Liquidity and the International Allocation of Economic Activity

Appendix — For Online Publication

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A The Model with Credit Frictions for Exporting

In contrast to the model in section 5, in which all firms with non-negative net profit opportunities from exporting—with productivities larger or equal than \( \hat{\phi}_X \) at Home and \( \hat{\phi}_X^* \) at Foreign—are exporters, in this extension only a fraction of these firms are able to export. Motivated by the recent literature on trade finance, I assume that firms need credit to cover their fixed costs of exporting and that for all potential exporters, credit opportunities arrive at a Poisson rate \( \frac{1}{\gamma > 0} \) at Home, and at a rate \( \frac{1}{\gamma^* > 0} \) at Foreign. Thus, a higher \( \gamma \) implies more credit frictions for Home potential exporters and the same for \( \gamma^* \) and Foreign potential exporters—if \( \gamma = \gamma^* = 0 \), so that there are no credit frictions in exporting, the model collapses to the original model in section in section 5.

A.1 The Composition of Firms

The description of preferences, demand, and production follow as in sections 5.1 and 5.2, with similar ZCP conditions and satisfying \( \hat{\phi}_X^* = \tau \hat{\phi}_D \) and \( \hat{\phi}_X = \tau \hat{\phi}_D^* \). Now, however, only a subset of Home firms with \( \varphi \geq \hat{\phi}_X \) are exporters, and the same is true for Foreign firms with \( \varphi \geq \hat{\phi}_X^* \). Therefore, the composition of firms is different to what is described in section 5.3. As before, the masses of sellers at Home and Foreign are respectively given by \( N = N_D + N_X^* \) and \( N^* = N_D^* + N_X \), where \( N_D = \left[ 1 - G(\hat{\phi}_D) \right] \frac{N_F}{\delta} \) and \( N_D^* = \left[ 1 - G(\hat{\phi}_D^*) \right] \frac{N_F^*}{\delta} \). For Home exporters, in steady state those that die (at rate \( \delta \)) are exactly replaced (at rate \( \frac{1}{\gamma} \)) by previously non-exporting...
firms with productivities larger than or equal to $\hat{\phi}_X$. That is,

$$\delta N_X = \frac{1}{\gamma} \left[ \left( 1 - G(\hat{\phi}_X) \right) \frac{N_E}{N_X} - N_X \right],$$

so that $N_X$ and analogously $N_X^*$ can be written as

$$N_X = \left( \frac{1}{\Lambda} \right) \left[ - \frac{1}{\gamma} \right] \left[ \left( 1 - G(\hat{\phi}_X) \right) \frac{N_E}{N_X} - N_X \right],$$

and

$$N_X^* = \left( \frac{1}{\Lambda^*} \right) \left[ - \frac{1}{\gamma} \right] \left[ \left( 1 - G(\hat{\phi}_X^*) \right) \frac{N_E^*}{N_X^*} - N_X^* \right],$$

where $\Lambda \equiv 1 + \delta \gamma > 1$ and $\Lambda^* \equiv 1 + \delta \gamma^* > 1$. Note that $\frac{1}{\Lambda}$ is the steady-state fraction of Home firms with productivities larger than or equal to $\hat{\phi}_X$ that export, and the same for $\frac{1}{\Lambda^*}$ in the case of Foreign firms.

To obtain $N_E$ and $N_E^*$, we follow the same steps as in section 5.3, but using instead the new expressions for $N_X$ and $N_X^*$ to get

$$N_E = \frac{\delta \eta \Lambda}{\sigma} \left[ \frac{\Lambda^* \Pi_D^* - \Pi_X}{\Lambda \Lambda^* \Pi_D \Pi_D^* - \Pi_D \Pi_D^* \Pi_X} \right],$$

(A-1)

$$N_E^* = \frac{\delta \eta \Lambda^*}{\sigma} \left[ \frac{\Lambda \Pi_D^* - \Pi_X^*}{\Lambda \Lambda^* \Pi_D \Pi_D^* - \Pi_D \Pi_D^* \Pi_X^*} \right],$$

