Money and Credit Redux*

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Abstract

Do we need both money and credit? In models with explicit roles for payment instruments, we show the answer is no. If credit is easy money is useless; if credit is tight money can be essential, but then credit is irrelevant, and changes in debt limits are neutral – real balances respond endogenously to leave total liquidity constant. This is true for exogenous or endogenous policy and debt limits, secured or unsecured credit, and fairly general preferences and pricing mechanisms. While we also show how to overturn some results, the benchmark model suggests credit conditions matter less than some people think.

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1 Introduction

In his Introduction to von Mises (1953), Lionel Robbins says "Of all branches of economic science, that part which relates to money and credit has probably the longest history and the most extensive literature." The issues are as vital today as they were then, and this project revisits them, by asking the following: Although one sees money and credit in everyday use, are they are both essential? Following Hahn (1973), an institution like money is said to be essential if the set of incentive-feasible allocations is bigger or better with it than without it.[1] In models where explicit frictions necessitate some instrument of intertemporal exchange, we show there is no need for both: if credit is easy, money is inessential; if credit is tight, money is essential, but credit is irrelevant, and changes in debt limits are neutral. Thus, within the bounds consistent with monetary equilibrium, tighter credit conditions lead to endogenous increases in real money balances, leaving total liquidity constant. This is true for exogenous or endogenous limits on debt and policy, secured or unsecured credit, and a fairly general class of trading mechanisms and preferences.

The results are relevant for several reasons. First, some people think that credit conditions are critical for macroeconomic performance (e.g., see Gertler and Kiyotaki 2010 and references therein). If their thinking is based on nonmonetary theories, they ought to check if their conclusions survive the introduction of currency. Second, as Wallace (2013) emphasizes, it is important to build models with both money and credit in order to understand certain aspects of monetary policy. Third, the issue is challenging, because obviously we need to deviate from the frictionless Arrow-Debreu paradigm. We use the now fairly standard New Monetarist approach.[2] The method involves describing an environment, including preferences, technologies and frictions

[1] See Townsend (1987,1988), Kocherlakota (1998), Wallace (2001,2010), Mills (2007), Aliprantis et al. (2007), Araujo and Minetti (2011), Araujo et al. (2012), Gu et al. (2013*a*), Araujo and Hu (2014) and Nosal et al. (2014) for applications to the essentiality of money, credit, banking and intermediation.

[2] Recent surveys of this literature and related material include Williamson and Wright (2010*a*,*b*), Wallace (2010), Nosal and Rocheteau (2011) and Lagos et al. (2014).

like spatial or temporal separation and imperfect information or commitment, where agents explicitly trade with each other, and not merely against their budget lines. In this context we can start to ask *how* they trade.

Proceeding along these lines, assumptions usually adopted to make money essential often make credit untenable, and vice versa, so it is not trivial to get both in the same model without ad hoc devices.[3] Consider Kocherlakota's (1998) formalization of the idea that money is a substitute for credit. He shows money cannot be essential if we can support credit using full commitment/enforcement. Nor can it be essential, even without commitment/enforcement, if we have perfect information (monitoring and record keeping, or what he calls memory) about agents' past actions. Money can be essential if we have neither commitment nor the requisite information, e.g., as in models along the lines of Kiyotaki and Wright (1989,1993), but then credit is completely ruled out. And while models along the lines of Kehoe and Levine (1993,2001), e.g., provide a foundation for endogenous credit conditions, they ignore money, with exceptions mentioned below. Our goals are to integrate elements from these microfounded models of money and of credit, and to study their interactions.

While our background environment is standard, we use a relatively general version. First, following Lagos and Wright (2005), quasi-linear utility is usually used in these models by those interested in analytic results (although not in computational work, e.g., Chiu and Molico 2010,2011). Taking advantage of recent developments by Wong (2012), we get analytic results for a larger class of preferences. This is relevant because, although our environment is not the most general imaginable, we want the results to be more than examples. Second, rather that picking a particular mechanism to determine the terms of trade, like bargaining or price taking, as used in previous

[3] For the issues at hand, it is not appropriate to *assume* missing markets, incomplete contracts etc., although something like that may *emerge* as an outcome. As Townsend (1988) says, "theory should explain why markets sometimes exist and sometimes do not, so that economic organization falls out in the solution to the mechanism design problem." As regards money and credit, in particular, Townsend (1989) asks, "Can we find a physical environment in which currency-like objects play an essential role in implementing efficient allocations? Would these objects coexist with ... credit?" To think seriously about these questions it is an obvious nonstarter to impose at the outset a partition of commodity space into cash goods and credit goods, as in Lucas and Stokey (1987).

work, we adopt a general specification that nests these plus more abstract formulations (e.g., Hu et al. 2009). Third, we consider exogenous and endogenous debt limits, as in Kehoe and Levine (1993), and we consider exogenous policy and endogenous limits to policy, as in Andolfatto (2013). We also consider secured credit, as in Kiyotaki and Moore (1997). These features are all relevant because we do not want the results to hinge on particular ways of determining prices, exogenous restrictions on the amount or type of credit, and arbitrary restrictions on policy.

As regards other related work, Townsend (1989) combines private information, spatial separation and limited monitoring/communication to get credit used by agents who know each and currency used by those who don't (see also Corbae and Ritter 2003 and Jin and Temzelides 2004). Our analysis is similar in spirit, but the environment is different on many dimensions. In particular, rather than agents knowing some people and not others, here opportunistic behavior is only detected randomly.[4] In a similar environment, Telyukova and Wright (2008) have multiple rounds of centralized trade where credit is available, alternating with a decentralized round where it is not because it is not monitored; we want to at least give credit a chance in decentralized trade. Lotz and Zhang (2013), Gomis-Porqueras and Sanches (2013), Araujo and Hu (2014) and Chiu et al. (2014) also study similar environments, with heterogeneity; we discuss in detail below how heterogeneity affects our results, but want to start with a representative-agent model.

Several papers following Shi (1996) have money and credit used in decentralized markets because they are complements, in the sense that debts are assumed to be settled in cash; here money and credit are substitutes as alternative ways to facilitate intertemporal trade.[5] Also, as mentioned, we endogenize credit limits so that

[4] This modeling device is taken from Gu et al. (2013a,b). Related papers with imperfect monitoring or record keeping include Kocherlakota and Wallace (1998), where agents are monitored with a lag; Cavalcanti and Wallace (1999a,b), where some agents are monitored while others are not; Sanches and Williamson (2010), where some meetings are monitored and other not; and Amandola and Ferraris (2013), where information sometimes gets lost.

[5] Li (2001), e.g., building on Shi (1996), has money and private debt, where the former is used as a medium of exchange and to repay debt or buy second-hand debt. See also Li (2007), Berentsen et al. (2007), Camera and Li (2008), Ferraris and Watanabe (2008), Ferraris (2010), Geromichalos

the substantive results do not hinge on arbitrary assumptions (e.g., money is clearly irrelevant when these limits are big). For similar reasons, we endogenize limits to policy (e.g., credit is usually irrelevant if we can run the Friedman rule). Also, we avoid imposing intrinsic properties favoring alternative payment instruments exogenously, since for the issues of interest here, that is too easy.[6] To summarize, we study money and credit as substitutes in decentralized exchange, where limits to debt and policy can be endogenous, in economies with commitment and information frictions, as well as classes of trading mechanisms and preferences that are fairly general, at least compared to some related work.

Section 2 presents the environment. Section 3 shows that, with homogeneous agents, an exogenous policy and exogenous debt limit, if the limit does not bind money is not valued, and if it binds money can be valued but credit is inessential and changes in debt limits are neutral. Section 4 discusses mechanisms. Section 5 endogenizes limits on debt and policy. Section 6 considers extensions. Given the same debt limit across buyer-seller pairs, credit is still inessential and changes in debt limits are still neutral with heterogeneous agents, as long as they are all credit constrained and at an interior solution for money demand; the results can be overturned if some households are credit constrained but at a corner solution for money demand. It also shows that when we add collateralized credit, changes in unsecured debt limits or pledgeability are neutral if the assets used as collateral are in fixed supply; the results also hold for debt limits but not for pledgeability if the collateral is instead a factor of production and reproducible. Second 6 also studies nonstationary equilibria. Section 7 concludes.

and Herrenbrueck (2011), Li and Li (2013) and He et al. (2014). In these models money and other payment instruments are complements; again, we model them as substitutes.

[6]Models of payment instruments with different attributes include He et al. (2005,2008) and Sanches and Williamson (2010), who assume that cash is subject to theft while credit and bank deposits are not, and Kahn et al. (2005) and Kahn and Roberds (2008), who assume the opposite. Also in environments similar to the one used here, Dong (2011), Bethune et al. (2014) and Liu et al. (2014) assume that the use of credit is costly, so cash may be better for small purchases. This approach follows a old tradition in reduced-form monetary economics of assuming costly credit; see Nosal and Rocheteau (2011, chapter 8) for citations to that branch of the literatute.

2 Environment

Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions as detailed below; then there is a frictionless centralized market, or CM. Each period in the CM, a large number of infinitely-lived households work, consume, adjust their portfolios and settle their debt/tax obligations – or renege on these obligations, as the case may be. In the DM, agents called *sellers*, denoted by s, can produce but do not want to consume, while others called *buyers*, denoted by b, want to consume but cannot produce. Buyers and sellers in the DM trade bilaterally in the baseline model, but we mention below how to have them trade multilaterally. So, for now, they meet pairwise and at random, with α denoting the probability that a buyer meets a seller in the DM.[7]

The period utility functions of buyers and sellers are

$$\mathcal{U}^b(q,x,\ell) = u(q) + U^b(x,\ell) \text{ and } \mathcal{U}^s(q,x,\ell) = -c(q) + U^s(x,\ell), \tag{1}$$

where q is the DM good, x is the CM good, and ℓ is leisure. Labor is $1-\ell$, and for now 1 unit of labor produces ω units of x, so ω is the CM real wage. The constraints $x \geq 0$, $q \geq 0$ and $\ell \in [0,1]$ are assumed not to bind, as can be guaranteed in the usual way. Also, U^j, u and c are twice continuously differentiable and strictly increasing, U is concave, $u'' \leq 0 \leq c''$ with one equality strict, and $u(0) = c(0) = 0$. The usefulness of the following restriction, adapted from Wong (2012), will be clear below:

Assumption 1 $|U^j| = 0$*, where* $|U| = U_{11}U_{22} - U_{12}^2$.

This is true for any quasi-linear function, $U = \tilde{U}(x) + \ell$ or $U = x + \tilde{U}(\ell)$, and for any that is homogeneous of degree 1, such as $U = x^a \ell^{1-a}$ or $U = (x^a + \ell^a)^{1/a}$.

[7] To endogenize α, it is not hard to specify a general matching technology, with or without entry/participation decisions on either side of the market. And, instead of saying buyers in the DM meet trading partners randomly, we can alternatively say α is the probability of a preference shock, and buyers hit with the shock visit sellers, either using directed or undirected search, at which point they trade, either bilaterally or multilaterally. The key results are the same. See the references in fn. 2 on implementing these and other extensions to the benchmark model.

There is discounting between the CM and DM according to $\beta = 1/(1+r)$, $r > 0$; any discounting between the DM and CM can be subsumed in the notation in (1). Goods q and x are nonstorable. There is an intrinsically worthless object called money that is storable. The money supply per buyer M changes over time at rate π, so that $M_{+1} = (1+\pi) M$, where the subscript $+1$ (or -1) on a variable indicates its value next (or last) period. Changes in M are accomplished by lump sum transfers if $\pi > 0$ or taxes if $\pi < 0$. We restrict attention to $\pi > \beta - 1$, or the limit $\pi \to \beta - 1$, which in this model is the Friedman rule; there is no monetary equilibrium with $\pi < \beta - 1$.

There are two standard ways to model money or credit. One is to assume a desire by individuals to smooth consumption in the presence of fluctuating resources. The other is to assume a desire to satisfy random consumption needs or opportunities. We use the latter, although any asynchronization of agents' resources and expenditures would work. In our DM, with probability α buyers have opportunities to get q from sellers, and the focus is on the payment instrument, cash or credit. Credit means a promise of numeraire in the next CM. Because there is no commitment or enforcement, generally, we need to incorporate punishments for those who renege on promises. As in Kehoe and Levine (1993), the nature of this punishment puts restrictions on debt. The same considerations apply to taxes: agents can renege on public obligations, like private obligations, with similar consequences. As in Andolfatto (2013), this puts restrictions on policy.

