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CLUBS, LOCAL PUBLIC GOODS AND TRANSPORTATION MODELS

A synthesis

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Clubs, local public goods, and transportation models are analyzed within a unified model. The emphasis is on the derivation of optimal allocation, pricing and the size of the sharing group. We derive the conditions under which optimal prices will yield surplus or deficit, as well as those under which competitive provision will be efficient. Given heterogeneous tastes we prove that segregation according to tastes is generally efficient although several cases where this result does not hold are also discussed. We show that the existing literature is unnecessarily restrictive and that the unified approach suggested here considerably extends the existing analysis of clubs' local public goods and the transportation problems.

1. Introduction

Clubs, local public goods and transportation models have a similar basic structure in the existing literature. Yet they have been treated as separate branches of economic theory, yielding sometimes conflicting results. Our purpose is to elaborate on these theories within the context of a unifying, more general model of resource allocation. The main issues are then reviewed and some general results are derived.

The basic unifying model in this paper differs from the classical conventional model of private goods by allowing for congestion-prone goods. These are goods which are consumed collectively by a group of consumers all of whom derive utility from sharing the services of a common facility (swimming pool, road, library, etc.) and disutility from the size of the sharing group.

We distinguish between private goods, club goods, and local public goods, according to the optimal size of the sharing group relatively to the size of the community. A private good is one for which the optimal sharing group is the

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smallest possible. A club good is one of which the optimal size of its sharing group is finite but still small relatively to the community size. The optimal sharing group of a local public good is the community itself. The classification is shown to be endogeneously determined rather than being inherent in the facility itself.

The model is used to derive the conditions for optimal allocation in the cases of clubs and local public goods for both homogeneous and heterogeneous population. We reiterate an earlier result — for an optimal sharing group the cost of providing the facility is fully recovered by the revenue from user charges — and we suggest a more rigorous proof to show that in this case the market can provide the service efficiently rendering nonmarket institutions unnecessary.

We provide a more general proof for the desirability of a system of segregated clubs when the population is heterogeneous, thus extending another earlier result to the case when the use of the facility is variable.

The optimal financing and the segregation issues are systematically examined for the case of a local public good. It is shown that in an optimal community optimal pricing can yield surpluses in the production of some goods (both private and local public goods) and deficits in the production of others. The optimal rule, however, is that the total sum of deficits should be equal to the total sum of surpluses (pure profits). The well-known Henry George rule for financing local public goods is implied as a special case.

Regarding segregation we show that in contrast to the case of clubs, in the case of an optimal community mixing different socioeconomic groups may be consistent with and even indispensible for optimal allocation of resources.

Using the results derived from the theories of clubs and local public goods we show that existing transportation models are unnecessarily restrictive. We offer therefore a more general formula for the relation between the revenue from user charges and the cost of providing road services. Using the results of the theories of clubs and local public goods we discuss the cases for providing people of different tastes with different facilities (e.g. neighborhood roads) or common facilities (e.g. main arteries).

The question of returns to scale plays an important role in the theory of nonpure public goods. A standard result is that efficient congestion tolls will exactly cover total cost in the case of constant returns, and lead to excess revenues (losses) in the case of decreasing (increasing) returns to scale. However, the literature does not sufficiently clarify what is meant by returns to scale: we define it in terms of homogeneity of optimal utility in population size and composition. This definition is sufficient to derive all the standard results. It includes all the cases that have appeared in the literature as special examples.

We sometimes introduce an analogue to the firm with a U-shaped average cost curve, thus dealing with returns to scale as a local rather than global characteristic. This proves very useful in solving simultaneously for the optimal number and other physical characteristics of roads (traffic lanes) connecting two regions.

The plan of the paper is the following. Section 2 presents the model and its main implication regarding the optimal pricing and financing. Section 3 presents the theory of clubs. Local public goods and optimal communities are the subject of section 4. In section 5 we apply the results of the two preceding sections to transportation.

2. The model

In this section we assume that all people have the same utility function and the same factor supply (income). We further restrict our analysis to the (optimal and market) solutions where identical (all) people end with identical utility.

Consider an isolated community with N individuals and a fixed amount of land, A. Each individual is assumed to supply one unit of labor services, to consume a private good x, and to use (with n other individuals, v times per period) a congestion-prone facility of size y. His utility increases with the consumption of the private good x, the frequency with which he uses the facility (v), and the size of the facility, (y). It declines with total use, that is, the combined usage frequency (nv) of all the individuals with whom he shares the facility. Accordingly, the individual's utility can be represented by[1]

$$u = u(x, v, y, nv). \tag{1}$$

Using subscripts to denote partial derivatives it is assumed that

$$u_1 \equiv u_x > 0; \quad u_3 \equiv u_y > 0,$$

$$u_2 \equiv u_v \geqq 0,$$

$$u_4 = u_{nv} \leqq 0.$$

Land and labor are used to produce two types of commodity: a private good, and a congestion-prone facility such as a road, a swimming pool, a park, or a library.

Labor is also required for servicing the congestion-prone facility. This requirement increases with total frequency of use, and may decrease with the size of the facility.

[1]This utility formulation is an extension of the Buchanan (1965) model which was introduced into the literature by Oakland (1972) and Berglas (1976b).

Combining these production functions with the resource constraints yields the following:

$$y = f(A_y, N_y), \tag{2}$$

$$Nx = g(A_x, N_x), \tag{3}$$

$$A_x + \frac{N}{n} A_y = A, \tag{4}$$

$$N_x + \frac{N}{n} N_y + \frac{N}{n} s(y, nv) = N, \tag{5}$$

where A_y and N_y are the land and labor input per facility y, and A_x and N_x are the land and labor used in producing Nx. Since all people are identical, x is the consumption of the private good by any one individual, and, the total consumption of the community, N, is Nx. N/n is the number of congestion-prone facilities, each of which is of size y, and s is the labor requirement for the servicing of each facility y.[2] This can represent labor used to maintain the swimming pool or the road. It could also represent consumer time spent on the road, in which case the costs are borne directly by the user. In order to simplify the analysis we assume that these labor services $s(\cdot)$ are hired by the firm that operates the facility. The operating firm may also require land services but this would burden the presentation unnecessarily.

