

# Monetary-Fiscal Policy Interactions under Implementable Monetary Policy Rules\*

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May 4, 2006

## Abstract

This paper examines the implications of forward and backward-looking monetary policy rules in an environment with monetary-fiscal interactions. We find that the unique Rational Expectations Equilibrium (REE) is always non-Ricardian under simple implementable monetary policy rules.

*JEL Classification:* E52; E62

*Keywords:* Taylor Rules, Fiscal theory, rational expectations, determinacy.

## 1 Introduction

A focus of recent research is the design of monetary policy rules under particular fiscal policy regimes. In most cases it is assumed that fiscal policy is Ricardian and so it is up to monetary policy to determine prices and inflation.<sup>1</sup> Papers that explicitly model monetary-fiscal interactions and highlight the role fiscal policy plays in price level determination include Leeper (1991) and Woodford (1995).<sup>2</sup> In their approach the interactions are studied under contemporaneous policy rules; however, some authors have questioned whether rules conditioning on current inflation or output are implementable (McCallum and Nelson (1999),

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\*We thank Eric Leeper for comments and suggestions. The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

<sup>1</sup>See Woodford (2003) for an overview.

<sup>2</sup>Recent papers on this topic include Leeper and Yun (2005) and Davig and Leeper (2005).

Clarida, Gali, and Gertler (2000), Orphanides (2001), Benhabib, Schmitt-Grohe, and Uribe (2003)).

This paper addresses an important gap in the literature on monetary policy rules. We examine the implications of simple, implementable monetary policy rules in an environment with monetary-fiscal interactions. In particular, we focus on a model where these interactions matter for price-level determination. We assume that monetary policy is characterized by an interest rate rule that is a linear feedback function of either lagged or expected inflation; this extends Leeper (1991), where the nominal interest rate is a function of the contemporaneous rate of inflation.

We find that the alternative policy rules produce new determinacy results. When monetary policy is forward looking, determinacy obtains provided fiscal policy is active. With a backward-looking interest rate rule, determinacy obtains provided the policy mix is active fiscal/passive monetary. As a corollary, we find that for both forward and backward-looking rules, determinacy implies that the unique Rational Expectations Equilibrium (REE) is non-Ricardian.

## 2 The Model

This paper adopts the representative agent endowment economy in Leeper (1991), also employed by Evans and Honkapohja (2004). This model has (linearized) reduced-form equations

$$E_t \pi_{t+1} = \beta R_t \tag{1}$$

$$b_t = -\phi_1 \pi_t - \phi_2 R_t - \phi_3 R_{t-1} + \beta^{-1} b_{t-1} - \tau_t, \tag{2}$$

where  $\pi_t$  is the inflation rate,  $b_t$  is real bond holdings,  $R_t$  is the nominal interest rate, and  $\tau_t$  is lump-sum taxes. Leeper (1991), and Evans and Honkapohja (2004), demonstrate that these equations can be derived from log-linearizations of the first-order conditions from the household problem in an endowment economy. Under this interpretation (1) is the (linearized) Fisher relation and (2) is the (linearized) intertemporal budget constraint. Here  $\beta$  is the discount rate and  $\phi_1, \phi_2, \phi_3$  are functions of the model's deep parameters. We refer the reader to these other papers for details on the derivations.

We use the endowment economy of Leeper (1991) because it is the most parsimonious model capable of illustrating our results. Woodford (1995) and Sims (1995) find that assuming a New Keynesian type model does not alter the basic qualitative fiscalist result. This is because the essential ingredient for modeling monetary-fiscal policy interaction is the intertemporal budget constraint, which must bind in an REE.<sup>3</sup>

To close the model, we assume that  $R_t$  and  $\tau_t$  are formed as simple, implementable

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<sup>3</sup>There is an extensive debate in the literature over whether satisfaction of the intertemporal budget constraint is an equilibrium condition. This debate is orthogonal to our aim of exploring the implications of implementable monetary policy rules for the fiscal theory of the price level.

