Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Suppose a household has preferences:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \]

\[ u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1 - \gamma} - \eta l_t \]
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What is the household’s coefficient of relative risk aversion?
Suppose a household has preferences:

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What is the household’s coefficient of relative risk aversion?

Answer: 0
Coefficient of Relative Risk Aversion

Suppose a household has preferences:

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\[ u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi} \]

What is the household’s coefficient of relative risk aversion?

Answer: \( \frac{1}{\frac{1}{\gamma} + \frac{1}{\chi}} \)
Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):
- Lottery winners reduce labor supply by $.11 per $1 of prize
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Cesarini, Lindqvist, Notowidigdo, and Östling (2017):
- Swedish lottery winners reduce labor supply by SEK .11/SEK won
- Spouses also reduce labor supply (but by less)
- Labor response is primarily due to reduction in *hours*
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- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market
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Large literature finds significantly negative wealth effect on labor supply (e.g., Pencavel 1986)
Frictional Labor Markets

No labor/perfectly rigid labor market:
- Arrow (1964), Pratt (1965)
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Frictional labor market:
- this paper
A Household

Household preferences:

\[ E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ U(c_{\tau}) - V(l_{\tau} + u_{\tau}) \right], \]
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\[ E_t \sum_{\tau=t}^{\infty} \beta^\tau - t [U(c_\tau) - V(l_\tau + u_\tau)], \]

Flow budget constraint:

\[ a_{\tau+1} = (1 + r_\tau)a_\tau + w_\tau l_\tau + d_\tau - c_\tau, \]
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No-Ponzi condition:

\[ \lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0, \]
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\{w_\tau, r_\tau, d_\tau\} are exogenous processes, governed by \( \Theta_\tau \)
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Labor market search:

\[ l_{\tau+1} = (1 - s) l_\tau + f(\Theta_\tau) u_\tau \]
The Value Function

State variables of the household’s problem are \((a_t, l_t; \Theta_t)\).

Let:

\[ c_t^* \equiv c^*(a_t, l_t; \Theta_t), \]
\[ u_t^* \equiv u^*(a_t, l_t; \Theta_t). \]
The Value Function

State variables of the household’s problem are \((a_t, l_t; \Theta_t)\).

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\]

\[
u_t^* \equiv u^*(a_t, l_t; \Theta_t).
\]

Value function, Bellman equation:

\[
\nabla(a_t, l_t; \Theta_t) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t \nabla(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}),
\]

where:

\[
a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,
\]

\[
l_{t+1}^* \equiv (1 - s)l_t + f(\Theta_t)u_t^*.
\]
Technical Conditions

**Assumption 1.** The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.

**Assumption 2.** A solution $V : X \rightarrow \mathbb{R}$ to the household’s generalized Bellman equation exists and is unique, continuous, and concave.

**Assumption 3.** For any $(a_t, l_t; \Theta_t) \in X$, the household’s optimal choice $(c_t^*, u_t^*)$ exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \Theta_t)$.

**Assumption 4.** For any $(a_t, l_t; \Theta_t)$ in the interior of $X$, the second derivatives of $V$ with respect to its first two arguments, $V_{11}(a_t, l_t; \Theta_t)$, $V_{12}(a_t, l_t; \Theta_t)$, and $V_{22}(a_t, l_t; \Theta_t)$, exist.
Assumptions about the Economic Environment

**Assumption 5.** *The household is infinitesimal.*

**Assumption 6.** *The household is representative.*

**Assumption 7.** *The model has a nonstochastic steady state, \( x_t = x_{t+k} \) for \( k = 1, 2, \ldots \), and \( x \in \{c, u, l, a, w, r, d, \Theta\} \).*
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Assumption 7’. *The model has a balanced growth path that can be renormalized to a nonstochastic steady state after a suitable change of variables.*
Arrow-Pratt in a Static One-Good Model (Review)

