Least Squares Learning? Evidence from the Laboratory*

Te Bao[†] Yun Dai[‡] John Duffy[§]

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Abstract

We report on an experiment testing the empirical relevance of least squares (LS) learning, a common way of modelling how individuals learn a rational expectations equilibrium (REE). Subjects are endowed with the correct perceived law of motion (PLM) for a price level variable they are seeking to forecast, but do not know the true parameterization of that PLM. Instead, they must choose and can adjust the parameters of this PLM over 50 periods. Consistent with the E-stability of the REE in the model studied, 97.8% of subjects achieve weak convergence to the REE in terms of their price level predictions. However, the number of participants that can be can be characterized as least squares learners via the adjustments they make to the parameterization of the PLM over time depends on properties of the data generating process of the dependent and independent variables. Participants learn the REE faster, and behave more like least square learners when there is greater variance in the independent variable of the model. We consider several alternatives to least squares learning and find evidence that many subjects employ a simple satisficing approach.

JEL Classification: C53, C91, D83, D84

Keywords: Rational Expectations Equilibrium, Least Squares Learning, Experimental Economics, Learning-to-Forecast Experiment, Behavioral Macroeconomics.

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[†]School of Social Sciences and NTU-WeBank Joint Research Center, Nanyang Technological University. Email: baote@ntu.edu.sg.

[‡]Department of Finance, Lingnan College, Sun Yat-Sen University. Email: daiy28@mail.sysu.edu.cn.

[§]Department of Economics, University of California, Irvine, and ISER, Osaka University. Email: duffy@uci.edu.

This is our key bounded rational assumption: we back away from the RE assumption, replacing it with the assumption that, in forecasting prices, firms act like econometricians.

– Evans and Honkapohja, 2001: page 28

1 Introduction

The rational expectations hypothesis (REH) (Muth, 1961; Lucas Jr, 1972) has been a dominant paradigm in macroeconomics since the 1970s. Nevertheless, many researchers remain interested in finding an evolutionary microfoundation justifying the REH (Arifovic (1994), Arifovic (1995), Arifovic (1996), Anufriev et al. (2013), Arifovic and Duffy (2018); Arifovic et al. (2019)). As pointed out by Sargent (2008), an important approach is to assume adaptive learning in combination with the "self-confirming equilibrium" (SCE) approach where the rational expectations equilibrium (REE) is considered as the possible destination of active learning by agents.

Adaptive learning models, including least squares learning models (Marcet and Sargent (1988), Guesnerie and Woodford (1991), Sargent (1993), Evans and Honkapohja (1999), Evans and Honkapohja (2001), Evans and Honkapohja (2003), Evans and Honkapohja (2009), Preston (2006), Branch et al. (2013), Branch and McGough (2016), Branch and McGough (2018)) usually assume that agents do not know the actual law of motion (ALM) of the economy. Instead, these learning agents use a *perceived* law of motion (PLM) and update the parameters of this PLM as new information arises. In the case of least squares learning, they minimize the sum of squared errors according to the least squares updating rule, just as econometricians do with their data.[1] Researchers in this field show that convergence to an REE can arise under certain conditions regarding the mapping between the perceived law of motion and the actual law of motion (i.e., E-stability).

In this paper, we take the theoretical prediction of least squares learning quite seriously and evaluate its predictions using an experiment based on the model of Bray and Savin (1986) and Fourgeaud et al. (1986). Differently from the many "learning-to-forecast" experiments (e.g., Marimon et al., 1993; Hommes et al., 2005a;

[1]Also, as in econometrics, the specification of the perceived law of motion can be correct or incorrect, the former including the REE as a special case. Here we concentrate on the simpler case where the PLM is correctly specified.

Hommes, 2011; Assenza et al., 2014; Petersen, 2014; Duffy, 2016; Bao et al., 2017; Arifovic and Duffy, 2018; Anufriev et al., 2019; Arifovic et al., 2019; Assenza et al., 2019; Hommes et al., 2019; Kryvtsov and Petersen, 2020; Bao et al., 2021; Rholes and Petersen, 2021; Evans et al., 2022; Petersen and Rholes, 2022; Arifovic et al., 2023; Hommes et al., 2023) where subjects make *point predictions*, subjects in our experiment submit parameterizations for the PLM directly. This design enables us to conduct the cleanest and most direct test of the *structural form* of the adaptive learning model that agents are using when learning. In particular, we can directly test whether individuals are adjusting the parameterization of their PLM *as if* they were running least squares regressions in their own minds, albeit without the assistance of computers and statistical software. In addition to least squares learning, we also consider the performance of several other learning models for explaining our results: past averaging models, constant gain learning, stochastic gradient descent learning and a satisficing model.

Our experiment involves three treatments that alter the variance and the shape of distribution of the independent variable of the model that agents are seeking to learn. In *Treatment A* of our experiment, the exogenous independent variable of our model, which we refer to as "weather", follows a simple AR(1) process. By contrast, in *Treatment U* and *Treatment U-small*, this same independent variable follows an i.i.d. uniform distribution. The difference between Treatments U and U-small is that, while the variance of the weather variable is the same in Treatment A and U-small, it is much greater in Treatment U.

As is well known (see, e.g., Greene, 2000), least squares estimates tend to be more accurate (that is, they have lower variance) the larger is the variance in the independent variables. Since the variance in realizations of the independent variable are greater in treatment U as compared with treatment U-small and this is the *only* change made to the model between treatments U and U-small, it follows from econometric theory that learning and convergence to the REE should be faster in Treatment U as compared with Treatment U-small. Further, if the shape of distribution does not matter very much, we should also see that participants' forecasting behavior and learning speed are similar in Treatment A and Treatment U-small.

We find that at the aggregate level, subjects' forecasts in *Treatment U* do indeed converge faster than subjects' forecasts in *Treatment A* and *Treatment U-small*. By the end of the 50 periods of the experiment, the average forecast in *Treatment U* has converged to the REE while the average forecast in *Treatment A* and *Treatment U-small* fails to do so within this timeframe. At the individual level, around 97.8% of the expectations satisfy our criterion for *weak convergence* to the REE. The fraction of individual expectations that converge to $REE \pm 5\% of the REE$ is 55.2% in *Treatment U*, 10% in *Treatment A* and 17.7% in *Treatment U-small*. Finally, just 12 out of 29 subjects (41.4%) in Treatment U, and 0 (0%) in Treatment A and Treatment U-small can be categorized as least squares learners in terms of the adjustment of their parameterization of the PLM over time. Still, some alternative models such as constant gain learning model and "learning by averaging" perform even *worse* than least squares learning in terms of their fit to the experimental data as measured by the mean squared error.

Our findings suggest that while the E-stability criterion provides a good characteriztion of stability under learning or "learnability" of rational expectations equilibrium at the aggregate level, individual subjects may update the parameters of their PLM in a heterogeneous way that deviates from the least-squares learning specification. Our results suggest that instead of searching for the least-squares minimizing combination of the two PLM parameters (a and b), many subjects seem to apply a "satisficing" heuristic (Simon, 1955,9) and stick with the "wrong" pair of parameters if that combination generates approximately the same point predictions as the true but unknown parameters. In other words, when faced with an unfamiliar and complex parameter search and updating problem in 2 dimensional (2-D) space, many subjects in our experiment appear to have reduced the problem to a simpler and more familiar single point prediction problem. This behavioral tendency to reduce a 2-D decision problem to its projection in 1-D space may also be found in theoretical models of "misspecified equilibrium" (Grandmont, 1998) and "(stochastic) consistent expectations equilibrium" (Hommes and Sorger, 1998; Hommes and Zhu, 2014). Note that the subjects in our experiment did not have access to statistical software or computational resources that would enable them to run the regressions associated with least square learning. We did not provide such access since we interpret the notion of adaptive learners-as-econometricians in the "as if" sense of Friedman (1953).[2] Still, we find that 20.3% of subjects do form and adjust their forecasts according to the predictions of the least squares learning model. However, the majority have to apply some simplification method to make the problem (seemingly) more tractable for them.

[2] Friedman (1953), p.21 argued that while expert billiard players might not know the complicated mathematical formulas underlying optimal play, they nevertheless behaved *as if* they knew those formulas. Here we are not supposing that subjects *optimally* form expectations but ask instead whether they form them in the manner prescribed by least squares learning in favorable conditions, i.e. given a PLM and the possibility to adjust the parameters of that PLM as new information arises. We would further add that it is unlikely that most members of the general public would have access to statistical software or be familiar with regression analysis.

Overall, this paper makes three main contributions to the literature.

First, to our knowledge, this is the first experiment where subjects submit structural expectations (model parameterizations) instead of simple point predictions of the variable they are learning about. This design allows us to observe precisely *how* individuals update the parameters of their PLM in real time. This is a particularly useful method for comparing competing models that predict the same qualitative outcome in terms of convergence, but which may differ in the way that individuals update the parameters of their forecasting models. Most surveys on expectation formation, like the Michigan Survey, only elicit point predictions or subjective probability distributions[3]. Data from our laboratory experiment are therefore particularly useful in answering questions regarding the structural path by which individuals update their expectations in real time and their weighting of different factors in forming those expectations. In the learning-to-forecast experiment literature, one study by Hommes et al. (2005b) also asks for forecasting strategies, instead of point predictions in each period. But the strategies they elicited were regarding how participants made their *point* predictions, not how they searched for or updated the parameters of their perceived law of motion as in our study.

Second, this paper presents the first experimental test specifically evaluating *least squares learning* as a behavioral primitive process. Bao and Duffy (2016) run an experiment to explore differences in theoretical predictions between adaptive and eductive learning (Binmore, 1987; Guesnerie, 1992; Evans and Guesnerie, 2005; Evans et al., 2019) models. But the adaptive learning model in that paper is a reduced form, point prediction version where the adaptive learning expectation degenerates to the sample average of all past realizations for prices. Therefore, those results do not reveal *how* people update the *parameters* of their perceived law of motion for the economy and in relation to a specific process such as least squares learning.

Third, our experiment also serves as a test of the capacity of humans to confront complex tasks without the help of computers. To this end, we also contribute to the literature on how the complexity of decision-making influences the accuracy of forecasting behavior (Charness and Levin, 2009; Mirdamadi and Petersen, 2018;

[3]For studies using this survey dataset, see Branch (2004), for studies that compare laboratory and field data on expectations, see Cornand and Hubert (2020) and Afrouzi et al. (2023). For evidence on how information rigidity leads to deviations from RE from survey data, see Coibion et al. (2018). For studies using Randomized Controlled Trials or field experiments, see e.g., Binder and Rodrigue (2018), Armona et al. (2019), Coibion et al. (2020b), Coibion et al. (2020a), Coibion et al. (2022).

Arifovic et al., 2019; Enke and Zimmermann, 2019; He and Kucinskas, 2019) and bounded rationality in expectation formation in macroeconomics in general (Honkapohja, 1995; Branch, 2004; Woodford, 2013).

The rest of the paper is organized as follows: Section 2 presents the experimental design, Section 3 reports on the experimental results, and finally, Section 4 provides a summary and conclusions.

2 Experimental Design

2.1 The Cobweb Model

Consider the cobweb model in Bray and Savin (1986), and Fourgeaud et al. (1986). There is a single market for a product that has a time lag in production (e.g., an agricultural product). The demand for this product depends negatively on the prevailing market price, p_t. The supply of the product is assumed to depend on both the average expectation across the homogeneous firms of the prices that will prevail in the current period, p_t^e, as well as the weather in the current period in the form of an observable shock, w_t.[4] The demand d_t, and supply s_t equations are given by:

$$d_t = m_I - m_p p_t + v_{1t}, \;\; m_p > 0$$

$$s_t = r_I + r_p p_t^e + r_w w_t + v_{2t}, \;\; r_p > 0$$

where m_I, m_p and r_I, r_p are the intercept and slope coefficients, respectively of the demand and supply functions, while v_{1t} and v_{2t} are random noise terms. Thus, in equilibrium, when $d_t = s_t$, the true law of motion for the price of the product is given by:

$$p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t. \tag{1}$$

In the above equation, $\mu = m_p^{-1}(m_I - r_I)$, $\alpha = -m_p^{-1} r_p < 0$, $\delta = -m_p^{-1} r_w$, $\eta_t = m_p^{-1}(v_{1t} - v_{2t})$, and $\eta_t \sim iid(0, \sigma_\eta^2)$. The distribution of the weather variable w_t is an i.i.d. process in Bray and Savin (1986). Alternatively, it may also follow a stationary exogenous VAR (vector autoregression) process driven by a multivariate white noise shock with bounded moments as assumed by Evans and Honkapohja (2001). In our paper, we experimented with both cases, that is, the weather follows

[4]Note that in the original Bray and Savin (1986) model, the current price level, p_t is assumed to depend on the lagged weather variable, w_{t-1}, as the supply in the current period will depend on the observable shock due to weather in the last period. In our experiment, we change this term to w_t in order to help subjects understand the setting more easily. This is a nominal change only and does not alter the results from the model because in the experiment, w_t is also realized and revealed to subjects *before* they make their decisions.

an i.i.d. distribution in two of the treatments, and an autocorrelated distribution in the other treatment.

Under adaptive learning, it is typically assumed that agents have a perceived law of motion (PLM) for prices. This law can be misspecified or correctly specified. Here we consider the case of a correctly specified PLM as it nests the REE as a special case. That is, the PLM we give to subjects implicitly takes account of the expectation feedback term, αp_t^e in the true law of motion.[5] For the model we consider, this perceived law of motion (PLM) is given by:

$$p_t = a + bw_t + \eta_t \tag{2}$$

An implication of the least squares learning approach (as well as variants such as weighted least squares) is that the estimates of the linear regression model will be more precise (i.e., have lower variance) when there is a larger variation in the independent variables[6]. Therefore, theoretically the variance of the estimated coefficients a, b should be smaller in the treatment where the exogenous weather variable has a larger variance.

The unique REE prediction for prices in the Cobweb model is as follows:[7]

$$p_t = \bar{a} + \bar{b}w_t + \eta_t,\ \bar{a} = (1-\alpha)^{-1}\mu,\ \bar{b} = (1-\alpha)^{-1}\delta$$

Given the PLM (2), the REE of the system is learnable only if the parameters of the model satisfy the expectational stability (or E-stability) criterion. Specifically, E-stability requires that $0 < \alpha < 1$.[8]

[5]Misspecified PLMs are also considered in Evans and Honkapohja (2001). Here we focus on the case where the PLM is *correctly* specified since our aim is to understand how agents update the parameters of a PLM that actually enables learning of the REE.

[6]In the simple linear regression model $y_i = \beta_1 + \beta_2 x_i + e_i$, an estimated model $\hat{y}_i = b_1 + b_2 x_i$ can be formed following the least squares principle, where $y_i = \hat{y}_i + \hat{e}_i$. $Var(b_1) = \frac{\sigma^2 N^{-1}\sum x_i^2}{\sum(x_i-\bar{x})^2}, Var(b_2) = \frac{\sigma^2}{\sum(x_i-\bar{x})^2}$. Thus, the wider the spread of the independent variable x, (i.e., a larger $\sum(x_i - \bar{x})^2$) will lead to a more precise estimate (i.e., smaller variance) of both of the parameters. Note that the spread of the independent variable does not affect the accuracy of the estimator because the expectation of the estimates derived using the least squares principle should always be unbiased, i.e., $E(b_1) = \beta_1, \quad E(b_2) = \beta_2$.

[7]A detailed derivation can be found in Appendix F).

[8]See Evans and Honkapohja (2001) Theorem 2.1.

2.2 Least Squares Learning in the Cobweb Model

A common way of modeling the learning of REE is to assume that agents are least squares learners.[9] Under a least squares learning (LSL) assumption, agents are assumed to start with some initial estimates for the parameters a and b of their PLM, e.g., $\hat{a}_0$, $\hat{b}_0$, and adjust these estimates over time so as to minimize the mean of the sum of squared errors between the linear PLM model predictions and actual realizations for prices, p. In our setting, agents regress p_t on a matrix x_t, where the latter is the combination of a vector of 1s, and w_t, a vector the contains the values of realized weather through period t.

$$x'_t = (1\ w'_t).$$

Thus, if agents are least squares learners, in each period t they will update their parameter estimates $\hat{a}_t$ and $\hat{b}_t$ for the PLM, (2) like econometricians so that

$$\begin{pmatrix} \hat{a}_t \\ \hat{b}_t \end{pmatrix} = \left(\sum_{i=1}^{t} x_i x'_i \right)^{-1} \left(\sum_{i=1}^{t} x_i p_t \right) \tag{3}$$

Using the LS estimates, $\hat{\theta}_t = \binom{\hat{a}_t}{\hat{b}_t}$, the learning agent forecasts the price level for period t:

$$p_t^e = \hat{\theta}'_{t-1} x_t \tag{4}$$

This forecast (4) is substituted into equation (1) to determine the actual value for p_t. The formula for determining the least squares estimates can also be written recursively as:

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{1}{t} R_t^{-1} x_t (p_t - \hat{\theta}'_{t-1} x_t)$$

$$R_t = R_{t-1} + \frac{1}{t}(x_t x'_t - R_{t-1})$$

where R_t is the variance-covariance matrix of this regression equation for period t.

A simple alternative to least squares learning that we will also consider is constant gain learning. In this case the gain term on the coefficient vector $\hat{\theta}$ and the moment matrix R is not $1/t$ (decreasing) as it is under least squares learning but is instead a constant value, $\lambda \in (0, 1)$, that bests fits the data.

[9] We understand that there is an important literature on how subjects switch between forecasting rules in learning to forecast experiments, e.g. Anufriev and Hommes (2012), Anufriev et al. (2019), Anufriev et al. (2022) based on Brock and Hommes (1998). Given the large mean squared error generated by the least squares learning model in our study, if we were to explore a heuristic switching model we might find that LS learning gets almost zero weight in favor of other approaches. But in that case, we would be mainly using the LSL forecast predictions and not the (a, b) vector elicited by our experiment.

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \lambda R_t^{-1} x_t (p_t - \hat{\theta}'_{t-1} x_t)$$

$$R_t = R_{t-1} + \lambda (x_t x'_t - R_{t-1})$$

Note that under constant gain learning, the parameter vector is updated by a constant γ times the prediction error in the last period. Therefore, the weight of the most recent past error will not decrease with t, and this algorithm exhibits more volatile dynamics. Indeed, if there is any source of noise in the model (as there is in our system), the constant algorithm will never quite settle down to the REE. Nevertheless, constant gain learning systems have been used by researchers to study learning dynamics, particularly in systems (unlike ours) that are subject to potential structural breaks in the variables being forecast, and so we also consider this specification.

2.3 Parameterization and Treatments

For the experiment, we chose to set $\mu = 9$, $\alpha = -0.5$ and $\delta = 0.9$, so that the market price is given by:

$$p_t = 9 - 0.5 p_t^e + 0.9 w_t + \eta_t, \quad \eta_t \sim i.i.d N(0, 1)$$

For each parameter tuple (a, b) submitted by subjects, the price expectation in each time period t is:

$$p_t^e = a + b \times w_t$$

Assuming that agents have rational expectations, i.e., $p_t^e = p_t$, the REE path for prices is given by[10]:

$$p_t = 6 + 0.6 w_t + \eta_t.$$

Our experiment consists of three treatmens that vary the process for the weather term, w_t. Under all three treatments, the long-run, expected value of the weather variable, $E_t(w_t) = 10$. The treatments differ mainly in the variance and persistence of the independent weather variable, w_t.