While we consider different punishments, as a benchmark, those caught reneging move to future autarky. As in Gu et al. (2013a,b), reneging is monitored, and hence punished, only stochastically. Here is one interpretation: If you fail to pay taxes, the fiscal authorities see this only if they audit you, which is random. Similarly, debtors pay into a common fund that is dispersed to lenders, and your failure to contribute is only noticed if the credit authorities audit you. Whatever the story, we need monitoring to be possible but not perfect to have a hope of getting both money and credit used in equilibrium (see Proposition 7).

3 Exogenous Policy and Debt Limits

We first study equilibrium for given limits to debt and deflation. This may be of interest in its own right, and is a stepping stone toward endogenizing these limits. For now the only asset is fiat currency, but this is generalized in Section 6.

3.1 The CM Problem

The state of an agent in the CM is his net worth, $A = \phi m - d - T$, where ϕ is the value of money in terms of numeraire x, d is debt and T is a tax. For convenience, only buyers, and not sellers, pay taxes if $T > 0$ or get transfers if $T < 0$. Debt, which comes from the previous DM, is paid off in the current CM with no loss of generality (without changing the results we could let agents roll it over, given the usual conditions to rule out Ponzi schemes). The value functions in the CM and DM are $W(A)$ and $V(\phi m)$. Until Section 6.4 we focus on stationary outcomes. This means real variables are constant, including ϕM, and hence $\phi/\phi_{+1} = 1 + \pi$ is the rate of inflation as well as monetary expansion. It also means $W(\cdot)$ and $V(\cdot)$ are time invariant.

The CM problem for an agent of type $j = b, s$ (buyer or seller) is

$$W_j(A) = \max_{x,\ell,\hat{m}} \left\{ U^j(x,\ell) + \beta V_j(\phi_{+1}\hat{m}) \right\} \text{ st } A + \omega(1-\ell) = x + \phi\hat{m}. \tag{2}$$

Let $x = x(A)$, $\ell = \ell(A)$ and $\hat{m} = \hat{m}(A)$ be a solution, satisfying the FOC's

$$\begin{aligned} -\omega U_1^j(x,\ell) + U_2^j(x,\ell) &= 0 && (3) \\ A + \omega(1-\ell) - \phi\hat{m} - x &= 0 && (4) \\ -\phi U_1^j(x,\ell) + \beta\phi_{+1} V_j'(\phi_{+1}\hat{m}) &\leq 0, \ = \text{ if } \hat{m} > 0. && (5) \end{aligned}$$

Sellers choose $\hat{m}_s = 0$, since they have no use for cash in the DM. For buyers, $\hat{m} = \hat{m}_b > 0$ in monetary equilibrium (this is defined more formally below, but for now think of a monetary equilibrium as a situation with $\phi > 0$).

Assumption 1 implies several results that greatly simplify the analysis:

Lemma 1 *Given an interior solution for $x(A)$ and $\ell(A)$, $\hat{m}_j'(A) = 0$.*

Lemma 2 *Let $\Lambda_j(A) = U_1^j[x(A), \ell(A)]$. Then $W_j'(A) = \Lambda_j(A)$ and $\Lambda_j'(A) = 0$. Let $U_0^j = U^j[\omega - \ell(0)\omega, \ell(0)]$. Then $U^j[\omega - \ell(A)\omega + A, \ell(A)] = U_0^j + \Lambda_j A$.*

Proofs are in the Appendix. In terms of substance, Lemma 1 says all buyers take the same $\hat{m}$ out of the CM, independent of A and hence independent of the m they brought in, so we do not have to track the distribution of $\hat{m}$ across buyers in the DM as a state variable. Lemma 2 says CM payoffs are linear.[8]

3.2 The DM Problem

A buyer trades with a seller in the DM with probability α, and they have to choose a quantity q and payment p subject to $p \leq L$ where $L = D + \phi m$ is the *liquidity position* of the buyer, his debt limit plus real balances. To determine the terms of trade (p, q), we adopt a *general trading mechanism*, denoted Γ, assuming only mild conditions. First, a trade (p, q) can depend on the trading surpluses,

$$\begin{aligned} S_b &= u(q) + W_b(A_b - p) - W_b(A_b) = u(q) - \Lambda_b p && (6) \\ S_s &= -c(q) + W_s(A_s + p) - W_s(A_s) = \Lambda_s p - c(q), && (7) \end{aligned}$$

which depend on the marginal utility of wealth (Λ_b, Λ_s), but not on wealth (A_b, A_s), by Lemma 2. Second, (p, q) can depend on L because of the constraint $p \leq L$.[9] In general, $p = \Gamma_p(L; \Lambda_b, \Lambda_s)$ and $q = \Gamma_q(L; \Lambda_b, \Lambda_s)$, but to reduce notation we often write $p = \Gamma_p(L)$ and $q = \Gamma_q(L)$.

Given (Λ_b, Λ_s), define the unconstrained efficient quantity q^* by

$$u'(q^*)/\Lambda_b = c'(q^*)/\Lambda_s. \tag{8}$$

[8] Versions of these results appear in Wong (2012), who also characterizes the class of functions U for which $|U| = 0$ holds, making his argument more involved. Hence, the Appendix includes a simpler proof. Without Lemma 1, we would have to track the distribution of $\hat{m}$ as in Molico (2006), Chiu and Molico (2010,2011) or Dressler (2011). However, separability in q can be relaxed – e.g., it is easy to check that the main results go through with $\mathcal{U}^j = U^j(q, x) + \ell$.

[9] We understand that this is not the most general possible scenario. Related to Arauojo and Hu (2014), one can imagine outcomes might depend on $(D, \phi m)$, not just $L = D + \phi m$, even though only the sum matters for buyer and seller payoffs in our environment.

Let $p^* = \inf\{L : \Gamma_q(L) = q^*\}$ be the minimum payment required by the mechanism for a buyer to get q^*. To guarantee $q^* \in (0, \bar{q})$ exists, where $\bar{q}$ is a natural upper bound, we assume DM gains from trade are positive but finite:

Assumption 2 *$u'(0)/\Lambda_b > c'(0)/\Lambda_s$ and $\exists \bar{q} > 0$ such that $u(\bar{q})/\Lambda_b = c(\bar{q})/\Lambda_s$.*

Then we focus on mechanisms of the form

$$\Gamma_p(L) = \begin{cases} L & \text{if } L < p^* \\ p^* & \text{otherwise} \end{cases} \text{ and } \Gamma_q(L) = \begin{cases} v^{-1}(L) & \text{if } L < p^* \\ q^* & \text{otherwise} \end{cases} \tag{9}$$

where v is some strictly increasing function with $v(0) = 0$ and $v(q^*) = p^*$ (like Γ, v depends on the Λ_j's but that is also often suppressed in the notation).

This class of mechanisms includes standard bargaining solutions, competitive price taking and many other specifications. Section 4 presents axioms that imply Γ must take the form in (9) and discusses examples. The focus here is on economic content: (9) says that a buyer gets the efficient quantity q^* and pays some amount $p^* = v(q^*)$ determined by the mechanism, if he can afford it in the sense that $p^* \leq L$; and if he cannot afford it, he pays $p = L$ and gets $q = v^{-1}(L) < q^*$. Thus, $v^{-1}(L)$ is the quantity a constrained buyer gets, while $v(q)$ is how much he has to pay to get q. We also assume that v is twice continuously differentiable almost everywhere.

Consider a seller, who as we said takes no money to the DM. If he does not trade, he gets continuation value $W_s(0)$. If he trades, he gets this plus surplus $\Lambda_s p - c(q)$, where $p = \Gamma_p(\bar{L})$ and $q = \Gamma_q(\bar{L})$ depend on the liquidity position of the buyer with whom he trades. For a buyer in the DM with real balances ϕm,

$$V_b(\phi m) = W_b(\phi m - T) + \alpha [u(q) - \Lambda_b p], \tag{10}$$

where $p = \Gamma_p(L)$ and $q = \Gamma_q(L)$ depend on his own liquidity. It is easy to show (see the Appendix) the following:

Lemma 3 *In stationary monetary equilibrium buyers are constrained: $q < q^*$.*

Given $q < q^*$, in monetary equilibrium buyers exhaust liquidity, $p = D + \phi m$. Substituting this into V_b, then V_b into W_b, after simplifying we get

$$W_b(A) = U_0^b + \Lambda_b(A - \beta T) + \beta W_b(0) + \beta\left\{-i\Lambda_b\phi_{+1}\hat{m} + \alpha\left[u(q_{+1}) - \Lambda_b v(q_{+1})\right]\right\}, \quad (11)$$

where i denotes the nominal interest rate defined by the Fisher equation $1 + i = (1+\pi)/\beta$.[10] From the Fisher equation, it is equivalent to fix i or π, so we take i as the policy instrument. Then rewrite (11) as

$$W_b(A) = \Theta + \alpha\beta J(q; i),$$

where $\Theta = \Lambda_b A + U_0^b - \beta\Lambda_b T + \beta W_b(0) + \beta i \Lambda_b D$ is irrelevant for the choice of $\hat{m}$, and hence the objective function can be taken to be

$$J(q; i) = u(q) - (1 + i/\alpha)\Lambda_b v(q). \quad (12)$$

This replaces buyers' choice of $\hat{m}$ with the choice of q. Without loss of generality we impose $q \in [0, q^*]$, and represent the problem by

$$q_i = \arg\max J(q; i) \text{ st } q \in [0, q^*]. \quad (13)$$

In a monetary equilibrium $\phi m > 0$ and $v(q_i) > D$. This and Lemma 3 imply $v^{-1}(D) < q_i < q^*$, and q_i satisfies the FOC

$$e(q) \equiv u'(q) - (1 + i/\alpha)\Lambda_b v'(q) = 0. \quad (14)$$

Given a solution to $e(q_i) = 0$ and $m = M$ (market clearing), real balances are $\phi M = v(q_i) - D$. Therefore, given q_i, we have $\phi M > 0$ iff $D < v(q_i)$.

[10] To derive this, notice that

$$\begin{aligned} W_b(A) &= U_0^b + \Lambda_b(A - \phi\hat{m}) + \beta\left\{W_b\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u(q) - \Lambda_b v(q)\right]\right\} \\ &= U_0^b + \Lambda_b(A - \beta T) + \beta W_b(0) - \Lambda_b\phi\hat{m} + \beta\left\{\Lambda_b\phi_{+1}\hat{m} + \alpha\left[u(q) - \Lambda_b v(q)\right]\right\}, \end{aligned}$$

then use the Fisher equation. For our purposes, the Fisher equation is an accounting identity defining i, but we can also say i is the nominal return that makes agents indifferent to borrowing and lending between one CM and the next CM. The Friedman rule is the limit $i \to 0$.

3.3 Equilibrium

Here is the definition of (for now, stationary) equilibrium:

Definition 1 *Given a mechanism* Γ*, debt limit* D*, and policy* i*, a (symmetric, stationary) monetary equilibrium is a CM allocation* (x, ℓ)*, a DM outcome* (p, q) *and real balances* ϕM *such that:*

1. q *solves (13),* $p = v(q)$ *and* $\phi M = p - D > 0$*;*
2. (x, ℓ) *solves (2) for all agents, with* $\hat{m} = 0$ *for sellers,* $\hat{m} = M_{+1}$ *for buyers, and* $\int x = \omega \int \ell$ *(market clearing).*

Definition 2 *A nonmonetary equilibrium is similar except* $\phi M = 0$*.*

Notice that (p, q) can be determined independently of (x, ℓ), and hence we can discuss some properties of the DM outcome without reference to the CM.[11] Our method is this: look for a solution $q_i \in [0, q^*]$ to (13); if $p_i = v(q_i) > D$ then $\phi M > 0$ and monetary equilibrium exists; otherwise, we must have $\phi = 0$ and $q = \min\{q^*, v^{-1}(L)\}$.