Compare:

\[ E u(c + \sigma \varepsilon) \quad \text{vs.} \quad u(c - \mu) \]
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Compute:

\[ u(c - \mu) \approx u(c) - \mu u'(c), \]
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Compute:

\[ u(c - \mu) \approx u(c) - \mu u'(c), \]

\[ E u(c + \sigma \varepsilon) \approx u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2} u''(c)\sigma^2 E[\varepsilon^2], \]
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\[ \mu = \frac{-u''(c)}{u'(c)} \frac{\sigma^2}{2}. \]
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Coefficient of absolute risk aversion is defined to be:

\[ \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2 = \frac{-u''(c)}{u'(c)}. \]
Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t) a_t + w_t l_t - c_t + \sigma \varepsilon_t + 1,$$

Note we cannot easily consider gambles over:

- $a_t$ (state variable, already known at $t$)
- $c_t$ (choice variable)

Note ($\ast$) is equivalent to gamble over asset returns:

$$a_{t+1} = (1 + r_t + \sigma \tilde{\varepsilon}_t + 1) a_t + w_t l_t - c_t,$$

or income:

$$a_{t+1} = (1 + r_t) a_t + w_t l_t + (d_t + \sigma \varepsilon_t + 1) - c_t,$$
Consider a one-shot gamble in period $t$:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$  \hspace{1cm} (*)
Arrow-Pratt in a Dynamic Model

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vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$
Arrow-Pratt in a Dynamic Model

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$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t + \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\nabla_1 (a_t, l_t; \Theta_t) \frac{\mu}{(1 + r_t)}$$
Consider a one-shot gamble in period $t$:

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$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\beta E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1}) \mu.$$
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vs.

$$a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from $\mu$:

$$\beta E_t V_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \mu.$$

Loss from $\sigma$:

$$\beta E_t V_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \frac{\sigma^2}{2}.$$
Definition 1. *The household’s coefficient of absolute risk aversion at* \((a_t, l_t; \Theta_t)\) *is given by* \(R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2.\)
Coefficient of Absolute Risk Aversion

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**Proposition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is well-defined and satisfies

\[
R^a(a_t, l_t; \Theta_t) = \frac{-E_t \nabla_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \nabla_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}.
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**Proposition 1.** The household’s coefficient of absolute risk aversion at \((a_t, l_t; \Theta_t)\) is well-defined and satisfies

\[
R^a(a_t, l_t; \Theta_t) = -\frac{E_t \nabla_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \nabla_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}.
\]

Evaluated at the nonstochastic steady state, this simplifies to:

\[
R^a(a, l; \Theta) = \frac{-\nabla_{11}(a, l; \Theta)}{\nabla_1(a, l; \Theta)}
\]

Solve for $V_1$ and $V_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_{\tau}) - V(l_{\tau} + u_{\tau})]$$
Solve for $V_1$ and $V_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_{\tau}) - V(l_{\tau} + u_{\tau})]$$

Benveniste-Scheinkman:

$$\nabla_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c^*_t). \quad (*)$$
Solve for $V_1$ and $V_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} [U(c_\tau) - V(l_\tau + u_\tau)]$$

Benveniste-Scheinkman:

$$\nabla_1(a_t, l_t; \Theta_t) = (1 + r_t) U'(c^*_t). \quad (\ast)$$

Differentiate $(\ast)$ to get:

$$\nabla_{11}(a_t, l_t; \Theta_t) = (1 + r_t) U''(c^*_t) \frac{\partial c^*_t}{\partial a_t}.$$
Solve for $\frac{\partial c_t^*}{\partial a_t}$
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Consumption Euler equation:

$$U'(c_t^*) = \beta E_t(1 + r_{t+1}) U'(c_{t+1}^*),$$
Solve for $\frac{\partial c^*_t}{\partial a_t}$

Consumption Euler equation:

$$U'(c^*_t) = \beta E_t (1 + r_{t+1}) U'(c^*_{t+1}),$$

implies, at steady state:

$$\frac{\partial c^*_t}{\partial a_t} = E_t \frac{\partial c^*_{t+1}}{\partial a_t} = E_t \frac{\partial c^*_{t+k}}{\partial a_t}, \quad k = 1, 2, \ldots$$
Solve for $\partial c_t^*/\partial a_t$

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Household’s budget constraint, no-Ponzi condition imply:

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r.$$
Solve for $\frac{\partial l^*_{t+k}}{\partial a_t}$

