, \pi_{\phi_{cj}}, \beta_{\phi_1}, \beta_{\phi_0}) = \frac{n_{\phi_{cj}} + \beta_{\phi_1}}{n_{all_{cj}} + \beta_{\phi_1} + \beta_{\phi_0}}
\]

where \( n_{\phi_{cj}} \) is the number of frames in category \( c_j \) that have the property, \( n_{all_{cj}} \) is all the frames in \( c_j \), \( \beta_{\phi_1} \) is the pseudocount for frames exhibiting the property, and \( \beta_{\phi_0} \) is the pseudocount for frames not exhibiting the property.

The probability for the entire set of frames \( F_j \) is the product of each individual frame token’s probability. We can use each individual frame’s frequency \( F_{ji} \) to calculate this, assuming \( \pi_{\phi} \) captures the complete distribution (both \( \phi_1 \) and \( \phi_0 \) probabilities) for property \( \phi \):

\[
p_{\phi_{cj}} = \prod_i p_{\phi_{i,cj}}^{F_{ji}}
\]

for all \( i \) frame types in verb \( v_j \). In particular, if the frame \( f_{ji} \) has the property and appears \( F_{ji} \) times, its contribution is \( p_{\phi_{i,cj}}^{F_{ji}} \); if frame \( f_{ji} \) doesn’t have the property and appears \( F_{ji} \) times, its contribution is \( (1 - p_{\phi_{i,cj}})^{F_{ji}} \).

The joint probability of all \( B \) binary properties is the product of \( p_{\phi_c} \) for all binary properties \( \phi \):

\[
p_{\text{binary}_{cj}} = \prod_{\phi=1}^{B} p_{\phi_{cj}}
\]

C.4 Sampling multinomial properties

This works similarly to the sampling of binary properties – it’s just that there are more than two options available. This would apply for the -exp-mapping modeled learner that infers the thematic-syntactic mappings for each verb class. Each underlying intermediate representation’s thematic role (e.g., UTAH: \textit{AGENT}-like or rUTAH: \textit{HIGHEST}) would map with some probability to a syntactic position (\textit{subject}, \textit{object}, \textit{oblique object}). So, there would be three probabilities for semantic arguments: \( p_{\text{subj}-\text{pos}} \), \( p_{\text{obj}-\text{pos}} \), and \( p_{\text{iobj}-\text{pos}} \). Each of these has three options of where to appear: syntactic \textit{subject}, syntactic \textit{object}, or syntactic \textit{oblique object}. The probabilities of appearing in those positions are a multinomial distribution.

Syntactic frames are another example that applies to all modeled learners, as a frame can be one of the F frame types available.
This property would be captured by the $\theta_{\psi_c}$ in the plate diagram.

\[
p_{\psi_{c_j}} = P(f_{ji}|c_j, c_{-j}, F_{c_{-j}}, \theta_{\psi_{c_j}}, \alpha_{\psi}) = \frac{n_{\psi_{c_j}} + \alpha_{\psi}}{n_{all_{c_j}} + O \ast \alpha_{\psi}}
\] (7)

where $f_{ji}$ is the $i$th frame token in the total frames $F_j$ for verb $v_j$, $c_j$ is the category of $v_j$, $\theta_{\psi_{c_j}}$ is the distribution over the $O$ options, $\alpha_{\psi}$ is the pseudocount for all options, $n_{\psi_{c_j}}$ is the number of frames in class $c_j$ that have this value, $n_{all_{c_j}}$ is all the frames in class $c_j$ that exhibit any of the values, and $O$ is the number of options of available (e.g., number of syntactic frame types or number of syntactic positions).

The probability for the entire set of frames $F_j$ is the product of each individual frame token’s probability. We can use each individual frame’s frequency $F_{ji}$ to calculate this, assuming $\theta_{\psi}$ captures the complete distribution for property $\phi$:

\[
p_{\psi_{c_j}} = \prod_{i} p_{F_{ji}}
\] (8)

for all $i$ frames in verb $v_j$.

The joint probability of all $M$ multinomial properties is the product of $p_{\psi_c}$ for all multinomial properties $\psi$:

\[
p_{\mathrm{multinomial}_{c_j}} = \prod_{\psi=1}^{M} p_{\psi_{c_j}}
\] (9)

### C.5 Complete sampling equation for new verb class for verb

Let $\lambda$ be the set of hyperparameters needed for this calculation, including the hyperparameters for category selection, binary properties, and multinomial properties. The complete equation for selecting a new verb class is:

\[
p_{c_j} = P(c_j|c_{-j}, \gamma_c, F_{-j}, \lambda) = p_{\text{cat}_j} \ast p_{\text{binary}_{c_j}} \ast p_{\text{multinomial}_{c_j}}
\]

Adjust these probabilities with an annealing temperature if desired (see section C.7 on annealing below), and then roll a C+1 sided die weighted according to the calculated probabilities. Then, select the category label that comes up.

### C.6 Hyperparameter sampling

We follow the approach in Goldwater and Griffiths (2007) to sample hyperparameters. In particular, there are priors over each of the hyperparameters (can assume an improper uniform prior), and use a single Metropolis-Hastings update (Gilks, Richardson, & Spiegelhalter, 1996) to resample the value of each hyperparameter after each iteration of the Gibbs sampler through all individual verbs.
To update the value of hyper parameter $\kappa$, we can sample a proposed new value $\kappa'$ from a normal distribution with $\mu = \kappa$ and $\sigma = .1\kappa$. Then, we calculate the probability of the data (all verbs), given $\kappa$ vs. given $\kappa'$.

**C.6.1 Verb category hyperparameter $\gamma_c$**

Let $V$ be the total number of observed verb types with their collection of observed frames, $C$ be the current verb classes, and $\lambda_{-\gamma}$ be the set of hyperparameters except for $\gamma_c$. The current hyperparameter value is $\gamma_c$ while the proposed one would be $\gamma'_c$. However, for purposes of the calculation (which we’ll have to do for both), we can represent both of them below as $\gamma$.

$$p(C|V, \lambda_{-\gamma}, \gamma) = \frac{p(V|C, \lambda_{-\gamma}, \gamma) * p(C|\gamma)}{P(V, \lambda_{-\gamma}, \gamma)}$$

We can disregard the denominator since we’re just comparing these two values for $\gamma_c$ and $\gamma'_c$, and they’ll have the same denominator.