***Treatment U* (Uniform Noise)**: In this treatment, the time t realization of w_t

[10]In our experiment, the subjects are told that $a \in [0, 10], b \in [0, 1]$. Our experience with previous forecasting experiments suggests that subjects are very likely to start with the midpoint of the interval, i.e. (5, 0.5). To test whether least squares learning will result in convergence to REE, we should choose a pair of a, b that are not (5, 0.5) but followin the learning literature in macroeconomics, not too far from the REE values either. We therefore choose (6, 0.6) so that the REE is learnable and subjects have sufficient incentives to learn.

is an i.i.d. uniform random draw over the interval $[0, 20]$, i.e. $w_t \sim U(10, 20)$. Thus, $E_t(w_t) = 10$ and the variance of weather in this treatment is given by:

$$\sigma_w^2 = \frac{(20-0)^2}{12} = \frac{100}{3} \approx 33.33.$$

In treatment U, the expected value of the market price is $E(p_t) = 6 + 0.6 \times 10 = 12$, which is the expected value of the REE for the market price in this treatment.

***Treatment A* (Autoregressive Noise)**: In this treatment, we suppose that w_t follows the auto-regressive process:

$$w_t = 2 + 0.8w_{t-1} + \epsilon_t, \ \epsilon_t \sim N(0, 1).$$

For treatment A, we add a constant term to the DGP for w_t to ensure that the long-run equilibrium expected value $E(w_t) = 10$, is the same as in treatment U to facilitate comparisons.[11] Therefore, the REE value of the market price in Treatment A is the same as in Treatment U: $E(p_t) = 6 + 0.6 \times 10 = 12$. According to the definition of the AR(1) process, the variance of the weather variable in this treatment is given by:

$$\sigma_w^2 = \frac{\sigma_\epsilon^2}{1-0.8^2} = \frac{25}{9} \approx 2.78.$$

The careful reader may note that our Treatments U and A differ in two dimensions: the size of the variance of the weather, and the shape of the distribution and autocorrelation. This may lead to a confounding factors issue: when the results from the two treatments differ, is it because of the size of variance, or the shape of the distribution? To address this issue, we added:

***Treatment U-Small* (Uniform Noise with the Same Variance as Treatment A)**: In this treatment, the weather variable follows an i.i.d. uniform distribution as in Treatment U, but the size of the variance is set to be $\frac{25}{9}$, which is the same level as in Treatment A. If results from Treatment U-small are closer to Treatment U (A), then that would suggest the differences between Treatments U and A are mainly driven by the shape of distribution (size of the variance).[12]

Specifically, the time t realization of w_t in Treatment U-Small is an i.i.d. uniform random draw over the interval $[10 - 5/\sqrt{3}, 10 + 5/\sqrt{3}]$. Thus, $E_t(w_t) = 10$ and the variance of the weather variable in this treatment is given by:

[11] We are aware that $E(w_t) = 2 + 0.8w_{t-1}$ is not a constant anymore, therefore, the REE in this system is no longer a point like in Treatment U. Detailed data on w_t and ϵ_t can be found in Table E.1.

[12] We do not run a treatment where the weather variable follows an AR(1) process and the variance is 33.33, since given our high AR(1) coefficient (0.8), having such a large variance could result in negative realizations for the weather variable.

$$\sigma_w^2 = \frac{((10 + 5/\sqrt{3}) - (10 - 5/\sqrt{3}))^2}{12} = \frac{100/3}{12} = \frac{25}{9} \approx 2.78$$

Figure 1 shows the time series realizations for w_t that were used in the three treatments of our experiment, Treatments A, U and U-small. The fourth, bottom right panel of Figure 1 shows a plot of η_t, the noise term realizations used in all three treatments. We used the same 50 realizations for w_t for all subjects who participated in Treatment A, U, or U-small in order to facilitate comparisons across subjects and not add further noise across treatments. As Figure 1 reveals, the variation in w_t is much greater in Treatment U as compared with Treatments A or U-small.

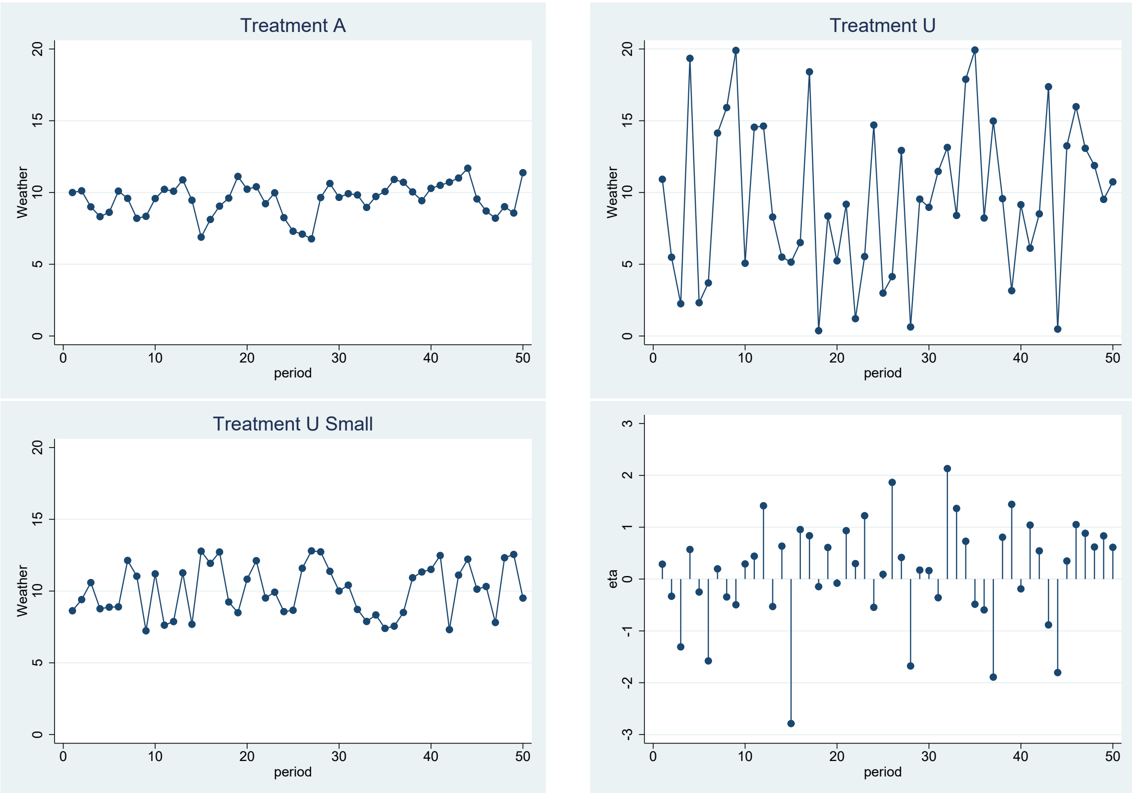

Figure 1. The time series realizations for w_t in Treatment A (top left panel), Treatment U (top right panel), Treatment U-Small (bottom left panel) and the error term, η_t (bottom right panel) as used in the experiment.

2.4 Testable Hypothesis

Figure 2 shows the simulated time series for the estimates of a, b assuming that individuals follow least squares learning (top panels) or constant gain learning (bottom panels). Since individuals choose values from $a \in [0, 10], b \in [0, 1]$, a natural guess would be that most of them would start from the midpoints of those intervals, i.e., $a_1 = 5, b_1 = 0.5$, and so we start all simulations at these points. The model updates the estimates $\hat{a}_t, \hat{b}_t$ using the realized p_t, w_t, in exactly the same way that the least squares learning or constant gain learning model does.

The simulated dynamics for least squares learning suggest that while the least squares estimates $\hat{a}$, $\hat{b}$ have a tendency to converge to the REE values in all three treatments, the convergence is quicker and more reliable in Treatment U as compared with either Treatment A or U-small.

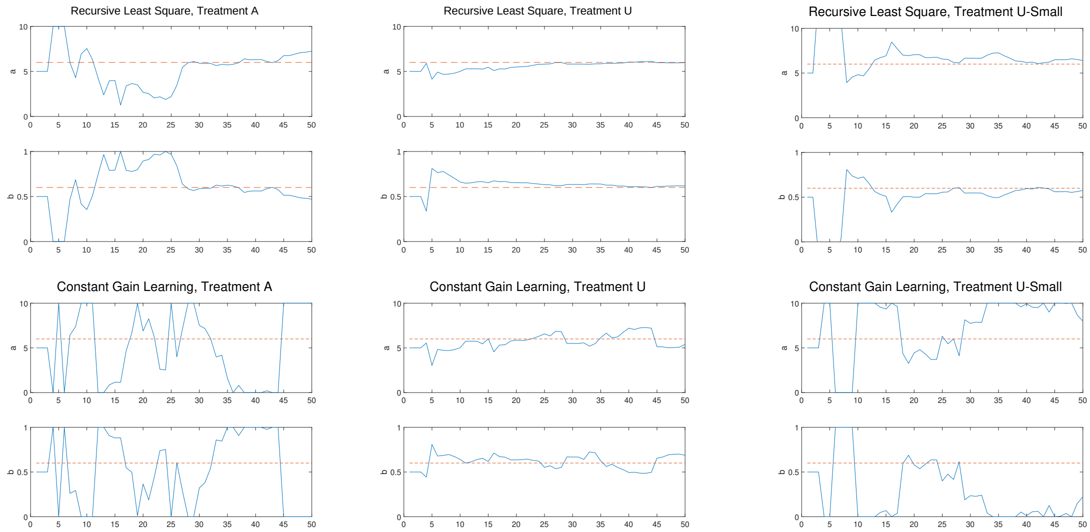

Figure 2. Least squares learning predictions for the paths of the parameter estimates a (top panels) and b (bottom panels) against the REE in each treatment. We initialize each simulation by setting $a = 5$, and $b = 0.5$, and we use the same realizations for w_t and η_t that were used in the experiment. The theoretical values are $a^* = 6, b^* = 0.6$.

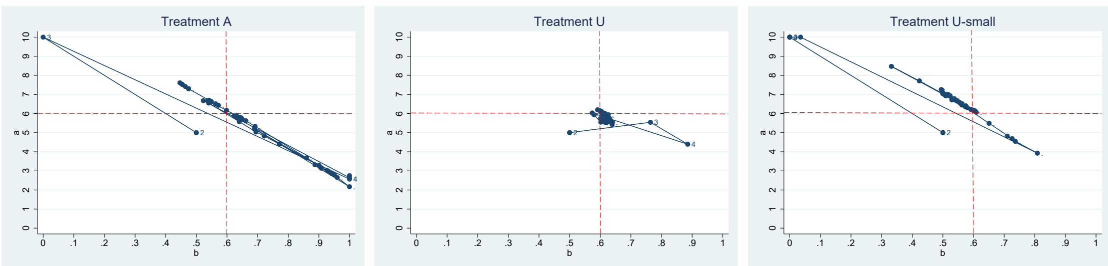

Figure 3. A scatter plot of the least squares learning prediction for the path of the parameters a and b against the REE in Treatment A (left panel), Treatment U (middle panel) and Treatment U-Small (right panel). The labels near the points indicate the period number. We initialize the simulation by setting $a = 5$, $b = 0.5$, and use the same realizations of w_t and η_t that were used in the experiment. The dashed lines are $a = 6, b = 0.6$

We also generated the simulated dynamics for a, b using the constant gain learning model. As suggested by Branch and Evans (2006); Pfajfar and Santoro (2010), the constant gain learning model that best fits the data is usually one with a small gain parameter, λ, e.g., between 0.01 and 0.02. We performed a grid search over λ values between 0.01 and 1 with a step length of 0.01, and selected the λ value (constant gain) that minimizes the MSE between the model's 1 period ahead forecast and each individual's forecast. The results suggest that the mean of the optimal values for the constant gain term λ is 0.0148 in Treatment A, 0.0132 in Treatment U and 0.0081 in Treatment U-Small. The bottom panel of Figure 2 shows the simulated a, b estimates over time. As the figure reveals, the simulated paths for a, b under constant gain learning model are not that different from those under least squares learning. This is because when λ is small, the "learning speed" of the constant gain learning model is not very high. The constant gain learning model also suggests that agents are able to learn the REE within 50 periods in Treatment U, but not within 50 periods in Treatment A and U-small.

Figure 3 shows a scatter plot of pairs of the least squares learning estimates $\hat{a}$, $\hat{b}$ over time in a 2-D plane for all three treatments. (Scatter plots for constant gain learning are similar and omitted). The labels (which are admittedly hard to read after the first few periods) indicate the period numbers. This figure shows that while the parameters seem to make large movements along a downward-sloping line in Treatment A and Treatment U-Small, they do not quite reach the REE by the end of the 50 period horizon. By contrast, in Treatment U, the parameter estimates follow a more compact spiral that does yield convergence to the REE within 50 periods.

Based on these theoretical predictions, we formulate the following main testable hypotheses:

Hypothesis 1 *(E-stability): Subjects can learn the REE, $\bar{a}, \bar{b}$ by the end of the experiment.*

Hypothesis 2 *(Least-squares learning): Subjects update the parameterization of the PLM (2) following the least squares principle, i.e., their estimates for $\hat{a}_t$ and $\hat{b}_t$ follow the predictions of (3), given the complete history of $\{p_s, w_s\}_{s=1}^t$. Subjects behave more like least square learners in the treatments where there is greater variation in the exogenous weather variable.*

Recall that the latter hypothesis is a property of the OLS estimator. Hypothesis 2 may not be supported if subjects use other forms of learning. For example, if subjects apply sample average learning, their performance may be worse when the exogenous weather variable is more volatile and has no auto-correlation.

2.5 Experimental Details

The experiment uses a between-subjects design in which individual subjects are assigned either to Treatment A, U or U-small; each subject only participates in a single treatment. Subjects must then repeatedly choose parameter vectors (a, b) for the PLM, equation (2) over the 50 periods of an experimental session.

Since this is an *individual decision-making* experiment where subjects do not interact with other subjects, we regard each subject as an *independent observation*. In total, 93 subjects were recruited from Nanyang Technological University (NTU, Singapore) to participate in our experiment, which was conduced in five experimental sessions. Subjects were recruited using ORSEE (Greiner, 2015).

We assigned 30 participants to *Treatment U*, 29 participants to *Treatment A* and 34 participants to *Treatment U-Small*. Table A.1 of Appendix A summarizes the number of observations in the different sessions and treatments. After completing 50 periods of parameterizing the PLM, subjects were asked to complete a survey.[13]

The experiment was computerized. At the start of each session, subjects were given written instructions explaining the decisions they would make, the computerized decision screens they would use in making those decisions, and how they earned money from their participation in the experiment. A copy of the experimental instructions is found in Appendix D and screenshots of the experimental interface are found in Appendix E. Before subjects could proceed on to the experiment, they had to correctly answer several control questions testing their understanding of the instructions. These questions are also found in Appendix D.

Subjects earned points during the experiment based on the accuracy of their price predictions. They are asked to predict the price of the commodity in the form of $Price = a + b \times Weather$. They see the realized value of $Weather$ in the same period when making the price prediction, but are not told explicitly the data generating process of the $Weather$ variable. They are told that the value of a is between 0 and 10, and the value of b is between 0 and 1. They are not told the range of $Weather$ variable as they could learn it from the realizations of this variable, and the current value of w_t is always revealed to them before they make any choices. The payoff function (in points) is a decreasing function of the price prediction *error*, and is denoted by:

$$\text{Payoff} = \frac{100}{1 + |p_t^e - p_t|}$$

Subjects were told that at the end of the experiment, points earned over all 50 periods would be converted into money earnings at a fixed and known rate (200 points = 1 SGD). Note that we did *not* incentivize subjects to choose pairs (a, b) to be as close as possible to the values that least squares learning would predict at any moment in time, as our interest was in *whether* subjects would in fact choose their parameter estimates in the LSL fashion. Incentivizing subjects to update the PLM parameters in the LSL manner would only *bias* behavior in the direction of the LSL model since the necessary incentivization scheme would require disclosing

[13] The survey asked them about their age, sex, and how many times they had participated in prior economic experiments. The survey also asked them to provide the strategies they used throughout the experiment. A copy of the survey can be found in Appendix E.

to subjects the LSL updating rule by way of explaining their payoff function.[14]

Subjects chose a and b using two slider bars on their decision screen with a parameter range of [0,10] for a and a parameter range of [0,1] for b. (See screenshots in Appendix E). Note that these ranges include the REE values, $\overline{a} = 0.6$ and $\overline{b} = 6$.[15] It is important to note that with a *linear* PLM, and a new i.i.d. realization of the exogenous variable, w_t, in each period, the subject's choice of the parameters a and b in every period t is the same situation that someone using least squares learning would face.

As subjects moved the sliders for either parameter, the computer program showed both the value of a and b and the implied price forecast, p_t^e that would result from their choices for a and b. By moving one slider at a time, they could see how a change in a or b affected their price forecast p_t^e. Subjects had unlimited time to move the sliders around to see what they implied for price forecasts *before* clicking on a submit button that finalized their choices for a and b in each period t. Thus, subjects were incentivized to think about their choices for the two parameters a and b of the PLM and what those choices implied for their price forecast, p_t^e.[16] Following each period, subjects received feedback in the form of an updated plot of all past prices together with their predictions. They also saw a table containing the history of all their prior period estimates for a, b, realizations of the weather variable w, their implied price forecast p^e the realized price, p their prediction error, $|p^e - p|$, and both their period and cumulative point totals.

Recall from the payoff function that the maximum payoff for a perfect forecast is 100 points per period. Subjects' final payoff is the sum of their 3 SGD show-up fee, and the money value of the points they earned from all 50 periods of the experiment. The experiment takes around two hours on average to complete (including instructions, quiz and the 50 period task and the exit survey) and the average total payment (including the show-up fee) is 20.83 SGD for *Treatment A*, 20.70 SGD for *Treatment U* and 21.51 SGD for *Treatment U Small*. The total average payoff of the experiment is 21.04 SGD.

[14]Similarly, we did not incentivize subjects to choose values for $(a,\ b)$ to be as close as possible to the REE values $(\overline{a},\ \overline{b})$ since subjects would have been able to discover these REE values by looking at their payoff point discrepancies alone.

[15]The midpoints of these parameter ranges, (0.5 and 5, respectively) are a natural first period guess for subjects and are not too far away from REE values. This choice of interval ranges was by design since most learning analyses (see, e.g. Evans and Honkapohja (2001)) study how agents learn in response to very small perturbations of expectations away from REE values.

[16]This design is similar in spirit to the "strategy method" of Selten (1965) that is used to elicit *strategies* as opposed to *actions* alone in game theory experiments.

3 Experimental Results

3.1 Convergence to REE

3.1.1 Convergence of the Market Price

Figure 4 shows the average deviation and the average absolute deviation of the actual price from the REE in Treatments U, A and U-small using all data for reach treatment. As Figure 4 reveals, on average, the deviation from the REE price is small in all three treatments. The difference between the average market price and the REE is usually less than 1. The results of a t-test suggest that the absolute difference between the market price and the REE is significant at the 5% level in Treatment U ($t = 11.626, p - value = 0.000$), Treatment U-Small ($t = 31.4834, p - value = 0.0000$) and Treatment A ($t = 10.257, p - value = 0.000$). On the other hand, the average difference between the market price and the REE is not significantly different from zero at the 5% level for either Treatment U ($t = -0.105, p-value = 0.381$), Treatment U-Small ($t = 1.2230, p-value = 0.2272$) or Treatment A ($t = -0.884, p - value = 0.299$).

We also performed a t-test on whether the difference between the actual market price and the REE price is significantly different from 0 at the 5% significance level for each *individual* subject and we report these results in Table A.2 in Appendix A. It turns out that we cannot reject the null hypothesis of no difference for all but one subject each in each of the three Treatments. That is, we cannot reject the null of no difference for 29 out of 30 subjects in Treatment A, and 28 out of 29 subjects in Treatment U and 33 out of 34 subjects in Treatment U-Small.

