

# International Trade and Transnational Insecurity: How Comparative Advantage and Power are Jointly Determined<sup>†</sup>

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**Abstract:** We augment the canonical neoclassical model of trade to allow for interstate disputes over land, oil, water, or other resources. The costs of such disputes in terms of arming depend on the trade regime in place. Under either autarky or free trade, the larger country (in terms of factor endowments) need not to be more powerful. Yet, under free trade, there is a stronger tendency for arming incentives to be equalized and thus for a “leveling of the playing field.” Depending on world prices, free trade can intensify arming incentives to such an extent that the additional security costs swamp the traditional gains from trade and thus render autarky more desirable for one or both rival states. Furthermore, contestation of resources can reverse a country’s apparent comparative advantage relative to its comparative advantage in the absence of conflict. And, where such conflict is present, comparisons of autarkic prices to world prices could be inaccurate predictors of trade patterns.

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## 1 Introduction

Thinking about international trade and trade policy is typically based on models that assume perfectly and costlessly enforced property rights. Especially in transnational settings, however, where there is no ultimate authority to enforce property rights, countries expend resources on their defense and diplomacy in order to secure their borders and interests or, to put it differently, to self-enforce their property rights. Moreover, sometimes countries engage in wars that induce destruction and, at the same time, imply a diversion of (otherwise) productive resources, beyond that observed during peacetime. These costs are considerable by several measures<sup>1</sup> and therefore, one would suspect, they are of economic relevance. Nevertheless, assessing the economic relevance of security costs is difficult, given the absence of such costs in models of international trade.<sup>2</sup>

As an example of the problems and issues we seek to understand, consider the countries along the Nile river, as discussed in Klare (2001). The economy of Egypt critically depends on the Nile flowing at the rate that it has flowed for millennia, and given Egypt's population growth that dependence is not likely to fall in the foreseeable future. The countries located upstream—i.e., Burundi, Congo, Ethiopia, Kenya, Rwanda, Sudan, Tanzania, and Uganda—are poorer than Egypt, and using the Nile for power-generation and irrigation would be a key factor in their economic development. Of course, such use of the Nile by these upstream countries would result in a reduced flow to Egypt and serious harm to its economy. When Ethiopia, with the help of the World Bank, was drawing plans to build dams in its territory, Egypt credibly threatened to destroy the dams using its air force. Debates over the usage of the Nile's water remain largely unresolved, with significant implications having both security and economic dimensions. The value of water to Egypt and the other up-stream countries depends on, among other factors, their degree of trade openness and the prices of traded goods that use water as an input. For example, the international price of Egyptian cotton, a good that uses water as a main input, also affects the value of

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<sup>1</sup>Military expenditures alone were about 2.6 percent of world GDP during 2004, varying from less than 1 percent for a few countries to more than 10 percent for Saudi Arabia (SIPRI, 2005). As for the overall costs of conflict (including civil war), Hess (2003) estimates a lower bound for the yearly cost of conflict of 8 percent of steady state consumption for the 1960 to 1992 period. For high-income countries like the United States and France the cost was a bit over 3 percent of consumption, whereas for Iraq and Iran, largely as a result of the war between them, it was 65 percent and 26 percent of their respective yearly consumptions.

<sup>2</sup>Political scientists have long been interested in the linkages between international trade and conflict. (See Barbieri and Schneider (1999), who have surveyed much of the theoretical and empirical literatures on the subject.) Economists, by contrast, have only begun to explore the relationship between security and trade. Examples include Anderson and Marcouiller (2005) and Anderton et al. (1999), who analyze Ricardian models in which traded commodities are insecure either because of the presence of pirates and bandits or because the contending sides influence the terms of trade through arming. Both approaches emphasize the important point that international trade can be hampered by the anarchic nature of international relations. Skaperdas and Syropoulos (2001, 2002) address some of the implications of insecure property in the context of simple exchange models.

the Nile's water flow to Egypt.

For this example of international contestation of resources and similar ones,<sup>3</sup> a number of questions naturally arise. How does insecurity of productive resources affect military expenditures and other related costs borne by the countries claiming ownership of such resources? Do these security costs vary with the trade openness of the countries involved? Does insecurity significantly distort the allocation of resources within each country towards different productive uses compared with the absence of insecurity? And, of course, how does insecurity affect overall economic well-being?

To tackle such questions, we take a micro-founded, economic approach that allows for the interdependence of security and trade policies. To be more precise, we extend the canonical neoclassical model of trade, with two factors of production and two consumption goods, in two substantive ways: First, we abandon the assumption that property rights on both factors are perfectly and costlessly enforced. Instead, at least part of the total endowment of one factor is considered insecure and subject to dispute.<sup>4</sup> We will be calling this factor "land." It could contain oil, diamonds, other natural resources, or water. Second, other than the two consumption goods, there is a third possible output that can be produced by combining the two factors of production. That output, "guns" or "arms," represents the direct costs of insecurity and is used solely to contest the insecure portion of the disputed factor of production. The regular neoclassical model (which we also call the "Nirvana" model, to use Demsetz's (1969) apt term) is the limiting case of our model when security is perfect and there is no arming. We also suppose the countries are "small," in the sense that they behave as price-takers in international markets for the two final goods.<sup>5</sup>

Our central findings offer a new perspective, one that differs significantly from that provided by the canonical neoclassical model of trade. Perhaps the most obvious and potentially the most empirically significant result is that, given arming is used only in contesting disputed resources and not directly in producing final goods for consumption, economic well-being is lower relative to the case where property rights are perfectly and costlessly

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<sup>3</sup>Territorial disputes or, more generally, competing claims over resources (notably land, oil, natural gas as well as water) have resulted in numerous conflicts throughout history and nowadays (Klare, 2001). Renner (2002, p.6) reports that about a quarter of the roughly 50 wars and armed conflicts that were active in 2001 had a strong resource dimension.

<sup>4</sup>While the extant trade theory has ruled out these problems by assumption, there are notable exceptions including Chichilnisky (1994), who argues that trade may reduce welfare in the South by accentuating the over-exploitation of an open-access resource in which it has a comparative advantage, and Brander and Taylor (1997a), who formally proved this idea. See also Hotte et al. (2000) who study the effects of trade in an open-access resource and extend the analysis to consider the evolution of private enforcement in dynamic environments. Margolis and Shogren (2002) consider a North-South trade model with enclosures. The key difference between these models and ours is that enforcement costs are due to active contestation of resources. There are also important differences in the models considered and most of the questions addressed.

<sup>5</sup>As described in more detail below, we suppose that there are three countries: two small countries that contest the resource and another, large country (the "rest of the world.")

enforced. The military planes that Egypt buys and the possible security countermeasures (perhaps the future deployment of Patriot missiles as well as planes) taken by Ethiopia in the absence of a lasting agreement of the Nile's water rights can be expected to lower the welfare of Egyptians or of Ethiopians or of both. And, as the degree of insecurity—defined as the portion of the total endowment of the contested factor of production that is in dispute—increases, the welfare of the affected populations can be expected to fall.

Furthermore, security costs and, thus, the equilibrium distribution of power vary with the trade regime. What is meant by “power” depends on the context, but here we define it as the share of the contested resource that each country receives, which depends on the ratio of the countries' arms. Arming under autarky can be very different from arming under free trade as the prices of factor inputs and outputs, in general, vary across these two regimes. In the autarkic regime, arms are equalized across countries only under certain special circumstances. Given the possibility of non-equalization of arms, one might naturally wonder which country will have the “upper hand” in arms competition and, thus, be more powerful. In general, the outcome depends on both the nature of technology and the distribution of secure factor endowments between countries. One interesting finding is that the relatively bigger economy (in terms of secure factor endowments) is not necessarily more “powerful” (i.e., does not necessarily produce more arms). In the free trade regime, there is a well known tendency for equalization of factor prices across countries, which in turn creates an even greater tendency towards equalization of guns across countries (relative to the autarkic regime) and thus towards a “leveling of the playing field.” In the extreme case where such trade results in complete factor-price equalization and the degree of insecurity is not too large, arming and thus power too are equalized across the two countries.<sup>6</sup>

Insecurity manifests itself not only in the diversion of resources away from the production of consumption goods, but also in distortions in comparative advantage, and it can do so in at least two ways: (i) to cause trade patterns to differ from the ones that would have emerged under Nirvana; and, (ii) to alter the information content of the difference between free-trade and autarkic prices, possibly leading to an erroneous prediction of the direction of trade flows. The key to understanding the logic underlying these effects is to note that, by inducing nations to arm, insecurity alters the resources remaining for the production

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<sup>6</sup>If the predicted tendency towards arms equalization is empirically relevant, then we would expect to observe larger countries (in terms of GDP) devoting a smaller percentage of their GDP to arming than their smaller adversaries. For three of four pairs of countries that have had a history of adversarial relationships, the smaller country (in terms of GDP) has a higher fraction of GDP in military expenditures (Argentina (1.3%) vs Chile (2.7%); India (2.5%) vs Pakistan (3%); Iran (2.5%) vs. Iraq (8.6%)). The one exception is the Turkey-Greece pair (Turkey (5.3%) vs. Greece (4.3%)). Nonetheless, one might argue that Turkey perceives itself to face battle on multiple fronts (Iran, Armenia, Syria, and most of all the Kurdish issue); thus, if Turkey viewed Greece as its only foreign policy concern, it could easily be expected to have a lower percentage of its GDP devoted to guns (the Kurdish insurgency is very expensive). [The above data are from the CIA World Factbook: <https://www.cia.gov/library/publications/the-world-factbook/>]

of consumption goods. Suppose, for example, the technology for arming makes intensive use of the perfectly secure resource (labor). Then, insecurity implies relatively less of that resource is left for the production of traded goods. Consequently, under autarky, the market clearing price of the consumption good that employs the secure resource intensively tends to exceed the price that would have prevailed under Nirvana. The net effect is that there exists a range of international prices for which a country is an importer of that consumption good when there is insecurity, whereas it would have been an exporter of the same good under Nirvana. Similarly, because the introduction of free trade in consumption goods alters product prices, factor prices, arming incentives, and thus resources left for the production of consumption goods, a country's true comparative advantage (given insecurity) can differ from that which is implied by a simple comparison of autarkic prices to world prices. Both of these distortions imply that the presence of insecurity plays an important role in the determination of a country's actual trade patterns that traditional theory fails to capture.

Given that autarky and free trade yield different security costs, autarky can be superior to trade. Such a welfare ranking obtains if the familiar gains from trade are outweighed by higher costs of security under trade compared to autarky. In the case of identical countries, if the world price of the consumption good produced intensively with the insecure resource is less than the price that prevails under autarky, then the introduction of free trade lowers the intensity of conflict, and thus reinforces the familiar gains from trade. However, if the world price of that same good is higher than the autarkic price, then the intensity of the conflict is higher; and, the added security costs under free trade relative to autarky can swamp the traditional gains from trade, to result in lower economic welfare. Although the added security costs under free trade increase with the world price above the autarkic price, the traditional gains from trade increase at a faster rate. As such, the possibility that autarky can Pareto dominate free trade arises only when the world price is not too far above the representative country's autarkic price. Even when the countries involved in conflict differ with respect to their secure factor endowments, there exists a range of world prices, one for each country, that renders free trade inferior to autarky from that individual country's perspective. These results suggest that, in the presence of insecurity, protectionist trade policies might be welfare-improving and thus preferable even for small countries, a prediction that does not follow from the canonical Nirvana model of trade. However, in the absence of trade policy coordination, the contesting countries might opt for free trade, thereby locking themselves in a prisoner's dilemma situation. Moreover, depending on the initial distribution of secure resources and on the world price, the countries' preferences and thus possibly their choices over trade regimes may very well diverge.<sup>7</sup>

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<sup>7</sup>That trade and security policies are somehow related may appear obvious. In fact, many embargoes, sanctions, and various other forms of trade restrictions that have been used throughout history and continue to be used today can be considered extensions of security policies (Hirschman, 1945). The British policy of

The substantive characterization results we have just outlined do not depend on specific functional forms of production or utility functions. Likewise, we provide rather general conditions for the existence and uniqueness of Nash equilibrium under autarky and under free trade. Thus, by providing a generalized treatment, we set the stage for both additional applications of the augmented canonical trade model that allows for insecure property rights and for extensions that include the important case of large countries, the latter being of great relevance to the fields of international relations and international political economy for studying the relationship between trade and security policies at the global level.

In the next section, we present the formal model and a preliminary analysis that proves useful in subsequent sections. Then, in Section 3, we investigate optimal security policies, conflict, and the balance of power, under autarky and free trade. In Section 4, we explore the implications of international conflict for trade patterns and trade volumes. In Section 5, we compare autarky and free trade in terms of their impact on arming and welfare. In Section 6, we briefly examine some additional issues, including the choice of trade regime, and the implications of commercial policies. Lastly, in Section 7, we offer several concluding comments. All technical arguments and proofs have been relegated to the Appendix.

## 2 Framework and Preliminary Analysis

Consider a global economy that consists of two countries, indexed by  $i = 1, 2$ , and the rest of the world (ROW), which for simplicity is treated as a single entity and taken as exogenous. Each country can produce two consumption goods (say “butter” and “oil”), indexed by  $j = 1, 2$ , using labor and land under conditions of constant returns to scale. In the spirit of the Heckscher-Ohlin-Samuelson (HOS) trade model, we assume the countries have access to the same production technology; consumers have identical and homothetic preferences defined over the two consumption goods; and, all markets are perfectly competitive. Each country  $i$  possesses  $L^i$  units of *secure* labor and  $K^i$  units of *secure* land. However, departing from the HOS trade model, we assume there exists additional land of  $K_0$  units. Although this additional land is divisible, its division between the two countries is subject to dispute.<sup>8</sup> Policymakers use arming to gain control of the disputed resource,  $K_0$ , with the ultimate goal of maximizing national welfare.<sup>9</sup>

Let country  $i$ 's “guns” be denoted by  $G^i$ , a variable most accurately viewed as a producible composite commodity that reflects country  $i$ 's military capability. Country  $i$ 's share

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appeasement towards Germany during the 1930s was based on classical liberal arguments about the use of economic carrots to avoid war (Kagan, 1995).

<sup>8</sup>The insecure resource could also be interpreted as a natural resource (e.g., oil, water) or simply physical capital.

<sup>9</sup>The assumption that leaders are benevolent could be relaxed. However, since our main objective here is to provide a unified framework for the analysis of arming incentives and trade openness, such an important modification will necessarily have to be addressed in future work.

of  $K_0$ , then, is determined by the contest success function (CSF):

$$\phi^i(G^i, G^j) = \begin{cases} \frac{f(G^i)}{f(G^1)+f(G^2)} & \text{if } \sum_{i=1,2} G^i > 0; \\ \frac{1}{2} & \text{if } \sum_{i=1,2} G^i = 0, \end{cases} \quad (1)$$

for  $i = 1, 2$  ( $j \neq i$ ), where  $f(\cdot) \geq 0$ ,  $f(0) = 0$ ,  $f'(\cdot) > 0$ ,  $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$ , and  $f''(\cdot) \leq 0$ .<sup>10</sup> According to (1), the fraction of the disputed resource a country secures in the contest depends on its own guns as well as those of its adversary. Specifically, it is increasing in the country's own guns ( $\phi_{G^i}^i \equiv \partial\phi^i/\partial G^i > 0$ ) and decreasing in the guns of its adversary ( $\phi_{G^j}^i \equiv \partial\phi^i/\partial G^j < 0$ ,  $j \neq i$ ).<sup>11</sup> Clearly, each country has an incentive to produce guns, whereby it can obtain a larger share of the contested land and thus more income. But, there is an opportunity cost of doing so—namely, the loss in income as resources are diverted away from the production of consumption goods. This trade-off, which is trade-regime dependent, plays a prominent role in the determination of the countries' security policies.<sup>12</sup>

The setting here is an anarchic one, so that writing enforceable (binding) contracts on the proliferation of arms and the division of  $K_0$  is not possible. Instead, we view guns as the “enforcement” variable that determines both the distribution of the contested resource and the endowments of labor and land that can be employed in useful production. Accordingly, the sequence of events is as follows:

- (i) Given the initial inter-country configuration of secure factor endowments ( $L^i$  and  $K^i$ ), the two countries ( $i = 1, 2$ ) simultaneously choose their (irreversible) production of guns  $G^i$ .
- (ii) Once these choices are made, the contested land is divided according to (1): each country  $i$  receives  $\phi^i K_0$  units of the contested resource.
- (iii) With the quantities of land and labor left for the production of consumption goods having thus been determined, private production and consumption decisions take

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<sup>10</sup>As revealed in the Appendix, the condition that  $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$  and the assumed concavity of  $f(\cdot)$  help establish existence and uniqueness of equilibrium. See Skaperdas (1996) for an axiomatization of (1), requiring only that  $f(\cdot)$  is a non-negative, increasing function. One functional form for  $f(\cdot)$  that has been widely used in the rent-seeking literature, as well as in the literatures on tournaments and conflict, is  $f(G) = G^\gamma$  where  $\gamma \in (0, 1]$  (Tullock, 1980). See Hirshleifer (1989) for a comparison of the properties of this form with those of  $f(G) = e^{\gamma G}$ .

<sup>11</sup>The influence of guns on a country's share,  $\phi^i(G^i, G^j)$ , could be taken literally or viewed as the reduced form of a bargaining process, in which relative arming figures prominently in the division of the contested resource. See Anbarci et al. (2002) for an analysis of this issue and how, in particular, different bargaining solution concepts lead to division rules that vary in their sensitivity to guns.

<sup>12</sup>While countries often build their own military constellation, they can, in practice, also buy or sell certain weapons in the world market, as well as hire mercenaries or foreign security experts. However, to highlight how pure trade in goods affects arming incentives, we abstract from such possibilities here, focusing instead on the case where guns can be produced only domestically.

place. Under autarky, prices adjust to clear domestic markets. Under free trade, the prices of consumption goods are fixed in the world market.

A conflictual equilibrium is the Nash equilibrium in guns, conditional on the prevailing trade regime.

To complete the basic model, we now specify the supply and demand sides of each economy. Starting with the supply side, let  $\psi^i \equiv \psi(w^i, r^i)$  and  $c_j^i \equiv c_j(w^i, r^i)$  represent respectively the unit cost functions of guns and goods  $j = 1, 2$  in country  $i$ , where  $w^i$  and  $r^i$  denote competitively determined factor prices—respectively, the wage paid to labor and the rental rate paid to landowners. These unit cost functions have the usual properties, including concavity and linear homogeneity in factor prices. By Shephard's lemma, the unit labor and land requirements in arms production are given respectively by  $\psi_w^i \equiv \partial\psi^i/\partial w^i > 0$  and  $\psi_r^i \equiv \partial\psi^i/\partial r^i > 0$ . Similarly,  $a_{Kj}^i \equiv \partial c_j^i/\partial r^i > 0$  and  $a_{Lj}^i \equiv \partial c_j^i/\partial w^i > 0$  represent the unit land and labor requirements in producing good  $j$ . Therefore, the land/labor ratio in guns is  $k_G^i \equiv \psi_r^i/\psi_w^i$ , and the corresponding ratio in industry  $j$  is  $k_j^i \equiv a_{Kj}^i/a_{Lj}^i$ . Industry 2 is land-intensive if  $k_2^i > k_1^i$  (or labor-intensive if  $k_2^i < k_1^i$ ) at all relevant factor prices. We follow much of the literature based on the HOS trade model in ruling out factor intensity reversals. Although we will show how the ranking of factor intensities across industries  $j = 1, 2$  matters, for specificity we emphasize throughout much of the discussion the case where good 2 is produced intensively with the insecure resource (i.e., land).

Taking good 1 as the numeraire, let  $p^i$  denote the relative price of good 2 in country  $i$ . With diversification in production, perfect competition requires

$$c_1(w^i, r^i) = a_{L1}^i w^i + a_{K1}^i r^i = 1 \quad (2)$$

$$c_2(w^i, r^i) = a_{L2}^i w^i + a_{K2}^i r^i = p^i, \quad (3)$$

for  $i = 1, 2$ . These equations, together with the assumption of identical technologies across countries and the properties of unit cost functions, imply that the wage/rental ratio,  $\omega^i \equiv w^i/r^i$ , can be written as a function of  $p^i$  (i.e.,  $\omega^i = \omega(p^i)$ ). By the Stolper-Samuelson (1941) theorem,  $\omega_p^i (\equiv \partial\omega/\partial p^i) \leq 0$  as  $k_2^i \geq k_1^i$  (see Lemma A1 in the Appendix).