Let’s look at each term in turn.

$$p(V|C, \lambda_{-\gamma}, \gamma) = \prod_{j=1}^{V} \text{binary}_{c_j} * \text{multinomial}_{c_j}$$

Note that none of these terms depend on $\gamma$ – so these will be the same for both $\gamma$ calculations. (Effectively, this term is a constant, and all the action happens in $p(C|\gamma)$.)

$$p(C|\gamma) = \prod_{i=1}^{C} p_{cat_i}$$

$$= \prod_{i=1}^{C} \frac{n_{c_i} + \gamma}{V + C * \gamma}$$

where $n_{c_i}$ is the number of verbs in class $i$ and $V$ is the total number of verbs.

**C.6.2 Binomial property hyperparameters $\beta_{\phi_1}, \beta_{\phi_0}$**

Let $V$ be the total number of observed verb types with their collection of observed frames, $C$ be the current verb classes, and $\lambda_{-\beta}$ be the set of hyperparameters except for the one $\beta$ value being sampled. The current hyperparameter value is $\beta_{\phi_x}$ while the proposed one would be $\beta'_{\phi_x}$. However, for purposes of the calculation (which we’ll have to do for both), we can represent both of them below as $\beta$.  

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\[
p(C|V, \lambda_{-\beta}, \beta) = \frac{p(V|C, \lambda_{-\beta}, \beta) \cdot p(C|\gamma)}{P(V, \lambda_{-\beta}, \beta)} \\
\propto p(V|C, \lambda_{-\beta}, \beta) \cdot p(C|\gamma)
\]

(We can disregard the denominator since we’re just comparing these two values for \(\beta_\phi\) and \(\beta'_{\phi_x}\), and they’ll have the same denominator.)

Let’s look at each term in turn.

\[
p(V|C, \lambda_{-\beta}, \beta) = \prod_{j=1}^{V} p_{\text{binary}_{c_j}} \cdot p_{\text{multinomial}_{c_j}}
\]

Since \(p_{\text{multinomial}_{c_j}}\) doesn’t depend on \(\beta\), it’s a constant for both \(\beta\) calculations. For \(p_{\text{binary}_{c_j}}\), only the calculation for binary property \(\phi\) will be affected by this \(\beta\) calculation – all the other binary properties will have different \(\beta_{\phi}\)s that will remain constant for this \(\beta\) calculation. So, the calculation for \(p_{\phi_i,c_j}\) is the one we pay attention to, calculating that for each verb in \(V\) and taking the product:

\[
p(V|C, \lambda_{-\beta}, \beta) = \prod_{j=1}^{V} p_{\phi_i,c_j}
\]

\[
= \prod_{j=1}^{V} \prod_{i=1}^{F_j} p_{\phi_i,c_j}^{F_{ji}}
\]

where \(F_j\) is the number of frame types in verb \(v_j\), \(F_{ji}\) is the frequency of the frame type \(i\) displaying a value of the desired property in class \(c_j\), and \(p_{\phi_i,c_j}\) is the distribution over the two options (so that the appropriate probability is used depending on which value the frame displays):

\[
p_{\phi_i,c_j} = \frac{n_{\phi_i,c_j} + \beta}{F + \beta + \beta_{\text{other}}}
\]

where \(n_{\phi_i,c_j}\) is the number of frames in class \(c_j\) that have the appropriate property value for the \(\beta\) being calculated, \(F\) is the total number of frames in class \(c_j\), and \(\beta_{\text{other}}\) is the \(\beta\) not being sampled.

\[
p(C|\gamma) = \prod_{i=1}^{C} p_{\text{cat}_i}
\]

Note that \(p_{\text{cat}_i}\) doesn’t depend on \(\beta\) – so these will be the same for both \(\beta\) calculations. (Effectively, this term is a constant, and all the action happens above in \(p(V|C, \lambda_{-\beta}, \beta)\).)
C.6.3 Multinomial property hyperparameters $\alpha_{\psi}$

This process is going to be very similar to the binary hyperparameter sampling above.

Let $V$ be the total number of observed verb types with their collection of observed frames, $C$ be the current verb classes, and $\lambda_{-\alpha}$ be the set of hyperparameters except for the $\alpha$ value being sampled. The current hyperparameter value is $\alpha_{\psi}$, while the proposed one would be $\alpha'_{\psi}$. However, for purposes of the calculation (which we’ll have to do for both), we can represent both of them below as $\alpha$.

\[
p(C | V, \lambda_{-\alpha}, \alpha) = \frac{p(V | C, \lambda_{-\alpha}, \alpha) \ast p(C | \gamma)}{P(V, \lambda_{-\alpha}, \alpha)} \propto p(V | C, \lambda_{-\alpha}, \alpha) \ast p(C | \gamma)
\]

(We can disregard the denominator since we’re just comparing these two values for $\alpha_{\psi}$ and $\alpha'_{\psi}$, and they’ll have the same denominator.)

Let’s look at each term in turn.

\[
p(V | C, \lambda_{-\alpha}, \alpha) = \prod_{j=1}^{V} p_{\text{binary}_{c_j}} \ast p_{\text{multinomial}_{c_j}}
\]

Since $p_{\text{binary}_{c_j}}$ doesn’t depend on $\alpha$, it’s a constant for both $\alpha$ calculations. For $p_{\text{multinomial}_{c_j}}$, only the calculation for multinomial property $\psi$ will be affected by this $\alpha$ calculation – all the other multinomial properties will have different $\alpha_{\psi}$s that will remain constant for this $\alpha$ calculation. So, the calculation for $p_{\psi_{i,c_j}}$ is the one we pay attention to, calculating that for each verb in $V$ and taking the product:

\[
p(V | C, \lambda_{-\alpha}, \alpha) = \prod_{j=1}^{V} p_{\psi_{i,c_j}} = \prod_{j=1}^{V} F_j \prod_{i=1}^{F_i} p_{\psi_{i,c_j}}
\]

where $F_j$ is the number of frame types in verb $v_j$, $F_{ji}$ is the frequency of the frame type $i$ displaying a value of the desired property in class $c_j$, and $p_{\psi_{i,c_j}}$ is the distribution over the various options (so that the appropriate probability is used depending on which value the frame displays):

\[
p_{\psi_{i,c_j}} = \frac{n_{\psi_{c_j}} + \alpha}{F + O \ast \alpha}
\]

where $n_{\psi_{c_j}}$ is the number of frames in class $c_j$ that have the appropriate property value for the $\alpha$ being calculated, $F$ is the total number of frames in class $c_j$, and $O$ is the number of multinomial options there are for this property.
\[ p(C|\gamma) = \prod_{i=1}^{C} p_{cat_i} \]

As with the binary properties, \( p_{cat_i} \) doesn’t depend on \( \alpha \) – so these will be the same for both \( \alpha \) calculations. (Effectively, this term is a constant, and all the action happens above in \( p(V|C, \lambda_{-\alpha}, \alpha) \).)