Labor search (unemployment) Euler equation:

$$\frac{V'(l_t + u^*_t)}{f(\Theta_t)} = \beta E_t \left[ w_{t+1} U'(c^*_{t+1}) - V'(l^*_{t+1} + u^*_{t+1}) \right. $$

$$\left. + (1 - s) \frac{V'(l^*_{t+1} + u^*_{t+1})}{f(\Theta_{t+1})} \right]$$
Solve for $\frac{\partial l_{t+k}^*}{\partial a_t}$

Labor search (unemployment) Euler equation:

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\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t \left[ w_{t+1} U'(c_{t+1}^*) - V'(l_{t+1}^* + u_{t+1}^*) 
+ (1 - s) \frac{V'(l_{t+1}^* + u_{t+1}^*)}{f(\Theta_{t+1})} \right]
$$

and transition equation

$$
l_{t+1} = (1 - s)l_t + f(\Theta_t)u_t
$$
Solve for $\frac{\partial l_{t+k}^*}{\partial a_t}$

Labor search (unemployment) Euler equation:

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+ (1 - s) \frac{V'(l_{t+1}^* + u_{t+1}^*)}{f(\Theta_{t+1})} \right]
$$

and transition equation

$$
l_{t+1} = (1 - s)l_t + f(\Theta_t)u_t
$$

imply, at steady state:

$$
E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[ 1 - (1 - s - f(\Theta))^k \right] \frac{\partial c_t^*}{\partial a_t}.
$$

where $\gamma \equiv -cU''(c)/U'(c)$, $\chi \equiv (l + u)V''(l + u)/V'(l + u)$.
Solve for $\frac{\partial c^*_t}{\partial a_t}$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1 + r)^k} E_t \left[ \frac{\partial c^*_{t+k}}{\partial a_t} - w \frac{\partial l^*_{t+k}}{\partial a_t} \right] = 1 + r$$
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Consumption Euler equation:

$$\frac{\partial c^*_t}{\partial a_t} = E_t \frac{\partial c^*_{t+1}}{\partial a_t} = E_t \frac{\partial c^*_{t+k}}{\partial a_t}, \quad k = 1, 2, \ldots,$$
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Labor Euler equation:

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l + u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[ 1 - (1 - s - f(\Theta))^k \right] \frac{\partial c^*_t}{\partial a_t} ,$$
Solve for $\partial c^*_t / \partial a_t$

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Labor Euler equation:

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Solution is a “modified permanent income hypothesis”:

$$\frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}.$$
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1(a, l; \theta) = (1 + r) U'(c), \]
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1(a, l; \theta) = (1 + r) U'(c), \]
\[ \nabla_{11}(a, l; \theta) = (1 + r) U''(c) \frac{\partial c^*_t}{\partial a_t}, \]
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\[ \frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + \gamma \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}, \]
Solve for Coefficient of Absolute Risk Aversion

\[ \nabla_1(a, l; \theta) = (1 + r) U'(c), \]
\[ \nabla_{11}(a, l; \theta) = (1 + r) U''(c) \frac{\partial c^*_t}{\partial a_t}, \]
\[ \frac{\partial c^*_t}{\partial a_t} = \frac{r}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}, \]

**Proposition 2.** Given Assumptions 1–7, the household’s coefficient of absolute risk aversion, \( R^a(a_t, l_t; \Theta_t) \), evaluated at steady state, satisfies

\[ R^a(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{r}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}. \]
Relative Risk Aversion

Compare: \[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \epsilon_{t+1} \]

vs.

\[ a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t. \]
Relative Risk Aversion

Compare: \[ a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1} \]

vs.

\[ a_{t+1} = (1 + r_t) a_t + w_t l_t + d_t - c_t - \mu A_t. \]

**Definition 2.** The households' coefficient of relative risk aversion, \( R^c(a_t, l_t; \Theta_t) = A_t R^a(a_t, l_t; \Theta_t) \), where \( A_t \) denotes the household's financial assets plus present discounted value of labor income.

