Liquidity and the International Allocation of Economic Activity

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Abstract

I present a model to study the linkages between the market for liquidity and the international allocation of economic activity. Private assets’ liquidity properties affect interest rates, with consequences on entry, production, and market capitalization. In a closed economy, the liquidity market increases the size and productivity of the sector that generates liquid assets. In an open economy, cross-country differences in financial development generate an allocation of economic activity favoring the country supplying the most liquid assets. Supporting the theory, I show that asset acceptability as collateral is negatively correlated with yields, and positively correlated with measures of economic activity.

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1 Introduction

Private assets such as equity, commercial paper, and corporate bonds provide liquidity services to the financial system because they can be used as media of exchange or as collateral in financial transactions. The money role of private assets not only expands the size of the financial sector by allowing more and larger financial transactions, but also affects real economic activity in sectors where the assets are generated. In particular, values of private assets include a liquidity premium that reflects their degree of moneyness in financial-sector activities; these augmented values in turn affect issuing firms’ production, entry and exit decisions, and aggregate-level outcomes such as aggregate prices and productivity. At an international level, cross-country differences in financial development—as measured by the degree of liquidity services provided by a country’s assets—potentially influence the organization of economic activity across borders, with consequences on international trade relationships.

The goal of this paper is to elucidate the links between the market for liquid assets and the international allocation of economic activity. Toward this goal, I introduce a theoretical model that shows how liquidity needs of the financial system affect the real-economy sector that generates liquid assets. At an international level, I look at how cross-country differences in asset liquidity affect the international allocation of economic activity, and study the effects of trade liberalization. The model introduces a market for liquid assets into the standard Melitz (2003) model of trade with heterogeneous (in productivity) firms. The market for liquidity—which follows Rocheteau and Rodriguez-Lopez (2014)—determines equilibrium interest rates for different types of liquid assets and the equilibrium amount of liquidity in the economy. The supply of liquidity is composed of claims on Melitz firms’ profits (private liquidity) and government bonds (public liquidity), while the demand for liquidity is determined by financiers who need liquid assets to be used as collateral in their financial activities. The end result is a Melitz-type model with endogenous interest rates driven by asset-liquidity considerations.

The market for liquid assets has positive spillovers on the real economy. To show this, I start by describing a closed economy with three types of agents: households, financiers, and heterogeneous firms. Financiers fund the entry of heterogeneous firms in exchange for claims on the firms’ future profits from their sales of differentiated-good varieties to households. In addition, financiers trade financial services in an over-the-counter (OTC) market; these transactions are backed by a collateral agreement, with claims on firms and government bonds playing the collateral role. Thus, claims on

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1 Liquidity is priced: the most liquid assets—those with high degree of moneyness (easily traded and highly acceptable as media of exchange)—have higher prices and lower interest rates. Abundant evidence on the liquidity premium appears in the cross-section and over time in equity markets (see, e.g., Pastor and Stambaugh, 2003 and Liu, 2006) and corporate bond markets (see, e.g., Lin, Wang, and Wu, 2011). In comparison with U.S. Treasury bonds, which are considered to be the most liquid financial assets in the world, Chen, Lesmond, and Wei (2007) and Bao, Pan, and Wang (2011) find that corporate bond yield spreads—the rate-of-return difference between corporate bonds and U.S. Treasuries—decline with corporate bond liquidity.
firms and government bonds are “liquid assets” because they serve as collateral in OTC financial transactions.

The model shows that the financiers’ demand for liquid assets is increasing in the assets’ interest rate: when the interest rate increases, the financiers’ cost of holding assets declines and hence they will hold more of them. In contrast, there is an inverse relationship between the supply of private liquidity and the interest rate. When the interest rate is equal to the rate of time preference, firms are priced at their “fundamental value”, which is the value that would prevail in the absence of liquidity services from private assets. For a lower level of the assets’ interest rate, the average value of firms increases, driving up total market capitalization \(i.e.,\) the supplied amount of private liquidity increases. In equilibrium, the interest rate is below the rate of time preference, and both the total market capitalization and the average productivity of firms are larger than at the fundamental-value outcome. Thus, and in accordance with a Mundell-Tobin effect (Mundell, 1963; Tobin, 1965), the liquidity of private assets increases the size (and productivity) of the real-economy sector that generates them.\(^2\)

The model is then expanded to a two-country setting with cross-country differences in asset liquidity. There are four categories of assets—Home and Foreign private assets, and Home and Foreign government bonds—with the liquidity of each country’s assets being determined by their acceptability as collateral in OTC transactions in the world financial system. The model determines interest rates, production, and the total capitalization of firms in both countries, as well as the amount of international trade. The model shows that more liquid assets yield lower interest rates and as a consequence, differences in asset liquidity across countries affect the international allocation of economic activity. In particular, if Home assets are more liquid than Foreign assets, then Home has a larger production sector, higher aggregate productivity, a lower aggregate price, and higher household welfare than Foreign. Moreover, although trade liberalization typically has conventional Melitz-type effects in both countries, the total capitalization of firms increases at Home but declines at Foreign; \(i.e.,\) trade liberalization widens the gap in economic activity between the countries.

I present two extensions to the model. The benchmark two-country model assumes that the acceptability of each asset type in the financial market is exogenous and depends exclusively on national origin. However, the acceptability of an asset is likely to be an endogenous outcome—resulting from

\(^2\)The mechanism linking the liquidity market to real economic activity also resembles the mechanism of the Bewley model of Aiyagari (1994), in which households accumulate claims on capital to self-insure against idiosyncratic labor income shocks. In that model \(i\) households’ precautionary savings are increasing in the interest rate—the holding cost of a claim on capital declines as its interest rate increases—up to the discount rate (at that point the holding cost of a claim on capital is zero and savings tend to infinity), and \(ii\) the amount of capital in the production sector is declining in the interest rate (a higher interest rate implies a higher marginal product of capital, which then implies a lower level of capital—as usual, the marginal product of capital is declining). In equilibrium, due to the role of capital as a self-insurance device, the interest rate is below the discount rate and the aggregate capital stock in the economy is above its certainty level. Hence, the model here can be interpreted as a tractable version of Aiyagari’s model in which instead of holding assets for precautionary-saving motives due to idiosyncratic income shocks, agents hold assets due to the liquidity services they provide in their random opportunities to trade in the OTC financial market.
comparing costs and benefits of using that asset as collateral—and assets from the same country can differ in their acceptability properties. Consequently, the first extension follows the recognizability approach to asset liquidity (Lester, Postlewaite, and Wright, 2012) to endogenize the acceptability of Foreign assets, showing that if Home assets are more liquid than Foreign assets to begin with, then trade liberalization further reduces Foreign-asset acceptability, exacerbating the effect of trade on the gap in economic activity between the countries. Meanwhile, the second extension allows for differences in acceptability between public and private assets within and between countries, and across private assets within each country, while also endogenizing the set of acceptable private assets from each country.

To shed light on the empirical relevance of the mechanisms proposed in the model, the last part of this paper provides an empirical assessment on the relationships between asset acceptability and both yields and economic activity. The theory in this paper shows that asset acceptability commands a liquidity premium that is reflected in lower yields and higher firm values, and that in an open economy, differences in acceptability across assets from different countries cause an unequal distribution of economic activity in favor of the firms that issue the most liquid assets. After obtaining a list of corporate bonds and stocks that are acceptable as collateral by a large U.S. clearinghouse for OTC derivatives, this paper presents evidence that is consistent with these results. First, after controlling for credit rating, I show that yields on acceptable corporate bonds are lower than yields on non-acceptable bonds. Second, I use worldwide firm-level Compustat data to show that in comparison to firms within the same industry whose assets are not acceptable, firms that issue acceptable assets have (1) higher shares in the industry’s worldwide sales, employment, R&D expenditure, book value, and market value, (2) higher ratios of profits to industry sales, and (3) higher market-to-book ratios.

The model in this paper is related to the two-country models of Geromichalos and Simonovska (2014), Geromichalos and Jung (2018), and Lee and Jung (2019), who introduce new monetarist frameworks to explain international finance phenomena such as the positive correlation between consumption and asset home bias, the link between the foreign-exchange market and international trade, and the uncovered interest parity puzzle. This paper is also related to models that try to explain global imbalances—which feature capital flows from emerging countries to rich countries (the so-called Lucas paradox)—as a result of cross-country differences in financial development. The OLG model of Caballero, Farhi, and Gourinchas (2008) has a definition of financial development that is similar to the one in this model, while the Bewley-type models of Angeletos and Panousi (2011) and Mendoza, Quadrini, and Rios-Rull (2009) relate financial development to financial contract enforceability in the insurance of idiosyncratic risks. In these three models, financially developed countries have higher autarky interest rates as a result of lower demands for financial assets; financial integration equalizes interest rates and thus drives capital flows towards the most financially developed countries. In
contrast, in this paper the demand for liquid assets is set in a world financial market (independently of each country’s financial development) and liquidity differences across assets are the main drivers of capital flows, with each asset yielding an equilibrium interest rate in accordance with its liquidity properties.

The paper is organized as follows. Section 1.1 describes some facts about the liquidity market. Section 2 introduces the closed-economy version of the model, highlighting the novel mechanisms of this framework. Section 3 presents the two-country model, and section 4 studies the effects of cross-country differences in asset liquidity on the allocation of economic activity, as well as the implications for trade liberalization. Section 5 presents the two extensions to the model. Section 6 presents the empirical evidence on acceptability, yields, and economic activity. Lastly, section 7 concludes.

1.1 Facts About Financial Markets and Their Liquidity Needs

In this paper, the financial sector is modeled to resemble wholesale financial markets that rely on collateral to perform their lending and trading activities. To gauge the importance of the liquidity needs in this type of markets, this section describes two prominent examples: the global OTC market for interest-rate derivatives and the U.S. tri-party repo market. Additionally, this section presents some facts about the global supply and demand for liquid assets.

With a notional amount outstanding of $524 trillion by June 2019, the OTC market for interest-rate derivatives (IRDs) accounts for 82% of the global OTC derivatives market (BIS, 2019)—foreign-exchange derivatives account for 15%, and the remaining 3% is split between commodity, equity, and credit derivatives. According to Bank of International Settlements (BIS) data, the average daily trading volume of OTC IRDs increased from $2.1 trillion during April 2010 to $6.5 trillion during April 2019. As discussed by Ehlers and Hardy (2019) and Wooldridge (2019), this increase was driven by (i) risk-reducing regulatory reforms by G20 countries requiring the clearing of OTC derivatives by central counterparties (see also section 6.1), and (ii) new electronic trading platforms that increased trading speed and reduced transaction costs. The BIS data also shows that 50% of the daily trading volume is denominated in U.S. dollars and 24% in Euros. In terms of location, during April 2019 32% of the daily trading volume took place in U.S. sales desks and 50% in the United Kingdom. Although the BIS does not provide collateral data, Berends and King (2015) report that the composition of collateral posted by U.S. life insurance companies in their OTC derivatives contracts by September 2013 was U.S. Treasuries (43%), U.S. government agency obligations (22%), corporate bonds (16%), cash (13%), and other assets (6%).

The U.S. tri-party repo market is a short-term funding market where security dealers receive cash from investors in exchange for securities that will be purchased back typically one day later (it is called “tri-party” because each transaction is settled by a clearing bank). It is essentially a market for one-
day collateralized loans, with the advantage that in the case of the dealer’s default, the cash investor can sell the collateral without an automatic-stay rule (Copeland, Duffie, Martin, and McLaughlin, 2012).³ This market is crucial for the implementation of U.S. monetary policy, with the New York Fed conducting “repo and reverse repo operations each day as a means to help keep the federal funds rate in the target range set by the Federal Open Market Committee (FOMC)”⁴. According to data published by the New York Fed, the daily trading volume—in terms of the market value of the collateral—in the tri-party repo market during the 2013-2019 period was $1.8 trillion (it has been more than $2 trillion since September 2018). The same data shows that the average composition of collateral during that period was U.S. Treasuries (45.6%), U.S. government agency obligations (36.2%), equities and corporate bonds (11.9%), asset-backed securities (2.4%), and other assets (3.9%).⁵

The essential role of liquid assets as collateral in financial markets as well as the growing demand for high-quality collateral—fueled by new regulations following the global financial crisis—are documented, among others, by the IMF (2012) and the BIS (2013). Regarding the value and composition of the world’s supply of liquid assets, the IMF (2012) estimates that by 2011 it was about $74.4 trillion and was composed of OECD-countries sovereign debt (56 percent), asset-backed securities (17 percent), corporate bonds (11 percent), gold (11 percent), and covered bonds (4 percent). With respect to their country of origin, the U.S. is the main supplier of liquid assets for the world financial system. According to estimations by the BIS (2013), in 2012 the U.S. accounted for about half of the supply of high-quality assets eligible as collateral in financial transactions (the U.S. is followed by Japan, the Euro area, and the U.K.). The importance of the U.S. as a world provider of liquidity is even higher in the production of more sophisticated financial instruments. For example, according to Cetorelli and Peristiani (2012), from 1983 to 2008 the U.S. accounted for 73.1 percent of the issuance of asset-backed securities (ABS).

On the other hand, based on holdings of sovereign debt, the IMF (2012) estimates that by the end of 2010 the demand for safe and liquid assets was coming from private banks (34 percent), central banks (21 percent), insurance companies (15 percent), pension funds (7 percent), sovereign wealth funds (1 percent), and other entities (22 percent). Hence, most of the demand for liquid assets arises from within the financial sector.⁶ Accordingly, the demand for liquid assets in this model stems from

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³The U.S. tri-party repo market is very concentrated. According to Brickler, Copeland, and Martin (2011), in 2011 the top 10 dealers (or collateral providers) accounted for 80 percent of the borrowing, while the top 10 cash investors accounted for 60 percent of the lending. The only two clearing banks in the market are JP Morgan Chase and the Bank of New York Mellon.

⁴This quote is from the New York Fed’s website at https://apps.newyorkfed.org/markets/autorates/temp.

⁵U.S. government agency obligations include mortgage-backed securities, collateralized mortgage obligations, and debt securities issued by Freddie Mac, Fannie Mae and Ginnie Mae, as well as debt securities issued by other federal agencies.

⁶See also Gourinchas and Jeanne (2012), who find that the demand for safe assets by the U.S. private real sector has been very stable over time (and also for the U.K., France, and Germany, but not for Japan), and hence attribute most of the increase in the demand for safe assets to the financial system.
financiers that need liquid assets to be used as collateral in their trading activities.

2 Liquidity in the Closed Economy

To describe the basic interactions between the market for liquid assets and the real economy, I describe first a closed economy.

2.1 The Environment

The model is in continuous time, \( t \in \mathbb{R}_+ \), and there are three categories of agents: a unit measure of households, a unit measure of financiers, and an endogenous measure of heterogeneous (in productivity) firms. There are three types of goods: a homogeneous good that is produced and consumed by households and financiers and that is taken as the numéraire, a heterogeneous good that is produced in many varieties by heterogeneous firms and that is consumed by households only, and a financial service that is produced and consumed by financiers only.

2.1.1 Households

Households are risk-neutral and discount future consumption at rate \( \rho > 0 \), with lifetime utility given by

\[
\int_{0}^{\infty} e^{-\rho t} C(t) dt,
\]

where \( C(t) \) is the household’s consumption index described as

\[
C(t) = H(t)^{1-\eta} Q(t)^{\eta}, \tag{1}
\]

where \( H(t) \) denotes the consumption of the homogeneous good, \( Q(t) = \left( \int_{\omega \in \Omega} q^c(\omega, t) \frac{1}{\sigma} d\omega \right) \frac{\sigma}{\sigma-1} \) is the CES consumption aggregator of differentiated-good varieties, and \( \eta \in (0, 0.5] \). In \( Q(t) \), \( q^c(\omega, t) \) denotes the consumption of variety \( \omega \), \( \Omega \) is the set of varieties available for purchase, and \( \sigma > 1 \) is the elasticity of substitution between varieties.

Each household is endowed with a unit of labor per unit of time devoted either to produce one unit of the homogeneous good (which is produced under perfect competition without any other costs), or to produce in the differentiated-good sector as an employee of a differentiated-good firm. In the absence of any frictions in the labor market, the wage of each household is 1 (in terms of the homogeneous good).

Given (1) and the unit wage, the representative household’s total expenditure on differentiated-good varieties is \( \eta \). It follows that each household’s demand for differentiated-good variety \( \omega \) is

\[
q^c(\omega, t) = \left[ \frac{p(\omega, t)^{-\sigma}}{P(t)^{1-\sigma}} \right] \eta, \tag{2}
\]
where \( p(\omega, t) \) is the price of variety \( \omega \) at time \( t \), and \( P(t) \equiv \left[ \int_{\omega \in \Omega} p(\omega, t)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \) is the price of the CES aggregator \( Q(t) \). Given that there is a unit mass of households, equation (2) also corresponds to the market demand for variety \( \omega \), and \( P(t)Q(t) \equiv \eta \) is the country’s total expenditure on differentiated-good varieties. Moreover, the total expenditure on the homogeneous good is \( 1 - \eta \), and thus, the indirect utility flow of households is

\[
W(t) = \frac{(1 - \eta)^{1-\eta} \eta^n}{P(t)^\eta} \propto \frac{1}{P(t)^\eta}.
\]

Hence, for a given \( \eta \), changes in household welfare only depend on changes in the aggregate price of differentiated goods.

### 2.1.2 Financiers

Financiers define their preferences over the consumption of financial services—traded in an over-the-counter market (which involves bilateral matching and bargaining)—and the consumption of the homogeneous good. A financier discounts time at rate \( \rho \) and its lifetime expected utility is

\[
E \left\{ \sum_{n=1}^{\infty} e^{-\rho T_n} \left\{ F(y(T_n)) - x(T_n) \right\} + \int_0^{\infty} e^{-\rho t} H(t) dt \right\},
\]

where the first term accounts for the utility from consumption of financial services, and the second term accounts for the utility from consumption of the homogeneous good.