### C.6.4 Metropolis-Hastings update

We can now do the Metropolis-Hastings update: the probability of accepting the new value depends on the ratio between \( p(C | V, \lambda_{-\kappa}, \kappa) \) and \( p(C | V, \lambda_{-\kappa}, \kappa') \), with a term correcting for the asymmetric proposal distribution.

1. Calculate \( a_1 \):

   \[ a_1 = \frac{p(C|V, \lambda_{-\kappa}, \kappa)}{p(C|V, \lambda_{-\kappa}, \kappa')} \]  
   
   \[ (10) \]

2. Calculate \( a_2 \):

   \[ a_2 = \frac{p(\kappa|\kappa')}{p(\kappa'|\kappa)} \]  
   
   \[ (11) \]

   where \( p(\kappa|\kappa') \) is the probability of drawing \( \kappa \) from a normal distribution with \( \mu = \kappa' \) and \( \sigma = .1\kappa' \), while \( p(\kappa'|\kappa) \) is the probability of drawing \( \kappa' \) from a normal distribution with \( \mu = \kappa \) and \( \sigma = .1\kappa \).

3. Calculate \( a = a_1a_2 \).

   If \( a \geq 1 \), accept \( \kappa' \).

   Otherwise, flip a weighted coin (and we can anneal with temperature \( T \) if desired). With probability \( \frac{a^T}{a^T + (1-a)^T} \), choose \( \kappa' \). With probability \( \frac{(1-a)^T}{a^T + (1-a)^T} \), keep the original \( \kappa \). If no annealing (or \( T=1 \) for that iteration), this defaults to probability \( a \) and \( 1-a \) for the two options.

### C.7 Annealing

To help the Gibbs sampler converge faster, a simulated annealing regime is typically used (e.g., in Goldwater & Griffiths 2007) to force more exploration early on by flattening the probabilities and less exploration later on by sharpening the probabilities. The way this is done is by raising the calculated probability to a power (which is the temperature \( T \)).

\[ \text{annealed probability} = \text{probability}^T \]  

\[ (12) \]
As the Gibbs sampler does more iterations, the temperature $T$ lowers so the calculated probabilities are sharpened. (Basically, the sampler is more confident about its calculated probabilities later on in learning.) We follow Goldwater and Griffiths (2007) and use a range of $T = 2$ lowered down to 0.8 over the course of all iterations.

**D Filtered verb classes**

Tables 11-16 show the filtered verb classifications for each strategy implemented by a computational-level Bayesian learner. Each modeled learner ran 10 times for each dataset (<3yrs, <4yrs, <5yrs), and these ten verb clusterings were aggregated into an aggregate verb clustering for each learner. Any verb pair together in more than $\frac{3}{4}$ of the learner runs (>7 out of 10) was put together in the aggregate verb clustering. Similarly, any verb that was in a class of its own (a singleton) for more than $\frac{3}{4}$ of the learner runs was put as a singleton in the aggregate verb clustering.
Table 11: <3yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that involve the use of surface morphology (+surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/-expect-mapping).

<table>
<thead>
<tr>
<th>Inferred: begin, confuse, dress, figure, fill, fold, knock, lean, mix, name, pick, play, rain, roll, seem, send, teach, tell, tip, wait, wake, wonder</th>
<th>UTAH</th>
<th>Surface-morphology</th>
<th>rUTAH</th>
<th>Expect-mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impletions: begin, confuse, dress, figure, fill, fold, knock, lean, mix, name, pick, play, rain, seem, send, teach, tell, tip, wait, wake, wonder</td>
<td>1: answer, beat, bite, blow, bother, break, bring, brush, build, burn, buy, carry, catch, change, chase, check, clean, close, color, cool, count, crack, cut, decide, do, draw, dry, dump, eat, feel, find, finish, fix, forget, guess, have, hear, help, hit, hold, hook, hope, hurt, keep, kick, kiss, know, leave, lose, love, make, match, mean, meet, melt, mind, miss, move, miss, need, open, paint, pass, pinch, poke, pour, pretend, pull, push, put, reach, read, recognize, record, remember, save, say, scare, scratch, see, shake, share, shoot, shut, sing, snap, spell, spell, spray, squeeze, squish, start, stir, stop, study, swing, take, tape, tear, think, throw, tickle, tie, touch, turn, understand, untie, use, wash, watch, wear, wipe, wish</td>
<td>1: answer, beat, bite, blow, bother, break, bring, brush, build, burn, buy, carry, catch, change, chase, check, clean, close, color, cool, count, crack, cut, decide, do, draw, dry, dump, eat, feel, find, finish, fix, forget, guess, have, hear, help, hit, hold, hook, hope, hurt, keep, kick, kiss, know, leave, lose, love, make, match, mean, meet, mind, miss, move, open, paint, pinch, pour, pull, push, reach, read, recognize, record, remember, rock, roll, save, say, scare, see, shake, share, shove, shut, sing, snap, spell, spell, spray, squeeze, square, stick, stir, stop, study, suppose, swing, tape, tease, test, tickle, touch, understand, use, watch, wear, wipe, wish</td>
<td>1: answer, beat, bite, blow, bother, break, bring, brush, build, burn, buy, carry, catch, change, chase, check, clean, close, color, cool, count, cover, crack, cut, decide, do, draw, drink, drop, dry, eat, feel, find, finish, fix, forget, guess, have, hear, help, hit, hold, hook, hope, hurt, keep, kick, kiss, know, leave, lose, love, make, match, mean, meet, mind, miss, move, move, need, open, paint, pass, pinch, pull, pretend, pull, put, reach, read, recognize, record, remember, save, say, scare, scratch, see, shake, share, shave, shoot, shut, sing, snap, spell, spell, spray, squeeze, squeal, squish, start, stir, stop, study, suppose, swing, take, tape, tear, think, throw, tickle, tie, touch, turn, untie, use, wash, watch, wear, wipe, wish</td>
<td>2: decide, know, love, mean, remember, understand</td>
</tr>
</tbody>
</table>