reaction functions. We follow Leeper (1991) in assuming that tax policy is set according to

$$\tau_t = \gamma b_{t-1} + \psi_t. \quad (3)$$

Leeper also assumed a contemporaneous interest rate rule of the form

$$R_t = \alpha \pi_t + \theta_t. \quad (4)$$

Here  $\psi_t$  and  $\theta_t$  are independent (mean-zero) white noise shocks. Interest rate rules of this form have been criticized by McCallum and Nelson (1999) among others as not being implementable. To address this concern, instead we assume that policy is implemented using rules conditional on observable data; specifically,

$$R_t = \alpha E_t \pi_{t+j} + \theta_t \quad j = -1, 1. \quad (5)$$

In particular, we impose that interest rates are set as either forward-looking or backward-looking rules.

Leeper provides the following definition.

**Definition.** *Monetary policy is active if  $|\alpha\beta| > 1$  and passive otherwise. Fiscal policy is active if  $|\beta^{-1} - \gamma| > 1$  and passive otherwise.*

Intuitively, monetary policy is active if a rise in inflation results in a more than one-for-one rise in the nominal rate and fiscal policy is active if taxes do not adjust to fully offset debt-financed tax changes.

A rational expectations equilibrium (REE) is any stochastic process satisfying (1)-(3) and (5). If there is a unique non-explosive REE then the model is said to be (locally) determinate; if there are multiple non-explosive REE then the model is said to be (locally) indeterminate; and if no non-explosive equilibria exist then the model is said to be explosive.

Using the definition of active monetary and fiscal policy<sup>4</sup>, Leeper (1991) identifies equilibrium determinacy with active/passive regimes. We summarize his findings as follows.

1. The model (1)-(4) has a (locally) unique non-explosive REE for all  $(\alpha, \gamma)$  which satisfy  $|\alpha\beta| > 1$  and  $|\beta^{-1} - \gamma| < 1$ . The unique REE satisfies the restriction  $\pi_t = -\alpha^{-1}\theta_t$  and is referred to as the monetarist solution because the inflation path depends only on monetary shocks.
2. The model (1)-(4) has a (locally) unique non-explosive REE for all  $(\alpha, \gamma)$  which satisfy  $|\alpha\beta| < 1$  and  $|\beta^{-1} - \gamma| > 1$ . The unique REE satisfies the restriction  $\pi_t = K_1 b_t + K_2 \theta_t$  for appropriately defined constants  $K_i$ ,  $i = 1, 2$ . This solution is referred to as the fiscalist solution because inflation depends on the path of real bonds.

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<sup>4</sup>In the monetary policy literature, activist policy has also been taken to have the precise opposite meaning. For instance, activist monetary policy has also been used to describe policy primarily concerned with output and not inflation stabilization. See Orphanides (2001) for a discussion.

These results emphasize a unique REE exists provided one policy authority takes an active stance and the other a passive stance. In the case of active monetary and passive fiscal policy, the inflation process is driven entirely by white noise monetary policy shocks. When fiscal policy is active and monetary policy is passive, the stochastic process for inflation depends on the stochastic process for bonds - in this instance, fiscal policy pins down the price level.

### 3 Implementable Policy Rules

We now analyze the determinacy properties of the model when closed with policy rules conditioning on observable proxies of inflation, and we compare our results with those in Leeper.

#### 3.1 A Forward-looking Interest Rate Rule

Consider the model (1)-(3) closed with the following forward-looking monetary policy rule:

$$R_t = \alpha E_t \pi_{t+1} + \theta_t. \quad (6)$$

We have the following result.

**Proposition 1** *Assume monetary policy is forward-looking and set according to (6). The model (1)-(3) is determinate if and only if policy  $(\alpha, \gamma)$  satisfies  $|(\beta^{-1} - \gamma)| > 1$ . The equilibrium inflation process is given by*

$$\pi_t = G_1 \theta_t + G_2 \theta_{t-1} + G_2 \psi_t, \quad (7)$$

where the determination of the  $G_i$ 's are described in the Appendix.

