The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

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April 16, 2010
1. Motivation and Background
2. Epstein-Zin Preferences in a Standard NK Model
3. Long-Run Risks
4. Model Implications
5. Conclusions
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The **UIP premium puzzle**: excess returns on high-interest-rate foreign currencies are much larger (and more variable) than can be explained by standard preferences in a DSGE model.
Motivation

Basic Model

Long-Run Risks

Model Implications

Conclusions

Kim-Wright Term Premium

Fig. 1 10-year Treasury bond yield and inflation expectations

Data are quarterly. The 10-year zero-coupon Treasury bond yield is the end-of-quarter yield from Gurkaynak, Sack, and Wright (2007). 10-year inflation expectations are from the Federal Reserve Board, which is from three sources: from 1991 onward, the data are inflation expectations from 5 to 10 years ahead from the Survey of Professional Forecasters; from 1981 to 1991, the data are inflation expectations from 5 to 10 years ahead from the Blue Chip Survey of forecasters; prior to 1981, this series was extended backward by Federal Reserve Board staff using multiple data sources and the FRB/US model.

Fig. 2 Affine, no-arbitrage model decomposition of 10-year bond yield

Data are quarterly, sampled at the end of each quarter. Source: Kim and Wright (2005).
Why Study the Term Premium?
Why Study the Term Premium in a DSGE Model?

- Relative to equity premium, the term premium:
  - applies to a larger volume of securities
  - is used by central banks to measure expectations of monetary policy
  - only requires modeling short-term interest rate, not dividends or leverage
  - provides an additional perspective on the model
  - tests nominal rigidities

- More generally:
  - many empirical questions about risk premia require a structural DSGE model to provide reliable answers
  - DSGE models widely used in macroeconomics; total failure to explain risk premia may signal flaws in the model
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We examine to what extent the Piazzesi-Schneider results generalize to the DSGE case
Related Strands of the Literature

The Bond Premium in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in an Endowment Economy:

Epstein-Zin Preferences in a DSGE Model:

Epstein-Zin Preferences and the Bond Premium in a DSGE Model:
Epstein-Zin Preferences in a Standard NK Model

- Epstein-Zin Preferences
- Standard New Keynesian Model
- Price Assets in the Model
- Solve the Model
- Results
Epstein-Zin Preferences

Standard preferences:

\[ V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1} \]
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Note:

- need to impose \( u \geq 0 \)
- or \( u \leq 0 \) and \( V_t \equiv u(c_t, l_t) - \beta (E_t(-V_{t+1})^{1-\alpha})^{1/(1-\alpha)} \)
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We’ll use standard NK utility kernel:

$$u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\gamma}}{1+\chi}$$
Epstein-Zin Preferences

Household optimality conditions with EZ preferences:

$$\mu_t U_1\big|_{(c_t, l_t)} = P_t \lambda_t$$

$$-\mu_t U_2\big|_{(c_t, l_t)} = W_t \lambda_t$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1 + r_{t+1})$$

$$\mu_t = \mu_{t-1} (E_{t-1} V_t^{1-\alpha})^{\alpha/(1-\alpha)} V_t^{-\alpha}, \quad \mu_0 = 1$$
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Household optimality conditions with EZ preferences:

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\begin{align*}
\mu_t \left. u_1 \right|_{(c_t, l_t)} &= P_t \lambda_t \\
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\lambda_t &= \beta E_t \lambda_{t+1} (1 + r_{t+1}) \\
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\end{align*}
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Recall: \[ V_t = u(c_t, l_t) + \beta \left( E_t V_t^{1-\alpha} \right)^{1/(1-\alpha)} \]
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\]

Recall: \( V_t = u(c_t, l_t) + \beta (E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)} \)

Stochastic discount factor:

\[
m_{t,t+1} \equiv \frac{\beta u_1 \bigg|_{(c_{t+1}, l_{t+1})}}{u_1 \bigg|_{(c_t, l_t)}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}}
\]
New Keynesian Model (Very Standard)

Continuum of differentiated firms:
- face Dixit-Stiglitz demand with elasticity $\frac{1+\theta}{\theta}$, markup $\theta$
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions $y_t = A_t\bar{k}^{1-\eta}l_t^\eta$
- have firm-specific capital stocks
- face aggregate technology $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters $\theta = .2$, $\rho_A = .9$, $\sigma_A^2 = .01^2$

Perfectly competitive goods aggregation sector
New Keynesian Model (Very Standard)

Government:
- imposes lump-sum taxes $G_t$ on households
- destroys the resources it collects

$$\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$$

Parameters $\bar{G} = .17 \bar{Y}$, $\rho_G = .9$, $\sigma^2_G = .004^2$
New Keynesian Model (Very Standard)

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- imposes lump-sum taxes $G_t$ on households
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Parameters $\bar{G} = .17 \bar{Y}$, $\rho_G = .9$, $\sigma^2_G = .004^2$