In the first term, \( \{T_n\} \) is a Poisson process with arrival rate \( \nu > 0 \) that indicates the times at which the financier is matched with another financier. After a match is formed, a financier is chosen at random to be either user or supplier of services. For a user, the utility from consuming \( y \) units of financial services is \( F(y) \), where \( F \) is strictly concave, \( F(0) = 0 \), \( F'(0) \rightarrow \infty \), and \( F'(\infty) = 0 \). For a supplier, the disutility from providing \( x \) units of financial services is \( x \). For a given financier, either \( y(T_n) > 0 \) (with probability 0.5) or \( x(T_n) > 0 \) (with probability 0.5). For any match, feasibility requires that \( y(T_n) \leq x(T_n) \)—the consumption of the user must be no greater than the production of the supplier.

At all \( t \notin \{T_n\}_{n=1}^{\infty} \) financiers can produce and consume the homogeneous good. The technology to produce/consume the homogeneous good is, however, not available at times \( \{T_n\} \) when financiers are matched in the OTC market. Therefore, the buyer of financial services requires a loan to finance its purchase. Assuming lack of commitment and monitoring, financiers rely on liquid assets (to be used as collateral) to secure their loans in the OTC market.

### 2.1.3 Firms

Producers of differentiated-good varieties are heterogeneous in productivity. Following Melitz (2003), after paying a sunk entry cost of \( f_E \) units of the homogeneous good, a firm draws its productivity from a probability distribution with support \([\varphi_{\min}, \infty)\), cumulative function \( G(\varphi) \), and density function
g(φ). Firms' entry costs are paid for by financiers in exchange for the ownership in the future profits of the firm. Crucially, these claims on firms' profits belong to the set of liquid assets that financiers can use as collateral in OTC trades.

The production function of a firm with productivity φ is \( q(φ, t) = φL(t) \), where \( L(t) \) denotes labor. There are also fixed costs of operation, with each producing firm paying \( f \) units of the homogeneous good per unit of time. In addition, all firms are subject to a random death shock, which arrives at Poisson rate \( δ > 0 \).

Given CES preferences for differentiated-good varieties, the profit maximization problem for a firm with productivity \( φ \) yields the usual pricing equation with a fixed markup over marginal cost:

\[
p(φ) = \left( \frac{η}{σf} \right)^{\frac{1}{σ}} p[ϕ(t)].
\]

Equation (4) can then be used to rewrite the gross profit function as

\[
π(φ, t) = \left[ \frac{φ}{ϕ(t)} \right]^{σ-1} f,
\]

which shows that firm-level profits are increasing in productivity and declining with the cutoff productivity level.

There is also a convenient expression for the mass of producing firms, \( N(t) \). Note first that the aggregate price of differentiated-good varieties, \( P(t) \), can be calculated as

\[
P(t) = \left[ N(t) \int_{ϕ(t)}^{∞} p(φ)^{1-σ} g[φ|φ ≥ ϕ(t)] dφ \right]^{\frac{1}{σ}}.
\]

It then follows from (4) and (6) that

\[
N(t) = \frac{η}{σf} \left[ \frac{ϕ(t)}{ϕ(t)} \right]^{σ-1},
\]

where

\[
ϕ(t) = \left[ \int_{ϕ(t)}^{∞} φ^{σ-1} g[φ|φ ≥ ϕ(t)] dφ \right]^{\frac{1}{σ-1}}
\]

is the average productivity of producing firms.

### 2.1.4 Government bonds

There is a supply \( B \) of pure-discount government bonds that pay one unit of the homogeneous good at the time of maturity. The terminal payment of bonds is financed through lump-sum taxation on
financiers.\textsuperscript{7} Along with claims on firms’ profits, government bonds can serve as collateral in the OTC market.

\section*{2.2 The Market for Liquidity}

In the absence of perfect commitment, financiers need liquidity to secure their debt obligations from their OTC transactions. This section describes the supply of private liquidity arising from differentiated-good firms, the demand of liquidity by financiers, and the determination of the real interest rate to clear the market for liquid assets. I focus on steady-state equilibria—the cutoff productivity level, the mass of firms, and the interest rate are constant over time—and hence, we can suppress the time index, \( t \), in some parts of this section.

\subsection*{2.2.1 Supply of Liquidity}

All claims on producing firms’ profits are part of the liquidity of the economy, and therefore, the amount of private liquidity available to financiers is equivalent to the aggregate capitalization of firms.\textsuperscript{8} This section determines the aggregate capitalization of firms as a function of the interest rate on liquid assets, \( r \).

A producing firm with productivity \( \varphi \) generates a flow dividend, \( \pi(\varphi) - f \), and dies at rate \( \delta \). The value of this firm is denoted by \( V(\varphi) \), which solves \( rV(\varphi) = \pi(\varphi) - f - \delta V(\varphi) \); that is,

\[ V(\varphi) = \frac{\pi(\varphi) - f}{r + \delta}, \tag{9} \]

so that the value of the firm is the discounted sum of its instantaneous profits, \( \pi(\varphi) - f \), with the effective discount rate given by the sum of the interest rate and the death rate. Therefore, the average value of producing firms is \( \bar{V} = \int_{\hat{\varphi}}^{\infty} V(\varphi) g(\varphi | \varphi \geq \hat{\varphi}) d\varphi \), which from equations (5), (8), and (9) can be written as

\[ \bar{V} = \frac{f}{r + \delta} \left[ \left( \frac{\hat{\varphi}}{\bar{\varphi}} \right)^{\sigma - 1} - 1 \right]. \tag{10} \]

Financiers fund the entry of each firm before the realization of the firm’s productivity. Thus, in equilibrium, the pre-entry expected value of a firm, \( V_E = \int_{\hat{\varphi}}^{\infty} V(\varphi) g(\varphi | \varphi \geq \hat{\varphi}) d\varphi \), is equal to the sunk entry cost, \( f_E \). Note that \( V_E = [1 - G(\hat{\varphi})] \bar{V} \) and therefore, the free-entry condition is given by

\[ \frac{f[1 - G(\hat{\varphi})]}{r + \delta} \left[ \left( \frac{\hat{\varphi}}{\bar{\varphi}} \right)^{\sigma - 1} - 1 \right] = f_E. \tag{11} \]

Equation (11) determines a unique \( \hat{\varphi} \) for each \( r \), and yields that \( \frac{d\hat{\varphi}}{dr} = -\frac{f_E \hat{\varphi}^{\sigma - 1}}{(\sigma - 1) [1 - G(\hat{\varphi})]} < 0 \): an increase in \( r \) negatively affects the value of firms and hence the value of entry, so that a decline in \( \hat{\varphi} \) (which

\textsuperscript{7}The lump-sum tax could also be paid by households, or by both households and financiers. If households also pay a lump-sum tax, we would need to subtract that amount to the unit wage they receive, but none of the paper’s results would change.

\textsuperscript{8}In section 5.2 I consider the case in which only a fraction of the total capitalization of firms is part of the liquidity available to financiers.
rises firm-level profits) is needed to restore the free-entry condition. Note also that the average value of producing firms can be written more compactly as \( \bar{V} = \frac{f_E}{1 - G(\hat{\phi})} \).

The private provision of liquidity is defined as \( A = NV \). Using (7), (10), and (11), it follows that

\[
A(r) = \frac{\eta f_E}{\sigma \left\{ f \left[ 1 - G(\hat{\phi}(r)) \right] + f_E (r + \delta) \right\}},
\]

where \( dA(r)/dr < 0 \): as the real interest rate increases, the average value of producing firms, \( \bar{V} \), declines and even though the mass of producing firms may increase or decrease (depending on the assumed productivity distribution), the private supply of liquidity shrinks. Moreover, from (11) I obtain that \( \hat{\phi}(-\delta) \rightarrow \infty \), so that \( G(\hat{\phi}(-\delta)) \rightarrow 1 \) and thus \( A(-\delta) \rightarrow \infty \); on the other hand, \( A(\rho) \) is positive and finite.

The aggregate liquidity supply of the economy, \( L_S(r) \), is given by the sum of the private provision of liquidity, \( A(r) \), and the public provision of liquidity, \( B \). As shown below, due to the liquidity services provided by private and public assets, their equilibrium interest rate, \( r \), will be smaller than the rate of time preference, \( \rho \), which is the interest rate on illiquid assets.

### 2.2.2 Demand for Liquidity

Financiers demand liquid assets to be used as collateral in their OTC transactions. This section describes the relationship between the financiers’ holdings of liquid assets and the interest rate. The relationship is straightforward: the higher the interest rate an asset yields, the lower the financier’s cost of holding this asset, and hence the higher the financier’s demand for this asset.

This section follows the OTC-market description of Rocheteau and Rodriguez-Lopez (2014), which is related to the OTC structures of Duffie, Garleanu, and Pedersen (2005) and Lagos and Rocheteau (2009). Importantly, this is not the only way to generate a positive relationship between the demand for liquidity and the interest rate: as long as financiers have a precautionary motive for holding some types of assets, a positive relationship between the demand for these assets and their interest rate will emerge even if financiers meet in a competitive market. I follow the OTC structure with bilateral matching and bargaining because of the predominance of OTC trades in financial transactions.

The financier’s problem can be written as

\[
W(a_0) = \max_{a(t), h(t)} \mathbb{E} \left\{ \int_0^{T_1} e^{-\rho t} h(t) dt + e^{-\rho T_1} Z [a(T_1)] \right\}
\]

subject to

\[
\dot{a} = ra - h - \Upsilon
\]

and \( a(t) \geq 0 \), with \( a(0) = a_0 \). From (13), the financier chooses asset holdings, \( a(t) \), and homogeneous-good consumption, \( h(t) \), that maximize the discounted cumulative consumption up to \( T_1 \)—the random time at which the financier is matched with another financier—plus the present continuation value of
a trading opportunity in the OTC market at $T_1$ with $a(T_1)$ holdings of liquid assets, $Z[a(T_1)]$. The financier’s budget constraint in (14) shows that the financier’s change in asset holdings ($\dot{a}$) should equal the interest on those assets ($r_a$) plus the financier’s production of the homogeneous good ($-h$) net of taxes ($\Upsilon$).

Given the assumption that $T_1$ is exponentially distributed with arrival rate $\nu$ (waiting times of a Poisson process are exponentially distributed), the maximization problem in (13)-(14) can be rewritten as

$$\max_{a(t), h(t)} \int_0^\infty e^{-(\nu+\rho)t} \{h(t) + \nu Z[a(t)]\} dt \quad \text{subject to} \quad \dot{a} = r_a - h - \Upsilon.$$ 

It will be made clear below that the strict concavity of $F(\cdot)$ and $F'(0) \to \infty$ ensure that the constraint $a(t) \geq 0$ never binds. The current-value Hamiltonian is then $H(h, a, \xi) = h + \nu Z(a) + \xi (r_a - h - \Upsilon)$, with state variable $a$, control variable $h$, and current-value co-state variable $\xi$. From the first necessary condition $H_h(h, a, \xi) = 0$, it follows that $\xi = 1$ for all $t$. From the second necessary condition, $H_a(h, a, \xi) = (\nu + \rho)\xi - \dot{\xi}$, and given that $\xi = 1$ and $\dot{\xi} = 0$, it follows that the demand for liquid assets is determined by

$$Z'(a) = 1 + \frac{\rho - r}{\nu}. \tag{15}$$

In (15), $Z'(a)$ is the financier’s benefit from an additional unit of liquid assets, which should be equal to the cost of purchasing the asset (which is 1 because liquid assets are in terms of the numéraire) plus the asset’s expected holding cost until the next OTC match, $(\rho - r)/\nu$ (the average time until the next OTC match is $1/\nu$).

Notice that the solution for $a$ from (15) does not depend on $a_0$, so that after a match is dissolved the amount of assets that the financier holds jumps immediately to its desired level. As discussed by Choi and Rocheteau (2020), who provide a more formal treatment of monetarist models in continuous time with jumps in the state variable, this property of the model is a consequence of the technology available to financiers to produce and consume the numéraire good both in flows and in discrete quantities at some countable instants of time.\(^9\)

When $T_1$ arrives, the financier has an equal chance of being a buyer or seller of financial services, and thus, $Z(a) = [Z^b(a) + Z^s(a)]/2$, where $Z^b$ is the value of being a buyer of financial services and $Z^s$ is the value of being a seller of those services. Once the roles of the financiers are established, the buyer sets the terms of the OTC contract with a take-it-or-leave-it offer to the seller.

The OTC contract, $(y, \alpha)$, includes the buyer’s consumption of financial services, $y$, and the transfer of liquid assets from the buyer to the seller, $\alpha$. If the buyer holds $a^b$ units of liquid assets, the buyer’s problem is

$$\max_{y, \alpha} \{F(y) - \alpha\} \quad \text{subject to} \quad \alpha \geq y \quad \text{and} \quad \alpha \in [0, a^b].$$

\(^9\)See also the working paper version of Rocheteau and Rodriguez-Lopez (2013).
Hence, the contract \((y, \alpha)\) maximizes the buyer’s surplus from trading, \(F(y) - \alpha\), subject to the participation constraint for the seller, \(\alpha \geq y\), and the feasibility condition for the buyer, \(\alpha \in [0, a^b]\).

If \(a^b \geq \hat{y}\), the solution is \(y = \alpha = \hat{y}\), where \(F'(\hat{y}) = 1\); otherwise, \(y = \alpha = a^b\). Intuitively, the buyer’s surplus-maximizing consumption of financial services is \(\hat{y}\), but that outcome only occurs if the buyer has enough liquid assets to transfer to the seller (i.e., if \(a^b \geq \hat{y}\)). If \(a^b < \hat{y}\), the buyer is liquidity constrained and the best she can do is to transfer all of her liquid assets to the seller and get in exchange an equivalent amount of financial services.

The value function for the buyer is \(Z^b(a) = \max_{y \leq a} \{F(y) - y\} + W(a)\), where the first term is the whole surplus of the match (which is equal to \(F(\hat{y}) - \hat{y}\) if \(a \geq \hat{y}\), and is equal to \(F(a) - a\) if \(a < \hat{y}\)), and \(W(a)\) is the financier’s continuation value. The seller’s surplus from the match is zero, and thus, \(Z^s(a) = W(a)\). It follows that

\[
Z(a) = \frac{1}{2}\max_{y \leq a} \{F(y) - y\} + W(a),
\]

which indicates that with probability 1/2 the financier is a buyer, in which case she will transfer up to \(a\) units of liquid assets in exchange for \(y\). Therefore, the financier’s benefit from an additional unit of liquid assets at the time of the match (but before knowing her buyer or seller role) is

\[
Z'(a) = \begin{cases} 
W'(a) & \text{if } a \geq \hat{y} \\
\frac{F'(a)-1}{2} + W'(a) & \text{if } a < \hat{y}.
\end{cases}
\]

(17)

Given that \(F'(y) > 0\), \(F''(y) < 0\), and \(F'(\hat{y}) = 1\), it follows that \(F'(a) - 1 > 0\) if \(a < \hat{y}\), and is exactly zero if \(a = \hat{y}\). Using these results along with the fact that \(W'(a) = \xi = 1\), (17) can be rewritten as

\[
Z'(a) = \frac{[F'(a) - 1]^+}{2} + 1,
\]

(18)

where \([x]^+ = \max\{x, 0\}\).

From (15) and (18) it follows that \((\rho - r)/\gamma = [F'(a) - 1]^+, \) where \(\gamma = \nu/2\) is the rate at which a financier is matched as a buyer. Given that \(F''(0) \to \infty\) and \(F''(\cdot) < 0\), the right-hand side of the previous equation strictly decreases from \(\infty\) to 0 if \(a \in [0, \hat{y})\), and is zero if \(a \geq \hat{y}\). The left-hand side is finite and non-negative, and thus, the solution for \(a\) is always greater than zero. In terms of the financier’s consumption of financial services, \(y = \min\{a, \hat{y}\}\), it solves

\[
F'(y) = 1 + \frac{\rho - r}{\gamma}
\]

(19)

for \(r \leq \rho\). If \(r < \rho\), so that \(F'(y) > 1\) and \(y = a < \hat{y}\), the financier’s demand for liquid assets is \(a^d = F^{-1}[1 + (\rho - r)/\gamma]\). If \(r = \rho\), so that the cost of holding liquid assets is zero and \(y = \hat{y}\), the financier’s demand for liquid assets takes any value in the range \([\hat{y}, \infty)\).
There is a unit measure of financiers, which implies that the aggregate demand for liquid assets, $L_D(r)$, is identical to the financier’s individual demand, and therefore

$$L_D(r) = A'(r) - 1 + \rho - r \gamma$$

if $r < \rho$,

$$[\hat{y}, \infty)$$

if $r = \rho$.

(20)

If $r < \rho$, there is a positive relationship between $L_D(r)$ and $r$: an increase in the interest rate on liquid assets reduces their holding cost, $(\rho - r)\gamma$, which drives financiers to hold more of them. When $r = \rho$, liquidity is costless to hold and hence financiers will hold any amount in the range $[\hat{y}, \infty)$.

2.2.3 Equilibrium

The equilibrium in the market for liquidity occurs at the intersection of supply and demand:

$$L_S(r) \equiv A(r) + B = L_D(r),$$

(21)

where $A(r)$ is given by (12) and $L_D(r)$ is given by (20). Figure 1 shows a graphical representation of the equilibrium in the market for liquid assets. The supply of private assets, $A(r)$, is downward sloping, with its lowest value being $A(\rho)$ and tending to infinity when $r$ approaches $-\delta$ from the right. The aggregate liquidity supply, $L_S(r)$, adds $B$ to $A(r)$, and hence it is simply a right-shifted version of $A(r)$. The demand for liquidity, $L_D(r)$, is upward sloping as long as $r < \rho$, and it becomes horizontal at $r = \rho$. The intersection of supply and demand gives a unique equilibrium, $(L^e, r^e)$. The formal definition of a steady-state equilibrium follows.