(A-2)

where

$$\Pi_i = \int_{\hat{\phi}_i}^{\infty} \pi_i(\varphi) g(\varphi) d\varphi \quad \text{and} \quad \Pi_i^* = \int_{\hat{\phi}_i^*}^{\infty} \pi_i^*(\varphi) g(\varphi) d\varphi$$

for $i \in \{D, X\}$. Similar to section 5.3, the usual assumption that exporting firms always sell for their domestic markets, $\hat{\phi}_D < \hat{\phi}_X$ and $\hat{\phi}_D^* < \hat{\phi}_X^*$, implies that $\Pi_D > \Pi_X$ and $\Pi_D^* > \Pi_X^*$, which ensures positive equilibrium values for $N_E$ and $N_E^*$.

### A.2 Free-Entry Conditions and the Supply of Private Liquidity

As in section 5, the interest rate on Home assets is $r$, and the interest rate on Foreign assets is $r^*$, with both being bounded above by the interest rate on illiquid assets, $\rho$. With firms in both countries dying at rate $\delta$ and accessing exporting credit at rates $\frac{1}{\gamma}$ at Home and $\frac{1}{\gamma^*}$ at Foreign, the free-entry conditions are given by

$$\int_{\hat{\phi}_D}^{\infty} \left[ \frac{\pi_D(\varphi) - f}{r + \delta} \right] g(\varphi) d\varphi + \frac{1}{\Lambda} \int_{\hat{\phi}_X}^{\infty} \left[ \frac{\pi_X(\varphi) - f}{r + \delta} \right] g(\varphi) d\varphi = f_E,$$

$$\int_{\hat{\phi}_D^*}^{\infty} \left[ \frac{\pi_D^*(\varphi) - f}{r^* + \delta} \right] g(\varphi) d\varphi + \frac{1}{\Lambda^*} \int_{\hat{\phi}_X^*}^{\infty} \left[ \frac{\pi_X^*(\varphi) - f}{r^* + \delta} \right] g(\varphi) d\varphi = f_E,$$

where the left-hand side in each equation is the pre-entry expected value of a firm in each country, $V_E$ at Home and $V_E^*$ at Foreign, and the right-hand side accounts for the entry cost, $f_E$, which is the same in both countries. In contrast to section 5, the pre-entry expected profits from exporting in each country depend on the fraction of time that the potential entrant expects to be exporting
during its lifetime ($\frac{1}{X}$ for Home entrants, and $\frac{1}{X^*}$ for Foreign entrants). Under the assumed Pareto distribution for productivity, the free-entry conditions can be rewritten as

$$\frac{\Gamma}{r + \delta} \left[ \frac{1}{\tilde{\varphi}_D^k} + \left( \frac{1}{\Lambda} \right) \frac{1}{\tilde{\varphi}_X^k} \right] = f_E,$$

(A-3)

$$\frac{\Gamma}{r^* + \delta} \left[ \frac{1}{\tilde{\varphi}_{D^*}^k} + \left( \frac{1}{\Lambda^*} \right) \frac{1}{\tilde{\varphi}_{X^*}^k} \right] = f_E,$$

(A-4)

where $\Gamma = \frac{(\sigma - 1)\varphi_{min}^k}{k - \sigma + 1}$.

From equations (21), (22), (A-3), and (A-4), we solve for the cutoff productivity levels in terms of Home and Foreign interest rates as

$$\tilde{\varphi}_D = \frac{1}{\tau} \left\{ \frac{\Gamma}{f_E \Lambda} \left[ \frac{\Lambda \Lambda^* r^{2k} - 1}{\Lambda^k (r + \delta) - (r^* + \delta)} \right] \right\}^{\frac{1}{\tau}},$$

(A-5)

$$\tilde{\varphi}_{D^*} = \frac{1}{\tau} \left\{ \frac{\Gamma}{f_E \Lambda^*} \left[ \frac{\Lambda \Lambda^* r^{2k} - 1}{\Lambda^k (r^* + \delta) - (r + \delta)} \right] \right\}^{\frac{1}{\tau}},$$