At steady state, \( A = c/r \), and

\[
R^c(a; \Theta) = \frac{-U''(c)}{U'(c)} \cfrac{c}{1 + \frac{\gamma}{\chi} \cfrac{w(l+u)}{c} \cfrac{f(\Theta)}{r + s + f(\Theta)}}.
\]
Numerical Example

Household period utility function:

\[
\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}
\]
Numerical Example

Household period utility function:

\[
\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}
\]

Economy is a simple RBC model with labor market frictions:

- Competitive firms,
- Cobb-Douglas production functions, \( y_t = Z_t k_t^{1-\phi} l_t^\phi \)
- AR(1) technology, \( \log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t \)
- Capital accumulation, \( k_{t+1} = (1 - \delta) k_t + y_t - c_t \)
- Labor market frictions, \( l_{t+1} = (1 - s) l_t + h_t \)
Numerical Example

Labor market search:

- Cobb-Douglas matching function, \( h_t = \mu u_t^{1-\eta} v_t^\eta \)
- Wage set by Nash bargaining with equal weights
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Equity security:
- Equity is a consumption claim
- Equity premium is expected excess return,

\[
\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)
\]
Numerical Example

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\]

Baseline calibration:
- Production: \( \phi = 0.7, \delta = .0083, \rho_z = 0.99, \sigma_\varepsilon = .005 \)
- Matching: \( s = .02, \eta = 0.5, v/u = 0.6, f(\Theta) = 0.28 \)
- Preferences: \( \beta = .996, \gamma = 100, \chi = 100, l + u = 0.3 \)
Figure 1: Risk Aversion and Equity Premium vs. $\chi$

- Fixed-labor measure of risk aversion (left axis)
- Relative risk aversion $R^c$ (left axis)
- Equity premium (right axis)
Figure 2: Risk Aversion and Equity Premium vs. $\gamma$
Figure 3: Risk Aversion and Equity Premium vs. $f(\Theta)$
Proposition 3. Let $f_1, f_2 : \Omega_\Theta \to [0, 1]$. Given Assumptions 1–8 and fixed values for the parameters $s$, $\beta$, $\gamma$, and $\chi$, let $(a_1, l_1; \Theta_1)$ and $(a_2, l_2; \Theta_2)$ denote corresponding steady-state values of $(a_t, l_t; \Theta_t)$. If $f_1(\Theta_1) < f_2(\Theta_2)$, then $R_c^1(a_1, l_1; \Theta_1) > R_c^2(a_2, l_2; \Theta_2)$. 
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Proof:

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} c \left( 1 + \frac{\gamma \omega l}{\chi c} \frac{s + f(\Theta)}{r + s + f(\Theta)} \right)
\]

is decreasing in \( f(\Theta) \).
Risk Aversion Is Higher in Recessions

Proposition 4. Given Assumptions 1–8 and fixed values for the parameters $s, \beta, \gamma, \text{ and } \chi$, $R^c(a, I; \Theta)$ is decreasing in $I/u$. 
**Proposition 4.** Given Assumptions 1–8 and fixed values for the parameters $s$, $\beta$, $\gamma$, and $\chi$, $R^c(a, l; \Theta)$ is decreasing in $l/u$.

Proof:

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r+s+f(\Theta)}}.
\]

Using $sl = f(\Theta)u$,

\[
R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s(1+1/u)}{r+s(1+1/u)}}.
\]
Risk Aversion Higher for Less Employable Households

Two types of households:
- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$
Risk Aversion Higher for Less Employable Households

Two types of households:
- Measure 1 of type 1 households
- Measure 0 of type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$

Then Proposition 4 implies $R^c_2(a_2, l_2; \Theta) > R^c_1(a_1, l_1; \Theta)$. 
<table>
<thead>
<tr>
<th></th>
<th>s</th>
<th>f(Θ)</th>
<th>Percentage of Households Owning Equities</th>
<th>Percentage of Households Owning Risky Financial Assets</th>
<th>Share of Household Portfolios in Currency and Deposits</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>.019</td>
<td>.282</td>
<td>48.9</td>
<td>49.2</td>
<td>12.4</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>.009</td>
<td>.056</td>
<td>31.5</td>
<td>32.4</td>
<td>26.0</td>
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<tr>
<td>Germany</td>
<td>.006</td>
<td>.035</td>
<td>18.9</td>
<td>25.1</td>
<td>33.9</td>
</tr>
<tr>
<td>France</td>
<td>.007</td>
<td>.033</td>
<td>–</td>
<td>–</td>
<td>29.1</td>
</tr>
<tr>
<td>Spain</td>
<td>.012</td>
<td>.020</td>
<td>–</td>
<td>–</td>
<td>38.1</td>
</tr>
<tr>
<td>Italy</td>
<td>.004</td>
<td>.013</td>
<td>18.9</td>
<td>22.1</td>
<td>27.9</td>
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### Table 2: International Comparison