Let  $X_j^i$  denote the output of good  $j$  and  $(K_X^i, L_X^i)$  denote the vector of residual quantities of resources left for the production of consumption goods in country  $i$  once labor and land resources for guns  $G^i$ , respectively  $\psi_w^i G^i$  and  $\psi_r^i G^i$ , have already been set aside and the distribution of the contested resource has been realized,  $\phi^i K_0$ . Factor market clearing and diversification in production (i.e.,  $X_j^i > 0$  for each good  $j = 1, 2$ ) require in each country  $i$ ,

$$a_{K1}^i X_1^i + a_{K2}^i X_2^i = K_X^i \quad (\equiv K^i + \phi^i K_0 - \psi_r^i G^i) \quad (4)$$

$$a_{L1}^i X_1^i + a_{L2}^i X_2^i = L_X^i \quad (\equiv L^i - \psi_w^i G^i). \quad (5)$$

Now, let  $k_X^i$  denote country  $i$ 's residual land/labor ratio:

$$k_X^i \equiv \frac{K_X^i}{L_X^i} = \frac{K^i + \phi^i K_0 - \psi_r^i G^i}{L^i - \psi_w^i G^i}, \quad \text{for } i = 1, 2. \quad (6)$$

Then, it is straightforward to verify, from (4) and (5) along with the linear homogeneity of unit cost functions and the fact that  $\omega^i = \omega(p^i)$ , that the relative supply of good 2 (oil),  $RS^i \equiv X_2^i/X_1^i$ , can be written as  $RS^i = RS(p^i, k_X^i)$ . In the Appendix (Lemma A1) we show (i)  $\partial RS^i/\partial p^i > 0$  due to increasing opportunity costs, and (ii)  $\partial RS^i/\partial k_X^i \geq 0$  as  $k_2^i \geq k_1^i$  by the Rybczynski (1955) theorem. Further, as can be seen from (6), when both goods are produced, the residual land/labor ratio can be written as a function of the relative price of good 2, the guns produced by the two countries, and resource endowments. To avoid cluttering of notation, we write this function as  $k_X^i = k_X^i(p^i, G^i, G^j)$ . Lemma A2 in the Appendix describes the dependence of  $k_X^i$  on its arguments. The important point to recognize at this stage is that the relative supply of good 2 can also be written as a function of the price and guns:  $RS^i = RS(p^i, G^i, G^j)$ . The exact nature of this relationship, which we characterize in the next section, is needed in the identification of market-clearing prices and in the analysis of conflict under autarky.

Turning to the demand side of each economy, we suppose that consumers' preferences, defined over the consumption of goods 1 and 2, are homothetic. Letting  $R^i$  and  $\mu^i \equiv \mu(p^i)$  denote respectively net national income and the marginal utility of income, country  $i$ 's indirect utility (aggregate welfare) function can be written as<sup>13</sup>

$$V^i \equiv V^i(p^i, G^i, G^j) = \mu(p^i)R^i(p^i, K_X^i, L_X^i), \quad \text{for } i = 1, 2 \ (j \neq i). \quad (7)$$

Equation (7) implicitly assumes that policymakers finance the cost of arming with nondistortionary income taxes.<sup>14</sup> This assumption together with that of perfect competition imply that country  $i$ 's net national income ( $R^i$ ) is the country's maximized value of domestic production of consumption goods and, at the same time, the minimized value of rental payments paid to residual labor and land owners. From equations (2)–(5), we have  $R^i = X_1^i + p^i X_2^i = r^i K_X^i + w^i L_X^i$  for  $i = 1, 2$ , which explains the arguments of  $R^i$  and  $V^i$  in (7).<sup>15</sup> As one can verify,  $R^i$  is increasing and convex in  $p^i$ , and increasing and concave in the residual factor inputs ( $K_X^i, L_X^i$ ). Furthermore, the supply of good 2 satisfies  $X_2^i = R_p^i$

<sup>13</sup>In this expression, we suppress the obvious dependence of  $V^i$  on resource endowments to avoid cluttering.

<sup>14</sup>We are also assuming that governments can adjust taxes to meet their security obligations in the absence of public resistance. In practice, fiscal objections to expenditures on security—for, say, economic, political or ideological reasons—can limit the government's budgetary flexibility. Moreover, we do not consider looting as a possible source of finance.

<sup>15</sup> $R^i$  should be identified with the familiar gross domestic product (GDP) or revenue function (Dixit and Norman, 1980), excluding arms expenditures.

( $\equiv \partial R^i / \partial p^i$ ) and  $\partial X_2^i / \partial p^i = R_{pp}^i \geq 0$ , while factor prices satisfy  $w^i = R_L^i$  ( $\equiv \partial R^i / \partial L_X^i$ ) and  $r^i = R_K^i$  ( $\equiv \partial R^i / \partial K_X^i$ ).

Using Roy's identity, country  $i$ 's demand function for good 2 can be written as a share of the country's net national income:  $D_2^i = \alpha_D^i R^i / p^i$ , where the associated expenditure share is given by  $\alpha_D^i \equiv \alpha_D(p^i) = -p^i \mu_p^i / \mu^i (> 0)$ .<sup>16</sup> Now, since the supply of good 2 is given by  $X_2^i = R_p^i$ , the excess demand for (or net imports of) good 2 is given by  $M^i \equiv D_2^i - X_2^i$ .<sup>17</sup> Then, holding fixed the secure resource endowments ( $K^i$  and  $L^i$ ) as well as the disputed resource ( $K_0$ ), total differentiation of (7) yields

$$dV^i = \mu(p^i) [-M^i dp^i + (r^i K_0 \phi_{G^i}^i - \psi^i) dG^i + r^i K_0 \phi_{G^j}^i dG^j] \quad \text{for } i = 1, 2 \ (j \neq i). \quad (8)$$

The first term inside the square brackets, weighted by the marginal utility of income  $\mu(p^i)$ , is a terms of trade effect. For net importers of good 2, an increase in  $p^i$  increases the domestic cost of good 2, and is thus welfare-reducing. By contrast, such an increase in the relative price is welfare-improving for net exporters of good 2.

The second term in the brackets (weighted by  $\mu(p^i)$ ) captures the welfare effect of a change in country  $i$ 's guns,  $G^i$ . Ceteris paribus, an increase in  $G^i$  increases country  $i$ 's share of the contested land and thus its national income (the first term in the parentheses). At the same time, however, the increase in  $G^i$  draws additional resources away from the production of consumption goods and thus reduces national income (the second term inside the parentheses).

The third term in the brackets (again weighted by  $\mu(p^i)$ ) captures the welfare effect of a change in arms by country  $i$ 's opponent,  $G^j$ . An increase in  $G^j$  reduces country  $i$ 's share of the contested resource and thus its income, and thereby adversely affects that country. Note that, for fixed product prices, an equi-proportionate expansion of both countries' guns, where  $G^1 = G^2$  initially, implies no change in the division of the contested land, while increasing the resource cost of guns, and thus necessarily leaves both countries worse off.

We now demonstrate how the above ideas inform the derivation of the optimizing security policies (arming) under alternative trade regimes. A key feature of the optimization problem facing each country  $i$  is that, with diversification in production, the corresponding first-order condition (FOC), given by

$$V_{G^i}^i(p^i, G^i, G^j) = \mu(p^i) r^i [K_0 \phi_{G^i}^i - \psi^i / r^i] = 0 \quad \text{for } i = 1, 2 \ (j \neq i), \quad (9)$$

<sup>16</sup>It can be shown that this share is decreasing or increasing in  $p^i$ , depending on whether the elasticity of substitution in consumption is larger or smaller than unity, respectively.

<sup>17</sup>We omit subscript "2" from " $M^i$ " to avoid cluttering of notation.

is independent of the trade regime being considered.<sup>18</sup> Under autarky, domestic product market clearing requires  $p^i$  to adjust so that  $M^i = 0$ , which implies that the first term inside the brackets in (8) vanishes, thereby yielding (9) as the relevant FOC. Under free trade in consumption goods, product prices are invariant to security policies for “small” countries; thus, the first term also vanishes under this trade regime.

Equation (9) shows that country  $i$ 's net marginal payoff from arming consists of two key components: (i) the marginal benefit of producing guns, which is given by  $MB^i \equiv K_0 \phi_{G^i}^i$  when measured in land units; and (ii) the marginal cost of producing guns, which is given by  $MC^i \equiv \psi^i / r^i$  (again measured in land units). Note that  $MB^i$  is independent of  $p^i$  and, as such, of the trade regime considered. By contrast, as shown below,  $MC^i$  is trade-regime dependent. Lemma 1 and the discussion to follow elaborate on these ideas further.

**Lemma 1.** *Each country  $i$ 's indirect utility function,  $V^i$ , has the following properties:*

- (a)  $V_{G^i G^i}^i < 0$ ;
- (b)  $V_{G^i G^j}^i \gtrless 0$  if  $G^i \gtrless G^j$ ,  $j \neq i$ ;
- (c)  $V_{G^i p^i}^i \gtrless 0$  if  $k_2^i \gtrless k_1^i$  when evaluated at the value of  $G^i$  that solves  $V_{G^i}^i = 0$ ;
- (d)  $V^i$  is strictly quasi-convex in  $p^i$ , and is minimized at the value of  $p^i$  that solves  $M^i = 0$ .

As shown in the Appendix, parts (a) and (b) follow from the properties of the CSF in (1). Part (a) establishes the strict concavity of  $V^i$  in country  $i$ 's guns ( $G^i$ ) given the guns chosen by its rival ( $G^j$ ), and thus explains the downward sloping shape of  $MB^i$  depicted in Fig. 1.<sup>19</sup> According to part (b), each country's net marginal payoff to arming rises or falls with its rival's arms (i.e.,  $MB^i$  in Fig. 1 shifts up or down) depending on which contender initially produces more guns. Part (c) states that, at an optimum, the influence of changes in the relative price ( $p^i$ ) on the net marginal payoff from country  $i$ 's arming depends on the ranking of the factor intensities in industries 1 and 2. Since the marginal benefit of guns ( $MB^i$ ) is independent of  $p^i$ , this influence is driven by the impact of a change in  $p^i$  on the country's marginal cost ( $MC^i$ ). To see this effect, first observe that the linear homogeneity of the unit cost function for guns ( $\psi^i$ ) and the fact that  $\psi_w^i > 0$  imply that country  $i$ 's marginal cost of arming can be written as  $MC^i = \psi^i / r^i = \psi(\omega^i, 1)$  and is increasing in the country's wage/rental ratio ( $\omega^i$ ). By the Stolper-Samuelson theorem (see also Lemma A1(a)

<sup>18</sup>Our discussion here and to follow is based on the assumption that, under autarky, the distribution of factor endowments between the adversaries is such that their production of arms is not constrained by their secure land holdings. Under free trade, we assume further that technology, the distribution of factor endowments, the quantity of the contested resource and the world price are such that production of consumption goods is diversified.

<sup>19</sup>Note that, from the definition of  $MB^i$  and the properties of the CSF,  $MB^i$  depends on the amount of land being disputed,  $K_0$ , as well as on the guns produced by both countries,  $G^i$  and  $G^j$ ; furthermore, the condition that  $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$  implies  $\lim_{G^i \rightarrow 0} MB^i = \infty$ .

in the Appendix), an increase in  $p^i$  decreases (increases)  $\omega^i$  and thus decreases (increases) the country's marginal cost of arming if  $k_2^i > k_1^i$  ( $k_2^i < k_1^i$ ).

Under free trade when production is diversified, the world price alone determines the wage/ rental ratio ( $\omega^i$ ). This ratio is thus invariant to changes in  $G^i$ . It follows that  $MC^i$  is a constant function under this regime, as indicated by the red dotted line in Fig. 1. Under autarky, by contrast, product (and thus factor) prices are endogenous. In the next section, we show that  $MC^i$  is generally increasing in  $G^i$  under this regime, as depicted by the blue dashed-line curve in Fig. 1.

Part (d) is a well-known property of indirect utility functions (Dixit and Norman, 1980), highlighting the important idea that, for given guns, a country's welfare is higher, the greater is the deviation of product prices from their autarkic levels.

Using an asterisk (\*) to indicate optimizing policies, the next lemma shows how, given diversification in production of consumption goods, technology in the consumption goods sectors and product price differences across the contending countries are related to the countries' optimizing guns choices.

**Lemma 2.** *Optimizing security policies by contending states satisfy the following:*

$$(k_2^i - k_1^i) (p^1 - p^2) \begin{matrix} \geq \\ \leq \end{matrix} 0 \implies G^{1*} \begin{matrix} \geq \\ \leq \end{matrix} G^{2*}.$$

According to Lemma 2, the country with the larger relative price of the insecure-resource-intensive good (i.e., good 2 if  $k_2^i > k_1^i$  or good 1 if  $k_1^i > k_2^i$ ) produces more guns. This finding proves useful in our characterization of conflictual equilibria and comparative advantage.<sup>20</sup>

### 3 Trade Regimes and Conflict

In this section, we explore the implications of the trade regime—autarky and free trade—for arming, the distribution of power, and welfare. We now use an asterisk (\*) to indicate Nash equilibrium values, differentiating between trade regimes with subscripts “A” for *autarky* and “F” for *free trade*. We first consider autarky and then free trade.

#### 3.1 Conflict under autarky

The first-order conditions in (9) reveal that the optimizing security policies ( $G^{i*}$ ,  $i = 1, 2$ ) depend on the product prices prevailing in the respective country,  $p^i$ . Thus, to close the model we need two additional conditions that determine the autarkic prices,  $p_A^i$  for  $i = 1, 2$ .

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<sup>20</sup>It is worth noting that, when product prices are endogenous (as in the case of autarky), the “if” part in Lemma 2 becomes “if and only if.”

These conditions require domestic markets to clear:  $M^i = 0$  or equivalently,

$$RD(p^i) = RS(p^i, k_X^i(p^i, G^i, G^j)) \quad \text{for } i = 1, 2, \quad (10)$$

where  $RD(p^i)$  denotes the relative demand for good 2. While the demand for good 2, as noted above, is given by  $D_2^i = \alpha_D^i R^i / p^i$ , the demand for good 1 is  $D_1^i = (1 - \alpha_D^i) R^i$ ; as such, the relative demand for good 2 is  $RD(p^i) \equiv D_2^i / D_1^i = \frac{1}{p^i} \frac{\alpha_D(p^i)}{1 - \alpha_D(p^i)}$ . Thus, homotheticity of consumer preferences implies  $RD(p^i)$  is uniquely determined by and decreasing in the relative price of good 2,  $p^i$ . In addition, as noted above, the relative supply of good 2 ( $RS^i$ ), shown in the right-hand side (RHS) of (10), is increasing in  $p^i$ .

We can now establish the following, which shows how the equilibrium price is influenced by exogenous changes in guns and resource endowments:

**Lemma 3.** *Under autarky, country  $i$ 's market clearing price of the non-numeraire good,  $p_A^i$ , and its residual land/labor ratio,  $k_X^i$ , are related as follows:*

$$\frac{\partial p_A^i}{\partial k_X^i} \leq 0 \quad \text{if } k_2^i \geq k_1^i.$$

We thus have for each country  $i = 1, 2$

- (a)  $\frac{\partial p_A^i}{\partial G^i} \leq 0$  if  $k_2^i \geq k_1^i$ ,  $\forall G^i$  that satisfy  $V_{G^i}^i + \varepsilon > 0$ , for some  $\varepsilon > 0$ ;
- (b)  $\frac{\partial p_A^i}{\partial G^j} \geq 0$  if  $k_2^i \geq k_1^i$  ( $j \neq i$ );
- (c)  $(k_2^i - k_1^i) \left( \frac{\partial p_A^i}{\partial G^i} + \frac{\partial p_A^i}{\partial G^j} \right) \leq 0$  for  $G^i = G^j$  ( $j \neq i$ ) if  $k_G^i \leq k^i \equiv \frac{K^i + \phi^i K_0}{L^i}$ ;
- (d)  $\frac{\partial p_A^i}{\partial L^i} \geq 0$ ,  $\frac{\partial p_A^i}{\partial K_0} \leq 0$ , and  $\frac{\partial p_A^i}{\partial K^i} \leq 0$  if  $k_2^i \geq k_1^i$ .

By the Rybczynski theorem, an increase in a country's residual land/labor ratio ( $k_X^i$ ) expands the relative supply of its land-intensive good (also see Lemma A1(b) in the Appendix), and thereby creates an excess supply that forces its autarkic price to fall. Parts (a)–(d) follow immediately from this relationship and the dependence of  $k_X^i$  on guns and resource endowments, as mentioned earlier and described in Lemma A2 of the Appendix.

Part (a) implies that a country's marginal cost of producing guns under autarky,  $MC_A^i = \psi(\omega(p_A^i), 1)$ , is increasing in the country's guns regardless of the ranking of factor intensities in the consumption goods industries. The logic here is as follows. In the neighborhood of the optimum implicitly defined by (9), an increase in  $G^i$  raises country  $i$ 's residual land/labor ratio,  $k_X^i$  (see Lemma A2(b) in the Appendix). Now, suppose  $k_2^i > k_1^i$ . Then, as shown in Lemma A1(b), this increase translates into an increase in the relative supply of good 2 ( $RS^i$ ), causing the autarkic price of good 2 ( $p_A^i$ ) to fall. Alternatively, if  $k_2^i < k_1^i$ , the increase in  $k_X^i$  causes  $p_A^i$  to rise. In both cases, by the Stolper-Samuelson theorem (also see Lemma A1(a)), these price adjustments, in turn, force the wage/rental ratio ( $\omega^i$ ) to rise and thus induce

$MC_A^i$  to rise with  $G^i$ . Similar reasoning establishes that  $MC_A^i$  increases with increases in the country's secure endowment of land ( $K^i$ ) and in the amount of disputed land ( $K_0$ ) and falls with increases in the country's secure endowment of labor ( $L^i$ ) and in the other country's guns ( $G^j$ ,  $j \neq i$ ). Furthermore, part (c) indicates that  $MC_A^i$  increases (decreases) with equi-proportionate increases of both countries' guns, where  $G^i = G^j$  initially, provided that the land/labor intensity ratio in the production of guns ( $k_G^i$ ) is less (greater) than the country's ratio of *ex post* secure land/labor holdings ( $k^i$ ). Obviously, the intersection of  $MC_A^i$  with  $MB^i$  at point  $A$  in Fig. 1 gives country  $i$ 's best-response function,  $B_A^i(G^j)$ .

The influence of technology, the degree of resource insecurity (institutions), and the size of factor endowments on autarkic prices and thus on the marginal cost of arming can have important implications for optimal security policies. To flesh out these implications, we first establish that a pure-strategy equilibrium in security policies exists. Since the dependence of conflictual equilibria on trade regimes and their comparison are central to our analysis, we also identify (sufficient) conditions for uniqueness of equilibrium, and then proceed to characterize it.

**Theorem 1.** (*Autarky*) *An interior Nash equilibrium in pure strategies (security policies) exists. Furthermore, the equilibrium is unique if the technology for arms is sufficiently labor-intensive or the inputs to arms are not very close complements.*

As indicated by the proof of Theorem 1, uniqueness of equilibrium can be assured under fairly general circumstances.<sup>21</sup> The proof also clarifies how the endogeneity of prices matters for the shapes of best-response functions,  $B_A^i(G^j)$  ( $i = 1, 2$ ,  $j \neq i$ ), which are illustrated in Fig. 2 by the solid-line curves.<sup>22</sup> As shown in figure, the best-response function of each country,  $B_A^i(G^j)$ , is positively related to the adversary's arming choice (reflecting strategic complementarity) up to and beyond its point of intersection with the 45° line; however, at some point beyond that intersection, the function becomes negatively related to  $G^j$ , (reflecting strategic substitutability).<sup>23</sup> The conflictual (Nash) equilibrium under

<sup>21</sup>The proof is based on the assumption that not all secure land supplies are absorbed into the production of guns, but can be amended to allow this possibility. One sufficient condition that precludes this complication from arising is that the degree of land insecurity (i.e., the fraction of contested land) is not too high. Another possible condition is that guns are produced with labor only ( $k_G^i = 0$ ). Note that, in any case, provided that both factors are essential to the production of consumption goods, labor will never be fully absorbed in the production of guns in the autarkic regime.

<sup>22</sup>Ignore the other curves drawn in the figure for now.

<sup>23</sup>The shape of a country's best-response function depends, in part, on how the marginal benefit of arming ( $MB^i$ ) is influenced the country rival's security policies (see Lemma 1(b)). However, where  $G^1 = G^2$  initially, a small increase in  $G^j$  has no effect on this marginal benefit (see (A.4) in the Appendix). Thus, the strategic complementarity of security policies along the 45° line is due instead to the indirect influence of the adversary's guns ( $G^j$ ,  $j \neq i$ ) on the marginal cost of arming ( $MC_A^i$ ), through the autarkic price ( $p_A^i$ ) as noted in Lemma 3(b). In particular, regardless of which consumption good is land-intensive, an increase in  $G^j$  ( $j \neq i$ ), through its impact on  $p_A^i$  and thus on  $\omega_A^i$ , reduces country  $i$ 's marginal cost of arming at each  $G^i$  and thus induces country  $i$  to respond more aggressively by producing more guns.

autarky is depicted by the intersection of best-response functions.<sup>24</sup> One such equilibrium—a symmetric one—is point  $A$ , where  $B_A^1$  and  $B_A^2$  intersect along the  $45^\circ$  line. Point  $A'$  (where the red dotted-line best-response functions intersect) depicts an asymmetric equilibrium.

Several important questions naturally arise. What determines the relative arming incentives of the two contenders and thus military superiority (power) under autarky? Clearly, the distribution of secure factor endowments between the two adversaries matters. But, how? Is economic might equivalent to military might? Or, in the words of Jack Hirshleifer (1991, p. 177), “Can initially weaker or poorer contenders end up gaining on initially stronger or wealthier opponents?”

We begin to address these questions, among others, with the help of Fig. 3, where for specificity we assume  $k_2^i > k_1^i > k_G^i$ . By construction, the sides of the outer rectangle depict the aggregate supplies of land (including  $K_0$ ) and labor that are available to the contenders. The sides of the inner rectangle  $A^1B^1A^2B^2$  identify the aggregate quantities of secure land and labor. Points in this inner rectangle give the distribution of secure resources across countries, with point  $A^i$  being country  $i$ 's origin. The vertical distance  $O^iA^i$  ( $i = 1, 2$ ), from the horizontal sides of the outer rectangle to those of  $A^1B^1A^2B^2$ , shows the quantity of  $K_0$  country  $i$  secures in the contest,  $\phi^i K_0$ .