This result shows that when the economy satisfies E-stability ($\alpha < 1$), the market price indeed converges to the REE. But it is important to remain aware that for the same realized w_t, there are infinitely many pairs of values of a and b that satisfy the equation $a + bw_t = 6 + 0.6w_t$. Therefore, we cannot rule out the possibility that individuals successfully predict the REE but are using a model that differs from the REE values for a and b or from what least squares learning would predict for the estimates of those parameter values at any point in time. In the next section, we will consider in more detail whether individuals indeed learn to choose the right combination of values for a and b.

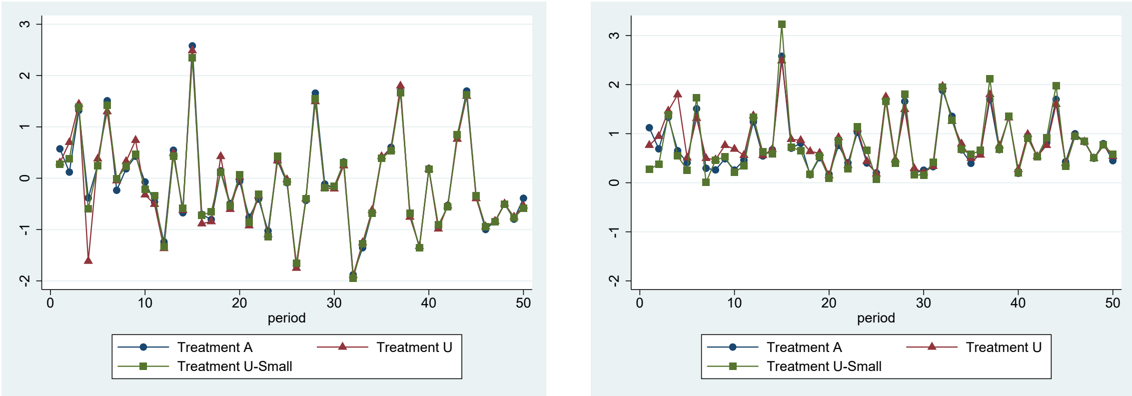

Figure 4. The average deviation (left panel) and average absolute deviation (right panel) of market price from the REE in Treatment A (triangles), Treatment U (circles) and Treatment U-Small (squares).

3.1.2 Sample Means for a, b

In this section, we investigate whether the mean parameter estimates for a, b converge to the REE values, $\overline{a}$, $\overline{b}$. We test two important characteristics, namely whether the parameter estimates are biased and whether they exhibit excess volatility, by comparing the mean and the variance of the coefficient estimates with the REE values in all treatments.

Figure 5 plots the average of all subject predictions of the parameters a, b in each of the 50 periods of the experiment against the REE values which are represented by the horizontal lines, where $\bar{a} = 6$, and $\bar{b} = 0.6$. The Figure reveals the rapid and dampened adjustment over time toward the REE in *Treatment U*. By contrast, in Treatment A, we observe a persistent upward bias in the average estimate for a and a corresponding downward bias in the average estimate for b, relative to REE predictions.[17] Similarly, (but opposite) in Treatment U-small, the estimate of a is persistently below the REE value while the estimate of b is persistently above the REE value.

[17] We also conduct tests on the speed of convergence, where following the analysis in Figure 1, subjects in *Treatment U* are able to reach the REE much faster than subjects in *Treatment A*. Details of this analysis can be found in Appendix A.

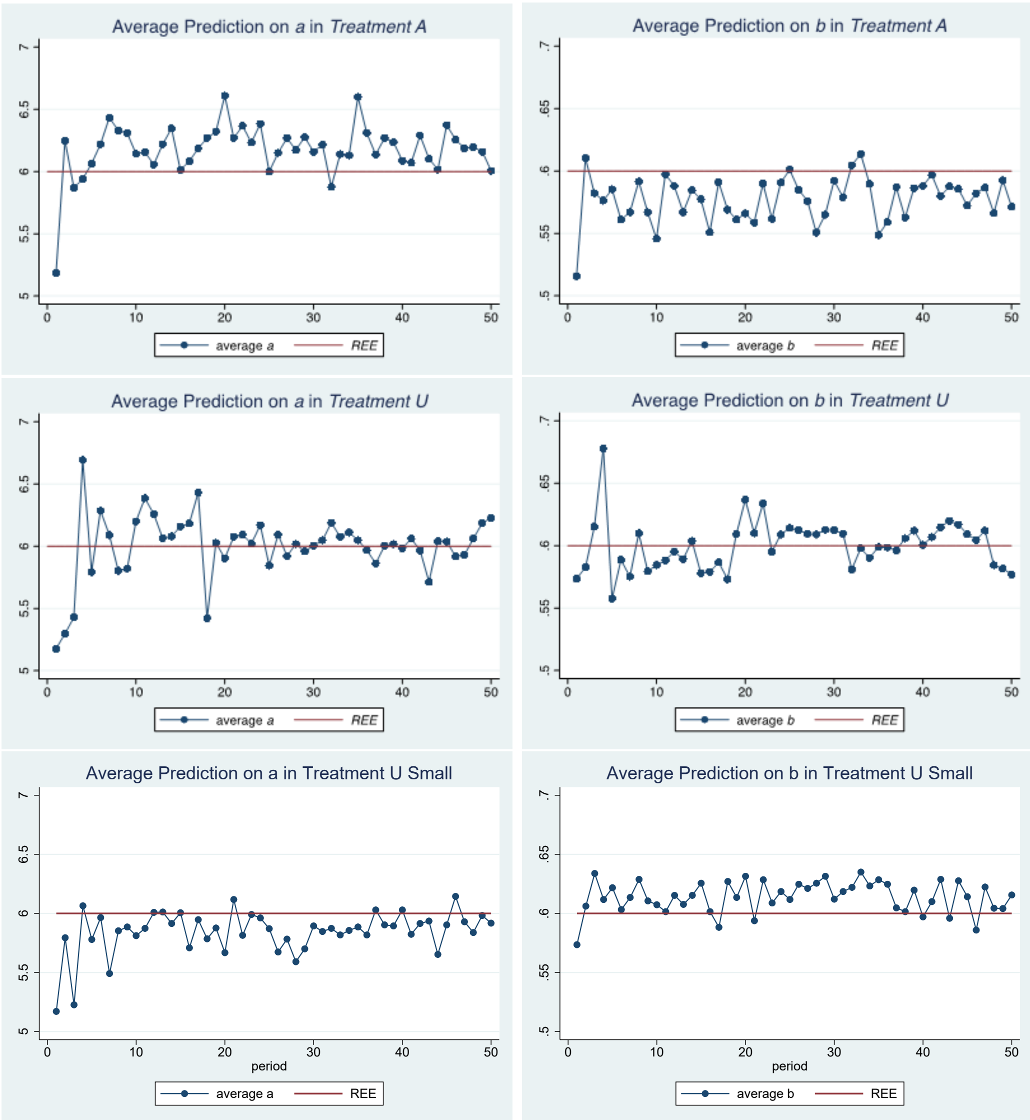

Figure 5. Average predictions for the parameters a and b for treatment A (upper panel), Treatment U and treatment U-Small (lower panel) against the REE values. The theoretical values are $a^* = 6, b^* = 0.6$.

We next make a more direct comparison between the mean values for a_t, b_t and the REE values. Table 1 shows the sample means and standard deviations of the parameter estimates for a and b in the first 25 periods, the last 25 periods, and the full sample of 50 periods for the two treatments. We note that the means for a, b in *Treatment U* are closest to the REE values, $a = 6$, $b = 0.6$ for all three intervals as compared with treatments *A* and *U-small*.

Table 1. Sample means and standard deviations for a, b, over the first 25 periods, the second 25 periods, and the full sample of 50 periods for the two treatments. The theoretical values are $a^* = 6, b^* = 0.6$.

	Periods 1-25		Periods 26-50		Periods 1-50	
	sample average	s.d	sample average	s.d	sample average	s.d
			Treatment A			
a	6.170	1.453	6.188	1.404	6.179	1.428
b	0.574	0.152	0.580	0.148	0.577	0.152
			Treatment U			
a	5.988	1.220	6.018	1.390	6.003	1.120
b	0.598	0.134	0.603	0.114	0.600	0.126
			Treatment U-Small			
a	5.824	0.041	5.865	0.038	5.845	0.028
b	0.612	0.004	0.616	0.004	0.614	0.003

Table 2 shows results from a t-test of the null hypothesis that the sample means of subjects' choices for a, b differ from the REE values over the first 25 periods, and the last 25 periods, and the full sample of 50 periods of all treatments. For *Treatment U* the t-tests reveal that the null hypothesis that the sample mean of the parameters equals the REE values *cannot be rejected* at the 5% significance level over each of the three time windows. However, for *Treatment A* and *Treatment U-Small*, the t-tests lead to *rejection* of the null that the sample means of both parameters are equal to the REE values over all three time windows. These results imply that both parameters in *Treatment U* converge to REE on average over all three samples, while the parameters fail to converge to the REE in *Treatment A* and *Treatment U-Small* in any time sample.

Table 2. Test of whether the means of the predicted parameters a, b, are different from the REE values ($a^* = 6, b^* = 0.6$) over the first 25, and last 25 periods, and all 50 periods for all treatments.

	Periods 1-25				Periods 26-50				Periods 1-50			
	$a = 6$		$b = 0.6$		$a = 6$		$b = 0.6$		$a = 6$		$b = 0.6$	
Treat	t-stat	p	t-stat	p	t-stat	p	t-stat	p	t-stat	p	t-stat	p
A	3.213	0.001	-4.676	0.000	3.433	0.000	-3.882	0.000	4.702	0.000	-8.790	0.000
U	-0.262	0.794	-0.437	0.662	0.660	0.509	0.626	0.531	0.229	0.819	0.152	0.880
U-Small	-4.339	0.000	2.936	0.003	-3.504	0.000	4.175	0.000	-5.561	0.000	4.998	0.000

Tables B.1 and B.2 in Appendix Appendix B show the 95% confidence intervals for a and b for each individual subject. If the confidence interval of a (b) contains 6 (0.6), it should imply that we cannot reject the null hypothesis that $a = 6$ ($b = 0.6$) for this individual. We count the number of subjects for whom we can reject neither $a = 6$ or $b = 0.6$ in each treatment, and find that we cannot reject that the means of a and b are equal to their REE level for 5 out of 30 subjects (16.67%) in Treatment

A, 3 out of 34 subjects (8.82%) in Treatment U-small and 14 out of 29 subjects (48.28%) in Treatment U.

3.1.3 Individual-level Analysis of the Convergence of a, b

The last section focused on whether the sample mean values for a, b are equal to the REE values. In this section, we examine the development of the elicited a, b *over time*, i.e., whether there is some tendency for estimates to converge to the theoretical values of $a = 6, b = 0.6$ with experience.

We use a very intuitive convergence criterion: a subjects' estimates for a, b are said to converge to the REE if they lie in a very small neighborhood ($\pm .3$ or .03) of the theoretical values, i.e., $a \in [5.7, 6.3]$, $b \in [0.57, 0.63]$ *and* do not leave that interval following the first period the interval is entered (a consistency requirement). We chose these intervals because they correspond to the $REE \pm 5\% \times REE$. This measure has also been used in previous learning-to-forecast experiments e.g., Bao et al. (2013). Table 3 reports the distribution of individuals in terms of the number of periods required for convergence to the REE. In treatment A, most subjects never learn the REE; only 3 out of 30 subjects (10%) learn the REE within the 50 periods allowed in the experiment. By contrast, in Treatment U, 6 out of 29 subjects managed to learn the REE within 10 periods, and 10 more managed to learn it within 50 periods. Thus, by the end of the experiment, more than half of subjects in Treatment U, 16 out of 29 (55%), have learned the REE. In Treatment U-small, only 6 out of 34 (18%) subjects converge within 50 periods.

Table 3. Distribution of the number of periods it takes for subjects to converge to both REE parameter values, $a \in [5.7, 6.3]$, $b \in [0.57, 0.63]$.

	Treatment					
	A		*U*		*U − Small*	
No. of Periods before Convergence	No. of subjects	Percentage	No. of subjects	Percentage	No. of subjects	Percentage
$T = 1$	0	0.0%	1	3.4%	0	0.0%
$T \in [2, 5]$	0	0.0%	4	13.8%	0	0.0%
$T \in [6, 10]$	0	0.0%	1	3.4%	2	5.9%
$T \in [11, 25]$	1	3.3%	4	13.8%	0	0.0%
$T \in [26, 49]$	2	6.7%	6	20.7%	4	11.8%
$T \geq 50$	27	90.0%	13	44.8%	28	82.35%
Total	30	100%	29	100%	34	100%

In addition to this simple convergence criterion, we also examine convergence using regression analysis. We use the convergence formula suggested by Bao et al. (2013) to find the number and percentage of subjects who successfully achieve convergence to the REE in each treatment.

The linear equation we use for testing whether convergence obtains is as follows. We assume the updating of the parameters follows an AR(1) process. For the parameter submitted by each subject i in period t, we test how that parameter depends on last period's $(t-1)$ submitted parameter, where ρ stands for the coefficient of that relationship, μ is a constant term, and ϵ is the error term.

$$a_{i,t} = \rho_{a_i} a_{i,t-1} + \mu_{a_i} + \varepsilon_{a_i}$$

$$b_{i,t} = \rho_{b_i} b_{i,t-1} + \mu_{b_i} + \varepsilon_{b_i}$$

We say there is *weak convergence* if the parameter submitted by subject i has an estimated value for ρ that is significantly smaller than 1, i.e., if

$$|\hat{\rho}_i| < 1$$

We find that the null hypothesis that $|\hat{\rho}_i| = 1$ *is rejected* for 97.8% of our sample (or 180 out of 184 predictions,[18] at the 5% significance level in favor of the alternative that $|\hat{\rho}_i| < 1$, implying that our sample exhibits some overall *weak convergence* when predicting the parameters.

We summarize our results to this point as follows:

Result 1 (E-stability): On average, subjects' predictions for both parameters converge to the REE values in treatment U but not in treatments A or U-small. At the individual level, 97.8% of all parameter choices (180 out of 184) satisfy a *weak* form of convergence to the REE values.

3.2 Fit of the Least Squares Learning Model to the Data

3.2.1 Aggregate Level

In this section, we test whether subjects update their parameter estimates in each of the periods in the manner predicted by least squares learning (3).

The least squares learning model states that subject i will update their parameter estimates for a, b in the current period t, based on the new realization for the weather variable w and past realized price information i.e., prices for periods 1 to $t-1$. For each subject i, in period t, the simple mathematical expression of these least squares learning estimates is given by:

$$\hat{b}_{i,t} = \frac{\sum_{s=1}^{t-1}(w_{i,s} - \overline{w_i})(p_{i,s} - \overline{p_i})}{\sum_{s=1}^{t-1}(w_{i,s} - \overline{w_i})^2}, \quad \hat{a}_{i,t} = \overline{p_i} - \hat{b}_{i,t}\overline{w_i}$$

where $\overline{w_i} = \frac{\sum_{s=1}^{t-1} w_{i,s}}{t-1}$, $\overline{p_i} = \frac{\sum_{s=1}^{t-1} p_{i,s}}{t-1}$. Thus, the parameter estimates $\hat{a}, \hat{b}$ are the ones that subject i should submit in time period t if he or she follows the LS learning rule.

[18]Note that there are two parameters a, b, so the total number of equations is $92 \times 2 = 184$.

In each period, the LS learning model uses the same information set as subjects had available to them in the experiment and makes a one period ahead forecast for subjects' choices, $\hat{a}_{i,t}$, $\hat{b}_{i,t}$. Note that this is different from the simulation we did in Figure 2 of Section 2 where the model makes 50 periods ahead forecasts for subject's choices for a, b in all 50 periods after we initialized the model using $a_1 = 5, b_1 = 0.5$. That is, while both simulations assume that the agent knows the history of the weather. They use the past prices that each subject actually faced when deciding how to update their estimates of a and b, but not on the past prices generated by the LS learning algorithm.

We ran the iterated LS regression for all three treatments, and recorded the predicted parameter estimates. Note that unlessotherwise stated, the results we present in this section start from period $t = 3$ ($T \in [1, 2]$). This is due to the sample size being too small in period $t = 2$ ($T \in [1, 1]$). The sample also ends at period $t = 50$ as we do not have data on subjects' submitted parameters for period 51.

Figure 6 plots the average estimated $\hat{a}, \hat{b}$ for the LS learning model in each treatment against the average a, b from the experimental data, and the REE. This figure reveals a striking difference between subjects' choices for a b and the least squares learning predictions in Treatment A and Treatment U-Small. The estimated $\hat{a}$ ($\hat{b}$) is downward (upward) biased while the experimental data is upward (downward) biased relative to the REE! Meanwhile, the least squares learning model tracks subjects' choices for a, b considerably better in Treatment U. In all treatments, the human subject estimates are more volatile than the least squares learning estimates for both parameters.

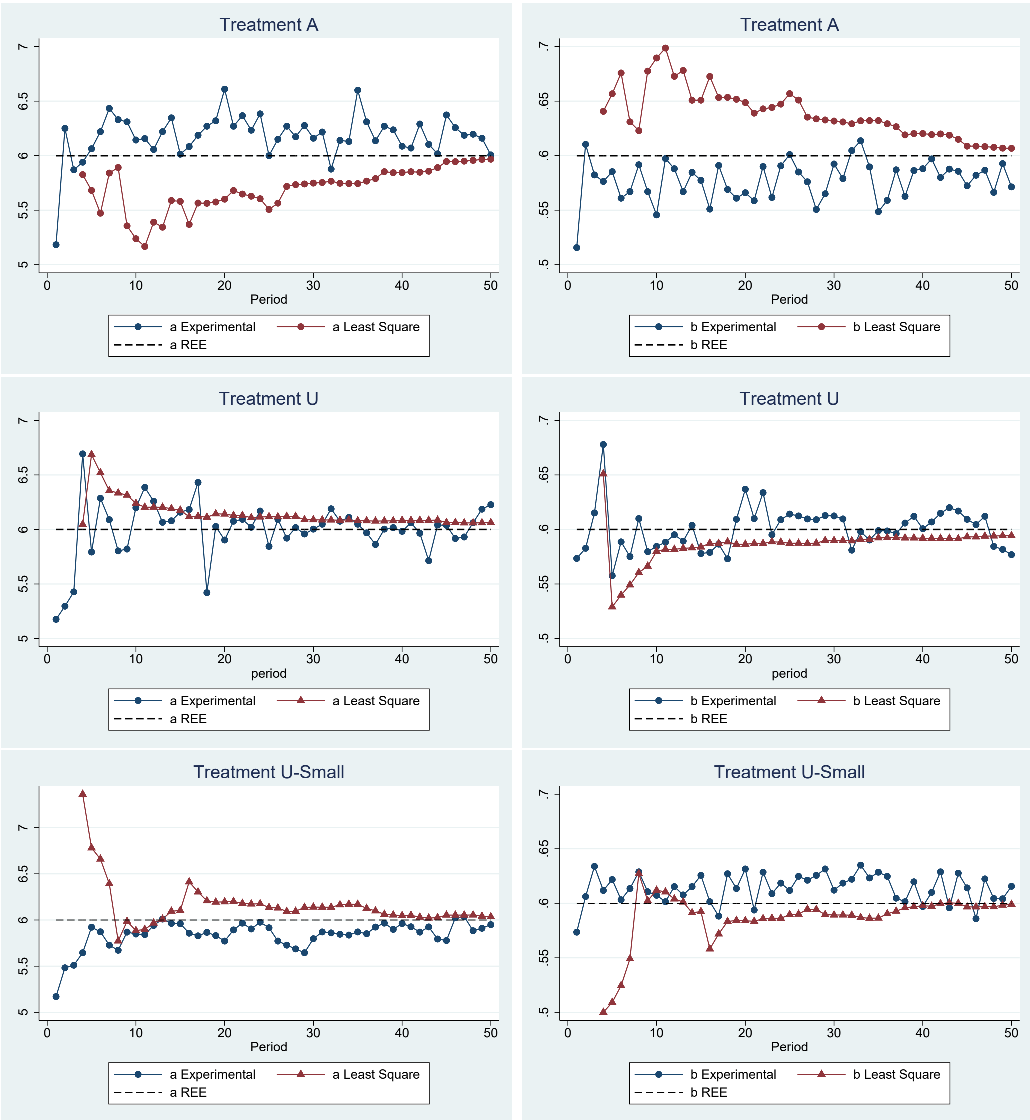

Figure 6. Mean experimental data versus the mean prediction of the 1 period least squares learning model for the parameters a (left panel) and b (right panel) against the REE in each treatment. The theoretical values are $a^* = 6, b^* = 0.6$.