To insure the solution to (13) is $q_i > 0$, impose:

Assumption 3 $\exists q > 0$ *such that* $\Lambda_b v(q) < u(q)$*.*

This holds automatically for any reasonable mechanism (e.g., it holds with competitive pricing, and with standard bargaining solutions iff the buyer has bargaining power $\theta > 0$). Given this, in the limit as $i \to 0$, $q_0 = \arg\max_q J(q; 0) > 0$. So $q_i > 0$ at least for i not too big. In the Appendix we also prove:[12]

Lemma 4 *For generic parameters, the solution to (13) is unique and* $\partial q/\partial i < 0$*.*

[11] This dichotomy is convenient but not critical for the results. The model does not dichotomize, e.g., when $\mathcal{U}^j = U^j(q, x) + \ell$, but the main results go through.

[12] Lemma 4 is similar to a result in Wright (2010); we include a proof since the setup is slightly different. Also, while the stationary monetary equilibrium happens to be generically unique, this is not crucial for our message: if multiple monetary equilibria were to exist, our results about credit would apply to all of them.

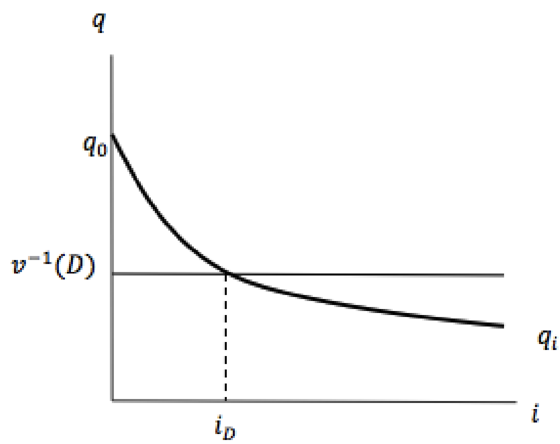

Figure 1: DM Quantity vs the Nominal Rate

Figure 1 plots q_i against i. The intercept is $q_0 \leq q^*$ (e.g., $q_0 = q^*$ with Walrasian pricing or with Kalai bargaining for any θ, and $q_0 < q^*$ with Nash bargaining iff $\theta < 1$). By Lemma 4, q_i is generically single-valued and strictly decreasing. Again, $\phi M > 0$ iff $v(q_i) > D$. Given a D such that $v^{-1}(D) < q_0$, as in Figure 1, there is a unique $i_D > 0$ such that monetary equilibrium exists iff $i < i_D$. Or, to state the results in terms of D:

Proposition 1 *There are three possible outcomes:*

1. *if $v(q^*) \leq D$ there is no monetary equilibrium and there is a nonmonetary equilibrium with $q = q^*$;*

2. *if $v(q_i) \leq D < v(q^*)$ there is no monetary equilibrium and there is a nonmonetary equilibrium with $q \in [q_i, q^*)$;*

3. *if $D < v(q_i)$ there is a (generically unique) monetary equilibrium with $q = q_i$ and there is a nonmonetary equilibrium with $q = v^{-1}(D) < q_i$.*

Proof: First suppose $D \geq v(q^*)$. Then buyers can get q^* on credit, and if we try to construct a monetary equilibrium we fail, since $\phi M = v(q_i) - D \leq v(q^*) - D \leq 0$. Now suppose $v(q^*) > D \geq v(q_i)$. Then buyers can only get $q = v^{-1}(D) < q^*$ on credit, but if we try to construct a monetary equilibrium we still fail, as $\phi M = v(q_i) - D \leq 0$.

Finally suppose $v(q_i) > D$. Then $\phi M = v(q_i) - D > 0$ and monetary equilibrium exists. It is generically unique by Lemma 4. ■

Now notice something interesting: in monetary equilibrium $q = q_i$ does not depend on D. This is because buyers acquire real balances up to the point where the marginal benefit equals i, or $e(q_i) = 0$, as given in (14). Hence, $\phi M = v(q_i) - D$ adjusts to guarantee that the liquidity provided by cash fills the gap between the p required to get q_i and the debt limit. This is not to say an individual's debt limit is irrelevant: if we keep everyone else the same and lower D for one agent he can be worse off; but if we lower D for everyone ϕ adjusts to keep $L = D + \phi M$ exactly the same. One manifestation of this is that welfare for the representative buyer at initial date $t = 0$, with M dollars and tax obligation T, does not depend on D:[13]

$$W_0 = W_b(\phi M - T) = \frac{U_0^b + \alpha\beta\left[u(q) - \Lambda_b v(q)\right]}{1-\beta}. \tag{15}$$

If D is low money is essential, since at least for some i there is a monetary equilibrium where welfare is higher than without money. But in monetary equilibrium credit is inessential and changes in debt limits are neutral. Now, if $v(q^*) > D > v(q_i)$ then D matters, but then equilibrium is nonmonetary. The neutrality of D in monetary equilibrium may be surprising, as one might have thought that higher D allows buyers to increase x or ℓ by cutting back on ϕm while staying equally liquid. That is incorrect. A change in desired real balances after a change in D is exactly offset by the change in the value of the currency buyers currently hold. There is *complete crowding out* of ϕM by D. If buyers start with different m, changes in D can have distributional effects; we come back to this in Section 6, while here for the straightforward representative-agent case, we summarize the results as follows:

[13] To derive this, notice that

$$\begin{aligned} W_b(\phi M - T) &= U_0^b + \Lambda_b(\phi M - T - \phi\hat{m}) + \beta\left\{W\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u(q) - \Lambda_b v(q)\right]\right\} \\ &= U_0^b + \beta\left\{W\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u(q) - \Lambda_b v(q)\right]\right\}, \end{aligned}$$

using $T = -\pi\phi M$ (the government budget) and $\hat{m} = (1+\pi)M$ (market clearing). Since $W\left(\phi_{+1}\hat{m} - T\right) = W_0$ in stationary monetary equilibrium, this reduces to (15).

Proposition 2 *Money is essential iff $D < v(q_i)$. In a stationary monetary equilibrium, both money and credit may be used, but credit is inessential and changes in D are neutral.*

4 Mechanisms

We do not want the results to depend on a particular way of determining the terms of trade. How general is our class of mechanisms? Consider the following:[14]

Axiom 1 *(Feasibility): $\forall L$, $0 \leq \Gamma_p(L) \leq L$, $0 \leq \Gamma_q(L)$.*

Axiom 2 *(Individual Rationality): $\forall L$, $u \circ \Gamma_q(L) \geq \Lambda_b \Gamma_p(L)$ and $\Lambda_s \Gamma_p(L) \geq c \circ \Gamma_q(L)$.*

Axiom 3 *(Monotonicity): $\Gamma_p(L_2) > \Gamma_p(L_1) \Leftrightarrow \Gamma_q(L_2) > \Gamma_q(L_1)$.*

Axiom 4 *(Bilateral Efficiency): $\forall L, \nexists (p', q')$ with $p' \leq L$ such that $u(q') - \Lambda_b p' > u \circ \Gamma_q(L) - \Lambda_b \Gamma_p(L)$ and $\Lambda_s p' - c(q) > \Lambda_s \Gamma_p(L) - c \circ \Gamma_q(L)$.*

Note Axiom 3 does not say S_b and S_s are increasing in L, only that one must pay more p to get more q, and so generalized Nash bargaining, e.g., satisfies this even though S_b may not be increasing in L (Aruoba et al. 2007). Also, while Axiom 4 seems reasonable, it is not critical for the main results about credit – e.g., they hold for the monopsony mechanism discussed below. Also, Axiom 4 is an ex post condition in the DM saying that we cannot make the parties better off conditional on L; it does not say the ex ante choice of L in the CM is efficient.

Proposition 3 *Any Γ satisfying Axioms 1-4 takes the form given in (9).*

[14] Again, $\mathbf{\Gamma}$ in general depends on not only L, but also (Λ_b, Λ_s), and hence on the wage ω. When the CM production function is linear, ω is fixed and we can suppress it in the notation; when we consider nonlinear production functions below we are more explicit. There are, however, special cases where this point is moot: if $\Lambda_b = \Lambda_s$, or if $U^j(x, \ell)$ is quasi-linear, then for many common mechanisms, including those discussed below, (Λ_b, Λ_s) and ω vanish from the surpluses S_b and S_s. And again, we maintain the assumption that $\mathbf{\Gamma}$ depends on $L = D + \phi m$, not $(D, \phi m)$.

The proof is in the Appendix; here we give examples. A simple one is Kalai's proportional bargaining solution, with θ the buyers' bargaining power. Kalai's solution in this context maximizes S_b wrt (p,q) subject to $S_b = \theta(S_b + S_s)$ and $p \leq L$. Let

$$v(q) = \frac{\theta c(q) + (1-\theta)\, u(q)}{\theta \Lambda_s + (1-\theta)\, \Lambda_b} \tag{16}$$

and $p^* = v(q^*)$. The solution is this: if $p^* \leq L$ then $(p,q) = (p^*, q^*)$; and if $p^* > L$ then $p = L$ and $q = v^{-1}(L)$ with v given by (16). Generalized Nash bargaining maximizes $S_b^{\theta} S_s^{1-\theta}$ wrt (p,q) subject to $p \leq L$, which gives a similar qualitative outcome, except[15]

$$v(q) = \frac{\theta u'(q)\, c(q) + (1-\theta)\, c'(q)\, u(q)}{\theta u'(q)\, \Lambda_s + (1-\theta)\, c'(q)\, \Lambda_b}. \tag{17}$$

We can also use Walrasian pricing, motivated by saying that agents trade in large groups, not bilaterally, but otherwise keeping the environment the same (Rocheteau and Wright 2005). This gives the same qualitative outcome with $v(q) = Pq$, where agents take P as given although in equilibrium $P = c'(q)/\Lambda_s$. An example that violates Axiom 4 is a monopsonist that takes as given $c'(q)/\Lambda_s$ rather than the price:

$$\max_q \left\{ u(q) - \Lambda_b q c'(q)/\Lambda_s \right\} \text{ st } qc'(q)/\Lambda_s \leq L. \tag{18}$$

Let $\tilde{q}$ be a solution without the liquidity constraint (the standard monopsonist outcome) and let $\tilde{p} = \tilde{q} c'(\tilde{q})/\Lambda_s$. This mechanism looks like (9), except the critical value is $\tilde{p} = v(\tilde{q})$ rather than $p^* = v(q^*)$. Still, our substantive results about money and credit hold for the monopsony mechanism.

In terms of less standard solution concepts, following Hu et al. (2009), consider trying to construct $v(q)$ so that equilibrium supports a desirable q^o, which could be q^* or something else. This is relevant because we want results that hold even when we try our best to achieve good outcomes with creatively-designed mechanisms. Given i, the next result describes what we can achieve using only money.

[15] The Nash and Kalai solutions are the same if $\theta = 1$ or $u(q) = c(q) = q$. If $\theta < 1$ and either $u'' < 0$ or $c'' > 0$, they are different when $p \leq L$ binds. But both take the form in (9).

Proposition 4 *Let $\hat{q}$ solve $u(\hat{q})/\Lambda_b = (1+i/\alpha)\, c(\hat{q})/\Lambda_s$. Then there exists a mechanism to support any $q^o \leq \min\{q^*, \hat{q}\}$. If $q^* \leq \hat{q}$ we can achieve $q_i = q^*$ even if $i > 0$. We cannot support $q^o > q^*$.*

The proof in the Appendix constructs the mechanism.[16] Intuitively, we want $v(q)$ to give buyers the incentive ex ante, in the CM, to choose the right $\hat{m}$. Given the objective function $J(q,i) = u(q) - (1+i/\alpha)\, \Lambda_b v(q)$, to get q^o we need $u'(q^o) = (1+i/\alpha)\, \Lambda_b v'(q^o)$, which tells us something about $v(q)$. But we also must ensure agents have incentives to trade q^o ex post, when they meet in the DM. A key observation is that $q > \hat{q}$ cannot be supported, and i big makes $\hat{q}$ small. When i is high, e.g., we cannot achieve q^* even with this mechanism. Hence, it can be desirable to reduce i as much as possible, which is why we want to endogenize policy limits.

5 Endogenous Policy and Debt Limits

If agents can renege then we need some monitoring. But for money to be essential credit cannot be too good, which means monitoring cannot be perfect (Proposition 7). At the same time, for credit to be essential it must not be possible to achieve desirable outcomes using money only. For some solution concepts, including Walrasian pricing and Kalai bargaining, $q_i = q^*$ iff $i = 0$. So at least for these mechanisms, one can ask, why not run the Friedman rule and forget about credit? One answer is that commitment problems might render $i = 0$ infeasible. While Proposition 4 shows it may be possible to achieve a desirable q^o even when $i > 0$, this is only true if $q^o < \hat{q}$, and high i makes $\hat{q}$ small, so we want to know how low i can go. Since $i < r$ entails deflation it necessitates taxation, $T > 0$, but individuals can choose to not pay T if it is too high. This is nice, we think, because the same frictions that hinder credit and hence can make money essential – imperfect commitment and monitoring – hinder the ability to tax and hence to achieve desirable outcomes using only money.

