A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt

Eric T. Swanson
University of California, Irvine

Impulse and Propagation Mechanisms Workshop
NBER Summer Institute
July 11, 2016
Motivation

Goal: Show that a simple macroeconomic model (with Epstein-Zin preferences) is consistent with a wide variety of asset pricing facts

- equity premium puzzle
- long-term bond premium puzzle (nominal and real)
- credit spread puzzle
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- financial intermediaries: Adrian-Etula-Muir (2013)
Motivation

Implications for Finance:

- unifying explanation for asset pricing puzzles
- structural model of asset prices (provides intuition, robustness to breaks and policy interventions)
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Two key ingredients:
- Epstein-Zin preferences
- nominal rigidities
Households

Period utility function:

\[ u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1 + \chi} \]

- additive separability between \( c \) and \( l \)
- SDF comparable to finance literature
- log preferences for balanced growth, simplicity

Flow budget constraint:

\[ a_{t+1} = e^i_t a_t + w_t l_t + d_t - c_t \]

Calibration: (IES = 1), \( \chi = 3 \), \( l = 1 \) (\( \eta = .54 \))
Generalized Recursive Preferences

Household chooses state-contingent \{ (c_t, l_t) \} to maximize

\[
V(a_t; \theta_t) = \max_{(c_t, l_t)} u(c_t, l_t) - \beta \alpha^{-1} \log [E_t \exp(-\alpha V(a_{t+1}; \theta_{t+1}))]
\]

Calibration: \( \beta = .992, \ RRA (R^c) = 60 \) \( (\alpha = 59.15) \)
Firms are very standard:

- continuum of monopolistic firms (gross markup $\lambda$)
- Calvo price setting (probability $1 - \xi$)
- Cobb-Douglas production functions, $y_t(f) = A_t k^{1-\theta} l_t(f)^\theta$
- fixed firm-specific capital stocks $k$

Random walk technology: \[ \log A_t = \log A_{t-1} + \varepsilon_t \]

- simplicity
- comparability to finance literature
- helps match equity premium

Calibration: $\lambda = 1.1$, $\xi = 0.8$, $\theta = 0.6$, $\sigma_A = 0.007$, $(\rho_A = 1)$, $\frac{k}{4Y} = 2.5$
No government purchases or investment:

\[ Y_t = C_t \]

Taylor-type monetary policy rule:

\[ i_t = r + \pi_t + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y (y_t - \bar{y}_t) \]

“Output gap” \((y_t - \bar{y}_t)\) defined relative to moving average:

\[ \bar{y}_t \equiv \rho_y \bar{y}_{t-1} + (1 - \rho_y) y_t \]

Rule has no inertia:

- simplicity

Calibration: \(\phi_\pi = 0.5, \phi_y = 0.75, \bar{\pi} = 0.008, \rho_y = 0.9\)
Solution Method

Write equations of the model in recursive form

Divide nonstationary variables \((Y_t, C_t, w_t, \text{etc.})\) by \(A_t\)

Solve using perturbation methods around nonstoch. steady state
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Solve using perturbation methods around nonstoch. steady state

- first-order: no risk premia
- second-order: risk premia are constant
- third-order: time-varying risk premia
- higher-order: more accurate over larger region

Model has 2 state variables \((\bar{y}_t, \Delta_t)\), one shock \((\varepsilon_t)\)
Impulse Responses

Technology $A_t$
Impulse Responses

Consumption $C_t$
Impulse Responses

Inflation $\pi_t$
Impulse Responses

Short-term nominal interest rate $i_t$

ann. pct.
Impulse Responses

Short–term real interest rate $r_t$
Nonlinear vs. Linear Impulse Response Responses

Price Dispersion $\Delta_t$

1st-order solution
5th-order solution

Consumption $C_t$
Nonlinear vs. Linear Impulse Response Responses

Price Dispersion $\Delta_t$

- 1st-order solution
- 5th-order solution

Graph showing the price dispersion $\Delta_t$ over time with two lines representing 1st-order and 5th-order solutions.
Nonlinear vs. Linear Impulse Response Responses

Consumption $C_t$

1st-order solution
5th-order solution

Consumption $C_t$
Nonlinear vs. Linear Impulse Response

Labor $L_t$

- **Percent**
- **Consumption $C_t$**

- **1st-order solution**
- **5th-order solution**

- **1st-order solution**
- **5th-order solution**
Equity: Levered Consumption Claim

Equity price

\[ p_t^e = E_t m_{t+1}(C_{t+1}^\nu + p_{t+1}^e) \]

where \( \nu \) is degree of leverage
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Calibration: \( \nu = 3 \)
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Equity Premium