(A-6)

$$\tilde{\varphi}_X = \left\{ \frac{\Gamma}{f_E \Lambda} \left[ \frac{\Lambda \Lambda^* r^{2k} - 1}{\Lambda^k (r + \delta) - (r^* + \delta)} \right] \right\}^{\frac{1}{\tau}},$$

(A-7)

$$\tilde{\varphi}_{X^*} = \left\{ \frac{\Gamma}{f_E \Lambda^*} \left[ \frac{\Lambda \Lambda^* r^{2k} - 1}{\Lambda^k (r^* + \delta) - (r + \delta)} \right] \right\}^{\frac{1}{\tau}}.$$  

(A-8)

From (A-5)-(A-8), the usual assumption that exporting firms always sell for their domestic markets, $\tilde{\varphi}_D < \tilde{\varphi}_X$ and $\tilde{\varphi}_{D^*} < \tilde{\varphi}_{X^*}$, requires that

$$\frac{\tau^k \Lambda (1 + \Lambda^*)}{\Lambda^* [\Lambda r^{2k} + 1]} < \frac{r^* + \delta}{r + \delta} < \frac{\Lambda [\Lambda^* r^{2k} + 1]}{\tau^k \Lambda^* (1 + \Lambda)},$$

(A-9)

which is a sufficient condition for an interior solution.

Financiers use claims on Home and Foreign firms’ profits as private liquidity and similar to section 5, the total supply of Home private liquidity, $A$, and the total supply of Foreign private liquidity, $A^*$, are equal to the total market capitalization of firms in each country. As in section 5.5.1, $A = \frac{N_E f_E}{r}$, $A^* = \frac{N^*_E f_E}{r}$, and therefore, we substitute equations (A-5)-(A-8) into (A-1) and (A-2) and rewrite the supplies of Home and Foreign private liquidity in terms of Home and Foreign interest rates, $r$ and $r^*$, as

$$A = \frac{(\sigma - 1)\eta}{\sigma k} \left[ \frac{\Lambda \tau^k}{\Lambda^k (r + \delta) - (r^* + \delta)} - \frac{1}{\Lambda^* \tau^k (r^* + \delta) - (r + \delta)} \right],$$

(A-10)

$$A^* = \frac{(\sigma - 1)\eta}{\sigma k} \left[ \frac{\Lambda^* \tau^k}{\Lambda^* \tau^k (r^* + \delta) - (r + \delta)} - \frac{1}{\Lambda^* \tau^k (r + \delta) - (r^* + \delta)} \right].$$

(A-11)

**A.2.1 Steady-State Equilibrium in the Model with Credit Frictions for Exporting**

With the exception of the new equations for $A$ and $A^*$ in (A-10) and (A-11), the description of the international market for liquidity is similar to section 5.5. Hence, a steady-state equilibrium solves
for the amounts of Home and Foreign private liquidity \((A, A^*)\) and Home and Foreign interest rates \((r, r^*)\) from equations (39), (40), (A-10), and (A-11). These solutions are then used in (37) and (38) to obtain the steady-state amounts of financial services traded in each type of match \((y_{HF}, y_H)\), and in (A-5)-(A-8) to obtain the steady-state cutoff productivity levels that indicate the tradability of Home and Foreign goods in each market \((\hat{\phi}_D, \hat{\phi}_X, \hat{\phi}_D^*, \hat{\phi}_X^*)\).

\section*{B The Model with Heterogeneous Liquidity Across Multiple Assets}

This extension introduces liquidity differences across assets by assuming that the different categories of Home and Foreign assets have different acceptability properties in OTC matches. In particular, I assume that (i) Home assets are acceptable as collateral in a larger fraction of OTC matches than Foreign assets, (ii) for each country’s assets, public liquidity is acceptable in a larger fraction of matches than private liquidity, and (iii) there is heterogeneity in acceptability across private assets, with firm-level productivity being positively correlated with collateral fitness. The description of preferences, production, demand, the cutoff productivity levels, and the composition of firms follow as in sections 5.1, 5.2, and 5.3.