[16] There is more than one way to do this, and while our approach is in the spirit of Hu et al. (2009), the details are quite different. Note in particular that our mechanism is linear over the range where the DM incentive conditions are slack.

To determine just how much debt and deflation are viable, we need to specify the sequence of events in the CM. First, a buyer decides whether to pay d, which is monitored with probability μ_D. If he pays d, or does not pay but is not caught, he then decides whether to pay T, which is monitored with probability μ_T. If he pays T, or does not pay but is not caught, he chooses $(x, \ell, \hat{m})$ as before. Deviators that default on debts or taxes, if caught, are banished to autarky here, but the Appendix allows them to continue trading in the DM using cash. In autarky agents produce x for themselves, and pay no taxes or get no transfers in the CM. Although they are banned from future markets, deviators spend any cash on hand in the CM in the period when they are caught. Since anyone excluded from the DM in the future chooses $\hat{m} = 0$, the autarky payoff is $\underline{W}(\phi m) = \Lambda_b \phi m + U_0 (1+r)/r$.

Working backwards from the tax-payment decision, compliance requires[17]

$$W_b(\phi m - T) \geq \mu_T \underline{W}(\phi m) + (1 - \mu_T) W_b(\phi m).$$

The RHS is the expected payoff from deviating, where there is a chance μ_T of getting caught. Inserting W_b and $\underline{W}$, we get

$$T \leq \frac{\mu_T}{r + \mu_T} \left\{ \alpha \left[u(q) / \Lambda_b - v(q) \right] - i\phi M \right\}. \tag{19}$$

Or, using $T = -\pi \phi M$ and the Fisher equation,

$$\alpha \left[u(q) - \Lambda_b v(q) \right] \geq \frac{r \left[r + \mu_T - i(1 - \mu_T) \right] \Lambda_b \phi M}{(1+r)\mu_T}. \tag{20}$$

If $i \geq r$ then $T \leq 0$ and this is satisfied trivially; otherwise policy must respect (20), and this imposes a lower bound on i.

Definition 3 *Policy i is feasible if a monetary equilibrium exists where (20) holds.*

Similarly, the debt-repayment constraint is

$$W_b(\phi m - T - d) \geq \mu_D \underline{W}(\phi m) + (1 - \mu_D) W_b(\phi m - T), \tag{21}$$

[17] Without loss in generality we restrict attention to one-shot deviations; for our purposes this is simply the unimprovability principle of dynamic programming (e.g., Kreps 1990).

where there is a chance μ_D of getting caught. This reduces to

$$d \leq \frac{\mu_D}{r} \left\{ \alpha \left[u(q)/\Lambda_b - v(q) \right] - r\phi M \right\}. \tag{22}$$

For feasible i, in monetary equilibrium, the RHS of (22) is strictly positive. We now endogenize D, adapting methods in Alvarez and Jermann (2000) or Gu et al. (2013b).

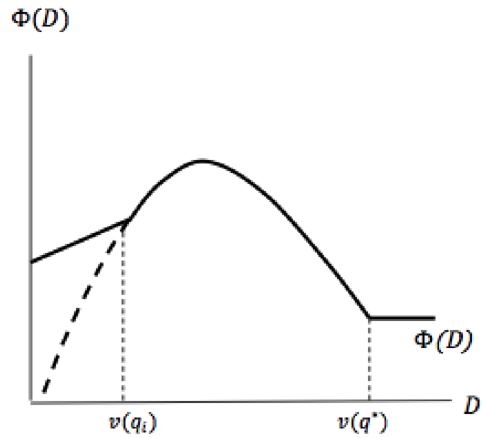

Figure 2: The Correspondence $\Phi(D)$.

First pick an arbitrary D. Generally, the endogenous variables and hence the RHS of (22) depend on D. From Proposition 1, this can be written

$$\Phi(D) \equiv \begin{cases} \xi \left[u(q_i)/\Lambda_b - (1 + r/\alpha)\, v(q_i) \right] + \mu_D D & \text{if } D < v(q_i) \\ \xi \left[u \circ v^{-1}(D)/\Lambda_b - D \right] & \text{if } v(q_i) \leq D < v(q^*) \\ \xi \left[u(q^*)/\Lambda_b - v(q^*) \right] & \text{if } v(q^*) < D \end{cases} \tag{23}$$

where $\xi \equiv \mu_D \alpha / r$. Each branch corresponds to one of the three cases in Proposition 1, and this assumes we select monetary equilibrium when it exists, in the first branch where $D < v(q_i)$. We can also select nonmonetary equilibrium, whence the first branch is the same as the second. In Figure 2 the solid curve is drawn selecting monetary equilibrium when it exists, and the dashed curve is drawn selecting nonmonetary equilibrium. With either selection Φ is continuous. Also, $\Phi(D) = \Phi^*$ is constant $\forall D \geq v(q^*)$, where $\Phi^* = \xi \left[u(q^*)/\Lambda_b - v(q^*) \right]$. If the debt limit is exogenously set to D, agents would actually be willing to honor an obligation d iff $d \leq \Phi(D)$. Therefore, we have:

Definition 4 *An endogenous debt limit is a nonnegative fixed point* $\hat{D} = \Phi(\hat{D})$.

Since the focus here is on money and credit, from now on we select the monetary equilibrium when it exists.[18] This does *not* mean the economy will end up in a monetary equilibrium however – it only means we would get a monetary equilibrium if D were low, but the endogenous D may not be low. Given this, the first branch of $\Phi(D)$ is linear with slope μ_D and intercept

$$\Phi(0) = \xi \left[u(q_i)/\Lambda_b - (1 + r/\alpha)\, v(q_i) \right].$$

For $i \geq r$, we have $\Phi(0) > \xi J(q_i; i) > 0$. For $i < r$, we have $\Phi(D) > 0\ \forall D > 0$ if i satisfies (20), and so we have $\Phi(0) > 0$ for feasible policies. Hence, $D = 0$ is not a fixed point: when money is valued, you would honor some $d > 0$ to avoid expulsion from the DM, even if there were no DM credit, because as long as you do not default off the equilibrium path, you can use cash.[19]

Since $\Phi(0) > 0$ and $\Phi(D) = \Phi^* < D$ for large D, $\exists \hat{D} = \Phi(\hat{D}) > 0$. Given $\hat{D}$, to see if it is consistent with monetary equilibrium, we must check $\phi M = v(q_i) - \hat{D} > 0$, which implies $\hat{D}$ is on the linear branch of Φ. Figure 3 shows several cases: the first panel has $\hat{D} > v(q^*)$, so the debt limit is not binding; the second has $v(q^*) > \hat{D} > v(q_i)$, so it is binding but money is not valued; the third has $\hat{D} < v(q_i)$, so there is a monetary equilibrium; the fourth shows multiple fixed points, one of each type. While multiplicity is interesting, we can give conditions to rule it out. Suppose

[18] For the record, if we instead select the nonmonetary equilibrium, one fixed point is $\hat{D} = 0$, which means the DM shuts down. Thus, you believe there will be no credit in the future, exclusion from the DM is painless, and you would renege on any $d > 0$. So $\hat{D} = 0$ is an endogenous debt limit. There coexist others if $u'(0)$ is not too small, in which given $\hat{D} > 0$, exclusion from the DM is painful, and therefore $\hat{D}$ is self enforcing. There can also be multiple fixed points, and $\hat{D}$ can be above or below $v(q^*)$, so the limit may or may not bind. See Gu et al. (2013*b*) for more on endogenous debt limits in nonmonetary economies, including nonstationary outcomes. See Carapella and Williamson (2014) for nonsymmetric outcomes, including cases with deafult in equilibrium. See Bethune et al. (2014) for extensions of these papers.

[19] Since a monetary equilibrium at low D precludes 0 as an endogenous debt limit, one can say money is good for credit. This is different from models where money is bad for credit (e.g., Aiyagari and Williamson 1999 or Berentsen et al. 2007) because the punishment here is autarky, not monetary trade, which we cover in the Appendix. That case is in some ways easier, but also has some restrictive implications (e.g., as in Berentsen et al. 2007, $\pi < 0$ is never feasible).

$S_b(D) = u \circ v^{-1}(D) - \Lambda_b D$ is concave, which is true for Walrasian pricing, and Nash or Kalai bargaining if θ is not too small. Then once can show for any feasible i there is a unique $\hat{D} = \Phi(D) > 0$.

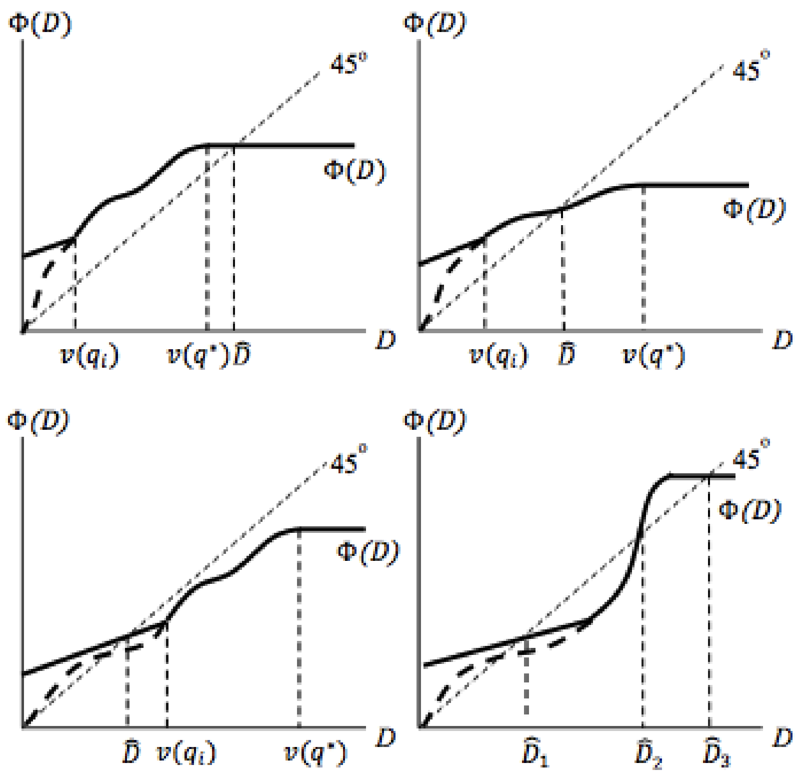

Figure 3: Endogenous Debt Limits

We summarize the main points in the above observations as follows:

Proposition 5 *Given a feasible policy* i*,* $\exists \hat{D} = \Phi(\hat{D}) \geq 0$*. There are three possible outcomes:*

1. *if* $v(q^*) \leq \hat{D}$ *there is no monetary equilibrium and there is a nonmonetary equilibrium with* $q = q^*$*;*

2. *if* $v(q_i) \leq \hat{D} < v(q^*)$ *there is no monetary equilibrium and there is a nonmonetary equilibrium with* $q \in [q_i, q^*)$*;*

3. *if* $\hat{D} < v(q_i)$ *there is a monetary equilibrium with* $q = q_i$*.*

We now combine the endogenous debt limit with the limit on feasible monetary policy. First, in a monetary equilibrium with an endogenous debt limit, since the fixed point is on the linear branch of $\Phi(D)$, we can solve explicitly for

$$\hat{D}_i = \frac{\mu_D}{1-\mu_D}\frac{\alpha}{r}\left[u(q_i)/\Lambda_b - (1+r/\alpha)\,v(q_i)\right], \tag{24}$$

where we indicate here that $\hat{D}_i$ depends on i. As when there were no constraints on policy, we have to check $v(q_i) > \hat{D}$, and now we also have to check whether $\hat{D}$ satisfies (20). Thus, monetary equilibrium requires: (i) $\hat{D} < v(q_i)$, so that the endogenous debt limit is tight enough for money to be valued; and (ii) condition (20), which says agents are willing to pay their taxes.