**Definition.** A steady-state equilibrium is a triple $(\hat{\varphi}, y, r)$ that solves (11), (19), and (21).
The steady-state equilibrium is unique: there is unique $r$ that clears the market for liquidity, $\hat{\varphi}$ is uniquely determined from (11), and $y$ is uniquely determined from (19). The equilibrium shows key relationships between the market for liquid assets and the real economy. In Figure 1, $A(\rho)$ denotes the market capitalization of firms that would prevail in the absence of liquidity services of private assets. I refer to $A(\rho)$ as the “fundamental-value” capitalization. Due to the liquidity services that private assets provide to the financial sector, the equilibrium total market capitalization of differentiated-good firms is $A^e > A(\rho)$. Moreover, $\hat{\varphi}(r^e) > \hat{\varphi}(\rho)$ (recall that $d\hat{\varphi}/dr < 0$), which implies from (6) and (8) that when compared to the fundamental-value outcome, the aggregate price, $P$, is lower and the average productivity, $\bar{\varphi}$, is higher when private assets provide liquidity services.

Note that if $B = 0$, the equilibrium in the market for liquidity would be given by the intersection of $A(r)$ and $L_D(r)$, which implies a lower equilibrium interest rate and a higher equilibrium level of private liquidity. As in Holmström and Tirole (2011) and Rocheteau and Rodriguez-Lopez (2014), this result highlights the crowding-out effect that public liquidity, $B$, has on private liquidity, $A$.

Note that if the government is interested in maximizing the surplus in the financial sector by increasing the amount of public liquidity (so that $\hat{y}$ can be reached), it would push the differentiated-good sector towards the fundamental-value outcome.

If the supply of liquidity is abundant, so that the equilibrium occurs in the horizontal part of the demand for liquidity, the interest rate equals the discount rate and hence the price of liquidity is zero (i.e., a liquidity premium does not exist)—as previously discussed by Holmström and Tirole (1998) and Rocheteau (2011), liquidity premia only emerge if liquid assets are in scarce supply.

3 Liquidity in the Open Economy

The closed-economy model highlights the benefits of the market for liquidity on the real economy. But how do differences across countries in their abilities to generate liquid assets affect the international allocation of economic activity? This section extends the previous model to a two-country setting that allows for heterogeneity in liquidity properties across different countries’ assets.

There are two countries, Home and Foreign, and two production sectors in each country: a homogeneous-good sector and a differentiated-good sector. The homogeneous good is traded costlessly and is produced under perfect competition, while each variety of the differentiated good is potentially tradable and is produced under monopolistic competition. Each country is inhabited by a unit measure of households, with each household providing a unit of labor per unit of time. Foreign variables are denoted with a star (*). As in the models of Chaney (2008), Helpman and Itskhoki

\[10\] In support to the crowding-out mechanism, Krishnamurthy and Vissing-Jorgensen (2015) find a strong inverse relationship between the supply of U.S. Treasuries and the amount of private assets—similar evidence is found by Gorton, Lewellen, and Metrick (2012).
(2010), and Helpman, Melitz, and Yeaple (2004), the assumption of a costlessly traded homogeneous good ensures that trade is balanced: any Home surplus in the trade of differentiated goods is met with an identical Home deficit in the trade of the homogeneous good.

There is an international OTC financial market in which Home and Foreign financiers trade financial services. There is a unit measure of financiers in the world. To secure their transactions, financiers may use as collateral four categories of assets: Home and Foreign private assets, and Home and Foreign government bonds. However, there is heterogeneity in the liquidity properties across Home and Foreign assets. I assume that Home and Foreign are identical but for the liquidity properties of their assets.

I start this section by describing the conventional Melitz’s two-country structure, then I discuss the international market for liquid assets and define the equilibrium.

### 3.1 Preferences, Demand, and Production

The description of preferences and demand for Home is similar to section 2.1.1. Analogous expressions hold for Foreign. Hence, the total expenditure on differentiated goods in Foreign is $\eta$, and the Foreign’s market demand for variety $\omega$ is $q^c(\omega, t) = \left[ p^*(\omega, t)^{-\sigma} \right] \eta$, where $p^*(\omega)$ is the Foreign price of variety $\omega$, and $P^*(t) = \left[ \int_{\omega \in \Omega^*} p^*(\omega, t)^{1-\sigma} d\omega \right]^{1-\sigma}$.

Home and Foreign have identical production structures. In each country, producers in the differentiated good sector are heterogeneous in productivity. After entry, each Home and Foreign firm draws its productivity from the same cumulative distribution function, $G(\varphi)$. Each firm then decides whether or not to produce for the domestic and export markets. The decision to produce or not for a market is determined by the ability of the firm to cover the fixed cost of selling in that market. Although there can be imbalances from trading differentiated goods, costless trade in the homogeneous good ensures overall trade balance.

As before, the production function of a Home firm with productivity $\varphi$ is given by $q(\varphi, t) = \varphi L(t)$, where $L(t)$ denotes Home labor. Analogously, the production function of a Foreign firm with productivity $\varphi$ is given by $q^*(\varphi, t) = \varphi L^*(t)$, where $L^*(t)$ denotes Foreign labor. Hence, the marginal cost of a Home firm with productivity $\varphi$ from selling in the Home market is $\frac{1}{\varphi}$. If the Home firm decides to export its finished good, its marginal cost from selling in the Foreign market is $\frac{\tau}{\varphi}$, where $\tau > 1$ accounts for an iceberg exporting cost—the Home firm must ship $\tau$ units of the good for one unit to reach the Foreign market.

Assuming market segmentation and given CES preferences, the prices that a Home firm with productivity $\varphi$ sets in the domestic ($D$) and export ($X$) markets are given by $p_D(\varphi) = \left( \frac{\sigma}{\sigma-\tau} \right)^{\frac{1}{\sigma}}$ and $p_X(\varphi) = \left( \frac{\sigma}{\sigma-\tau} \right)^{\frac{\tau}{\sigma}}$, respectively. Using these pricing equations and the market demand functions, this
firm’s gross profit functions—before deducting fixed costs—from selling in each market are

\[
\pi_D(\varphi) = \frac{1}{\sigma} \left[ \frac{P}{p_D(\varphi)} \right]^{\sigma-1} \eta \quad \text{and} \quad \pi_X(\varphi) = \frac{1}{\sigma} \left[ \frac{P^*}{p_X(\varphi)} \right]^{\sigma-1} \eta,
\]

which are increasing in productivity (i.e., \(\pi'_D(\varphi) > 0\) and \(\pi'_X(\varphi) > 0\)). Similarly, the marginal cost for a Foreign firm with productivity \(\varphi\) is \(\frac{1}{\sigma} \varphi \) from selling domestically, and \(\tau \frac{\varphi}{\sigma} \) from selling in the Home market, so that the prices set by a Foreign firm with productivity \(\varphi\) are \(p^*_D(\varphi) = \left( \frac{\sigma}{\sigma-1} \right) \frac{\varphi}{\sigma} \) in its domestic market, and \(p^*_X(\varphi) = \left( \frac{\sigma}{\sigma-1} \right) \frac{\tau \varphi}{\sigma} \) in its export market. This firm’s gross profit functions from selling in each market are then

\[
\pi^*_D(\varphi) = \frac{1}{\sigma} \left[ \frac{P^*}{p^*_D(\varphi)} \right]^{\sigma-1} \eta \quad \text{and} \quad \pi^*_X(\varphi) = \frac{1}{\sigma} \left[ \frac{P^*}{p^*_X(\varphi)} \right]^{\sigma-1} \eta.
\]

### 3.2 Cutoff Productivity Levels and the Composition of Firms

There are fixed costs of selling in each market. These fixed costs along with the CES demand system imply the existence of cutoff productivity levels that determine the tradability of each differentiated good in each market. For Home firms there are two cutoff productivity levels: one for selling in the domestic market, \(\hat{\varphi}_D\), and one for selling in the export market, \(\hat{\varphi}_X\). Then, for example, if a Home firm’s productivity is between \(\hat{\varphi}_D\) and \(\hat{\varphi}_X\), the firm produces for the domestic market (as it will be able to cover the fixed cost of selling domestically), but not for the export market (as it will not be able to cover the fixed cost of exporting). Similarly, \(\hat{\varphi}^*_D\) and \(\hat{\varphi}^*_X\) denote the cutoff productivity levels for Foreign firms.

As before, all fixed costs are in terms of the homogeneous good. To simplify notation I assume that the fixed cost of selling in each market is equal to \(f\) for both Home and Foreign firms. Therefore, the cutoff productivity levels satisfy the zero-cutoff-profit (ZCP) conditions \(\pi_i(\hat{\varphi}_i) = f\) and \(\pi^*_i(\hat{\varphi}^*_i) = f\), for \(i \in \{D, X\}\). Using the gross profit functions from the previous section, the ZCP conditions can be written as

\[
\frac{1}{\sigma} \left[ \frac{P}{p_D(\hat{\varphi}_D)} \right]^{\sigma-1} \eta = f, \quad \frac{1}{\sigma} \left[ \frac{P^*}{p_X(\hat{\varphi}_X)} \right]^{\sigma-1} \eta = f, \quad \frac{1}{\sigma} \left[ \frac{P^*}{p_D^*(\hat{\varphi}^*_D)} \right]^{\sigma-1} \eta = f, \quad \text{and} \quad \frac{1}{\sigma} \left[ \frac{P^*}{p_X^*(\hat{\varphi}^*_X)} \right]^{\sigma-1} \eta = f.
\]

Combining the first and fourth conditions, the second and third conditions, and given the pricing equations from the previous section, it follows that

\[
\hat{\varphi}^*_X = \tau \hat{\varphi}_D, \quad (22)
\]

\[
\hat{\varphi}_X = \tau \hat{\varphi}^*_D. \quad (23)
\]

These equations indicate the relationship between the cutoff productivity levels for firms selling in the same market. Moreover, using the ZCP conditions we substitute out \(P\) and \(P^*\) in the gross profit
functions to rewrite them as

\[ \pi_i(\varphi) = \left( \frac{\varphi}{\bar{\varphi}_i} \right)^{\sigma-1} f, \quad (24) \]

\[ \pi_i^*(\varphi) = \left( \frac{\varphi}{\bar{\varphi}_i^*} \right)^{\sigma-1} f, \quad (25) \]

for \( i \in \{D, X\} \).

Let \( N \) and \( N^* \) denote, respectively, the masses of sellers of differentiated goods in Home and Foreign. In Home, \( N \) is composed of a mass of \( N_D \) Home firms and a mass of \( N_X^* \) Foreign firms, so that \( N = N_D + N_X^* \). Similarly, \( N^* = N_D^* + N_X^* \), where \( N_D^* \) is the mass of Foreign producers selling domestically, and \( N_X^* \) is the mass of Home exporters. As before, firms in each country are subject to a random death shock arriving at Poisson rate \( \delta > 0 \). In steady state, the firms that die are exactly replaced by successful entrants so that

\[ \delta N_i = [1 - G(\bar{\varphi}_i)] N_E, \]

\[ \delta N_i^* = [1 - G(\bar{\varphi}_i^*)] N_E^*, \]

where \( N_E \) and \( N_E^* \) denote the masses of Home and Foreign entrants per unit of time, \( G(\varphi) \) is the cumulative distribution function from which Home and Foreign firms draw their productivities after entry, and \( i \in \{D, X\} \). Thus, to obtain expressions for \( N_D, N_X, N_D^*, \) and \( N_X^* \) in terms of the cutoff productivity levels, I need to derive first the expressions for \( N_E \) and \( N_E^* \).

To obtain \( N_E \) and \( N_E^* \), note first that the aggregate price equations of Home and Foreign are given by

\[ P = \left[ N_D \bar{p}^{1-\sigma}_D + N_X^* \bar{p}^{1-\sigma}_X \right]^{\frac{1}{1-\sigma}}, \quad (26) \]

\[ P^* = \left[ N_D^* \bar{p}^{1-\sigma}_D + N_X^* \bar{p}^{1-\sigma}_X \right]^{\frac{1}{1-\sigma}}, \quad (27) \]

where \( \bar{p}_i = p_i(\bar{\varphi}_i) \) is the average price in market \( i \) of Home-produced differentiated goods, and \( \bar{p}_i^* = p_i^*(\bar{\varphi}_i^*) \) is the average price in market \( i \) of Foreign-produced goods, with average productivities given by

\[ \bar{\varphi}_i = \left[ \int_{\bar{\varphi}_i}^{\infty} \varphi^{\sigma-1} g(\varphi|\varphi \geq \bar{\varphi}_i) d\varphi \right]^{\frac{1}{\sigma-1}} \quad \text{and} \quad \bar{\varphi}_i^* = \left[ \int_{\bar{\varphi}_i^*}^{\infty} \varphi^{\sigma-1} g(\varphi|\varphi \geq \bar{\varphi}_i^*) d\varphi \right]^{\frac{1}{\sigma-1}}, \]

for \( i \in \{D, X\} \). Substituting the expressions for \( \bar{p}_i, \bar{p}_i^*, N_i, \) and \( N_i^* \), for \( i \in \{D, X\} \), into equations (26) and (27), and using the ZCP conditions to substitute for \( P \) and \( P^* \) along with equations (24)-(25), I obtain the system of equations that solves for \( N_E \) and \( N_E^* \) as

\[ N_E = \frac{\delta \eta}{\sigma} \left[ \frac{\Pi^*_D - \Pi^*_X}{\Pi_D \Pi_D^* - \Pi_X^* \Pi_X} \right], \quad (28) \]

\[ N_E^* = \frac{\delta \eta}{\sigma} \left[ \frac{\Pi^*_D - \Pi^*_X}{\Pi_D \Pi_D^* - \Pi_X^* \Pi_X} \right], \quad (29) \]
where

\[ \Pi_i = \int_{\hat{\varphi}_i}^{\infty} \pi_i(\varphi)g(\varphi)d\varphi \quad \text{and} \quad \Pi_i^* = \int_{\hat{\varphi}_i^*}^{\infty} \pi_i^*(\varphi)g(\varphi)d\varphi \]

for \( i \in \{D, X\} \). Notice that \( \Pi_i \) is the unconditional expected gross profit for a Home potential entrant from selling in market \( i \), and \( \Pi_i^* \) is the unconditional expected gross profit for a Foreign potential entrant from selling in market \( i \).

As is usual in Melitz-type heterogeneous-firm models, I assume that exporting costs are large enough so that \( \hat{\varphi}_D < \hat{\varphi}_X \) and \( \hat{\varphi}_D^* < \hat{\varphi}_X^* \); that is, exporting firms always produce for their domestic market. This assumption implies that \( \Pi_D > \Pi_X \) and \( \Pi_D^* > \Pi_X^* \), which guarantees an interior solution for \( N_E \) and \( N_E^* \).

### 3.3 Free-Entry Conditions, Average Productivity, and Household Welfare

As venture capitalists, financiers fund the entry of differentiated-good firms in both countries in exchange for claims on firms’ profits. These private assets serve as liquidity in financial-sector transactions, but Home and Foreign assets may differ in their liquidity properties. As a consequence, the interest rate on Home assets, \( r \), and the interest rate on Foreign assets, \( r^* \), may be different. Both \( r \) and \( r^* \) are bounded above by the interest rate on illiquid assets, \( \rho \).

Firms in both countries die at rate \( \delta \). Letting \( \mathbb{1}\{\varphi \geq \hat{\varphi}_i\} \) denote an indicator function taking the value of 1 if \( \varphi \geq \hat{\varphi}_i \) (and zero otherwise), for \( i \in \{D, X\} \), the value of a Home firm with productivity \( \varphi \) is given by

\[
V(\varphi) \equiv \left[ \pi_D(\varphi) - f \right] \mathbb{1}\{\varphi \geq \hat{\varphi}_D\} + \left[ \pi_X(\varphi) - f \right] \mathbb{1}\{\varphi \geq \hat{\varphi}_X\},
\]

where \( \pi_D(\varphi) - f \) is the net profit flow from domestic sales, and \( \pi_X(\varphi) - f \) is the net profit flow from exporting. As a firm knows its productivity only after entry, the pre-entry expected value of a firm for Home potential entrants is \( V_E \equiv \int_{\hat{\varphi}_D}^{\infty} V(\varphi)g(\varphi)d\varphi \). With similar expressions for Foreign firms, and assuming identical entry costs of \( f_E \) for Home and Foreign entrants, the free-entry conditions for differentiated-good firms at Home and Foreign are, respectively, \( V_E = f_E \) and \( V_E^* = f_E^* \).

To gain tractability in our two-country version of the model, I further assume a Pareto distribution of productivity, so that \( G(\varphi) = 1 - \left( \frac{\varphi_{\min}}{\bar{\varphi}} \right)^k \) and \( g(\varphi) = \frac{k\varphi^{k-1}/\varphi_{\min}^k}{\bar{\varphi}^{k+1}} \), where \( k > \sigma - 1 \) is a parameter of productivity dispersion—higher \( k \) means lower heterogeneity. \(^{12}\) Under a Pareto distribution, the free-

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\(^{11}\)To prove that \( \Pi_D > \Pi_X \) when \( \hat{\varphi}_D < \hat{\varphi}_X \), and that \( \Pi_D^* > \Pi_X^* \) when \( \hat{\varphi}_D^* < \hat{\varphi}_X^* \), I simply use the result that \( \int_{a}^{\infty} \varphi^{-1}g(\varphi)d\varphi > \int_{b}^{\infty} \varphi^{-1}g(\varphi)d\varphi \) if \( a \) and \( b \) are positive and \( a < b \).