Table 4 reports the average mean squared error (MSE) of the least squares model relative to the experimental data and the average (square) root MSE (σ) for all treatments over all 50 periods, the first 25 periods and the second 25 periods. In general, the theoretical prediction of the least squares learning model is not far from the experimental data. The average root MSE is always between 1 and 2 for parameter a, and between 0.01 and 0.05 for parameter b, corresponding to approximately $16\% - 67\%$ of the value of the REE. The MSE for the LS model higher in Treatment A than in Treatment U or Treatment U-small in all intervals except for parameter a in periods 26-50. Generally we find that the least squares model fits the data better in Treatment U and U-small than in Treatment A.

Table 4. MSE and root MSE (σ) of the least squares learning model relative to the experimental data in the first 25 periods, the second 25 periods, and the full sample of 50 periods for all three treatments. The theoretical values are $a^* = 6, b^* = 0.6$.

	Period 1-50		Period 1-25		Period 26-50	
	MSE	σ	MSE	σ	MSE	σ
			Treatment A			
a	4.536	1.984	6.733	2.479	2.428	1.219
b	0.050	0.207	0.073	0.257	0.055	0.132
			Treatment U			
a	1.837	1.121	2.470	1.343	1.228	0.774
b	0.020	0.117	0.025	0.136	0.016	0.088
			Treatment U-small			
a	1.350	1.162	1.430	1.196	1.271	1.127
b	0.013	0.116	0.014	0.119	0.013	0.112

We further investigate whether the null hypothesis that $a_{i,t} = \hat{a}_{i,t}$, $b_{i,t} = \hat{b}_{i,t}$, holds on average over the aggregate level. We claim a successful adoption of the LS learning rule if the null hypothesis that agent update parameter estimates according to least squares learning predictions *cannot be rejected* at the 5% significance level. The test results using t-tests, can be found in Table 5. It turns out that the estimates differ significantly from the LS learning rule at the aggregate level when we use the data over all 50 periods. The null hypothesis is also *rejected* even if we restrict the sample to last 25 periods (periods 26-50) with the sole exception of parameter a in Treatment U. In summary, we find almost no supportive evidence that subjects update their parameter estimates following the LS learning model at the aggregate level.

Table 5. Results of t-tests of the null hypothesis that $a_{i,t} = \hat{a}_{i,t}, b_{i,t} = \hat{b}_{i,t}$ holds on average over the full sample of subjects.The theoretical values $a = 6, b = 0.6$.

Treatment A				*Treatment U*				*Treatment U-small*			
a		b		a		b		a		b	
				All 50 Periods							
z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value
7.237	0.0000	-8.891	0.0000	-3.424	0.0006	3.6058	0.0003	-6.9882	0.0000	6.5729	0.0000
				Second 25 Periods							
z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value	z-stat	p-value
6.347	0.0000	-6.969	0.0000	-1.526	0.1274	2.423	0.0156	-4.9525	0.0000	4.6081	0.0000

3.2.2 Individual Predictions

In this section, we examine how many subjects follow least squares learning at the *individual level*. When we conduct the estimation for each individual, we follow the "1 period ahead" method, namely, in each period, the individual updates the parameters a and b following the least squares learning rule as specified on page 9, and we load the realized values of p_i from the experimental data. We only show the fitted values a and b after Period 4 because we need at least 3 data points (periods) to run a meaningful OLS estimation.

We calculate the mean squared error (MSE) between the least squares learning model prediction and each subject's choice for a, b in Treatments A, U and U-Small. We consider the person a user of the least squares learning model if their MSE is sufficiently small, i.e., if $MSE < 0.36$ (i.e. a Root MSE less than 0.6, or 10% of the REE) for a and a $MSE < 0.036$ for b (a Root MSE less than 0.06, or 10% of REE) for b. The results can be found in Table C.1 in Appendix C.

Our results reveal that while 12 out of 29 (41.4%) subjects in Treatment U (Subjects 1, 2, 6, 12, 15, 16, 17, 18, 20, 21, 23 and 28) can be categorized as least squares learners, there are *no subjects* in Treatment A or Treatment U-Small who can be categorized as a least squares learners using the same approach.

Result 2 (Least Squares Learning): We *reject* the hypothesis that subjects update the parameters of the PLM following the LS learning rule on average in the aggregate. Yet at the individual level, around 41.4% of subjects in Treatment U and no subject in Treatment A or Treatment U-Small appear to update their beliefs following the LS learning rule (3).

3.3 Other Learning Models

Since least squares learning does not seem to characterize very well what most subjects were doing in terms of parameterizing the PLM (2) over time, in this section we ask whether other models might do a better job of rationalizing the behavior of the subjects in our experiment. Specifically, we consider four alternatives to least squares learning: (1) constant gain learning model, (2) the least mean squares (or stochastic gradient) learning model, and finally (3) a model of satisficing.

3.4 Constant Gain Learning

We also estimate the constant gain learning model (discussed earlier in section 2.2) for each individual and report the results in Table C.2 in Appendix C. Similar to the exercise reported on in the previous section, we consider the person a user of the constant gain learning model if the MSE between their parameterization of the PLM and the constant gain learning model predictions are sufficiently small, i.e., if $MSE < 0.36$ (Root of MSE less than 0.6) for a and $MSE < 0.036$ for b (Root of MSE less than 0.06) for b. It turns out that there are only 8 out of 29

(27.6%) subjects in Treatment U (Subjects 1, 2, 12, 15, 18, 20, 23 and 28) who can be categorized as users of the constant gain learning model, and no subject in Treatment A or Treatment U-Small who can be categorized as a user of constant gain learning model. For the 8 subjects in Treatment U, the mean squared error of the constant gain learning model is larger than for the least squares learning model. In general, though the constant gain learning model is usually assumed to converge "faster" to an REE than least squares learning and is more suitable in the context where the price dynamics are more volatile, we do not find stronger evidence for constant gain learning in our individual-level analysis even though the constant gain learning model has a free parameter γ estimated for each subject that helps to best fit the experimental data.

3.5 Least Mean Squares Learning

The time and memory complexities of RLS and constant gain learning are both $O(m)^2$, where m is the dimension of x. A much simpler learning algorithm is the least mean squares (LMS) learning model which is also known as stochastic gradient descent learning (e.g. Evans et al. (2010)). In this case, only the parameter vector $\hat{\theta}$ is updated according to the gradient of the error term; the variance covariance matrix is not used.

This algorithm is also derived from the objective of minimizing the mean of squared errors, but it does not rely on cross-correlations or auto-correlations, i.e., on the variance-covariance matrix R. Thus, the time and memory complexities of LMS learning are $O(m)$. On the other hand, convergence to the global minimum is not assured under least mean squares learning unless the gain parameter λ is gradually reduced over time as in RLS.

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \lambda x_t (p_t - \hat{\theta}'_{t-1} x_t) \tag{5}$$

We estimate the least means squares learning model (λ) for each subject and report the MSE between those predictions and subjects' parameter choices in Table 6 in Section 4.6 and C.3 of Appendix C. The MSE of the least mean squares learning model turns out to be larger than for the least squares and constant gain learning models. If we consider a subject to be a user of the least mean square learning model if the MSE between the data and that model's predictions is sufficiently small, i.e., if $MSE < 0.36$ (root MSE less than 0.6) for a and $MSE < 0.036$ for b (root MSE less than 0.06) for b, then it turns out that there are only 2 out of 29 (6.9%) subjects in Treatment A (Subjects 15 and 21), 2 out of 30 (Subjects 2 and 20) subjects in Treatment U (6.7%), and 2 out of 34 (Subjects 7 and 9) subjects in Treatment U (5.9%) who can be categorized as users of the least mean square learning model.

We think the main reason is that the stochastic nature of this learning process has difficulty in capturing the convergence to REE that we observe among most of our subjects.

3.6 A Satisficing Model

A final plausible explanation for the behavior of subjects in our experiment is that they were using some type of "simple satisfying heuristic" or satisficing rule as suggested by Simon (Simon, 1955,9). In particular, subjects might stay with a prediction rule, or a specific combination of a, b so long as those parameter choices kept their prediction error small, or reached a close enough neighborhood of the REE. Thereafter, they do not engage in any further updating of the parameter vector $(a,\ b)$.

In our experiment, the unique REE of the economy is $p^e = 6 + 0.6w_t$. If we ignore the variation in w_t and simply use the expected value, $E[w_t] = 10$, the numerical value of the price point prediction associated with the REE is $6+0.6E(w_t) = 6 + 0.6 \times 10 = 12$.

If the variation in w_t is small, then *any* combination of a, b that satisfies the equation $a + 10b = 12$ should generate a point prediction that is not very far from the REE of the economy, and hence yield only a small prediction error. If subjects learn via experimenting with different combinations of a, b and adjust their choices to minimize the prediction errors, this process may lead them to choose any pair of values for a, b that are not too far away from $a = 6, b = 0.6$ but which also satisfy the equation $a + 10b = 12$, for example, $a = 7$ and $b = 0.5$ or $a = 4.8$ and $b = 0.72$ would work.[19]

Figures 9-**??** show the dynamics of $a_i + 10b_i$ for each subject i in Treatments A, U, and U-small, respectively. Indeed, though many subjects fail to learn the REE values for a and b, most of them are able to choose a combination of the a, b parameters that satisfies the equation $a + 10b = 12$.

Table C.4 in Appendix C reports on a 95% confidence interval for $a + 10b$ in all treatments. It turns out that this confidence interval includes the REE value for 12 out of 30 subjects in Treatment A, 25 out of 29 subjects in Treatment U (all subjects

[19]Indeed, if a subject starts from either the midpoint of the domain of a or b and only updates the other parameter, we should observe many subjects choosing $a = 5, b = 0.7$ or $a = 7, b = 0.5$. It turns out we cannot reject this type of behavior for 6 subjects in Treatment A (Subject 10,12,13,15,21 and 30), and 2 subjects in Treatment U (Subject 6 and 9).

but 3, 6, 13 and 26), and all 34 out of 34 subjects in Treatment U-small. For these subjects, a t-test indicates that we cannot reject the null that these subjects chose a and b so as to satisfy the equation $a + 10b = 12$ at the 95% level. In other words, across all treatments, most subjects are able to come up with a point prediction that is close to the REE point prediction, even without learning the true REE values for a and b.

Figures 10-12 show scatter-plots of a and b over time for each individual in Treatments A, U, and U-small, respectively. These figures reveal a substantial level of heterogeneity in the ways that people update the parameter vector of the PLM over time. While the behavior of some subjects (Subjects 1, 8, 14, 16, 24 in Treatment A, Subjects 16 and 24 in Treatment U and Subjects 13, 17 and 21 in Treatment U-small) seem to behave in a similar manner to the simulated path of a, b from the least squares learning model as shown in Figure 3, other subjects behave very differently. For example, some subjects (Subjects 11, 17, and 21 in Treatment A, Subjects 12, 18 and 20 in Treatment U, and Subjects 28, 30 and 34 in Treatment U-small) seem to experiment with different values for b while keeping the value of a fixed. Some subjects (Subjects 11, 20, 27 and 29 in Treatment A, and Subjects 7, 9, 16 and 25 in Treatment U-small) are also able to reach a small neighborhood of the REE fairly quickly. Some subjects (Subjects 3, 6, 8, 10, 25 in Treatment A, 26 in Treatment U and Subjects 18 and 33 in Treatment U-small) explored a large range of values for the parameters before they settled down in a region that was usually not far from the REE values.

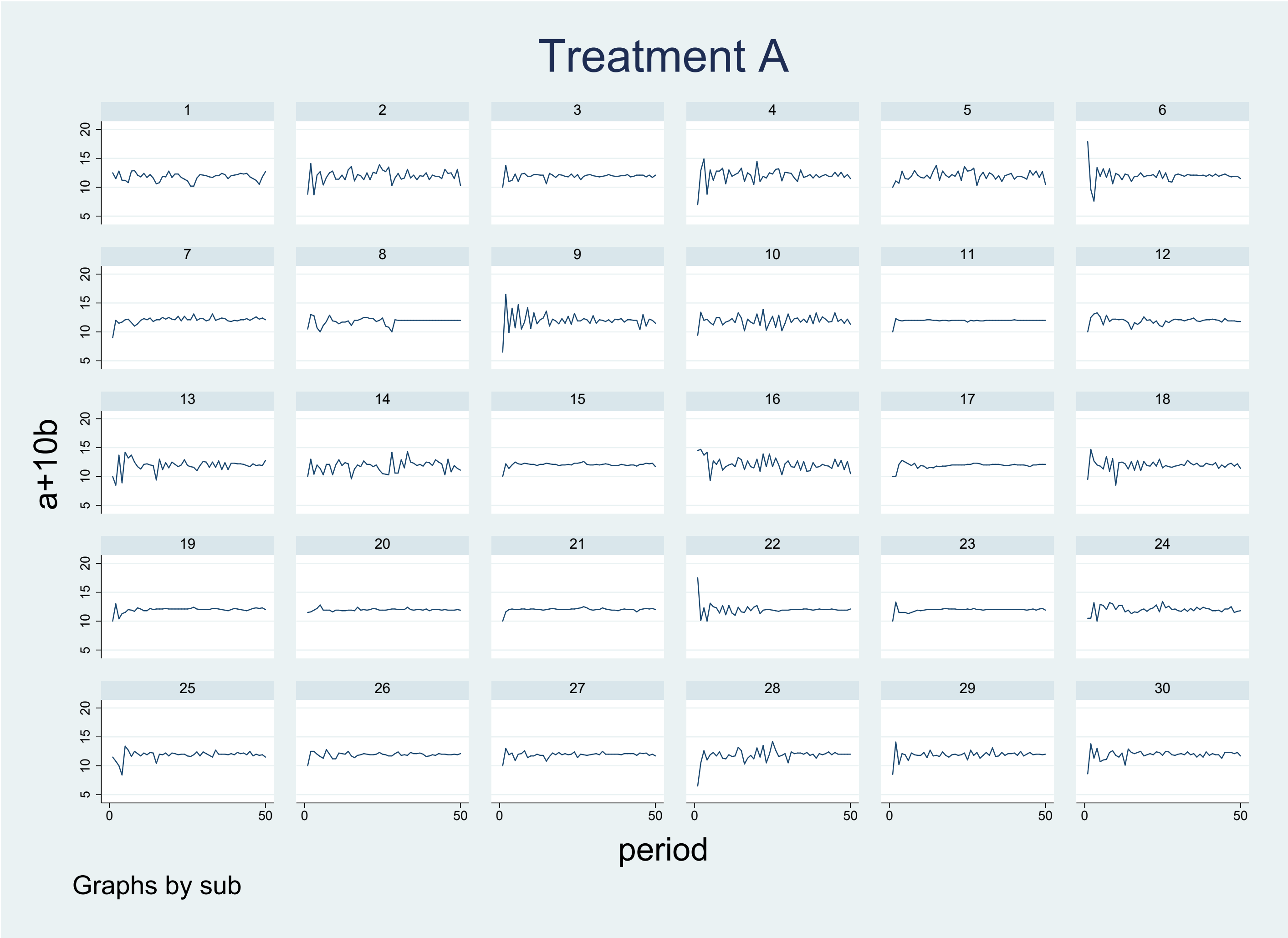

Figure 7. The value of $a + 10b$ for each individual in Treatment A. We report the 95% confidence intervals of $a + 10b$ for each subject in Table C.4.

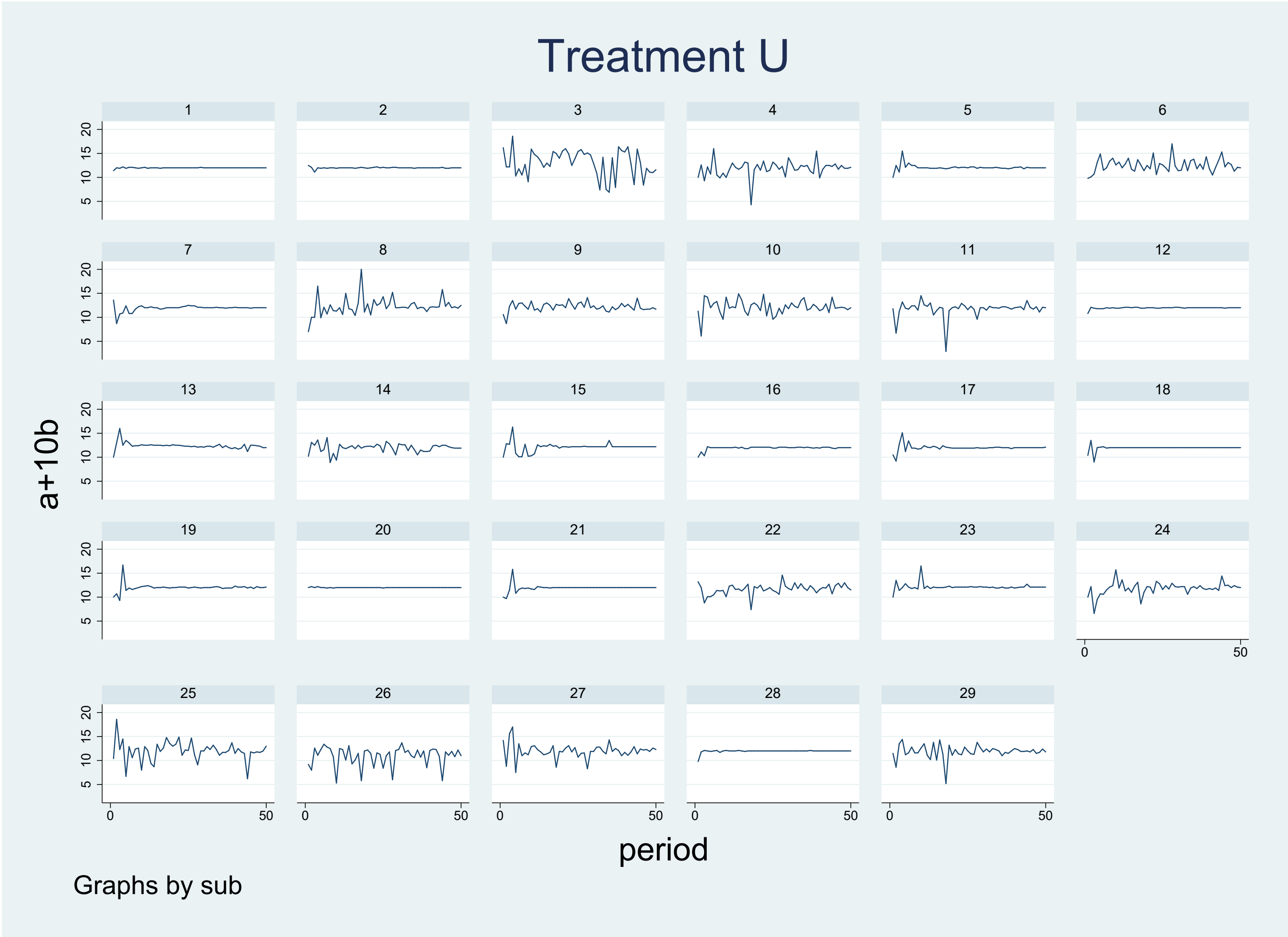

Figure 8. The value of $a + 10b$ for each individual in Treatment U. We report the 95% confidence intervals of $a + 10b$ for each subject in Table C.4. The confidence interval includes 12 for all subjects except Subject 3, 6, 13 and 26 in Treatment U.