We assume that the g function exhibits constant returns to scale, but no such restriction is necessary for the congestion-prone good (the function f).[3] More specifically, there is no need to require that doubling the size of a swimming pool or the width of the road will double its costs, a requirement that is frequently in contrast to empirical observations [see, for example, Mohring (1976)]. We assume that the two production functions f and g are twice differentiable, and all marginal products are positive and decreasing.

In deriving the conditions for optimal allocation using the assumption that equals should be treated equally,[4] we maximize (1) subject to (2) through (5). Maximization is done in two stages. First, we assume that the consumption group size n is given, and $u(\cdot)$ is maximized over x, v, y, L_x, N_x, L_y and N_y.

[2]The servicing function is introduced separately in order to facilitate comparison with other models. We could instead introduce nv as an additional variable in the function f.

[3]As in the classical analysis of competitive general equilibrium all the following propositions can be derived in the case of decreasing returns. This in our model means that g exhibits decreasing returns, and f can be written as $y = \bar{f}(A_y, N_y, N/n)$, where $f_3 < 0$. In a more general formulation, y can represent a vector of characteristics. In this case (2) should be replaced by $\phi(A_y, N/n, y_1, y_2, \ldots, y) = 0$.

[4]This assumption is implied, for example, by the Rawlsian social welfare function and is required for stable market equilibrium.

The resulting necessary conditions can be reduced to:

$$u_2/u_1 = g_2 s_2 - n u_4/u_1, \tag{6}$$

$$n u_3/u_1 = g_2/f_2 + g_2 s_1, \tag{7}$$

$$g_2/g_1 = f_2/f_1. \tag{8}$$

In order to facilitate the interpretation of (6) and (7), we define x as the numeraire and fix its price, $P_x = 1$. It follows that g_2, the marginal product of labor in x, is the wage rate. The RHS of (6) represents the social cost of increasing the use of the facility by one unit (v). The first term $g_2 s_2$ is the additional labor input requirement multiplied by the wage rate. The second term, nu_4/u_1, is the decrease in total utility as a result of increased congestion (all social costs are measured in terms of the private good). Thus, according to (6), marginal subjective evaluation of the use of the facility must equal its marginal social cost.

Eq. (7) is the well-known Samuelson condition. The LHS represents the sum of the marginal evaluations of the facility while the RHS represents its marginal cost, made up of the marginal cost of the facility g_2/f_2, and of its effect on maintenance costs, $g_2 s_1$. Eq. (8) is the usual production efficiency condition.

Now turn to the second stage of optimization. Let $u^*(n)$ be the optimal utility level as a function of n where (2) through (8) are satisfied. Using the envelope theorem,

$$\frac{\mathrm{d}u^*(n)}{\mathrm{d}n} = \frac{u_1}{Nn}\left[\frac{N}{n}(g_1 A_y + g_2 N_y + g_2 s) - Nv(g_2 s_2 - n u_4/u_1)\right]. \tag{9}$$

With x as a numeraire, g_1 is the rent per unit of land and g_2 is the wage rate. It follows that $(N/n)(g_1 A_y + g_2 N_y + g_2 s)$ is the total cost of providing the club service. Furthermore, it follows from (6) that in order to induce utility-maximizing consumers to consume the optimal consumption mix, the price per unit of use of the facility should be $P_v = g_2 s_s - n u_4/u_1$. The last term in (9) is thus total revenue of the club services. $u_1/(Nn)$ is a positive number, α. Given these (shadow) prices, eq. (9) says that $\mathrm{d}u^*(n)/\mathrm{d}n = \alpha$(cost-revenue) or,[5]

$$\text{Cost} \gtreqless \text{Revenue} \Leftrightarrow \mathrm{d}u^*(n)/\mathrm{d}n \gtreqless 0.$$

[5]Boadway (1980) has claimed that full coverage of costs requires a linear transformation curve. This is clearly not the case in the present formulation. Furthermore, the reader may be worried about the case where f in (2) exhibits increasing returns to scale; Berglas (1981) provides a numerical example of the existence of an optimum with costs exactly covered by user charge, given economies of scale.

Some alternative configurations of $u^*(n)$ are illustrated in fig. 1. The optimal consumption group size for the congestion-prone good $\underline{a}$ consists of one consumer. This implies that it is best to use facility $\underline{a}$ privately. In other words, there is no reason to distinguish between $\underline{a}$ and x.

The optimal size of the consumption group for $\underline{b}$ is n_b^*. It follows from (9) that the optimal user charge is just sufficient to cover the cost of providing the service. If, in addition N/n_b^* is a (large) integer, $\underline{b}$ is defined as a 'market club good', for in this case (as we later show) the service can be efficiently provided through the market.

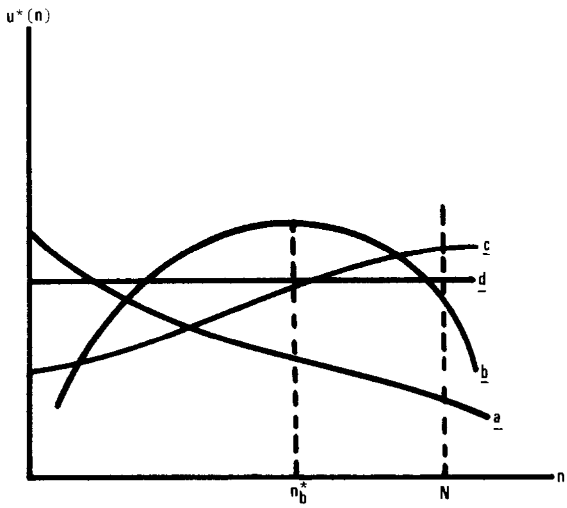

Fig. 1

The optimal consumption group of facility $\underline{c}$ is N, i.e. the total community. We define a commodity the sharing group of which is the total community as a 'local public good'. Observe that this definition covers the case where n^* is somewhat smaller than N but where (given N) one sharing group is nevertheless optimal. Later, when we let N vary, this case of local public good where $n^* < N$ will prove to be very important. Given eq. (9), it follows that whenever $n^* \neq N$ the provision of local public goods will generate deficits or surpluses — these will later be analyzed in detail.[6]

Finally, the optimal consumption-group size of congestion-prone facility good d is indeterminate. The services can be provided as a 'private good', a 'market club good', or a 'local public good'. In all these cases, optimal user charges will exactly cover cost.