2: get, smell, suppose, test
3: rock, taste, tease
4: belong, bob, bounce, care, cook, crawl, cry, dance, drive, fit, fly, grow, hammer, hang, hide, hop, jump, laugh, lay, listen, live, look, peek, play, ride, ring, row, run, sit, sleep, slip, sound, square, stand, stay, step, swim, talk, visit, walk, wind, work, write

5: fall, hurry
6: bump, call, drop, serve
7: dig, juggle
8: entertain, excuse, thank
9: freeze, smile
10: learn, worry
11: like, want
12: bless, pet, press
13: cool, set
14: chirp, climb, come, go, let, lie, peep, point, pop, speak, try, whisper
15: feed, show
16: lick, peel
Table 12: <3yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that don't involve the use of surface morphology (-surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/-expect-mapping).

<table>
<thead>
<tr>
<th>Singletons:</th>
<th>begin, figure, seem, tip</th>
<th>Singletons:</th>
<th>begin, figure, seem, tip</th>
<th>Singletons:</th>
<th>begin, figure, seem, tip</th>
<th>Singletons:</th>
<th>begin, figure, seem, tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: answer, ask, bite, bother, bring, brush, build, bump, burn, buy, call, carry, catch, change, chase, check, chop, close, color, count, cover, crack, cut, dig, do, draw, drink, drop, dry, dump, eat, feed, find, finish, fix, fold, get, give, have, hear, hit, hold, hook, juggles, keep, kick, kiss, knock, lose, make, match, mix, move, mow, name, need, offer, open, pass, peel, pet, pick, pinch, plug, poke, pour, press, pull, push, put, reach, read, recognize, record, record, rock, roll, save, say, scare, scratch, send, serve, shake, share, shoot, show, shut, sign, sing, smash, spell, spray, stir, stop, study, swing, take, taste, teach, tear, tell, test, throw, tickle, tie, touch, turn, untie, visit, wash, watch, wear, wind, wipe</td>
<td>1: answer, ask, bite, blow, break, build, build, bump, burn, buy, call, carry, catch, change, chase, close, color, count, crack, cut, dig, do, draw, drink, drop, dry, dump, eat, find, finish, fix, get, have, hear, hit, hold, hook, juggles, keep, kick, kiss, knock, lose, make, match, meet, mind, mix, move, mow, need, open, paint, peel, pick, pinch, plug, poke, pour, press, pull, push, put, reach, read, recognize, record, record, rock, roll, save, say, scare, scratch, send, serve, shake, share, shoot, show, shut, sign, sing, smash, spell, spray, stir, stop, swing, take, taste, tickle, touch, untie, visit, wash, wear, wind, wipe</td>
<td>1: answer, ask, bite, build, bump, burn, buy, call, carry, catch, change, chase, close, count, crack, cut, dig, do, draw, drink, drop, dry, dump, eat, find, finish, fix, get, have, hear, hit, hold, hook, juggles, keep, kick, kiss, knock, lose, make, mix, move, mow, need, offer, open, pass, peel, pet, pick, pinch, plug, poke, pour, press, pull, push, put, reach, read, recognize, record, record, rock, roll, save, say, scare, scratch, send, serve, shake, share, shoot, shut, sign, sing, spell, spray, squeeze, stir, swing, take, taste, throw, tickle, touch, untie, wash, wash, wear, wipe</td>
<td>1: ask, beat, blow, bring, check, close, chop, clean, cover, fold, hook, knock, lick, match, mix, pass, pick, plug, pour, pull, put, roll, save, shoot, slide, smash, take, tear, throw, tie, turn, wash, wind</td>
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<tr>
<td>2: beat, lick</td>
<td>2: bang, cool, cook, hammer, listen, play, ride, set, write</td>
<td>2: bang, belong, climb, come, cook, crawl, feel, fill, grow, hammer, hang, lay, lean, listen, look, peek, play, point, ride, run, set, sit, slide, smell, sound, stand, step, stick, talk, wake, worry, write</td>
<td>2: bang, cook, cool, hammer, listen, play, ride, set, write</td>
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<tr>
<td>3: bang, belong, climb, come, cook, crawl, feel, fill, grow, hammer, hang, lay, lean, listen, look, peek, play, point, ride, run, set, sit, slide, smell, sound, stand, step, stick, talk, wake, worry, write</td>
<td>3: bang, belong, bounce, climb, cool, cook, crawl, drive, fall, feel, fill, grow, hammer, hang, jump, lay, lean, lie, listen, look, peek, prepe, play, point, pop, ride, run, set, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, wake, worry, write</td>
<td>3: bang, belong, bounce, climb, cool, cook, crawl, drive, fall, feel, fill, fly, grow, hammer, hang, jump, lay, lean, lie, listen, look, peek, prepe, play, point, pop, ride, run, set, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, wake, worry, write</td>
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<tr>
<td>4: bet, confuse, guess, hope, pretend, think, wish</td>
<td>4: bet, confuse, guess, hope, pretend, think, wish</td>
<td>4: bet, confuse, guess, hope, pretend, think, wish</td>
<td>4: bet, knock</td>
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<tr>
<td>5: bless, entertain, excuse, thank</td>
<td>5: bless, entertain, excuse, thank</td>
<td>5: bless, entertain, excuse, thank</td>
<td>5: bless, entertain, excuse, thank</td>
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<tr>
<td>6: blow, chew, clean</td>
<td>6: blow, chew, clean</td>
<td>6: blow, chew, clean</td>
<td>6: blow, chew, clean</td>
<td></td>
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<tr>
<td>7: bob, bounce, care, chirp, cool, cry, dance, dress, drip, drive, fall, fit, fly, happen, hide, jump, laugh, live, melt, prepare, pop, ring, sleep, slip, speak, squeeze, stay, swim, walk, work</td>
<td>7: bob, bounce, care, chirp, cry, dance, drip, happen, laugh, live, melt, ring, smile, squeeze, walk, work</td>
<td>7: bob, care, chirp, cry, dance, drip, happen, laugh, live, melt, ring, squeeze, walk, work</td>
<td>7: bob, care, chirp, cry, dance, drip, happen, laugh, live, melt, ring, squeeze, walk, work</td>
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<tr>
<td>8: break, mind, paint, squeeze, squash</td>
<td>8: come, learn</td>
<td>8: come, learn</td>
<td>8: come, learn</td>
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<tr>
<td>9: decide, know, mean, remember, reuse, understand, wonder</td>
<td>9: decide, know, mean, remember, understand, wonder</td>
<td>9: decide, know, mean, remember, understand, wonder</td>
<td>9: decide, know, mean, remember, understand, wonder</td>
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<tr>
<td>10: forget, go, help, let, like, start, suppose, try, use, want</td>
<td>10: forget, go, learn, let, like, start, suppose, try, use, want</td>
<td>10: forget, go, learn, let, like, start, suppose, try, use, want</td>
<td>10: forget, go, help, learn, let, like, start, suppose, try, use, want</td>
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<td></td>
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<tr>
<td>11: hurt, meet, see, tape</td>
<td>11: hurt, meet, see, tape</td>
<td>11: hurt, meet, see, tape</td>
<td>11: hurt, know, mean, mind, remember, see, study, tape, understand, wonder</td>
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<tr>
<td>12: manage, smile</td>
<td>12: manage, shave</td>
<td>12: manage, shave</td>
<td>12: manage, shave</td>
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<tr>
<td>14: freeze, rain</td>
<td>14: freeze, rain</td>
<td>14: freeze, rain</td>
<td>14: freeze, rain</td>
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</tbody>
</table>
Table 13: <4yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that involve the use of surface morphology (+/expect-morphology, an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/expect-mapping)).