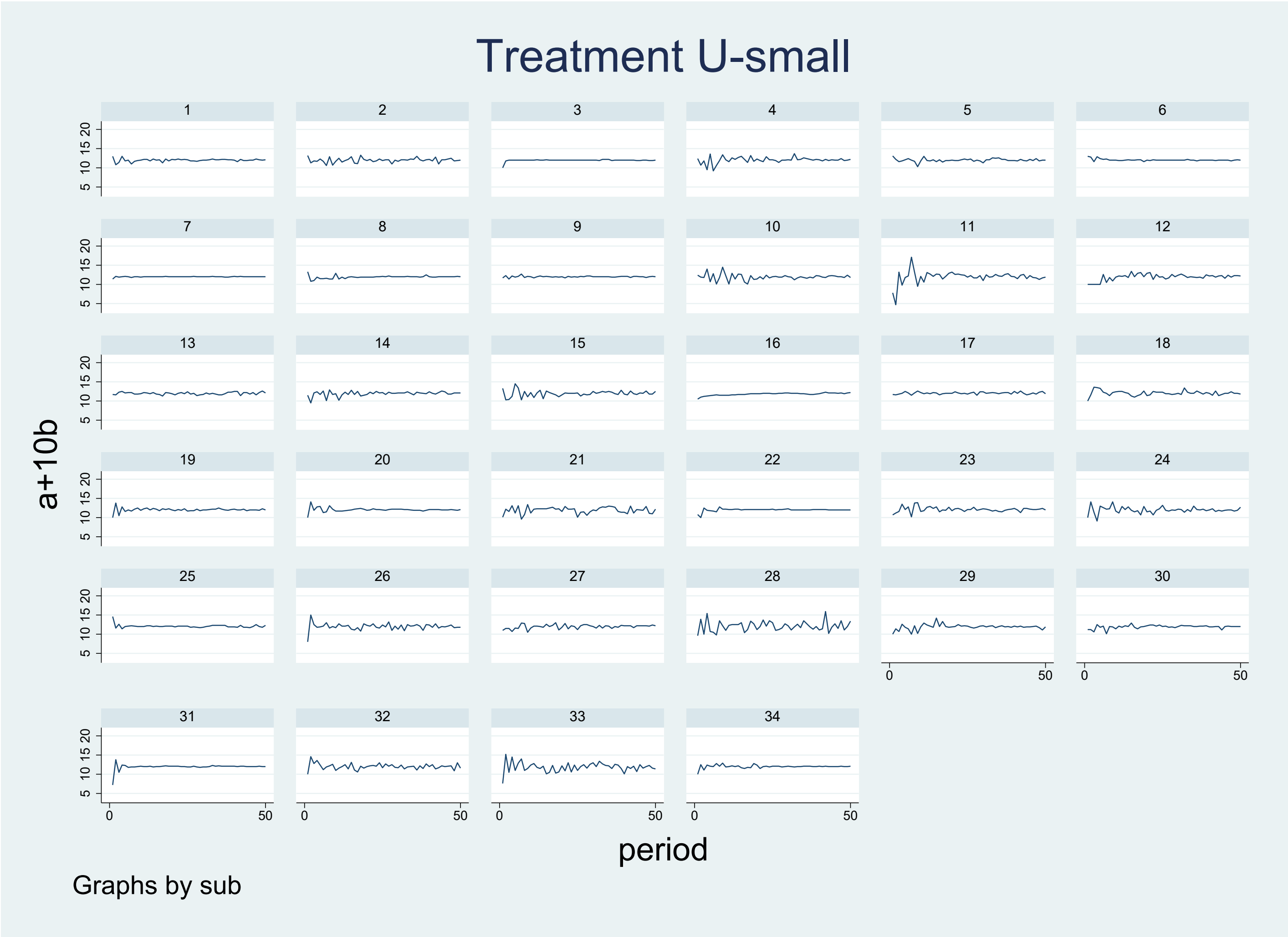

Figure 9. The value of $a + 10b$ for each individual in Treatment U-small. We report the 95% confidence intervals of $a + 10b$ for each subject in Table C.4.

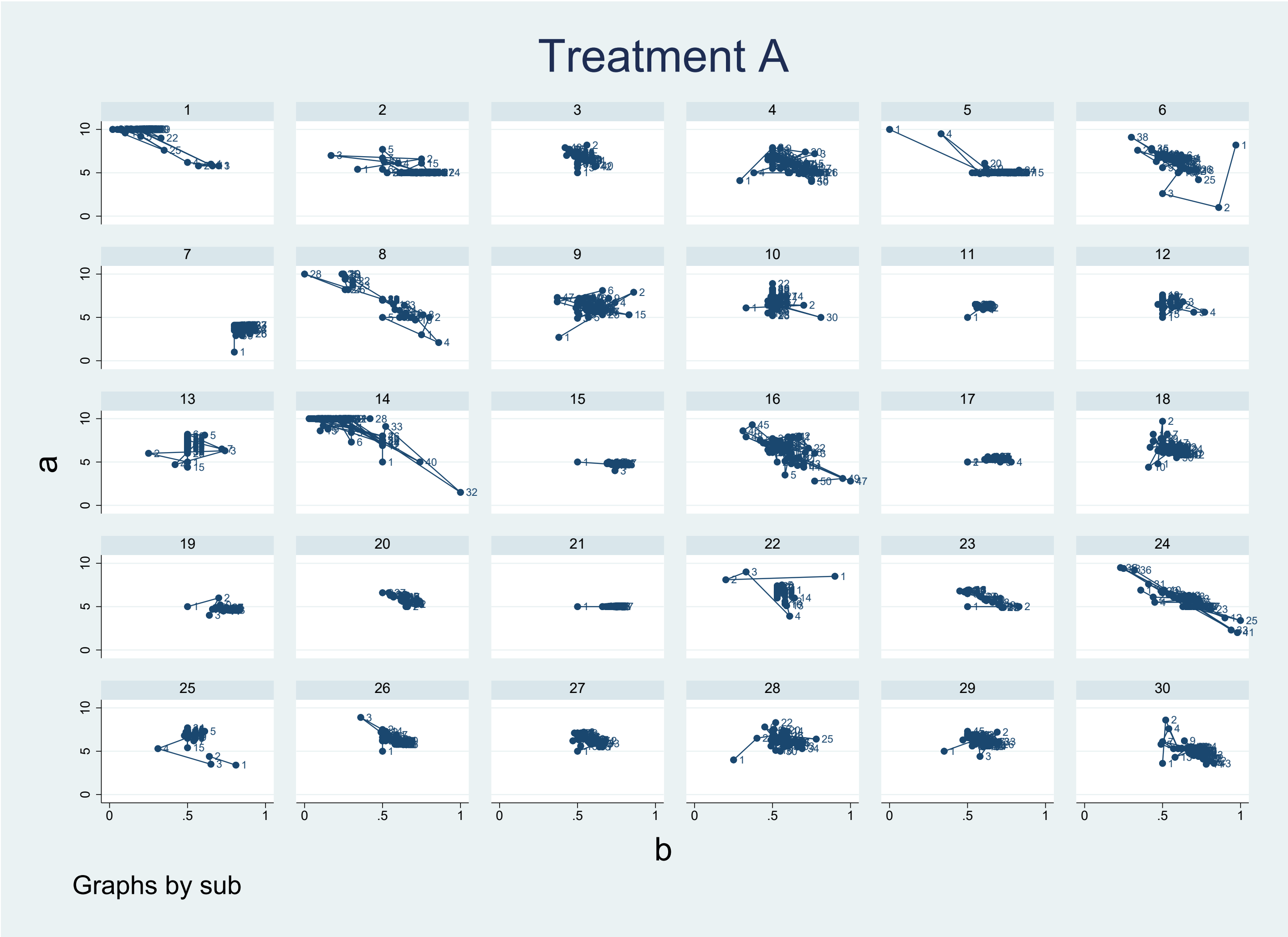

Figure 10. A scatterplot of a and b for each individual subject in Treatment A.

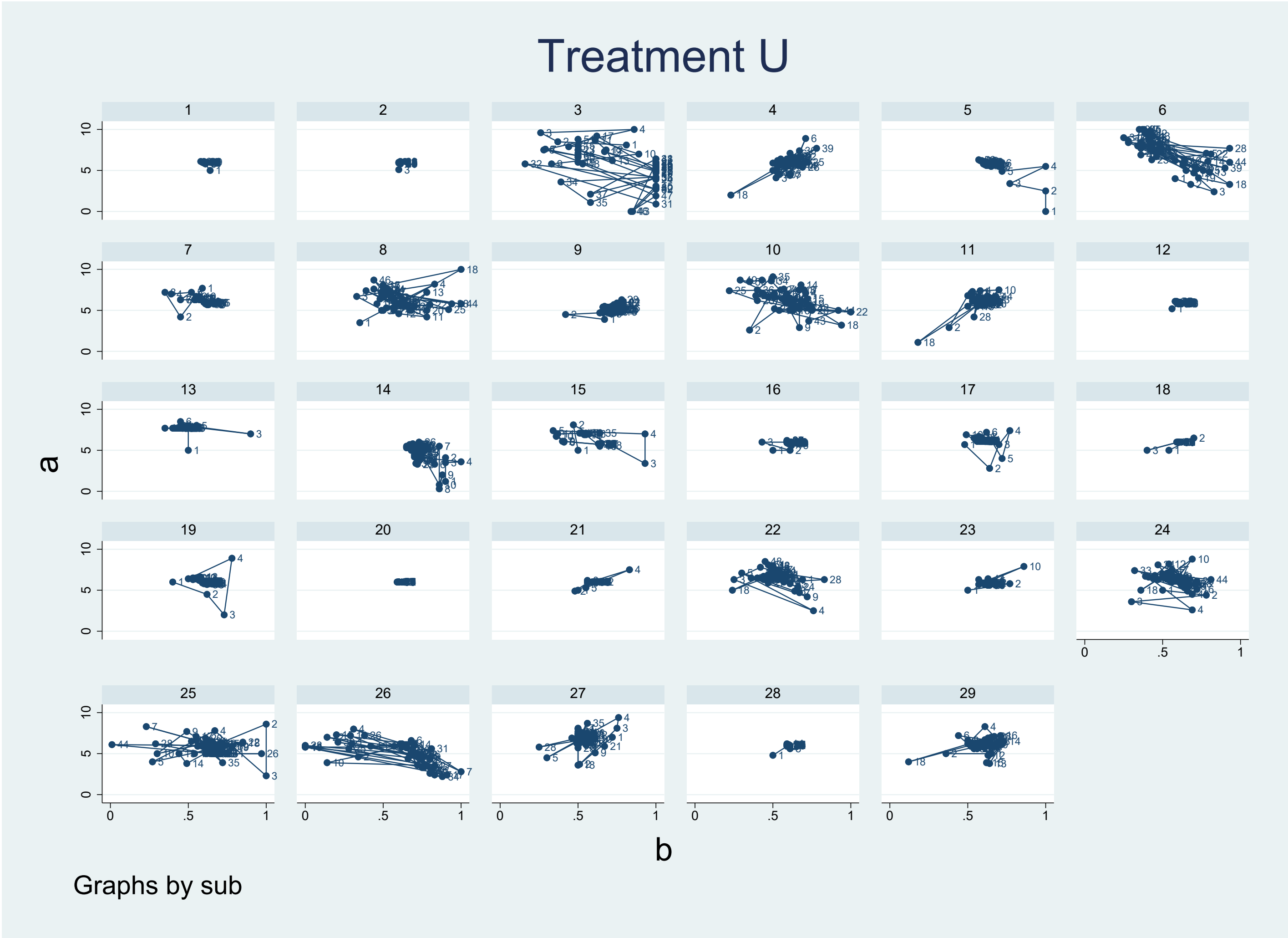

Figure 11. A scatterplot of a and b for each individual subject in Treatment U.

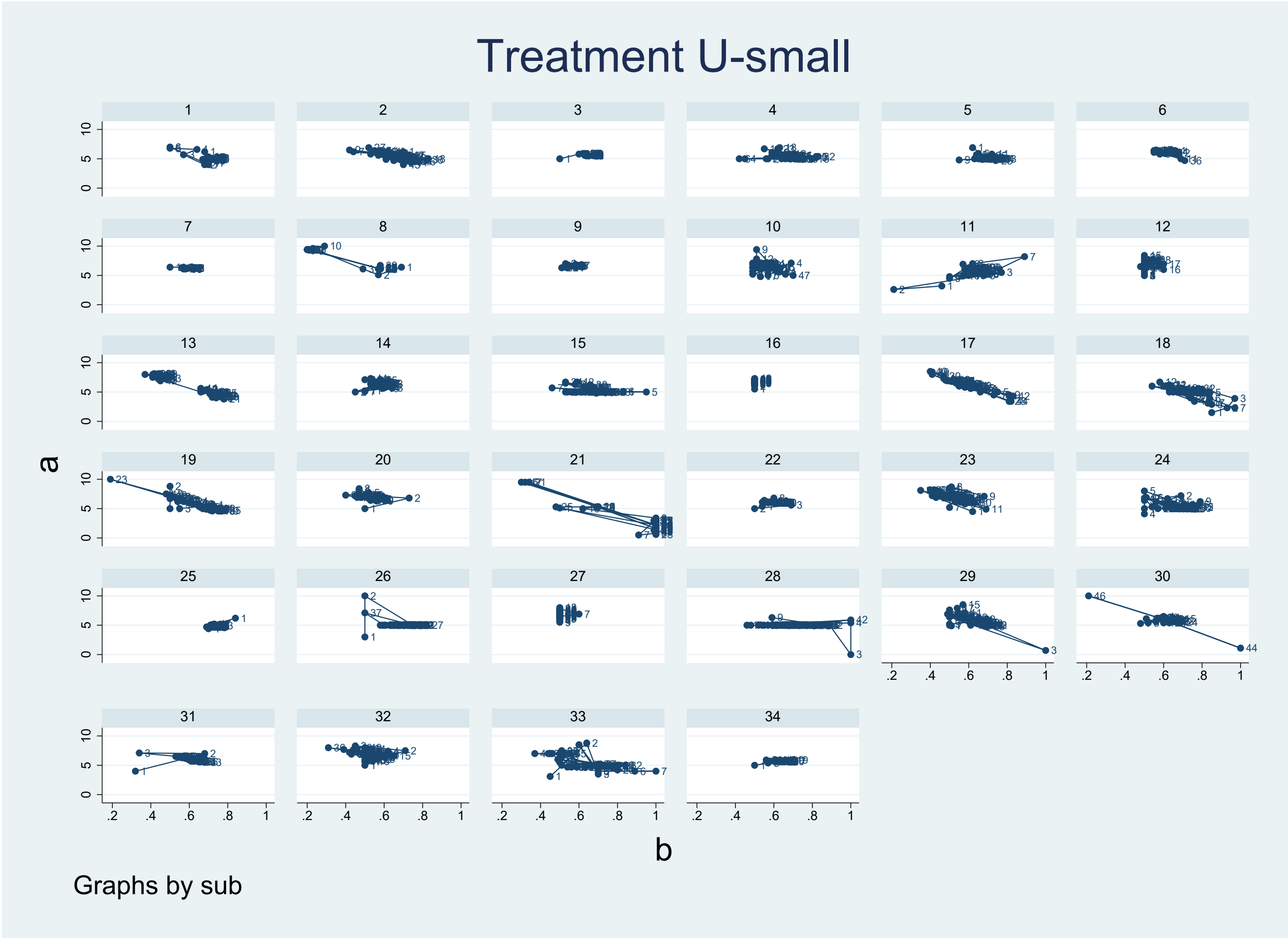

Figure 12. A scatterplot of a and b for each individual subject in Treatment U-small.

3.7 Comparison of Model Fits to the Experimental Data

Table 6 summarizes the average mean squared errors across the different learning models relative to the experimental data.[20] MSE_a refers to the mean squared error between the experimental data and the model for parameter a, while MSE_b refers to the mean squared error between the experimental data and the model for parameter b and finally MSE_p refers to the mean squared error between the data and the model price forecast p. Note that the satisficing (SF) model assumes that agents choose a combination of a, b that satisfies $a + 10b = 12$. Thus it is not possible to calculate MSE_a, MSE_b for those two models, and the MSE_p of the SF model will be the same as the RE model.

Table 6. The mean squared error for different learning models in terms of fitting experimental data. MSE_a is the squared error between the data and model prediction for parameter a, MSE_b is the mean squared error between the data and the model for parameter b and MSE_p is the mean squared error between the data and the model for the price forecast, p. The models we consider include RE (rational expectations), RLS (recursive least squares learning), CGL (constant gain learning), LMS (least mean squares learning) and SF (a satisficing rule).

	Treatment A			Treatment U			Treatment U-small		
	MSE_a	MSE_b	MSE_p	MSE_a	MSE_b	MSE_p	MSE_a	MSE_b	MSE_p
RE	2.069	0.023	0.969	1.254	0.016	1.168	1.662	0.019	1.119
RLS	1.837	0.020	1.345	4.536	0.050	0.979	2.671	0.032	1.245
CGL	5.076	0.061	0.989	1.779	0.019	1.280	3.034	0.032	0.518
LMS	1.767	0.210	5.934	1.521	0.169	5.284	1.235	0.135	4.362
SF	NA	NA	0.969	NA	NA	1.168	NA	NA	1.119

In general, there is not a large difference between the average MSEs of the different learning models (particularly least squares and constant gain learning) for all treatments, and the average MSEs for most models are generally greater than that of the rational expectations model where people constantly choose $a = 6, b = 0.6$. A likely explanation for this finding is that most of the learning models have long memory and put heavy weight on past observations. These models therefore predict slower learning speeds than the subjects' actual learning speed in the experiment, and therefore underperform relative to the RE model/satisficing rule.

If we consider a subject to be a user of a model if the MSE_p is sufficiently small, i.e., if $MSE_p < 1.44$ (root MSE less than 1.2, 10% of the REE price prediction), then the numbers of subjects who can be categorized as users of the different can-

[20] Note that different from the RLS, CGL and LMS models, the satisficing rule uses a linear combination of a and b instead of the exact values of them. Therefore, caution needs to be taken in drawing any comparison between the satisficing rule and other model.

didate models is reported in Table 7.

Table 7. The number of subjects who can be categorized as users of different models based on the size of MSE_p. The models we consider include RE (rational expectations), RLS (recursive least square), CGL (constant gain learning), LMS (least mean square learning) and SF (a satisficing rule).