Consider first the benchmark case, where countries 1 and 2 have identical secure endowments, as indicated by the midpoint of the diagonal  $A^1A^2$  ( $D$ ). To differentiate the resulting equilibrium values from others, we place a tilde ( $\sim$ ) over the associated variables. Since the countries are identical, they face identical arming incentives; therefore,  $G_A^{i*} = \tilde{G}_A^*$ , with each country thus receiving one half of the contested resource,  $K_0$ . Now thinking of point  $O^i$  as country  $i$ 's origin of overall endowments, we have  $O^1A^1 = O^2A^2 = \frac{1}{2}K_0$ , with vectors  $O^iC^i$  and  $C^iD$  depicting the resources country  $i$  devotes to the production of arms and consumption goods, respectively.<sup>25</sup> By symmetry,  $O^1C^1 = O^2C^2$  and  $C^1D = C^2D$ . Furthermore, by Lemma 2,  $p_A^{i*} = \tilde{p}_A^*$  and thus  $\omega_A^{i*} = \tilde{\omega}_A^*$  and  $k_X^{i*} = \tilde{k}_X^*$  for each  $i$ . The sides of parallelogram  $C^1J^1C^2J^2$  capture the aggregate sectoral distribution of resources in the production of good 1 (sides  $C^1J^1 = C^2J^2$ ) and the production of good 2 (sides  $C^1J^2 = C^2J^1$ ).<sup>26</sup>

One might naturally ask if there are other distributions of secure endowments that similarly yield a symmetric equilibrium in guns (i.e.,  $G^{1*} = G^{2*}$ ) and thus imply the possible emergence of the “paradox of power” under autarky. To proceed, we define the following sets in the context of Fig. 3:  $\mathcal{S}^0 \equiv F^1F^2$ ,  $\mathcal{S}^1 \equiv F^1F^2A^2B^2$  (excluding  $\mathcal{S}^0$ ), and  $\mathcal{S}^2 \equiv F^2F^1A^1B^1$

<sup>24</sup>Note that, because each country always has an incentive to produce a small (but positive) quantity of arms when its rival produces none,  $(0, 0)$  is not a Nash equilibrium.

<sup>25</sup>One can confirm, from the FOCs shown in (9), that the quantity of land employed in the production of arms in country  $i$ ,  $K_G^i \equiv \psi_r^i G^i$ , does not exceed  $\frac{1}{4}K_0$ .

<sup>26</sup>Consistent with the ranking of factor intensities assumed for the graph (i.e.,  $k_2^i > k_1^i > k_G^i$ ), the slope of  $C^1J^1$  ( $k_2^i$ ) is greater than that of  $C^1J^2$  ( $k_1^i$ ), which is greater than the slope of  $O^iC^i$  ( $k_G^i$ ).

(again excluding  $\mathcal{S}^0$ ).<sup>27</sup> We can now establish the following:

**Proposition 1.** (*Arming and Autarkic Prices*) *Under autarky, there exists a non-empty set of asymmetric factor distributions under which contending states face identical market-clearing prices, produce identical quantities of arms, and are equally powerful. All other asymmetric distributions generate different prices and unequal arming and power. Specifically, for each country  $i = 1, 2$  ( $j \neq i$ ),*

- (a)  $G_A^{i*} = \tilde{G}_A^*$  and  $p_A^{i*} = \tilde{p}_A^*$  for factor distributions in  $\mathcal{S}^0$ ;
- (b)  $G_A^{i*} > G_A^{j*}$  and  $p_A^{i*} \geq p_A^{j*}$  as  $k_2^i \geq k_1^i$  for factor distributions in  $\mathcal{S}^i$ .

Proposition 1 indicates that the symmetric equilibrium  $(\tilde{G}_A^*, \tilde{p}_A^*)$  arises not only when the contending sides are identical, but also when they differ with respect to their secure resource endowments. In particular, part (a) indicates that the symmetric equilibrium arises for all secure factor distributions in  $\mathcal{S}^0$ , shown as a thick blue line (set) going through point  $D$  in Fig. 3. The key property of these distributions is that they all imply the same residual land/labor ratio:  $k_X^1 = k_X^2 = \tilde{k}_X^*$ . To see this, consider the following experiment. Starting at the symmetric point  $D$ , transfer both land and labor resources from country 2 to country 1 so that we move along  $\mathcal{S}^0$  in the direction of point  $E$ . For constant guns and prices, such a redistribution of resources leaves the value of the residual land/labor ratios unchanged at  $\tilde{k}_X^*$ .<sup>28</sup> Thus, the countries' relative supply and relative demand functions do not shift; and, there is no pressure for autarkic prices to change. But, given prices do, in fact, remain fixed at  $\tilde{p}_A^*$  for all secure resource distributions in  $\mathcal{S}^0$ , arming incentives remain unchanged in both countries, and the parallelogram  $C^1J^1C^2J^2$  will continue to capture the overall allocation of resources into goods 1 and 2.

Part (b) shows the conditions under which a country arms more heavily than its adversary, and identifies the implications for the relative ranking of autarkic prices in two countries. For some intuition, consider an initial secure resource distribution in  $\mathcal{S}^0$ , say point  $E$  in Fig. 3, which assumes  $k_2^i > k_1^i$ . By Proposition 1(a), the conflictual equilibrium is initially on the 45° line of Fig. 2, at point  $A$ , where  $B_A^1$  and  $B_A^2$  intersect. Now arbitrarily transfer labor from country 2 to country 1, such as that implied by the move from point  $E$  to point  $H \in \mathcal{S}^1$  in Fig. 3. According to Lemma 3(d) with  $k_2^i > k_1^i$ , this transfer of labor raises country 1's autarkic price, which in turn decreases its marginal cost of arming (Lemma 1(c)); at the same, the loss of labor for country 2 reduces its autarkic price, and thus increases its marginal cost of arming. As a consequence, country 1 (2) will behave more (less) aggressively, as shown in Fig. 2 by the clockwise rotation of  $B_A^1$  to  $B_A^{1'}$  and that

<sup>27</sup>For endowment distributions in the shaded regions of Fig. 3, the initial secure land holdings are binding in the production of guns. The analysis could be amended to consider these distributions. But, for simplicity and clarity, we focus on secure factor distributions in  $\{\mathcal{S}^0 \cup \mathcal{S}^1 \cup \mathcal{S}^2\}$ , and assume that that the degree of insecurity is not too high, such that  $\mathcal{S}^0$  contains the symmetric distribution,  $D$ , as well as asymmetric ones.

<sup>28</sup>Note that along  $\mathcal{S}^0$  in Fig. 3,  $\frac{dk_X^i}{k_X^i} \Big|_{dG^i=dp^i=0} = \frac{dK_X^i}{K_X^i} - \frac{dL_X^i}{L_X^i} = 0$ , which requires  $\frac{dK_X^i}{dL_X^i} = \frac{K_X^i}{L_X^i} = \tilde{k}_X^*$ .

of  $B_A^2$  to  $B_A^{2'}$ . The properties of the best-response functions described earlier (and in the proof to Theorem 1) ensure that the new equilibrium point lies below the 45° line, such as point  $A'$  in the figure, where clearly country 1 arms more heavily than its adversary. By Lemma 2, under the maintained assumption that  $k_2^i > k_1^i$ , this divergence in the two countries' guns production,  $G^{1*} > G^{2*}$ , is consistent with the divergence in their autarkic prices described above,  $p_A^{1*} > p_A^{2*}$ .<sup>29</sup>

Proposition 1 suggests generally that the relationship between factor abundance, arming, and military superiority in the autarkic regime is complex and multi-faceted. Differences in secure resource endowments across countries need translate into differences in their arming and thus unequal power. Consistent with the “paradox of power” (Hirschleifer, 1991), the initially resource disadvantaged side could obtain an equal share of the contested pie, as illustrated in the case of unequal distributions of secure land and labor endowments in  $\mathcal{S}^0$ . However, the possible emergence of the paradox of power is not limited to such distributions. Consider, for example, point  $Q \in \mathcal{S}^1$  in Fig. 3. Although country 1's secure factor endowments (and income) are smaller than its opponent's at  $Q$ , country 1 arms more heavily and commands a bigger share of the contested resource than its rival. It might be tempting to conclude that country 1 arms more heavily in this case because it is relatively less well endowed in the contested resource (land):  $K^1/L^1 < K^2/L^2$  at point  $Q$ . However, this reasoning would be incorrect, as illustrated by point  $H \in \mathcal{S}^1$ , where country 1 again produces more guns, but is relatively more abundant in land (i.e.,  $K^1/L^1 > K^2/L^2$ ). What part (b) (with Fig. 3) makes clear is that, for any given distribution of the secure land between the contending states, the more powerful country is that which has a sufficiently large (though not necessarily larger) endowment of the uncontested resource, labor.

In addition, Proposition 1 implies that, in the presence of insecure property, the relationship between a country's autarkic price and secure land/labor endowment ratio is not necessarily monotonic. Specifically, in contrast to the neoclassical world where all property is secure, the country with the largest *ex ante* or *ex post* secure land/labor ratio (respectively,  $K^i/L^i$  and  $k^i \equiv (K^i + \phi^i K_0)/L^i$ ) need not enjoy the lowest relative price of the good that employs land intensively. Compare, for example, different distributions in  $\mathcal{S}^0$  where the countries' secure land/labor endowment ratios differ, but  $p_A^{i*} = \tilde{p}_A^*$  at the same time.<sup>30</sup> What matters for both military superiority and the ranking of the two countries' autarkic prices is how the countries' residual factor endowments (i.e.,  $k_X^{1*}$  and  $k_X^{2*}$ ), which are them-

<sup>29</sup>To be sure, given the changes in autarkic prices and guns induced by a hypothetical change in the distribution of secure resources from  $\mathcal{S}^0$  to  $\mathcal{S}^i$ , Fig. 3's depictions of (i) the distribution the insecure land ( $K_0$ ) and (ii) the aggregate allocation of factors to the production of consumption goods and guns (parallelogram  $C^1J^1C^2J^2$ ) no longer apply.

<sup>30</sup>This point can also be seen by comparing different endowment configurations along  $A^1A^2$  where both countries have identical *ex ante* secure land/labor ratios. For these points, market clearing prices would be identical across countries if all property were secure but they are not when some property is insecure.

selves endogenous in the presence of conflict, compare with  $\tilde{k}_X^*$ .<sup>31</sup> For distributions in  $\mathcal{S}^i$ , we have  $k_X^{i*} < \tilde{k}_X^* < k_X^{j*}$ , regardless of the ranking of factor intensities across industries.

By now, it should be clear that the international distribution of secure resources has important implications for arming and power. The next proposition, which outlines the implications of secure endowment reassignments, proves useful in our upcoming comparison of trade regimes.

**Proposition 2.** (*Transfers of Secure Resources*) *For initial factor distributions in  $\mathcal{S}^0$ , a small transfer of a secure resource from country  $j$  to its adversary  $i$  ( $\neq j$ ) has the following implications for arming and welfare:*

- (a)  $\frac{dG_A^{i*}}{dL^i} = -\frac{dG_A^{j*}}{dL^i} > 0$  but  $\frac{dG_A^{i*}}{dK^i} = -\frac{dG_A^{j*}}{dK^i} < 0$ ;
- (b)  $\frac{dV_A^{i*}}{dL^i} = -\frac{dV_A^{j*}}{dL^i} > \mu(\tilde{p}_A^*)w(\tilde{p}_A^*)$  but  $\frac{dV_A^{i*}}{dK^i} = -\frac{dV_A^{j*}}{dK^i} < \mu(\tilde{p}_A^*)r(\tilde{p}_A^*)$ .

Part (a) shows that the effects of a transfer of secure resources on equilibrium arming depend on the type of resource transfer considered.<sup>32</sup> Interestingly, while the country with the increased (reduced) labor endowment produces more (less) guns in the new equilibrium, exactly the opposite is true for redistributions of secure land. The driving force behind these effects is the impact of endowment changes on autarkic prices, which influence factor prices and ultimately the contestants' marginal costs of producing guns. Thus, under autarky, the composition of secure assets in the contending states has important implications for relative arming and the emergence of the paradox of power we referred to earlier.<sup>33</sup>

To understand part (b), we extend the decomposition of welfare effects in (8) to include the effect of changes in the countries' secure holdings of resources. Focusing on labor redistributions, invoking the envelope theorem and using the fact that  $M^i = 0$  under autarky yield

$$\frac{dV_A^{i*}}{dL^i} = \mu(p_A^{i*}) \left[ w(p_A^{i*}) + r(p_A^{i*})K_0\phi_{G^j}^i \frac{dG_A^{j*}}{dL^i} \right] \text{ for } i = 1, 2 \ (j \neq i), \quad (11)$$

<sup>31</sup>The endogeneity of  $k_X^{i*}$  is also important for predicting trade patterns, as shown below.

<sup>32</sup>Note that while Proposition 1 provides a global sort of result in identifying the shifts in the best-response functions for small changes in secure endowments and the implications for how  $G^1$  and  $G^2$  compare, this proposition provides local results for small changes in secure endowments in the neighborhood of  $\mathcal{S}^0$ . As shown in the proof (in the Appendix) this comparative static exercise necessarily factors in the strategic effects of such changes.

<sup>33</sup>In part (a), the offsetting changes in the contestants' guns imply that the resource redistributions considered leave the aggregate production of guns unchanged. It can be shown that, if the initial resource distribution is in  $\mathcal{S}^i$ , redistributions of the type considered in part (a) typically generate a reduction in the aggregate production of guns. Even more interestingly, when the initial distribution is sufficiently unequal across countries, resource redistributions may induce both to produce less guns. Thus, the extent to which secure endowments differ across countries has important implications for the intensity of conflict between them.

where  $p_A^{i*} = \tilde{p}_A^*$  for initial distributions in  $\mathcal{S}^0$ . As shown in (11), a transfer of labor generates two effects. The first term in the brackets (weighted by the marginal utility of income) represents a wage-income adjustment to the change in the country's labor endowment, which is positive (negative) for the recipient (donor) country. The second term (also weighted by the marginal utility of income) represents a strategic effect arising from the rival's response in arming. By Proposition, (2a), this effect is similarly positive (negative) for the recipient (donor) country. Not surprisingly, then, the overall welfare effect of a transfer of secure labor resources is positive for the recipient country and negative for the donor country. Analogous reasoning can be used to sort out the welfare effects of land redistributions. In contrast to the case of labor redistributions, the welfare effects of land redistributions are smaller in magnitude than the factor price related changes in national income.<sup>34</sup>

Next, we consider, for the case of where arms are initially equalized across countries, the effects of an increase in the degree of land insecurity, in which  $K_0$  increases and each country's secure resources  $K^i$  decrease at the same time, so as to leave the aggregate quantity of land unchanged:  $dK^i = -\phi^i dK_0$  for  $i = 1, 2$ :

**Proposition 3.** (*Degree of Insecurity under Autarky*) *For initial factor distributions in  $\mathcal{S}^0$ , an increase in the degree of insecurity has the following effects on arming and welfare:*

- (a)  $\frac{dG_A^{i*}}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} = \frac{dG_A^{j*}}{dK_0} \Big|_{dK^j = -\frac{1}{2}dK_0} > 0$ ;
- (b)  $\frac{dV_A^{i*}}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} = \frac{dV_A^{j*}}{dK_0} \Big|_{dK^j = -\frac{1}{2}dK_0} < 0$ .

As shown in the Appendix, increasing the degree of insecurity increases each contenders' marginal benefit of arming ( $MB^i$ ). Yet, since  $dK^i = -\frac{1}{2}dK_0$  for  $i = 1, 2$ , the change in the composition of land endowments has no direct influence on the autarkic price in the two countries, and hence no direct influence on their marginal cost of arming ( $MC_A^i$ ). The overall effect can be illustrated in the context of Fig. 2 as a clockwise rotation in  $B_A^1(G^2)$  and a counterclockwise rotation in  $B_A^2(G^1)$ . For an initial distribution of secure resources in  $\mathcal{S}^0$ , where  $G_A^{i*} = \tilde{G}_A^*$  and  $p_A^{i*} = \tilde{p}_A^*$  for  $i = 1, 2$ , these effects on the two countries' best response functions are identical, such that they intersect along the 45° line, but to the northeast of the intersection of the original functions. The welfare implications should be clear. Since the total amount of land and labor and their distribution among the two countries remain unchanged, an increase in the degree of insecurity induces more arming, which draws additional resources away from the production of consumption goods, and thus lowers income. Not surprisingly, welfare is maximized where the degree of insecurity is 0 (as presumed in the traditional trade model), and decreases as the degree of insecurity rises.

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<sup>34</sup>This difference is due to the fact that the strategic effect in the case of a land transfer is negative (positive) for the recipient (donor) country.

### 3.2 Conflict under free trade

Turning to trade, we suppose the contending countries are “small” in world markets and that there are no trade costs. Letting  $\pi$  denote the international price of the non-numeraire good, free trade in consumption goods requires  $p^i = \pi$ , for  $i = 1, 2$ . Since  $\pi$  is given by world markets and thus independent of national security policies, a country’s payoff function can be identified with its indirect utility function,  $V^i$ . Depending on fundamentals, the degree of land insecurity, and the international price level, it is possible, as in the case of autarky, for arms production to be constrained by the countries’ secure land holdings.<sup>35</sup> It is also possible now for one or both contestants to specialize completely in the production of one consumption good.

But, to highlight the factor price effects of opening borders up to free trade and the striking implications this can have for arming incentives, we focus on the case of diversified production where the production of arms is not constrained by the countries’ secure land holdings. With parts (a) and (b) of Lemma 1, we first obtain the following:

**Theorem 2.** (*Free Trade*) *If the world price, technology, the distribution of secure endowments and the degree of land insecurity are such that (i) free trade in consumption goods leads to international factor price equalization, and (ii) the production of arms does not exhaust either country’s secure land endowment, an interior Nash equilibrium in security policies will exist, and will be unique and symmetric.*

To understand Theorem 2, suppose for now that  $\pi = \tilde{p}_A^*$  and that the two countries have identical secure resource endowments. Provided the conditions of the theorem are fulfilled, the intersection of  $MB^i$  and  $MC_F^i$  (as illustrated in Fig. 1 at point  $A$ ) determines country  $i$ ’s best-response,  $B_F^i(G^j)$ . Since under free trade product and thus factor prices are independent of either country’s security policy, the shapes of best-response functions are determined solely by the properties of the CSF,  $\phi^i$ —that is,  $\partial B_F^i / \partial G^j = -V_{G^i G^j}^i / V_{G^i G^i}^i = -\phi_{G^i G^j}^i / \phi_{G^i G^i}^i \gtrless 0$  when  $G^i \gtrless G^j$ . Thus, as illustrated in Fig. 2, the best-response functions under free trade are upward-sloping (strategic complementarity) up to their point of intersection with the 45° line, and downward sloping (strategic substitutability) thereafter.<sup>36</sup> When secure resources are identically distributed across the two countries, they face identical marginal benefit and marginal cost functions for guns, thereby yielding the symmetric equilibrium, point  $A$  in Figure 2 where  $G_F^{i*} = \tilde{G}_A^*$  for  $i = 1, 2$ .<sup>37</sup> What about when secure endowments are unevenly distributed across the two countries? Provided that

<sup>35</sup>However, note that, as in the case of autarky, countries will not use their entire labor endowments in the production of guns, provided that both factors are essential in the production of consumption goods.

<sup>36</sup>The absence of strategic complementarity at the 45° line here, in contrast to the case of autarky (see footnote 23 and the expression for  $\partial B_A^i / \partial G^j$  shown in (A.15) in the Appendix), is due to the previously noted absence of any indirect influence of guns by an adversary through the relative price,  $p^i = \pi$ . In the case of free trade,  $\pi$  is given by world markets.

<sup>37</sup>As before, point  $(0, 0)$  is not an equilibrium. That the level of arming by both countries under free trade,

the distribution is such that free trade in consumption goods implies international factor price equalization<sup>38</sup> and such that the production of guns does not exhaust either country's secure land resources, the contending states will continue to face identical marginal benefit and marginal cost functions for guns, thus again yielding the symmetric Nash equilibrium:  $G_F^{i*} = \tilde{G}_A^*$ .

Let us now reflect on the conditions that induce this type of equilibrium.<sup>39</sup> Continue to assume that  $\pi = \tilde{p}_A^*$  and that the degree of land insecurity is sufficiently small so that  $\mathcal{S}^0$  in Fig. 3 contains unequal distributions of secure resources across the two countries as well as the equal distribution, and the parallelogram  $C^1J^1C^2J^2$  in Fig. 4 (which is similar to Fig. 3) captures the joint sectoral decomposition of inputs in the consumption goods sectors of the two contestants. Since  $G_F^{i*} = \tilde{G}_A^*$  for  $i = 1, 2$  at  $\pi = \tilde{p}_A^*$ , we can temporarily abstract from resource constraints in the production of guns, and view  $C^1C^2$  as the vector of residual factor supplies of an integrated economy (i.e., an economy in which both goods and factors are tradable); this integrated economy replicates the equilibrium that arises when where there is free trade in goods only, countries 1 and 2 are considered separately, and their residual factor endowments are in  $C^1J^1C^2J^2$  (Dixit and Norman, 1980; Helpman and Krugman, 1985). Under these circumstances, trade in consumption goods causes factor prices to be equalized internationally for all residual resource distributions in  $C^1J^1C^2J^2$ . But, country  $i$ 's secure factor endowments must, at the same time, cover the vector of resources that are absorbed in its arms sector,  $O^1C^1 = O^2C^2$ .<sup>40</sup> Therefore, the set of secure endowment distributions that are consistent with international factor price equalization and unconstrained production of guns is the shaded hexagon in Fig. 4, which we label the “arms equalization set” (*AES*).<sup>41</sup> An immediate consequence of Theorem 2 is

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$G_F^{i*} = G_F^*$  for  $i = 1, 2$ , equals that which emerges under autarky assuming identical adversaries,  $\tilde{G}_A^*$ , is due to our (benchmark) assumption that  $\pi = \tilde{p}_A^*$ . Below we show how changes in  $\pi$  influence the level of arming by both countries.