<table>
<thead>
<tr>
<th></th>
<th>ex-put-mapping</th>
<th>ex-put-mapping</th>
<th>ex-put-mapping</th>
<th>ex-put-mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>singletons:</td>
<td>begin, confuse, dress, figure, land, rain, seem, wonder</td>
<td>singletons:</td>
<td>begin, confuse, dress, figure, land, rain, seem, wonder</td>
<td>singletons:</td>
</tr>
<tr>
<td>1: answer, attach, bear, believe, bend, bite, blow, bother, break, bring, build, bump, burn, buy, carry, catch, change, chase, check, chk, chop, close, color, count, cover, crack, cross, cut, decide, do, draw, drink, drop, dry, drink, eat, examine, feel, find, fin, fix, fold, forget, get, hang, have, hear, help, hit, hold, hook, hurt, keep, kick, kiss, know, leave, lose, make, match, mean, meet, mind, miss, move, mov, open, paint, park, pinch, poke, poor, press, pretend, pull, push, put, reach, read, recognize, record, remem, roll, save, say, scare, scratch, screw, see, sell, send, set, shake, share, shoot, shut, sing, smell, snap, spell, spill, squish, stick, stir, study, surprise, swing, take, tape, tear, thank, throw, tickle, tie, touch, trade, turn, untie, use, visit, wash, wear, wind, wipe</td>
<td>1: answer, attach, bear, believe, bite, bother, break, bring, build, bump, burn, buy, call, catch, change, chase, check, chop, close, color, count, cover, crack, cross, cut, do, draw, drink, drop, dry, eat, examine, feel, find, fin, fix, forget, get, have, hear, help, hit, hold, hurt, keep, kick, kiss, know, leave, lose, make, match, mean, meet, mind, miss, move, mov, open, paint, park, peel, pinch, pinch, poor, press, pull, punch, put, push, record, roll, scare, scratch, screw, self, set, shake, share, shoot, shut, sing, smell, snap, spell, spill, squish, stick, stir, study, suppose, surprise, test, test, touch, trade, understand, untie, use, visit, wash, wear, win, wind, wipe</td>
<td>1: answer, attach, bear, believe, bite, bother, break, bring, build, bump, burn, buy, call, catch, change, chase, check, chop, close, color, count, cover, crack, cross, cut, decide, do, draw, drink, drop, dry, dump, eat, examine, feed, feel, find, finish, fix, fold, forget, get, hang, have, hear, help, hit, hold, hurt, keep, kick, kill, kiss, know, leave, love, make, match, mean, meet, mind, miss, move, mov, open, paint, park, peel, pinch, pinch, poor, press, pull, punch, put, reach, record, roll, scare, scratch, screw, self, set, shake, share, shoot, shut, sing, smell, snap, spell, spill, squish, stick, stir, study, suppose, surprise, test, test, touch, trade, understand, untie, use, visit, wash, wear, win, wind, wipe</td>
<td>1: answer, attach, bear, believe, bite, bother, break, bring, build, bump, burn, buy, call, catch, change, chase, check, chop, close, color, count, cover, crack, cross, cut, decide, do, draw, drink, drop, dry, dump, eat, examine, feed, feel, find, finish, fix, fold, forget, get, hang, have, hear, help, hit, hold, hurt, keep, kick, kill, kiss, know, leave, love, make, match, mean, meet, mind, miss, move, mov, open, paint, park, peel, pinch, pinch, poor, press, pull, punch, put, reach, record, roll, scare, scratch, screw, self, set, shake, share, shoot, shut, sing, smell, snap, spell, spill, squish, stick, stir, study, suppose, surprise, test, test, touch, trade, understand, untie, use, visit, wash, wear, win, wind, wipe</td>
<td></td>
</tr>
</tbody>
</table>
Table 14: <4yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that don’t involve the use of surface morphology (-surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/expect-mapping).