Model	Treatment A	Percentage	Treatment U	Percentage	Treatment U-small	Percentage
RE	30	100.0%	29	100.0%	34	100.0%
RLS	19	63.3%	8	27.6%	25	73.5%
CGL	29	96.7%	23	79.3%	33	97.1%
LMS	5	16.7%	3	10.3%	13	38.2%
SF	30	100.0%	29	100.0%	34	100.0%

Among the learning models, the RE/SF learning model generates the smallest MSE in terms of fitting the price data in Treatment A, the recursive least squares learning model generates the smallest MSE in terms of fitting the price data in Treatment U, and the recursive least squares learning model generates the smallest MSE in terms of fitting the price data in Treatment U-small. Still, as we have seen, the satisficing rule provides the best description of the overall pattern of subjects' prediction behavior in all treatments, and like the RE prediction it has the lowest overall MSE across all treatments as well as the largest number of users according to the criteria of sufficiently small MSE_p.

4 Conclusion

In this paper, we have conducted the first ever structural test of the seminal least squares learning model using a simple Cobweb model economy. The subjects in our experiment submit predictions for two unknown parameters in a linear PLM that nests the REE as a special case. We observe how subjects update these parameters over time. Since the slope coefficient on the expectations term, α, is less than 1, our experimental economy satisfies the E-stability condition, and so learning agents should converge to the REE.

In general, all of our markets converge to a *neighborhood* of the REE, which is supportive of the E-stability prediction. We find that around 97% of the individual predictions satisfy a *weak convergence* criterion. On average, the predictions by subjects in *Treatment U* converge faster than the predictions of subjects in *Treatment A* and *Treatment U-small*. Treatment U-small, which keeps the uniform nature of the i.i.d. random draws for the weather term but lowers the variance, delivers findings that are closer to treatment A than to treatment U, which suggests that the size of the variance rather than the persistence of the exogenous variable is the more important factor in explaining departures from learning the REE.

Our results suggest that the least squares learning model yields correct predictions at the aggregate level in terms of convergence or near convergence to the REE. However, at the individual level, it does not seem to be a good descriptor of how individual agents update their expectations over time. Least squares learners are found only in treatment U where they comprise 41.4% of the subjects in that treatment; we find no subject employing least squares learning in Treatment A or U-small, and only one such subject in our treatment U-small.

For those who deviate from least squares learning, many of them seem to adopt some kind of dimension-reducing strategy focusing on price point prediction accuracy alone. This behavior is consistent with the "satisficing" approach of Simon (Simon, 1955,9), and a simple satisficing heuristic appears to explain our experimental data better than does least squares learning.

The environment we have studied is a very simple individual-decision making environment. Our lack of evidence in support of least squares updating of parameters of the PLM doesn't necessarily constitute a robust case against the use of recursive least squares updating to model learning behavior. Our findings may simply reflect current limitations in our ability to gather data that would validate the use of the least squares learning approach. We hope that others can build upon our design in ways that make the case for or against least squares learning more compelling.

Indeed, in future research, it would be of interest to study how agents update the parameters of their PLM in settings where there is a group of n subjects whose forecasts matter for realizations of the variables being forecasted as in an n-player learning–to–forecast experiments. It would further be of interest to elicit subjects' PLMs, rather than impose a PLM on them that explicitly nests the REE solution as a special case and asking how they would parameterize that particular PLM. In cases where subjects did not use a PLM that nested the REE, it might be the case that they converge to some kind of self-confirming equilibrium (Cho and Sargent (2008)). Finally it would also be of interest to give subjects access to statistical software or to provide them with a choice of forecasting models to form forecasts of future prices on their behalf that included least squares learning as one possibility among others. We view the present study as a first, *small step* in the direction of developing a more structural approach to understanding the manner in which agents form expectations and so we leave the study of these more complex environments to future research.

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Appendices

Appendix A: Supplementary Information

Appendix A.1: Session Information

Table A.1. Characteristics of Experimental Design.

Session Number	Treatment Type	Number of participants *(N)*
1	*A*	13
2	*A*	17
3	*U*	14
4	*U*	10
5	*U*	5
6	*U-Small*	17
7	*U-Small*	17

Appendix A.2: t-test on the Price Level

We perform a t-test on the equality between the realized market price and the REE price level and report the results in Table A.2.

Table A.2. t-test on the price level. The null hypothesis is that the average price is equal to the REE, i.e. $p_t^* = 6 + 0.6w_t$.

	Treatment A		***Treatment U***		***Treatment U-small***	
Sub	t-statistic	p-value	t-statistic	p-value	t-statistic	p-value
1	0.050	0.971	1.150	0.256	0.35	0.744
2	1.000	0.316	1.100	0.280	0.45	0.641
3	0.650	0.521	-1.750	0.086	1.3	0.202
4	0.150	0.873	1.150	0.247	0.1	0.91
5	1.350	0.189	-0.200	0.840	0.6	0.555
6	0.150	0.886	1.800	0.078	-0.15	0.891
7	0.950	0.353	0.850	0.411	1.3	0.194
8	1.500	0.135	-0.050	0.951	1.55	0.125
9	0.250	0.823	0.300	0.782	0.7	0.502
10	0.300	0.769	0.000	0.985	0.95	0.337
11	1.200	0.236	1.600	0.116	0.25	0.812
12	0.900	0.368	1.750	0.084	1	0.316
13	0.400	0.690	-2.800	0.007	0.4	0.675
14	0.250	0.790	0.400	0.702	1.05	0.289
15	1.250	0.213	-0.600	0.550	0.55	0.572
16	-0.900	0.385	1.350	0.185	4.65	0
17	2.250	0.030	0.300	0.752	0.35	0.718
18	0.150	0.881	1.300	0.198	-0.35	0.711
19	1.650	0.105	0.100	0.911	0.4	0.676
20	1.200	0.238	0.950	0.358	-0.3	0.76
21	1.750	0.083	0.250	0.803	1.3	0.199
22	-0.200	0.840	0.350	0.729	0.35	0.742
23	1.500	0.138	-0.450	0.656	-0.85	0.391
24	0.800	0.416	0.850	0.400	0.45	0.638
25	1.200	0.235	0.450	0.666	-0.75	0.456
26	1.350	0.185	1.250	0.212	0.35	0.718
27	1.600	0.114	-0.600	0.556	0.85	0.392
28	1.200	0.237	1.400	0.171	0.1	0.937
29	1.100	0.276	0.650	0.517	1.05	0.31
30	1.300	0.205			1.25	0.223
31					0.85	0.392
32					-0.25	0.813
33					1.1	0.271
34					0.45	0.667

Appendix A.3: Comparison of the Demographic Characteristics of Participants in the Two Treatments

As a balance check to rule out the possibility of selection bias, we conduct a regression analysis of differences in demographic characteristics between the group of subjects assigned to Treatment A and the group assigned to Treatment U. Two sample t-tests are used to compare the demographic characteristics and participation experience between the two groups. The results indicate that there is no statistically significant difference at the 5% significance level between the groups on the recorded factors. It confirms that our randomization was successful and gives us more freedom to conclude that the observed differences with predictions are brought about by differences in the treatment conditions alone.

Appendix B: Testing Convergence Using Linear Estimation

Table B.1. Mean, standard error and 95% confidence interval (CI) of a in Treatment A, U and U-Small. The theoretical values are $a^* = 6, b^* = 0.6$.

	Treatment A				***Treatment U***				***Treatment U-Small***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI	
1	9.333	0.242	8.847	9.819	5.976	0.020	5.936	6.017	5.104	0.077	4.949	5.259
2	5.339	0.126	5.087	5.592	5.894	0.018	5.857	5.931	5.456	0.086	5.284	5.628
3	6.849	0.086	6.676	7.022	5.557	0.340	4.874	6.240	5.616	0.019	5.578	5.654
4	5.798	0.144	5.509	6.087	5.902	0.141	5.618	6.186	5.440	0.062	5.315	5.565
5	5.320	0.162	4.995	5.644	5.573	0.140	5.290	5.855	5.110	0.048	5.013	5.207
6	6.269	0.182	5.902	6.635	7.133	0.269	6.594	7.673	6.096	0.044	6.008	6.184
7	3.700	0.068	3.563	3.837	6.124	0.070	5.984	6.263	6.206	0.007	6.191	6.221
8	6.804	0.236	6.329	7.278	6.335	0.171	5.991	6.679	6.762	0.183	6.395	7.129
9	6.194	0.121	5.950	6.438	5.124	0.061	5.001	5.246	6.588	0.018	6.551	6.625
10	6.892	0.125	6.640	7.144	6.204	0.203	5.797	6.611	6.498	0.112	6.272	6.724
11	6.304	0.032	6.240	6.368	6.041	0.142	5.757	6.326	5.690	0.119	5.450	5.930
12	6.827	0.097	6.632	7.023	5.939	0.018	5.903	5.976	6.844	0.113	6.617	7.071
13	6.875	0.107	6.659	7.090	7.692	0.059	7.574	7.811	5.940	0.222	5.495	6.385
14	8.920	0.278	8.361	9.478	4.598	0.179	4.238	4.958	6.314	0.067	6.178	6.450
15	4.800	0.045	4.710	4.890	6.075	0.103	5.869	6.280	5.288	0.070	5.148	5.428
16	6.412	0.211	5.987	6.837	5.931	0.029	5.873	5.990	6.798	0.047	6.704	6.892
17	5.322	0.019	5.284	5.359	6.090	0.089	5.912	6.269	6.194	0.148	5.896	6.492
18	6.563	0.119	6.324	6.801	5.971	0.029	5.912	6.030	4.880	0.146	4.588	5.172
19	4.729	0.034	4.662	4.797	6.000	0.107	5.785	6.215	5.866	0.157	5.550	6.182
20	5.835	0.061	5.712	5.958	6.000	0.000	6.000	6.000	6.904	0.069	6.765	7.043
21	5.000	0.000	5.000	5.000	5.976	0.044	5.888	6.065	3.074	0.312	2.446	3.702
22	6.645	0.105	6.434	6.856	6.578	0.143	6.291	6.865	6.034	0.033	5.967	6.101
23	6.147	0.094	5.959	6.335	5.835	0.049	5.737	5.933	6.910	0.118	6.672	7.148
24	5.592	0.195	5.201	5.983	6.159	0.148	5.862	6.456	5.406	0.101	5.204	5.608
25	6.725	0.121	6.483	6.968	5.735	0.155	5.424	6.046	4.908	0.035	4.838	4.978
26	6.247	0.081	6.084	6.410	5.002	0.201	4.597	5.407	5.102	0.116	4.869	5.335
27	6.418	0.071	6.275	6.560	6.757	0.146	6.463	7.051	6.952	0.072	6.808	7.096
28	6.235	0.104	6.027	6.444	5.976	0.026	5.925	6.028	4.950	0.106	4.737	5.163
29	6.131	0.130	5.870	6.393	6.016	0.110	5.796	6.236	5.786	0.160	5.465	6.107
30	5.022	0.125	4.771	5.272					5.890	0.133	5.623	6.157
31									6.058	0.062	5.933	6.183
32									6.890	0.098	6.694	7.086
33									5.466	0.183	5.097	5.835
34									5.698	0.019	5.660	5.736

Table B.2. Mean, standard error and 95% confidence interval (CI) of b in Treatment A, U and U-Small. The theoretical values are $a^* = 6, b^* = 0.6$.

	Treatment A				***Treatment U***				***Treatment U-Small***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI	
1	0.227	0.021	0.185	0.269	0.601	0.001	0.599	0.604	0.690	0.006	0.677	0.703
2	0.679	0.019	0.641	0.717	0.610	0.001	0.609	0.611	0.653	0.012	0.630	0.677
3	0.506	0.005	0.495	0.516	0.750	0.038	0.674	0.826	0.634	0.003	0.628	0.640
4	0.622	0.017	0.589	0.656	0.593	0.011	0.570	0.615	0.660	0.011	0.638	0.682
5	0.683	0.020	0.643	0.722	0.649	0.013	0.622	0.676	0.689	0.005	0.679	0.699
6	0.564	0.017	0.530	0.598	0.537	0.027	0.483	0.592	0.596	0.005	0.586	0.606
7	0.841	0.005	0.831	0.851	0.580	0.009	0.562	0.597	0.578	0.002	0.575	0.582
8	0.504	0.023	0.459	0.550	0.595	0.021	0.552	0.638	0.516	0.020	0.476	0.556
9	0.573	0.014	0.545	0.600	0.705	0.009	0.688	0.722	0.541	0.002	0.538	0.545
10	0.519	0.011	0.497	0.540	0.593	0.022	0.550	0.636	0.542	0.007	0.528	0.557
11	0.565	0.002	0.561	0.570	0.569	0.010	0.549	0.590	0.626	0.013	0.601	0.651
12	0.527	0.012	0.503	0.551	0.602	0.001	0.599	0.605	0.507	0.003	0.501	0.513
13	0.495	0.013	0.469	0.521	0.469	0.010	0.450	0.488	0.604	0.021	0.561	0.646
14	0.264	0.027	0.209	0.319	0.735	0.011	0.713	0.756	0.562	0.005	0.552	0.571
15	0.714	0.015	0.684	0.745	0.602	0.016	0.571	0.634	0.668	0.012	0.644	0.692
16	0.571	0.021	0.528	0.613	0.599	0.004	0.591	0.608	0.500	0.000	.	.
17	0.659	0.006	0.647	0.671	0.590	0.006	0.578	0.603	0.585	0.014	0.557	0.614
18	0.542	0.008	0.525	0.559	0.597	0.005	0.588	0.606	0.718	0.013	0.691	0.744
19	0.721	0.007	0.707	0.736	0.599	0.007	0.585	0.613	0.617	0.015	0.586	0.648
20	0.611	0.007	0.597	0.624	0.600	0.001	0.599	0.602	0.515	0.006	0.503	0.528
21	0.700	0.005	0.691	0.709	0.595	0.006	0.584	0.607	0.886	0.029	0.828	0.945
22	0.537	0.011	0.515	0.559	0.512	0.015	0.481	0.543	0.597	0.004	0.589	0.605
23	0.578	0.012	0.555	0.601	0.630	0.006	0.617	0.643	0.521	0.010	0.502	0.540
24	0.642	0.022	0.597	0.687	0.568	0.015	0.537	0.598	0.658	0.011	0.636	0.680
25	0.514	0.009	0.497	0.531	0.626	0.025	0.575	0.677	0.717	0.003	0.711	0.723
26	0.567	0.007	0.553	0.581	0.592	0.036	0.520	0.664	0.688	0.010	0.668	0.709
27	0.538	0.013	0.512	0.564	0.525	0.012	0.502	0.549	0.502	0.002	0.498	0.506
28	0.561	0.011	0.539	0.582	0.598	0.002	0.594	0.602	0.714	0.018	0.678	0.751
29	0.564	0.009	0.546	0.581	0.585	0.013	0.560	0.611	0.612	0.013	0.586	0.638
30	0.694	0.012	0.669	0.719					0.603	0.012	0.578	0.628
31									0.589	0.009	0.571	0.607
32									0.514	0.009	0.495	0.532
33									0.639	0.020	0.600	0.678
34									0.631	0.005	0.622	0.640

Table B.3. Linear Estimation on parameter a in *Treatment A*.

Number	$\hat{\rho}_a$	p-value ($\lvert\hat{\rho}_a\rvert \geq 1$)	μ_a	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	0.5466	0.0128	4.3842	0.3543	0.9517	2.3364	9.6777	0.0000
2	0.4304	0.0039	2.9828	0.1849	0.5606	2.3223	5.2333	0.0000
3	-0.1747	0.0000	8.1052	0.0366	0.5417	1.5705	6.8979	0.0000
4	-0.3225	0.0000	7.7630	0.1032	0.9402	1.8359	5.8677	0.2001
5	-0.0306	0.0000	5.2887	0.0020	0.6678	2.0131	5.1312	0.0000
6	0.1060	0.0005	5.5985	0.0117	1.2809	1.3254	6.2617	0.2007
7	0.0808	0.0000	3.4494	0.0173	0.3027	1.3677	3.7522	0.0000
8	0.7153	0.0017	2.0166	0.5706	1.0794	2.4043	7.0772	0.0488
9	-0.4139	0.0000	8.8678	0.2615	0.6072	1.7516	6.2716	0.0000
10	-0.2500	0.0000	8.5551	0.0630	0.7695	1.8870	6.8441	0.0000
11	0.2287	0.0000	4.8879	0.1569	0.1242	1.3604	6.3398	0.0000
12	0.3446	0.0000	4.4695	0.1542	0.4376	1.9248	6.8244	0.0000
13	-0.1271	0.0000	7.7926	0.0179	0.7325	2.0651	6.9148	0.0000
14	0.0083	0.0000	9.0764	0.0001	1.6261	2.0598	9.1515	0.0000
15	0.2406	0.0000	3.6101	0.0614	0.1371	1.6955	4.7563	0.0000
16	-0.0897	0.0000	6.8856	0.0072	1.4441	1.8947	6.3192	0.0947
17	0.435	0.0053	3.0136	0.2129	0.1158	2.2905	5.3345	0.0000
18	-0.0959	0.0000	7.2044	0.0098	0.8156	1.3068	6.5736	0.0000
19	-0.1225	0.0002	5.2974	0.0155	0.2384	1.6636	4.7168	0.0000
20	0.4822	0.0000	3.0389	0.2509	0.3745	2.0254	5.8668	0.2072
21				Omitted because of collinearity				
22	-0.0630	0.0004	7.0351	0.0045	0.7205	2.1160	6.6181	0.0000
23	0.5436	0.0002	2.8251	0.3116	0.5551	2.0980	6.1952	0.2744
24	0.1987	0.0000	4.4617	0.0400	1.3907	1.9089	5.5705	0.0818
25	0.5415	0.0028	3.1490	0.4200	0.5665	2.4158	6.8756	0.0000
26	0.4306	0.0081	3.5829	0.2045	0.5081	1.2646	6.2970	0.0263
27	0.1050	0.0000	5.7847	0.0132	0.4664	2.0765	6.4637	0.0000
28	0.0094	0.0000	6.1970	0.0001	0.6662	1.9393	6.2556	0.0087
29	-0.1564	0.0000	7.2344	0.0264	0.5354	1.7212	6.2578	0.0001
30	0.0495	0.0000	4.8307	0.0025	0.8623	1.3318	5.0826	0.0000

Table B.4. Linear estimation on parameter b in *Treatment A*. μ is the intercept of the regression while ρ is the slope.