\(^{12}\)Since Chaney (2008), the assumption of a Pareto distribution for productivity is extensively used in trade models with heterogeneous firms.
entry conditions can be conveniently written as
\[ \frac{\Gamma}{r + \delta} \left( \frac{1}{\hat{\varphi}_D^k} + \frac{1}{\hat{\varphi}_X^k} \right) = f_E, \]  
(30)
\[ \frac{\Gamma}{r^* + \delta} \left( \frac{1}{\hat{\varphi}_D^k} + \frac{1}{\hat{\varphi}_X^k} \right) = f_E, \]  
(31)
where \( \Gamma \equiv \frac{\left(\sigma - 1\right) \varphi_{m}^{k}}{k - \sigma + 1} f. \) Therefore, from equations (22), (23), (30), and (31), I solve for the cutoff productivity levels in terms of Home and Foreign interest rates as
\[ \hat{\varphi}_D = \frac{1}{\Gamma} \left( \frac{\Gamma}{f_E} \left[ \frac{\tau^{2k} - 1}{\tau^{k}(r + \delta) - (r^* + \delta)} \right] \right)^{\frac{1}{\pi}}, \]
\[ \hat{\varphi}_D^* = \frac{1}{\Gamma} \left( \frac{\Gamma}{f_E} \left[ \frac{\tau^{2k} - 1}{\tau^{k}(r + \delta) - (r + \delta)} \right] \right)^{\frac{1}{\pi}}, \]
\[ \hat{\varphi}_X = \frac{\Gamma}{f_E} \left( \frac{\tau^{2k} - 1}{\tau^{k}(r^* + \delta) - (r + \delta)} \right), \]
\[ \hat{\varphi}_X^* = \frac{\Gamma}{f_E} \left( \frac{\tau^{2k} - 1}{\tau^{k}(r^* + \delta) - (r^* + \delta)} \right)^{\frac{1}{\pi}}. \]
(32)
The usual assumption that exporting firms always sell for their domestic markets (\( \hat{\varphi}_D < \hat{\varphi}_X \) and \( \hat{\varphi}_D^* < \hat{\varphi}_X^* \)) requires that \( \frac{2k}{\tau^{k+1}k + 1} < \frac{r + \delta}{r^* + \delta} < \frac{\tau^{2k} - 1}{2\tau^{k+1}}, \) which is a sufficient condition for an interior solution.

From the equations in (32), note that \( \frac{\partial \hat{\varphi}_D}{\partial r} < 0, \frac{\partial \hat{\varphi}_X}{\partial r} > 0, \frac{\partial \hat{\varphi}_D^*}{\partial r^*} > 0, \) and \( \frac{\partial \hat{\varphi}_X^*}{\partial r^*} < 0. \) Keeping everything else constant, the effect of a decline in \( r \) at Home is similar to the effect of a reduction in the death rate (\( \delta \)) in the conventional Melitz model: the value of Home firms increases, causing an increase in entry at Home and thus creating a tougher competitive environment in the Home market, driving up the survival cutoff \( \hat{\varphi}_D. \) As entry is propelled at Home, Foreign firms’ profit opportunities in the Home market decline, reducing entry at Foreign and thus making the Foreign market less competitive. As a consequence, the survival cutoff for Foreign firms, \( \hat{\varphi}_D^*, \) declines and less productive Home firms start exporting (\( \hat{\varphi}_X \) falls). There is a positive relationship between average productivity levels and cutoff productivity levels, and in particular, they are directly proportional under a Pareto distribution for productivity: \( \varphi_i = \left[ \frac{k}{k - \sigma + 1} \right]^{\frac{1}{\tau}} \tilde{\varphi}_i \) for \( i \in \{D, X\}. \) Therefore, a reduction in \( r \) increases Home average productivity (\( \hat{\varphi}_D \))—reflecting the tougher competitive environment in the Home market—and reduces Foreign average productivity (\( \hat{\varphi}_D^* \))—reflecting the softer competitive environment in the Foreign market. The opposite happens after a reduction in \( r^*. \)

The analysis of household welfare is straightforward. From the first ZCP condition in section 3.2, note that
\[ P = \left[ \frac{\eta}{\sigma f} \right]^{\frac{1}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} \right) \frac{1}{\hat{\varphi}_D^k}, \]
so that \( P \) and \( \hat{\varphi}_D \) are inversely proportional. Given the previous expression and equation (3), it follows that Home household welfare, \( W \), is directly proportional to \( \hat{\varphi}_D^0. \) Thus, an advantage of this model is that for a change in a parameter of interest (other than \( k, \sigma, f, \) and \( \eta \)), it is sufficient to look at the response of the domestic cutoff productivity level, \( \hat{\varphi}_D, \) to infer changes in Home average productivity in the differentiated-good sector, \( \hat{\varphi}_D, \) in the Home aggregate price level for differentiated goods, \( P, \) and in Home household welfare, \( W. \) The same holds for \( \hat{\varphi}_D^*, P^*, \) and \( W^* \) with respect to \( \hat{\varphi}_D^*. \)
3.4 Output Value and Trade Flows in the Differentiated-Good Sector

Each country spends $\eta$ on differentiated goods; hence, the world’s output value of the differentiated-good sector is $2\eta$, which is split between Home and Foreign producers. For a Home firm with productivity $\varphi$, its output value from selling in market $i$ is $v_i(\varphi) = p_i(\varphi)q_i(\varphi)$ if $\varphi \geq \bar{\varphi}_i$, for $i \in \{D, X\}$. Therefore, the total output value of Home firms from selling in market $i$ is given by $Y_i = N_i\bar{v}_i$, where $\bar{v}_i = \int_{\bar{\varphi}_i}^{\varphi} v(\varphi)g(\varphi|\varphi \geq \bar{\varphi}_i)d\varphi$ is the average output value of Home firms selling in market $i$. Given CES preferences, the gross-profit function of a firm with productivity $\varphi$ from selling in market $i$, $\pi_i(\varphi)$, is equivalent to $\frac{p_i(\varphi)q_i(\varphi)}{\sigma}$, so that $v_i(\varphi)$ can be rewritten as $v_i(\varphi) = \sigma \pi_i(\varphi)$. Using $\delta N_i = [1 - G(\bar{\varphi}_i)] N_E$ from section 3.2, it follows that $Y_i = N_i\sigma \pi_i(\bar{\varphi}_i) = \frac{\sigma}{\delta} \bar{\Pi}_i N_E$.

The total output value of Home firms is then given by $Y = Y_D + Y_X = \frac{\sigma}{\delta} (\bar{\Pi}_D + \bar{\Pi}_X) N_E$, which collapses to

$$Y = \eta + \left(\frac{\tau^k(r^* - r)(r + r^* + 2\delta)}{[\tau^k(r^* + \delta) - (r + \delta)][\tau^k(r + \delta) - (r^* + \delta)]}\right)^{\frac{1}{\tau}}$$

after using (28), (32), (33), and the Pareto distribution for productivity. The second term in equation (34), denoted by $T$, is the trade balance for Home in the differentiated-good sector. Note that $T$, which can also be calculated as $Y_X - Y_X^* = \frac{\sigma}{\delta}(\bar{\Pi}_X N_E - \bar{\Pi}_X^* N_E^*)$, is positive if $r^* > r$, is negative if $r^* < r$, and is zero if $r^* = r$.\(^\text{13}\) From the sufficient condition for an interior solution described in the previous section, if $\frac{r^* + \delta}{r + \delta}$ approaches its upper limit $(\frac{2k+1}{2r+1})$, then $T \to \eta$ and thus $Y \to 2\eta$, so that Home firms are the only producers in the differentiated-good sector. Conversely, if $\frac{r^* + \delta}{r + \delta}$ approaches its lower limit $(\frac{2k}{2r+1})$, then $T \to -\eta$ and thus $Y \to 0$, so that Home does not produce differentiated goods. As mentioned before, costless trade of the homogeneous good ensures balanced trade and hence, if Home has a trade surplus of $T > 0$ in the differentiated-good sector, it will also have a trade deficit of $T$ in the homogeneous-good sector.

3.5 The International Market for Liquidity

Liquidity differences across countries’ assets arise from different acceptability properties of Home and Foreign assets in OTC financial matches. In particular, in a fraction $\mu$ of OTC matches only Home assets are acceptable as collateral, and in the remaining $\mu^* = 1 - \mu$ fraction of matches both Home and Foreign assets are acceptable. Analogously, Foreign assets are acceptable in a fraction $\mu^*$ of OTC matches, and Home assets are acceptable in all matches.

Thus, differences between Home and Foreign will only span from the value that $\mu$ takes. Focusing only on $\mu$ allows us to clearly elucidate the strong effects that liquidity differences across countries.

\(^{13}\) Analogous expressions hold for Foreign, with $Y_i^*$ denoting Foreign firms’ output value from selling in market $i$, for $i \in \{D, X\}$, and $Y^* = Y_D^* + Y_X^* = 2\eta - \bar{Y} = \eta - T$ denoting the total output value of Foreign firms in the differentiated-good sector.
can have on the international allocation of economic activity. There are two extreme cases: on the one hand, if \( \mu = 0 \) all Home and Foreign assets are acceptable in all OTC matches and thus there are no liquidity differences across countries; on the other hand, if \( \mu = 1 \) Home assets are acceptable in all OTC matches, but Foreign assets are totally illiquid (\( \mu^* = 0 \) and thus Foreign assets are never accepted in OTC transactions). Thus, as \( \mu \) rises the relative liquidity differences between Home and Foreign assets become larger in favor of Home.

3.5.1 Supply of Home and Foreign Liquidity

Financiers use claims on Home and Foreign firms’ profits as private liquidity (i.e., as collateral in their OTC financial transactions). Similar to the closed-economy case, the total supply of Home private liquidity, \( A \), and the total supply of Foreign private liquidity, \( A^* \), are equal to the total market capitalization of firms in each country.

The total market capitalization of Home firms is given by \( A = N_D \bar{V} \), where \( N_D = [1 - G(\hat{\varphi}_D)] \frac{N_E}{\delta} \) and \( \bar{V} \equiv \int_{\hat{\varphi}_D}^{\infty} V(\varphi)g(\varphi|\varphi \geq \hat{\varphi}_D)d\varphi \) is the average value of Home firms. From the free-entry condition \( (\int_{\hat{\varphi}_D}^{\infty} V(\varphi)g(\varphi)d\varphi = f_E) \) notice that \( \bar{V} \) is equivalent to \( \frac{f_E}{1-G(\hat{\varphi}_D)} \), and therefore \( A = \frac{N_E f_E}{\delta} \). Analogously, \( A^* = \frac{N^*_E f_E}{\delta} \). Hence, the total market capitalization of firms in each country is directly proportional to the mass of entrants. Using (28), (29), the cutoff levels in (32) and (33), and given the Pareto distribution for productivity, it follows that the supplies of Home and Foreign private liquidity in terms of Home and Foreign interest rates, \( r \) and \( r^* \), are given by

\[
A = \left( \frac{\sigma - 1}{\sigma k} \right) \frac{\tau^k}{\tau^k(r + \delta) - (r^* + \delta)} - \frac{1}{\tau^k(r + \delta) - (r + \delta)}, \tag{35}
\]

\[
A^* = \left( \frac{\sigma - 1}{\sigma k} \right) \frac{\tau^k}{\tau^k(r^* + \delta) - (r^* + \delta)} - \frac{1}{\tau^k(r^* + \delta) - (r^* + \delta)}. \tag{36}
\]

Equations (35) and (36) yield that \( \frac{\partial A}{\partial r} < 0, \frac{\partial A}{\partial r^*} > 0, \frac{\partial A^*}{\partial r} > 0, \) and \( \frac{\partial A^*}{\partial r^*} < 0 \), so that a reduction in \( r \) increases the supply of Home private liquidity (\( A \)) at the expense of Foreign private liquidity (\( A^* \)), while the opposite happens after a reduction in \( r^* \).

In addition to private liquidity, financiers may use Home and Foreign government bonds—public liquidity—to meet their collateral needs. I assume a fixed supply of Home government bonds, \( B \), and a fixed supply of Foreign government bonds, \( B^* \).

3.5.2 Demand for Home and Foreign Liquid Assets

Let \( a \) and \( a^* \) denote the financier’s holdings of Home and Foreign private assets, and let \( b \) and \( b^* \) denote the financier’s holdings of Home and Foreign government bonds. There are no liquidity differences between public and private assets of the same country, and thus, the interest rates of Home and Foreign
government bonds are also $r$ and $r^*$.

Hence, the budget constraint of a Home financier is

$$\dot{a} + \dot{a}^* + \dot{b} + \dot{b}^* = ra + r^*a^* + rb + r^*b^* - h - \Upsilon.$$  

The left-hand side presents the change in the financier’s wealth, which is given by the financier’s total investment in private assets and government bonds from both Home and Foreign. The right-hand side shows the interest payments on the financier’s portfolio net of homogeneous-good consumption ($h$) and taxes ($\Upsilon$). The budget constraint of a Foreign financier is identical to a Home financier’s budget constraint, with the exception of the last term, which changes to $\Upsilon^*$ (taxes in Foreign).

Following similar steps to those in section 2.2.2 to obtain (16), the continuation value of a financier upon being matched (but before realizing its buyer or seller role), $Z(a, a^*, b, b^*)$, is given by

$$Z(a, a^*, b, b^*) = \mu \max_{y_1 \leq a + b} \{ F(y_1) - y_1 \} + \mu^* \max_{y_2 \leq a + a^* + b + b^*} \{ F(y_2) - y_2 \} + W(a, a^*, b, b^*),$$  

This expression shows that with probability 1/2 the financier is the buyer in the OTC match, in which case she can make a take-it-or-leave-it offer to the seller in order to maximize her surplus, $F(y_j) - y_j$, where subscript $j \in \{1, 2\}$ indicates the type of OTC match: in a type-1 meeting only Home assets are acceptable as collateral, while in a type-2 meeting both Home and Foreign assets are acceptable. With probability $\mu$ the financier is in a type-1 meeting so that she can transfer up to $a + b$ to purchase $y_1$ financial services. On the other hand, with probability $\mu^* = 1 - \mu$, the financier is in a type-2 meeting and thus, she can transfer up to $a + a^* + b + b^*$ in exchange for $y_2$ financial services.

Similar to the derivation of equation (19) in the closed-economy model, the financier’s optimal portfolio solves

$$\frac{\rho - r^*}{\gamma} = \mu^* \left[ F'(y_2) - 1 \right] \quad (37)$$  

$$\frac{\rho - r}{\gamma} = \mu^* \left[ F'(y_2) - 1 \right] + \mu \left[ F'(y_1) - 1 \right]. \quad (38)$$  

Equations (37) and (38) define the optimal choice of each type of asset. In (37), the left-hand side shows the financier’s holding cost of a public or private Foreign asset, while the right-hand side shows the expected marginal surplus from holding an additional unit of public or private Foreign assets—Foreign assets can only be used in a fraction $\mu^*$ of all matches, in which case the marginal surplus of the financier is $F'(y_2) - 1$. In (38), the left-hand side shows the holding cost of a Home asset, while the right-hand side is its marginal surplus from its use in all matches in the financial market.

Similar to the closed-economy case, the quantity of financial services traded in an OTC match is the minimum between the value of the buyer’s liquidity in that match and the surplus-maximizing quantity, $\hat{y}$. Given that financiers hold identical portfolios and that there is a unit measure of financiers
in the world, market clearing implies that \( a = A, a^* = A^*, b = B, \) and \( b = B^* \). Thus,

\[
y_2 = \min \{ A + A^* + B + B^*, \hat{y} \},
\]
\[
y_1 = \min \{ A + B, \hat{y} \}.
\]

(39) \hspace{2cm} (40)

Note from (37)-(40) that we can have different scenarios. Suppose, for example, that liquidity is scarce in matches that only accept Home assets, but is abundant in matches that also accept Foreign assets; i.e., \( A + B < \hat{y} \) but \( A + A^* + B + B^* > \hat{y} \). Hence, \( y_2 = \hat{y} \) and \( y_1 = A + B \), so that \( F'(y_2) - 1 = 0 \), but \( F'(y_1) - 1 > 0 \). Therefore, it follows from (37)-(38) that Foreign assets pay the maximum interest rate, \( r^* = \rho \), while Home assets yield an interest rate of \( r < \rho \).

Alternatively, combining (37), (38), (39), and (40), Foreign and Home interest rates can be written as

\[
r^* = \rho - \mu^* \gamma \left[ F'(A + A^* + B + B^*) - 1 \right]^+, \]
\[
r = \rho - \mu^* \gamma \left[ F'(A + A^* + B + B^*) - 1 \right]^+ - \mu \gamma \left[ F'(A + B) - 1 \right]^+,
\]

(41) \hspace{2cm} (42)

where \( [x]^+ = \max\{x, 0\} \). Note that Foreign assets dominate Home assets in their interest rate, \( r^* > r \), if and only if \( \mu > 0 \) and \( A + B < \hat{y} \). If liquidity is scarce in all matches (\( A + A^* + B + B^* < \hat{y} \)) and \( \mu > 0 \), it is the case that \( \rho > r^* > r \). Thus, more liquid assets yield a lower interest rate.

3.5.3 Steady-State Equilibrium in the Two-Country Model

The steady-state equilibrium solves for the amounts of Home and Foreign private liquidity \((A, A^*)\) and Home and Foreign interest rates \((r, r^*)\) from equations (35), (36), (41), and (42). These solutions are then used in (39) and (40) to obtain the steady-state amounts of financial services traded in each type of match \((y_1, y_2)\), and in (32) and (33) to obtain the steady-state cutoff productivity levels that indicate the tradability of Home and Foreign goods in each market \((\hat{\phi}_D, \hat{\phi}_X; \hat{\phi}^*_D, \hat{\phi}^*_X)\).