[6]This classification is incomplete since it does not cover the case where the optimal number of sharing groups is larger than one, but is a small number; and the case where N/n^* is not an integer. The role of these goods will become clear as we proceed.

It is worth noting that classification of a good in one of these categories is not necessarily inherent in the good itself but may be endogeneously determined. For example, a swimming pool may optimally be a private good in an affluent community, where the demand for privacy is high. It may be a market club good in a middle-class community, while in a relatively poor community it may even be a local public good.

To be more realistic the model (1)–(5) can be extended to include congestion prone facilities of different types. Each facility may service a different optimal consumption group n_i^* and may be of different type. Thus within a community we may have simultaneously commodities of types $\underline{a}$, $\underline{b}$, $\underline{c}$, and $\underline{d}$. The optimal size and consumption group of each facility will be determined endogenously.

The above analysis can be reformulated using cost functions. This will both considerably simplify the following analysis and will make it easier to compare results with the existing literature.

Given factor prices P_N and P_A, the cost function of the provision of club services can be written as $c=\bar{c}(y,nv,P_N,P_A)$. Taking prices as given ($P_x=1$, $P_N=g_2$, and $P_A=g_1$), the cost function can be written as $c=c(y,nv)$.

Furthermore, the assumption of identical consumers implies a given income, $I=g_1A/N+g_2$. The maximization problem now reduces to selecting y, v, and n to maximize (1) s.t.

$$nx+c(y,nv)=nI. \tag{10}$$

The solution to this problem (n^*,y^*,v^*) is, of course, the same as that derived for problems (1) through (5). The necessary conditions are

$$u_2/u_1=c_2-nu_4/u_1, \tag{6'}$$

$$nu_3/u_1=c_1, \tag{7'}$$

$$\mathrm{d}u^*(n)/\mathrm{d}n=\frac{u_1}{n^2}[c-nv(c_2-nu_4/u_1)]. \tag{9'}$$

The explanations of these necessary conditions are exactly those for (6), (7) and (9). Costs are exactly covered when $\mathrm{d}u^*(n)/\mathrm{d}n=0$, or when there exists an optimal n^* that maximizes utility.

Several studies of congestion-prone services impose restrictions on the homogeneity of the cost and/or utility function. To understand why, it will be useful to reformulate (9′) in terms of the homogeneity parameters of the utility and cost functions. Define r_u and r_c as the degree of homogeneity of the utility and cost functions in variables y and nv respectively. Using (7′), we can then rewrite (9′) as

$$\mathrm{d}u^*(n)/\mathrm{d}n = \frac{u_1}{n^2} c(1 - r_c) + \frac{u}{n} r_u. \tag{11}$$

A typical restriction in the literature is to assume that congestion is a function of the number of users divided by the size of the facility; in the terms used here, utility (or service cost) is a function of y/nv, and the cost per unit of y is constant. This implies that $r_c = 1$ and $r_u = 0$, regardless of n. Given (11), $\mathrm{d}u^*(n)/\mathrm{d}n$ in this case is identically zero regardless of group size. This restriction implies that we are dealing with commodity type d in fig. 1; group size is irrelevant for optimality and the product could be efficiently provided as a private good. This formulation will prove useful in the analysis of the transportation problem.

3. The club theory

Club theory occupies a very important place in recent economic literature. [See Sandler and Tschirhart (1980).] We shall prove the basic theorems of club theory using a different, somewhat more rigorous approach than that used in the literature. Our proof of the optimality of market provision of club goods will be a variant of the model of market provision of goods with variable quality. In the case of the optimality of segregation we introduce for the first time the proof for the case where individuals change the intensity with which they use the club facility. The usefulness of this approach will be apparent in the subsequent analysis of local public goods and of the transportation problem.

Given that $u^*(n)$ assumes a maximum at a finite nonzero consumption group of size n^*, it is possible to prove the following proposition [see also Berglas (1976b)].

Proposition 1. If there exists an optimal consumption group n^*, and if the total population N can be divided into an integer number k of optimal consumption groups of size n^*, then there exists a price system that supports the optimal allocation. An outline of the proof is provided below [for a detailed proof see Berglas and Pines (1978)].

Consider the service of a club commodity, distinguished by two characteristics y and nv, and let the price of the service be defined as the solution to

$$\max_{p,x,v} p$$

$$\text{s.t.} \quad u(x, v, y, nv) = u^*(n^*),$$

$$x + vp = I,$$

where $u^*(n^*)$ is the optimal utility determined by the solution of the maximization of (1) s.t. (10). The solution of this problem yields $p=p(y,nv)$, i.e. a price function of quality towards which consumers and producers react as price takers.[7] If each agent maximizes his objective function (utility in the case of consumers, profits in the case of producers) the resulting allocation will be optimal.[8]

We can conclude, therefore, that if the divisibility assumption is satisfied, and if, in addition, exclusion is possible, congestion-prone goods need not be supplied by clubs [as Buchanan (1965) implied];[9] rather, the market can support the optimal allocation.

The club commodity may be characterized by several quality attributes. Swimming pools may be of different sizes, offer different services, or differ in water temperature. All these parameters can be introduced into the model by allowing y to be a vector. The necessary condition (7′) must then hold for each characteristic separately.

The preceding analysis assumed a homogeneous population both in terms of preferences and initial holdings. This allowed unique definition of an optimum resource allocation. We now consider a heterogeneous population and Pareto-efficient allocations.