Number	$\hat{\rho}_b$	p-value ($\lvert\hat{\rho}_b\rvert \geq 1$)	μ_b	R^2	Root MSE	Durbin Watson	Equilibrium	Wald test p-value
1	0.4512	0.0015	0.1188	0.2443	0.1194	2.2080	0.2168	0.0000
2	0.0503	0.0000	0.6452	0.0029	0.1204	1.9117	0.6792	0.0000
3	0.1641	0.0000	0.4227	0.0253	0.0384	1.9796	0.5060	0.0000
4	-0.2733	0.0000	0.7973	0.0869	0.1064	2.1886	0.6261	0.0267
5	0.1110	0.0000	0.6146	0.0260	0.0918	2.0267	0.6918	0.0000
6	0.2191	0.0000	0.4358	0.0629	0.0996	2.2610	0.5583	0.0156
7	0.3970	0.0004	0.5060	0.1576	0.0256	2.2699	0.8391	0.0000
8	0.7118	0.0560	0.1403	0.5312	0.1123	2.5591	0.4861	0.0572
9	-0.3416	0.0000	0.7795	0.1291	0.0835	1.4754	0.5805	0.0321
10	-0.1863	0.0000	0.6114	0.0428	0.0545	1.4801	0.5152	0.0000
11	0.2933	0.0116	0.4007	0.1307	0.0124	0.7740	0.5672	0.0000
12	0.6334	0.0949	0.1897	0.4013	0.0419	1.7072	0.5177	0.0000
13	-0.4799	0.0000	0.7472	0.2303	0.0552	1.3843	0.5047	0.0000
14	0.1943	0.0000	0.2117	0.0384	0.1909	2.0552	0.2630	0.0000
15	0.0551	0.0000	0.693	0.0133	0.0179	0.9173	0.7334	0.0000
16	-0.1969	0.0000	0.6941	0.0374	0.1296	1.9066	0.5798	0.1911
17	0.3991	0.0247	0.3992	0.2293	0.0309	2.0119	0.6639	0.0000
18	0.2261	0.0000	0.4220	0.0521	0.0586	2.1020	0.5452	0.0000
19	0.2941	0.0000	0.5170	0.2679	0.0197	1.7159	0.7323	0.0000
20	0.5458	0.0003	0.2777	0.3017	0.0395	2.0957	0.6123	0.3659
21	0.2621	0.0000	0.5207	0.3197	0.0127	1.4197	0.7060	0.0000
22	-0.1632	0.0042	0.6175	0.0462	0.0609	0.6189	0.5305	0.0000
23	0.3711	0.0004	0.3658	0.1396	0.0780	1.7425	0.5819	0.3005
24	0.1422	0.0000	0.5569	0.0216	0.1568	2.0167	0.6492	0.0598
25	0.1140	0.0014	0.4493	0.0250	0.0449	2.5970	0.5068	0.0000
26	0.6788	0.0376	0.1838	0.4738	0.0367	2.6100	0.5732	0.0513
27	-0.0552	0.0000	0.5799	0.0031	0.0532	1.7854	0.5497	0.0000
28	0.3800	0.0000	0.3551	0.2181	0.0565	2.3680	0.5726	0.0330
29	-0.2709	0.0000	0.7230	0.0962	0.0522	1.7406	0.5688	0.0000
30	0.6067	0.0022	0.2753	0.4106	0.0578	2.4363	0.6997	0.0000

Appendix C: Modelling the Forecasting Strategy at the Individual Level

Table C.1. The mean squared error of the recursive least squares learning learning model for each subject in Treatment A, U and U-Small.

Treatment A	MSE_a	MSE_b	Treatment U	MSE_a	MSE_b	Treatment U-Small	MSE_a	MSE_b
1	20.2748	0.2272	1	0.0038	0.0001	1	1.670	0.018
2	1.4377	0.0234	2	0.0293	0.0003	2	0.701	0.010
3	1.4622	0.0138	3	5.5731	0.0993	3	2.033	0.023
4	2.6421	0.0361	4	0.9670	0.0074	4	1.241	0.015
5	1.6777	0.0277	5	1.1212	0.0211	5	1.336	0.014
6	3.5342	0.0475	6	4.1360	0.0437	6	0.733	0.005
7	10.8363	0.1156	7	0.2593	0.0063	7	0.123	0.001
8	4.7650	0.0498	8	1.8924	0.0238	8	7.812	0.094
9	2.0785	0.0288	9	1.8127	0.0272	9	0.541	0.005
10	4.1401	0.0440	10	1.7940	0.0226	10	5.012	0.053
11	1.0212	0.0129	11	0.9237	0.0054	11	9.938	0.160
12	3.6883	0.0383	12	0.0100	0.0003	12	0.832	0.007
13	3.9192	0.0437	13	7.7283	0.0537	13	3.734	0.037
14	15.2220	0.1851	14	6.8075	0.0571	14	0.613	0.006
15	1.2980	0.0138	15	0.2685	0.0064	15	2.162	0.025
16	4.0759	0.0513	16	0.0255	0.0003	16	0.939	0.013
17	1.5917	0.0229	17	0.3302	0.0029	17	1.208	0.011
18	1.7316	0.0173	18	0.0026	0.0001	18	4.629	0.080
19	5.7752	0.0532	19	0.4103	0.0050	19	1.116	0.013
20	0.6736	0.0084	20	0.0034	0.0000	20	1.796	0.016
21	1.0703	0.0121	21	0.3498	0.0028	21	16.529	0.153
22	2.9990	0.0392	22	1.3446	0.0183	22	1.053	0.012
23	0.4317	0.0061	23	0.1309	0.0022	23	3.574	0.028
24	3.0107	0.0370	24	1.0970	0.0112	24	2.093	0.026
25	1.9935	0.0158	25	1.2195	0.0292	25	2.168	0.025
26	1.5810	0.0198	26	3.8584	0.0668	26	5.431	0.079
27	0.4781	0.0080	27	1.2913	0.0077	27	0.744	0.003
28	5.1372	0.0770	28	0.0145	0.0006	28	0.493	0.023
29	1.3162	0.0164	29	0.5750	0.0073	29	1.293	0.013
30	1.3409	0.0194				30	1.583	0.020
						31	0.660	0.011
						32	2.423	0.031
						33	4.044	0.053
						34	0.559	0.007

Table C.2. The mean squared error of the constant gain learning model for each subject in Treatment A, U and U-SMall.

Treatment A	MSE_a	MSE_b	γ	Treatment U	MSE_a	MSE_b	γ	Treatment U-Small	MSE_a	MSE_b	γ
1	25.1347	0.2460	0.01	1	0.0384	0.0002	0.27	1	1.7787	0.0162	0.38
2	2.4421	0.0369	0.01	2	0.0729	0.0002	0.01	2	2.0724	0.0287	0.02
3	3.1365	0.0340	0.3	3	7.9946	0.0577	0.15	3	1.7150	0.0186	0.3
4	3.5402	0.0502	0.26	4	1.5429	0.0086	0.22	4	0.9662	0.0134	0.05
5	2.5643	0.0398	0.32	5	0.5393	0.0195	0.32	5	3.1067	0.0352	0.05
6	4.1357	0.0445	0.5	6	5.9779	0.0528	0.2	6	1.6670	0.0184	0.03
7	12.1870	0.1652	0.01	7	0.5267	0.0128	0.31	7	0.3516	0.0034	0.31
8	6.4276	0.0793	0.31	8	2.2658	0.0274	0.62	8	3.1137	0.0362	0.3
9	4.1499	0.0576	0.3	9	0.8746	0.0210	0.3	9	0.8678	0.0092	0.19
10	6.0884	0.0674	0.28	10	2.2412	0.0216	0.17	10	2.4762	0.0267	0.11
11	2.1186	0.0318	0.37	11	1.3765	0.0047	0.08	11	2.3548	0.0406	0.18
12	2.9955	0.0459	0.33	12	0.0352	0.0006	0.31	12	3.2162	0.0305	0.25
13	3.3969	0.0427	0.37	13	6.8487	0.0457	0.41	13	2.7470	0.0277	0.01
14	13.7778	0.1480	0.22	14	5.9884	0.0551	0.29	14	0.9377	0.0096	0.17
15	4.9380	0.0680	0.39	15	1.0784	0.0267	0.37	15	2.4906	0.0238	0.26
16	6.3307	0.0558	0.21	16	0.0579	0.0005	0.32	16	3.2395	0.0349	0.27
17	3.7499	0.0543	0.39	17	0.5174	0.0072	0.44	17	1.3869	0.0124	0.19
18	4.5664	0.0514	0.3	18	0.1464	0.0007	0.25	18	2.1843	0.0304	0.36
19	3.5210	0.0524	0.01	19	1.0154	0.0130	0.34	19	2.4193	0.0260	0.31
20	1.3215	0.0161	0.32	20	0.0014	0.0000	0.02	20	4.1917	0.0315	0.3
21	4.1122	0.0571	0.37	21	0.1139	0.0050	0.49	21	18.9455	0.2038	0.79
22	3.9143	0.0472	0.38	22	1.4359	0.0226	0.36	22	0.6075	0.0040	0.31
23	2.4223	0.0310	0.01	23	0.4680	0.0046	0.28	23	4.9157	0.0367	0.38
24	6.1098	0.0748	0.5	24	1.6112	0.0116	0.19	24	2.4792	0.0220	0.24
25	3.5249	0.0370	0.45	25	2.1439	0.0415	0.26	25	5.6081	0.0666	0.92
26	2.7143	0.0301	0.32	26	3.9461	0.0669	0.3	26	3.7881	0.0338	0.29
27	2.2466	0.0316	0.37	27	1.9689	0.0141	0.31	27	2.0800	0.0142	0.21
28	5.3577	0.0572	0.34	28	0.0965	0.0017	0.34	28	2.5506	0.0339	0.3
29	2.4703	0.0360	0.29	29	0.6744	0.0090	0.01	29	2.5459	0.0226	0.3
30	2.8794	0.0327	0.01					30	3.2130	0.0332	0.31
								31	2.1399	0.0236	0.31
								32	4.2473	0.0325	0.3
								33	5.3634	0.0632	0.35
								34	1.3906	0.0182	0.31

Table C.3. The mean squared error of the least mean square learning model for each subject in Treatment A and U.

Treatment A	MSE_a	MSE_b	λ	Treatment U	MSE_a	MSE_b	λ	Treatment U-Small	MSE_a	MSE_b	λ
1	8.2152	0.3560	0.21	1	0.2004	0.1436	0.03	1	1.4709	0.0132	0.01
2	0.3995	0.0234	0.01	2	0.0520	0.0137	0.01	2	0.8986	0.0155	0.01
3	0.7293	0.2409	0.14	3	7.4191	0.3794	0.18	3	0.3666	0.0041	0.01
4	2.0970	0.2693	0.11	4	1.0352	0.1852	0.06	4	0.4756	0.0127	0.01
5	5.4008	0.2731	0.25	5	3.2267	0.2507	0.15	5	3.3066	0.2499	0.03
6	2.7783	0.2374	0.17	6	4.8819	0.2525	0.13	6	0.1793	0.0023	0.01
7	2.0770	0.1385	0.06	7	2.5854	0.2315	0.14	7	0.0495	0.0245	0.02
8	2.1135	0.2537	0.17	8	2.6754	0.3079	0.16	8	1.7753	0.0234	0.01
9	1.8980	0.2597	0.15	9	0.4979	0.1606	0.03	9	0.0520	0.0052	0.02
10	0.6168	0.2511	0.08	10	2.1554	0.1417	0.02	10	0.7797	0.0387	0.02
11	0.2400	0.2457	0.06	11	1.4309	0.1534	0.04	11	1.5957	0.2483	0.11
12	0.4615	0.2401	0.12	12	0.1391	0.1718	0.04	12	0.4420	0.2419	0.11
13	0.9391	0.2307	0.13	13	2.4645	0.1985	0.1	13	1.2950	0.2532	0.05
14	8.3568	0.3701	0.27	14	3.1031	0.2396	0.08	14	0.2666	0.0040	0.01
15	0.0898	0.0023	0.01	15	0.9297	0.1530	0.03	15	0.2372	0.0726	0.02
16	2.6797	0.2681	0.11	16	0.2568	0.1596	0.04	16	0.1110	0.2400	0.08
17	0.1063	0.0032	0.01	17	0.3953	0.1181	0.03	17	1.1030	0.2126	0.03
18	1.4588	0.2394	0.13	18	0.2022	0.1634	0.04	18	2.9536	0.2529	0.09
19	0.1457	0.0128	0.02	19	0.5795	0.0218	0.01	19	1.2072	0.2221	0.03
20	0.5006	0.2298	0.04	20	0.0003	0.0030	0.01	20	1.0021	0.2402	0.14
21	0.0006	0.0008	0.01	21	0.2596	0.1495	0.03	21	8.9788	0.0835	0.01
22	1.0577	0.2457	0.13	22	0.8757	0.2029	0.05	22	0.1521	0.1976	0.03
23	0.8560	0.2364	0.04	23	0.3875	0.1692	0.03	23	1.3497	0.2362	0.13
24	3.6128	0.2627	0.03	24	1.0726	0.2220	0.05	24	0.6300	0.0656	0.02
25	0.9682	0.2456	0.2	25	1.3857	0.1760	0.02	25	1.6762	0.0186	0.01
26	1.0451	0.2336	0.04	26	3.4554	0.1029	0.01	26	2.1347	0.1931	0.04
27	0.3987	0.2337	0.07	27	1.1324	0.0595	0.01	27	0.2979	0.2382	0.08
28	1.4380	0.2433	0.11	28	0.3684	0.2021	0.07	28	0.5588	0.0300	0.01
29	0.4659	0.2496	0.05	29	0.9378	0.1781	0.03	29	1.6615	0.2044	0.03
30	1.9773	0.1055	0.03					30	0.9091	0.0104	0.01
								31	1.1888	0.2564	0.06
								32	1.0631	0.2397	0.11
								33	1.6643	0.2407	0.05
								34	0.1680	0.1951	0.03

Table C.4. Mean, standard error and 95% confidence interval (CI) of $a + 10b$ in Treatments A, U and U-small. The theoretical values are $a^* = 6, b^* = 0.6$.

	Treatment A				***Treatment U***				***Treatment U-small***			
Sub	Mean	Std. Err.	95% CI		Mean	Std. Err.	95% CI					
1	11.81	0.09	11.63	12.00	11.99	0.01	11.96	12.02	12.00	0.05	11.90	12.11
2	11.97	0.15	11.66	12.28	11.99	0.02	11.95	12.04	11.99	0.08	11.83	12.15
3	11.93	0.07	11.78	12.08	13.04	0.39	12.26	13.83	11.96	0.04	11.88	12.04
4	12.05	0.17	11.70	12.40	11.83	0.24	11.36	12.31	12.04	0.11	11.81	12.27
5	11.99	0.11	11.76	12.22	12.07	0.09	11.89	12.24	12.00	0.06	11.89	12.12
6	12.00	0.17	11.65	12.34	12.51	0.20	12.11	12.91	12.05	0.04	11.98	12.13
7	12.07	0.08	11.91	12.24	11.92	0.09	11.74	12.10	11.99	0.01	11.96	12.02
8	11.84	0.09	11.67	12.02	12.30	0.27	11.76	12.84	11.92	0.05	11.81	12.03
9	12.00	0.19	11.61	12.39	12.18	0.13	11.93	12.44	12.00	0.03	11.95	12.05
10	11.95	0.13	11.70	12.21	12.11	0.22	11.68	12.55	11.92	0.11	11.70	12.15
11	11.95	0.04	11.87	12.04	11.73	0.23	11.26	12.19	11.95	0.23	11.50	12.40
12	11.94	0.08	11.77	12.10	11.96	0.03	11.91	12.01	11.91	0.12	11.68	12.14
13	11.93	0.15	11.63	12.23	12.38	0.10	12.17	12.58	11.98	0.04	11.89	12.06
14	11.76	0.14	11.47	12.05	11.94	0.14	11.67	12.22	11.93	0.09	11.76	12.11
15	12.05	0.05	11.95	12.15	12.10	0.14	11.83	12.37	11.97	0.11	11.75	12.19
16	12.16	0.15	11.85	12.47	11.92	0.06	11.81	12.04	11.80	0.05	11.70	11.89
17	11.91	0.07	11.78	12.04	11.99	0.10	11.79	12.19	12.05	0.04	11.97	12.13
18	11.97	0.13	11.71	12.23	11.94	0.07	11.79	12.09	12.06	0.09	11.88	12.23
19	11.98	0.06	11.86	12.11	11.99	0.12	11.74	12.23	12.04	0.07	11.90	12.18
20	11.97	0.03	11.91	12.02	12.00	0.01	11.99	12.02	12.06	0.07	11.91	12.20
21	12.00	0.05	11.91	12.09	11.93	0.10	11.72	12.14	11.94	0.12	11.70	12.18
22	12.03	0.14	11.75	12.30	11.68	0.16	11.35	12.00	12.01	0.05	11.90	12.12
23	11.94	0.05	11.83	12.05	12.14	0.11	11.92	12.35	12.12	0.09	11.94	12.31
24	12.03	0.09	11.84	12.21	11.83	0.19	11.44	12.21	11.99	0.12	11.75	12.22
25	11.87	0.10	11.66	12.07	12.01	0.29	11.43	12.59	12.08	0.06	11.96	12.20
26	11.92	0.06	11.80	12.03	10.92	0.28	10.35	11.49	11.98	0.13	11.73	12.24
27	11.92	0.06	11.79	12.04	11.99	0.22	11.54	12.44	11.97	0.07	11.82	12.12
28	11.83	0.15	11.53	12.13	11.95	0.04	11.86	12.04	12.09	0.18	11.72	12.46
29	11.89	0.10	11.68	12.10	11.87	0.20	11.46	12.27	11.91	0.10	11.71	12.10
30	11.93	0.11	11.72	12.15					11.92	0.06	11.79	12.05
31									11.95	0.11	11.73	12.17
32									12.03	0.11	11.81	12.24
33									11.86	0.17	11.51	12.20
34									12.01	0.06	11.89	12.13

Table C.5. The mean squared error of the learning by REE or the satisficing rule for each subject in Treatment A and U.

Treatment A	MSE	Treatment U	MSE	Treatment U-Small	MSE
1	1.1678	1	0.8700	1	0.8768
2	1.1804	2	1.1339	2	0.8416
3	1.1930	3	0.9752	3	0.8222
4	1.1112	4	0.9592	4	0.9800
5	1.1018	5	1.0387	5	0.8822
6	1.0945	6	1.3506	6	0.8372
7	1.0930	7	0.8944	7	0.8182
8	1.1017	8	0.9949	8	0.9219
9	1.0912	9	1.0806	9	0.8211
10	1.0946	10	1.0315	10	0.8587
11	1.0912	11	0.8232	11	1.4193
12	1.0824	12	0.9767	12	0.9640
13	1.0980	13	1.1182	13	0.8380
14	1.1111	14	1.0963	14	0.8082
15	1.0827	15	0.8329	15	0.9061
16	1.0571	16	1.1173	16	0.8912
17	1.0739	17	0.8391	17	0.8077
18	1.0765	18	0.9648	18	0.9728
19	1.0964	19	0.8823	19	0.8802
20	1.0775	20	0.8348	20	0.8845
21	1.1064	21	0.8500	21	0.8268
22	1.1059	22	1.0253	22	0.8569
23	1.1325	23	0.8478	23	0.8662
24	1.1662	24	0.8899	24	0.8505
25	1.1777	25	0.9673	25	0.8310
26	1.1307	26	0.8259	26	0.9869
27	1.0961	27	0.9214	27	0.9124
28	1.0038	28	0.9248	28	1.1404
29	1.1821	29	0.9820	29	0.8679
		30	1.0278	30	0.8444
				31	0.9603
				32	0.8245
				33	1.3522
				34	0.8719

Appendix D: Experimental Instructions and Quiz

Appendix D.1: Experimental Instructions

Welcome to this experiment in economic decision-making. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's experiment. There is no talking for the duration of this session. If you have a question at any time, please raise your hand and your question will be answered in private.

General information

Imagine you are an advisor to a farm that is the only supplier of a product in a local market. In each time period the owner of the farm needs to decide how many units of the product he will produce. To make an optimal decision each period, the owner requires a good prediction of the market price of the product in each period. As the advisor to the farm owner, you will be asked to predict the market price, p_t of the product during 50 successive time periods, t=1,2,. . .,50. Your earnings from this experiment will depend on the accuracy of your price predictions alone. The smaller are your prediction errors, the greater will be your earnings.

About the prediction of the market price

Your firm will use the following model to predict the market price for the product in each time period, t.

$$p_t^e = a + b \times w_t$$

where a is a positive number that is usually between 0 and 10, w_t is the a measure of how good the weather is for producing the agricultural product, and b is the coefficient that measures how sensitive the product is to the change of weather.