4 Financial Development, the Allocation of Economic Activity, and Trade

I now investigate the effects of cross-country differences in financial development on the allocation of economic activity, and study the model’s implications for the effects of trade liberalization. This paper defines financial development as a country’s ability to generate assets that are acceptable as collateral or means of payment in financial transactions. In the current framework, financial development is captured by the acceptability parameter \( \mu \).

Figure 2 shows the responses of several variables to changes in \( \mu \) under three scenarios: (1) “Abundant Liquidity” corresponds to the case when there is a large enough supply of liquid assets so that
OTC matches always reach the surplus-maximizing consumption of financial services, \( \hat{y} \); (2) “No Trade” corresponds to a case with scarce liquidity and no international trade in differentiated goods \((\tau \to \infty)\); and (3) “Trade” corresponds to the case with scarce liquidity and \( \tau \) sufficiently small to allow for international trade flows in differentiated goods. The rest of this section describes this figure under each scenario.
4.1 Abundant Liquidity

Consider the case of abundant liquidity in every match in the OTC financial market (i.e., $A + B \geq \hat{y}$) so that liquidity is not valued: $[F'(A + B) - 1]^+ = [F'(A + A^* + B + B^*) - 1]^+ = 0$ and thus $r = r^* = \rho$. This case is represented by the “Abundant Liquidity” lines in Figure 2. From equations (35) and (36), the total capitalization of firms is the same in both countries, $A = A^* = \frac{(\sigma - 1) \eta}{\sigma k (\rho + \delta)} = A_0$, which is independent of $\mu$ and $\tau$. This is the “fundamental-value” capitalization outcome of the conventional Melitz model with two identical countries, a Pareto distribution of productivity, and no liquidity services of private assets. In this case, equation (34) yields a zero trade balance in the differentiated-good sector and thus, output value is $\eta$ in each country.

With abundant liquidity, trade liberalization—a decline in $\tau$—does not affect the total capitalization of firms nor entry ($N_E = N_E^* = \frac{\delta A}{f_E}$), but the standard results of Melitz (2003) hold: trade liberalization induces reallocation of market shares towards more productive firms, and increases average productivity. To see this, note that with abundant liquidity ($r = r^* = \rho$) the cutoff productivity levels in (32)-(33) become

$$\hat{\phi}_D = \frac{\hat{\phi}_D^*}{\tau} \left[ \frac{\Gamma(\tau k + 1)}{f_E(\rho + \delta)} \right]^{\frac{1}{\tau}}$$

and

$$\hat{\phi}_X = \frac{\hat{\phi}_X^*}{\tau} \left[ \frac{\Gamma(\tau k + 1)}{f_E(\rho + \delta)} \right]^{\frac{1}{\tau}},$$

so that $\frac{d\hat{\phi}_D}{d\tau} = \frac{d\hat{\phi}_D^*}{d\tau} < 0$ and $\frac{d\hat{\phi}_X}{d\tau} = \frac{d\hat{\phi}_X^*}{d\tau} > 0$. Therefore, a decline in $\tau$ increases $\hat{\phi}_D$ and $\hat{\phi}_D^*$, wiping out the least productive firms, and increasing both countries' average productivities, $\hat{\phi}_D$ and $\hat{\phi}_D^*$ (recall that $\hat{\phi}_D$ and $\hat{\phi}_D^*$ are directly proportional, and the same for $\hat{\phi}_X$ and $\hat{\phi}_X^*$). Given that the productivity cutoff for accessing the domestic market is inversely proportional to the aggregate price in each country, it follows that a decline in $\tau$ reduces aggregate prices, $P$ and $P^*$, and thus increases household welfare in each country. At the same time, $\hat{\phi}_X$ and $\hat{\phi}_X^*$ decline when $\tau$ falls, so that less productive firms start to export after trade liberalization.

4.2 Scarce Liquidity

Let us now consider the case with scarce liquidity in all OTC matches (i.e., $A + A^* + B + B^* < \hat{y}$) so that there are liquidity premiums for Home and Foreign assets—the only exception is when $\mu = 1$, in which case Foreign assets do not yield a liquidity premium.

If $\mu = 0$, so that Home and Foreign assets are equally liquid, the right-hand sides of (41) and (42) are identical and thus interest rates are the same in both countries, $r = r^* = r_0 < \rho$. From (35) and (36) it follows that $A = A^* = \frac{(\sigma - 1) \eta}{\sigma k (r_0 + \delta)} = A_0$, which is independent of $\tau$. From (34), it is also the case that $T = 0$ and output value is $\eta$ in each country. Hence, similar to the abundant-liquidity case, trade liberalization does not affect the total capitalization of firms (nor output value) in each country, but it has conventional Melitz’s effects on aggregate productivity, prices, and household welfare. It is the
case, however, that \( A_0 > A \), with their ratio given by
\[
\frac{A_0}{A} = \frac{\rho + \delta}{r_0 + \delta} = 1 + \frac{\gamma[F'(2A_0 + B + B^*) - 1]}{r_0 + \delta} > 1.
\]

Therefore, as in the closed-economy case, the liquidity role of private assets in the financial market causes an expansion in the differentiated-good sector in each country, which translates to higher entry, higher average productivity, and lower aggregate prices. Note also that an increase in the supply of government bonds in either country (an increase in \( B \) or \( B^* \)) crowds out economic activity in both countries: \( F'(2A_0 + B + B^*) \) declines towards 1, \( r_0 \) rises towards \( \rho \), and \( A_0 \) gets closer to the abundant-liquidity outcome, \( A \).

If \( \mu \in (0,1] \), so that Home assets are more liquid than Foreign assets, equations (41) and (42) imply that \( r < r^* \leq \rho \). This scenario yields the main results of the paper, which are summarized in the following proposition.

**Proposition 1. (Essential liquidity, economic activity, and differentiated-good trade)**
If liquidity is scarce in all financial matches, and Home assets are more liquid than Foreign assets, then:

1. Home has a larger capitalization, is more productive, and runs a surplus in differentiated-good trade;

2. An increase in \( \mu \) increases the total capitalization and average productivity of Home firms, while the opposite happens for Foreign firms, driving an increase in Home’s differentiated-good trade surplus;

3. Trade liberalization increases the total capitalization of Home firms and reduces the capitalization of Foreign firms, increasing Home’s surplus in differentiated-good trade;

4. Increases in Home or Foreign public liquidity crowd out Home private liquidity, but may increase Foreign private liquidity.

For the first part of the Proposition 1, note from the ratio of (35) and (36) that
\[
\frac{A}{A^*} = 1 + \frac{(\tau^k + 1)^2(r^* - r)}{(\tau^{2k} + 1)(r + \delta) - 2\tau^k(r^* + \delta)}.
\]

In an interior solution, the denominator in the second term of the right-hand side is always positive and therefore, \( A > A^* \) when \( r < r^* \). From (34), Home runs a trade surplus in the differentiated-good sector, \( T > 0 \), so that Home’s output value is larger than Foreign’s output value, \( Y > Y^* \). Thus, even though Home and Foreign have identical production structures, the allocation of economic activity favors Home because of its lower interest rate, which results from Home’s better ability to generate
<table>
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<th>endogenous→</th>
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<th>A*</th>
<th>r</th>
<th>r*</th>
<th>(\hat{\varphi}_D)</th>
<th>(\hat{\varphi}_X)</th>
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Table 1: Comparative statics with scarce liquidity in all matches and \(\mu \in (0, 1)\) — Each cell indicates the sign of the derivative of the endogenous variable in the column with respect to the exogenous variable in the row.

liquid assets for the financial market. From (32) and (33), note that \(\hat{\varphi}_D > \hat{\varphi}_D^*\) if and only if \(r < r^*\); therefore, Home enjoys a higher average productivity \((\bar{\varphi}_D > \bar{\varphi}_D^*)\), a lower aggregate price \((P < P^*)\), and higher household welfare \((W > W^*)\).

For the rest of Proposition 1, Table 1 reports the equilibrium responses of \(A, A^*, r, r^*\), the cutoff levels, and \(T\) to changes in \(\mu, \tau, B,\) and \(B^*\) for the case of \(\mu \in (0, 1)\). The comparative statics are similar when \(\mu = 1\)—so that Foreign assets are never accepted as collateral in the financial market \((i.e., \mu^* = 0)\)—with two exceptions: the column for \(r^*\) and the row for \(B^*\). In the first case, equation (41) shows that \(r^*\) is constant and equal to \(\rho\) when \(\mu^* = 0\), and thus the responses of \(r^*\) to \(\tau, B,\) and \(B^*\) are zero.14 In the second case, if Foreign assets are totally illiquid then \(B^*\) becomes irrelevant for the model’s equilibrium—note that \(B^*\) disappears from equations (41)-(42) when \(\mu^* = 0\)—and hence changes in \(B^*\) do not affect any of the endogenous variables in Table 1.

For the second part of Proposition 1, the first row of Table 1 shows that as \(\mu\) increases—so that Foreign assets become less liquid—the total capitalization of Home firms increases, while the total capitalization of Foreign firms declines. To understand this result, let us start from a state of commercial autarky \((\tau \to \infty)\), represented by the “No Trade” lines in Figure 2. In that case, \(\frac{d\rho}{d\mu}\) is negative, and (35)-(36) collapse to \(A = \frac{(\sigma-1)\eta}{\sigma k (\tau + \delta)}\) and \(A^* = \frac{(\sigma-1)\eta}{\sigma k (\tau^* + \delta)}\), so that there is a straightforward inverse relationship between a country’s interest rate and its total capitalization of firms. A rise in \(\mu\) increases the financiers’ relative demand for Home assets, causing a reduction in the Home interest rate (the liquidity premium of Home assets rises), which in turn drives up entry of Home firms and their total capitalization. Along the way, \(\hat{\varphi}_D\) increases and hence average productivity increases, the aggregate price declines, and household welfare increases at Home. The opposite happens at Foreign as \(\mu\) increases, with \(r^*\) increasing towards \(\rho\), and \(A^*\) declining from \(A_0\) towards the fundamental-value outcome, \(A\).

With trade in differentiated goods—represented by the “Trade” lines in Figure 2—the impact of cross-country liquidity differences on the allocation of economic activity are magnified. An increase in \(\mu\) increases entry at Home more than in commercial autarky because Home firms now have access

\[14\] This is equivalent to a case with \(\mu \in (0, 1)\) in which liquidity is abundant in OTC matches that accept Foreign assets \(i.e., A + A^* + B + B^* > \hat{y}\), but scarce in matches that only accept Home assets \(i.e., A + B < \hat{y}\).
to the Foreign market. Meanwhile, entry at Foreign declines more than under commercial autarky because in addition to a higher interest rate, Foreign firms are subject to tougher competition from new Home entrants. As a consequence, Home’s trade surplus—and thus output value—increases. The sign of \( \frac{dr}{d\mu} \) becomes ambiguous because the increase in \( A \) implies that more liquidity is available in OTC financial transactions that only accept Home assets \( (F'(A + B) \text{ declines}) \), and with the magnifying effects of trade the increase in \( A \) can be sufficiently large that \( r \) could even go up for high levels of \( \mu \).

For the third part of Proposition 1, the second row in Table 1 shows the unequal effects of trade liberalization: a decline in \( \tau \) increases \( A \), reduces \( A^* \), and improves Home’s trade surplus \( (T \text{ increases}) \), exacerbating the gap in the allocation of economic activity across countries. To understand this, note first that in the initial steady state with \( \mu \in (0, 1] \), entry is lower at Foreign and

\[
\hat{\phi}_D^* < \hat{\phi}_D < \hat{\phi}_X < \hat{\phi}_X^*,
\]

where \( \hat{\phi}_X = \tau \hat{\phi}_D^* \) and \( \hat{\phi}_X^* = \tau \hat{\phi}_D^* \), so that average productivity is higher at Home \( (\hat{\phi}_D > \hat{\phi}_D^*) \), but Foreign exporters are on average more productive than Home exporters \( (\hat{\phi}_X^* > \hat{\phi}_X) \). A low \( \hat{\phi}_D^* \) and a high \( \hat{\phi}_X^* \) indicate that entry incentives for Foreign firms are mostly driven by profit opportunities from selling domestically, as their export market is more competitive and thus more difficult to access. On the other hand, the smaller gap between \( \hat{\phi}_D \) and \( \hat{\phi}_X \) shows that entry incentives at Home are more balanced between domestic sales and export opportunities. In that scenario, the decline in \( \tau \) has a negative effect on Foreign firms’ profit opportunities: due to their higher dependence on domestic sales, Foreign firms’ new entry incentives from exporting are weak compared to the reduction in entry incentives due to tougher competition in the domestic market (from new Home exporters). In contrast, for Home firms the better profit opportunities from exporting dominate their weaker outlook from domestic sales. In the end, trade liberalization increases entry at Home and reduces entry at Foreign, which translates to a higher \( A \) and a lower \( A^* \).

As mentioned before, in the conventional Melitz model, trade liberalization wipes out the least productive firms in both countries \( (\hat{\phi}_D \text{ and } \hat{\phi}_D^* \text{ increase}) \), but allows less productive firms to export \( (\hat{\phi}_X \text{ and } \hat{\phi}_X^* \text{ decline}) \). In the current model, the second row of Table 1 shows that a decline in \( \tau \) indeed reduces \( \hat{\phi}_X \) and \( \hat{\phi}_X^* \), but has an ambiguous effect on the domestic cutoff levels, \( \hat{\phi}_D \) and \( \hat{\phi}_D^* \). Although theoretically possible, however, reductions in \( \hat{\phi}_D \) or \( \hat{\phi}_D^* \) after a fall in \( \tau \) may only occur under strict parametric and functional-form conditions.\(^{15}\) Thus, as shown in Figures 2c and 2d, the typical outcome of a reduction in \( \tau \) is an increase in average productivity and a decline in aggregate prices in both countries, so that household welfare typically increases in Home and Foreign.

\(^{15}\)For example, \( \frac{d\hat{\phi}_D}{d\tau} > 0 \) requires an almost depleted Foreign differentiated-good sector (as a consequence of both a level of \( \mu \) close to 1 and a low iceberg trade cost), and a small value for \( |F''(A + B)| \) so that \( r \) barely increases after a reduction in \( \tau \). On the other hand, \( \frac{d\hat{\phi}_D^*}{d\tau} > 0 \) requires low levels of \( \mu \) and \( \tau \), and \( \frac{dr}{d\mu} < 0 \). In numerical simulations, these conditions are difficult to satisfy using Cobb-Douglas or logarithmic functional forms for \( F(y) \) under standard parameter values used in Melitz-type models.
The second row of Table 1 also shows that a decline in $\tau$ has an ambiguous effect on the interest rate of Foreign assets, $r^*$. Given $\mu \in (0, 1)$, from (41) note that the response of $r^*$ to $\tau$ depends on world liquidity, $A + A^* + B + B^*$. If the contraction at Foreign dominates the expansion at Home so that $A + A^*$ declines, available liquidity is lower in OTC financial matches that accept Foreign assets, and thus $F'(A + A^* + B + B^*)$ increases and $r^*$ declines (the liquidity premium on Foreign assets rises). The opposite happens if $A + A^*$ increases. On the other hand, given $\mu \in (0, 1)$, trade liberalization always increases the interest rate on Home assets, $r$; i.e., the liquidity premium on Home assets declines. From (42), this result implies that the increase in liquidity in matches that only accept Home assets is sufficiently large (so that the decline in $F'(A + B)$ is sufficiently strong) to offset the downward pressure on $r$ from a possible reduction in liquidity in OTC matches that also accept Foreign assets.

For the last part of Proposition 1, the last two rows in Table 1 show that an increase in $B$ or $B^*$ increases the amount of available liquidity in financial matches, driving an increase in interest rates of both Home and Foreign assets, and crowding out Home private liquidity ($A$ declines). However, the response of Foreign private liquidity, $A^*$, is ambiguous. From the third row of Table 1, note that an increase in $B$ not only reduces $A$ and Home’s trade surplus ($T$ declines), but also reduces average productivity ($\bar{\psi}_D \propto \bar{\psi}_D$ declines), increasing $P$ and reducing Home’s household welfare. Given these outcomes, in this model the only reason why the Home government would increase public liquidity is to benefit financiers so that the amount of traded financial services is closer to the surplus maximizing level, $\hat{\gamma}$. Alternatively, if the Home government wants to stimulate production in the differentiated-good sector (and increase household welfare at the same time), it can reduce the amount of public liquidity at the expense of financiers’ surpluses. Hence, the Home government faces a trade-off between maximizing the surplus in the OTC financial market and maximizing household welfare (see Rocheteau and Rodriguez-Lopez, 2014 for a discussion of the normative implications of a similar type of trade-off).

Regarding the ambiguous response of $A^*$, an increase in $B$ has a positive effect on $A^*$ if $\tau$ is sufficiently small (if $\tau \to \infty$, so that there is no trade in differentiated goods, then $\frac{dA^*}{d\tau} < 0$) and $\mu$ is sufficiently high. To see this, note that if $\mu = 1$ (so that Foreign assets are totally illiquid), then an increase in $B$ has no impact on $r^*$ because the latter is already equal to its upper bound $\rho$. In contrast, the increase in $B$ increases $r$, which crowds out Home private liquidity, $A$. With entry and average productivity declining at Home, Foreign firms’ export incentives increase and therefore entry, average productivity, and total capitalization increase at Foreign. On the other hand, the conditions for an increase in Foreign government bonds, $B^*$, to be a useful policy instrument to increase $A^*$ are harder to satisfy. In addition to a sufficiently small $\tau$, $\mu$ should be neither too low nor too high (as $\mu$

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16Given the free-entry condition for firms in the differentiated-good sector, the positive economic profits of successful producers exactly cancel out with the entry costs of unsuccessful entrants. Therefore, differentiated-good firms can be ignored in welfare analysis.
increases, Foreign public bonds lose relevance in the liquidity market), and \(|F''(A + B)|\) must have a low value. Moreover, an increase in \(B^*\) always reduces \(\hat{\varphi}_D^*\) so that even if \(A^*\) increases, Foreign will have lower average productivity, a higher aggregate price, and lower household welfare.