<sup>38</sup>The conditions for international factor price equalization include, as in the standard Heckscher-Ohlin trade model, constant returns to scale in production, the absence of factor intensity reversals, identical technologies across countries, diversification in production, absence of market failures or distortions, no trade barriers, and the existence of at least as many productive factors in the tradable goods sectors as there are traded goods (Samuelson, 1949).

<sup>39</sup>To be sure, existence and uniqueness of equilibrium arise under less restrictive conditions than the ones stated in Theorem 2. Indeed, the analysis could be extended to entertain the possibilities that factor prices are not equalized internationally and the production of guns is constrained by the countries' secure land holdings. However, since these possibilities only complicate the analysis without altering the key insights of our comparison of conflict under autarky and free trade, we choose not to treat them formally here.

<sup>40</sup>Note that the upper boundaries of the (shaded) sets of distributions for which the countries' secure land holdings bind are humped shaped under autarky (see Fig. 3), since different distributions imply different product and thus factor prices, which then change each country's incentive to arm. By contrast, the upper boundaries of the analogous sets under free trade, which cross over the parallelogram ( $C^1J^1C^2J^2$ ), are flat (though similarly passing through  $F^1$  and  $F^2$ ), since world product prices pin down factor prices and thus incentives to arm.

<sup>41</sup>For distributions of secure resources outside the intersection of  $A^1B^1A^2B^2$  and  $C^1J^1C^2J^2$ , at least one country will specialize in the production of one consumption good. Such specialization precludes the pos-

**Corollary 1.** *Under free trade, redistributions of secure endowments across the two contending countries within the AES have no effect on their arming. Thus, the welfare effects of such redistributions coincide with the factor price related changes in national income weighted by the marginal utility of income.*

As before, the welfare effects of endowment redistributions can be visualized by considering a transfer of labor or land from country  $j$  to country  $i$  ( $\neq j$ ). A welfare decomposition similar to that in (11), noting that the strategic effect of such a transfer vanishes since there is no effect on equilibrium arming, yields the following for the case of a transfer of labor:  $dV_F^{i*}/dL^i = -dV_F^{i*}/dL^i = \mu(\pi)w(\pi)$ . Similarly one can verify that  $dV_F^{i*}/dK^i = -dV_F^{i*}/dK^i = \mu(\pi)r(\pi)$ .

The next proposition outlines the effects of an increase in the degree of land insecurity—namely, an increase in  $K_0$  and a decrease in the countries' secure land endowments, such that  $dK^i = -\phi^i dK_0$ —in the free trade regime, where initially arms are equalized across the contending nations.

**Proposition 4.** (*Degree of Insecurity under Free Trade*) *For initial factor distributions in the AES, an increase in the degree of insecurity has the following effects on arming and welfare:*

- (a)  $\frac{dG_F^{i*}}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} = \frac{dG_F^{j*}}{dK_0} \Big|_{dK^j = -\frac{1}{2}dK_0} > 0$ ;
- (b)  $\frac{dV_F^{i*}}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} = \frac{dV_F^{j*}}{dK_0} \Big|_{dK^j = -\frac{1}{2}dK_0} < 0$ .

Increasing the degree of insecurity in the free trade regime adds, as in the autarkic regime, to the marginal benefit of arming ( $MB^i$ ). Furthermore, as in the autarkic regime, there is no effect on the marginal cost of arming, ( $MC_F^i$ ). However, the reason here is somewhat different. In particular, because product prices alone determine factor prices under free trade,  $MC_F^i$  is a constant function; a change in  $K_0$  has no effect on  $MC_F^i$ .<sup>42</sup> For an initial distribution in the AES, the positive effect on  $MB^i$  is the same for both countries. Hence, each country's best-response function increases, and by the same amount, resulting in a higher equilibrium level of arming. Since total endowments have not changed, the amount of resources diverted from production of consumption goods falls, and so does

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sibility of international factor price equalization and renders a country's marginal cost of producing guns independent of the world price, but increasing in its arms. For distributions of secure resources in  $C^1J^1C^2J^2$  but outside the AES, free trade in consumption goods leads to factor price equalization. Although one country's secure land constraint binds in the production of guns, once the disputed land is divided, both countries diversify in their production of the two goods. Nonetheless, due to that binding constraint, the marginal benefit of producing more arms is not equalized across countries; accordingly, free trade does not lead to arms equalization.

<sup>42</sup>Recall that, under autarky, the effect of an increase in the degree of insecurity has no direct effect on the price, since the condition that  $dK^i = -\phi^i dK_0$  implies no change in the countries' residual land endowments. Thus, there is no effect on the positioning of  $MC_A^i$ .

income. Thus, the effects of increasing the degree of insecurity are qualitatively the same under the two trade regimes, where the distribution of secure resources implies equalized arming incentives ( $\mathcal{S}^0$  under autarky and the *AES* under free trade).<sup>43</sup> This discussion suggests further that increasing the degree of insecurity shrinks the *AES*.

The size of the *AES* relative to the set of possible secure endowment distributions provides a measure of the likelihood that free trade will equalize arms internationally. This measure depends not only on the degree of land insecurity, but also on the world price and the nature of technology. *Ceteris paribus*, the greater is the degree of similarity between the consumption goods technologies (i.e., the smaller is  $|k_2^i - k_1^i|$ ) and the larger is the deviation of the international price ( $\pi$ ) from the autarkic price, ( $\tilde{p}_A^*$ ), the smaller is the factor price equalization set and thus the smaller is the *AES*. Indeed, it is possible for the *AES* to contain only the equal distribution.<sup>44</sup>

But, for unequal distributions in the *AES*, free trade in consumption goods levels the playing field in arms competition and equalizes power internationally. We thus have again, as in the case of autarky, the emergence of (the “strong version” of) the paradox of power (Hirshleifer, 1991)—where the relatively less affluent adversary devotes a larger fraction of its income to arms and obtains an equal share of the disputed resource—but for a potentially larger set of factor distributions, particularly when  $\pi = \tilde{p}_A^*$ .

What are the implications of changes in international prices for arming, trade flows and welfare?<sup>45</sup> We address these questions in the following three propositions. To proceed, note

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<sup>43</sup>However, for distributions in the *AES* subset of  $\mathcal{S}^0$ , the effects are more pronounced in the free trade regime, as can be illustrated in the context of Fig. 1, which shows the representative country’s marginal benefit and cost schedules, assuming  $\pi = \tilde{p}_A^*$ , so that initially  $G_F^{i*} = \tilde{G}_A^*$ . Since the marginal benefit schedule of arming is trade-regime independent, an increase in insecurity shifts this schedule upward by the same amount under both regimes, and at the same time leaves the countries’ marginal cost schedules under the two regimes unchanged. However, because  $MC_A^i$  is increasing in  $G^i$ , while  $MC_F^i$  is independent of  $G^i$ , each country’s response under autarky is less than that under free trade. As a result, an increase in insecurity increases the Nash equilibrium quantities of guns by more under free trade (where  $\pi$  is unchanged) than under autarky. Since the change in insecurity has no effect on the countries’ *ex post* secure endowments, only the strategic effect matters, implying that welfare will fall by more under free trade than under autarky for initial distributions in  $\mathcal{S}^0$ .

<sup>44</sup>Nonetheless, there always exist combinations of technologies, secure factor distributions, degrees of land insecurity, and world prices that preclude this possibility.

<sup>45</sup>It is important to note that, when  $\pi$  differs from  $\tilde{p}_A^*$ , Fig. 4 will change considerably. To get some sense of these changes, suppose that  $\pi$  increases above  $\tilde{p}_A^*$ , but only by a small amount to ensure continued diversification in production. By Lemma 1(a), under the maintained assumption that  $k_2^i > k_1^i$ , the increase in  $\pi$  reduces the wage/rental rate ratio,  $\omega^i$ , in each country  $i$ . The implied changes in factor prices, in turn, reduce the land intensity (i.e., the land/labor ratio demanded) in each sector (including guns), thereby flattening the sides of the hexagon and the vectors  $O^i C^i$  for  $i = 1, 2$ . Furthermore, the increase in  $\pi$ , by Proposition 5(a) below, increases each country’s incentive to produce more guns, to increase the set of distributions for which the countries’ secure land resources bind, as well as guns production by both countries. At the same time, with guns being labor-intensive, by Lemma A2(d),  $k_X^*$  increases above  $\tilde{k}_X^*$ . Thus, the line  $C^1 C^2$  going through point  $D$  becomes steeper and no longer coincides with  $\mathcal{S}^0$  in Fig. 3 (for autarky). The only common point will be point  $D$ .

that, for each country  $i$ , there exists a range of prices  $(\underline{\pi}^i, \bar{\pi}^i)$  that ensures diversification in the production of consumption goods for country  $i$ . Then, for world prices  $\pi \in (\underline{\pi}, \bar{\pi})$ , where  $\underline{\pi} = \max\{\underline{\pi}^1, \underline{\pi}^2\}$  and  $\bar{\pi} = \min\{\bar{\pi}^1, \bar{\pi}^2\}$ , production is diversified in both countries  $i$ . By contrast, for world prices  $\pi \notin (\underline{\pi}, \bar{\pi})$ , where  $\underline{\underline{\pi}} \equiv \min\{\underline{\pi}^1, \underline{\pi}^2\} \leq \underline{\pi}$  and  $\bar{\bar{\pi}} \equiv \max\{\bar{\pi}^1, \bar{\pi}^2\} \geq \bar{\pi}$ , both countries specialize in production.<sup>46</sup> Using these definitions, we have the following:

**Proposition 5.** (*International Prices and Arming*)

- (a) Assuming that secure land endowments are not exhausted in the production of guns, equilibrium guns are increasing in the world price of the land-intensive good for world prices  $\pi \in (\underline{\pi}, \bar{\pi})$  (i.e.,  $dG_F^{i*}/d\pi \geq 0 \forall \pi \in (\underline{\pi}, \bar{\pi})$  if  $k_2^i \geq k_1^i$ , for  $i = 1, 2$ ).
- (b) Equilibrium guns are invariant to price changes for all world prices  $\pi \notin (\underline{\pi}, \bar{\pi})$  (i.e.,  $dG_F^{i*}/d\pi = 0, \forall \pi \notin (\underline{\pi}, \bar{\pi})$ ).

To fix ideas, suppose good 2 is land-intensive ( $k_2^i > k_1^i$ ) and that the conditions specified in part (a) are satisfied. Thus, factor prices and arms are equalized across countries. Now, let the world price of good 2 ( $\pi$ ) rise. While this price change has no effect on either country's marginal benefit of arming ( $MB^i$ ), by the Stolper-Samuelson theorem (Lemma A1(a)) the wage/rental ratio in each contending country ( $\omega^i$ ) will fall; and, as discussed previously in connection with Lemma 1(c), this factor price adjustment causes each country  $i$ 's marginal cost of arming ( $MC_F^i$ ) to fall, thereby inducing both to arm more heavily. Analogous reasoning establishes that, in the case good 1 is land-intensive ( $k_1^i > k_2^i$ ), an increase in  $\pi \in (\underline{\pi}, \bar{\pi})$  induces less arming. Turning to part (b), note that, from equations (2)–(5), price changes outside the relevant range for country  $i$ ,  $\pi \notin (\underline{\pi}^i, \bar{\pi}^i)$ , force all factor prices to rise proportionately in that country and thus have no effect on the marginal cost of arming. Part (b), then, follows from the condition imposed that imply neither country diversifies in production. But, for our purposes, the important point is that changes in the world price can have important implications for security policies when production is diversified. Furthermore, the qualitative linkage hinges on the nature of technology. For simplicity and clarity, henceforth we maintain the assumption that good 2 is land-intensive—i.e.,  $k_2^i > k_1^i$ .

Proposition 5 can be illustrated with the help of Fig. 5, which assumes identical adversaries. The positively sloped blue dashed-line curve identifies the equilibrium gun quantity as implicitly defined by (9),  $G^*(\pi)$  (read along the horizontal axis), that each adversary will produce as a function of  $\pi$  (read along the vertical axis) under free trade.<sup>47</sup> The extensions of  $G^*(\pi)$  in the shaded regions of specialization show the independence of equilibrium guns

<sup>46</sup>Of course, for identical adversaries,  $\underline{\underline{\pi}} = \underline{\pi}$  and  $\bar{\bar{\pi}} = \bar{\pi}$ . But, for adversaries having different secure factor endowments, there also exist price ranges for which one country's gun choices depend on prices while the other country's do not—namely,  $(\underline{\underline{\pi}}, \underline{\pi})$  and  $(\bar{\pi}, \bar{\bar{\pi}})$ . However, these complications are not relevant for our arguments.

<sup>47</sup>Ignore the other curves for now.

from the world price noted in part (b).<sup>48</sup>

Naturally, the direction of a country's trade flows will depend on world prices. To explore this issue, define  $\pi_A^i$  as the world price level that eliminates country  $i$ 's trade. The next proposition describes the relationship between trade eliminating prices and autarkic prices and the direction of trade flows.

**Proposition 6.** (*International Prices and Trade Flows*) *For each adversary  $i$ , there is a world price,  $\pi_A^i$ , such that  $M_F^{i*}(\pi) \leq 0$  if  $\pi \geq \pi_A^i$ . Furthermore, for factor distributions in the AES subset of  $\mathcal{S}^0$ , we have  $\pi_A^i = \tilde{p}_A^*$ ; but, under the maintained assumption that good 2 is land-intensive, for factor distributions in the AES subset of  $\mathcal{S}^i$ , we have  $\pi_A^i > p_A^{i*}$  and  $\pi_A^j < p_A^{j*}$  ( $j \neq i$ ).*

The interesting part of Proposition 6 is that, in the presence of differences in secure factor ownership, a contending country's trade eliminating price,  $\pi_A^i$ , differs from the price that emerges under autarky,  $p_A^{i*}$ . The key to understanding this result is to note that the introduction of free trade, through its impact on arming incentives, alters the countries' residual factor endowments and therefore their excess demand functions. To see the logic here, suppose  $\pi = p_A^{2*}$  and consider a distribution of secure resources in the AES subset of  $\mathcal{S}^1$ , where country 1 is relatively more aggressive under autarky. By Proposition 1(b), the autarkic conflictual equilibrium can be identified with point  $A'$  in Fig. 2. Since free trade levels the playing field in arms competition and, as a result, induces country 2 to become more aggressive and country 1 to become less aggressive (compare the free trade equilibrium at point  $A$  with the autarkic equilibrium at point  $A'$ ), country 2's residual land/labor ratio,  $k_X^2$ , increases with a move to free trade (Lemmas A2(b) and A3).<sup>49</sup> By Lemma A1(b), assuming that  $k_2^i > k_1^i$ , country 2's excess supply of the non-numeraire good, when evaluated at the autarkic price,  $p_A^{2*}$ , is strictly positive. As such, the world price that eliminates trade,  $\pi_A^2$ , must be below  $p_A^{2*}$ . Similar logic shows that, at the same time,  $\pi_A^1 > p_A^{1*}$ .<sup>50</sup> These results prove useful in the next section, where we discuss the broader implications of conflict for trade patterns.

Turning to welfare, the decomposition in (8) together with the envelope theorem yield

$$\frac{dV_F^{i*}}{d\pi} = \mu(\pi) \left[ -M_F^{i*} + r(\pi)K_0\phi_{G^j}^i \frac{dG_F^{j*}(\pi)}{d\pi} \right], \quad \text{for } i = 1, 2 \text{ and } j \neq i. \quad (12)$$

The first term inside the brackets, weighted by the marginal utility of income ( $\mu(\pi)$ ), captures the direct welfare effect of a price change, and its sign is determined by the country's

<sup>48</sup>Note the equality of guns produced by the two countries in these shaded regions arises because the figure is drawn under the assumption that the countries are identical in their secure resource endowments.

<sup>49</sup>Lemma A2(b) shows  $\partial k_X^i / \partial G^i > 0$  in the neighborhood of an optimum; Lemma A3 shows  $\partial k_X^i / \partial G^j < 0$  always in the neighborhood of an optimum when  $G^i \leq G^j$  and nearly always otherwise.

<sup>50</sup>Note, assuming instead that  $k_1^i > k_2^i$  implies a reversal in the ranking of the  $\pi_A^i$  and  $p_A^{i*}$  for each country.

trade pattern. It is positive for net exporters of the non-numeraire good ( $M_F^{i*} < 0$ ), and negative for net importers ( $M_F^{i*} > 0$ ). The second term (again weighted by  $\mu(\pi)$ ) captures the strategic welfare effect of a price change. By the properties of the CSF and Proposition 5(a) under the maintained assumption that  $k_2^i > k_1^i$ , this indirect effect is negative when arms are equalized internationally; but, by Proposition 5(b), the effect vanishes when the world price rises above  $\bar{\pi}$  or falls below  $\underline{\pi}$ . The next proposition takes these ideas one step further.

**Proposition 7.** (*International Prices and Welfare*) *A contending country  $i$ 's welfare is*

- (a) *decreasing in the world price, in the neighborhood of  $\pi = \pi_A^i$  (i.e.,  $dV_F^{i*}(\pi_A^i)/d\pi < 0$ ); and,*
- (b) *minimized at a world price,  $\pi = \pi_{\min}^i (> \pi_A^i)$ .*

Part (a) points out that an improvement in a contending country's terms of trade is necessarily "immiserizing" if the country exports the disputed-resource-intensive product and provided that  $\pi$  does not differ considerably from its trade eliminating price,  $\pi_A^i$ . The idea here is simple, building on the two welfare effects of an increase in the world price revealed by equation (12). Specifically, in the neighborhood of  $\pi_A^i$ , the direct, positive effect of a terms of trade improvement for country's  $i$  income is swamped by the loss in income due to its opponent's increased aggressiveness.

Part (b) indicates that a country's welfare is minimized at some price,  $\pi_{\min}^i > \pi_A^i$ , where the beneficial, direct effect of a terms of trade improvement equals the adverse strategic effect that results from increased arms production by the rival country. This finding establishes the existence of a range of prices,  $(\pi_A^i, \pi_{\min}^i)$ , under which terms of trade improvements are welfare-reducing for a country immersed in conflict. Thus, international conflict over resources can expose contending countries with an apparent comparative advantage in the contested-resource-intensive products to the "resource curse" problem. While others have shown that the problem can be attributed to domestic rent-seeking (e.g., Torvik, 2002; Mehlum et al. 2006), redistributive politics (e.g., Robinson et al., 2005), and domestic conflict (Garfinkel et al., 2008), our finding suggests that the absence and/or ineffectiveness of international institutions aimed at managing international conflict has a bearing on this problem as well.<sup>51</sup>

Although country size is inconsequential in the determination of the quantity of guns that adversaries produce under free trade for initial factor distributions in the *AES*, country size can play an important role in the determination of the range of international prices

<sup>51</sup>Our focus here is on the case where  $k_2^i > k_1^i$ , but analogous results emerges when the factor intensity ranking is reversed. The general point is that increases in the relative price of the good produced intensively with land is welfare reducing for prices close to the trade eliminating price.

for which the resource curse problem arises. Consider, for example, an uneven distribution of secure resources in the *AES* subset of  $\mathcal{S}^0$ —or, more precisely, a point such as  $E$  in Fig. 4, where country 1 is larger than country 2 and where initially  $\pi = \pi_A^i = \widehat{p}_A^*$ . From (12), the strategic welfare effect of an increase in the world price,  $\pi$ , above  $\widehat{p}_A^*$  (the second term in (12)) will not differ across adversaries. However, the marginal benefit from such a price increase (the first term in (12)) will differ. Since, by construction, country 1 is larger than country 2, country 1 will be relatively more involved in trade than its rival (i.e.,  $-M_F^{1*} > -M_F^{2*} > 0$ ) for  $\pi > \widehat{p}_A^*$ ; therefore,  $\pi_{\min}^1 < \pi_{\min}^2$ , and we have the following:

**Corollary 2.** *For uneven factor distributions in  $\mathcal{S}^0$ , the relatively smaller adversary will experience the resource curse problem over a larger range of international prices.*

## 4 Trade Patterns and Trade Volumes

By inducing countries to allocate resources in the production of arms, international contestation of resources alters a country’s residual factor endowments ( $k_X^i$ ) and can thus affect its observed comparative advantage. In this section, we contribute two ideas to the literature. First, we illustrate how the presence of conflict can distort a contending country’s comparative advantage. Second, we show that a simple comparison of international and autarkic prices need not provide an accurate prediction of trade patterns in contending countries.