For assumptions (i) and (ii), in a fraction \(\mu_g\) of OTC matches only Home government bonds are acceptable as collateral, in a fraction \(\mu_p\) of matches both public and private Home assets are acceptable, in a fraction \(\mu_p^*\) of matches Home assets and Foreign government bonds are acceptable, and in the remaining \(\mu_p^*\) fraction of matches all categories of assets are acceptable. Analogously, Foreign private assets are acceptable in a fraction \(\mu_p^*\) of OTC matches, Foreign bonds are acceptable in a fraction \(\mu_p^* + \mu_g^*\) of matches, Home private assets in a fraction \(\mu_p^* + \mu_g^* + \mu_p\) of matches, and Home bonds are acceptable in all matches \((\mu_p^* + \mu_g^* + \mu_p + \mu_g = 1)\).

Regarding (iii), to each producing Home firm \((\phi \geq \hat{\phi}_D)\) we associate a loan-to-value ratio, \(\lambda(\phi) \in [0, 1]\), that specifies the fraction of the asset value that can be pledged as collateral in an OTC transaction: a financier can obtain a loan of size \(\lambda(\phi) a(\phi)\) if she commits \(a(\phi)\) assets of type \(\phi\) as collateral. The function \(\lambda(\phi)\) satisfies \(\lambda'(\phi) > 0\) for all \(\phi \geq \hat{\phi}_D\), \(\lambda(\hat{\phi}_D) = 0\), \(\lambda(\infty) \rightarrow 1\), and \(\frac{d\lambda(\phi)}{d\phi} < 0\). Hence, firm-level productivity is positively related to collateral fitness, which captures the idea that low-productivity firms are seen by financiers as more volatile and sensitive to shocks than more productive firms and thus they get lower loan-to-value ratios. Note that a firm at the cutoff \(\hat{\phi}_D\) is illiquid and hence must yield a return of \(\rho\) — financiers know that this firm will die for any minimal shock causing an increase in \(\hat{\phi}_D\), so they are unwilling to accept assets of type \(\hat{\phi}_D\) in OTC transactions. Analogous properties hold for loan-to-value ratios of Foreign private assets, which are described by the function \(\lambda^*(\phi)\).
Although the analysis below only requires \( \lambda(\varphi) \) and \( \lambda^*(\varphi) \) to meet the properties described above, I assume a useful functional form that depends on a single parameter:

\[
\lambda(\varphi) = 1 - \left( \frac{\varphi_D}{\varphi} \right)^\beta \quad \text{and} \quad \lambda^*(\varphi) = 1 - \left( \frac{\varphi^*_A}{\varphi} \right)^{\beta^*}
\]

where \( \varphi \geq \varphi_D \) for Home firms, \( \varphi \geq \varphi^*_D \) for Foreign firms, \( \beta > 0 \), and \( \beta^* > 0 \). If \( \beta \to \infty \), then \( \lambda(\varphi) \to 1 \) for all \( \varphi > \varphi_D \), which approximates the case in which all claims on producing firms are equally liquid. Note also that \( \frac{d\lambda(\varphi)}{d\beta} > 0 \) for all \( \varphi > \varphi_D \), so that a decline in \( \beta \) is useful to analyze the effects of a liquidity crisis affecting loan-to-value ratios of Home private assets.

Furthermore, I assume that for a Home or Foreign private asset to be part of the available liquidity to financiers, the asset must be certified by a rating agency that makes public the asset’s underlying productivity. Each private asset’s certification process involves a sunk cost of \( f_A \) (in terms of the homogeneous good), which implies the existence of two more cutoff productivity levels, \( \varphi_A \) and \( \varphi_A^* \), that separate assets into “non-certified” and “certified” categories. Non-certified assets have underlying productivities in the range \( [\varphi_D, \varphi_A) \), they are illiquid, and hence pay the illiquid interest rate, \( \rho \). Certified assets have underlying productivities in the range \( [\varphi_A, \infty) \), they are liquid, and hence pay an interest rate below \( \rho \).