The weather variable is randomly drawn in each period, and you will see the realisation of it at the beginning of each period. Suppose in one period, w_t=8, you estimates are a=3, b=0.5, your implied prediction will be:

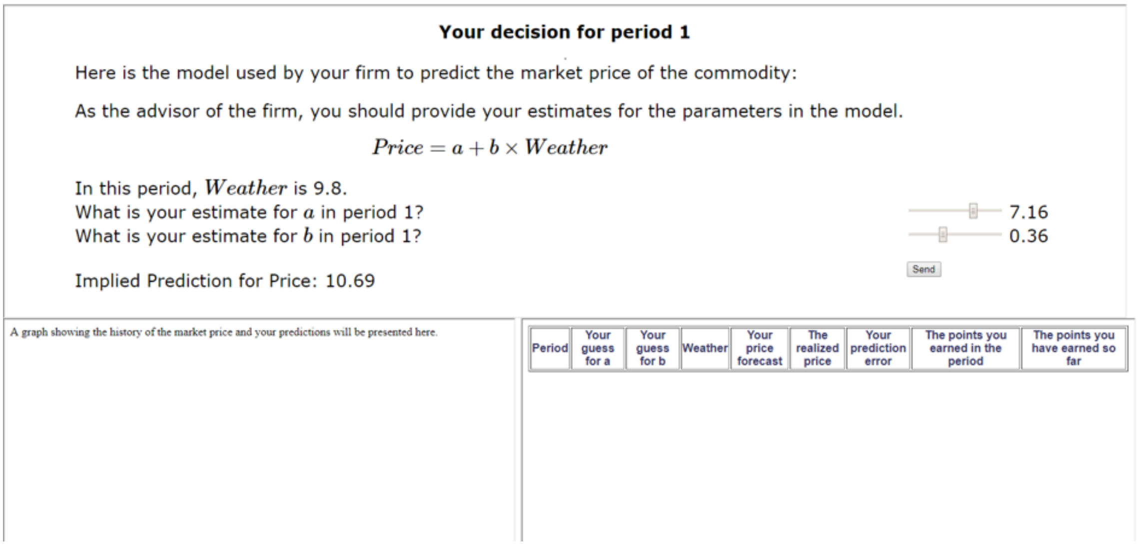
$$p_t^e = 3 + 0.5 \times 8 = 7$$

Suppose the market price in this period turns out to be $p_t = 4.9$. Your forecast error, $|p_t - p_t^e| = |7 - 4.9| = 2.1$. This forecast error of 2.1would determine your points for the period as discussed below.

Please also note that this example is for illustration purposes only. The value of the weather in your local market may be different from 8. The price determination function in this example may also be different from the price determination function in your local market. The precise value of weather in your market in each period will be given on your decision page.

Your task

Your only task in this experiment is to correctly predict the market price in each time period as accurately as possible. You need to choose the value of a and b using the slider bar on the computer screen. The value of a is between 0 and 10, and the value of b is between 0 and 1. The slider bar starts at the midpoint of the interval, and you can feel free to move it to any value that you want to choose. You can see your implied prediction $p_t^e = a + b \times w_t$ in real time in the line below. When you have decided on your choice of the parameters, you can press "send" to submit your decision.



At the beginning of the experiment you are asked to give a prediction for the price of your farm's product in period 1. Note that, while there are several farms being advised by a forecaster like you in each period, these different local markets are totally separate from your own so what happens in other markets does not have any influence on your market. After all forecasters have submitted their choice of parameters (and hence implied predictions) for the first period, the local market price for period 1 will be determined and will be revealed to you. Based the accuracy of your prediction in period 1, your earnings will be calculated. Subsequently, you are asked to enter your prediction for period 2. When all forecasters have submitted their predictions for the second period, the market price for that period in your local market will be revealed to you and your earnings will be calculated, and so on, for all 50 consecutive periods.

Information

Following the first period, you will see information on your computer screen that consists of 1) a plot of all past prices together with your predictions and 2) a table containing the history of the past prices, your past estimates of a, b, the implied price forecasts and payoffs.

About your payoff

Your payoff depends on the accuracy of your price forecast. The earnings shown on the computer screen will be in terms of points. When your prediction is p_t^e and the market price p_t your payoff is a decreasing function in your prediction error, namely the distance between your forecast and the realised price.

$$\text{payoff} = \frac{100}{1 + |p_t^e - p_t|}$$

Recalling the example above, if your forecast error for the period, $|p_t - p_t^e|$, was 2.1, then according to the payoff function you would earn 100/3.1=32.26 points for the period.

Notice that the maximum possible payoff in points you can earn from the forecasting task is 100 for each period, and the larger is your prediction error, $|p_t^e - p_t|$, the fewer points you earn. There is a Payoff Table on your desk, which shows the points you can earn for various different prediction errors.

At the end of the experiment your total points earned from all 50 periods will be converted into dollars at the rate of 1 dollar for every 200 points that you earned. Thus, the more points you earn, the greater are your Euro earnings.

Questions?

If you have questions about any part of these instructions at any time, please raise your hand and an experimenter will come to you and answer your question in private.

Appendix D.2: Quiz

We want to make sure that you understand the instructions. Therefore, we ask a few questions. You can only go to the decision page after you have answered all the questions correctly.

Question 1: Suppose in one period, the Weather is equal to 6, your estimates for the parameters in the model are a=3.6, b=0.5. What is your implied prediction for the price (a+b*Weather) in this period? (Answer: 6.6)

Question 2: If your forecast error for a period is 1, what is your payoff in this period? (Answer: 50)

Question 3: Is the price in our market influenced by other participants' price forecasts? (Answer: No)

Appendix E: Experimental Interface, Respondent Questionnaire, and Weather List

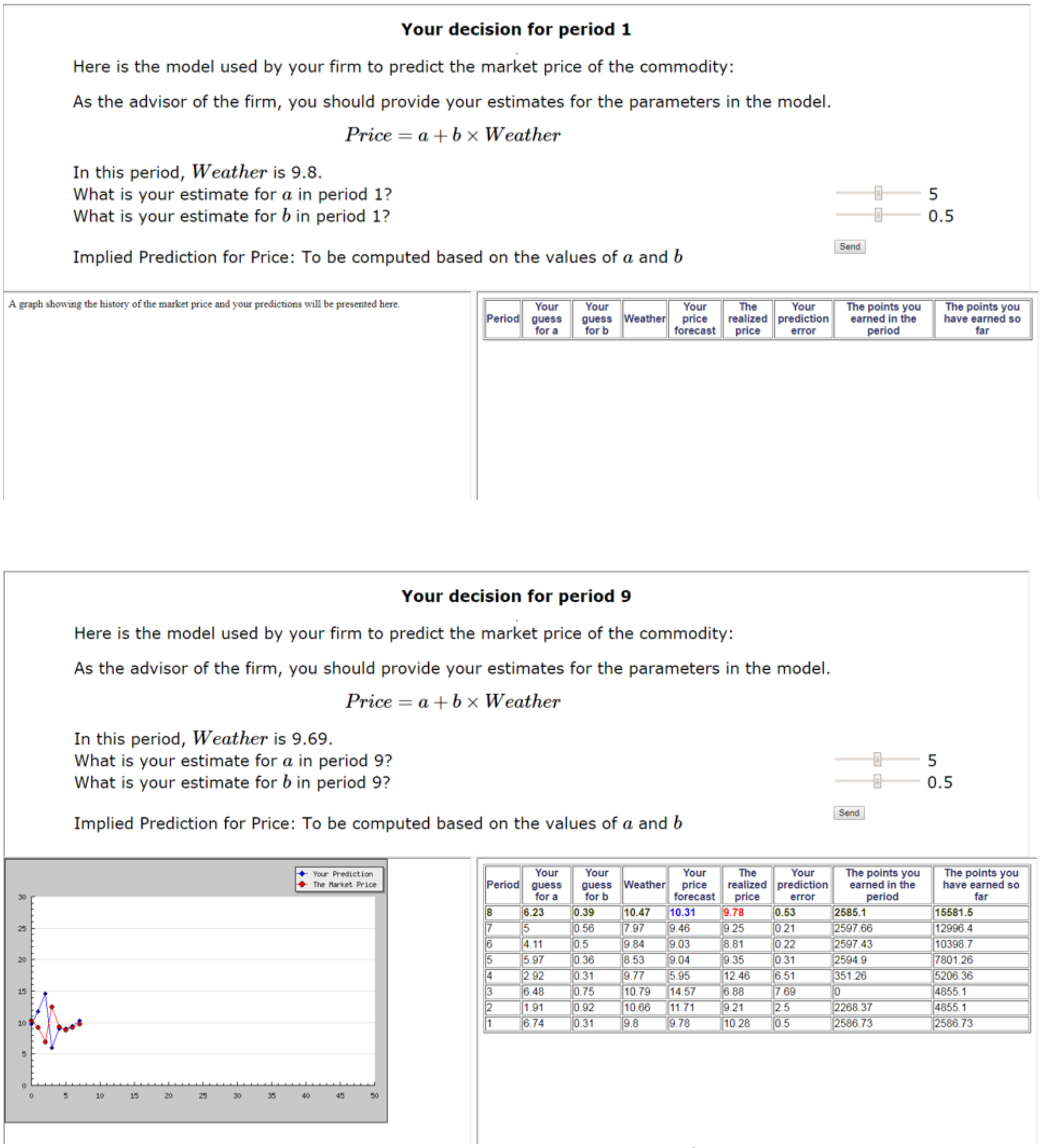


Figure E.1. Experimental interface.

Respondent Questionnaire

You have made your prediction for all periods! Here is a questionnaire to complete on your backgrounds. Please answer the questions and press “send” to submit. After that you will see the payment page.

1. Age: ____________

2. Gender:
☐ Male
☐ Female

3. Study Program: choose from list
☐ Faculty of Economics and Econometrics
☐ Faculty of Social and Behavioural Sciences: Psychology
☐ Faculty of Social and Behavioural Sciences: other than Psychology
☐ Faculty of Science, Mathematics and Computer Science
☐ Faculty of Law
☐ Faculty of Humanities
☐ Faculty of Medicine
☐ Another University
☐ Others

4. Have you come to an economics experiment before?
☐ No
☐ Yes, only once
☐ Yes, more than once

5. How do you describe your strategy in this experiment?

____

Table E.1. Distribution of weather w_t in Treatments A and U and $\epsilon_t \sim N(0,1)$.

period	Weather A	Weather U	ϵ_t
1	10	10.92	0.2872
2	10.12	5.49	-0.3316
3	9	2.25	-1.3085
4	8.32	19.35	0.5709
5	8.62	2.32	-0.2499
6	10.1	3.7	-1.5791
7	9.59	14.14	0.1971
8	8.2	15.92	-0.3451
9	8.34	19.9	-0.4954
10	9.58	5.07	0.2918
11	10.22	14.55	0.442
12	10.09	14.63	1.4143
13	10.88	8.29	-0.5298
14	9.46	5.5	0.6355
15	6.89	5.15	-2.7869
16	8.12	6.51	0.9556
17	9.05	18.41	0.8365
18	9.61	0.37	-0.1459
19	11.12	8.36	0.6092
20	10.23	5.24	-0.0798
21	10.4	9.18	0.9335
22	9.22	1.21	0.2988
23	9.98	5.54	1.2221
24	8.25	14.7	-0.5452
25	7.3	2.99	0.0912
26	7.1	4.14	1.8649
27	6.77	12.93	0.4169
28	9.65	0.63	-1.6766
29	10.63	9.54	0.1737
30	9.66	8.97	0.1636
31	9.92	11.47	-0.3594
32	9.83	13.14	2.132
33	8.96	8.4	1.3624
34	9.72	17.89	0.7295
35	10.07	19.93	-0.4855
36	10.91	8.22	-0.5949
37	10.71	14.98	-1.891
38	10.04	9.57	0.8072
39	9.43	3.16	1.4417
40	10.29	9.15	-0.189
41	10.5	6.12	1.0415
42	10.72	8.51	0.5437
43	11.01	17.37	-0.8844
44	11.69	0.48	-1.8038
45	9.55	13.25	0.3485
46	8.71	15.98	1.0516
47	8.21	13.08	0.8825
48	9.01	11.88	0.6185
49	8.57	9.52	0.8332
50	11.38	10.74	0.6123

Appendix F: Omitted Proof on Cobweb Model

For completeness of explanation, we repeat what we have mentioned in Section 2.1.

Consider the cobweb model in Evans and Honkapohja (2001) based on the analysis of Bray and Savin (1986), and Fourgeaud et al. (1986). It consists of a single competitive market with a time lag in production (e.g. agricultural product), where demand depends negatively on the prevailing market price; supply is assumed to depend on both the average expectation across the homogeneous firms of the price of the product in the current period, as well as the weather in the current period in the form of an observable shock. [21], denoted as:

$$d_t = m_I - m_p p_t + v_{1t}, \;\; m_p > 0$$

$$s_t = r_I + r_p p_t^e + r_w w_t + v_{2t}, \;\; r_p > 0$$

d_t, s_t represents the demand and supply of the product, m_I and r_I denotes the intercept, v_{1t} and v_{2t} are the random variables of unobserved random noise. Thus, at the market clearing price where $d_t = s_t$, the reduced form of price determination function is:

$$d_t = s_t$$

$$m_I - m_p p_t + v_{1t} = r_I + r_p p_t^e + r_w w_t + v_{2t}$$

$$p_t = \frac{m_I - r_I}{m_p} + \left(\frac{-r_p}{m_p}\right) p_t^e + \left(\frac{-r_w}{m_p}\right) w_t + \left(\frac{v_{1t} - v_{2t}}{m_p}\right)$$

Thus,

$$p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t$$

In the above equation, $\mu = m_p^{-1}(m_I - r_I)$, $\alpha = -m_p^{-1} r_p < 0$, $\delta = -m_p^{-1} r_w < 0$, $\eta_t = m_p^{-1}(v_{1t} - v_{2t})$, and $\eta_t \; iid(0, \sigma_\eta^2)$. The distribution of the weather w_t can be followed by an iid process as was assumed in Bray and Savin (1986). Alternatively, it can follow a stationary exogenous VAR (vector autoregression) process driven by a multivariate white noise shock with bounded moments as the setting in Evans and Honkapohja (2001).

According to the least squares principle, prediction of estimators of a simple linear regression model will be more precise (i.e. with lower variance) when there is a larger variation of independent variables[22]. Therefore theoretically, the variance

[21]Note that the weather in the original setting is assumed to based on the weather in the previous term w_{t-1}, assuming that the supply in the current period will depend on the observable shock brought from weather in the last period. However, we change the source of this observable shock into w_t. This is to help the subjects understand the setting more easily, and the change in the term will not change the quantitative results from the model.

[22]In the simple linear regression model $y_i = \beta_1 + \beta_2 x_i + e_i$, an estimated model $\hat{y} = b_1 + b_2 x_i$ can be formed using least squares principle, where $y_i = \hat{y}_i + \hat{e}_i$. $Var\,(b_1) = \frac{\sigma^2 N^{-1} \sum x_i^2}{\sum (x_i - \overline{x})^2}$, $Var\,(b_2) = \frac{\sigma^2}{\sum (x_i - \overline{x})^2}$. Thus, the wider spread of independent variable weather (i.e. a larger $\sum (x_i - \overline{x})^2$) will lead to a more precise estimate (i.e. smaller variance) on both of the parameters. Note that the spread of the independent variable does not affect the accuracy on the estimator because the expectation of the estimates following least squares principle should always be unbiased, i.e.

of the estimates from Bray and Savin (1986) should be smaller than in the setting of Evans and Honkapohja (2001). We design two separate treatments to verify this hypothesis.

If we assume that subjects form a rational belief following the adaptive expectation or any other fixed-weight distributed lag formula, that is, the expected price in the current terms is to be based on (or conditional on) the information of information available in the previous term, then the expectation price in the current term can be written as:

$$p_t^e = E_{t-1}p_t$$

Operating with E_{t-1} on both sides and solve for $E_{t-1}p_t$, and combining with the equation of $p_t = E_{t-1}p_t + \eta_t$ we have:

$$E(p_t) = E(\mu + \alpha p_t^e + \delta w_t + \eta_t)$$
$$E_{t-1}p_t = \mu + \alpha E_{t-1}p_t + \delta w_t$$
$$E_{t-1}p_t = \frac{\mu}{1-\alpha} + \frac{\delta}{1-\alpha}w_t$$
$$p_t = \frac{\mu}{1-\alpha} + \frac{\delta}{1-\alpha}w_t + \eta_t$$

Thus,

$$p_t = \bar{a} + \bar{b}w_t + \eta_t, \quad \bar{a} = (1-\alpha)^{-1}\mu, \; \bar{b} = (1-\alpha)^{-1}\delta$$

The equation above states the unique REE of the cobweb model, and it is said to have unique REE because p_t does not depend on the expected future prices.

Though the firms may have difficulty in obtaining the real value of REE, the process is still learnable using LS learning according to Evans and Honkapohja (2012) since LS learning assumes that firm to have a subjective model of the relationship between p_t and the observable shock, namely the perceived law of motion, denoted as:

$$p_t = a + bw_t + \eta_t$$

Subsequently, under the assumption that firms have data on the evolution of the economy from periods $i = 0,\ldots,t-1$, they will update their belief on the parameters of a, b repeatedly in each period, using the information from the past. Letting (a_{t-1}, b_{t-1}) denote the estimation through time $t-1$, using the information set $\{p_i,\ w_i\}_{i=0}^{t-1}$. Thus, their prediction for period t would be the expectation of p_t using the price information from period 0 to period $t-1$:

$$E_{t-1}p_t = p_t - \eta_t = a + bw_t$$

$$p_t^e = a_{t-1} + b_{t-1}w_t$$

In this approach, the rationality is implied through the process of a continuous update on the parameters in the model instead of the immediate formation of

$E(b_1) = \beta_1, E(b_2) = \beta_2.$

expectation. Agents are to update the model like econometricians or statisticians using LS learning, with the formula denoted as the equation of:

$$\begin{pmatrix} a_{t-1} \\ b_{t-1} \end{pmatrix} = \left(\sum_{i=1}^{t-1} z_{i-1} z'_{i-1} \right)^{-1} \left(\sum_{i=1}^{t-1} z_{i-1} p_i \right)$$

where

$$z'_i = (1 \; w'_i)$$

The fully specified dynamic system is: at the beginning of time t, subjects form the expectation based on $p_t^e = a_{t-1} + b_{t-1} w_t$, and update their parameter according to $\binom{a_{t-1}}{b_{t-1}} = \left(\sum_{i=1}^{t-1} z_{i-1} z'_{i-1}\right)^{-1} \left(\sum_{i=1}^{t-1} z_{i-1} p_i\right)$, where $z'_i = (1 \; w'_i)$. On top of it, given the w_t and the random noise η_t, the time t price is determined by $p_t = \mu + \alpha p_t^e + \delta w_t + \eta_t$. This result could thus be used by the agent to update the parameters again, through adding $(p_t, \; w_t)$ to the data set and computing the revised estimates a_t and b_t, and subsequently to forecast p_{t+1}^e using w_{t+1} in the beginning of time $t+1$. This process continues repeatedly over time.

Meanwhile according to the E-stability principle (as the basic required concept governing the stability of equilibria that mapping from PLM to ALM from learning), in order for a_t and b_t to exhibit an asymptotic stability of an REE under LS learning (i.e. PLM is gradually converged towards ALM), the condition of $\alpha < 1$ must be satisfied to let $a_t \to \overline{a}, b_t \to \overline{b}$.

In other words, for a cobweb model, it must meet the condition of a downward-sloping demand curve as well as $|m_p| > r_p$, to reach an expectational stability or "E-stability" to let $\alpha = -m_p^{-1} r_p < 1$.