<table>
<thead>
<tr>
<th>Table 14: &lt;4yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that don’t involve the use of surface morphology (-surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/expect-mapping).</th>
<th>UTAH</th>
<th>-surface-morphology</th>
<th>rUTAH</th>
<th>-surface-morphology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singletons: begun, figure, row, som, want</td>
<td>Singletons: begun, figure, row, som, want</td>
<td>Singletons: begun, figure, som, tip, want</td>
<td>Singletons: begun, figure, row, som, tip, want</td>
<td>Singletons: begun, figure, row, som, want</td>
</tr>
<tr>
<td>1: answer, bite, bother, build, bump, catch, chase, check, close, dig, drink, dry, eat, frighten, kill, loss, measure, meet, move, mow, open, peel, reach, read, scare, scratch, shake, shout, sing, spell, spray, squeeze, tinkle, touch, trade, visit</td>
<td>1: answer, bite, bother, build, bump, catch, chase, check, close, dig, drink, dry, eat, frighten, kill, loss, measure, meet, move, mow, open, peel, reach, read, scare, scratch, shake, shout, sing, spell, spray, squeeze, tinkle, touch, trade, visit</td>
<td>1: answer, bite, bother, build, bump, catch, chase, check, close, dig, drink, dry, eat, frighten, kill, loss, measure, meet, move, mow, open, peel, reach, read, scare, scratch, shake, shout, sing, spell, spray, squeeze, tinkle, touch, trade, visit</td>
<td>1: answer, bite, bother, build, bump, catch, chase, check, close, dig, drink, dry, eat, frighten, kill, loss, measure, meet, move, mow, open, peel, reach, read, scare, scratch, shake, shout, sing, spell, spray, squeeze, tinkle, touch, trade, visit</td>
<td></td>
</tr>
<tr>
<td>2: ask, bake, break, bring, brush, buy, call, carry, change, cook, crack, do, draw, drop, find, fix, get, have, hear, hit, keep, love, make, mean, mind, name, need, paint, park, recognize, say, see, share, shoot, smash, spill, spit, stop, study, tape, test, use, watch, wear, win</td>
<td>2: attach, beat, bend, bring, cover, cut, fill, hold, hook, knock, lick, mix, pass, pick, plug, pour, press, pull, punch, push, put, save, spank, spell, spray, squeeze, stir, stop, study, test, tinkle, touch, trade, unite, use, visit, watch, wear, win</td>
<td>2: attach, beat, bend, bring, cover, cut, fill, hold, hook, knock, lick, mix, pass, pick, plug, pour, press, pull, punch, push, put, save, spank, spell, spray, squeeze, stir, stop, study, test, tinkle, touch, trade, unite, use, visit, watch, wear, win</td>
<td>2: attach, beat, bend, bring, hook, knock, lick, mix, pass, pick, pour, press, punch, push, put, save, spank, spell, spray, squeeze, stir, stop, study, test, tinkle, touch, trade, use, watch, wear, wipe</td>
<td></td>
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<tr>
<td>3: attach, beat, bend, blow, bump, chop, clean, color, count, cover, cut, dump, fold, hang, hold, hook, kick, knock, leave, lick, match, mix, pass, pick, plug, poke, press, pull, push, punch, put, record, roll, save, screw, slide, spank, swing, take, tear, throw, tie, turn, wash, wind, wipe</td>
<td>3: bite, carry, color, count, dump, hit, kick, leave, park, poke, screw, shoot, swing, wipe</td>
<td>3: blow, chew, chop, clean, cover, drive, fold, hang, match, paint, pull, rock, roll, slide, turn</td>
<td>3: blow, chew, chop, clean, cover, drive, fold, hang, match, paint, pull, rock, roll, slide, turn</td>
<td></td>
</tr>
<tr>
<td>4: back, hurry</td>
<td>4: bear, live</td>
<td>4: bear, live</td>
<td>4: bear, live</td>
<td></td>
</tr>
<tr>
<td>5: bang, bounce, climb, cool, crash, crawl, dance, dress, fall, feel, light, fit, fly, grow, hammer, hop, jump, lay, lean, lie, listen, look, peek, person, play, point, pop, ride, run, set, shave, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td>5: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
<td>5: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
<td>5: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
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<tr>
<td>6: back, hurry</td>
<td>6: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
<td>6: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
<td>6: bang, light, grow, hammer, play, ride, set, shave, taste, write</td>
<td></td>
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<tr>
<td>7: bounce, climp, cool, crash, crawl, dance, feel, fall, fit, fly, grow, hammer, happen, hop, hurry, jump, lay, lean, lie, listen, look, person, peek, person, pop, person, pop, run, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td>7: bounce, climp, cool, crash, crawl, dance, feel, fall, fit, fly, grow, hammer, happen, hop, hurry, jump, lay, lean, lie, listen, look, person, peek, person, pop, person, pop, run, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td>7: bounce, climp, cool, crash, crawl, dance, feel, fall, fit, fly, grow, hammer, happen, hop, hurry, jump, lay, lean, lie, listen, look, person, peek, person, pop, person, pop, run, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td>7: bounce, climp, cool, crash, crawl, dance, feel, fall, fit, fly, grow, hammer, happen, hop, hurry, jump, lay, lean, lie, listen, look, person, peek, person, pop, person, pop, run, sit, sleep, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td></td>
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<tr>
<td>8: believe, decide, know, remember, surprise, understand, wonder</td>
<td>8: believe, decide, hurt, know, mind, remember, see, surprise, tape, understand, wonder</td>
<td>8: believe, decide, hurt, know, mind, remember, see, surprise, tape, understand, wonder</td>
<td>8: believe, decide, hurt, know, mind, remember, see, surprise, tape, understand, wonder</td>
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<tr>
<td>10: bless, entertain, excuse, pay, thank</td>
<td>10: bless, entertain, excuse, pay, thank</td>
<td>10: bless, entertain, excuse, pay, thank</td>
<td>10: bless, entertain, excuse, pay, thank</td>
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</tr>
<tr>
<td>11: bob, chup, ski</td>
<td>11: bob, chup, ski</td>
<td>11: bob, chup, ski</td>
<td>11: bob, chup, ski</td>
<td></td>
</tr>
<tr>
<td>12: care, cry, disappear, dream, drip, laugh, manage, march, melt, melt, smile, work</td>
<td>12: care, cry, disappear, dream, drip, laugh, manage, march, melt, melt, smile, work</td>
<td>12: care, cry, disappear, dream, drip, manage, march, melt, ring, smile, work</td>
<td>12: care, cry, disappear, dream, drip, manage, march, melt, ring, smile, work</td>
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<tr>
<td>13: cross, examine, kiss, pet, pinch, sell, sign, snap, squash</td>
<td>13: cross, examine, kiss, pet, pinch, sell, sign, snap, squash</td>
<td>13: cross, examine, kiss, pet, pinch, sell, sign, snap, squash</td>
<td>13: cross, examine, kiss, pet, pinch, sell, sign, snap, squash</td>
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</tr>
<tr>
<td>14: feed, give, offer, send, serve, show, teach, tell</td>
<td>14: feed, give, offer, send, serve, show, teach, tell</td>
<td>14: feed, give, offer, send, serve, show, teach, tell</td>
<td>14: feed, give, offer, send, serve, show, teach, tell</td>
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<tr>
<td>15: finish, hate, hurt, miss</td>
<td>15: forget, like</td>
<td>15: forget, like</td>
<td>15: forget, like</td>
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<tr>
<td>16: forget, help, like</td>
<td>16: give, go, land, learn, let, start, suppose, try, want</td>
<td>16: give, go, land, learn, let, start, suppose, try, want</td>
<td>16: give, go, land, learn, let, start, suppose, try, want</td>
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<tr>
<td>17: freeze, rain</td>
<td>17: freeze, rain</td>
<td>17: freeze, rain</td>
<td>17: freeze, rain</td>
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<tr>
<td>18: go, land, learn, let, start, suppose, try, want</td>
<td>18: go, land, learn, let, start, suppose, try, want</td>
<td>18: go, land, learn, let, start, suppose, try, want</td>
<td>18: go, land, learn, let, start, suppose, try, want</td>
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</tr>
</tbody>
</table>
Table 15: 5yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that involve the use of surface morphology (+surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/expect-mapping).