Let \( r(\varphi) \) denote the rate of return of Home private assets with underlying productivity \( \varphi \), so that \( r(\varphi) = \rho \) if \( \varphi \in [\varphi_D, \varphi_A) \) and \( r(\varphi) < \rho \) if \( \varphi \in [\varphi_A, \infty) \). Similarly, let \( r^*(\varphi) \) denote the rate of return of Foreign assets with underlying productivity \( \varphi \). To pin down \( \varphi_A \) and \( \varphi_A^* \), note that an asset with underlying productivity \( \varphi \) will be certified if and only if the value of the firm when certified minus the sunk certification cost, is no less than the value of the firm when not certified; this condition holds with equality for a firm at the cutoff. Thus, \( \varphi_A \) and \( \varphi_A^* \) solve

\[
\left[ [\pi_D(\varphi_A) - f] \mathbb{1}\{\varphi_A \geq \varphi_D\} + [\pi_X(\varphi_A) - f] \mathbb{1}\{\varphi_A \geq \varphi_X\} \right] \left[ \frac{1}{r(\varphi_A) + \delta} - \frac{1}{\rho + \delta} \right] = f_A \quad \text{(B-1)}
\]

\[
\left[ [\pi_D^*(\varphi_A^*) - f] \mathbb{1}\{\varphi_A^* \geq \varphi_D^*\} + [\pi_X^*(\varphi_A^*) - f] \mathbb{1}\{\varphi_A^* \geq \varphi_X^*\} \right] \left[ \frac{1}{r(\varphi_A^*) + \delta} - \frac{1}{\rho + \delta} \right] = f_A, \quad \text{(B-2)}
\]

where \( \mathbb{1}\{\cdot\} \) is an indicator function taking the value of 1 if the condition inside the brackets is satisfied (and is zero otherwise). The left-hand side in (B-1) and (B-2) shows the difference between the discounted sum of instantaneous profits when certified (with an effective discount rate of \( r(\varphi_A) + \delta \)) and the discounted sum of instantaneous profits when not certified (with an effective discount rate of \( \rho + \delta \)). The right-hand side in (B-1) and (B-2) shows the sunk certification cost.

**B.1 Supply of Private Liquid Assets**

Financiers fund the entry of differentiated-good firms in both countries in exchange for claims on firms’ profits. As mentioned before, financiers may use these claims as private liquidity (i.e., as
collateral in their financial transactions). In contrast to the benchmark case, however, the total market capitalization of firms is no longer equivalent to the amount of private liquidity available. In particular, in the presence of loan-to-value ratios below 1 and certification costs that give rise to the cutoffs $\hat{\phi}_A$ and $\hat{\phi}_A^*$, the total supply of Home private liquidity, $A$, and the total supply of Foreign private liquidity, $A^*$, are now a fraction of the total market capitalization of firms in each country.

At Home, the value of a firm with productivity $\varphi$ is defined as

$$ V(\varphi) = \frac{[\pi_D(\varphi) - f] \mathbb{1}\{\varphi \geq \hat{\phi}_D\} + [\pi_X(\varphi) - f] \mathbb{1}\{\varphi \geq \hat{\phi}_X\}}{r(\varphi) + \delta} $$

where $r(\varphi) = \rho$ if $\varphi \in [\hat{\phi}_D, \hat{\phi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\phi}_A, \infty)$, and $\mathbb{1}\{\cdot\}$ is the indicator function. As a firm knows its productivity only after entry, the pre-entry expected value of a firm for Home potential entrants is $V_E = \int_{\hat{\phi}_D}^{\infty} V(\varphi) g(\varphi) d\varphi$. With similar expressions for Foreign firms, and assuming identical entry costs for Home entrants and Foreign entrants, $f_E$, the free-entry conditions for differentiated-good firms at Home and Foreign are

