The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

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Rudebusch, Sack, Swanson (2006): term premium in standard NK DSGE models is far too small, stable relative to the data.
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Part of a Broader Project

- Rudebusch, Sack, Swanson (2006): term premium in standard NK DSGE models is far too small, stable relative to the data.

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- this paper: Epstein-Zin preferences in a NK DSGE model
Why Study the Term Premium?
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Relative to equity premium, the term premium:
- only requires modeling short-term interest rate, not dividends or leverage
- is used by central banks to measure expectations of monetary policy
- applies to a larger volume of securities
- provides an additional perspective on the model
- tests nominal rigidities

DSGE model: many empirical questions about risk premia require a structural DSGE model to provide reliable answers

DSGE models widely used in macroeconomics; total failure to explain risk premia may signal flaws in the model
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Epstein-Zin Preferences

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We’ll use standard NK utility kernel:

\[ u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \]
Epstein-Zin Preferences

Household optimality conditions with EZ preferences:

\[ \mu_t u_1 \big|_{(c_t, l_t)} = P_t \lambda_t \]

\[ -\mu_t u_2 \big|_{(c_t, l_t)} = w_t \lambda_t \]

\[ \lambda_t = \beta E_t \lambda_{t+1} (1 + r_{t+1}) \]

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Recall: \[ V_t = u(c_t, l_t) + \beta (E_t V_t^{1-\alpha})^{1/(1-\alpha)} \]
The DSGE Model

- Continuum of households with Epstein-Zin preferences
  - consume output, supply labor

- Continuum of Dixit-Stiglitz differentiated firms
  - set prices in Calvo contracts with avg. duration 4 quarters
  - identical Cobb-Douglas production functions
  - face aggregate technology: \( \log A_t = \rho_A \log A_{t-1} + \varepsilon^A_t \)

- Government
  - purchases \( G_t \), financed by lump-sum taxes
  - \( \log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon^G_t \)

- Monetary Authority
  - sets short-term nominal interest rate using a Taylor-type rule
  - monetary policy shock
The Term Premium in the Model

Asset pricing:

\[ p_t = d_t + E_t[m_{t+1}p_{t+1}] \]
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Zero-coupon bond pricing:

\[ p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}] \]

\[ i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)} \]
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Risk-neutral bond price:

\[ \hat{p}^{(n)}_t = e^{-i_t} E_t[\hat{p}^{(n-1)}_{t+1}] \]

Term premium:

\[ \psi^{(n)}_t \equiv i^{(n)}_t - \hat{i}^{(n)}_t \]
Solving the Model

State variables of the model:

\[ A_{t-1}, G_{t-1}, i_{t-1}, \bar{\pi}_{t-1}, \Delta_{t-1}, \varepsilon^A_t, \varepsilon^G_t, \varepsilon^i_t \]
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We solve the model by perturbation methods

- We compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes \( n \)th order approximations
Table 2: Empirical and Model-Based Unconditional Moments

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<thead>
<tr>
<th>Variable</th>
<th>U.S. Data 1961–2007</th>
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memo: IES .5 .5 .5
quasi-CRRA 2 75 90
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Coefficient of Relative Risk Aversion

- Epstein-Zin preferences:

\[
m_{t,t+1} \equiv \beta u_1 \bigg|_{(c_{t+1}, l_{t+1})} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}}
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Long-Run Inflation Risk

Long-run inflation risk makes long-term bonds more risky: the same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk. Long-term inflation expectations are more observable than long-term consumption growth. Other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary.
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Long-Run Inflation Risk

10-year zero-coupon Treasury yield

10-year inflation expectations from SPF, Blue Chip
Long-Run Inflation Risk and the Term Premium

Expected Utility Preferences, no long-run risk
Long-Run Inflation Risk and the Term Premium

Epstein-Zin Preferences, no long-run risk

Expected Utility Preferences, no long-run risk

mean term premium (basis points)

quasi-CRRA
Result: Nominal Yield Curve is Upward-Sloping

- if interest rates are low in recessions
- then bond prices rise in recessions
- $\Rightarrow$ the term premium should be negative
- the yield curve slopes downward

Note: Backus et. al intuition still applies to real yield curve
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This paper:
- technology/supply shocks imply inflation is high in recessions
- then nominal bond prices \textit{fall} in recessions
  \[ \implies \text{the nominal yield curve slopes upward} \]
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Model-Implied Nominal Yield Curve (left axis)

Model-Implied Real Yield Curve (right axis)
Result: Nominal Yield Curve is Upward-Sloping

UK Nominal Yield Curve, 1994-2007 (left axis)

UK Real Yield Curve, 1994-2007 (right axis)
Result: Nominal Yield Curve is Upward-Sloping

US Nominal Yield Curve, 1994-2007 (left axis)

US Real Yield Curve, 2004-2007 (right axis)
Result: Model Term Premium is Countercyclical
Result: Model Generates Endogenous Heteroskedasticity

\[ p_{t}^{(2)} - \hat{p}_{t}^{(2)} = E_{t} m_{t+1} p_{t+1}^{(1)} - E_{t} m_{t+1} E_{t} p_{t+1}^{(1)} = Cov_{t}(m_{t+1}, p_{t+1}^{(1)}) \]

time-varying term premium ⇐⇒ conditional heteroskedasticity
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time-varying term premium $\iff$ conditional heteroskedasticity

Second-order solution:

\[x_t = \mu_x + \sum \alpha_x d_{x_{t-1}} + \sum \alpha_\varepsilon \varepsilon_t + \sum \alpha_{xx} d_{x_{t-1}} d_{x_{t-1}} + \sum \alpha_{x\varepsilon} d_{x_{t-1}} \varepsilon_t + \sum \alpha_{\varepsilon\varepsilon} \varepsilon_t \varepsilon_t + \ldots\]
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<th>term premium std dev (bp)</th>
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<tr>
<td>baseline model</td>
<td>86.5</td>
<td>11.0</td>
</tr>
<tr>
<td>log-linear log-normal</td>
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Conclusions

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3. Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework: agents are risk-averse and cannot offset long-run real or nominal risks.

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