One of the main issues of club theory with respect to heterogeneous populations is whether Pareto-efficient allocations require a segregated system of consumption groups, i.e. whether a consumption group should be composed of members with identical income and preferences, whom we identify as belonging to the same class. Turning to this question we assume that the congestion effect is identical across classes, i.e. $u(\cdot)$ and $c(\cdot)$ of each class depend on the total frequency of use of a given facility $\sum_j n^j v^j$ (where a superscript denotes the class), rather than on the socioeconomic composition of the user population. The assumption that congestion is a function of the number of users regardless of their personal characteristics is the most appropriate for the discussion of segregation according to tastes. It is straightforward that if blacks and whites prefer not to share the same facility, segregation is optimal; similarly, if men and women derive utility from sharing a mixed swimming pool, segregation is not optimal. By adopting the congestion variable $\sum_j n^j v^j$, we prove that when people are indifferent about co-users, segregation according to taste is optimal.[10] Under this assumption it is possible to prove the following two propositions.

[7]This approach is an application of Rosen (1974).

[8]In this approach agents are assumed to react to a given *price system* whereas Berglas (1976b) and Boadway (1980) assume maximization of profits by producers knowing the consumer's *utility function*, an approach consistent with Nash equilibrium, but not with competitive equilibrium in the strict sense.

[9]Buchanan (1965, footnotes 7 and 8) mentions the possibility of market provision, but develops his paper claiming that these commodities will be supplied by clubs.

[10]This proof is unaffected if $u(\cdot)$ and $c(\cdot)$ depend on $\sum_j \alpha^j n^j v^j$, where α^j are positive coefficients. More complicated cases can also be allowed for, although not the case in which one socioeconomic group derives positive utility from association with other groups.

Proposition 2. It is suboptimal to have two or more identical mixed consumption groups. (This proposition assumes two classes, but may easily be extended to any number of classes.)

Proof. Suppose there are two identical mixed consumption groups. Pareto-efficient allocation within each consumption group can be shown to imply:

$$u_2^a/u_1^a = u_2^b/u_1^b = c_2 - n^a u_4^a/u_1^a - n^b u_4^b/u_1^b, \tag{6''}$$

$$n^a u_3^a/u_1^a + n^b u_3^b/u_1^b = c_1, \tag{7''}$$

where superscripts a and b represent the two classes. Eqs. (6″) and (7″) are straightforward extensions of (6′) and (7′) respectively.[11]

It can be shown that the population mix of the two clubs (consumption groups) may be changed without affecting the consumption of any individual or the total frequency with which the facility is used.[12] Hence, both the level of utility of each individual and the cost of providing the facility in each consumption group remain constant. However, since the composition of the population is now different, conditions (6″) and (7″) no longer hold, which means that resources within each club can now be reallocated so as to

[11]Eqs. (6″) and (7″) are derived from the following maximization problem, where x^a, x^b, v^a, v^b, and y are the optimization variables:

$$\max u^a(x^a, v^a, y, n^a v^a + n^b v^b)$$

$$\text{s.t.} \quad -u^b(x^b, v^b, y, n^a v^a + n^b v^b) + \bar{u}^b = 0,$$

$$n^a x^a + n^b x^b + c(y, n^a v^a + n^b v^b) - I^a n^a - I^b n^b = 0.$$

This model does not assume that types a and b pay the same fee for the club service; it is the conditions necessary for maximization that dictate that the price per unit of service v will be the same. Sandler and Tschirhart (1980) fail to understand this result. They claim that optimal segregation proofs presuppose that all consumers pay the same fee and are thus second-best results. This assumption is clearly not required for our proofs of propositions 2 and 3.

[12]Let n_i^k and v_i^k be, respectively, the number of club members and the rate of facility use of class k in club i $(i=1,2)$ in two identical clubs. Initially, $n_1^k = n_2^k$ and $v_1^k = v_2^k$ where $(k=a,b)$. Transfer one member of class a from club 1 to club 2, and v^b/v^a members of class b from club 2 to club 1. The resulting total use of the facility in club 1 is

$$(n^a - 1)v^a + (n^b + v^a/v^b)v^b = n^a v^a + n^b v^b,$$

and in club 2

$$(n^a + 1)v^a + (n^b - v^a/v^b)v^b = n^a v^a + n^b v^b.$$

Even though the composition of the users in each of the clubs has changed, there is no change in the use of the facility in each of the two clubs. The initial transfer of one individual of type a was arbitrary; we could have shifted two or more. In the case where initially $n^a v^a < n^b v^b$ we could even shift all type a individuals to club 2.

increase the utility of each member in the community. Therefore the original clubs are necessarily suboptimal.[13] Q.E.D.

Proposition 2 can be used to prove that mixed clubs are suboptimal in the case where utilities are homogeneous of degree zero in club size. Let $u^{*a}(n^a, n^b, \bar{u}^b)$ represent the maximum utility of a, given that $u^b = \bar{u}^b$, and that the club is composed of n^a consumers of type a, and n^b consumers of type b. Defining u^{*a} it is assumed that v^a, v^b, and y change as we change n^a and n^b. Then if $u^{*a}(\cdot)$ is zero homogeneous in n^a and n^b, it can be divided into two identical, half-size clubs without affecting the utilities of either a or b. By proposition 2, these two clubs are not optimal. Thus, given the homogeneity assumption, mixed clubs are not efficient. As we later show, several well-known transportation models, as well as Oakland (1972), assume that the use of the same facility by consumers whose tastes differ is optimal even though the underlying model implies that $u^{*a}(\cdot)$ is homogeneous of degree zero in the composition of population.

This result can be extended to the case where optimal club sizes are finite. It can be shown that provided some integer conditions hold, a system of segregated clubs is superior to a system in which there are mixed clubs. Suppose that total income is divided arbitrarily between types a and b, such that $I^a = \bar{I}^a$ and $I^b = \bar{I}^b$. By the earlier procedure it is then possible to define two types of segregated clubs that will maximize utility for types a and b. These clubs will have optimal membership n^{*a} and n^{*b}, and yield utilities u^{*a} and u^{*b}, respectively. The integer assumption requires that N^a/n^{*a} is an integer, and N^b/n^{*b} is an integer. If this is the case, we claim that the allocation is optimal. In order to prove this result suppose that the allocation is not optimal; we can then solve

$$\max u^b(\cdot)$$

$$\text{s.t.} \quad (1) \ u_a(\cdot) = u^{*a},$$

$$(2) \ n^a x^a + n^b x^b + c(y, v^a n^a + v^b n^b) = n^a \bar{I}^a + n^b \bar{I}^b.$$

Denoting the solution to this problem $\bar{u}^b$, it follows that $\bar{u}^b > u^{*b}$. Ignoring the integer problem, consider the case of two identical clubs that yield utility $\bar{u}^b$ and u^{*a}. By proposition 2, utility can be increased beyond $\bar{u}^b$ (holding $u^a = u^{*a}$) which is a contradiction since $\bar{u}^b$ is the maximum, given u^{*a}. For an alternative proof see Berglas and Pines (1978). We formulate this result as follows.