To consider the effects of conflict on a contending country’s comparative advantage, it is instructive to focus on identical adversaries and show how their trade pattern (with conflict) compares to the (hypothetical) trade pattern we would observe if property were secure and there were no arming. We illustrate the key ideas with the help of Fig. 5. As noted earlier, the curve  $G^*(\pi)$  depicts the equilibrium choice of guns under free trade as a function of price. The downward sloping solid-line curve,  $p_A(G)$ , shows the negative dependence of the representative country’s autarkic price on the common quantity of guns,  $G$ . This relationship follows from Lemma 3(c), with the assumptions that good 2 is land-intensive and that the ratio of a country’s *ex post* endowment of land to its secure endowment of labor,  $k^i \equiv \frac{K^i + \phi^i K_0}{L^i}$ , exceeds its land/labor ratio used in the production of guns,  $k_G^i(\omega)$ . Alternatively, if  $k^i < k_G^i(\omega)$  while  $k_2^i > k_1^i$ , then  $p_A(G)$  would be increasing in  $G$ .<sup>52</sup>

Point  $A$ , where  $G^*(\pi)$  and  $p_A(G)$  intersect, identifies the conflictual equilibrium under autarky. Clearly, if  $\pi = \widehat{p}_A^*$ , point  $A$  would also describe the conflictual equilibrium under free trade. By Proposition 6 and our assumption that the contenders are identical, the

<sup>52</sup>See again Lemma 3(c). The effect of the ranking of  $k^i$  and  $k_G(\omega)$  on the qualitative relation between  $G$  and  $p_A(G)$  has to do with its effect on qualitative relation between the representative country’s guns and its residual land/labor ratio,  $k_X^i$ . By Lemma A2(d), when  $k^i > k_G^i$  ( $k^i < k_G^i$ ), an exogenous equi-proportionate increase in guns produced by both countries, where initially  $G^i = G^j$ , increases (decreases)  $k_X^i$ , which in turn implies (by Lemma 3(c) with the maintained assumption that  $k_2^i > k_1^i$ ) a decrease (an increase) in the trade eliminating price  $\pi_A$ .

international price that eliminates a contending country's trade flows equals the equilibrium price that obtains under autarky (i.e.,  $\pi_A^i = \tilde{p}_A^*$ ); therefore, under conflict and trade, both adversaries will export the land-intensive good (2) if  $\pi > \tilde{p}_A^*$  and will import it if  $\pi < \tilde{p}_A^*$ .

Consider now the hypothetical case of no arming. From the specification of the CSF in (1), in this case each adversary would receive  $\frac{1}{2}$  of the contested resource  $K_0$ , and the autarkic price would coincide with price  $p_A^n = p_A(0)$  (superscript “n” stands for “no conflict” or “Nirvana”). Thus, in the absence of conflict, the representative country would export the land-intensive good (i.e., good 2) if  $\pi > p_A^n$  and would import it if  $\pi < p_A^n$ . Bringing these ideas together, the following proposition clarifies how conflict and the nature of technology in arms interact to determine trade patterns.

**Proposition 8.** (*Trade Patterns with Identical Adversaries*) *Suppose  $k_2^i > k_1^i$  and free trade in goods leads to international arms equalization. Then, conflict over land reverses the contending countries' comparative advantage for  $\pi \in (\tilde{p}_A^*, p_A^n)$  if  $k^i > k_G^i(\omega)$ , or for  $\pi \in (p_A^n, \tilde{p}_A^*)$  if  $k^i < k_G^i(\omega)$ , relative to what would be observed in the hypothetical case of no conflict.*

As suggested in the proposition, the nature of the distortion depends on the relative ranking of  $k^i$  and  $k_G^i(\omega)$ . The reason is that this relative ranking determines the relative ranking of autarkic prices under conflict and under no conflict,  $\tilde{p}_A^* = p_A(\tilde{G}_A^*)$  and  $p_A^n = p_A(0)$ . Suppose, for example,  $k^i > k_G^i(\omega)$ . In this case, as noted earlier and illustrated in Fig. 5,  $p_A(G)$  is decreasing in  $G$ , so that  $p_A^n > \tilde{p}_A^*$ . Now suppose  $\pi \in (\tilde{p}_A^*, p_A^n)$ —for example, the world price  $\pi = \pi'$  shown in Fig. 5. Obviously, since  $\pi' < p_A^n$ , under no conflict both contestants would import good 2. But, since  $\pi' > \tilde{p}_A^*$  at the same time, under conflict each country exports good 2, the good that employs the resource disputed resource. If instead  $k^i < k_G^i(\omega)$ , then as noted earlier  $p_A(G)$  would be increasing in  $G$ , and as a result  $p_A^n < \tilde{p}_A^*$ . In this case, if  $\pi \in (p_A^n, \tilde{p}_A^*)$ , then each country would export good 2 in the hypothetical case of no conflict, and import the good under conflict.<sup>53</sup>

Two observations are now in order. First, resource insecurity and the conflict it engenders can cause a country's natural comparative advantage to diverge from its comparative advantage that would be observed in the hypothetical case of no conflict—and as would be predicted in the standard trade model.<sup>54</sup> As we have just seen, the nature of technology

<sup>53</sup>An additional possibility (not considered in Proposition 8) is that an increase in guns production eventually reverses the ranking of between  $k^i$  and  $k_G^i(\omega)$ . Suppose, for example that initially  $k_G(\omega^i) < k^i$ , so that  $p_A(G)$  is initially decreasing in  $G$ . Then, as  $G$  rises, the induced decrease in the relative product price (by Lemma A1(a) with the assumption that  $k_2^i > k_1^i$ ) implies an increase in the wage/rental ratio,  $\omega^i$ , which in turn induces the countries to increase their land intensities in each sector. Since  $G^1 = G^2$  implies  $k^i$  is fixed,  $k_G(\omega^i)$  could eventually rise above  $k^i$ . (This possibility does not arise if production functions are Cobb-Douglas.) Nevertheless, the main message survives: as long as  $\tilde{p}_A^* \neq p_A^n$ , there will exist price ranges under which the presence of conflict reverses a country's comparative advantage.

<sup>54</sup>The idea that a country's apparent comparative advantage depends on the nature of property rights is reminiscent of a related point in Brander and Taylor (1997b) who show that, over time, the depletion of a

and the world price play key roles in this regard. Second, it might be inappropriate to view international conflict over productive resources as a type of trade cost that necessarily reduces the size of a contending country's trade flows. Suppose, for example,  $\pi = p_A^n$ . Then, in the absence of conflict, the contestants would not engage in trade. But, as we have just seen, in the presence of conflict and under free trade, the contending countries will be net exporters of the land-intensive good if  $k^i > k_G^i(\omega)$ ; consequently, depending on technology, international conflict may very well expand, rather than shrink, trade volumes.<sup>55</sup>

From the perspective of neoclassical trade theory, a country's trade pattern is determined by comparing the world price to its autarkic price. However, in the world of insecure property and international conflict, an unqualified application of this logic can lead to erroneous inferences about trade patterns. In other words, insecure property and conflict over resources might not only alter trade patterns but also the standard logic we normally use in trade theory to predict trade patterns.

**Proposition 9.** (*Trade Patterns with Different Sized Adversaries*) *If the distribution of secure factors differs across countries, then, in the presence of international conflict,*

- (a) *it might not be possible to determine a contending country's trade pattern simply by comparing the international price to its autarkic price;*
- (b) *a contending country need not export the good that employs intensively its relatively abundant factor.*

We establish the validity of Proposition 9 informally and with the help of Proposition 6, which shows the price that eliminates a country's trade differs from its autarkic price (i.e.,  $\pi_A^i \neq p_A^{i*}$ ) for factor distributions in the AES subset of  $\mathcal{S}^i$ . Consider such a distribution of resources. Clearly, if arming did not depend on the trade regime considered and if  $\pi = p_A^{i*}$ , country  $i$  would not engage in trade. However, as was argued above, the trade regime does influence arming that, in turn, drives a wedge between each country's autarkic price and its trade eliminating price. In short, the direction of trade flows is **not** determined by how the international price ( $\pi$ ) differs from the country's autarkic price,  $p_A^{i*}$ , but rather how it differs from  $\pi_A^i$ .

To see the logic behind part (b), suppose  $\pi = \tilde{p}_A^*$  and consider point  $H$  in Fig. 4. At this distribution, since  $\pi = \tilde{p}_A^* < \pi_A^1$ , country 1 will export the labor-intensive good; and, since  $\pi = \tilde{p}_A^* > \pi_A^2$ , country 2 will export the land-intensive good. But, at point  $H$ , country 1 is land-abundant and country 2 is labor-abundant, both in terms of their secure endowments

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common-pool resource in a country with ill-defined property rights can reverse its comparative advantage. In our setting, residual factor endowments (and thus comparative advantage) can change because the dissipation of resources in conflict is trade-regime dependent.

<sup>55</sup>The relationship between the volume of trade and conflict has been addressed empirically in the political science literature (e.g., Barbieri, 2002), which appears to find support for the idea that conflict might stimulate trade.

and in terms of their final endowments under free trade, which confirms part (b).<sup>56</sup>

## 5 Comparison of Trade Regimes

In this section, we illustrate the following two ideas: (i) the effects of resource insecurity and competition to establish property rights on arming incentives depend on the trade regime in place; and, (ii) the costs of arming can overwhelm a country's traditional gains from trade. The analysis not only clarifies how international conflict generates a trade-regime dependent distortion (Bhagwati, 1971), but also sheds light on the conditions under which free trade intensifies this distortion. To substantiate these points we consider two possibilities: (i) when adversaries are identical, which unveils the gist of the argument and the circumstances under which autarky dominates trade; and, (ii) when adversaries have different endowment profiles, which sheds some light on the conditions under which national preferences over trade regimes can diverge.

**Proposition 10.** (*Relative Appeal of Free Trade with Identical Adversaries*) *If free trade in consumption goods induces adversaries with identical endowment profiles to*

- (a) **export** the land-intensive commodity, then the adversaries will arm more heavily under free trade than under autarky, and free trade will be Pareto dominated by autarky for a certain range of international prices;
- (b) **import** the land-intensive product, there will be less arming under free trade than under autarky, and free trade will Pareto dominate autarky.

Proposition 10 holds regardless of the ranking of the relative land-intensities of the two goods. But, for clarity we again assume  $k_2^i > k_1^i$ , and illustrate the proposition with the help of Fig. 5. Recall point  $A$  depicts the conflictual equilibrium under autarky and the corresponding equilibrium under free trade is on  $G^*(\pi)$ . Then, as illustrated in the figure, for  $\pi > \widehat{p}_A^*$ , where the adversaries export the land-intensive good (2), a move from autarky to free trade intensifies the conflict between them, inducing more arming. By contrast, for  $\pi < \widehat{p}_A^*$ , where the adversaries import the land-intensive good, a discrete move to free trade from autarky weakens the conflict, and thus induces less arming.<sup>57</sup>

Turning to payoffs, we begin by characterizing the shape of welfare contours in the  $(G, \pi)$  space. First note that, regardless of the trade regime considered, equi-proportionate arms

<sup>56</sup>It is also worth noting that, since  $p_A^{1*} > p_A^{2*}$  at point  $H$ , the land- (labor-) intensive commodity does not necessarily command the smaller (larger) autarkic price in the land- (labor-) abundant country.

<sup>57</sup>Note the difference between this result and that of Hirshleifer (1991), who explored the linkages between conflict over output, identifying market integration with the degree of complementarity between the inputs in useful production. Specifically, he observed that the diversion of resources into arms falls with the degree of market integration, although the size of this effect is small. Our approach suggests that, when conflict is over resources and market integration takes the form of a move from more protected (autarky) to less protected (free trade) trade regimes, the severity of conflict (measured by the level of arming) can rise or fall depending on, among other things, technology, the degree of resource insecurity and international prices.

increases do not alter the division of the contested land but raise resource costs. As such, the representative country's welfare is decreasing in guns,  $G$ . Now observe that, for given guns, a country's welfare increases with the deviation of the world price from its autarky level  $p_A(G)$  (Lemma 1(d)). It follows, then, that a contestant's welfare contours will have the shape indicated in Fig. 5, with contours further to the right indicating lower welfare levels for the two contestants. Noting that each contestant exports the contested-resource-intensive product at prices  $\pi > \tilde{p}_A^*$ , Fig. 5 illustrates that autarky will Pareto dominate free trade for all international prices within the  $(\tilde{p}_A^*, \pi')$  interval, but not for prices beyond  $\pi'$ .<sup>58</sup> The figure also shows that free trade is Pareto superior to autarky when countries import the land-intensive product.

Now consider adversaries with different endowment profiles. Arbitrary factor distributions in  $\mathcal{S}^i$  can complicate the welfare ranking of trade regimes for at least two reasons. First, because adversaries begin to specialize in production at different international prices, it becomes necessary to investigate arming incentives outside the *AES* for one country initially and eventually for both. Second, the endogeneity of trade patterns together with the fact that  $V_F^{i*} \neq V_A^{i*}$  at  $\pi = p_A^{i*}$  for arbitrary distributions in  $\mathcal{S}^i$  make it difficult to identify workable benchmarks for comparison purposes. Still, as the next proposition illustrates, there exist two noteworthy asymmetries that yield tractable comparisons.

**Proposition 11.** (*Relative Appeal of Free Trade with Different Sized Adversaries*)

- (a) *For any uneven factor distribution in the AES subset of  $\mathcal{S}^0$ , there exists a range of international prices that render autarky Pareto superior to free trade.*
- (b) *If  $\pi = \tilde{p}_A^*$ , there exist subsets  $\mathcal{D}^i \subseteq \mathcal{S}^i$  of factor distributions adjacent to the AES of  $\mathcal{S}^0$  such that one country prefers autarky over free trade while its adversary does not.*

Part (a) is an extension of Proposition 10 to uneven distributions in the *AES* subset of  $\mathcal{S}^0$ . Part (b) clarifies how countries' preferences over trade regimes might differ when more general factor endowment asymmetries are considered. As shown in the proof (presented in the Appendix), the key to the divergence in preferences is the presence of a strategic effect when redistributing resources under autarky (Proposition 2), and the absence of such a strategic effect under free trade (Corollary 1).

## 6 Trade Policies

Thus far, we have studied the consequences of trade regimes, taking those regimes as exogenous and, in particular, abstracting from the possibility that policymakers can choose to intervene with trade policy. In this section, we illustrate how a country might use its trade

<sup>58</sup>As shown in the figure,  $\pi' (> \tilde{p}_A^*$  if  $k_2^i > k_1^i$ ) is defined as the international price that satisfies  $V_F^{i*}(\pi) = \tilde{V}_A^*$ .

policy to tilt the distribution of power to its favor. We also highlight the welfare effects of alternative trade policy choices.

Suppose country  $i$  intervenes in trade with an ad valorem trade tax,  $\tau^i$ , on good 2—i.e., requiring  $\tau^i > 0$  when  $M^i > 0$ , and  $\tau^i < 0$  when  $M^i < 0$ . Under these circumstances,  $p^i = (1 + \tau^i)\pi > 0$  for  $i = 1, 2$ . Assuming that, in each country  $i$ , tariff revenues,  $\tau^i\pi M^i = (p^i - \pi)M^i$ , are redistributed to consumers in lump-sum fashion, the country's income for consumption purposes, denoted by  $Y^i$ , equals the revenue generated from producing consumption goods plus tariff revenues,  $Y^i = R^i + (p^i - \pi)M^i$ ; and, the indirect utility function is now given by  $V(p^i, Y^i) = \mu(p^i)[R^i + (p^i - \pi)M^i]$ . Observe that  $D^i = -V_p^i/V_Y^i = -\mu'(p^i)Y^i/\mu(p^i)$  and (as before)  $X_2^i = R_p^i$ . Accordingly, we can extend the decomposition of country  $i$ 's welfare as follows:

$$dV^i = \mu(p^i) [-M^i d\pi + \tau^i \pi dM^i + (r^i K_0 \phi_{G^i}^i - \psi^i) dG^i + r^i K_0 \phi_{G^j}^i dG^j], \quad (13)$$

for  $i = 1, 2$  ( $j \neq i$ ). The above equation delineates the channels through which security and trade policies affect welfare. The first and second terms inside the brackets reflect the familiar terms of trade and volume of trade effects, respectively. By our assumption that countries 1 and 2 are “small” in world markets, the first term will vanish when the effects of trade and security policies are considered. However, if country  $i$  participates in world trade, the second term will not vanish; instead, since country  $i$ 's excess demand for good 2 can be written as  $M^i \equiv M^i(\pi, \tau^i, G^i, G^j)$ , this effect will depend on country  $i$ 's trade and security policies as well as on its adversary  $j$ 's security (but not trade) policy. The third and fourth terms in (13) capture the direct effects of security policies discussed earlier.

Now suppose country  $i$  chooses its trade and security policies simultaneously. Since  $\partial\pi/\partial\tau^i = 0$  by virtue of the fact that country  $i$  is “small,” we have  $dM = (\partial M^i/\partial\tau^i)d\tau^i$ , where  $\partial M^i/\partial\tau^i < 0$ . If  $M_i > 0$ , an increase in  $\tau^i$ , where initially  $\tau^i > 0$ , reduces tariff revenues and therefore income. By the same token, if  $M^i < 0$ , a decrease in  $\tau^i$  where initially  $\tau^i \leq 0$ , reduces tax revenues. It follows, then, that country  $i$ 's optimal trade policy is necessarily free trade (i.e.,  $\tau^{i*} = 0$ ), which implies the second term in (13) also vanishes. As such, country  $i$ 's optimal security policy coincides with the one described earlier in the context of free trade. Since this argument is true for both adversaries, a policy of free trade coupled with the security policies of previous sections constitute the Nash equilibrium of the extended game of trade and security policies. Hence, when free trade is an option yet Pareto dominated by autarky, countries might find themselves locked into a “prisoner's dilemma” situation, relative to the case where trade is foreclosed. One can also verify that, if security policies are determined prior to trade policies, this result remains intact.

Do trade policy commitments prior to the implementation of security policies alter the

analysis in a substantive way? To explore this possibility, consider a two-stage game, in which countries choose their trade policies in stage 1 and their security policies in stage 2. The key difference here from our analysis of security policies alone in the previous sections is that, in the presence of trade taxes ( $\tau^i$ ), country  $i$ 's optimal security policy will factor in its possible impact on the volume of trade. Starting with stage 2, at an interior solution, country  $i$ 's FOC for welfare maximization becomes

$$\frac{\partial V^i}{\partial G^i} = \mu(p^i) \left[ \tau^i \pi \frac{\partial M^i}{\partial G^i} + r^i K_0 \phi_{G^i}^i - \psi^i \right] = 0, \quad \text{for } i = 1, 2. \quad (14)$$

Keeping in mind that we can write  $M^i \equiv M^i(\pi, \tau^i, G^i, G^j)$ , the effects of trade policy on arming and power can be identified with standard comparative statics exercises performed on (14). Such an analysis, however, would take us off track. For our purposes, the key point is that trade policy precommitments can strategically affect the security policies of small countries.

To proceed, identify with an asterisk (\*) the solution to the above system of equations and let subscript  $T$  indicate the presence of tariffs/taxes. For simplicity and clarity, suppose country  $i$  imports good 2, so that trade policy takes the form of tariffs. Now consider country  $i$ 's stage-1 problem. Appealing to the envelope theorem, the welfare effects of a change in country  $i$ 's trade policy can be summarized as follows:

$$\frac{\partial V_T^i}{\partial \tau^i} = \mu(p^i) \left[ \tau^i \pi \frac{\partial M^i}{\partial \tau^i} + \left( \tau^i \pi \frac{\partial M^i}{\partial G^j} + r^i K_0 \phi_{G^j}^i \right) \frac{\partial G_T^{j*}}{\partial \tau^i} \right] \quad \text{for } i = 1, 2 \quad (j \neq i). \quad (15)$$

The direct effect of an increase in country  $i$ 's tariffs, shown in the first term inside the square brackets of (15), is to reduce its volume of imports (i.e.,  $\partial M^i / \partial \tau^i < 0$ ), which adversely affects income and thus welfare for  $\tau^i > 0$ . However, there also exists a strategic effect, arising from the possible impact of the country's trade policy on the rival country's security policy. Represented by the second term inside the square brackets, this strategic effect has two components. To fix ideas, suppose as before that  $k_2^i > k_1^i$ . Then, an increase in  $G^j$ , which reduces the quantity of land appropriated by country  $i$ , tends to raise its reliance on imports of the land-intensive good 2:  $\partial M^i / \partial G^j > 0$  for  $i = 1, 2$  ( $i \neq j$ ). But, assume further that a more restrictive trade policy by country  $i$  (i.e., an increase in  $\tau^i > 0$ ) induces its adversary to behave less aggressively in security competition (i.e.,  $\partial G_T^{j*} / \partial \tau^i < 0$ ).<sup>59</sup> In this case, the first component of the strategic effect is negative. At the same time,

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<sup>59</sup>Characterizing the commitment effects of trade agreements (multilateral, as under the WTO, or preferential, as under the EU and Mercosur), on security policies is an interesting and important research question in its own right. While the terms of trade effects of such agreements have been emphasized in the literature (e.g., Bagwell and Staiger (2002)), the commitment effects of trade agreements, particularly in the context of security policies could be relevant as well. We leave this issue for future research.

however, the second component is positive because  $\phi_{Gj}^i < 0$  (a direct income effect). While identifying precisely the implications for tariffs requires additional work, one thing is clear. If the strategic welfare effect of trade policy is positive overall, then there exists an additional rationale for precommitted intervention in trade (e.g., trade agreements) for small countries.

## 7 Concluding Remarks

In the decades leading up to World War I, the proportion of world trade to world GDP reached unprecedented magnitudes (O'Rourke and Williamson, 2000). Yet, international conflict ensued with much ferocity and despite expectations to the contrary.<sup>60</sup> Similarly, the expansion of trade in the post World War II era has been spectacular. Still, while interstate conflict might have subsided over this latter period, insecurity and contention continue to flare up in many parts of the world. Whereas not all disputes can be considered to have material causes, there is no doubt that contestation of water resources, land, oil, diamonds and other resources by different countries has at least some role to play in many international disputes and drives the military expenditures and security policies of the countries that are involved in such disputes. Moreover, as Findlay and O'Rourke (2007) show in their magisterial survey of the Eurasia's economic history, military competition for resources and the expansion of world trade were inextricably linked over the whole of the past millenium, and it would be unlikely that such links will be absent in the future.