<table>
<thead>
<tr>
<th>Relative Risk Aversion $R^c$</th>
<th>$\gamma = 2$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>$f(\Theta)$</td>
<td>$\frac{s+f(\Theta)}{r+s+f(\Theta)}$</td>
<td>$\chi = 1.5$</td>
<td>$\chi = 0.5$</td>
</tr>
<tr>
<td>Theoretical labor market benchmarks:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>perfect rigidity</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>near-perfect flexibility</td>
<td>1</td>
<td>1</td>
<td>.997</td>
<td>.86</td>
</tr>
</tbody>
</table>

International comparison, $r = .004$:

| United States | .019 | .282 | .977 | .87 | .46 | 2.04 | 6.77 |
| United Kingdom | .009 | .056 | .903 | .91 | .50 | 2.17 | 7.13 |
| Germany       | .006 | .035 | .854 | .94 | .52 | 2.26 | 7.38 |
| France        | .007 | .033 | .851 | .94 | .53 | 2.27 | 7.40 |
| Spain         | .012 | .020 | .821 | .96 | .54 | 2.34 | 7.57 |
| Italy         | .004 | .013 | .7081 | 1.03 | .62 | 2.61 | 8.28 |
### Table 3: Cyclical Variation in Risk Aversion

<table>
<thead>
<tr>
<th>Relative Risk Aversion $R^c$</th>
<th>$\gamma = 2$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi = 1.5$</td>
<td>$\chi = 0.5$</td>
<td>$\chi = 2.5$</td>
<td>$\chi = 10$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$s$</th>
<th>$f(\Theta)$</th>
<th>$r$</th>
<th>$\frac{s+f(\Theta)}{r+s+f(\Theta)}$</th>
<th>$\chi = 1.5$</th>
<th>$\chi = 0.5$</th>
<th>$\chi = 2.5$</th>
<th>$\chi = 10$</th>
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</thead>
<tbody>
<tr>
<td>U.S., expansion</td>
<td>.017</td>
<td>.35</td>
<td>.003</td>
<td>.995</td>
<td>0.86</td>
<td>0.46</td>
<td>2.01</td>
<td>6.70</td>
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<tr>
<td>U.S., recession</td>
<td>.022</td>
<td>.20</td>
<td>.011</td>
<td>.953</td>
<td>0.88</td>
<td>0.47</td>
<td>2.08</td>
<td>6.88</td>
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<tr>
<td>rigid lab mkt, expan</td>
<td>.0036</td>
<td>.016</td>
<td>.003</td>
<td>.868</td>
<td>0.93</td>
<td>0.52</td>
<td>2.24</td>
<td>7.31</td>
</tr>
<tr>
<td>rigid lab mkt, recess</td>
<td>.0046</td>
<td>.009</td>
<td>.011</td>
<td>.557</td>
<td>1.15</td>
<td>0.76</td>
<td>3.10</td>
<td>9.45</td>
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</tbody>
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### Other International Evidence: Campbell (1999)

<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>United States</td>
<td>0.9%</td>
<td>4.5%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>2.6%</td>
<td>6.2%</td>
</tr>
<tr>
<td>Germany</td>
<td>2.5%</td>
<td>4.8%</td>
</tr>
<tr>
<td>France</td>
<td>2.1%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Netherlands</td>
<td>2.8%</td>
<td>9.0%</td>
</tr>
<tr>
<td>Switzerland</td>
<td>2.2%</td>
<td>10.1%</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.9%</td>
<td>6.4%</td>
</tr>
<tr>
<td>Italy</td>
<td>1.7%</td>
<td>−1.5%</td>
</tr>
</tbody>
</table>
Conclusions

General conclusions:
- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:
- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher in recessions
- Risk aversion is higher for households that are less employable

Quantitative findings:
- Frictions can play a contributing role to higher risk aversion in Europe
- Risk aversion formulas in Swanson (2012) still a good approximation