[13]A similar argument appears in McGuire (1974) and Berglas (1976b). However, their demonstration refers to cases in which the congestion effects depend on the number of users rather than on the total use of the facility, i.e. the use of the facility does not vary among groups.

Proposition 3. If there exists a finite optimal club size n^{*i} for each consumer class (as defined above), and if the total population of each class, N^i, can be divided into an integer number, k^i, of optimal clubs of size n^{*i}, then a segregated club system (i.e. with each club having a homogeneous population) is Pareto-superior to any nonsegregated one (i.e. a system which includes mixed clubs).

By satisfying the conditions of proposition 3 we create a system of segregated consumption groups which can be supplied in a competitive market. The price of each club good will be specified as a function of quality characteristics, i.e. capacity y and intensity of use $n^i v^i$, and the different prices will induce different individuals to purchase services of different clubs. There is therefore no need for a nonmarket economic institution to guarantee efficient resource allocation.

The club model theory was developed for the case where N/n^* is a 'large' integer. The problem where N/n^* is large but not an integer is essentially not different from the problem of a conventional industry which consists of identical firms with U-shaped average cost curve. Suppose in this industry minimum average cost is obtained at output x^*, and let $P^* = \min AC$. Let the quantity demanded at P^* be equal to X. If X/x^* is not an integer, we have the same integer problem that we encounter in the club model. As is customary, we disregard this problem.

Moving to the theory of local public goods and optimal communities we further consider the case for segregation between classes. We prove that in the case of local public goods mixed consumption groups are sometimes optimal.

4. Optimal communities

Thus far, we have assumed that the supply of the services of the congestion-prone facilities is made within communities. No attempt has been made to justify the existence of communities, nor to explain their size and socioeconomic composition. Without additional assumptions it would seem that the optimal community is identical to the total population of the country. Accordingly the production of the private good and the supply of the congestion-prone facilities would be determined according to their optimal size. Any individual would belong to a sharing group for each congestion-prone facility. In the case of nationwide public goods all individuals would form a single consumption group. But such an allocation would be extremely inefficient, if not infeasible. The reason is that the production of goods requires space, i.e. a physical location. The consumption of goods therefore requires either that goods must be shipped to people, or people to goods. The optimal allocation of resources should therefore take into account these transportation costs as a function of the spatial distribution of the population and the supply of goods. The resulting allocation could then be formulated in terms of market areas for the various goods. This would require simultaneous solution of the problems introduced by Lösch (1954), Tiebout (1956), Buchanan (1965), and others, which is, of course, beyond the scope of this paper.

Once we let N vary, we are immediately facing the question of what determines the optimal size of the community. One straightforward explanation is spatial considerations. We shall approximate this case when we assume that land per community is fixed (the model given by eqs. (1)–(5)). In order to clarify the underlying relationship it will prove useful to start with a model without land thus confronting other considerations restricting the optimal size of the community. The model to be discussed is a variant of the congestion-prone goods model. The simplest model is the one used in Oates (1972) where communities provide just one pure public good and that there are costs of concentration (such as congestion and pollution, etc.) which are a function of the size of the community. In short, the utility function of this case using our notation can be written as $u(x, y, N)$.[14] The same idea applies to our model (when there is no land) with non-pure public goods. Suppose the only local public good is a road, the utility of which is given by (1) and suppose that the optimal user group of a road is n^*. Now if there are no externalities, i.e. if all the benefits accrue to the user of the road one can say that the community size is indeterminate or that communities of the sizes n^*, $2n^*$, kn^* are equally efficient. But suppose now that there are externalities among users of the different roads, e.g. smoke. Then, formally, we have to add the variable N (or Nv) to our utility function (1). However, observe that in our example it will be obvious that the optimal community is $n^* = N^*$, i.e. there will be just one facility, and the optimal consumption group determines the size of the community. Thus we can proceed with the analysis using (1) by substituting N for n.

We start the analysis with the simplest case where each community provides a single local public good. This model proves to be very useful in illustrating the Tiebout hypothesis and in demonstrating its analogy with club theory. Later we introduce several local public goods and show that this change considerably affects the Tiebout hypothesis.

We start by specifying the assumptions required for the proof of the Tiebout hypothesis. The last two assumptions will be relaxed later.

A1. Mobility among communities of utility maximizing consumers is costless.

A2. City developers or local governments are profit maximizing agents.

[14]The implications of this model and its relationship to the more general model of this paper are fully analyzed in Berglas (1976b).

A3. The population of optimal communities (to be defined immediately) are small relative to the size of national population. Such that the optimal allocation requires many communities.

A4. Individual income is independent of the size of the community. This is Tiebout's assumption that income is received as dividends. In our formulation this can cover more interesting cases, e.g. the supply of land for each community may be infinitely elastic at its alternative price (say, the marginal product in agriculture is fixed); or perfectly mobile, capital can be substituted for land in the model (1)–(5).

A5. At the optimal allocation we have just one congestion-prone public good.

Given these assumptions, it is straightforward that maximizing (1) subject to (10) with N substituted for n can be used to solve for the optimal community size N^* and the optimal local public facility y^*. If all individuals are identical it is optimal to divide the national population $\bar{N}$ into $\bar{N}/N^*$ (assumed an integer) identical communities. These communities will provide facility of size y^* which will be exactly financed by the optimal toll $P_v = c_2 - N_{u4}/u_1$. This is, of course, in complete analogy with our club model. It follows that competition among communities will result in the optimal allocation. Furthermore, once we introduce different economic classes, by complete analogy to the club model the nation can be divided into $\bar{N}^a/N^{*a} = k^a$ and $\bar{N}^b/N^{*b} = k^b$ segregated communities. If k^a and k^b are integers this allocation can be supported by a competitive price system. These results are essentially the Tiebout (1956) hypothesis.