Equity premium $\psi_t^e$
Real Government Debt

Real $n$-period zero-coupon bond price:

$$p_t^{(n)} = E_t m_{t+1} p_{t+1}^{(n-1)}.$$
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Real $n$-period zero-coupon bond price:

$$p_t^{(n)} = E_t m_{t+1} p_{t+1}^{(n-1)},$$

$$p_t^{(0)} = 1, \quad p_t^{(1)} = e^{-r_t}$$
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Real yield:

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## Real Yield Curve

### Table 3: Real Zero-Coupon Bond Yields

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*Footnotes:*

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- "Evans (1999)"
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Supply shocks make nominal long-term bonds risky: inflation risk.
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<td>UK gilts, 1970–2015&lt;sup&gt;b&lt;/sup&gt;</td>
<td>6.92</td>
<td>7.10</td>
<td>7.26</td>
<td>7.51</td>
<td>7.70</td>
<td>7.89</td>
<td>0.96</td>
</tr>
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<td>6.38</td>
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<td>6.50</td>
<td>6.48</td>
<td>0.28</td>
</tr>
<tr>
<td>macroeconomic model</td>
<td>5.35</td>
<td>5.59</td>
<td>5.80</td>
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</tr>
</tbody>
</table>

<sup>a</sup>Gürkaynak, Sack, and Wright (2007) online dataset

<sup>b</sup>Bank of England web site

Supply shocks make nominal long-term bonds risky: inflation risk
Nominal Term Premium

Nominal term premium $\psi_t^{(40)}$

ann. bp

0

10

20

30

40

50

0

-10

-8

-6

-4

-2

0

-10

-8

-6

-4

-2
Defaultable Debt

Default-free depreciating nominal consol:

\[ p_t^c = E_t m_{t+1} e^{-\pi_{t+1}} (1 + \delta p_{t+1}^c) \]
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\[ p^c_t = E_t m_{t+1} e^{-\pi_{t+1}} (1 + \delta p^c_{t+1}) \]

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Nominal consol with default:

\[ p_t^d = E_t m_{t+1} e^{-\pi_{t+1}} \left[ (1 - 1_{t+1}^d)(1 + \delta p_{t+1}^d) + 1_{t+1}^d \omega_{t+1} p_t^d \right] \]
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The credit spread is \( i_t^d - i_t^c \)
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<tr>
<th>average ann. default prob.</th>
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If default isn’t cyclical, then it’s not risky.
### Table 5: Credit Spread

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If default isn’t cyclical, then it’s not risky.
Default Rate is Countercyclical

A. Default rates and credit spreads

- Moody's Recovery Rates
- Altman Recovery Rates

Figure 1. Default rates, credit spreads, and recovery rates over the business cycle.

Panel A plots the Moody's annual corporate default rates during 1920 to 2008 and the monthly Baa-Aaa credit spreads during 1920/01 to 2009/02. Panel B plots the average recovery rates during 1982 to 2008. The "Long-Term Mean" recovery rate is 41.4%, based on Moody's data. Shaded areas are NBER-dated recessions.

For annual data, any calendar year with at least 5 months being in a recession as defined by NBER is treated as a recession year.

The default component of the average 10-year Baa-Treasury spread in this model rises from 57 to 105 bps, whereas the average optimal market leverage of a Baa-rated firm drops from 50% to 37%, both consistent with the U.S. data.

Figure 1 provides some empirical evidence on the business cycle movements in default rates, credit spreads, and recovery rates. The dashed line in Panel A plots the annual default rates over 1920 to 2008. There are several spikes in the default rates, each coinciding with an NBER recession. The solid line plots the monthly Baa-Aaa credit spreads from January 1920 to February 2009. The spreads shoot up in most recessions, most visibly during the Great Depression, the savings and loan crisis in the early 1980s, and the recent financial crisis in 2008. However, they do not always move in lock-step with default rates (the correlation at an annual frequency is 0.65), which suggests that other factors, such as recovery rates and risk premia, also affect the movements in spreads.

Next, business cycle variation in the recovery rates is evident in...
Recovery Rate is Procyclical

A. Default rates and credit spreads

B. Recovery rates

source: Chen (2010)
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Discussion

1. IES ≤ 1 vs. IES > 1
2. Volatility shocks
3. Endogenous conditional heteroskedasticity
4. Monetary and fiscal policy shocks
5. Financial accelerator
Intertemporal Elasticity of Substitution

Long-run risks literature typically assumes IES > 1, for two reasons:

- ensures equity prices rise (by more than consumption) in response to an increase in technology
- ensures equity prices fall in response to an increase in volatility
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Equity premium $\psi_t^e$
Impulse Responses to Technology Shock

Equity price $p_t^e$
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Extend model above to include volatility shocks:

\[
\log \sigma_{A,t} = (1 - \rho_\sigma) \log \bar{\sigma}_A + \rho_\sigma \log \sigma_{A,t-1} + \varepsilon_t^\sigma
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Calibration: $\rho_\sigma = .98$, $\text{Var}(\varepsilon_t^\sigma) = (0.1)^2$
Impulse Responses to Volatility Shock

Volatility $\sigma_{A,t}$
Impulse Responses to Volatility Shock

![Graph showing the impulse response of consumption C_t to volatility shocks.