### 4.3 Discussion on the Role of Public Liquidity

In the model there is no mechanism through which an increase in Home public liquidity could increase Home private liquidity. However, given that entry in the differentiated-good sector is made possible by financiers, it makes sense that the size of the OTC financial sector may help promote entry of differentiated-good firms. Following this premise, a previous version of this paper developed an extension of the model in which entrepreneurs in each country face searching costs for funds—to be used to cover the entry cost in the differentiated-good sector—and where the rate at which they get access to those funds depends on the measure of financiers, which is endogenous in each country.\(^17\)

That extension highlights another mechanism through which public liquidity can affect real economic activity (besides the crowding-out mechanism): more public liquidity increases the expected surplus from participating in the OTC financial market, increasing the measure of financiers and thus making it easier for entrepreneurs to find funds. In a closed economy, this new mechanism offsets the crowding-out effect. In an open economy, however, the new mechanism offsets the crowding-out effect for the country whose access to funds improve relatively more, and reinforces the crowding-out effect for the other country.

There are other channels, not considered in this paper, through which an increase in available liquidity may increase real economic activity. For example, in recent two-country monetary-search models, liquid assets serve to facilitate and expand international trade either because they are used directly as media of exchange to purchase domestic and foreign goods (Geromichalos and Simonovska, 2014), or because they can be exchanged for foreign currency that is necessary to purchase foreign goods (Geromichalos and Jung, 2018). Moreover, this type of mechanism can be embodied in frameworks that also include the crowding-out effect. Along those lines, Herrenbrueck (2014) and Geromichalos and Herrenbrueck (2017) introduce closed-economy models with money, bonds, and capital that include both a Mundell-Tobin effect (which highlights the substitutability across assets) and a real balance effect (which highlights complementarities across assets in the production process).

### 5 Model’s Extensions

In the open-economy model in sections 3 and 4, financiers secure their transactions by using as collateral Home and Foreign private assets, and Home and Foreign government bonds. Up to now I have assumed that differences in liquidity properties across assets—which stem from differences in the acceptability

\(^{17}\)That version is available at http://www.socsci.uci.edu/~jantonio/Papers/liquidity_economicactivity_Dec2016.pdf.
of assets in financial transactions—are exogenous and depend exclusively on national origin; i.e., μ is
given, and there are no liquidity differences between public and private assets of the same country. This
section presents two extensions that relax these assumptions: in the first extension the difference in
liquidity properties between Home and Foreign assets arises endogenously, while the second extension
allows for differences in liquidity properties between public and private assets within and between
countries, and across private assets within each country.

5.1 Endogenous Acceptability

In the model Home assets are acceptable in all OTC matches, but Foreign assets are only accepted
in a fraction \( \mu^* = 1 - \mu \) of matches. But what determines \( \mu^* \)? Foreign assets are less acceptable than
Home assets if their use in financial markets is subject to costs of acquiring information or costs of
uncertainty; these include costs associated with verification of financial information of Foreign firms,
lack of confidence in Foreign institutions (due, for example, to weak contract enforcement), and a higher
likelihood of fraud. Hence, whether or not an asset is acceptable as collateral is likely an endogenous
outcome resulting from comparing the benefits and costs of using this asset. Therefore, if the benefits
of accepting Foreign assets in OTC matches decline (due, for example, to weaker Foreign economic
conditions) while costs remain the same, then sellers of financial services become even less willing to
accept them, so that \( \mu^* \) declines. Along these lines, the following sections show how to endogenize \( \mu^* \)
and present an analysis of the effects of trade liberalization under endogenous acceptability.

5.1.1 The Recognizability Approach to Asset Liquidity

This extension follows the recognizability approach of Lester, Postlewaite, and Wright (2012), who
endogenize the acceptability of an asset as a medium of exchange by assuming that agents have
to invest in information in order to distinguish good claims from counterfeits.\(^{18}\) The information
cost that agents must pay to recognize good claims of an asset can be interpreted in a general way,
capturing all knowledge, institutional, and technology factors that affect an asset’s acceptability. In
this extension, Home assets are always recognizable but Foreign assets are only recognized in informed
matches (i.e., in matches where the seller of financial services paid the information cost). Financiers
are heterogeneous with respect to the information cost they must pay to recognize claims on Foreign
assets. In particular, I assume that each financier is born with an information flow cost \( \psi \geq \psi_{\text{min}} \) drawn
from a distribution with cumulative function \( \Lambda(\psi) \), where \( \psi_{\text{min}} \geq 0 \) and \( \Lambda'(\psi) > 0 \). It follows that a
financier is informed if and only if her surplus—when matched as a seller—from an informed match
net of the recognizability cost is no less than her surplus from an uniformed match. The fraction of

\(^{18}\)A related body of literature endogenizes acceptability by introducing informational asymmetries in asset values
either under adverse selection (see Guerrieri, Shimer, and Wright, 2010, Guerrieri and Shimer, 2012, and Rocheteau,
2011) or moral hazard (Li, Rocheteau, and Weill, 2012).
informed financiers is then $\mu^*$.

So far the model assumes that a buyer of financial services makes a take-it-or-leave-it offer to the seller, so that the seller’s surplus from the match is always zero. Under that protocol, financiers never pay the recognizability cost and thus Foreign assets are never accepted in OTC matches. Consequently, this extension changes the bargaining protocol so that sellers obtain a share of the match’s surplus, giving financiers an incentive to pay the recognizability cost. Letting $\theta \in [0, 1]$ denote the buyer’s bargaining power, I follow Aruoba, Rocheteau, and Waller (2007) and Lester, Postlewaite, and Wright (2012) and use Kalai’s proportional bargaining solution. This implies that for a type-$j$ match, for $j \in \{1, 2\}$, the transfer of assets from the buyer to the seller is $\theta y_j + (1 - \theta) F(y_j)$ in exchange for $y_j$ financial services, the surplus of the buyer is $\theta[F(y_j) - y_j]$, and the surplus of the seller is $(1 - \theta)[F(y_j) - y_j]$.

Under proportional bargaining, the continuation value of a financier upon being matched (but before realizing its buyer or seller role) is

$$Z(a, a^*, b, b^*) = \frac{\mu}{2} \max \{\theta[F(y_1) - y_1]\} + \frac{\mu^*}{2} \max \{\theta[F(y_2) - y_2]\} + K + W(a, a^*, b, b^*),$$

subject to

$$\theta y_1 + (1 - \theta) F(y_1) \leq a + b$$
$$\theta y_2 + (1 - \theta) F(y_2) \leq a + a^* + b + b^*.$$ 

In $Z(a, a^*, b, b^*)$, the first two terms indicate the expected surplus from being a buyer, and $K$ is the expected surplus from being a seller. Sellers of financial services do not have any incentives to hold assets, and thus $K$ is independent of $a$, $a^*$, $b$, and $b^*$. The two feasibility conditions state respectively that in an uninformed (type-1) match, the buyer can transfer up to $a + b$ in assets, while in an informed (type-2) match she can transfer up to $a + a^* + b + b^*$.

As in the derivation of equations (37) and (38), the financier’s optimal portfolio solves

$$\frac{\rho - r^*}{\gamma} = \frac{\mu^* \theta [F'(y_2) - 1]}{\theta + (1 - \theta) F'(y_2)}$$

(43)
$$\frac{\rho - r}{\gamma} = \frac{\mu \theta [F'(y_1) - 1]}{\theta + (1 - \theta) F'(y_1)} + \frac{\mu \theta [F'(y_1) - 1]}{\theta + (1 - \theta) F'(y_1)}.$$ (44)

The left-hand side in equation (43) shows the financier’s holding cost of a Foreign asset, and the right-hand side shows the expected marginal surplus from holding that asset—equation (44) shows the same for a Home asset. Moreover, under market clearing ($a = A$, $a^* = A^*$, $b = B$, and $b = B^*$), the transfer of assets for each type of match can be written as

$$\theta y_2 + (1 - \theta) F(y_2) = \min \{A + A^* + B + B^*, \theta \hat{y} + (1 - \theta) F(\hat{y})\},$$

(45)
$$\theta y_1 + (1 - \theta) F(y_1) = \min \{A + B, \theta \hat{y} + (1 - \theta) F(\hat{y})\},$$

(46)

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where \( \hat{y} \) solves \( F'(\hat{y}) = 1 \). Note that if sellers do not have any bargaining power (so that \( \theta = 1 \)), equations (43)-(46) become (37)-(40).

Given \( \mu^* \), equations (35), (36), (43), (44), (45), and (46) solve for \( A, A^*, r, r^*, y_1, \) and \( y_2 \). However, the main contribution of this section is that \( \mu^* \) is endogenous, so I still need to describe the equations that pin it down. For a given \( \mu^* \) and before being matched, a financier’s expected surplus from being a seller in a type-\( j \) match is \( \gamma(1-\theta)\{F[y_j(\mu^*)] - y_j(\mu^*)\} \), for \( j \in \{1,2\} \). Therefore, the financier’s expected benefit from being an informed seller, labeled as \( \Xi(\mu^*) \), is defined as the difference between the expected surplus in an informed (type 2) meeting and the expected surplus in an uninformed (type 1) meeting, that is,

\[
\Xi(\mu^*) \equiv \gamma(1-\theta)\{F[y_2(\mu^*)] - y_2(\mu^*) - F[y_1(\mu^*)] + y_1(\mu^*)\}.
\] (47)

From (47), I prove in section A.1 in the online Appendix that \( \Xi'(\mu^*) > 0 \) as long as \( y_1 < \hat{y} \). Thus, the higher the fraction of matches that accepts Foreign assets, the higher the net benefit for a financier from accepting them.

A financier with information cost \( \psi \) decides to be informed if and only if \( \psi \leq \Xi(\mu^*) \). For the equilibrium \( \mu^* \), however, we also need to consider the distribution of information costs across financiers. Given \( \Lambda(\psi) \) and \( \mu^* \), I define the maximum information cost that financiers are willing to pay to be informed as

\[
\Psi(\mu^*) \equiv \Lambda^{-1}(\mu^*),
\] (48)

where \( \Psi'(\mu^*) > 0 \) and \( \Psi(0) = \psi_{\text{min}} \).

Figure 3 shows how the interaction between equations (47) and (48) determine the equilibrium \( \mu^* \). For expositional purposes, I assume that information costs follow a uniform distribution in the interval \([\psi_{\text{min}}, \psi_{\text{max}}]\), so that \( \Lambda(\psi) = \frac{\psi - \psi_{\text{min}}}{\psi_{\text{max}} - \psi_{\text{min}}} \) and thus \( \Psi(\mu^*) = \psi_{\text{min}} + (\psi_{\text{max}} - \psi_{\text{min}})\mu^* \). Each panel in Figure 3 shows the same \( \Xi(\mu^*) \) but a different \( \Psi(\mu^*) \), with \( \psi_{\text{min},1} < \psi_{\text{min},2} < \psi_{\text{min},3} < \psi_{\text{min},4} \) and \( \psi_{\text{max},1} < \psi_{\text{max},2} < \psi_{\text{max},3} < \psi_{\text{max},4} \). Figure 3a shows the low-information-costs equilibrium, with \( \Xi(\mu^*) > \Psi_1(\mu^*) \) for every \( \mu^* \), which then yields \( \mu^* = 1 \) as the unique solution. As shown by Lester, Postlewaite, and Wright (2012), the interaction between (47) and (48) can yield multiple equilibria. This is depicted in Figure 3b, which shows three equilibria for \( \mu^* \): \( \mu^*_1 \), \( \mu^*_2 \) and 1. Figure 3c shows that if information costs go further up so that \( \Psi \) shifts to \( \Psi_3(\mu^*) \), there is a unique equilibrium at \( \mu^*_3 \). This highlights the dramatic effects that small changes in information costs can have on asset liquidity. If the initial equilibrium is \( \mu^* = 1 \) (so that Foreign assets are as liquid as Home assets) but then information costs slightly increase causing a small shift from \( \Psi_2(\mu^*) \) to \( \Psi_3(\mu^*) \), the final outcome is a large reduction in the acceptability of Foreign assets. Lastly, Figure 3d shows the high-information-costs equilibrium, with \( \Xi(\mu^*) < \Psi_4(\mu^*) \) for every \( \mu^* \) so that \( \mu^* = 0 \) is the unique solution.

5.1.2 Trade Liberalization with Endogenous Liquidity of Foreign Assets

This section studies the effects of trade liberalization under endogenous acceptability/liquidity of Foreign assets. For clarity, I focus on the case in which the equilibrium for $\mu^*$ is unique.

If liquidity is abundant in all OTC matches (i.e., $\theta \hat{y} + (1 - \theta) F(\hat{y}) \leq A + B$), we reach the same outcome in the differentiated-good sector as in section 4.1: $r = r^* = \rho$ and the total capitalization of firms in each country ($A = A^* = \frac{(\sigma - 1)\eta}{\sigma k (\rho + \delta)} = \mathcal{A}$) is independent of $\mu = 1 - \mu^*$ and $\tau$. Given that $y_1 = y_2 = \hat{y}$, it follows from (47) that $\Xi(\mu^*) = 0$ for every $\mu^*$, which then implies that in equilibrium $\mu^* = 0$ ($\Psi(0) > \Xi(\mu^*)$ for every $\mu^* > 0$, and $\Psi(0) \geq \Xi(0)$, with equality only if $\psi_{\text{min}} = 0$). Hence, when
Home assets are abundant in OTC matches there is no benefit for sellers from accepting also Foreign assets, and thus no financier will pay the information cost.

With scarce liquidity in all OTC matches (i.e., \( \theta \hat{y} + (1 - \theta) F(\hat{y}) > A + A^* + B + B^* \)), liquidity is valued and thus \( r < r^* < \rho \). As in Figure 3a, a unique equilibrium with \( \mu^* = 1 \) (so that \( r = r^* \)) occurs if \( \Psi(\mu^*) < \Xi(\mu^*) \) for \( \mu^* < 1 \) and \( \Psi(1) \leq \Xi(1) \). In such a case, similar results to those discussed in section 4.2 for the \( \mu = 0 \) case hold: the capitalization of firms is the same in both countries (\( A = A^* \)), which is greater than the fundamental-value capitalization (\( A \)) and is not affected by trade liberalization. Given that \( A \) and \( A^* \) are independent of \( \tau \) when \( \mu^* = 1 \), it follows from (45) and (46) that \( y_1 \) and \( y_2 \) are also independent of \( \tau \), which then implies from (47) that \( \frac{d\Xi(1)}{d\tau} = 0 \).

I now turn to the case of an interior solution for \( \mu^* \), so that Home assets are more liquid than Foreign assets and thus \( r < r^* < \rho \). As before, as a consequence of the lower liquidity of Foreign assets, the differentiated-good sector in the Home country is larger, more productive, and sets lower prices. But what is the effect of a decline in \( \tau \)? In section A.2 in the online Appendix I show that \( \frac{d\Xi(\mu^*)}{d\tau} > 0 \) if \( \mu^* < 1 \). This finding leads to the following proposition, which lays out the most important result from this extension.

**Proposition 2. (Trade liberalization and endogenous liquidity)**

If liquidity is scarce in all financial matches and Home assets are more liquid than Foreign assets in an initial interior equilibrium, then trade liberalization further reduces Foreign-asset liquidity (\( \mu^* \) declines).

Figure 4 presents a graphical description of Proposition 2. Starting from an equilibrium at \( \mu_0^* \), a decline in \( \tau \) shifts \( \Xi(\mu^*) \) down from \( \Xi_0(\mu^*) \) to \( \Xi_1(\mu^*) \), reducing the fraction of OTC meetings that accept Foreign assets to \( \mu_1^* \). As shown in the Appendix, for a given \( \mu^* < 1 \), the result that \( \frac{d\Xi(\mu^*)}{d\tau} > 0 \) follows from \( \frac{dA}{d\tau} < 0 \) and \( \frac{dA^*}{d\tau} > 0 \). With trade liberalization (a decline in \( \tau \)) increasing \( A \) and reducing \( A^* \), the amount of liquidity available in type-1 meetings \( (A + B) \) increases and hence \( y_1 \) and \( F(y_1) - y_1 \) increase, while the amount of liquidity available in type-2 meetings \( (A + A^* + B + B^*) \) can increase or decrease, with the same being true for \( y_2 \) and \( F(y_2) - y_2 \). The key point is that even if \( F(y_2) - y_2 \) increases after a decline in \( \tau \)—so that the increase in \( A \) is larger than the reduction in \( A^* \)—the increase in \( F(y_1) - y_1 \) is larger, causing a reduction in a financier’s benefit of being informed, \( \Xi(\mu^*) \). The last result is a consequence of \( F''(y) < 0 \), which implies that \( F'(y_1) > F'(y_2) \), so that the positive impact of an extra unit of available liquidity on the total surplus of an OTC match is larger in a type-1 meeting than in a type-2 meeting.

In comparison with the model with exogenous acceptability, in this version of the model the adverse effects of trade liberalization on the differentiated-good sector at Foreign—and its positive effects at Home—are compounded, with the increasing gap between \( A \) and \( A^* \) being exacerbated by
5.2 Heterogeneous Liquidity Across Multiple Assets

The second model’s extension introduces heterogeneity in the liquidity properties across the four categories of assets, and across private assets within each country. To preserve space, the details of the extended model are presented in section B in the online Appendix. The following sections discuss the main insights from this model.