From (24), $\hat{D} < v(q_i)$ iff $H(q_i) < 0$, where $H(q) \equiv u(q)/\Lambda_b - (1+r/\alpha\mu_D)\,v(q)$. For $i \geq r$, (20) always holds, and $H(q_i) < 0$ is the only condition for monetary equilibrium. For $i < r$, a calculation indicates that (20) holds iff $K(q_i;i) \geq 0$, where $K(q;i) \equiv u(q)/\Lambda_b - (1+\Omega_i)\,v(q)$ and

$$\Omega_i = \frac{r}{\alpha}\frac{\mu_T + r - (1-\mu_T)\,i}{\mu_T + [\mu_D + (1-\mu_D)\,\mu_T]\,r - \mu_D\,(1-\mu_T)\,i}.$$

Hence, for $i < r$ monetary equilibrium requires $H(q_i) < 0 \leq K(q_i;i)$. Summarizing:

Proposition 6 *Given $i > 0$, consider a candidate monetary equilibrium, with q_i solving (14), and an endogenous debt limit $\hat{D}_i$ given by (24). Then we have:*

1. *if $H(q_i) \geq 0$ then i is infeasible (inconsistent with monetary equilibrium);*

2. *if $H(q_i) < 0$ then monetary equilibrium exists and i is feasible iff (2a) $i \geq r$ or (2b) $i < r$ and $K(q_i;i) \geq 0$.*

It is hard to characterize the set of i's that satisfy the conditions in Proposition 6 in general, but if $K(q;i)$ and $H(q)$ are concave in q one can say more.[20] One thing we

[20] In general $K(q;i)$ is not concave, but it is in some cases. With Kalai bargaining, e.g., it is for θ close to 1; for a general θ and $u(q) = q^\sigma$ it is for σ close to 1. Concavity of K implies a unique $\bar{q}_i > 0$ with $K(\bar{q}_i;i) \gtreqless 0$ as $\bar{q}_i \gtreqless q_i$. At $i = r$, we know $q_i < \bar{q}_i$. As i decreases, q_i increases and $\bar{q}_i$ decreases, so there is a unique $\underline{i}$ where they meet. Concavity of $H(q)$ means there is an upper bound $\bar{\imath}$ such that $i \gtreqless \bar{\imath}$ as $H(q) \gtreqless 0$. If $\underline{i} < \bar{\imath}$, then $i < r$ is feasible, and $\underline{i}$ is a lower bound on i. If $\underline{i} > 0$ the Friedman rule $i = 0$ is infeasible. A calculation implies $i = 0$ is feasible if $(1 + r\Omega_0/\alpha)\,v(q_0) \leq u(q_0)/\Lambda_b < (1 + r/\alpha\mu_D)\,v(q_0)$.

can always say is that Proposition 6 reduces in a special case to Kocherlakota's (1998) result that money is not essential with perfect monitoring. Thus, since (24) implies $\hat{D}_i \to \infty$ as $\mu_D \to 1$, with perfect monitoring the endogenous debt limit is large enough that money cannot be valued. Heuristically, when $\mu_D = 1$ in Kocherlakota's setup, the planner does not need money, given what he can do with credit; here the endogenous (market) debt limit is sufficiently big that agents do not need money, given what they can do with credit. Summarizing:

Proposition 7 *With $\mu_D = 1$ and endogenous D, $\nexists$ monetary equilibrium.*

When $\mu_D < 1$, if monetary equilibrium exists, it is easy to describe. Given i, q_i satisfies (14) and $p_i = v(q_i)$. Then $\hat{D}$ in terms of q_i is given by (24). After some algebra, one can write real balances in terms of q_i as

$$\phi_i M = v(q_i) - \hat{D}_i = \frac{\alpha u(q_i)/\Lambda_b - (\alpha + r\mu_D)\, v(q_i)}{r(1-\mu_D)}. \tag{25}$$

This shows explicitly how $\phi_i M$ depends on the monitoring probability μ_D. All one has to do is check the above conditions to verify that monetary equilibrium exists and policy i is feasible.

Figure 4 shows some examples.[21] The left panel depicts $H(q)$, $K(q)$ and $e(q)$, where $e(q) = 0$ gives a candidate equilibrium. For $i = 0.04$ the solution to $e(q) = 0$ is such that $H(q) < 0 < K(q)$, and so it is a monetary equilibrium. For $i' = 0.01$ the solution to $e(q) = 0$ violates $0 < K(q)$, and so it is not an equilibrium – agents won't pay T. The right panel shows the effect of i on $\hat{D}$, and on real balances scaled by output, $B = \phi M / [\alpha v(q) + x]$, a standard measure of money demand used in Lucas (2000), Lagos and Wright (2005), and elsewhere. When μ_D rises, $\hat{D}$ increases and B decreases. As i rises, eventually $B = 0$ and monetary equilibrium breaks down. It also breaks down here for $i \leq 0.04$, where $K(q) > 0$ is violated. Feasible monetary policy requires that i is neither too high nor too low. Also notice money demand

[21] The left panel uses Kalai bargaining with $\theta = 0.85$, $u(q) = 2\sqrt{q}$, $c(q) = q$, $\Lambda_j = \omega = 1$, $r = \alpha = 0.1$, $\mu_D = 0.5$ and $\mu_T = 1$ (here we do not need U^j). In the other panel, $U^j(x, \ell) = 2\log(x) + \ell$, $r = \alpha = 0.25$ and μ_D is either 0.4 or 0.1.

is steeper here than it would be if D were exogenous: as i rises B falls, but also $\hat{D}$ rises, causing B to fall further. This is another instance of how one can go wrong by imposing exogenous restrictions like CIA constraints.

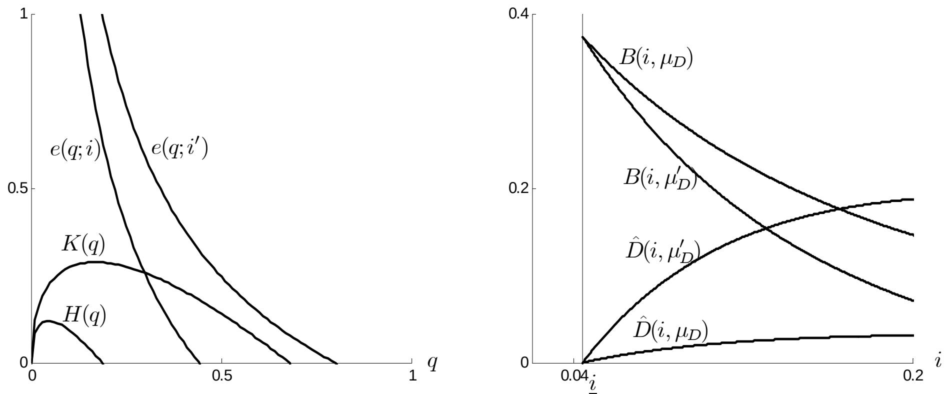

Figure 4: Endogenous Debt and Deflation Limits

For present purposes the key point is this: As was the case with exogenous policy and debt limits, in monetary equilibrium credit is inessential and changes in debt limits are neutral because they affect real balances endogenously so that total liquidity stays the same. Of course, with exogenous debt limits we can change D directly, but here it is endogenous, so changes in credit conditions now mean changes in the parameters affecting $\hat{D}$, like μ_D. Such changes are neutral. Therefore, just as in Section 5, if one were to ask "what is the appropriate policy response to changes in credit conditions?" the right answer might be "nothing." But now there is a caveat: if an underlying parameter like μ_D changes, it may be that $\underline{i}$ and $\bar{i}$ (the bounds on feasible policy) change, which could free up some options. Hence, with endogenous policy limits, it is possible that i should respond to underlying conditions. Indeed, it may be necessary that policy responds, if the old i is no longer consistent with the bounds $\underline{i}$ and $\bar{i}$.

6 Extensions

We now consider robustness. For simplicity, we focus mainly on the case with D and T exogenous, but they can be endogenized (see fn. 25).

6.1 Heterogeneity

We begin with heterogenous preferences, which implies the trading mechanism $v(\cdot)$ and DM purchases can differ across meetings. One might expect that people use money for small and credit for big purchases (recall fn. 6). It seems worth asking, could money and credit both be essential if q is sometimes small and sometimes large? More generally, what might heterogeneity in DM meetings do to the results?

First consider the case where D is exogenous and constant across matches. Let $\mathcal{U}_j^b(q,x,\ell) = u_j(q) + U_j^b(x,\ell)$ be the preferences for a type j buyer and $\mathcal{U}_h^s(q,x,\ell) = -c_h(q) + U_h^s(x,\ell)$ the preferences for a type h seller. Let $F(j)$ be the distribution function of buyer types, and $G(h|j)$ be the distribution function of seller types a type j buyer might meet in DM. Also suppose for now that buyers when they choose $\hat{m}$ do not know the type of seller they will meet in the next DM. Let $\mathcal{C}_j(L_j) = \left\{h : L_j < v_{j,h}\left(q_{j,h}^*\right)\right\}$, where $q_{j,h}^*$ solves $u_j(q)/\Lambda_j = c_h(q)/\Lambda_h$, be the set of sellers where the buyer is constrained. A buyer's objective function is[22]

$$J(L_j) = \int_{\mathcal{C}_j(L_j)} \left[u_j \circ v_{j,h}^{-1}(L_j) - \Lambda_j L_j\right] dG_j(h|j) - \Lambda_j L_j i/\alpha. \tag{26}$$

As long as $L_j > D$, or equivalently $\hat{m}_j > 0$, changes in D do not affect L_j or the DM allocation $q_{j,h} = \min\left\{v_{j,h}^{-1}(L_j), q_{j,h}^*\right\}$. One can also check that D does not affect the CM allocation. So changes in D are still neutral.

Now suppose a buyer alternatively does know the type of seller he will meet in the next DM while still in the CM. If he brings $\hat{m}_{j,h} > 0$, the DM quantity $q_{j,h}^i$ solves

$$u_j'(q) = (1 + i/\alpha)\, v_{j,h}'(q)\, \Lambda_j, \tag{27}$$

which does not depend on D. Again one can check changes in D also do not affect the CM, and hence they are neutral. Summarizing these observations, we have the following:

[22] Here it is more natural to frame buyers' choice as L, rather than q, since the latter generally depends on the meeting.

Proposition 8 *If D is the same in all meetings, credit is inessential and changes in D are neutral in stationary equilibrium where every buyer chooses $\hat{m} > 0$, whether or not they know the type of seller they will meet in the DM while in the CM.*

The results in Proposition 8 may not hold if some of buyers choose $\hat{m} = 0$. Suppose buyers know who they will meet in the next DM (the other case is similar). For those who choose $\hat{m} > 0$ a change in D is neutral. The same is true for those who can get q^*. However, for those who choose $\hat{m} = 0$ even though $q < q^*$ in some DM meetings, changes in D matter. Also, D affects the set of the buyers who choose $\hat{m} > 0$ and the set that get q^*. This overturns the result, as is no surprise. We already know that in the homogeneous-agent economy, in constrained nonmonetary equilibrium, q strictly increases in D. With heterogeneity, for buyers who do not hold money but are credit constrained, the situation is similar to that of a representative agent in constrained nonmonetary equilibrium. This did not affect the results above because they concern monetary equilibria. With heterogeneity, some agents can be in a situation where $q < q^*$ and still not use cash, while others do use cash so the equilibrium is still monetary. What matters for nonneutrality, therefore, is not heterogeneity per se, but having some agents choose $\hat{m} > 0$, while others choose $\hat{m} = 0$ even though $q < q^*$ in some meetings.[23]

Another way to make credit matter is to let D_j differ across buyers, which generally will be the case when it is endogenous and they are heterogeneous (although note that a given buyer has the same D in all meetings, since it is determined by future payoffs, not current partners). Interestingly, q_j does not depend on type j's debt limit, for the usual reasons, but his CM allocation might. This can be seen formally from the CM budget equation, after substituting the government budget equation, but it is simple enough to rely on intuition. When D_j differs, buyers generally choose different $\hat{m}_j$, and hence bear the distorting effect of inflation unequally. With homogeneous

[23]Similar results apply to a homogeneous-agent economy with shocks to preferences. If the shocks are realized in the DM, all buyers choose again choose the same L and D is neutral. If instead shocks are realized before buyers choose L, and if some buyers choose $\hat{m} = 0$ even though $q < q^*$ in some meetings, D is not neutral.

buyers, all bear it equally, and in fact it is partially offset by the lump sum transfer $-T$ used to inject money. Now those with higher D_j choose lower $\hat{m}_j$, but still get the same transfer $-T$. So a change in credit conditions has redistributive effects. This is no different than saying, e.g., a cigarette tax with proceeds rebated lump sum redistributes wealth between smokers and nonsmokers – it's true, if obvious, but it can also be neutralized with additional taxes.