Because of its expectational structure, this model is never explosive; therefore, if  $|(\beta^{-1} - \gamma)| < 1$  then there exists multiple REE, many exhibiting dependence on extrinsic, "sunspot" processes. Notice also that in case of determinacy, the unique non-explosive REE takes the 'fiscalist' form in part 2 of Leeper's result; but unlike the contemporaneous interest rate rule, there does not exist a 'monetarist' solution. In particular, with a forward-looking policy rule, the unique REE (7) exhibits the fiscal theory of the price level regardless of the stance of monetary policy.

The intuition behind this result is straightforward, as even if  $|\alpha\beta| > 1$  monetary policy is not active in any meaningful sense. To see this, notice that according to (1) if policy directs the nominal rate to rise by  $\alpha$ , then expected inflation rises by  $\alpha/\beta > \alpha$  implying that the change in  $R_t - E_t \pi_{t+1}$  is negative. Since it is not possible to have activist monetary policy, a monetarist solution does not exist.

### 3.2 A Backward-looking Interest Rate Rule

We now turn to policy rules of the form

$$R_t = \alpha\pi_{t-1} + \theta_t. \quad (8)$$

We have the following result.

**Proposition 2** *Assume monetary policy is backward-looking and set according to (8).*

1. *The model (1)-(3) is determinate if and only if policy  $(\alpha, \gamma)$  satisfies  $|\alpha\beta| < 1$  and  $|\beta^{-1} - \gamma| > 1$ .*
2. *The model (1)-(3) is indeterminate if and only if policy  $(\alpha, \gamma)$  satisfies  $|\alpha\beta| < 1$  and  $|\beta^{-1} - \gamma| < 1$ .*
3. *The model (1)-(3) is explosive if and only if policy  $(\alpha, \gamma)$  satisfies  $|\alpha\beta| > 1$ .*

*In case of determinacy, the unique equilibrium path satisfies*

$$\pi_t = H_1 b_t + H_2 \theta_{t-1} + H_3 \theta_t, \quad (9)$$

*where the determination of the  $H_i$ 's are described in the Appendix.*

As above, Proposition 2 shows that in the case of determinacy the inflation rate is determined, in part, by fiscal policy. Moreover, active fiscal policy is again a necessary condition for determinacy. Unlike the forward-looking case, monetary policy is restricted to be passive, as in Leeper (1991). Because of the backward-looking behavior of the policy rule, active monetary policy leads to explosive behavior, and so we again find that a monetarist solution does not exist.

A monetary authority that responds aggressively to lagged inflation will place the economy on an explosive inflationary path. A thought experiment provides an intuitive interpretation to our results: suppose there is an unanticipated positive shock to the nominal interest rate. This change in the relative return of nominal bonds to nominal money balances will induce the representative agent to substitute nominal bonds for nominal money. For the government's budget constraint to be satisfied, outstanding nominal debt will be revalued through a discrete jump in the price level so that discounted future primary surpluses and seigniorage equal current real debt. Since the monetary authority reacts to inflation with a lag, no action is taken to counter this contemporaneous increase in inflation. Next period, however, the monetary authority will react aggressively and increase nominal interest rates more than inflation. However, this leads to another substitution of bonds for money and places inflation on an explosive path. Therefore, to ensure non-explosiveness the monetary authority must react passively to lagged inflation. This contrasts with the case of the forward-looking rule, where monetary policy does not face a restriction on its response to expected inflation. In this case, because the central bank responds to projected inflation this explosive path does not exist regardless of the value of  $\alpha$ .