I assume that (i) Home assets are acceptable as collateral in a larger fraction of OTC matches than Foreign assets, (ii) for each country’s assets, public liquidity is acceptable in a larger fraction of matches than private liquidity, and (iii) there is heterogeneity in acceptability across private assets, with firm-level productivity being positively correlated with collateral fitness. In contrast, the model in section 3 only considers (i).

Figure B-1 in the Appendix presents a description of assumptions (i) and (ii). In a fraction $\mu_g$ of OTC matches only Home government bonds are acceptable as collateral, in a fraction $\mu_p$ of matches both public and private Home assets are acceptable, in a fraction $\mu^*_g$ of matches Home assets and Foreign government bonds are acceptable, and in the remaining $\mu^*_p$ fraction of matches all categories of assets are acceptable. Analogously, Foreign private assets are acceptable in a fraction $\mu^*_p$ of OTC matches, Foreign bonds are acceptable in a fraction $\mu^*_p + \mu^*_g$ of matches, Home private assets in a
fraction $\mu^*_p + \mu^*_g + \mu_p$ of matches, and Home bonds are acceptable in all matches ($\mu^*_p + \mu^*_g + \mu_p + \mu_g = 1$).\footnote{As in section 5.1, the model can be further extended to endogenize each $\mu$ by assuming different information costs for each type of match. To simplify the exposition, this extension abstracts from this feature.}

Regarding (iii), each producing Home firm (with $\varphi \geq \hat{\varphi}_D$) has an associated loan-to-value ratio, $\lambda(\varphi) \in [0,1]$, that specifies the fraction of the asset value that can be pledged as collateral in an OTC transaction: a financier can obtain a loan of size $\lambda(\varphi)a(\varphi)$ if she commits $a(\varphi)$ assets of type $\varphi$ as collateral. The function $\lambda(\varphi)$ satisfies $\lambda(\varphi) > 0$ for all $\varphi \geq \hat{\varphi}_D$, $\lambda(\hat{\varphi}_D) = 0$, $\lambda(\infty) \to 1$, and $\frac{d\lambda(\varphi)}{d\varphi} < 0$. Hence, firm-level productivity is positively related to collateral fitness, which captures the idea that low-productivity firms are seen by financiers as riskier and more sensitive to shocks than more productive firms and thus they get lower loan-to-value ratios. Note that a firm at the cutoff $\hat{\varphi}_D$ is illiquid and hence must yield a return of $\rho$—financiers know that this firm will die for any minimal shock causing an increase in $\hat{\varphi}_D$, so they are unwilling to accept assets of type $\hat{\varphi}_D$ in OTC transactions. Analogous properties hold for loan-to-value ratios of Foreign private assets, which are described by the function $\lambda^*(\varphi)$.

Although the analysis below only requires $\lambda(\varphi)$ and $\lambda^*(\varphi)$ to meet the properties described above, I assume a useful functional form that depends on a single parameter:

$$\lambda(\varphi) = 1 - \left(\frac{\hat{\varphi}_D}{\varphi}\right)^\kappa$$

and

$$\lambda^*(\varphi) = 1 - \left(\frac{\hat{\varphi}_D^*}{\varphi}\right)^\kappa$$

where $\varphi \geq \hat{\varphi}_D$ for Home firms, $\varphi \geq \hat{\varphi}_D^*$ for Foreign firms, $\kappa > 0$, and $\kappa^* > 0$. If $\kappa \to \infty$, then $\lambda(\varphi) \to 1$ for all $\varphi > \hat{\varphi}_D$, which approximates the case in which all claims on producing firms are equally liquid. Note also that $\frac{d\lambda(\varphi)}{d\kappa} > 0$ for all $\varphi > \hat{\varphi}_D$, so that a decline in $\kappa$ is useful to analyze the effects of a liquidity reduction stemming from lower loan-to-value ratios of Home private assets.

I further assume that for a Home or Foreign private asset to be part of the available liquidity to financiers, the asset must be certified by a rating agency (such as Fitch, Moody’s, or S&P) that makes public the asset’s underlying productivity. Each private asset’s certification process involves a sunk cost of $f_A$ (in terms of the homogeneous good), which implies the existence of two more cutoff productivity levels, $\hat{\varphi}_A$ and $\hat{\varphi}_A^*$, that separate assets into “non-certified” and “certified” categories. Non-certified assets have underlying productivities in the range $[\hat{\varphi}_D, \hat{\varphi}_A)$, they are illiquid, and hence pay the illiquid interest rate, $\rho$. Certified assets have underlying productivities in the range $[\hat{\varphi}_A, \infty)$, they are liquid, and hence pay an interest rate below $\rho$.

Let $r_h$ and $r_h^*$ denote, respectively, the interest rates of Home and Foreign government bonds. As well, $r(\varphi)$ is the interest rate of Home private assets with underlying productivity $\varphi$—so that $r(\varphi) = \rho$ if $\varphi \in [\hat{\varphi}_D, \hat{\varphi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\varphi}_A, \infty)$—and $r^*(\varphi)$ is the interest rate of Foreign private assets with underlying productivity $\varphi$. To pin down $\hat{\varphi}_A$ and $\hat{\varphi}_A^*$, notice that an asset with underlying productivity $\varphi$ will be certified if and only if the value of the issuing firm when certified minus the sunk certification
cost, is greater than or equal to the value of the firm when not certified; this condition holds with equality for a firm at the cutoff. Given that not all private assets are part of the liquidity available to financiers, and that only a fraction of the value of certified assets can be pledged as collateral, if follows that—in contrast to the model in section 3—a country’s total capitalization of firms is not equal to the country’s supply of private liquidity.

In the end, the steady-state equilibrium in this model’s extension solves for the cutoff productivity levels that indicate the tradability of Home and Foreign goods in each market ($\hat{\phi}_D, \hat{\phi}_X, \hat{\phi}_D^*, \hat{\phi}_X^*$), the cutoff productivity levels that separate certified and non-certified firms in each country ($\hat{\phi}_A, \hat{\phi}_A^*$), the amounts of Home and Foreign private liquidity ($A, A^*$), the amount of financial services traded in each type of match, and the structure of interest rates ($r^*(\phi), r^*_b, r(\phi), r_b$).

Figure 5 shows the full structure of interest rates when liquidity is scarce in every match in the financial market; i.e., when $A + A^* + B + B^* < \hat{y}$. Being the most liquid assets, Home government bonds yield the lowest interest rate, $r_b$.\(^{21}\) Also, although Home private assets are acceptable in more OTC matches than Foreign government bonds, there may be Home private assets issued by the least-productive certified firms whose interest rates are higher than $r^*_b$ due to their small loan-to-value ratios.

In section B.3 in the Appendix I discuss the effects of a permanent reduction in the acceptability of Home private assets, which can be captured either with a reduction in $\mu_p$ or with a reduction in the parameter $\kappa$ of the loan-to-value function. The first case resembles a financial system’s general rejection

\(^{21}\)As a by-product, note that this liquidity framework is useful to help explain the equity-premium puzzle, which refers to the observation that rates of return on equities are much higher than rates of return on government bonds. Lagos (2010) explores this venue in a related setting.
of Home private assets, while the second case resembles downgrades of ratings for Home private assets. In any of these cases, interest rates decline for Home and Foreign government bonds, and for most Foreign private assets (the exceptions are assets from low-productivity firms that become illiquid due to an increase in in $\phi^\ast_A$). On the other hand, although interest rates of most Home private assets increase, other types of financial phenomena can emerge depending on the origin of the acceptability reduction. If $\mu_p$ falls, financiers compensate for the reduction in Home liquidity by accepting previously illiquid Home private assets, and hence the average quality of Home collateral declines. In contrast, if $\kappa$ falls, the average quality of Home collateral increases, and additionally, there is flight-to-quality towards assets issued by the most productive Home firms, which drives a reduction in these assets’ interest rates.

6 Evidence on Acceptability, Yields, and Economic Activity

In the model above, an asset’s liquidity is determined by its acceptability as collateral in financial transactions. The model shows that (i) more acceptable assets have higher liquidity premiums and thus lower yields, and (ii) differences in acceptability across assets of different countries drive economic activity in favor of the firms that issue the most liquid assets. Applying both a corporate-bond yield analysis and a firm-level analysis, this section presents evidence that is consistent with these results.

6.1 Data

The data in this analysis covers the post Great Recession period: monthly data from April 2014 to March 2019 for the corporate-bond yield analysis, and yearly data for the fiscal years 2011-2017 for the firm-level analysis. This is a relevant period to study, as it witnessed an expansion in the demand for collateral-acceptable assets as a result of regulations intending to reduce systemic risk in financial markets (IMF, 2012). In particular, the Dodd-Frank Act (implemented in the U.S. since July 2010) introduced regulation requiring OTC derivative transactions such as interest-rate and credit-default swaps to be cleared by collateral-hungry central counterparties (CCPs). According to global OTC BIS data, by end-June 2019 78% of interest-rate derivatives and 54% of credit-default swaps were cleared by CCPs. In the U.S., 89% of interest-rate derivatives were cleared by CCPs in June 2019 (ISDA, 2019b).

Given that CCPs are the decision-makers regarding the specific assets that are accepted as collateral in their clearing activities—which can include, among others, cash, treasuries, gold, stock, and corporate bonds—I classify an asset (and its issuer) as “acceptable” if it appears on the April 2019

---

22 The BIS data shows that CCPs participation in global OTC derivatives markets is mainly in the clearing of interest-rate derivatives: of the $416 trillion in notional value cleared by CCPs by end-June 2019, 98% (408 trillion) was in the clearing of IRDs.
Table 2: Composition of corporate bonds by Moodys’s credit rating and CME acceptability

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Acceptable</th>
<th>Non-acceptable</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>64</td>
<td>28</td>
<td>92</td>
</tr>
<tr>
<td>AA1</td>
<td>42</td>
<td>16</td>
<td>58</td>
</tr>
<tr>
<td>AA2</td>
<td>89</td>
<td>108</td>
<td>197</td>
</tr>
<tr>
<td>AA3</td>
<td>77</td>
<td>171</td>
<td>248</td>
</tr>
<tr>
<td>A1</td>
<td>232</td>
<td>367</td>
<td>599</td>
</tr>
<tr>
<td>A2</td>
<td>229</td>
<td>595</td>
<td>824</td>
</tr>
<tr>
<td>A3</td>
<td>238</td>
<td>822</td>
<td>1,060</td>
</tr>
<tr>
<td>BAA1</td>
<td>12</td>
<td>1,419</td>
<td>1,431</td>
</tr>
<tr>
<td>BAA2</td>
<td>4</td>
<td>1,106</td>
<td>1,110</td>
</tr>
<tr>
<td>BAA3</td>
<td>0</td>
<td>870</td>
<td>870</td>
</tr>
<tr>
<td>Total</td>
<td>987</td>
<td>5,502</td>
<td>6,489</td>
</tr>
</tbody>
</table>

list of acceptable collateral of CME Clearing, which is the world’s second largest CCP for the clearing of OTC interest-rate derivatives (Ehlers and Hardy, 2019; ISDA, 2019a). The list, available in the online Data Supplement, includes 1,056 corporate bonds and 417 stocks identified by their CUSIP number. The 1,056 bonds were issued by 174 corporations, but concentration is large, with 25 firms issuing more than 50 percent of the bonds. The 417 stocks are a subset of the S&P 500. Although the list of corporate bonds can change over time due mainly to the issuance of new bonds and the maturity of others, the set of corporate-bond issuers as well as the list of stocks are likely to be relatively stable during our period of study.

From Fidelity, I obtain a list of 6,489 investment-grade corporate bonds along with their Moody’s credit ratings. Table 2 shows the composition of these bonds by rating and CME acceptability. The list aims to include all investment-grade corporate bonds issued in the U.S. that are active in March 2019, but all bonds missing a Moody’s rating are excluded. Note that 93.5% (987 out of 1,056) of CME acceptable bonds are included in the Fidelity list. Out of the 6,489 bonds, 9.2% are in high-grade categories (AAA to AA3), 38.3% are in medium-grade categories (A1 to A3), and 52.6% are in categories BAA1 to BAA3. On the other hand, out of the 987 acceptable bonds, 27.6% are in high-grade categories, 70.8% are in medium-grade categories, and only 1.6% are in categories below A3. Lastly, 15.2% of all bonds (987 out of 6,489) are acceptable as collateral by CME, but the likelihood of acceptability varies by rating: while the acceptability rates are 45.7% for high-grade bonds and 28.2% for medium-grade bonds, it is only 0.5% for bonds rated below A3.

\[23^\text{The most up-to-date CME’s collateral list is available at the CME Clearing’s website. CME only provides its current list, and thus, I cannot obtain CME’s lists for earlier time periods. According to FSB (2019), CME operates in nine jurisdictions including the U.S., the European Union, Canada, Japan, and China; in addition to interest-rate derivatives, CME clears foreign exchange, commodity, and equity derivatives (it stopped clearing credit derivatives in March 2018). The largest CCP in the world is the London Clearing House (LCH), but in contrast to CME, LCH does not provide a public detailed list of acceptable collateral.}\]

\[24^\text{Apple is the top issuer of acceptable corporate bonds, accounting for 43 of them (4.07 percent). Other top issuers include Comcast, Microsoft, Oracle, Toyota, Walmart, IBM, Home Depot, and Amazon.}\]
Using the CUSIP identifiers, I use TRACE (Trade Reporting and Compliance Engine)—a dissemination service providing real-time OTC corporate bond market transactions that is available through Wharton Research Data Services (WRDS)—to obtain daily close yields for the corporate bonds in the Fidelity list from April 2014 to March 2019. The daily data is then converted to a monthly yields database by obtaining each bond’s mean yield over the month. The last month includes yields for all 6,489 bonds, but the first month includes yields for only 2,573 bonds. The decline for earlier periods is mainly due to bonds not being issued yet, but also there are bonds that do not report transactions for some months. I also create a balanced sample that includes the 1,789 bonds that report yields for every month. Out of 1,789 bonds, 288 (16.1%) are acceptable by CME; importantly, the composition of bonds by credit rating and acceptability in the balanced panel is very similar to the composition of bonds in the full Fidelity list.\textsuperscript{25}

Economic activity data at the firm level is obtained from the annual Compustat North America and Global Databases—Compustat attempts to capture all publicly listed companies and is available through WRDS. Each firm reports information in the final month of its fiscal year, which varies from firm to firm.\textsuperscript{26} For the fiscal-year period 2011-2017, I obtain for each firm its yearly sales, profits (operating income), employment, R&D expenditure, book value, market value, headquarters country, and SIC (Standard Industrial Classification) four-digit industry. All nominal variables are reported in the currency of the headquarters country, and thus, I convert them to U.S. dollars using the Compustat monthly exchange rate file.\textsuperscript{27} After this, I use the monthly PCEPI (Personal Consumption Expenditure Price Index) of the Bureau of Economic Analysis to convert nominal U.S. values to real values.

This paper is interested on the effects of the liquidity market on economic activity in non-financial sectors, and thus, I remove all financial firms (with SIC codes between 6000 and 6799). In the end, the Compustat firm-level database contains 36,930 firms from 47 countries and 416 industries, with five countries accounting for almost 50% of all firms: U.S. (14.7%), China (11.6%), India (9.2%), Japan (8.9%), and Taiwan (5.4%). Also, not all the variables of interest are reported by all firms. While about 98% of firms report sales, profits, and book value, 68% report employment, 51% report R&D expenditure, and only 18% report market value (market value is only reported in the Compustat North America database).

\textsuperscript{25}Out of the 1,789 bonds in the balanced sample, 7.7% are in high-grade categories, 38.5% are in medium-grade categories, and 53.9% are in categories below A3. Out of the 288 acceptable bonds, 24.3% are in high-grade categories, 74.7% are in medium-grade categories, and 1% are in categories below A3. Lastly, the acceptability rates are 51.1% for high-grade bonds, 31.3% for A1 to A3 bonds, and 0.3% for bonds rated below A3.

\textsuperscript{26}Compustat assigns the same year to all firms that end their fiscal year between June in the current year and May in the following year; for example, if a firm ends its fiscal year in August 2015 and another ends its fiscal year in May 2016, both are assigned 2015 as their fiscal year.

\textsuperscript{27}For each firm, each nominal value is converted to U.S. dollars using the exchange rate on the month the firm files the report (in the end of the firm’s fiscal year). The Compustat exchange rate file is updated until April 2018. To be able to use all firms in fiscal year 2017, I use the April 2018 exchange rate for all those firms that end their fiscal year in May 2018.
6.2 Acceptability and Corporate Bond Yields

This section compares yields between acceptable and non-acceptable corporate bonds. Credit rating is an important determinant of bond yields, but as described above, it is also strongly correlated with asset acceptability. Thus, when analyzing the relationship between acceptability and yields, it is crucial to control for credit rating. Along these lines, for March 2019 Figure 6 shows the average yields by Moody’s credit rating and CME acceptability of the 6,489 corporate bonds described in Table 2. The solid markers present the point estimates, while the hollow markers indicate the share of each credit-rating category in the total of either acceptable or non-acceptable bonds. Out of 9 categories (there are no BAA3 acceptable bonds), acceptable bonds have lower yields than non-acceptable bonds in eight of them, and only a barely larger yield in the AA1 category.

I now assort bonds into two groups, a *high-grade group* that includes categories from AAA to AA3, and a *medium-grade group* that includes categories from A1 to A3.\(^{28}\) For the full period of study...
Figure 7: Average yields by investment-grade group for acceptable (red) and non-acceptable (blue) bonds: Unbalanced sample (solid) and balanced sample (dashed)

(April 2014 to March 2019), Figure 7 shows average yields by investment-grade group for acceptable and non-acceptable bonds. The solid lines use the unbalanced sample (which increases in size through time), and the dashed lines use the balanced sample. Throughout the entire period and no matter what sample is used, average yields of acceptable bonds are smaller than average yields of non-acceptable bonds.