$$ \int_{\hat{\phi}_D}^{\infty} \left[ \frac{\pi_D(\varphi) - f}{r(\varphi) + \delta} \right] g(\varphi) d\varphi + \int_{\hat{\phi}_X}^{\infty} \left[ \frac{\pi_X(\varphi) - f}{r(\varphi) + \delta} \right] g(\varphi) d\varphi = f_E + [1 - G(\hat{\phi}_A)] f_A, \quad (B-4) $$

$$ \int_{\hat{\phi}_D^*}^{\infty} \left[ \frac{\pi_D^*(\varphi) - f}{r^*(\varphi) + \delta} \right] g(\varphi) d\varphi + \int_{\hat{\phi}_X^*}^{\infty} \left[ \frac{\pi_X^*(\varphi) - f}{r^*(\varphi) + \delta} \right] g(\varphi) d\varphi = f_E + [1 - G(\hat{\phi}_A^*)] f_A. \quad (B-5) $$

In (B-4), the left-hand side is $V_E$, with the first term showing the expected discounted profits from selling domestically, and the second term showing the expected discounted profits from exporting; the right-hand side shows the sunk entry cost plus the expected certification cost (which is only paid if the entrant’s productivity draw is $\hat{\phi}_A$ or higher). Equation (B-5) has an analogous interpretation for Foreign potential entrants.

Let $A(\varphi)$ denote the supply of liquidity stemming from Home firms with productivity $\varphi$, and let $A^*(\varphi)$ denote the supply of liquidity stemming from Foreign firms with productivity $\varphi$. The aggregate value of Home firms with productivity $\varphi$ is given by $N_A V(\varphi) g(\varphi | \varphi \geq \hat{\phi}_A)$, where $N_A = [1 - G(\hat{\phi}_A)] N_E / \delta$ denotes the measure of certified Home firms. Analogously, $N^*_A V^*(\varphi) g(\varphi | \varphi \geq \hat{\phi}_A^*)$ is the aggregate value of Foreign firms with productivity $\varphi$ for all $\varphi \geq \hat{\phi}_A^*$, where $N^*_A = [1 - G(\hat{\phi}_A^*)] N_E^* / \delta$ is the measure of certified Foreign firms. Given that only fractions $\lambda(\varphi)$ and $\lambda^*(\varphi)$ of the value of these firms can serve as collateral in the OTC market, it follows that

$$ A(\varphi) = \lambda(\varphi) N_A V(\varphi) g(\varphi | \varphi \geq \hat{\phi}_A) $$

$$ A^*(\varphi) = \lambda^*(\varphi) N^*_A V^*(\varphi) g(\varphi | \varphi \geq \hat{\phi}_A^*) $$

for all $\varphi \geq \hat{\phi}_A$ at Home and $\varphi \geq \hat{\phi}_A^*$ at Foreign. The supplies of Home private liquidity, $A =$
\int_{\hat{\varphi}_A}^{\infty} A(\varphi)d\varphi, and Foreign private liquidity, \( A^* = \int_{\hat{\varphi}_A}^{\infty} A^*(\varphi)d\varphi \), can then be written as

\[ A = N_A \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi) \left[ \frac{[\pi_D(\varphi) - f] + [\pi_X(\varphi) - f]}{r(\varphi) + \delta} \right] g(\varphi \mid \varphi \geq \hat{\varphi}_A) d\varphi, \quad (B-6) \]

\[ A^* = N_A^* \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi) \left[ \frac{[\pi_D^*(\varphi) - f] + [\pi_X^*(\varphi) - f]}{r^*(\varphi) + \delta} \right] g(\varphi \mid \varphi \geq \hat{\varphi}_A^*) d\varphi. \quad (B-7) \]