The extent to which disputed resources or the goods they produce are tradable can thus have implications for the security policies that countries pursue and the costs those countries realize as a result. In turn, these security costs can vary with the trade regime in such a way so as to possibly outweigh the gains from trade. We have examined the issues emanating from such a setting using the neoclassical trade model, augmented by a disputed resource that is costly to contest.

To assess the implications of trade openness for arming, power, trade patterns, and welfare, we have analyzed two polar trade regimes: autarky and free trade. The key difference between these regimes for small countries is that prices are endogenously determined under autarky but not under free trade. Under either regime, the most affluent country need not be the most powerful one. Still, arming incentives are trade-regime dependent. One striking result is that there exist circumstances under which free trade equalizes arming incentives across contending states and thereby levels the playing field in arms competition.

Depending on the level of world prices, free trade in consumption goods might intensify arming incentives to generate additional *security costs that swamp the traditional gains from*

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<sup>60</sup>The prediction before World War I, for example, that war was impossible or unthinkable—because Britain and Germany had become so economically interdependent that conflict was viewed as "commercial suicide" (Angell, 1933)—was flatly contradicted by experience.

*trade and thus render autarky* more desirable for one or both rival states. Furthermore, in the presence of international conflict, a country's apparent comparative advantage can differ from its natural comparative advantage (absent insecurity) and comparisons of autarkic prices to world prices could be inappropriate predictors of trade patterns. This finding suggests that empirical work aiming to relate trade volumes to fundamentals would be incomplete if it did not include insecurity and contestation of resources.

With a focus on small countries, the analysis ignores the possible terms of trade effects in security policy that would be especially relevant for those countries having monopoly or monopsony power in world trade. Accordingly, it would be worthwhile to extend the analysis in that direction. One important difference from the current setting is that free trade in consumption goods need not equalize arming incentives even when factor prices are equalized and guns production is unconstrained. Another difference is that trade and security policies could be used simultaneously, the former to balance the terms of trade effects with the volume of trade effects, and the latter for security considerations. Such an extension of the basic model would be a rich and promising environment within which to explore the implications of policy interactions, including the economics of free trade agreements and their possible interactive effects with national security.

The model presented here could be fruitfully extended in a number of other ways. For example, the analysis could assign an active role to the rest of the world (ROW). Furthermore, the analysis could be generalized to situations where trade does not necessarily result in the equalization of factor prices, and thus give a meaningful role to the possibility of trade in arms. Last but not least, policy objectives could be specified to consider the role of politics.

Ultimately, solving the problem of insecurity entails the design and development of commitment devices that can reduce, and possibly eliminate, the need to arm. Such commitment devices, however, are not easy to come by and, judging from particular historical instances, they take a long time to develop. Europe is a good example of this. After the experience of the two world wars, the original six members of the European Community slowly began to develop mechanisms of economic integration that were, in large part, institutions of conflict management. This twin process of economic integration and conflict resolution through bureaucratic and political struggle, instead of conflict in the battlefield, is ongoing and far from complete, even after a century of tribulations. Trade openness and, more generally, economic interdependence might help to ameliorate conflict, but it would be naive to think that promoting such interdependence could achieve this by itself.

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## A Appendix

We first present several useful properties of the CSF in (1). For convenience, define  $f_i \equiv f(G^i)$ , where from our previous assumptions  $f'_i > 0$  and  $f''_i \leq 0$ . Now, differentiate  $\phi^i(G_i, G_j)$  with respect to its arguments,  $G^i$  and  $G^j$  for  $i = 1, 2$  ( $j \neq i$ ), to obtain

$$\phi_{G^i}^i = \frac{f'_i f_j}{(f_1 + f_2)^2} > 0 \quad (\text{A.1})$$

$$\phi_{G^j}^i = -\frac{f'_j f_i}{(f_1 + f_2)^2} < 0 \quad (\text{A.2})$$

$$\phi_{G^i G^i}^i = \frac{f_j}{(f_1 + f_2)^3} [f''_i (f_1 + f_2) - 2(f'_i)^2] < 0 \quad (\text{A.3})$$

$$\phi_{G^i G^j}^i = \frac{(f_i - f_j) f'_i f'_j}{(f_1 + f_2)^3} \begin{cases} \leq 0 & \text{if } G^i \geq G^j \\ \geq 0 & \text{if } G^i < G^j \end{cases} \quad (\text{A.4})$$

**Lemma A1.** *If production in a country is diversified (i.e.,  $X_j^i > 0$ , for both countries  $i = 1, 2$  and both goods  $j = 1, 2$ ), then*

- (a)  $\frac{\partial \omega^i / \partial p^i}{\omega^i / p^i} \leq 0$  if  $k_2^i \geq k_1^i$ ;
- (b)  $\frac{\partial RS^i / \partial k_X^i}{RS^i / k_X^i} \geq 0$  if  $k_2^i \geq k_1^i$ ;
- (c)  $\frac{\partial RS^i / \partial p^i}{RS^i / p^i} > 0$ .

**Proof:** Following Jones (1965), we denote the shares of factor  $h = K, L$  in the cost of producing good  $j = 1, 2$  by  $\theta_{hj}^i$ :  $\theta_{Kj}^i = r^i a_{Kj}^i / c_j^i$  and  $\theta_{Lj}^i = w^i a_{Lj}^i / c_j^i$ . Similarly,  $\theta_{KG}^i \equiv r^i \psi_r^i / \psi^i$  and  $\theta_{LG}^i \equiv w^i \psi_w^i / \psi^i$  indicate the corresponding cost shares in guns. Now denote the amount of land and labor employed in industry  $j = 1, 2$  respectively by  $K_j^i$  and  $L_j^i$ . Then, these quantities as a fraction of resources remaining once land and labor for producing guns have been set aside are respectively indicated by  $\lambda_{Kj}^i \equiv K_j^i / K_X^i$  and  $\lambda_{Lj}^i \equiv L_j^i / L_X^i$ . Finally, let a percentage change be indicated by a hat ( $\hat{\cdot}$ ) over the associated variable (e.g.,  $\hat{x} = \frac{dx}{x}$ ).

*Part (a):* Noting that  $c_1 = 1$  and  $c_2 = p$ , differentiation of (2) and (3) totally gives

$$\begin{aligned} \frac{\partial c_1^i}{\partial w^i} dw^i + \frac{\partial c_1^i}{\partial r^i} dr^i = 0 & \implies a_{L1}^i \frac{w^i}{c_1^i} \frac{dw^i}{w^i} + a_{K1}^i \frac{r^i}{c_1^i} \frac{dr^i}{r^i} = 0 \\ \frac{\partial c_2^i}{\partial w^i} dw^i + \frac{\partial c_2^i}{\partial r^i} dr^i = dp^i & \implies a_{L2}^i \frac{w^i}{c_2^i} \frac{dw^i}{w^i} + a_{K2}^i \frac{r^i}{c_2^i} \frac{dr^i}{r^i} = \frac{dp^i}{p^i}. \end{aligned}$$

With the definitions given above, we can write this system of equations as

$$\begin{pmatrix} \theta_{L1}^i & \theta_{K1}^i \\ \theta_{L2}^i & \theta_{K2}^i \end{pmatrix} \begin{pmatrix} \hat{w}^i \\ \hat{r}^i \end{pmatrix} = \begin{pmatrix} 0 \\ \hat{p}^i \end{pmatrix}. \quad (\text{A.5})$$

Now, since  $\sum_{h=K,L} \theta_{hj} = 1$  for  $j = 1, 2$  by definition, the determinant of the coefficient matrix above, denoted by  $|\theta^i|$ , can be written as

$$|\theta^i| \equiv \theta_{K2}^i - \theta_{K1}^i = \theta_{L1}^i - \theta_{L2}^i = \frac{\omega^i (k_2^i - k_1^i)}{(\omega^i + k_1^i)(\omega^i + k_2^i)} \geq 0 \text{ if } k_2^i \geq k_1^i.$$

Then, solving (A.5) for the  $p^i$ -induced changes in factor prices yields

$$\frac{p^i w_p^i}{w^i} = -\frac{\theta_{K1}^i}{|\theta^i|} \quad \text{and} \quad \frac{p^i r_p^i}{r^i} = \frac{\theta_{L1}^i}{|\theta^i|} \quad (\text{A.6})$$

From (A.6), then, we have  $p^i \omega_p^i / \omega^i = p^i w_p^i / w^i - p^i r_p^i / r^i = -1/|\theta^i| \leq 0$  when  $k_2^i \geq k_1^i$ , which completes the proof to part (a).

*Parts (b) and (c):* Note first that we can combine (4) and (5) to obtain  $\lambda_{L1}^i k_1^i + \lambda_{L2}^i k_2^i = k_X^i$ . Then, following the strategy above in part (a), we differentiate (4) and (5) totally and solve the resulting system of equations to obtain

$$\begin{aligned} \widehat{X}_1^i &= \frac{1}{|\lambda^i|} \left( -\lambda_{L2}^i \widehat{K}_X^i + \lambda_{K2}^i \widehat{L}_X^i \right) - \frac{1}{|\lambda^i| |\theta^i|} (\lambda_{L2}^i \delta_K^i + \lambda_{K2}^i \delta_L^i) \widehat{p}^i \\ \widehat{X}_2^i &= \frac{1}{|\lambda^i|} \left( +\lambda_{L1}^i \widehat{K}_X^i - \lambda_{K1}^i \widehat{L}_X^i \right) + \frac{1}{|\lambda^i| |\theta^i|} (\lambda_{L1}^i \delta_K^i + \lambda_{K1}^i \delta_L^i) \widehat{p}^i, \end{aligned}$$

where  $\delta_K^i \equiv \lambda_{K1}^i \theta_{L1}^i \sigma_1^i + \lambda_{K2}^i \theta_{L2}^i \sigma_2^i > 0$  and  $\delta_L^i \equiv \lambda_{L1}^i \theta_{K1}^i \sigma_1^i + \lambda_{L2}^i \theta_{K2}^i \sigma_2^i > 0$ , with  $\sigma_j^i = c_j^i \frac{\partial^2 c_j^i}{\partial w^i \partial r^i} / \frac{\partial c_j^i}{\partial w^i} \frac{\partial c_j^i}{\partial r^i}$  being the (absolute value of the) elasticity of substitution between land and labor in industry  $j$ ;  $|\lambda^i|$  denotes the determinant of the coefficient matrix obtained from differentiating (4) and (5); recalling  $\sum_{j=1,2} \lambda_{hj}^i = 1$  for  $h = K, L$ , we have

$$|\lambda^i| \equiv \lambda_{K2}^i - \lambda_{L2}^i = \lambda_{L1}^i - \lambda_{K1}^i = \frac{(k_2^i - k_X^i)(k_X^i - k_1^i)}{k_X^i (k_2^i - k_1^i)} \geq 0 \text{ if } k_2^i \geq k_1^i.$$

Now, observe that  $\widehat{k}_X^i = \widehat{K}_X^i - \widehat{L}_X^i$ , which implies

$$\widehat{RS}^i = \widehat{X}_2^i - \widehat{X}_1^i = \frac{1}{|\lambda^i|} \widehat{k}_X^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \widehat{p}^i. \quad (\text{A.7})$$

Inspection of (A.7) confirms parts (b) and (c).  $\parallel$

In Lemma A1 the residual land/labor ratio,  $k_X^i$ , is treated as exogenous. However, from (6) it is clear that  $k_X^i$  depends on price, guns, and factor supplies. The next lemma clarifies this dependence.

**Lemma A2.** Let  $k^i \equiv \frac{K^i + \phi^i K_0}{L^i}$  and suppose the production of consumption goods is

diversified. Then  $k_X^i = k_X^i(p^i, G^i, G^j; K_0, K^i, L^i)$  and

- (a)  $\frac{\partial k_X^i}{\partial p^i} \geq 0$  if  $k_2^i \geq k_1^i$ ;
- (b)  $\frac{\partial k_X^i}{\partial G^i} > 0$ ,  $\forall G^i$  that satisfy  $K_0 \phi_{G^i}^i - \psi^i/r^i + \varepsilon > 0$ , for some  $\varepsilon > 0$ ;
- (c)  $\frac{\partial k_X^i}{\partial G^j} < 0$ ,  $\forall i \neq j$ ;
- (d)  $\frac{\partial k_X^i}{\partial G^i} + \frac{\partial k_X^i}{\partial G^j} \geq 0$  if  $k^i \geq k_G^i$  whenever  $G^i = G^j$ ,  $\forall i \neq j$ ;
- (e)  $\frac{\partial k_X^i}{\partial L^i} < 0$  and  $\frac{\partial k_X^i}{\partial K_0} > 0$ ,  $\frac{\partial k_X^i}{\partial K^i} > 0$ .

**Proof:** Denote country  $i$ 's land and labor shares in total net income  $R^i$  by with  $s_K^i \equiv r^i K_X^i/R^i$  and  $s_L^i \equiv w^i L_X^i/R^i$ , and let  $\sigma_G^i = \psi^i \psi_{wr}^i / \psi_w^i \psi_r^i$  be the (absolute value of the) elasticity of substitution between land and labor in the guns sector. Total differentiation of (6), using the linear homogeneity of  $\psi^i$ , yields

$$\begin{aligned} \widehat{k}_X^i &= \left( \frac{\psi^i \theta_{LG}^i \theta_{KG}^i \sigma_G^i G^i}{|\theta^i| R^i s_K^i s_L^i} \right) \widehat{p}^i + \frac{\psi^i}{R^i s_K^i s_L^i} \left[ \frac{r^i s_L^i}{\psi^i} \left( K_0 \phi_{G^i}^i - \frac{\psi^i}{r^i} \right) + \theta_{LG}^i \right] dG^i \\ &\quad + \frac{r^i K_0 \phi_{G^j}^i}{R^i s_K^i} dG^j + \frac{\phi^i dK_0}{K_X^i} + \frac{dK^i}{K_X^i} - \frac{dL^i}{L_X^i}. \end{aligned} \quad (\text{A.8})$$

Parts (a)–(c) & (e): The proofs follow from (A.8).

Part (d): Suppose  $G^i = G^j$  so that  $\phi_{G^i}^i = -\phi_{G^j}^i$ . Now use  $dG^i = dG^j$  in (A.8) to obtain

$$\frac{\partial k_X^i / \partial G^i}{k_X^i} = \frac{\psi^i (\theta_{LG}^i - s_L^i)}{R^i s_K^i s_L^i}.$$

Using the definitions of  $\theta_{LG}^i$ ,  $s_L^i$ , and  $R^i$ , along with those for  $K_X^i$  and  $L_X^i$  in (4) and (5), tedious algebra verifies the following transformation of this relationship:

$$\frac{\partial k_X^i / \partial G^i}{k_X^i} = \frac{\psi^i (\theta_{LG}^i - s_L^i)}{R^i s_K^i s_L^i} = \frac{\psi_w^i (k_X^i - k_G^i)}{K_X^i} = \frac{\psi_w^i L^i (k^i - k_G^i)}{K_X^i L_X^i}. \quad (\text{A.9})$$

Inspection of this expression confirms part (d).  $\parallel$

### Proof of Lemma 1.

Part (a): Differentiate (9) with respect to  $G^i$  and use (A.3) to obtain

$$V_{G^i G^i}^i = \mu^i r^i K_0 \phi_{G^i G^i}^i < 0. \quad (\text{A.10})$$

Part (b): Differentiation of (8) with respect to  $G^j$  and use of (A.4) gives

$$V_{G^i G^j}^i = \mu^i r^i K_0 \phi_{G^i G^j}^i \geq 0 \quad \text{if} \quad G^i \geq G^j. \quad (\text{A.11})$$

Part (c): Recall that  $\psi^i/r^i = \psi(\omega^i, 1)$ , implying that  $\partial(\psi^i/r^i)/\partial p^i = \psi_{\omega^i}^i \omega_p^i$ . Then, differentiating (8) with respect to price and evaluating the resulting expression at the optimum gives (by Lemma A1(a))

$$\begin{aligned} V_{G^i p^i}^i &= -\mu^i r^i \frac{\partial(\psi^i/r^i)}{\partial p^i} = -\mu^i \left( \frac{\psi^i}{p^i} \right) \left( \frac{w^i \psi_w^i}{\psi^i} \right) \left( \frac{p^i \omega_p^i}{\omega^i} \right) \\ &= \mu^i \left( \frac{\psi^i}{p^i} \right) \frac{\theta_{LG}^i}{|\theta^i|} \geq 0 \quad \text{if } k_2^i \geq k_1^i. \end{aligned} \quad (\text{A.12})$$

Part (d): This is a standard property of indirect (trade) utility functions.  $\parallel$

**Proof of Lemma 2.** In (9) we can use (A.1) and (A.2) and the fact that  $MC^i = \psi^i/r^i = \psi(\omega(p^i), 1)$  to obtain

$$\frac{MB^1}{MB^2} = \frac{f'(G^1)/f(G^1)}{f'(G^2)/f(G^2)} = \frac{\psi(\omega(p^1), 1)}{\psi(\omega(p^2), 1)} = \frac{MC^1}{MC^2}$$

where, for simplicity, we have omitted stars. Now if  $k_2^i > k_1^i$ , then by Lemma A1(a),  $\psi(\omega(p^i), 1)$  is decreasing in  $p^i$ ; therefore, if  $p^1 \geq p^2$ ,  $MC^1/MC^2 \leq 1$ , which by the above equation requires  $MB^1/MB^2 \leq 1$ ; in turn, the concavity of  $f(\cdot)$  implies  $G^1 \geq G^2$ . Alternatively, if  $k_2^i < k_1^i$ ,  $\psi_{\omega^i}^i \omega_p^i > 0$  (Lemma A1(a)), which implies  $MC^1/MC^2 \geq 1$  if  $p^1 \geq p^2$ . But then  $MB^1/MB^2 \geq 1$  which requires  $G^1 \leq G^2$ .  $\parallel$

**Proof of Lemma 3.** Let  $\sigma_D^i > 0$  be the elasticity of substitution in consumption. Focusing on percentage changes, note that  $\widehat{RD}^i = -\sigma_D^i \widehat{p}^i$  and that the expression for  $\widehat{RS}^i$  is given in (A.7). Totally differentiating (10) and rearranging terms gives

$$\widehat{RD}^i = \widehat{RS}^i \implies \left( \sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \right) \widehat{p}^i + \frac{1}{|\lambda^i|} \widehat{k}_X^i = 0.$$

The above relation and the definition of  $|\lambda^i|$  reveal that  $p^i$  is decreasing (increasing) in  $k_X^i$  if  $k_2^i > k_1^i$  ( $k_2^i < k_1^i$ ). Combining (A.8), which gives an expression for  $\widehat{k}_X^i$ , with the above expression gives

$$\widehat{p}_A^i = -\frac{1}{\Delta^i |\lambda^i|} \left[ \frac{\partial k_X^i / \partial G^i}{k_X^i} dG^i + \frac{\partial k_X^i / \partial G^j}{k_X^i} dG^j + \frac{\phi^i dK_0}{K_X^i} + \frac{dK^i}{K_X^i} - \frac{dL^i}{L_X^i} \right] \quad (\text{A.13})$$

where  $\Delta^i \equiv \sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} + \frac{\psi^i \theta_{LG}^i \theta_{KG}^i \sigma_G^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} G^i > 0$ . The proofs to parts (a)–(d) now follow from (A.13) and (A.8).  $\parallel$

**Proof of Theorem 1.**

*Existence:* We establish existence of equilibrium in pure strategies, by showing that every country  $i$ 's payoff function  $V_A^i$  is strictly quasi-concave in its strategy,  $G^i$ . To do so, it is sufficient to show either that  $V_A^i$  is strictly monotonic in  $G^i$  or that  $V_A^i$  is first strictly increasing and then strictly decreasing over the agent's strategy space.

Let  $F(K_G^i, L_G^i)$  be the production function for guns that is dual to the unit cost function  $\psi(w^i, r^i)$  and define  $\bar{G}^i \equiv F(K^i, L^i)$  as the level of guns produced with the country's entire secure endowments of land and labor. Country  $i$ 's strategy space is  $[0, \bar{G}^i]$ . For any  $G^j \in [0, \bar{G}^j]$ , if  $G^i = \bar{G}^i$ , country  $i$  ( $\neq j$ ) will not be able to produce either of the consumption goods; therefore,  $V_A^i(\bar{G}^i, G^j) < V_A^i(G^i, G^j)$  for any  $G^i \in [0, \bar{G}^i)$  which implies that, under autarky, no country will use all of its resources to produce arms. Furthermore, since  $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$  by assumption, we must have  $\partial V_A^i / \partial G^i > 0$  as  $G^i \rightarrow 0$ . By the continuity of  $V_A^i$  in  $G^i$ , there will exist a best response function for each country  $i$ ,  $B_A^i(G^j) \equiv \min\{G^i \in (0, \bar{G}^i) \mid \partial V_A^i / \partial G^i = 0\}$ , with the property that  $\partial V_A^i / \partial G^i > 0 \forall G^i < B_A^i(G^j)$ . Thus, to establish strict quasi-concavity of  $V_A^i$  in  $G^i$  we need only to prove that  $\partial V_A^i / \partial G^i < 0, \forall G^i > B_A^i(G^j)$ .