<table>
<thead>
<tr>
<th>singletons: begin, confine, align, desire, figure, hurry, land, rain, rinse, seem, tip, want, wonder</th>
<th>expect-mapping</th>
<th>UTAH expect-mapping</th>
<th>surface-morphology</th>
<th>rUTAH expect-mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: answer, attach, bear, beat, bite, blow, bother, break, bring, build, burn, buy, carry, catch, change, chase, check, chew, chop, clean, close, color, count, cover, crack, cross, cut, do, draw, drill, drink, drive, dump, eat, find, finish, fix, fold, follow, frighten, grind, hang, hit, hold, hook, hurt, keep, kick, kill, kiss, knock, leave, lick, lose, match, measure, move, now, open, paint, park, pass, peel, pinch, plug, poke, pour, press, pull, punch, put, push, read, record, rewind, rock, roll, scare, scratch, screw, sell, set, shake, share, shave, shoot, shut, sing, slide, smack, spell, squeeze, squash, stick, stir, straighten, stretch, swallow, sweep, swing, take, tape, tear, throw, tickle, tie, touch, trade, turn, twist, untie, visit, wash, wear, wind, wipe, wrap</td>
<td>1: answer, attach, bear, believe, bite, blow, bother, break, bring, build, burn, buy, carry, catch, change, chase, check, chew, chop, clean, close, color, count, cover, crack, cross, cut, do, draw, drill, drink, drive, dump, eat, examine, find, finish, fit, follow, forget, get, have, hear, help, hit, hold, hurt, keep, kick, kill, kiss, know, leave, lose, make, match, mean, meet, mind, miss, move, open, paint, park, peel, pinch, poke, present, read, reach, recognize, record, remember, rewind, save, say, scan, scratch, see, sell, send, shake, share, sharpen, shave, shoot, sign, sing, smash, smell, snap, spell, spill, spray, squeeze, squash, stick, stir, straighten, suppose, surprise, swallow, swing, tape, test, tickle, touch, trade, untie, use, visit, watch, wear, wonder</td>
<td>1: answer, attach, bear, believe, bite, blow, bother, break, bring, build, burn, buy, carry, catch, change, chase, check, chop, clean, close, color, count, cover, crack, cross, cut, decide, do, draw, drink, drive, drop, dry, dump, eat, examine, find, finish, fit, follow, forget, get, have, hear, help, hit, hold, hurt, keep, kick, kill, kiss, know, leave, lose, make, match, mean, meet, mind, miss, move, open, paint, park, peel, pin, poke, pretend, pull, read, reach, recognize, record, remember, rewind, roll, rock, scramble, scratch, screw, sell, set, shake, share, sharpen, shave, shoot, shut, sing, sign, slide, smell, spell, spell, squeeze, squash, stick, stir, straighten, stretch, study, suppose, swallow, swing, take, tape, test, throw, tickle, tie, touch, trade, turn, twist, untie, use, visit, watch, wear, wind, wipe, wrap</td>
<td>2: bake, pet</td>
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</tr>
</tbody>
</table>
Table 16: <5yrs dataset: Aggregate inferred classes over 10 runs, given 4 strategies that don’t involve the use of surface morphology (-surface-morphology), an intermediate representation (UTAH/rUTAH), and an expectation of a mapping between the intermediate representation and observable syntactic positions (+/expect-mapping).