### B.2 Demand for Multiple Liquid Assets

Let \( a(\varphi) \) and \( a^*(\varphi) \) denote the financier’s holdings of Home and Foreign assets of type \( \varphi \), and let \( b \) and \( b^* \) denote the financier’s holdings of Home and Foreign government bonds. In addition to Home and Foreign private assets’ interest rates, \( r(\varphi) \) and \( r^*(\varphi) \), the interest rates of Home and Foreign government bonds are \( r_b \) and \( r_b^* \). Hence, the budget constraint of a Home financier is

\[ \int_{\hat{\varphi}_D}^{\infty} \hat{a}(\varphi)d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} \hat{a}^*(\varphi)d\varphi + b + b^* = \int_{\hat{\varphi}_D}^{\infty} r(\varphi)a(\varphi)d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} r^*(\varphi)a^*(\varphi)d\varphi + r_b b + r_b^* b^* - h - \Upsilon. \]

The left-hand side presents the change in the financier’s wealth, which is given by the financier’s total investment in private assets and government bonds from both Home and Foreign. The right-hand side shows the interest payments on the financier’s portfolio net of homogeneous-good consumption \( h \) and taxes \( \Upsilon \). In contrast to the closed-economy model, the financier’s portfolio is now composed of assets with different liquidity properties and interest rates. The budget constraint of a Foreign financier is identical to Home financier’s budget constraint, with the exception of the last term, which changes to \( \Upsilon^* \) (taxes in Foreign).

The total amounts of Home private liquidity, \( a \), and Foreign private liquidity, \( a^* \), held by a financier are given by

\[ a = \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi)a(\varphi)d\varphi \quad \text{and} \quad a^* = \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi)a^*(\varphi)d\varphi, \]

which weight the holdings of each asset by its loan-to-value ratio, and take into account that liquid assets have underlying productivities no less than \( \hat{\varphi}_A \) and \( \hat{\varphi}_A^* \).

Following similar steps to those in section 4.2 to obtain (15), we find that the continuation value of a financier upon being matched (but before realizing its buyer or seller role), \( Z(a, a^*, b, b^*) \), is given by

\[ Z(a, a^*, b, b^*) = \frac{\mu_p}{2} \max_{y_p \leq a + a^* + b + b^*} \{ F(y_p^*) - y_p^* \} + \frac{\mu_q}{2} \max_{y_q \leq a + a^* + b + b^*} \{ F(y_q^*) - y_q^* \} \]

\[ + \frac{\mu_p}{2} \max_{y_p \leq a + b} \{ F(y_p) - y_p \} + \frac{\mu_q}{2} \max_{y_q \leq b} \{ F(y_q) - y_q \} + W(a, a^*, b, b^*). \]

This expression shows that with probability 1/2 the financier is the buyer in the OTC match, in which case she can make a take-it-or-leave-it offer to the seller in order to maximize her surplus,
Equations (B-8)-(B-11) define the optimal choice of each type of asset. With probability \( \mu_p^* \), the financier is in a match in which private and public assets from both Home and Foreign are acceptable as collateral and thus, she can transfer up to \( a + a^* + b + b^* \) in exchange for \( y_p^* \). With probability \( \mu_a^* \) Home assets and Foreign government bonds are acceptable, so that the financier can transfer up to \( a + b + b^* \) to purchase \( y_p^* \). With probability \( \mu_p \) only Home assets are acceptable, so that the financier can transfer up to \( a + b \) to purchase \( y_p \). Lastly, with probability \( \mu_a \) only Home government bonds are acceptable, so that the financier can transfer up to \( b \) to purchase \( y_a \).