Monetary Authority:
- $i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[ \frac{1}{\beta} + \pi_t + g_y (y_t - \bar{y}) + g_\pi (\bar{\pi}_t - \pi^*) \right] + \varepsilon^i_t$

Parameters $\rho_i = .73$, $g_y = .53$, $g_\pi = .93$, $\pi^* = 0$, $\sigma^2_i = .004^2$
Asset Pricing

Asset pricing:

\[ p_t = d_t + E_t[m_{t+1}p_{t+1}] \]
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Zero-coupon bond pricing:

\[ p_t^{(n)} = E_t[m_{t+1}p_{t+1}^{(n-1)}] \]

\[ i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)} \]

Notation: let \( i_t \equiv i_t^{(1)} \)
The Term Premium in the Standard NK Model

In DSGE framework, convenient to work with a default-free consol, a perpetuity that pays $1, $\delta^c$, $\delta^2c$, $\delta^3c$, ... (nominal)

Price of the consol:

$\tilde{p}(n)_t = 1 + \delta^c E_t m_t + 1 \tilde{p}(n)_t + 1$

Risk-neutral consol price:

$\hat{p}(n)_t = 1 + \delta^c e^{-it} E_t \hat{p}(n)_t + 1$

Term premium:

$\psi(n)_t \equiv \log(\delta^c \tilde{p}(n)_t / \tilde{p}(n)_t) - 1 - \log(\delta^c \hat{p}(n)_t / \hat{p}(n)_t - 1)$
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\]
Solving the Model

The standard NK model above has a relatively large number of state variables: \( A_{t-1}, G_{t-1}, i_{t-1}, \Delta_{t-1}, \bar{\pi}_{t-1}, \varepsilon^A_t, \varepsilon^G_t, \varepsilon^i_t \)
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We solve the model by approximation around the nonstochastic steady state (perturbation methods)
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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a third-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes $n$th order approximations
## Empirical and Model-Based Unconditional Moments

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<th>Variable</th>
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Coefficient of Relative Risk Aversion

Arrow-Pratt:

\[-C \frac{u''(C)}{u'(C)}\]
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Here:

\[
V_t = \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} + \beta (E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}
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\[
\text{CRRA} = \frac{-W \ V''(W)}{V'(W)} + \alpha \frac{W \ V'(W)}{V(W)}
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Coefficient of Relative Risk Aversion

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CRRA

\[
= -\frac{W V''(W)}{V'(W)} + \alpha \frac{W V'(W)}{V(W)}
\]

\[
= -\frac{u_{11} + \lambda u_{12}}{u_1} \frac{c}{1 + w \lambda} + \alpha \frac{c u_1}{u}
\]
Coefficient of Relative Risk Aversion

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\]

see “Risk Aversion, the Labor Margin, and Asset Pricing in a DSGE Model”
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\[ m_{t,t+1} \equiv \frac{\beta u_1|_{(c_{t+1},l_{t+1})}}{u_1|_{(c_t,l_t)}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \]
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- Barillas-Hansen-Sargent (2008):

\[ m_{t,t+1} = \frac{\beta u_1(c_{t+1}, l_{t+1})}{u_1(c_t, l_t)} \frac{\psi_{t+1}}{\psi_t} \frac{P_t}{P_{t+1}} \]
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\[ m_{t,t+1} \equiv \frac{\beta u_1|_{(c_{t+1},l_{t+1})}}{u_1|_{(c_t,l_t)}} \left( \frac{V_{t+1}}{(E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}} \]

- Barillas-Hansen-Sargent (2008):

\[ m_{t,t+1} \equiv \frac{\beta u_1|_{(c_{t+1},l_{t+1})}}{u_1|_{(c_t,l_t)}} \frac{\psi_{t+1}}{\psi_t} \frac{P_t}{P_{t+1}} \]

Risk Aversion and the Term Premium

Expected Utility Preferences, no long-run risk
Risk Aversion and the Term Premium

The diagram shows the relationship between risk aversion (quasi-CRRA) and the term premium (basis points) for two different preference models:

1. **Epstein-Zin Preferences, no long-run risk**
   - The blue line represents this scenario.

2. **Expected Utility Preferences, no long-run risk**
   - The red line represents this scenario.