An interesting insight is gained by relaxing the assumption that in the optimum we have just one local public good. Thus, we shall now explicitly consider the case of two public goods; the extension to three or more goods is straightforward. Extending the model (1) and (10) to the case of two local public goods, using the same notation and adding superscripts to denote the two public commodities yields:

$$\max_{x, v^1, v^2, y^1, y^2, n^1, n^2} \quad u(x, v^1, y^1, n^1 v^1, v^2, y^2, n^2 v^2)$$

$$\text{s.t.} \quad Nx + Nc^1(y^1, n^1 v^1)/n^1 + Nc^2(y^2, n^2 v^2)/n^2 - NI = 0, \tag{12}$$

$$n^1 = n^2 = N. \tag{13}$$

If there exists an internal solution to the above problem, the necessary conditions are:

$$u_2/u_1 = c_2^1 - Nu_4/u_1, \tag{14}$$

$$u_5/u_1 = c_2^2 - Nu_7/u_1, \tag{15}$$

$$Nu_3/u_1 = c_1^1, \tag{16}$$

$$Nu_6/u_1 = c_1^2, \tag{17}$$

$$[Nv^1(c_2^1 - Nu_4/u_1) - c^1] + [Nv^2(c_2^2 - Nu_7/u_1) - c^2] = 0. \tag{18}$$

Hence, where $N = n_1 = n_2$, either the cost of each facility is just covered by the corresponding congestion toll, or the provision of one facility generates a deficit while the other generates a surplus of equal amounts. The surplus is thus used to finance the deficit. Alternatively a tax of 100 percent on the pure profit of the surplus generating service is called for.[15] Since in practice it is difficult to collect these taxes it may lead to the conclusion that the service should be provided publicly although it can be profitably supplied by a private firm.

It follows immediately from (18) that if one of the goods, say commodity 1, is a pure public good such that $u_4 = c_2^1 = 0$, then good 2 must be a congestion-prone public good. In general, Tiebout's (1956) community of finite optimal size implies that at least one of the goods supplied locally must be congestion-prone.

It can also be shown that

$$\partial u^*(n^1, n^2)/\partial n^1 \big|_{n^1 = n^2} = u_1[c^1 - nv^1(c_2^1 - nu_4/u_1)], \tag{19}$$

$$\partial u^*(n^1, n^2)/\partial n^2 \big|_{n^1 = n^2} = u_1[c^2 - nv^2(c_2^2 - nu_7/u_1)]. \tag{20}$$

Thus, at the optimum community size, either n^1 and n^2 are both optimal consumption groups, or one of them (say n_1) is too small, so that $\partial u^*(n^1, n^2)/\partial n^1 > 0$ and the other (say n_2) is too large, such that $\partial u^*(n^1, n^2)/\partial n^2 < 0$.

If an optimum exists at N^*, then dividing the nation into identical communities of size N^* is a Pareto optimum, and can be supported by a competitive equilibrium. Competitive equilibrium is thus consistent even with several pure public goods, provided there exists at least one congestion-prone good. Thus this part of the Tiebout hypothesis holds with more than one public good. However, increasing the number of public goods may make segregated communities suboptimal. Using the example above, assume that public good 1 yields a deficit. By (18), public good 2 yields surplus revenues. Turning to (19) and (20) it follows that utility could be increased if there were more consumers of commodity 1 and less consumers of commodity 2.

[15]This result is closely related to the Henry George Theorem that will be discussed later. Observe that in the derivation of these results it was not necessary to introduce land rents.

This cannot be done if all consumers are identical. But now suppose that there exists another class of consumers, type b, who do not consume commodity 2 at all. If their tastes with regard to the quality of commodity 1 are identical (similar) to those of the first group,[16] we (essentially) can increase n_1 and decrease n_2, thereby increasing utility. It is possible to derive conditions where this case of mixed communities is consistent with competitive equilibrium, but this is beyond the scope of this paper. We can summarize this result as follows: the larger the number of local public goods, the larger the difference between classes in the quantity of different services demanded, and the smaller the difference in the quality demanded, the more likely that mixed communities will be optimal.

In order to make our results comparable to those of Flatters, Henderson and Mietzkowski (1974), Stiglitz (1977), Arnott and Stiglitz (1979) and Arnott (1979) we reintroduce the assumption of A5 of only one public good and relax assumption A4 by introducing fixed land. Thus our problem is reduced to that of section 2, eqs. (1) through (5), where N is substituted for n.[17] Applying the envelope theorem, we obtain

$$\mathrm{d}u^*(N)/\mathrm{d}N = \frac{u_1}{N}[g_2 - x - v(g_2 s_2 - N u_4/u_1)]. \tag{21}$$

Optimal community size N^* requires, of course, $\mathrm{d}u^*(N)/\mathrm{d}N = 0$. Thus, in the optimum the contribution of the marginal consumer to the financing of the provision of the public facility, $g_2 - x$, should equal his marginal social cost, $v(g_2 s_2 - N u_4/u_1)$. Consider first the case of pure public good $u_4 = s_2 = 0$. It follows from (21) that $g_2 = x$; thus wages exactly match private consumption, which means that land rents (the only other source of income) should exactly cover the cost of the public service, or that optimal tax is 100% on rent, and this tax should exactly cover costs of the public service. This result is termed the Henry George rule [see Flatters et al. (1974), Stiglitz (1977), and Stiglitz and Arnott (1979)].

Turning to the case where the public good is congestion prone, using the assumption of linear homogeneous $g(\cdot)$ and some simple manipulations of (21) we get the following condition for optimum:

$$[g_1 A_y + g_2 N_y + g_2 s - N v(g_2 s_2 - N u_4/u_1)] - A g_1 = 0. \tag{22}$$

[16]Identical in tastes with regard to quality means that if each class were to choose the quality of each services (given by y and nv) in a segregated consumption group of unrestricted n it would select the same pair y, nv. The quantity demanded by each consumer type (v^a and v^b) are not necessarily identical.

[17]Observe that since land per community is fixed, the marginal product of labor and per capita income charges with the size of the community. This may also represent the case where incresing the size of community increases local transportation costs.