Similar to the derivation of equation (18) in the closed-economy model, the financier’s optimal portfolio solves

\[
\frac{\rho - r^*(\varphi)}{\theta} = \mu_p^* \lambda^*(\varphi) \left[ F'(y_p^*) - 1 \right] \quad \text{(B-8)}
\]

\[
\frac{\rho - r^*(\varphi)}{\theta} = \mu_p^* \left[ F'(y_p^*) - 1 \right] + \mu_a^* \left[ F'(y_a^*) - 1 \right] \quad \text{(B-9)}
\]

\[
\frac{\rho - r^*(\varphi)}{\theta} = \mu_p^* \lambda(\varphi) \left[ F'(y_p^*) - 1 \right] + \mu_a^* \lambda(\varphi) \left[ F'(y_a^*) - 1 \right] + \mu_p \lambda(\varphi) \left[ F'(y_p) - 1 \right] \quad \text{(B-10)}
\]

\[
\frac{\rho - r^*(\varphi)}{\theta} = \mu_p^* \left[ F'(y_p^*) - 1 \right] + \mu_a^* \left[ F'(y_a^*) - 1 \right] + \mu_p \left[ F'(y_p) - 1 \right] + \mu_a \left[ F'(y_a) - 1 \right]. \quad \text{(B-11)}
\]

Equations (B-8)-(B-11) define the optimal choice of each type of asset.1 On one extreme, the left-hand side of equation (B-8) is the holding cost of Foreign private asset of type \( \varphi \), while the right-hand side indicates the expected marginal surplus from holding an additional unit of that asset. That Foreign asset can only be used in a fraction \( \mu_p^* \) of all matches with a pledgeability ratio of \( \lambda^*(\varphi) \), in which case the marginal surplus of the financier is \( F'(y_p^*) - 1 \). On the other extreme, the left-hand side of (B-11) shows the holding cost of a Home government bond, while the right-hand side is its marginal surplus from its use in all matches in the financial market.

Similar to the closed-economy case, the quantity of financial services traded in an OTC match is the minimum between the value of the buyer’s liquidity in that match and the surplus-maximizing quantity, \( \hat{y} \). The difference is that in the closed-economy case only domestic private and public assets are used, and they are all fully acceptable in all matches. Given that financiers hold identical portfolios and that there is a unit measure of financiers in the world, market clearing implies that \( a = A, a^* = A^*, b = B, \) and \( b = B^* \). Thus, we get

\[
y_p^* = \min \{ A + A^* + B + B^*, \hat{y} \}, \quad \text{(B-12)}
\]

\[
y_a^* = \min \{ A + B + B^*, \hat{y} \}, \quad \text{(B-13)}
\]

\[
y_p = \min \{ A + B, \hat{y} \}, \quad \text{(B-14)}
\]

\[
y_a = \min \{ B, \hat{y} \}. \quad \text{(B-15)}
\]

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1Note that the closed-economy equation (18) can be obtained from (B-10) and (B-11) by assuming that \( \mu_p^* = \mu_a^* = \mu_p = \mu_a = 0, \mu_p = 1, \) and \( \beta \to \infty \) so that \( \lambda(\varphi) \to 1 \) for all \( \varphi \geq \varphi_D \).
Note from (B-8)-(B-11) and (B-12)-(B-15) that we can have several scenarios. Suppose, for example, that liquidity is scarce in matches that only accept Home assets, but is abundant in matches that also accept Foreign assets; i.e., \( A + B < \hat{y} \) but \( A + B + B^* > \hat{y} \). This implies that \( F'(y^*_p) - 1 = F'(y^*_g) - 1 = 0 \), but \( F'(y_g) - 1 > F'(y_p) - 1 > 0 \). Therefore, from (B-8)-(B-11) we obtain that all Foreign assets pay the maximum interest rate, \( r^*_b = r^*(\varphi) = \rho \), while Home liquid assets give interest \( r_b < r(\varphi) < \rho \).

The definition of a steady-state equilibrium in the two-country model follows.

**Definition.** A steady-state equilibrium in the two-country model is a list

\[
\langle \hat{\phi}_D, \hat{\phi}_X, \hat{\varphi}^*_D, \hat{\varphi}^*_X, \hat{\varphi}_A, \hat{\varphi}^*_A, A, A^*, y_p^*, y_g^*, y_p, y_g, r^*(\varphi), r^*_b, r(\varphi), r_b \rangle
\]

that solves (21), (22), (B-1), (B-2), (B-4)-(B-7), and (B-8)-(B-15).