A final way to make credit matter is to let monitoring be heterogeneous across sellers, say because they have different μ_D. Denote the distribution across DM meetings by $\tilde{F}(D)$. Assuming buyers in the CM do not know the D they will realize in the DM, all choose the same $\hat{m}$. In the DM, $q = q^*$ if $\phi m + D \geq v(q^*)$ and $q = v^{-1}(\phi m + D)$ otherwise. Hence, there is a D^* below which buyers are constrained. Then increase average D, or some other change in $F(D)$, affects which the set of meetings that are constrained or unconstrained. As with the other examples, this shows how certain, but not all, types of heterogeneity can make credit conditions matter.[24]

6.2 Real Pledgeable Assets

In addition to cash, consider a real asset a, in fixed supply normalized to 1, that has price ψ and pays dividend $\gamma > 0$ in numeraire in the CM. To avoid a minor technicality discussed in Geomichalos et al. (2007) and Lagos and Rocheteau (2008), assume in monetary equilibrium $q_0 = q^*$ at $i = 0$, as is always true for, e.g., Walrasian pricing or Kalai bargaining. Also, here we start without, and then reintroduce, fiat money. Then the CM budget constraint is $x = \omega(1-\ell) + \gamma a + \psi(a - \hat{a}) - d$. In the DM, $v(q) \leq D + \chi(\psi + \gamma)\hat{a}$, where $\chi \leq 1$ denotes the fraction of assets that can be used in DM trade. As in Kiyotaki and Moore (1997,2005), think of χ as the fraction of a that is pledgeable as collateral on DM debt (the usual interpretation is that if a

[24]The reasoning in these situations is similar to results in Sanches and Williamson (2010), Lotz and Zhang (2013), Gomis-Porqueras and Sanches (2013) and Araujo and Hu (2014). While we acknowledge their contributions, we think it is good to have benchmark results for homogeneous agents. It is interesting to see how credit matters with certain types of heterogeneity, but also important to know that credit does not matter with other types of heterogeneity, and certainly not with homogeneous buyers.

debtor defaults, off the equilibrium path, we can punish him by seizing a fraction χ of his assets while he absconds with the rest). Hence, there is both unsecured credit, limited by D, and secured credit, limited by $\chi(\psi+\gamma)\hat{a}$.

The DM constraint binds iff $\chi\gamma$ is low (Geromichalos et al. 2007; Lester et al. 2012). When it does not bind, $q=q^*$ and the asset price is its fundamental value $\psi=\psi^*\equiv\gamma/r$. Hence, suppose it binds. Then the Euler equation is

$$\psi=\beta\left(\psi_{+1}+\gamma\right)\left\{1+\alpha\chi\left[\frac{u'(q)-\Lambda_b v'(q)}{\Lambda_b v'(q)}\right]\right\}.$$

In stationary equilibrium this can be rearranged as

$$u'(q)=\left[1+\frac{r\psi-\gamma}{\alpha\chi(\psi+\gamma)}\right]\Lambda_b v'(q). \tag{28}$$

Generalizing Proposition 4, there is a unique equilibrium $(q,\psi)\in(0,q^*)\times(\psi^*,\infty)$ solving (28) and $v(q)=D+\chi(\psi+\gamma)$, and raising D or χ increases q. So credit conditions are not neutral.[25]

However, this does *not* overturn the main result, that credit is irrelevant in monetary economies, because the above analysis has no money. Bringing cash back, the Euler equations for $\hat{m}$ and $\hat{a}$ are

$$\phi = \beta\phi_{+1}\left[1+\alpha\frac{u'(q)-\Lambda_b v'(q)}{\Lambda_b v'(q)}\right] \tag{29}$$

$$\psi = \beta\left(\psi_{+1}+\gamma\right)\left[1+\alpha\chi\frac{u'(q)-\Lambda_b v'(q)}{\Lambda_b v'(q)}\right]. \tag{30}$$

In stationary equilibrium (29) reduces to $u'(q)=(1+i/\alpha)\Lambda_b v'(q)$, identical to (14) in the baseline model. Again, as long as money is valued, q does not depend on D or χ. Hence adding Kiyotaki-Moore credit, with real assets in fixed supply, does not

[25] To see how one endogenizes D with a real asset, consider the analog to (23):

$$\Phi(D)=\begin{cases}\xi J\circ q(D)/\Lambda_b+\dfrac{\mu_D}{r}\dfrac{(1+r-\chi)\psi(D)-\chi\gamma}{\chi[\psi(D)+\gamma]}D & \text{if } D<v(q^*)-\chi\gamma(1+r)/r\\ \xi\left[u(q^*)/\Lambda_b-v(q)\right] & \text{if } D\geq v(q^*)-\chi\gamma(1+r)/r\end{cases}$$

Now $\Phi(D)$ only has two branches; the middle branch in the benchmark model, where D is not big enough to get q^* but the asset is still not valued, only occurs with fiat money.

affect the key results. Note that χ does affect the asset price $\psi = \gamma(1+\chi i)/(r-\chi i)$, but that is irrelevant for the allocation, as it simply crowds out real balances to leave total liquidity L the same.

6.3 Reproducible Capital

Consider now introducing capital K, with ρ and δ denoting the rental and depreciation rates. The CM production function is $f(N,K)$, where N is total employment, and it displays constant returns. Profit maximization implies $\omega = f_1(N,K)$ and $\rho = f_2(N,K)$. We focus here on monetary equilibria, which exist under conditions given in Venkateswaran and Wright (2013).

The CM budget equation is $x + \phi\hat{m} + \hat{k} = A + \omega(1-\ell)$, where $A = \phi m + (\rho+1-\delta)k - d - T$ and k is individual capital. The DM constraint is $p \leq D + \phi m + \chi(\rho+1-\delta)k$, again including the pledgeability parameter χ. The Euler equations for $\hat{m}$ and $\hat{k}$ are

$$\begin{aligned}\Lambda_b \phi &= \beta\Lambda_{b,+1}\phi_{+1}\left[1+\alpha\frac{u'(q_{+1})-\Lambda_{b,+1}v'(q_{+1})}{\Lambda_{b,+1}v'(q_{+1})}\right] && (31)\\ \Lambda_b &= \beta\Lambda_{b,+1}\left(\rho_{+1}+1-\delta\right)\left[1+\alpha\chi\frac{u'(q_{+1})-\Lambda_{b,+1}v'(q_{+1})}{\Lambda_{b,+1}v'(q_{+1})}\right]. && (32)\end{aligned}$$

Note that even in stationary equilibrium, outside of steady state, K and other variables vary over time. In particular, Λ_j can depend on ω and hence on K, which may or may not mean that q depends on K, as we now show.

It is instructive to consider two examples, one with and one without quasi-linear utility. For the first, let $U^j(x,\ell) = \tilde{U}(x) + \ell$, which implies $\Lambda_j = \tilde{U}'(x) = 1/\omega$. Assume Kalai bargaining, $v(q) = [\theta c(q) + (1-\theta)u(q)]\omega$. Given K_0, equilibrium consists of paths for (q, x, K_{+1}, N) satisfying

$$\begin{aligned}u'(q) &= (1+i/\alpha)\left[\theta c'(q) + (1-\theta)u'(q)\right] && (33)\\ 1 &= f_1(N,K)\tilde{U}'(x) && (34)\\ \tilde{U}'(x) &= \beta\tilde{U}'(x_{+1})\left[f_2(N_{+1},K_{+1})+1-\delta\right](1+\chi i) && (35)\\ 2x &= f(N,K) + (1-\delta)K - K_{+1}. && (36)\end{aligned}$$

where (36) is the usual feasibility condition given a measure 1 each of buyers and sellers, and (35) comes from (32) for buyers (sellers do not hold k, as the return is too low, given they do not value liquidity). Notice on the RHS of (33) the ω in $v'(q)$ cancels with Λ_b. In this quasi-linear economy q does not depend on ω or K.

Moreover, D does not affect (q, x, K_{+1}, N), since it does not appear in (33)-(36). Again, changes in D lead to an endogenous response in real balances that keeps L constant. Changes in χ, however, are not neutral: in steady state, $\partial K/\partial\chi > 0$, and $\partial x/\partial\chi > 0$ if K and N are normal inputs, while $\partial N/\partial\chi$ is ambiguous due to wealth and substitution effects. Changes in χ do not affect q in this specification, but they affect the CM allocation, because when K is better able to relax the liquidity constraint investment increases. That did not happen in Section 6.2 for two reasons: the asset was in fixed supply; and it was not a factor of production.

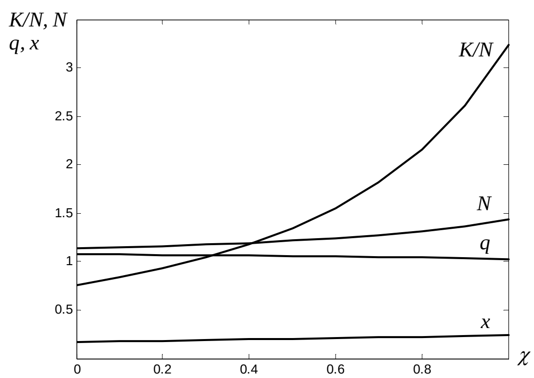

Figure 5A: Effects of χ

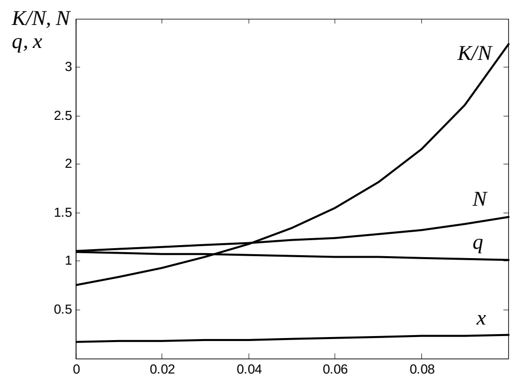

Figure 5B: Effects of i

The second example uses $U^b(x,\ell) = x^\sigma \ell^{1-\sigma}$ and $U^s(x,\ell) = \tilde{U}(x) + \ell$, and $\theta = 1$ so that $v(q) = c(q)/\Lambda_s$. Then (14) becomes $u'(q) = (1 + i/\alpha)\, c'(q)\, \Lambda_b/\Lambda_s$, but Λ_b/Λ_s does not cancel since buyers do not have quasi-linear utility. The FOC's from the CM imply $\Lambda_b = \omega^{\sigma-1}\sigma^\sigma (1-\sigma)^{1-\sigma}$ and $\Lambda_s = \omega^{-1}$, and hence

$$u'(q) = (1 + i/\alpha)\, \omega^\sigma \sigma^\sigma (1-\sigma)^{1-\sigma} c'(q). \tag{37}$$

Now q is decreasing in ω and therefore χ; if we were to switch buyer and seller preferences, then q would be increasing in ω and χ. The intuition is simple: When b

transfers purchasing power to s, the parties value it according to Λ_b and Λ_s. Changes in χ affect K, and hence ω, and if ω appears differently in Λ_b and Λ_s this tilts the terms of trade. The Appendix solves for steady state, and for an example Figures 5A and 5B show K/N, N, q and x as functions of χ and i. This is different from the quasi-linear case, where q is independent of χ. Hence, extending the model along the lines of Wong (2012) is not only interesting for the sake of generality, it affects the qualitative and quantitative results.

6.4 Dynamics

Here we characterize the dynamics in the benchmark specification, where money is the only asset, with exogenous policy and debt limits. The FOC wrt $\hat{m}$ evaluated at $m = M$ is now written as follows: If $\phi_{+1}M_{+1} + D < v(q^*)$ then

$$\phi = \beta\phi_{+1}\left\{\alpha\left[\frac{u'(q_{+1})}{v'(q_{+1})} - 1\right] + 1\right\} \text{ and } q_{+1} = v^{-1}(\phi_{+1}M_{+1} + D); \tag{38}$$

and if $\phi_{+1}M_{+1} + D \geq v(q^*)$ then

$$\phi = \beta\phi_{+1} \text{ and } q_{+1} = q^*. \tag{39}$$

Note q can never exceed q^*, but if next period real balances are enough to get q^* then the liquidity premium vanishes and $\phi = \beta\phi_{+1}$. In this case buyers may spend $p < \phi_{+1}M_{+1} + D$.