## 4 Conclusion

This paper extends Leeper (1991) by studying the implications of simple implementable interest rate rules for the fiscal theory of the price level. Specifically, we analyzed the local determinacy results in a model where the interaction of fiscal and monetary policy matters and monetary policy is described by a forward or backward-looking nominal interest rate rule. We showed that interest rate rules which respond to expected inflation yield existence of a locally unique non-explosive REE if and only if fiscal policy is active. Under a backward-looking rule, a locally unique REE exists when fiscal policy is active and monetary policy is passive. We also showed that there does not exist a monetarist REE, that is a locally unique non-explosive REE where inflation does not depend on fiscal shocks. These results are fiscalist in the sense that the path of equilibrium inflation depends on bonds and taxes.

## Appendix

### Proof of Proposition 1

The policy rule and Fisher relation combine to imply  $R_t = \frac{1}{1-\alpha\beta}\theta_t$ . Writing  $\xi_t = \pi_t - E_{t-1}\pi_t$  yields  $\pi_t = \frac{\beta}{1-\alpha\beta}\theta_{t-1} + \xi_t$ . Imposing this and the expressions for  $R_t, \tau_t$  into the intertemporal constraint provides

$$b_t = \delta b_{t-1} - A_1\theta_t - A_2\theta_{t-1} - \phi_1\xi_t - \psi_t,$$

where  $\delta = \beta^{-1} - \gamma$ ,  $A_1 = \frac{\phi_2}{1-\alpha\beta}$ , and  $A_2 = \frac{\phi_1\beta + \phi_3}{1-\alpha\beta}$ . If  $|\delta| < 1$  then this expression for  $b_t$  is non-explosive for all martingale difference sequences  $\xi_t$ : this is the indeterminate case. If  $|\delta| > 1$  then  $\xi_t$  must be chosen so that the state of the dynamic system lies in the associated contracting eigenspace: this is the determinate case. Here, notice that  $b_t = \Omega\theta_t$  for some  $\Omega$ . The lag structure of the intertemporal constraint then implies that  $\Omega = \frac{A_2}{\delta}$ ; whence  $A_1\theta_t - \phi_1\xi_t - \psi_t = \Omega\theta_t$ . It follows that  $\xi_t = -\frac{1}{\phi_1}((\Omega + A_1)\theta_t + \psi_t)$ . This may be used to compute the  $G_i$ 's.

### Proof of Proposition 2

The policy rule and reduced form equations may be stacked as  $y_t = My_{t-1} + N\eta_t$ , where  $y = (b_t, \pi_t, \pi_{t-1}, \theta_t)'$ ,  $\eta_t = (\xi_t, \theta_t, \psi_t)'$  and  $\xi_t$  is the inflation forecast error. Now diagonalize  $M$ :  $M = S\Lambda S^{-1}$ , where  $\Lambda$  is the diagonal matrix of eigenvalues, which are assumed ordered in decreasing magnitude. Changing coordinates to  $z = S^{-1}y$ , the dynamic system decouples as  $z_t = \Lambda z_{t-1} + \tilde{\eta}_t$ , where  $\tilde{\eta}_t = S^{-1}N\eta_t$ . The eigenvalues of  $M$  are  $\{\delta, \sqrt{\alpha\beta}, \sqrt{\alpha\beta}, 0\}$ . Determinacy obtains when precisely one of these eigenvalues has modulus larger than one. Since  $\sqrt{\alpha\beta}$  is a repeated eigenvalue, determinacy can only obtain if  $|\delta| > 1$  and  $|\sqrt{\alpha\beta}| < 1$ . If  $|\delta| < 1$  and  $|\sqrt{\alpha\beta}| < 1$  then the dynamic system is non-explosive for all  $\xi_t$ : this is the indeterminate case; and if  $|\sqrt{\alpha\beta}| > 1$  then the dynamic system will be explosive for all  $\xi_t$ : this is the explosive case. In case of determinacy,  $\xi_t$  must be chosen so that  $\tilde{\eta}_t = 0$ . This allows  $z_{1t} = 0$ , which can then be solved for  $\pi_t$  to obtain the relation in the proposition.

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