Suppose, on the contrary, that  $\partial V_A^i / \partial G^i \geq 0$ . Since  $V_A^i$  must eventually fall to  $V_A^i(\bar{G}^i, G^j)$ , this supposition implies that  $V_A^i$  must attain a local minimum at some  $G^i > B_A^i(G^j)$ , which would imply that  $\partial^2 V_A^i / (\partial G^i)^2 > 0$ . We now establish that this is not possible. Recalling that  $p_A^i = p_A^i(G^i, G^j)$  under autarky and that  $\omega^i = \omega(p^i)$ , we differentiate (9) with respect to  $G^i$  and apply (9) to the resulting expression to obtain

$$\frac{\partial^2 V_A^i}{(\partial G^i)^2} = [V_{G^i G^i}^i]_{p^i=p_A^i}^{(-)} + [V_{G^i p^i}^i]_{p^i=p_A^i}^{(\pm)} \left( \frac{\partial p_A^i}{\partial G^i} \right)^{(\mp)} < 0. \quad (\text{A.14})$$

By Lemma 1(a), the first term in the RHS of the above expression is negative regardless of the ranking of factor intensities. Furthermore, by Lemmas 1(c) and 3(b), the product of the expressions in the second term will also be negative.<sup>61</sup> It follows that  $\partial^2 V_A^i / (\partial G^i)^2 < 0$  at any  $G^i$  where  $\partial V_A^i / \partial G^i = 0$  regardless of the ranking of factor intensities. This proves  $B_A^i(G^j)$  is unique and establishes the existence of a pure-strategy equilibrium.

*Uniqueness:* Having already established that guns production is bounded (i.e.,  $B_A^i(G^j) \in (0, \bar{G}^i)$ ) for  $i = 1, 2$  ( $j \neq i$ ), we can now establish uniqueness of equilibrium by showing that, at any equilibrium point, the determinant of the Jacobian of the net marginal payoffs in (9) is positive—i.e.,  $|J| = \frac{\partial^2 V_A^1}{(\partial G^1)^2} \frac{\partial^2 V_A^2}{(\partial G^2)^2} - \frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} \frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} > 0$  (Kolstad and Mathiesen, 1987).

Consider an equilibrium point where  $G_A^{1*} = B_A^1(G_A^{2*})$  and  $G_A^{2*} = B_A^2(G_A^{1*})$ . From the expression for  $|J|$ , it can be seen that, if the product of the slopes of the two countries'

<sup>61</sup>In (A.14) and below, the top signs in “ $\pm$ ” and “ $\mp$ ” apply when  $k_2^i > k_1^i$  and the bottom signs apply when  $k_2^i < k_1^i$ .

best response functions,  $(\partial B_A^1/\partial G^2)(\partial B_A^2/\partial G^1)$ , is less than 1 at  $(G_A^{1*}, G_A^{2*})$ , then  $|J| > 0$ , implying that this equilibrium is unique. The slope of country  $i$ 's best-response function can be written as

$$\frac{\partial B_A^i}{\partial G^j} = -\frac{\partial^2 V_A^i/\partial G^i \partial G^j}{\partial^2 V_A^i/(\partial G^i)^2} = -\frac{[V_{G^i G^j}^i]_{p^i=p_A^i} + [V_{G^i p^i}^i]_{p^i=p_A^i} \left(\frac{\partial p_A^i}{\partial G^j}\right)^{(\pm)}}{[V_{G^i G^i}^i]_{p^i=p_A^i} + [V_{G^i p^i}^i]_{p^i=p_A^i} \left(\frac{\partial p_A^i}{\partial G^i}\right)^{(\pm)}}. \quad (\text{A.15})$$

Since  $\partial^2 V_A^i/(\partial G^i)^2 < 0$  as shown above, the sign of  $\partial B_A^i/\partial G^j$  is determined by the sign of  $\partial^2 V_A^i/\partial G^i \partial G^j$  shown in the numerator of (A.15). Now, by Lemmas 1(c) and 3(b), the second term of the numerator of the RHS of this expression is always positive. By Lemma 1(b), the first term in the numerator is positive if  $B_A^i(G^j) > G^j$  (also see (A.11)), in which case  $G^i$  is a strategic complement for  $G^j$ . However, if  $B_A^i(G^j) < G^j$ , then the first term is negative. Thus, when  $B_A^i(G^j)$  is sufficiently smaller than  $G^j$ ,  $G^i$  can become a strategic substitute for  $G^j$ . Furthermore, since  $\phi_{G^1 G^2}^1 = -\phi_{G^2 G^1}^2$  (see (A.4)), it follows from (A.11) that  $\text{sign}[V_{G^1 G^2}^1]_{p^1=p_A^1} = -\text{sign}[V_{G^2 G^1}^2]_{p^2=p_A^2}$ . Therefore, we have two possibilities to consider. Either (i)  $\partial B_A^i/\partial G^j > 0$  and  $\partial B_A^j/\partial G^i \leq 0$  for  $i = 1, 2$  ( $j \neq i$ ); or, (ii)  $\partial B_A^i/\partial G^j > 0$  for  $i = 1, 2$  ( $j \neq i$ ). It is easy to check that, in case (i),  $(\partial B_A^1/\partial G^2)(\partial B_A^2/\partial G^1) < 1$  and therefore  $|J| > 0$ . Turning to case (ii), we now establish the existence of (sufficient) conditions that ensure  $(\partial B_A^1/\partial G^2)(\partial B_A^2/\partial G^1) < 1$  and thus  $|J| > 0$ .<sup>62</sup>

To proceed, use (A.8) and (A.13), applying (9), to obtain

$$\frac{\partial p_A^i}{\partial G^i} = -\frac{p_A^i \psi^i \theta_{LG}^i}{\Delta^i |\lambda^i| R^i s_K^i s_L^i} \quad \text{and} \quad \frac{\partial p_A^i}{\partial G^j} = \frac{p_A^i \psi^i}{\Delta^i |\lambda^i| R^i s_K^i s_L^i} \left( -\frac{\phi_{G^j}^i}{\phi_{G^i}^i} \right) s_L^i.$$

The above expressions together with (A.10), (A.11), and (A.12) can be substituted into (A.15) to obtain  $\partial B_A^i/\partial G^j = -(\phi_{G^j}^i/\phi_{G^i}^i)\Gamma_A^i$ , where

$$\Gamma_A^i = \left[ -\frac{\phi_{G^i G^j}^i \Delta^i}{\phi_{G^j}^i} + \frac{\psi^i \theta_{LG}^i s_L^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} \right] \Bigg/ \left[ -\frac{\phi_{G^i G^i}^i \Delta^i}{\phi_{G^i}^i} + \frac{\psi^i (\theta_{LG}^i)^2}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} \right]. \quad (\text{A.16})$$

From equations (A.1) and (A.2), we have  $(\phi_{G^2}^1/\phi_{G^1}^1)(\phi_{G^1}^2/\phi_{G^2}^2) = 1$ , implying  $(\partial B_A^1/\partial G^2) \cdot (\partial B_A^2/\partial G^1) = \Gamma_A^1 \Gamma_A^2$ ; therefore, if  $\Gamma_A^i \in (0, 1)$  for  $i = 1, 2$ , then  $|J| > 0$ . In case (ii), both the numerator and the denominator of  $\Gamma_A^i$  are positive, so  $\Gamma_A^i > 0$ . Now define  $\eta^i \equiv G^i [-\phi_{G^i G^i}^i/\phi_{G^i}^i + \phi_{G^i G^j}^i/\phi_{G^j}^i]$ . From (A.1)–(A.4),  $\eta^i = G^i [f'_i/f_i - f''_i/f'_i] > 0$ . Then, subtracting the numerator of  $\Gamma_A^i$  from its denominator while using the definition of  $\Delta^i$

<sup>62</sup>Note that, in case (ii),  $|J| > 0$  is also the condition for local stability of equilibrium.

shown below (A.13) gives the following:

$$\frac{\eta^i}{G^i} \left( \sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \right) + \frac{\psi^i \theta_{LG}^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} (\theta_{LG}^i + \theta_{KG}^i \sigma_G^i \eta^i - s_L^i). \quad (\text{A.17})$$

Clearly, a sufficient condition for  $\Gamma_A^i < 1$  is that (A.17) is positive. Inspection of (A.17) reveals that this is almost always true. Since the first term and the coefficient in front of the second term are unambiguously positive, a sufficient (but hardly necessary) condition for  $\Gamma_A^i < 1$  is  $\theta_{LG}^i + \theta_{KG}^i \sigma_G^i \eta^i - s_L^i \geq 0$  or equivalently  $\theta_{LG}^i (1 - s_L^i) + \theta_{KG}^i (\sigma_G^i \eta^i - s_L^i) \geq 0$ . This condition is satisfied under a wide range of circumstances,<sup>63</sup> including: (i)  $\sigma_G^i \eta^i \geq s_L^i$ , which requires arms inputs not to be close complements; and (ii)  $\theta_{LG}^i \geq s_L^i$  (or, by (A.9),  $k^i > k_G^i$ ), which requires the guns sector to be sufficiently labor-intensive, regardless of the degree of substitutability between inputs in arms. Either condition, along with the boundary conditions established above, ensure uniqueness of equilibrium.  $\parallel$

**Proof of Proposition 1.** Since the logic behind part (a) was outlined in the main text, here we prove part (b). A redistribution of a secure resource from country  $j$  to country  $i (\neq j)$  expands (contracts) the “recipient” (“donor”) country’s resource endowment. Differentiating country  $i$ ’s FOC condition in (9) appropriately gives

$$\frac{\partial^2 V_A^i}{(\partial G^i)^2} dB_A^i + \frac{\partial^2 V_A^i}{\partial G^i \partial H^i} dH^i = 0 \implies \frac{dB_A^i}{dH^i} = -\frac{\partial^2 V_A^i / \partial G^i \partial H^i}{\partial^2 V_A^i / (\partial G^i)^2}$$

for  $H = L, K$ . Since  $\partial^2 V_A^i / (\partial G^i)^2 < 0$ , we have  $\text{sign}[dB_A^i / dH^i] = \text{sign}[\partial^2 V_A^i / \partial G^i \partial H^i]$ . Differentiation of (9) yields

$$\frac{\partial^2 V_A^i}{\partial G^i \partial H^i} = \left[ V_{G^i p^i}^i \right]_{p^i = p_A^i} \frac{dp_A^i}{dH^i}. \quad (\text{A.18})$$

From Lemma 1(c) and Lemma 3(d), it follows that, regardless of the ranking of  $k_1^i$  and  $k_2^i$ ,  $dB_A^i / dL^i > 0$  whereas  $dB_A^i / dK^i < 0$ . The signs of these derivatives imply that a transfer of labor from one country to another increases (decreases) arms production by the recipient (donor) for any given arms choice by the rival; yet, a transfer of land decreases (increases) arms production by the recipient (donor). Thus, if we start with an arbitrary secure endowment configuration in  $\mathcal{S}^0$  and transfer a small amount of labor from country  $j$  to country  $i$  or land from country  $i$  to country  $j$ , so that we end up somewhere in  $\mathcal{S}^i$ , the properties of best-response functions ensure that at the new equilibrium we will necessarily

<sup>63</sup>If, for example, the production function for guns is Cobb-Douglas and the CSF assumes the Tullock form (i.e.,  $f(G^i) = (G^i)^\gamma$ ,  $\forall \gamma \in (0, 1]$ ), then  $\sigma_G^i = 1$  and  $\eta^i = 1$ , thus implying that the sufficient condition simplifies to  $1 - s_L^i \geq 0$ , which is always satisfied.

have  $G_A^{i*} > G_A^{j*}$ . By Lemma 2, we must also have  $p_A^{i*} \geq p_A^{j*}$  if  $k_2^i \geq k_1^i$ .  $\parallel$

**Proof of Proposition 2.** To identify the effects of endowment changes on equilibrium security policies we differentiate the FOCs in (9) and solve the resulting system of equations to obtain

$$\begin{pmatrix} dG_A^{1*} \\ dG_A^{2*} \end{pmatrix} = \frac{1}{|J|} \begin{pmatrix} \frac{\partial^2 V_A^2}{(\partial G^2)^2} & -\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} \\ -\frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} & \frac{\partial^2 V_A^1}{(\partial G^1)^2} \end{pmatrix} \begin{pmatrix} -\frac{\partial^2 V_A^1}{\partial G^1 \partial H^1} dH^1 \\ -\frac{\partial^2 V_A^2}{\partial G^2 \partial H^2} dH^2 \end{pmatrix} \quad (\text{A.19})$$

for  $H^i = L^i, K^i$  where  $|J| > 0$  and all expressions are evaluated at the equilibrium. Start with an endowment distribution in  $\mathcal{S}^0$ , so that  $G_A^{i*} = \tilde{G}_A^{i*}$  and  $p_A^{i*} = \tilde{p}_A^{i*}$  for  $i = 1, 2$ . At such a distribution and prices, we have the following:

- (i)  $\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} = \frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} > 0$  since by Lemma 1(b)  $V_{G^1 G^2}^1 = V_{G^2 G^1}^2 = 0$  (see (A.15));
- (ii)  $\frac{\partial^2 V_A^1}{(\partial G^1)^2} = \frac{\partial^2 V_A^2}{(\partial G^2)^2} < 0$  by (A.14);
- (iii)  $\frac{\partial B_A^1}{\partial G^2} = -\frac{\partial^2 V_A^1 / \partial G^1 \partial G^2}{\partial^2 V_A^1 / (\partial G^1)^2} = -(\phi_{G^2}^1 / \phi_{G^1}^1) \Gamma_A^1 = \Gamma_A^1 \in (0, 1)$  by (A.15), (A.16) and the related discussion in the proof of Theorem 1; and,
- (iv)  $\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1} = \frac{\partial^2 V_A^2}{\partial G^2 \partial L^2} > 0$  whereas  $\frac{\partial^2 V_A^1}{\partial G^1 \partial K^1} = \frac{\partial^2 V_A^2}{\partial G^2 \partial K^2} < 0$  by (A.18) and the related discussion.

*Part (a):* Consider a small transfer of labor from country 2 to country 1, so that  $-dL^2 = dL^1 > 0$ . Using the above observations with (A.19) yields

$$\frac{dG_A^{1*}}{dL^1} = -\frac{dG_A^{2*}}{dL^1} = \frac{(+)}{|J|} \left[ -\frac{(+)}{(\partial G^1)^2} \right] \left( 1 - \frac{(+)}{\partial G^2} \right) \left( \frac{(+)}{\partial G^1 \partial L^1} \right) > 0.$$

Similar logic for land redistributions shows that  $dG_A^{1*}/dK^1 = -dG_A^{2*}/dK^1 < 0$ .

*Part (b):* This part is established by invoking symmetry and applying part (a) to (11) and an analogous expression for the welfare effects of a change in land.  $\parallel$

### Proof of Proposition 3.

*Part (a):* Following the approach taken in the proof to Proposition 1, we differentiate (9) to identify the effects on the countries' best-response functions:

$$\frac{\partial^2 V_A^i}{(\partial G^i)^2} dB_A^i + \frac{\partial^2 V_A^i}{\partial G^i \partial K_0} dK_0 = 0 \implies \frac{dB_A^i}{dK_0} = -\frac{\partial^2 V_A^i / \partial G^i \partial K_0}{\partial^2 V_A^i / (\partial G^i)^2}, \text{ for } i = 1, 2.$$

Since  $\partial^2 V_A^i / (\partial G^i)^2 < 0$ , we have  $\text{sign}[dB_A^i / dK_0] = \text{sign}[\partial^2 V_A^i / \partial G^i \partial K_0]$ . Now, differentiat-

ing (9) with respect to  $K_0$  and  $K^i$  where  $dK^i = -\phi^i dK_0$  gives

$$\frac{\partial^2 V_A^i}{\partial G^i \partial K_0} \Big|_{dK^i = -\phi^i dK_0} = \left[ V_{G^i K_0}^i \right]_{p^i = p_A^i} + \left[ V_{G^i p^i}^i \right]_{p^i = p_A^i} \frac{dp_A^i}{dK_0} \Big|_{dK^i = -\phi^i dK_0}, \quad (\text{A.20})$$

for  $i = 1, 2$ . The first term, which simplifies as  $\mu(p_A^i) r(p_A^i) \phi_{G^i}^i$ , captures the effect an increase in the degree of insecurity on country  $i$ 's marginal benefit of arming, and is clearly positive. The second term represents the indirect effect through autarkic prices on the country's marginal cost of arming. However, from (A.13) with the constraint that  $dK^i = -\phi^i dK_0$ , this effect is zero. Hence,  $\text{sign}[\partial^2 V_A^i / \partial G^i \partial K_0] = \text{sign}[dB_A^i / dK_0] > 0$ . Moreover, for an initial distribution of secure resources in  $\mathcal{S}^0$ , the effect on the two countries' best response functions will be the same, with a higher level of arming for both. To pin down this effect, we follow the approach taken in the proof to Proposition 2, using (A.20), to find

$$\begin{aligned} \frac{dG_A^{1*}}{dK_0} \Big|_{dK^1 = -\frac{1}{2}dK_0} &= \frac{dG_A^{2*}}{dK_0} \Big|_{dK^2 = -\frac{1}{2}dK_0} \\ &= \frac{(+)}{|J|} \left[ -\frac{\partial^2 V_A^1}{(\partial G^1)^2} \right] \left( 1 - \frac{\partial B_A^1}{\partial G^2} \right) \left[ \frac{\partial^2 V_A^1}{\partial G^1 \partial K_0} \right]_{dK^1 = -\frac{1}{2}dK_0} > 0. \end{aligned} \quad (\text{A.21})$$

*Part (b):* To identify the welfare implications, we extend the welfare decomposition in (8) to include changes in  $K_0$  and  $K^i$ . Imposing the condition that  $dK^i = -\frac{1}{2}dK_0$  and invoking the envelope condition, while noting that  $M^i = 0$  under autarky, gives:

$$\frac{dV_A^i}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} = \mu(\tilde{p}_A^*) r(\tilde{p}_A^*) K_0 \phi_{G^i}^i \frac{dG^j}{dK_0} \Big|_{dK^i = -\frac{1}{2}dK_0} \quad \text{for } i = 1, 2 \ (j \neq i),$$

By (A.21) and the properties of the CSF, this expression is the same for both  $i$  and is negative.  $\parallel$

**Proof of Theorem 2.** The proofs of existence and uniqueness of equilibrium are similar to those in Theorem 1 and are omitted. The proof of symmetry is outlined in the text.  $\parallel$

**Proof of Proposition 4.** The proof is similar to that for Proposition 3 and is thus omitted.  $\parallel$

**Lemma A3.** For given  $\pi$  and a factor distribution in the AES subset of  $\mathcal{S}^i$ , a country's residual land/labor ratio,  $k_X^i = k_X^i(\pi, B_F^i(G^j), G^j; \cdot)$ , will change as follows along its free

trade best-response function,  $B_F^i(G^j)$ , for  $i \neq j$ :

$$\widehat{k}_X^i = \frac{\psi^i}{R^i s_K^i s_L^i} \frac{f_j' f_i}{f_j f_i'} \left[ \left( \frac{f_i - f_j}{f_i + f_j} \right) \theta_{LG}^i - s_L^i \right] dG^j. \quad (\text{A.22})$$

- (a) If  $G^i \leq G^j$ , then  $dk_X^i/dG^j|_{G^i=B_F^i(G^j)} < 0$ ;
- (b) If  $G^i > G^j$ , then  $dk_X^i/dG^j|_{G^i=B_F^i(G^j)} < 0$  almost always. A sufficient (but hardly necessary) condition is  $\theta_{LG}^i < 2s_L^i$ .

**Proof.** Recall from our discussion in connection with Theorem 2, since free trade pins down product and, thus, factor prices,  $dB_F^i/dG^j = -\phi_{G^i G^j}^i / \phi_{G^i G^i}^i$ . Furthermore, observe that country  $i$ 's FOC (9) implies (i)  $r^i K_0 \phi_{G^i}^i = \psi^i$  and (ii)  $r^i K_0 \phi_{G^j}^i = \psi^i \phi_{G^j}^i / \phi_{G^i}^i$ . Then, these applications of (9) to (A.8) with (A.1)–(A.4) and the simplified expression for  $dB_F^i/dG^j$  gives (A.22). Parts (a) and (b) follow from (A.22).  $\parallel$

### Proof of Proposition 7.

*Part (a):* Note that, by the definition of  $\pi_A^i$ ,  $M_F^{i*}(\pi_A^i) = 0$  and observe that the strategic welfare effect (the second term in the RHS of (12)) is negative when  $k_2^i > k_1^i$ .