Let $z = \phi M$ and rewrite (38) and (39) as $z = g(z_{+1}; D)$ where:

$$g(z_{+1}; D) \equiv \begin{cases} \dfrac{\beta z_{+1}}{1+\pi}\left\{\alpha\left[\dfrac{u' \circ v^{-1}(z_{+1}+D)}{v' \circ v^{-1}(z_{+1}+D)} - 1\right] + 1\right\} & \text{if } z_{+1} + D < v(q^*) \\ \dfrac{\beta z_{+1}}{1+\pi} & \text{if } z_{+1} + D \geq v(q^*) \end{cases}$$

Given policy, which here we take to be π, a monetary equilibrium is a (nonnegative, bounded) sequence $\{z_t\}$ satisfying this dynamical system, where at every date $q = v^{-1}(z + D)$ if $z + D < v(q^*)$, and $q = q^*$ otherwise. Assume $1 + \pi > \beta$ and $D < v(q_i)$, as required for monetary equilibrium, where $q_i \in (0, q^*)$ is the unique monetary steady state established above and $z_i = v(q_i) - D$. There is of course also a nonmonetary steady state with $q = v^{-1}(D)$ and $z = 0$.

We can also write the dynamic system in terms of total liquidity, $L = z + D$, as $L = \tilde{g}(L_{+1}; D)$ where:

$$\tilde{g}(L_{+1}; D) \equiv \begin{cases} \frac{\beta(L_{+1} - D)}{1+\pi}\left\{\alpha\left[\frac{u' \circ v^{-1}(L_{+1})}{v' \circ v^{-1}(L_{+1})} - 1\right] + 1\right\} + D & \text{if } L_{+1} < v(q^*) \\ \frac{\beta(L_{+1} - D)}{1+\pi} & \text{if } L_{+1} \geq v(q^*) \end{cases}$$

At the steady state $L_i = v(q_i)$ and

$$\left.\frac{\partial L}{\partial L_+}\right|_{L_i} = 1 + \frac{v(q_i) - D}{1+i}\frac{\alpha u''(q_i) - (\alpha + i)\, v''(q_i)}{v'(q_i)^2}, \tag{40}$$

where we use the Fisher equation. Notice g crosses the 45^o line from above and g^{-1} crosses it from below, as shown in Figure 6A.[26] Similarly for $\tilde{g}$ in Figure 6B. Also shown is what happens as we vary D. Notice in Figure 6A that $g(z_{+1}; D_1) < g(z_{+1}; D_0)$ when $D_1 > D_0$, and similarly for $\tilde{g}$ in Figure 6B.

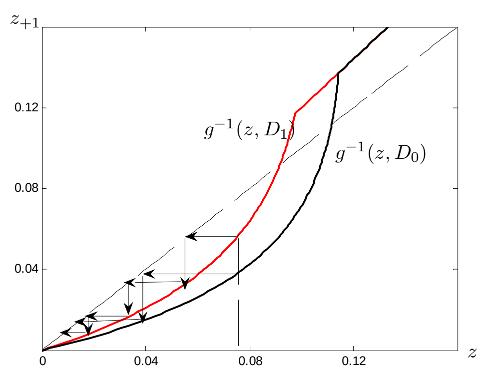

Figure 6A: Dynamics of z

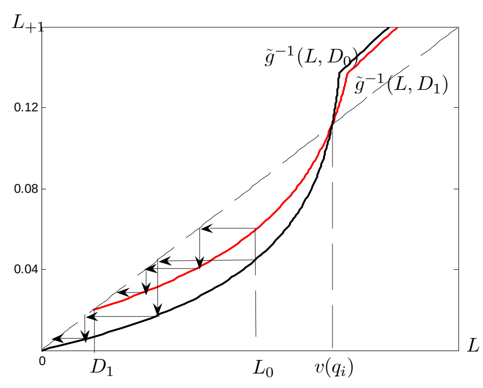

Figure 6B: Dynamics of L

In Figure 6A starting from any $z_0 \in (0, z_i)$, there is an equilibrium converging to the nonmonetary equilibrium; there is no equilibrium starting at $z_0 > z_i$. Similarly for Figure 6B, from which it is also clear that if we start at the same $L_0 < v(q_i)$, the

[26]This example uses $v(q) = c(q) = q^{1+\sigma}/(1+\sigma)$, $u(q) = A\left[(q+b)^{1-\gamma} - b^{1-\gamma}\right]/(1-\gamma)$, where $\sigma = 0$, $\gamma = 1.6$, $A = 0.1$, $b = 0.1$, $\alpha = 1$ and $(1+\pi)/\beta = 1.2$. While g and $\tilde{g}$ happen to be monotone here, that is not generally the case.

path for L generated by D_1 is above the path generated by $D_0 < D_1$, and so welfare is higher with D_1. However, there is still an equilibrium where credit does not matter, the steady state q_i. Hence we can still say that credit is inessential, but we can only say D is neutral in the stationary monetary equilibrium. The reason credit is not neutral in nonstationary equilibria is simple: in the long run, the value of money goes to 0, and since D matters in a nonmonetary equilibrium, it matters on the transition to a nonmonetary equilibrium.[27]

7 Conclusion

For many specifications we found the economy does not need money and credit: although both may be used, if money is valued credit is inessential and changes in debt limits are neutral. Ergo, the appropriate policy response to changes in credit conditions may be to simply let real balances adjust endogenously. However, if changes in fundamentals like μ_D affect the endogenous bounds on feasible i, it may be that monetary policy can, should or even must respond. The results hold for general pricing mechanisms, for secured or unsecured lending, for exogenous or endogenous policy and debt limits, and for somewhat if not completely general preferences. They hold for heterogeneous agents if buyers constrained by D are at interior solutions for $\hat{m}$, but not if they are at $\hat{m} = 0$. Some results are overturned by heterogeneous monitoring. With secured credit, pledgeability can matter if collateral is reproducible and a factor of production, but even then the unsecured debt limit is neutral.

We think we learn a lot from these results. And, while we do not claim they are in a class with famous irrelevancy propositions, like Modigliani-Miller in finance,

[27] For more complex dynamics, suppose $u''(q_i) - (1 + i/\alpha) v''(q_i)$ is sufficiently small so that $\partial L / \partial L_+ |_{L_i} < -1$. Then there are two periodic points, and hence cycles, when $D = 0$. As D increases, cycles eventually disappear. So again D affects nonstationary equilibria, but not the monetary steady state. In the above example, except $A = 0.25$, $\gamma = 3.5$ and $b = 0.07$, we have $q_i = 0.5688$ and $q^* = 0.6030$. Also there is a three-period cycle in which $z^1 = 0.5056$, $z^2 = 0.6067$ and $z^3 = 0.7280$. In this cyclic equilibrium, $q^1 = 0.5060$ and $q^2 = q^3 = q^*$. Given a thee-cycle, there exist cycles of all orders plus chaotic dynamics. One can also construct stochastic (sunspot) equilibria. See Azariadis (1993) for a textbook discussion of the methods, and Lagos and Wright (2003) for more on dynamics in a pure-currency version of this model.

Ricardian equivalence in macro, or Kareken-Wallace indeterminacy in exchange rates, there is something in common: even if one can find “loopholes” that overturn some results, they still contain grains of truth. It was interesting to see exactly what can and cannot overturn the results here. An operational point is this: if someone wants to argue that credit conditions matter, they might check if this is true in their models once money is introduced, which might require they incorporate the right kinds of heterogeneity or corner solutions. Many macro models used in theory and policy analysis these days ignore monetary considerations, but our results indicate that this is not innocuous. More generally, details concerning how agents trade, and in particular how they pay, can be critical for understanding positive and normative economic issues.

Appendix

Here we provide proofs of results that are not obvious, sketch the model with endogenous policy and debt limits when punishment involves allowing deviators to continue in the DM but only using cash, and solve the example in Section 6.3.

Proof of Lemma 1 and 2: Consider first buyers. They are constrained, $q < q^*$, in stationary monetary equilibrium. Differentiating (3)-(4), we get

$$\begin{bmatrix} -\omega U_{11}^b + U_{21}^b & -\omega U_{12}^b + U_{22}^b & 0 \\ -\phi U_{11}^b & -\phi U_{12}^b & \phi_{+1}\beta V_b'' \\ 1 & \omega & \phi \end{bmatrix} \begin{bmatrix} dx \\ d\ell \\ d\hat{m}_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ dA \end{bmatrix}$$

where V_b'' is well defined from (10) and the assumptions on v. The determinant is $\Delta_1 = \phi_{+1}\beta \left(\omega^2 U_{11}^b - 2\omega U_{21}^b + U_{22}^b\right) V_b'' > 0$, and $\partial \hat{m}_b / \partial A = \Delta_1^{-1} \phi \left| U^b \right| = 0$, since $\left| U^b \right| = 0$ by Assumption 1. Hence $\hat{m}_b$ is independent of A.

Let $\Lambda_b (A) = U_1^b [x (A), \ell (A)]$. Then

$$\frac{\partial U_1^b}{\partial A} = U_{11} \frac{\partial x}{\partial A} + U_{12} \frac{\partial \ell}{\partial A} = \Delta_1^{-1} \left[U_{11}^b \left(-\omega U_{11}^b + U_{21}^b \right) + U_{12}^b \left(-\omega U_{12}^b + U_{22}^b \right) \right] = 0.$$

By (3), $U_2^b (\cdot) = \Lambda_b \omega$. By the envelope theorem, $W_b' (\cdot) = \Lambda_b$. That takes care of buyers in monetary equilibrium. In a nonmonetary equilibrium, $\partial U_1^b / \partial A = -\Delta_0^{-1} \left| U^b \right| = 0$ where $\Delta_0 = -\left(\omega^2 U_{11}^b - 2\omega U_{21}^b + U_{22}^b \right) > 0$. Again, $U_1^b (\cdot) = \Lambda_b$ etc. This completes the argument for buyers. The argument for sellers is similar. ■

Proof of Lemma 3: Suppose $L \geq p^*$. Then $V_b' (\cdot) = W_b' (\cdot) = 1$, because the terms of trade $(p, q) = (p^*, q^*)$ are independent of L when the constraint is slack. By the FOC for $\hat{m}$ at equality, $\phi = \beta \phi_{+1}$. Since $\phi / \phi_{+1} = 1 + \pi$, this contradicts $\pi > \beta - 1$. In the limiting case of the Friedman rule, $\pi = \beta - 1$, money can be held even if the constraint is slack, but in this case money does not accomplish anything – payoffs would be the same if $M = 0$. ■

Proof of Lemma 4: In (12), the buyer's problem is $\max_q J (q; i)$ st $q \in [0, q^*]$, where $J (q; i) = u (q) - \Lambda_b (1 + i/\alpha) v(q)$ is twice continuously differentiable by assumption, and $J(0; i) = 0$. Clearly there exists a solution. By Assumption 3, $q = 0$ is not a solution. Hence for i not too big there exists a $q_i > 0$ maximizing $J (q; i)$. At any such q_i, the FOC $J_q (q_i; i) = 0$ holds, although there might be multiple local maximizers, as shown in Figure 7. The higher curve is $J(q;i)$ for $i = i_1$ and the lower curve is $J(q;i_2)$ for $i_2 > i_1$.