The expression in the brackets is the deficit associated with the provision of the public facility. Ag_1 is the aggregate land rent. Eq. (22) represents therefore an extended version of the Henry George suggested by Arnott (1979).

We can now also remove restriction A5 (only one public good). It follows from the above analysis that the deficits of the local public goods should be financed by 100% tax on all pure profits, including land rent and the surpluses accumulated in the provision of some of the public goods.

Furthermore, the principle that the deficits should be financed by the total sum of the pure profits is valid even when the production function of the private good $g(\cdot)$ does not exhibit constant returns to scale, as assumed so far. In this more general case the deficits associated with the provision of public goods and perhaps production of the private good should be financed by the surpluses associated with the provision of other public goods and perhaps the production of the private good. It follows that even the extended Henry George rule is but a special case of the more general principle of financing losses and deficits in the optimal community.

The introduction of land into the model has just minor effect on our discussion of the Tiebout hypothesis. If all potential locations are identical and the single local public good is a pure public good then a competitive equilibrium can exist and the population will segregate itself according to tastes. Once we introduce several public goods some of which are congestion-prone, then it follows from our analysis above that mixing of different classes may be optimal.

Before concluding, it will be useful to discuss some other limitations of the Tiebout hypothesis. Individuals may differ not only in tastes but also in skills. If different skills are complementary in production this can lead to Pareto superiority of mixed communities.[18] The implications of this case for market structure is discussed in Berglas (1976a).

The optimality of competitive allocation will not hold if we relax two of the above assumptions. If the size of the optimal community population is large relative to the size of the total population and $\bar{N}/N^*$ is not an integer then a stable competitive equilibrium is not possible. This case is discussed in Stiglitz (1977) and Helpman (1978). Another difficulty may arise if the potential areas for accommodating the communities differ one from the other. As shown by Buchanan and Goetz (1972), Flatters, Henderson and Mieszkowski (1974) and Stiglitz (1977), in this case optimal allocation requires a transfer of income from one community to another. Equilibrium without such transfers is suboptimal.

[18]This statement does not do full justice to Tiebout. Berglas (1976a) and our analysis in this paper show that the distribution of people of different tastes among optimal communities is not random, and that people of similar tastes do tend to bunch together, though this process falls short of complete segregation.

5. Applications to transportation

Roads are, by their nature, congestion-prone facilities. The utility of a road user depends on his use of the road (number of trips), the road's attributes y (width, separation of traffic from different directions, surface quality, geometry, etc.), and total road use, nv (volume of traffic). Similarly, the costs of providing transportation services depends on road characteristics y, and the volume of traffic. Thus, the theory of congestion-prone goods, elaborated in sections 2 through 4, is directly applicable to the issues of resource allocation and road financing. We therefore use our findings to review and criticize some of the arguments in the existing literature.

Transportation models customarily distinguish between user costs and costs borne by the authority that operates the road. This distinction can be introduced into our model by separating the cost function into two parts. With no private costs the optimal toll was given by (6′) regardless of n. When some of the costs are borne by the users, optimal toll is equal to total marginal cost minus private marginal cost. This is a straightforward extension. In order to facilitate comparisons with the rest of the paper we retain the formulation used in eqs. (1) and (10) and ignore private cost.[19] Once we allow n to change we can derive a theory of the optimal number of roads (N/n^*). For the case of a given number of users n we get, by (9′), that

$$\text{cost} - \text{revenue} \gtreqless 0 \Leftrightarrow n^* \gtreqless N.$$

Furthermore by (11) we can relate the case of surpluses and deficits to the homogeneity of the utility and cost functions with respect to y and nv.

It is useful to compare these results with existing transportation models. First, the above formulation emphasizes the importance of congestion in the analysis (and thereby of n). Thus, Mohring's (1970) approach of lumping together 'transportation, electricity generation and distribution, and most other facilities which are commonly classed as public utilities' seems to obscure the fact that transportation is a congestion-prone good while the other commodities are not. Thus, in the case of transportation, it may be optimal to build different roads for different individuals [this is recognized in Mohring (1976)] which is not the case for other public utilities.

The theory of congestion-prone goods tends to emphasize the advantages of adjusting characteristics to the tastes of the user and thus provide different roads and perhaps different transportation modes to people with different tastes. It is interesting, however, to investigate the conditions under which it will be optimal to provide a common facility to people of different tastes. To abstract from problems of optimal communities consider the case of

[19]Observe that the cost of time is taken care of by introducing y and nv into the utility function.

highways. Assume two economic classes a and b and let utility increase with the size of population. In terms of our analysis, this means that $u^{*a}(n^a, n^b, \bar{u}^b)$ is homogeneous of degree $k > 0$ in n^a and n^b,[20] and in the case of determinate size clubs that $n^{*a} > N^a$ and $n^{*b} > N^b$. In these two cases, mixing allows for increased utility as a result of the benefits of scale, but at the same time reduces utility as the two groups are forced to share the same road y. However, the benefits of economies of scale may be dominant; in this case, mixed roads will be optimal. Observe that if mixing is optimal then (6″) and (7″) are the necessary conditions. This implies that the optimal toll, $p_v = c_2 - (n^a u^a_n / u^a_1 + n^b u^b_n (u^b_1)$, should be equal for every user [see also Oakland (1972) and Berglas (1976b)].

Furthermore, suppose again that $n^{*a} > N^a$ and $n^{*b} > N^b$, and suppose that $u^{*a}(n^a, n^b, \bar{u}^b)$ exhibits locally homogeneity of degree zero. In this case the optimal toll exactly covers the total cost. It may still be the case that two roads servicing homogeneous consumers will yield greater utility for both a and b. This causes the planner considerable problems: separate facilities whose optimal tolls do not cover their total cost may be preferable to one common facility whose toll does cover its total cost. The cost–benefit analysis of these two alternatives may be extremely difficult.[21]

To summarize this argument one might say that the larger the differences in tastes and the smaller the degree of increasing returns, the more likely it is that segregated roads are optimal. This case for mixed consumption groups may be appropriate also for local public goods once we allow for mixed communities.