The x-axis represents time in weeks (10 to 50), and the y-axis represents the percentage change in consumption.

The graph shows a positive trend in consumption over time following a volatility shock.](image-url)
Impulse Responses to Volatility Shock

Inflation $\pi_t$

ann. pct.

-0.5
-0.4
-0.3
-0.2
-0.1
0.0

10 20 30 40 50
Impulse Responses to Volatility Shock

Equity premium $\psi_t^e$
Impulse Responses to Volatility Shock

Equity price $p_t^e$

percent

0 10 20 30 40 50

-5

-4

-3

-2

-1

0
Impulse Responses to Volatility Shock

Nominal term premium $\psi_t^{(40)}$

Time (ann. bp) vs. Nominal term premium $\psi_t^{(40)}$
### Endogenous Conditional Heteroskedasticity

Note that:

\[ \psi_t = E_t \left( \nu_{t+1} + \rho_{t+1} \right) \]

\[ \rho_t = E_t \left( \nu_{t+1} + \rho_{t+1} \right) E_{t+1} m_{t+1} \]

\[ E_t m_{t+1} \left( \nu_{t+1} + \rho_{t+1} \right) - E_{t+1} m_{t+1} \left( \nu_{t+1} + \rho_{t+1} \right) E_t \]

\[ = - \text{Cov}_t \left( m_{t+1}, \text{Re}_{t+1} \right) \]

\[ = - \text{Cov}_t \left( m_{t+1}, E_{t+1} \right) \text{Re}_{t+1} \]
Endogenous Conditional Heteroskedasticity

Note that

$$
\psi_t^e \equiv E_t R_{t+1}^e - e_{rt}^e
$$
Note that

\[ \psi_t^e \equiv E_t R_{t+1}^e - e_{t}^r \]

\[ = \frac{E_t (C_{t+1}^\nu + p_{t+1}^e)}{p_t^e} - e_{t}^r \]

\[ = \frac{E_t (C_{t+1}^\nu + p_{t+1}^e)}{E_t m_{t+1} (C_{t+1}^\nu + p_{t+1}^e)} - \frac{1}{E_t m_{t+1}} \]

\[ = \frac{E_t m_{t+1} E_t (C_{t+1}^\nu + p_{t+1}^e) - E_t m_{t+1} (C_{t+1}^\nu + p_{t+1}^e)}{E_t m_{t+1} p_t^e} \]

\[ = -\text{Cov}_t (m_{t+1}, R_{t+1}^e) \]

\[ = -\text{Cov}_t \left( \frac{m_{t+1}}{E_t m_{t+1}}, r_{t+1}^e \right) \]
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Traditional finance approach: assume shocks are heteroskedastic.

Here, conditional heteroskedasticity is **endogenous**.

Nonlinear solution contains terms of form

$$x_{t+1} \in x_t$$

so covariance $\text{Cov}_t$ depends on state $x_t$. 
Impulse Responses for Conditional Variance

Conditional Variance $\text{Var}_t[(C_{t+1}/C_t)^{-1}]$
Impulse Responses for Conditional Variance

Conditional Variance $\text{Var}_t[\exp(-\alpha V_{t+1})/E_t\exp(-\alpha V_{t+1})]$
Impulse Responses to Pos. and Neg. Tech. Shocks

Price Dispersion $\Delta_t$

Consumption $C_t$

---

- **Price Dispersion**: Two lines represent different conditions:
  - Blue dashed line: No previous shock in period 0.
  - Green line: Previous shock of 0.007 in period 0.

- **Consumption**: Two lines represent different conditions:
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Rudebusch and Swanson (2012) consider similar model with
- technology shock
- government purchases shock
- monetary policy shock
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All three shocks help the model fit macroeconomic variables
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- technology shock
- government purchases shock
- monetary policy shock

All three shocks help the model fit macroeconomic variables.

But technology shock is most important (by far) for fitting asset prices:
- technology shock is more persistent
- technology shock makes nominal assets risky
No Financial Accelerator

With model-implied stochastic discount factor $m_{t+1}$, we can price any asset.

Economy affects $m_{t+1} \Rightarrow$ economy affects asset prices.

However, asset prices have no effect on economy.
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...but not in this paper.
Conclusions

1. The standard textbook New Keynesian model (with Epstein-Zin preferences) is consistent with a wide variety of asset pricing facts/puzzles.

2. Unifies asset pricing puzzles into a single puzzle—Why does risk aversion in macro models need to be so high? (Literature provides good answers to this question).

3. Provides a structural framework for intuition about risk premia.

4. Suggests a way to model feedback from risk premia to macroeconomy.