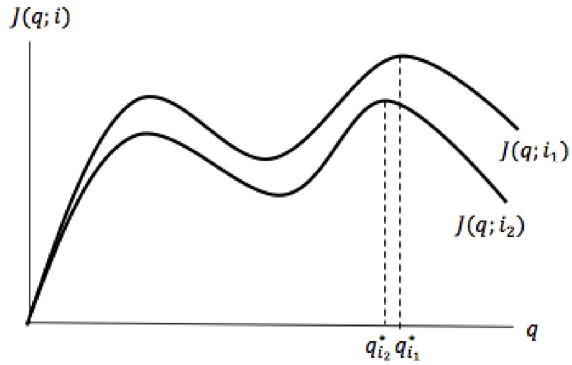

Figure 7: Uniqueness and Monotonicity

At any local maximum $J_q(q;i) = u' - \Lambda_b(1+i/\alpha)v' = 0$ and $J_{qq} = u'' - \Lambda_b(1+i/\alpha)v'' < 0$. We claim the global maximizer is unique for generic i. To see this suppose $J(q_1^*;i) = J(q_2^*;i) = \max_q J(q;i)$ with $q_2^* > q_1^*$. Increase i to $i+\varepsilon$. Since $J_i(q;i) = -\Lambda_b v(q) < 0$ and $v(q)$ is increasing, $J_i(q_2^*;i) < J_i(q_1^*;i)$. Thus $J(q_1^*;i+\varepsilon) > J(q_2^*;i+\varepsilon)$, and now the global maximizer is unique at a q near q_1^*.

Increasing i_1 to i_2, J shifts down, as shown. Each local maximizer shifts to the left, $\partial q/\partial i < 0$. In particular, the global maximizer shifts to the left, and q_i is decreasing in i when it is single valued. If we continue to increase i, we could reach a nongeneric point $\widetilde{i}$ where there are multiple global maximizers, say q_1^* and $q_2^* > q_1^*$. By the argument used above, for $\widetilde{i}+\varepsilon$ the unique global maximizer is close to q_1^* and for $\widetilde{i}-\varepsilon$ the unique global maximizer is close to q_2^*. So q_i is a continuously decreasing and single-valued function except possibly for i in a set of measure 0, where it is multiple valued and jumps to the left as i increases. Since for generic i there is a unique q_i, there cannot be multiple equilibria. ■

Proof of Proposition 3. First consider $L < p^*$. By A3 and the definition of p^*, we have $q = \Gamma_q(L) < q^*$. We prove $p = L$ by contradiction. Suppose $p \neq L$. We cannot have $p > L$, by A1, so $p < L$. Consider $p' = p + \varepsilon_p < L$ and $q' = q + \varepsilon_q < q^*$, which is feasible for small $(\varepsilon_p, \varepsilon_q)$. If $\varepsilon_p = [u(q') - u(q)]/\Lambda_b$, one can easily check that the buyer's surplus S_b does not change, while for the seller

$$dS_s = \Lambda_s p' - c(q') - \Lambda_s p + c(q) = \frac{\Lambda_s}{\Lambda_b}u(q') - c(q') - \frac{\Lambda_s}{\Lambda_b}u(q) + c(q). \tag{41}$$

Since $q^* > q' > q$, $\Lambda_s u(q)/\Lambda_b - c(q)$ is increasing in q. Therefore S_s increases, contradicting A4.

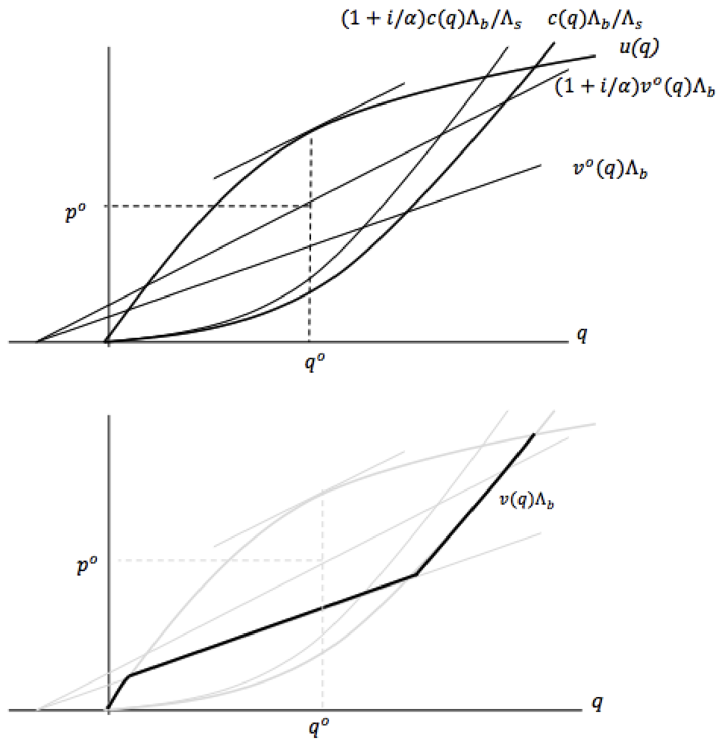

Figure 8: The HKW-like Mechanism

Next, consider $L \geq p^*$. We prove $q = q^*$ by contradiction. Suppose $q = \Gamma_q(L) < q^*$. We know $p = \Gamma_p(L) < p^*$ by A3. Let $p' = p + \varepsilon_p$ and $q' = q + \varepsilon_q$. As in the previous step, one can check (p', q') dominates (p, q), contradicting A4. Suppose instead $q > q^*$. Let $p' = p - \varepsilon_p$ and $q' = q - \varepsilon_q > q^*$, where $\varepsilon_p = [u(q) - u(q')]/\Lambda_b$, which satisfies A1 and A3 for small $(\varepsilon_p, \varepsilon_q)$. One can check that S_b does not change while the change in S_s is the same as (41). Since $q > q' > q^*$, $\Lambda_s u(q)/\Lambda_b - c(q)$ is decreasing in q. Therefore S_s increases, contradicting A4. Hence, $L \geq p^*$ implies $q = q^*$, which implies $p = p^*$ by the definition of p^* and A3. Hence, $v^{-1}(p^*) = q^*$. By A3, v^{-1} is strictly increasing. By A2, $v^{-1}(0) = 0$. By definition, $v^{-1}(p^*) = q^*$. ■

Proof of Proposition 4. Figure 8 shows $u(q)$ and $(1 + i/\alpha)c(q)\Lambda_b/\Lambda_s$. First consider $q^o \leq \min\{q^*, \hat{q}\}$. Pick p^o such that $(1 + i/\alpha)c(q^o)\Lambda_b/\Lambda_s < p^o\Lambda_b < u(q^o)$, which is possible given $q^o \in [0, \hat{q}]$. Draw a line through (q^o, p^o) with slope $u'(q^o)$, labelled in the graph $(1 + i/\alpha)v^o(q)\Lambda_b$. Now define $v(q)$ on $[0, \bar{q}]$ by first rotating $(1+i/\alpha)\, v^o(q)\, \Lambda_b$ to get $v^o(q)\, \Lambda_b$, then truncating it above by $u(q)$ and below by $c(q)\Lambda_b/\Lambda_s$. Since $v(\cdot)$ is strictly increasing, $v^{-1}(\cdot)$ is well defined, and a trading mechanism is given by (9). This mechanism is consistent with trading q^o and $p^o = v(q^o)$ ex post, in the DM, because $c(q^o)\Lambda_b/\Lambda_s \le p^o\Lambda_b \le u(q^o)$. And it is consistent with the ex ante decision to bring enough liquidity out of the CM, because q^o is the global maximizer of a buyer's objective function $J(q;i) = u(q) - (1+i/\alpha)\, v(q)\Lambda_b$. So we can support $q^o \le \min\{q^*, \hat{q}\}$.

Now consider $q^o > q^*$. We claim that such a q^o cannot be supported by a Pareto efficient mechanism. Although the buyer has the ex ante incentive to take enough liquidity out of the CM to pay p^o and get q^o, ex post in a DM meeting there is an alternative (p,q) that Pareto dominates (p^o, q^o), involving a reduction in q from q^o toward q^* combined with a reduction in p^o. Given A4, we cannot support $q^o > q^*$. Note that this is not an issue for $q^o \le q^*$, since when a buyer only brings enough to get q^o, renegotiation towards $q^* > q^o$ violates A1. So we can construct mechanisms that deliver any $q^o \le q^*$ in the case with $\hat{q} > q^*$. If $\hat{q} < q^*$ we cannot support q^*, as the ex ante expected utility is negative for $q = q^*$ while choosing $q = 0$ yields zero expected utility, and $\hat{q}$ is the highest q that is incentive feasible. ■

Alternative punishment: Suppose now that if an agent is caught reneging, he is banned from using credit in the DM, but can continue using cash. The punishment payoff is

$$\underline{W}(\phi m) = \max_{x,\ell,\hat{m},q} \left\{ U^b(x,\ell) + \beta\alpha[u(q) - \Lambda_b v(q)] + \beta \underline{W}\left(\phi_+ \hat{m}\right) \right\}$$
$$\text{st } \phi m + \omega(1-\ell) = x + \phi\hat{m} \text{ and } v(q) \le \phi_{+1}\hat{m},$$

In monetary equilibrium, this reduces to

$$\underline{W}(\phi m) = \frac{1+r}{r} U_0 + \phi m \Lambda_b - \Lambda_b \frac{i}{r} v(q_i) + \frac{\alpha}{r}[u(q_i) - \Lambda_b v(q_i)].$$

The policy constraint reduces to $(r+\mu_T)\, T \le \mu_T i D$. Given an incentive-feasible policy, the debt repayment constraint is again $d \le \Phi(D)$, where now

$$\Phi(D) \equiv \begin{cases} \mu_D D + \mu_D \dfrac{i-r}{r} v(q_i) & \text{if } D < v(q_i) \\ \xi\left[u \circ v^{-1}(\Lambda_b D)/\Lambda_b - D\right] & \text{if } v(q_i) \le D < v(q^*) \\ \xi\left[u(q^*)/\Lambda_b - v(q^*)\right] & \text{if } v(q^*) \le D \end{cases}$$

if we select the monetary equilibrium when it exists. A fixed point admitting monetary equilibrium solves

$$D = \frac{\mu_D}{1-\mu_D}\frac{i-r}{r}v(q_i),$$

which satisfies $0 \leq D < v(q_i)$ iff $r \leq i < r/\mu_D$. Substituting this into the constraint $(r+\mu_T)T \leq \mu_T iD$, we get $r \leq i$. Therefore $r \leq i < r/\mu_D$ is necessary and sufficient for a monetary equilibrium. In this case, deflation is simply not feasible. ■

Example in Section 6.3. For b, the FOC's imply

$$x = \sigma(A+\omega-\phi\hat{m}-\hat{k}) \text{ and } \omega\ell = (1-\sigma)(A+\omega-\phi\hat{m}-\hat{k}),$$

where $A = (1-\chi)(1+\rho-\delta)k - D - T$ for buyers that traded in the previous DM, and $A = (1+\rho-\delta)k + \phi m - T$ for those that did not. For sellers,

$$x = x^*(\omega) \text{ and } \omega\ell = \omega + A - \phi\hat{m} - \hat{k} - x^*(\omega),$$

where $x^*(\omega)$ solves $U'(x) = 1/\omega$, $A = \chi(1+\rho-\delta)k + \phi m + D$ for sellers that traded in the previous DM, and $A = 0$ for those that did not. Using $c(q)\omega = \chi(1+\rho-\delta)k + \phi m + D$, $T = -\pi\phi m$, and capital market clearing condition, in steady state

$$\begin{aligned}
x_1^b &= \sigma[\omega - \omega c(q) + (\rho-\delta)K] \text{ and } x_0^b = \sigma[\omega + (\rho-\delta)K] \\
\omega\ell_1^b &= (1-\sigma)[\omega - \omega c(q) + (\rho-\delta)K] \text{ and } \omega\ell_0^b = (1-\sigma)[\omega + (\rho-\delta)K] \\
x_1^s &= x_0^s = x^*(\omega) \\
\omega\ell_1^s &= \omega + \omega c(q) - x^*(\omega) \text{ and } \omega\ell_0^s = \omega - x^*(\omega),
\end{aligned}$$

where subscripts 1 and 0 denote those who traded and those who did not in the previous DM.

Assuming a measure 1 of buyers and of sellers, from the goods market clearing condition, we get the total measure of labor as

$$N = 2 - \int \ell = \frac{\sigma\omega[1-\alpha c(q)] + x^*(\omega)}{f(1,\kappa) - [\delta + \sigma(\rho-\delta)]\kappa}.$$

where κ is the capital/labor ratio. Using (**??**), (37), $\rho = f_2(1,\kappa)$ and $\omega = f(1,\kappa) - f_2(1,\kappa)\kappa$, We can solve for $(\kappa, \rho, \omega, q, N)$. From this we get $K = N\kappa$ and (x,ℓ) for all agents. ■

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