In the existing transportation literature we find several additional restrictions that considerably affect the analysis. We turn now to the discussion of these restrictions:

R1. A congestion variable appears either in the utility function [as in Strotz (1965)], or the cost function [as in Mohring and Harwitz (1962)], but not in both.

R2. The congestion function is defined to be homogeneous of degree zero in y and nv; more explicitly, the congestion variable is defined as nv/y.

R3. The cost function is further restricted to the form[22]

$$c(y, nv) = p_y y + nv \cdot t(nv/y). \tag{23}$$

[20]It is worthwhile to emphasize that increasing returns to scale here do not necessarily imply or are implied by increasing returns in the production of the club service.

[21]This issue is associated with the problem of optimal product differentiation in the face of increasing returns to scale [see, for example, Lancaster (1975)].

[22]The restricted cost function (where y and nv do not appear in the utility function) is common to Harwitz and Mohring (1962) and Mohring (1976), as well as to a group of urban models by Solow and Vickrey (1971), Hochman and Pines (1971), Mills and de Ferranti (1971), Dixit (1973), and others.

In a few cases decreasing costs are allowed for, in which case $p_y y$ is replaced by $c^0(y)$, where $c^{0\prime} > 0$, $c^{0\prime\prime} < 0$.

R4. Consumers of different types (once they are allowed for) are forced to use the same road.

R5. The discussion of the optimal consumer group is overlooked in the transport literature.

Analysis of these restrictions helps to demonstrate the additional insight and results derived from the use of our model. Restriction R1 is an unnecessary abstraction of reality. It tends to obscure the fact that results with regard to both optimal price system and the analysis of surpluses and deficits depend on the characteristics of both utility and cost functions.[23]

Consider now the restrictions of the congestion function R2 together with the cost function (23). In this case $r_u = 0$ and $r_c = 1$, where, as before, r_u and r_c are the homogeneity of the utility function and the cost function with respect to y and nv. It follows from (11) that in this case the size of the optimal consumer group n is indeterminate. The optimal toll will then cover total cost regardless of n. Furthermore, by considering the restricted congestion and cost functions, it follows that individual utility is unaffected if there are n roads of size y/n and each individual uses his own road. It follows that these restrictions imply that roads can be considered as private goods, no utility is derived from sharing roads.[24] (Roads in this case belong to category $\underline{d}$ in terms of fig. 1).

If, together with R2, we allow the cost of y to be decreasing in y (the cost per unit of width of the road falls with its width), it follows that $r_c < 1$ everywhere, while $r_u = 0$. This implies that $\mathrm{d}u^*(n)/\mathrm{d}n > 0$. It may be advantageous for the whole population to share the road, and revenues will always fall short of cost. This denies the possibility of an optimal population size n^*, and the possibility that revenues may cover or even exceed costs in the case of decreasing costs. It seems to us that models assuming that either constant or decreasing average costs everywhere and imply that utility is always increasing (or at least not decreasing) in n are inconsistent with empirical observations. Therefore, it may be useful to consider models with variable returns to scale, as we have done.

Turning to restriction R4, it is necessary to redefine congestion for a nonhomogeneous population. This is done by substituting $\sum n^i v^i$ for nv. Consider now the case of two types, a and b. It follows from R2 and (23) that $u^{*a}(n^a, n^b, \bar{n}^b)$ is zero homogeneous in n^a and n^b and therefore, by the analysis in section 3, it is optimal to have separate roads for type a and type

[23]One case should be perhaps emphasized in the case of transportation models. If working time is institutionally fixed, and congestion affects commuting time, then it follows that the congestion variable should be included in the utility function.

[24]This was observed and used by Hochman and Pines (1971). See also Berglas (1981).

b. This is generally not recognized in the transportation literature.[25] Furthermore, it follows from our analysis that separation according to tastes and income may still be optimal where R2 and (23) do not hold, and even where $r_u = 0$, $r_c < 1$.

The distinction made in this paper between club and local public goods also applies to the analysis of transportation. For example, neighborhood roads can be dealt with within the context of the club model. Thus it follows from section 3 that in the optimal allocation the toll can be expected to cover the cost of the infrastructure. Main arteries are local public facilities. Therefore it follows from our discussion in section 4 that they should be considered in conjunction with other local public goods. A priori their toll revenue minus cost can yield either surpluses or deficits.[26]

The extension of the analysis to cover peak load problems is straightforward [see Berglas and Pines (1978)]. It can be demonstrated in this case that when roads are of the optimal configuration and the size of the sharing group is optimal then tolls will exactly cover deficits. Furthermore, given the appropriate conditions segregation according to preferences remains optimal.

To sum up: we have shown that the transportation model is an application of the model of congestion-prone goods and that all the results of this model carry through. We have demonstrated that various restrictions imposed on the analysis of the transportation models in previous studies are not necessary for the analysis of the problem. Furthermore, these restrictions prevent the possibility of a full analysis of the effect of both size of population and diversity of tastes and incomes on the optimal transportation network and its financing. Although the analysis has relied heavily on utility functions, the results have been carefully interpreted in terms of demand curves, optimal prices, surpluses and deficits generated by optimal tolls. More specifically, we have shown that utility increases (falls) if optimal tolls generate surpluses (deficits). Thus the use of our results in cost–benefit studies does not seem insurmountable.

[25]See, for example, Harwitz and Mohring (1962) and Strotz (1965) who claim that roads with mixed population are optimal when their underlying model assumes both R2 and (23). Oakland's (1972) more general model suffers the same weakness. Mohring (1976, ch. 4) provides a numerical example where it may be optimal to build separate roads for people who differ in their valuation of time. In his example he just minimizes total cost. Though his example is considerably less general than our analysis his example does illustrate the general principle derived in this paper.

[26]Strotz (1965), who uses a model similar to ours and assumes that roads are the only public good and that the production of the private good does not exhibit increasing returns to scale also conjectures that '.... in improving the urban road network, we encounter *adverse* economies of scale. If so, road expenditures should be less than toll receipts, such as in an industry of decreasing returns, costs net of rents should be less than sales receipts (rents positive).' If this is the case, it follows from our analysis in section 4 that the community cannot be at its optimal size.

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