*Part (b):* By part (a), we have  $dV_F^{i*}(\pi_A^i)/d\pi < 0$ . Furthermore, by Proposition 5, there exists a sufficiently high price,  $\bar{\pi} > \pi_A^i$ , such that  $dG_F^{j*}/d\pi = 0 \forall \pi > \bar{\pi}$ . But then by (12) and the definition of  $\pi_A^i$ , which implies  $M_F^{i*}(\pi) < 0$  when evaluated at  $\pi > \pi_A^i$ , we must have  $dV_F^{i*}/d\pi > 0 \forall \pi > \bar{\pi}$ . Thus, there must exist a price,  $\pi_{\min}^i$ , that minimizes country  $i$ 's welfare and is such that the country exports the land-intensive product:  $\pi_{\min}^i > \pi_A^i$ .  $\parallel$

**Proof of Proposition 10.** Given the focus on identical adversaries (point  $D$  in Figs. 3 and 4), we have the following:

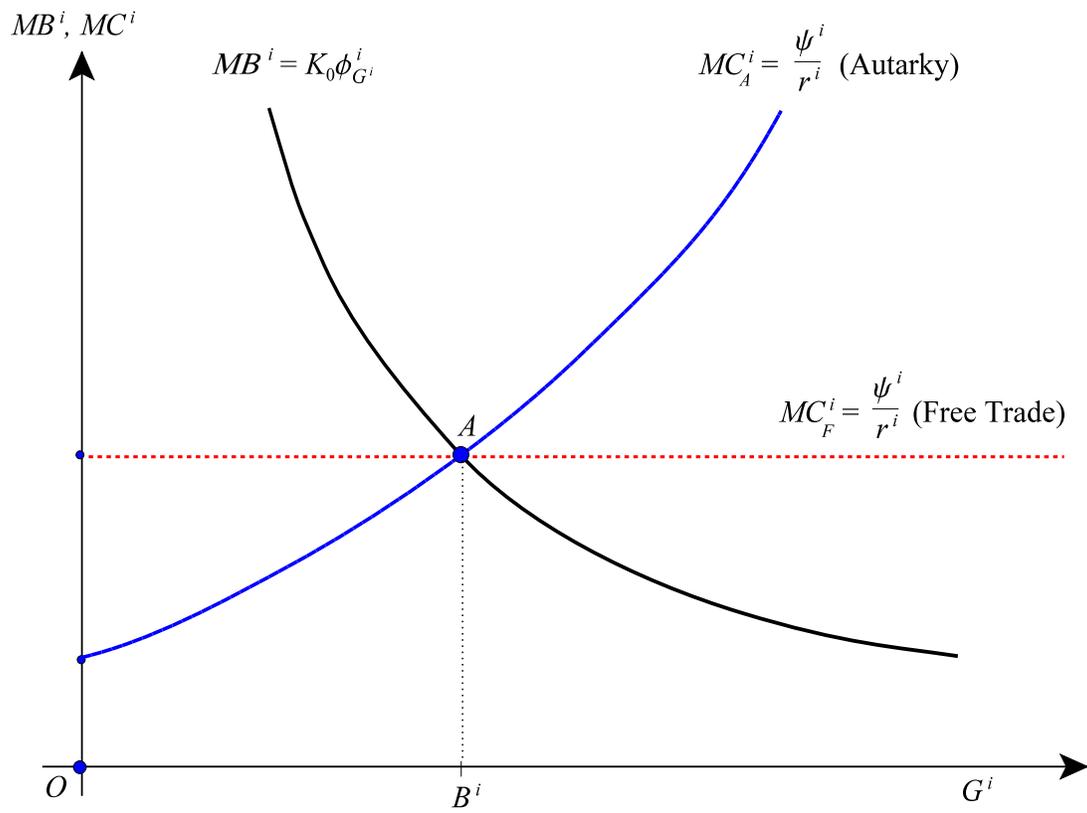
- (i)  $G_F^{i*} = G_F^*$ ,  $V_F^{i*} = V_F^*$ ,  $\pi_A^i = \widetilde{p}_A^* \in (\underline{\pi}, \bar{\pi})$  and  $\pi_{\min}^i = \pi_{\min}$  for  $i = 1, 2$ ;
- (ii) if  $\pi = \widetilde{p}_A^*$ , then  $G_F^* = \widetilde{G}_A^*$  and  $V_F^* = \widetilde{V}_A^*$ ;
- (iii) all adversaries export the land-intensive good if  $\pi \geq \widetilde{p}_A^*$  when  $k_2^i \geq k_1^i$ .

The arming comparisons follow immediately from Proposition 5(a). To establish the welfare results, first note that, because  $\widetilde{p}_A^* \in (\underline{\pi}, \bar{\pi})$ , Proposition 5(a) implies  $dG_F^{i*}(\widetilde{p}_A^*)/d\pi > 0$  for  $i = 1, 2$ . Then, by Proposition 7(b), there exists a  $\pi'$  ( $\geq \widetilde{p}_A^*$  as  $k_2^i \geq k_1^i$ ) that solves  $V_F^{i*}(\pi) = \widetilde{V}_A^*$ . To complete the proof, note that all contestants export the land-intensive good and  $V_F^*(\pi) < \widetilde{V}_A^*$ ,  $\forall \pi \in (\widetilde{p}_A^*, \pi')$  if  $k_2^i > k_1^i$ , or  $V_F^*(\pi) < \widetilde{V}_A^*$ ,  $\forall \pi \in (\pi', \widetilde{p}_A^*)$  if  $k_2^i < k_1^i$ .  $\parallel$

**Proof of Proposition 11.** For concreteness, suppose  $k_2^i > k_1^i$ . Since we consider secure factor distributions in the AES subset of  $\mathcal{S}^0$ , it will necessarily be the case that  $p_A^{i*} = \widetilde{p}_A^*$  for  $i = 1, 2$ , and thus  $\pi_A^i = \widetilde{p}_A^*$ .

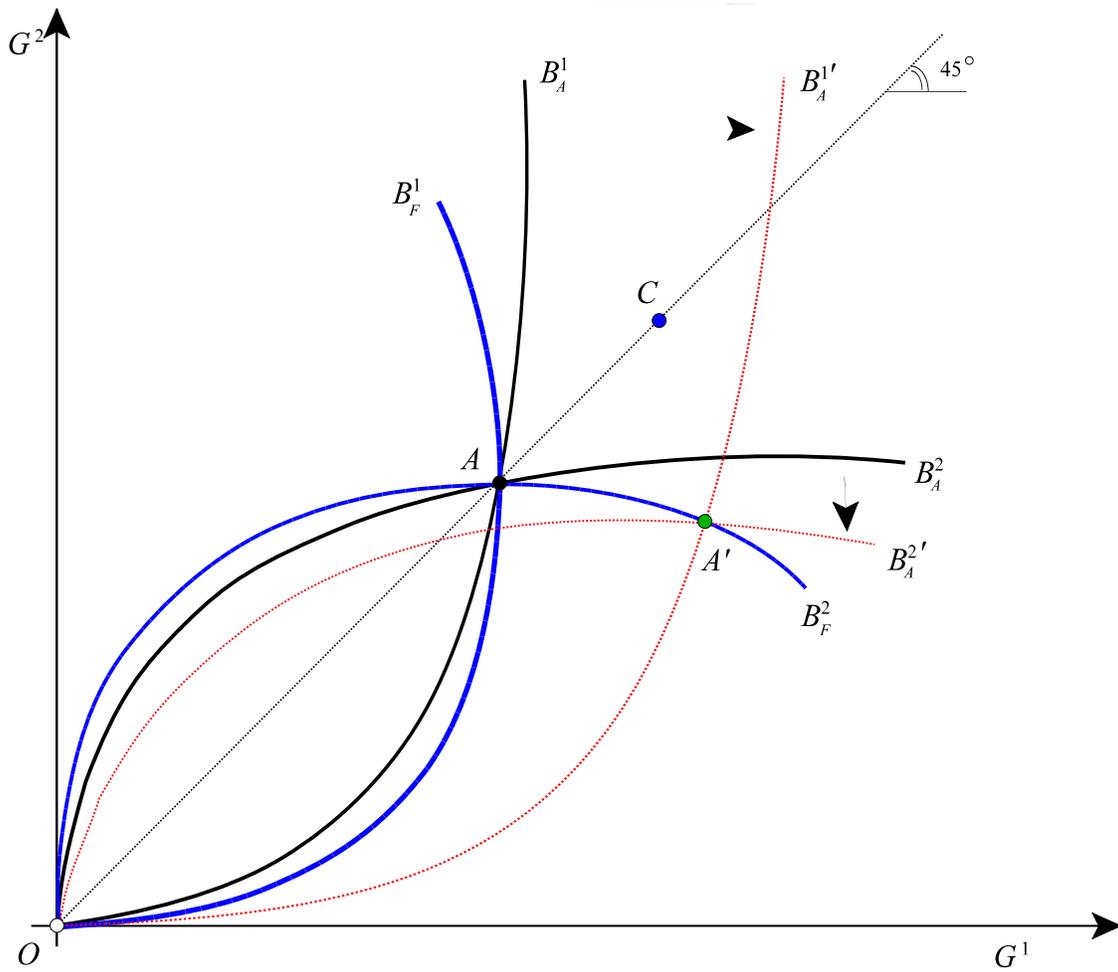
*Part (a):* Since  $\pi_A^i = \tilde{p}_A^*$  for  $i = 1, 2$ ,  $dG_F^{i*}(\tilde{p}_A^*)/d\pi > 0$  as in Proposition 10. By Proposition 7(b), there will thus exist a price  $\pi^{i'}$ ,  $i = 1, 2$  such that  $V_F^{i*}(\pi) < \tilde{V}_A^{i*}$ ,  $\forall \pi \in (\tilde{p}_A^*, \pi^{i'})$ . Now define  $\pi'' = \min\{\pi^{1'}, \pi^{2'}\}$ . It follows that  $V_F^{i*}(\pi) < \tilde{V}_A^{i*}$  for  $i = 1, 2$ ,  $\forall \pi \in (\tilde{p}_A^*, \pi'')$ .

*Part (b):* Starting at an arbitrary distribution in the *AES* subset of  $\mathcal{S}^0$ , transfer a small quantity of labor from country 2 to country 1 (i.e.,  $-dL^2 = dL^1 > 0$ ), so that the final distribution is in the *AES* subset of  $\mathcal{S}^1$ , as indicated by the move from point *E* to point *H* in Fig. 4. Proposition 2(b) and Corollary 1 imply that  $dV_F^{1*}/dL^1 < dV_A^{1*}/dL^1$  and  $dV_F^{2*}/dL^1 > dV_A^{2*}/dL^1$ . Since  $\pi = \tilde{\pi}_A^*$  implies  $V_F^{i*} = V_A^{i*}$  initially, we will have  $V_F^{1*} < V_A^{1*}$  and  $V_F^{2*} > V_A^{2*}$  after the transfer. By continuity, there exists additional labor transfers with the just described preferences over trade regimes. ||



**Figure 1**

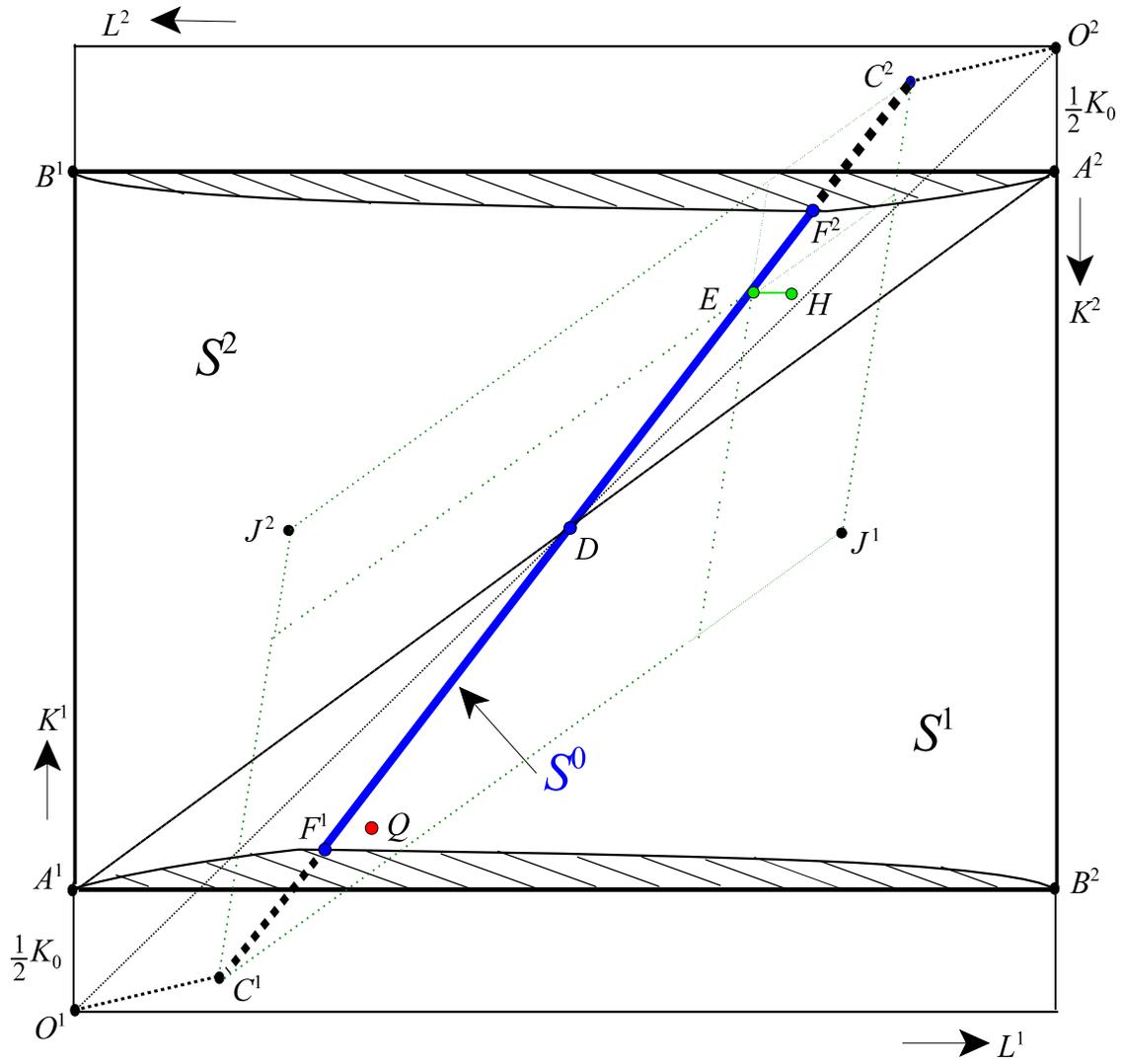
Individually Optimal Security Policies



**Figure 2**

Best-Response Functions in Security Policies

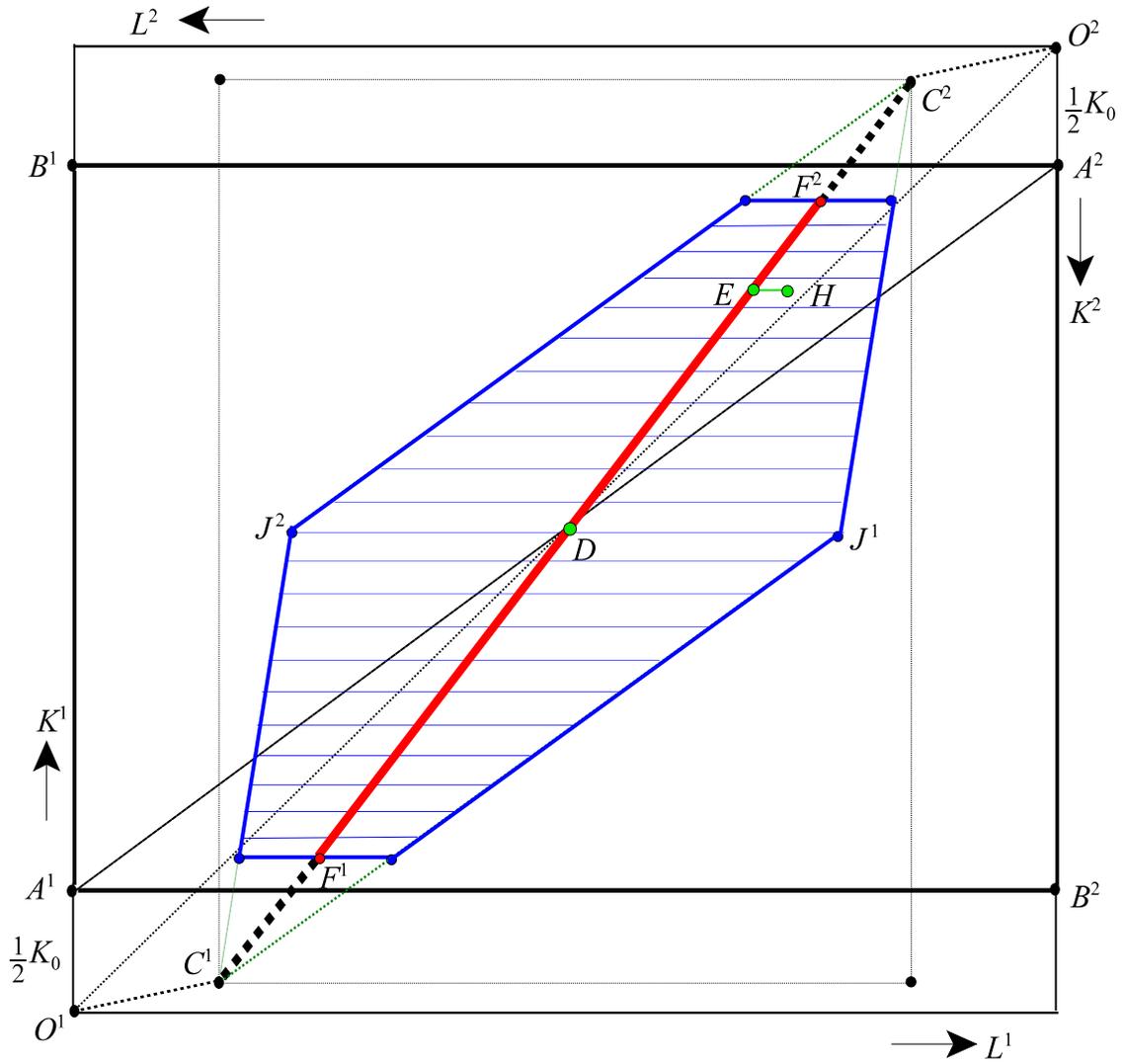
$$k_2^i > k_1^i > k_G^i, \quad i=1,2$$



**Figure 3**

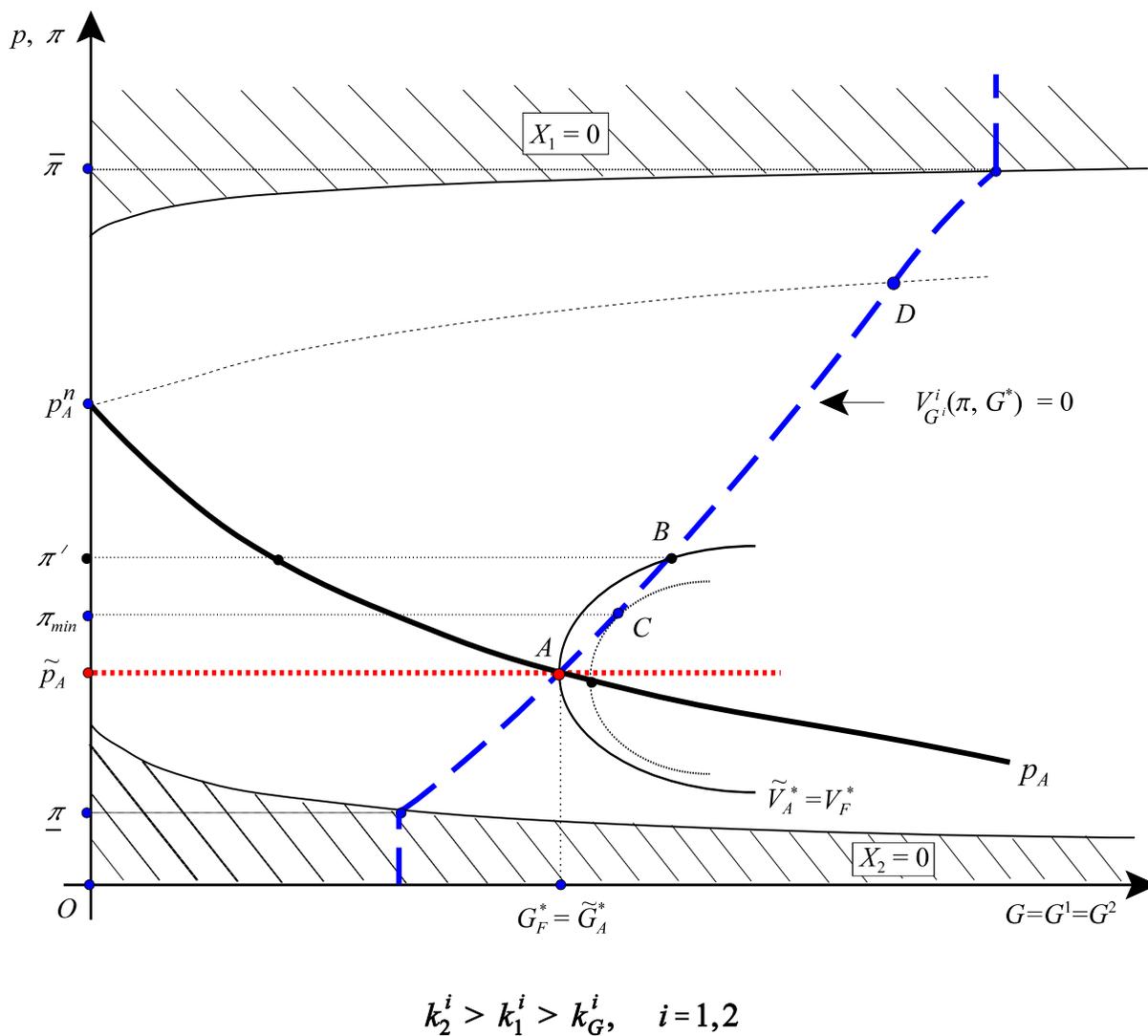
The Distribution of Factor Endowments, Sectoral Decomposition of Production, and Arming Incentives under Autarky

$$k_2^i > k_1^i > k_G^i, \quad i=1,2$$



**Figure 4**

Arms Equalization Region under Free Trade



**Figure 5**

Welfare, Patterns of Trade, and Equilibrium Security Policies  
with Identical Adversaries