To assess the statistical significance of the yield difference between acceptable and non-acceptable bonds while controlling for credit rating, I estimate the econometric models

\[
\bar{y}_{ij} = \zeta_1 \mathds{1}_i\{B\} + \vartheta_j + u_{ij} \quad \text{and} \quad \bar{y}_{ij} = \zeta_H \mathds{1}_i\{B\} \times \mathds{1}_j\{H\} + \zeta_M \mathds{1}_i\{B\} \times \mathds{1}_j\{M\} + \vartheta_j + u_{ij},
\]

(49)

where \(\bar{y}_{ij}\) is the average yield over time of corporate bond \(i\) with Moody’s credit rating \(j\), \(\mathds{1}_i\{B\}\) is a variable taking the value of 1 if corporate bond \(i\) is CME acceptable and 0 otherwise, and \(\mathds{1}_j\{k\}\) takes the value of 1 if credit rating \(j\) is in investment-grade group \(k\) and 0 otherwise, for \(k \in \{H, M\}\) — \(H\) indicates a high-grade bond and \(M\) indicates a medium-grade bond. The term \(\vartheta_j\) denotes a credit-rating fixed effect and \(u_{ij}\) is the error term. The parameters of interest are \(\zeta\), \(\zeta_H\), and \(\zeta_M\), which indicate the yield difference between acceptable and non-acceptable bonds overall and for each of the investment-grade groups.\(^{29}\)

Table 3 presents the results from the estimation of the specifications in (49) using three different samples: March 2019, the 2014-2019 unbalanced sample, and the 2014-2019 balanced sample. All

The smallest share of acceptable bonds (16 out of 987). Given this difference, the comparison between acceptable and non-acceptable low-grade bonds would not be reliable.

\(^{29}\)In a balanced panel, the econometric models above are equivalent to models that include time fixed effects and use \(y_{ijt}\) as the dependent variable, where \(t\) denotes month. That is, \(\bar{y}_{ij} = \zeta_1 \mathds{1}_i\{B\} + \vartheta_j + u_{ij}\) is the same model as \(y_{ijt} = \zeta_1 \mathds{1}_i\{B\} + \vartheta_j + \varpi_t + u_{ijt}\), where \(\varpi_t\) denotes a time fixed effect.
Table 3: Relationship between Acceptability and Corporate Bond Yields

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>By Investment-Grade Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>March 2019 (1)</td>
<td>2014-2019 (2)</td>
</tr>
<tr>
<td>$I_i {B}$</td>
<td>-0.247*** (0.025)</td>
<td>-0.248*** (0.027)</td>
</tr>
<tr>
<td>$I_i {B} \times I_j {H}$</td>
<td>-0.312*** (0.049)</td>
<td>-0.283*** (0.057)</td>
</tr>
<tr>
<td>$I_i {B} \times I_j {M}$</td>
<td>-0.230*** (0.029)</td>
<td>-0.242*** (0.031)</td>
</tr>
</tbody>
</table>

Sample Observations | 6,489 | Unbalanced | 6,489 | Balanced | 1,789 | Unbalanced | 6,489 | Balanced | 1,789 |

Notes: This table reports $\hat{\zeta}$, $\hat{\zeta}_H$, and $\hat{\zeta}_M$ from the estimation of the specifications in (49). The dependent variable is the bond-level average yield over the period indicated at the top of the column. In the unbalanced sample, the average yield for each bond is calculated over the months the bond appears in the data. All regressions include Moody’s credit-rating fixed effects (one dummy variable for each of the ten Moody’s categories). Robust standard errors in parentheses. The coefficients are statistically significant at the *10%, **5%, or ***1% level.

The columns show negative and statistically significant (at the 1% level) coefficients, showing that acceptability is strongly negatively correlated with yields. Overall, columns 1 to 3 show that after controlling for credit rating, acceptable bonds’ yields are on average between 25 and 29 basis points lower than yields on non-acceptable bonds. By investment-grade group, columns 4 to 6 show yields on acceptable bonds that are on average between 28 and 34 basis points lower for high-grade bonds, and between 23 and 29 basis points lower for medium-grade bonds.

6.3 Acceptability, Economic Activity, and Market-to-Book Ratios

Using the 2011-2017 annual Compustat data, here I look into the relationship between acceptability and the allocation of economic activity. To capture the international dimension of this relationship, the firm-level measures of economic activity are relative to the “world” level of a firm’s four-digit SIC industry. In particular, I use a firm’s share in the world’s four-digit industry; for example, the share in sales of firm $i$ from industry $j$ and country $k$ at fiscal year $t$ is calculated as

$$\frac{\text{sales}_{ijkt}}{\sum_k \sum_i \text{sales}_{ijkt}},$$

where the denominator takes the sum across the 47 countries over all industry-$j$ firms that report sales during that fiscal year. Besides the share in the industry’s worldwide sales, the other measures of firm-level economic activity are the shares in the industry’s worldwide employment, R&D expenditure, book value, market value, as well as the ratio of profits to industry sales.\(^{30}\)

\(^{30}\)Note that instead of the profits share, I use the ratio of profits to industry sales. I do not use the profits share because firms can have losses (negative profits), which obscures the meaning of $\sum_k \sum_i \text{profits}_{ijkt}$. 

44
Let $1_i\{S/B\}$ denote an indicator variable taking the value of 1 if firm $i$ issues CME acceptable stock or bonds, and 0 otherwise. As well, let $e_{ijkt}$ denote the firm-level measure of economic activity, which can be any of the six share measures mentioned above. The specification to estimate the relationship between acceptability and economic activity is given by

$$
\bar{e}_{ijk} = \beta 1_i\{S/B\} + \zeta_J + \nu_k + \varepsilon_{ijk},
$$

where $\bar{e}_{ijk}$ is the average of the economic-activity variable over the years firm $i$ appears in the data, $\zeta_J$ is a two-digit SIC industry fixed effect ($J$ indicates the two-digit industry), $\nu_k$ is a headquarters-country fixed effect, and $\varepsilon_{ijk}$ is the error term. The coefficient of interest is $\beta$, which captures the difference in the economic-activity variable between acceptable and non-acceptable firms.

The first row of Table 4 shows the estimates of $\beta$ for each of the six measures of economic activity. To verify whether stock acceptability and bond acceptability yield different results, the second and third rows of coefficients show the estimates of $\beta$ that result from replacing the acceptability dummy variable $1_i\{S/B\}$ with either $1_i\{S\}$ or $1_i\{B\}$; $1_i\{S\}$ is a dummy variable taking the value of 1 if firm $i$ issues acceptable stock, and $1_i\{B\}$ is a dummy variable taking the value of 1 if firm $i$ issues one or more acceptable corporate bonds.\(^{31}\) Note that all the coefficients in Table 4 are positive and statistically significant at a 1% level; thus, acceptability is positively and strongly correlated with all measures of economic activity. After controlling for headquarters country and two-digit industry, the coefficients in the first row indicate that stock/bond acceptability is associated with larger shares in worldwide industry sales (by 10 p.p.—percentage points), employment (by 9.6 p.p.), R&D expenditure (by 12 p.p.), book value (by 9.2 p.p.), and market value (by 20 p.p.), and with a larger ratio of profits to industry sales (by 2.1 p.p.).\(^{32}\) Using instead $1_i\{S\}$ or $1_i\{B\}$ does not alter the coefficients in a meaningful way, with the largest changes occurring for bond acceptability in R&D expenditure (change from 0.12 to 0.161) and market value (change from 0.2 to 0.242).

The market value of a firm is calculated as the stock price times the number of shares, while the book value is calculated as the firm’s assets minus liabilities. A result from Table 4 is that the coefficient on acceptability is more than twice larger in the market-value regressions than in the book-value regressions.\(^{33}\) An explanation for this finding is that the market value of a firm directly incorporates the liquidity premium as well as the indirect effects of providing liquid assets (such as capturing a larger market share and obtaining higher profits), while the book value only captures the

31 From the end of section 6.1, $1_i\{S/B\}$ is 1 for 415 firms, $1_i\{S\}$ is 1 for 363 firms, and $1_i\{B\}$ is 1 for 126 firms.

32 Regarding the employment variable, Compustat reports the worldwide employment of the firm, and thus, the estimated coefficient does not necessarily imply a strong positive relationship between acceptability and employment in the headquarters country.

33 Importantly, this result is not due to the smaller sample in the market-value regressions, which only includes firms in the Compustat North America database. The estimation of the three book-value regressions for a restricted sample that includes only the 6,745 observations used in the market-value regressions yields coefficients 0.095, 0.099, and 0.116. These three estimated coefficients are statistically significant at a 1% level.
Table 4: Acceptability and Economic Activity, 2011-2017

<table>
<thead>
<tr>
<th></th>
<th>sales</th>
<th>profits</th>
<th>employment</th>
<th>R&amp;D</th>
<th>book value</th>
<th>market value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>$1_i{S/B}$</td>
<td>0.100***</td>
<td>0.021***</td>
<td>0.096***</td>
<td>0.120***</td>
<td>0.092***</td>
<td>0.200***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.002)</td>
<td>(0.008)</td>
<td>(0.013)</td>
<td>(0.007)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$1_i{S}$</td>
<td>0.109***</td>
<td>0.023***</td>
<td>0.103***</td>
<td>0.123***</td>
<td>0.100***</td>
<td>0.207***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.002)</td>
<td>(0.009)</td>
<td>(0.014)</td>
<td>(0.008)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>$1_i{B}$</td>
<td>0.111***</td>
<td>0.026***</td>
<td>0.103***</td>
<td>0.161***</td>
<td>0.097***</td>
<td>0.242***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.003)</td>
<td>(0.016)</td>
<td>(0.024)</td>
<td>(0.014)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Observations</td>
<td>36,037</td>
<td>36,036</td>
<td>25,129</td>
<td>18,832</td>
<td>36,162</td>
<td>6,745</td>
</tr>
</tbody>
</table>

Notes: The first row in this table reports $\hat{\beta}$ from the estimation of the specification in (50), with each column indicating a different economic-activity variable. With the exception of column 2, each dependent variable is calculated as the average over time of the firm’s share in the world’s total of the four-digit industry. For column 2, the dependent variable is calculated as the average over time of the firm’s ratio of profits to the world’s total sales of the four-digit industry. The second and third row of coefficients change the acceptability variable to $1_i\{S\}$ and $1_i\{B\}$, respectively. All regressions include two-digit industry and headquarters-location dummy variables. Robust standard errors in parentheses. The coefficients are statistically significant at the *10%, **5%, or ***1% level.

indirect effects. In this context, the analysis of market-to-book ratios can serve as an additional tool to study the relationship between acceptability and the liquidity premium. Given that market value is only in reported in the Compustat North America database, the sample for the market-to-book ratio analysis is limited to 6,745 firms. This reduced sample mainly contains firms from the U.S. and Canada, but it also includes firms headquartered in 42 other countries (accounting for 1,017 of the 6,745 firms) that are publicly listed in North America.

To account for heterogeneous market, production, and financial conditions across industries—which may lead to different distributions of market-to-book ratios across industries—I construct a measure that compares the market-to-book ratio of a firm against the average market-to-book ratio of the firm’s four-digit industry. Let $R_{ijkt}$ denote the market-to-book ratio of firm $i$ from industry $j$ and country $k$ in year $t$, calculated as $R_{ijkt} = \frac{\text{market value}_{ijkt}}{\text{book value}_{ijkt}}$, and let $\bar{R}_{jt}$ denote the average market-to-book ratio in industry $j$ in year $t$ (weighted by book value). Hence, I define firm $i$’s market-to-book ratio log deviation from its industry mean as $d_{ijkt} = \ln \left( \frac{R_{ijkt}}{\bar{R}_{jt}} \right)$. Taking the average of $d_{ijkt}$ over time, denoted by $\bar{d}_{ijk}$, the specification to estimate the relationship between acceptability and market-to-book ratios is given by

$$\bar{d}_{ijk} = \beta 1_i\{S/B\} + \chi \Delta_{ijk} + \varsigma_j + \nu_k + \varepsilon_{ijk}, \quad (51)$$

where the terms on the right-hand side are defined as in equation (50), and the additional term $\Delta_{ijk}$ is a vector of firm-level characteristics. The variables included in $\Delta_{ijk}$ are the firm’s average share in the industry sales and the average ratio of profits to industry sales, which control for the effects that
Table 5: Acceptability and Market-to-Book Ratios, 2011-2017

<table>
<thead>
<tr>
<th></th>
<th>Average M/B log deviation from industry mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>$I_i(S/B)$</td>
<td>0.298***</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
</tr>
<tr>
<td>$I_i(S)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_i(B)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales share</td>
<td>-0.372**</td>
</tr>
<tr>
<td></td>
<td>(0.184)</td>
</tr>
<tr>
<td>Profits ratio</td>
<td>2.476***</td>
</tr>
<tr>
<td></td>
<td>(0.957)</td>
</tr>
<tr>
<td>Observations</td>
<td>6,745</td>
</tr>
</tbody>
</table>

Notes: Columns 1 and 2 report $\hat{\beta}$ from the estimation of the specification in (51) without and with firm-level controls. The dependent variable is the average over time of the firm’s market-to-book ratio log deviation from its industry mean. The firm-level controls are the average (over time) of the firm’s share in the four-digit industry sales, and the average (over time) of the ratio of the firm’s profits to the four-digit industry sales. Columns 3 and 4 use $I_i(S)$ as the acceptability variable, and columns 5 and 6 use $I_i(B)$. All regressions include two-digit industry and headquarters-location dummy variables. Robust standard errors in parentheses. The coefficients are statistically significant at the *10%, **5%, or ***1% level.

market share and performance have on market-to-book ratios.

Columns 1 and 2 in Table 5 show the results from the estimation of (51) without and with firm-level controls. Columns 3 to 6 present similar estimations, but replace the acceptability variable with either $I_i(S/B)$ or $I_i(B)$. The coefficient on acceptability in each of the six regressions is positive and statistically significant at a 1% level, showing a strong positive relationship between acceptability and market-to-book ratios; column 2, for example, shows that a firm that issues acceptable stock or bonds has a market-to-book ratio that is 27.6% larger than a similar firm that does not issue acceptable assets. Comparing odd and even columns, note that adding firm-level controls barely reduces the acceptability coefficient when using $I_i(S/B)$ or $I_i(S)$, while it is more responsive when using $I_i(B)$ (the coefficient changes from 0.315 to 0.248). The coefficients on the firm-level controls in the even columns are also statistically significant, showing that firms with larger market shares have smaller market-to-book ratios, and that firms with larger ratios of profits to industry sales have larger market-to-book ratios.

Finally, as a robustness check I perform a propensity-score-matching analysis to estimate the average treatment effect on the treated (ATET) of acceptability on the market-to-book ratio log deviation from the industry mean, while using the share of sales and profits as covariates. The ATET
estimates are 0.331 when using $I_i\{S/B\}$, 0.349 when using $I_i\{S\}$, and 0.291 when using $I_i\{B\}$, with all of them being statistically significant at a 1% level.\footnote{The Abadie-Imbens robust standard errors are respectively 0.054, 0.064, and 0.103. Each estimation uses 6,697 observations.} Therefore, compared to a counterfactual group, acceptability is associated with market-to-book ratios that are between 29 and 35 percent larger. To sum up, acceptability is not only strongly positively correlated with various measures of economic activity, but it is also associated with larger market-to-book ratios, suggesting the existence of large liquidity premiums.

7 Conclusion

The United States is by far the most important provider of public and private liquidity for the world’s financial system. But how does this benefit U.S. firms and consumers? This paper showed that a country’s capacity to generate liquid assets is a source of comparative advantage in international trade. It increases the size and average productivity of the real-economy sectors that generate liquid assets, lowers aggregate prices, and increases households’ welfare. This happens at the expense of competing foreign sectors that are not as good at providing liquidity. This is shown with a model that combines a new monetarist framework for the liquidity market and the standard Melitz’s model of international trade. In addition to the benchmark model, I demonstrate the model’s tractability by developing one version that endogenizes asset acceptability, and another version that accounts for heterogeneous liquidity properties across public and private assets within and between countries, and across private assets within each country.

In the model, the liquidity of an asset is determined by the asset’s acceptability as collateral in financial markets. The model shows that acceptability is associated with lower yields (and thus higher liquidity premiums) and better measures of economic activity. Using yield data of corporate bonds issued in the U.S. and firm-level data from 47 countries, along with a list of acceptable assets from a large U.S. central counterparty for the clearing of OTC derivatives, I find strong empirical support for the relationships implied by the model. In particular, I find (i) that yields on acceptable bonds are on average between 25 and 29 basis points lower than yields on non-acceptable bonds; (ii) that acceptability is associated with larger firm-level shares in the industry’s worldwide sales (by 10 p.p.), employment (by 9.6 p.p.), R&D expenditure (by 12 p.p.), book value (by 9.2 p.p.), and market value (by 20 p.p.), and with a larger ratio of profits to industry sales (by 2.1 p.p.); and (iii) that a firm that issues acceptable stock or bonds has a market-to-book ratio that is on average 27.6% larger than a similar non-acceptable firm.

The model can be extended to accommodate multinationals or offshoring possibilities. The current setting yields that differences in financial development translate to more economic activity in the most
developed country, and that trade liberalization exacerbates the economic activity gap. Accounting for production process fragmentation or horizontal FDI will still yield the same implications regarding total firm capitalization based on country’s ownership, but may allow for increases in economic activity in the least developed country through births and expansions of multinational subsidiaries or arm’s-length providers.

References