<table>
<thead>
<tr>
<th>UTAH</th>
<th>-surface-morphology</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>+expect-mapping</strong></td>
<td><strong>+expect-mapping</strong></td>
</tr>
<tr>
<td><strong>singletons:</strong> bear, begin, figure, row, seem, tip, wait</td>
<td><strong>singletons:</strong> bear, begin, figure, row, seem, tip, wait</td>
</tr>
<tr>
<td>1. answer, bite, build, butcher, buy, catch, change, change, check, close, count, cover, dig, draw, drop, drink, eat, fix, frighten, kill, match, move, new, open, peel, pull, reach, record, rewrite, scratch, screw, set, shake, shoot, shut, sing, spray, stir, swallow, swing, take, touch, trade, visit</td>
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</tr>
<tr>
<td>2. ask, feed, give, offer, pay, send, serve, show, smack, teach, tell</td>
<td>2. ask, call, feed, give, order, offer, pay, send, serve, show, smack, teach, tell</td>
</tr>
<tr>
<td>3. attach, beat, bend, bring, chew, chop, clean, color, cut, drill, dump, fill, fold, hold, hook, kick, knock, leave, lick, lift, mix, park, pass, pick, plug, poke, pour, press, push, pull, push, put, run, rise, save, slide, slip, spank, take, throw, turn, wash, wind, wipe, wrap</td>
<td>5. attach, beat, bend, bring, chop, clean, drill, dump, fill, fold, hold, hook, knock, lick, lift, mix, park, pass, pick, plug, poke, pour, press, pull, push, put, run, rise, save, slide, slip, spank, take, throw, turn, wash, wind, wipe, wrap</td>
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<tr>
<td>4. back, bang, bounce, climb, cool, crash, crawl, dance, end, fall, feel, fit, fly, grow, hammer, hop, jump, lay, lie, listen, look, peek, play, pop, rest, ride, run, shave, sit, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write</td>
<td>6. back, bounce, climb, come, cool, crash, crawl, dance, dress, end, fall, feel, fit, fly, grow, hop, hurry, jump, lay, lie, listen, look, peek, play, pop, rest, ride, run, shave, sit, slip, smell, sound, speak, stand, step, stick, swim, talk, taste, wake, walk, worry, write</td>
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<tr>
<td>5. bake, bother, buy, call, carry, crack, do, drop, find, follow, get, have, hear, help, hit, keep, lose, love, make, measure, name, need, recognize, sell, share, spell, spill, tape, test, use, watch, wear</td>
<td>7. back, bang, blow, bounce, climb, cool, crash, crawl, dance, end, fall, feel, fit, fly, grow, hammer, hop, jump, lay, lie, listen, look, match, peek, play, pop, rest, ride, rock, run, shave, sit, slip, smell, sound, speak, stand, step, stick, swim, talk, taste, wake, walk, worry, write</td>
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<tr>
<td>6. believe, decide, finish, hurt, know, mean, mind, miss, pretend, remember, say, see, study, surprise, understand, wish, wonder</td>
<td>8. believe, decide, hurt, know, mean, mind, miss, pretend, remember, say, see, study, surprise, understand, wish, wonder</td>
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<tr>
<td>7. belong, trip</td>
<td>9. belong, trip</td>
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<tr>
<td>8. bet, guess, hope, think</td>
<td>10. bet, confuse, guess, hope, think</td>
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<tr>
<td>9. bless, entertain, excuse, thank</td>
<td>11. bless, entertain, excuse, thank</td>
</tr>
<tr>
<td>10. blow, rock, sweep</td>
<td>12. blow, match, rock, sweep</td>
</tr>
<tr>
<td>11. bob, chirp, ski, sneeze</td>
<td>13. bob, chirp, ski, sneeze</td>
</tr>
<tr>
<td>12. break, cook, dry, hide, meet, paint, scare, sing, squeeze, tickle, untie</td>
<td>14. break, cook, finish, paint, reach, reward, shake, sing, spray, stop, win</td>
</tr>
<tr>
<td>13. care, cry, dream, drip, happen, laugh, live, March, melt, ring, sleep, smile, work</td>
<td>15. care, cry, disappear, dream, drip, happen, laugh, live, March, melt, ring, sleep, smile, squawk, work</td>
</tr>
<tr>
<td>14. come, dress, go, land, lean, learn, let, peepee, shop, start, stop, try</td>
<td>16. go, learn, lean, let, try, whisper</td>
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<tr>
<td>15. cross, examine, kiss, pet, pinch, sharpen, snap, squash, stretch, twist</td>
<td>17. cross, examine, kiss, pet, pinch, sharpen, stretch, twist</td>
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<tr>
<td>16. dig, draw, drop, drink, eat, fix, frighten, kill, match, move, new, open, peel, pull, reach, record, rewrite, scratch, screen, set, shake, shoot, shut, sing, spray, stir, swallow, swing, take, touch, trade, visit</td>
<td>18. dig, draw, drop, drink, eat, fix, frighten, kill, match, move, new, open, peel, pull, reach, record, rewrite, scratch, screen, set, shake, shoot, shut, sing, spray, stir, swallow, swing, take, touch, trade, visit</td>
</tr>
<tr>
<td>17. dress, feel, look, shop, sound</td>
<td>19. dress, feel, look, shop, sound</td>
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<tr>
<td>18. grind, straighten</td>
<td>20. grind, straighten</td>
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<tr>
<td>19. joggle, tease</td>
<td>21. joggle, tease</td>
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<tr>
<td>20. pet, sharpen</td>
<td>22. pet, sharpen</td>
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<tr>
<td>21. suppose, want</td>
<td>23. suppose, want</td>
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<td>22. suppose, want</td>
<td>24. suppose, want</td>
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<tr>
<td>23. suppose, want</td>
<td>25. suppose, want</td>
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<tr>
<td>24. go, learn, let, start, suppose, try, want</td>
<td>26. go, learn, let, start, suppose, try, want</td>
</tr>
</tbody>
</table>

1: answer, bake, bite, build, butcher, buy, catch, change, change, check, close, color, count, cover, dig, draw, drop, drink, eat, fix, frighten, kill, match, move, new, open, peel, pull, reach, record, rewrite, scratch, screw, set, shake, shoot, shut, sing, spray, stir, swallow, swing, take, touch, trade, visit

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4: back, bang, bounce, climb, cool, crash, crawl, dance, end, fall, feel, fit, fly, grow, hammer, hop, jump, lay, lie, listen, look, peek, play, pop, rest, ride, run, shave, sit, slip, smell, sound, speak, stand, stay, step, stick, swim, talk, taste, wake, walk, worry, write

5: bake, bother, buy, call, carry, crack, do, drop, find, follow, get, have, hear, help, hit, keep, lose, love, make, measure, name, need, recognize, sell, share, spell, spill, tape, test, use, watch, wear

6: believe, decide, finish, hurt, know, mean, mind, miss, pretend, remember, say, see, study, surprise, understand, wish, wonder

7: belong, trip

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22: suppose, want

23: suppose, want

24: go, learn, let, start, suppose, try, want