The x-axis represents the quasi-CRRA values ranging from 0 to 100, while the y-axis represents the mean term premium in basis points ranging from -20 to 140.
Risk Aversion and the Term Premium

- Epstein-Zin Preferences, with long-run inflation risk
- Epstein-Zin Preferences, no long-run risk
- Expected Utility Preferences, no long-run risk
Long-Run Risks

- Long-Run Inflation Risk
- Long-Run Real Risk
<table>
<thead>
<tr>
<th>Motivation</th>
<th>Basic Model</th>
<th>Long-Run Risks</th>
<th>Model Implications</th>
<th>Conclusions</th>
</tr>
</thead>
</table>

**Long-Run Inflation Risk**

Long-run inflation risk makes long-term bonds more risky: same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk. Long-term inflation expectations are more observable than long-term consumption growth.
Long-Run Inflation Risk

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- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary
Motivation

Basic Model

Long-Run Risks

Model Implications

Conclusions

Long-Run Inflation Risk

Suppose:

\[ \pi_t^* = \rho \pi_{t-1}^* + \varepsilon_t^* \]
Long-Run Inflation Risk

Suppose:

\[ \pi_t^* = \rho^* \pi_{t-1}^* + \varepsilon_t^* \]

Then:

- inflation is volatile, but not risky
- in fact, long-term bonds act like insurance: when \( \pi^* \uparrow \), then \( C \uparrow \) and \( p^{(40)} \downarrow \)
- result: term premium is *negative*
Consider instead:

\[ \pi_t^* = \rho_{\pi}^* \pi_{t-1}^* + (1 - \rho_{\pi}^*) \theta_{\pi^*} (\bar{\pi} t - \pi_t^*) + \varepsilon_t^{\pi^*} \]
Consider instead:

\[ \pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + (1 - \rho_{\pi^*}) \theta_{\pi^*} (\bar{\pi}_t - \pi_t^*) + \varepsilon_t^* \]

- \( \theta_{\pi^*} \) describes pass-through from current \( \pi \) to long-term \( \pi^* \)
- Gürkaynak, Sack, and Swanson (2005) found evidence for \( \theta_{\pi^*} > 0 \) in U.S. bond response to macro data releases
- makes long-term bonds act less like insurance: when technology/supply shock, then \( \pi \uparrow, C \downarrow, \) and \( p^{(40)} \downarrow \)
- supply shocks become very costly
- The term premium is *positive*, closely associated with \( \theta_{\pi^*} \)
### Model-Based Moments with Long-Run Inflation Risk

<table>
<thead>
<tr>
<th>Variable</th>
<th>U.S. Data</th>
<th>EU Preferences &amp; LR $\pi^*$ Risk</th>
<th>EZ Preferences &amp; LR $\pi^*$ Risk</th>
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memo: IES .5 1.1

quasi-CRRA 2 90
Motivation

Basic Model

Long-Run Risks

Model Implications

Conclusions

Long-Run Productivity Risk

Following Bansal and Yaron (2004), introduce long-run real risk to make the economy more risky:

Assume productivity follows:

\[ \log A_t = \log A_t^* + \epsilon_t^A \]

\[ \log A_t^* = \rho_{A^*} \log A_{t-1}^* + \epsilon_{t}^{A^*} \]

- makes the economy much riskier to agents
- increases volatility of stochastic discount factor
<table>
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quasi-CRRA 90 90
Model Implications

- Nominal Yield Curve is Upward-Sloping
- Term Premium is Countercyclical
- Model Is Nonhomothetic, Heteroskedastic
Nominal Yield Curve is Upward-Sloping


- if interest rates are low in recessions
- then bond prices rise in recessions
- $\Rightarrow$ the term premium should be negative
- the yield curve slopes downward
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This paper:
- technology shocks imply that inflation is high in recessions
- then nominal bond prices \textit{fall} in recessions
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This paper:
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Note: Backus et. al intuition still applies to real yield curve
Nominal Yield Curve is Upward-Sloping

Model-Impplied Nominal Yield Curve (left axis)

Model-Impieved Real Yield Curve (right axis)

US Nominal Yield Curve, 1994-2007 (left axis)

US Real Yield Curve, 2004-2007 (right axis)
Model Term Premium is Countercyclical

Response to Technology Shock

Response to Government Spending Shock

Response to Monetary Policy Shock

Inflation

Long-Term Bond Price

Consumption
Model Is Nonhomothetic, Heteroskedastic

\[ p_t^{(2)} - \hat{p}_t^{(2)} = E_t m_{t+1} p_{t+1}^{(1)} - E_t m_{t+1} E_t p_{t+1}^{(1)} = \text{Cov}_t(m_{t+1}, p_{t+1}^{(1)}) \]
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time-varying term premium ⇐⇒ conditional heteroskedasticity
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time-varying term premium ⇐⇒ conditional heteroskedasticity

Second-order solution:

\[ x_t = \mu_x + \sum \alpha_x dx_{t-1} + \sum \alpha_\xi \varepsilon_t \]
\[ + \sum \alpha_{xx} dx_{t-1} dx_{t-1} + \sum \alpha_x \xi dx_{t-1} \varepsilon_t + \sum \alpha_\xi \xi \varepsilon_t \varepsilon_t + \ldots \]
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<td>baseline model</td>
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<td>log-linear log-normal</td>
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</table>
Conclusions

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2. Habit-based preferences can solve bond premium puzzle in endowment economy, but fail in NK DSGE framework: although agents are risk-averse, they can offset that risk.
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3. Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework: agents are risk-averse and cannot offset long-run real or nominal risks.

4. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments.