
ROMANELL LECTURE

A NATURALISTIC LOOK AT LOGIC

PENELOPE MADDY

UNIVERSITY OF CALIFORNIA-IRVINE

There's a tendency to suppose that a naturalist is automatically, by virtue of her naturalism, committed to some particular view of logic. These days, for example, the classical Quinean picture is sometimes taken to be the naturalistic standard: logic lies at the center of the web of belief; remote from sense experience, but widely confirmed by its role in all our successful theorizing; a posteriori like the rest, but also the most resistant to change, given the principle of minimum mutilation; and thus apparently, or even practically, a priori.¹ But others, at other times, have held that other views of logic followed directly from naturalism, say psychologism, or simple inductivism, or some form of linguistic conventionalism. The trouble is that 'naturalism' means something different in each case, or that it comes encumbered with various inessential add-ons (like holism).

What I hope to do here is outline an alternative for the naturalist, an alternative if not more appealing, at least more robust than the others in a sense I hope to make clear. To do this, I begin by sketching the version of naturalism I'll be using as a backdrop. But I should admit up front that I will not assume the proper philosopher's role and attempt to defend either of these positions. Along the way, I'll gesture at what might be viewed as the shortcomings of the alternatives and the appeal of the position I'm describing, but for now, I'll be satisfied if I succeed in sketching it clearly enough for further evaluation.²

I. NATURALISM

Strictly speaking, to my way of thinking, there is no explicit doctrine answering to the name 'naturalism'. There is, rather, a distinctive way of approaching questions about the world, a distinctive method of inquiry, practiced, most often unselfconsciously, by a figure I'll call 'the naturalist'. My job is to describe the naturalist's behavior.³ In the course of doing so, I will use forms of expression like 'the naturalist begins all inquiry within the context of her best science,' but it's important to insist that the naturalist herself says nothing like this; she simply behaves in a way that seems best described by such phrases. The point is that the naturalist does not begin by announcing 'I use only the methods of science!'; she begins instead with observation, experimentation, theory formation, etc., methods of the sort we observers tend to classify as scientific.

This point is crucial for two reasons. One is to separate my naturalist from a closely-related, perhaps more familiar figure. To see the difference, suppose we ask our subject, 'Why do you believe in atoms?' This other quasi-naturalist might reply, 'Because science tells me that there are atoms and I believe the utterances of science.' My naturalist says no such thing, appealing instead to the actual scientific evidence: 'because of Einstein's calculations a, b, c, and Perrin's experiments d, e, f,' and so on.⁴ It isn't some general meta-thesis about the reliability of science that impresses her, but the detailed scientific evidence specific to each individual case.

A second important consequence of this approach to naturalism is that it doesn't rest on any official demarcation criterion for what counts as science. The quasi-naturalist who holds to the principle 'believe only the utterances of science' might well be expected to specify what distinguishes those utterances from the rest. My naturalist takes no such global position. She is convinced by particular arguments and methods, as they come along; at a more reflective level, she will have reasons for thinking these arguments and methods to be reliable, given the context of her other beliefs, but she needn't espouse any global account of precisely what all these particulars have in common or any general principle on which to rule other things out. When I describe this naturalist, as any sociologist might, my best bet is to use rough-and-ready labels like 'scientific methods', but this usage reflects only our ordinary understanding of the words, not any preferred analysis in terms of necessary and sufficient conditions.⁵ The real work comes in describing the naturalist's reaction in particular cases, in understanding what specific types of evidence are found compelling and why (as, for example, in the case of atoms).

To a first approximation, then, my naturalist coincides with Quine's:

The naturalistic philosopher begins his reasoning within the inherited world theory as a going concern. He tentatively believes all of it, but believes also that some unidentified portions are wrong. He tries to improve, clarify, and understand the system from within. He is the busy sailor adrift on Neurath's boat. (Quine [1975], p. 72)

For such a naturalist, ontological questions — are there atoms? are there numbers? — epistemological questions — how do these humans, as described by physiology, psychology, biology, etc., come to know about this world, as described by physics, chemistry, astronomy, botany, etc.? — all these are treated as broadly scientific questions. This includes a scientific study of science itself, which examines and classifies scientific methods and norms, undertakes to determine which are most effective in finding out how the world is, and strives in this way to improve science from within.⁶

The most salient difference between my naturalist and Quine's comes in the results of this scientific study of science.⁷ Here the Quinean concludes that scientific confirmation is holistic, and that it

can be codified, most simply as ‘observation and the hypothetico-deductive method’ (Quine [1975], p. 72), or in more detail by a list of five theoretical virtues (simplicity, familiarity of principle, scope, fecundity, consistency with empirical tests (Quine [1955], p. 247)). My naturalist holds that this account doesn’t do justice to the actual complexities of scientific practice. Any careful examination of the process of science will yield a more subtle and intricate case-by-case account of its methods and their justification: not every part of our best theory is confirmed or even available for confirmation; many sorts of idealizations and approximations are used; and so on.⁸ From this disagreement follows another, about the status of mathematics,⁹ but I won’t belabor these matters here.

To cast light on my naturalist’s posture from another direction, consider her response to those philosophical controversies in which her opponent insists that scientific answers are not what is called for.¹⁰ So, for example, a van Fraassenite counsels that we might remain agnostic about atoms. Faced with such skepticism, our naturalist will review the Einstein/Perrin evidence, asking if there is some defect here that she has failed to notice. No, the van Fraassenite replies, this evidence is perfectly good for scientific purposes, but (he continues) there is another, ‘epistemic’ point of view, for whose philosophical purposes this evidence is not enough, indeed for which no evidence for the existence of unobservables could ever be enough. At this point, the quasi-naturalist described a moment ago might reply that van Fraassen has gone beyond scientific methods, that his worry is unscientific, and dismiss his concerns on those grounds. In contrast, my naturalist will simply want to know the source and justification of the blanket rejection of entities not observable by her naked eye. Unless the van Fraassenite manages to present her with sound scientific reasons for doubting the existence of atoms — something he himself does not pretend to do — she will be left confused about what other purposes he has in mind, but without reason as yet to question the ordinary evidence for atoms that she takes to be compelling.

Let me summarize: my naturalist begins her investigations as a native of the current scientific theory, which is, in turn, an extension of common sense; presented with a question, she follows methods and norms most commonly described as ‘scientific’. Among these questions are some traditionally philosophical ones, about ontology, epistemology, etc., which she also approaches in her clear-eyed scientific way. If the philosopher insists that this is not appropriate, that different standards apply, she will ask after the source and justification of those new standards, and she will evaluate the answers as she does everything else, by her own lights. At some point, she will undertake an investigation of her own methods and norms, which she will evaluate as critically as any others and improve as best she can. In this way, she steadily pursues her comprehensive study of the world around her.

II. A FEW NATURALISTIC POSSIBILITIES

Let's now take a moment to review a sampling of views on the nature of logic that have traditionally been associated with naturalism of one sort or another. This should counteract the impression that the naturalist is forced in one particular direction by her naturalism alone, and provide a comparison class for the alternative I'll be sketching later.

One early, proto-naturalistic school of thought was the scientific materialism of 19th century Germany.¹¹ The movement began as a reaction against theology and speculative metaphysics (especially Hegel), and quickly grew to include the idea that¹²

...philosophers can no longer be viewed in opposition to natural scientists, because any philosophy worthy of its name laps up the best sap of the tree of knowledge, and by the same token produces only the ripest fruit of that tree.

In plainer language, philosophy uses the methods and results of science and what it produces is again science, broadly construed.

The leading materialists were physiologists — concerned primarily with issues like vitalism, spontaneous generation, and the development of species — but they were also widely-read popularizers of the burgeoning natural science of the time and opponents of all things supernatural. All knowledge springs from sense experience, on their view, but this is not the sense data of the British empiricists: sensations are understood physiologically, which comes down to matters of chemistry and physics. Logic, for the materialists, is a record of the laws of human thought, ultimately understood in physiological terms.

These same thinkers were attracted to Mill's simple inductivism (though they also insisted that the goal of science is not just facts, but also explanation).¹³ This raises the possibility of a purely inductivist account of logic: we trust *modus ponens* because it has worked so well in so many cases. Though a view of this sort is often attributed to Mill, I think this is inaccurate;¹⁴ in fact, Bolzano comes closest:

...these sciences [logic, arithmetic, geometry, and pure physics] enjoy such a high degree of certainty only because they have the advantage that their most important doctrines can be easily and variously tested by experience, and have been so tested... The only reason why we are so certain that the rules *barbara*, *celarent*, etc., are valid is because they have been confirmed in thousands of arguments in which we have applied them. (Bolzano [1837], §315.4)

Whether or not Bolzano's position as a whole can be understood as a form of proto-naturalism,¹⁵ this simple inductivist position on logic clearly imports no extra-scientific machinery.

The immediate worry about simple inductivism is that the very process of confirming a generalization by examination of instances already presupposes at least some logic (e.g., universal instantiation,

the law of non-contradiction, etc.). Quine's neo-empiricist account suffers from a related problem: how can logical truths be revisable items among others in the web of belief when at least some laws of logic are needed in the very process of evaluating and revising the web itself? This doesn't mean there is no sense in which logical truth might be empirical; I come back to this question in section V below.

Another conspicuous possibility is conventionalism: the truths of logic are true by convention. This view is forcefully expressed by the positivist Hahn.¹⁶ If I stipulate that a certain tree may be referred to as a 'Australian Myrtle' or as a 'Peppermint Willow', then the statement that every Australian Myrtle is a Peppermint Willow appears as a 'universally valid proposition'. Hahn continues:

It tells me nothing about the plant... The statement merely expresses a convention concerning the way we wish to talk about the plant in question... Similar considerations apply to the principles of logic. (Hahn [1933], pp. 152-3)

The problem, as Quine pointed out,¹⁷ is that the truths of logic can only be obtained from a list of explicit conventions by presupposing logic, thus defeating the purpose. Lewis has since initiated a program for describing implicit conventions,¹⁸ but these methods once again seem to presuppose logic, this time in the form of 'common knowledge'. Working in this tradition, Skyrms has suggested an evolutionary account of how the meanings of the logical particles might develop without explicit convention or common knowledge, but logical truths then seem to depend on the structure of the world (revealed in the form of evolutionary hard knocks) in addition to those conventional meanings.¹⁹ So the viability of a purely conventionalist account of logic remains an open question.

It seems to me that each of these accounts — psychologism, simple inductivism, Quinean empiricism, conventionalism — can be understood as a broadly scientific theory of logical truth, fully open to the naturalist for consideration. I confess that none of them seems to me a particularly likely candidate, partly for the reasons I've touched on briefly above,²⁰ and partly for reasons based in the philosophy of mathematics that I won't try to summarize here. My goal in what follows will be to sketch an alternative. I begin with a return to Kant, as his position, with an update from Frege, will provide the backbone for the new naturalistic option.

III. KANT ON LOGIC

We want to know what kind of statements logical truths are (what makes them true) and how we come to know them. I have in mind here examples of the most garden variety, like the familiar syllogism — if all men are mortal and Socrates is a man, then Socrates must be mortal — or statements with the form of simple sentential tautologies — if it's either red or green and it's not red, then it must be green.²¹ To sketch Kant's position on these questions, we need to begin from the wider context of the critical philosophy as a whole.²²

Kant's goal is to account for our a priori knowledge of the world, and his method is transcendental analysis:

This transcendental reflection is a duty from which no one can escape if he would judge anything about things *a priori*. (A263/B319)

The first step in the application of this method is to distinguish two levels of inquiry: empirical and transcendental. At the empirical level, we function as ordinary natural scientists, guided by theory, observation, and experiment; such things as space, time, and ordinary physical objects related by causal interactions are objectively real features of the world. Viewed from the transcendental level, however, the scene is very different.

To see how, recall that for Kant, human knowledge is generated by the cooperation of two faculties: sensibility and understanding. The sensibility is passively receptive; it combines fleeting sensations under the forms of intuition (space and time) to produce intuitions. The understanding is actively spontaneous; it combines intuitions and concepts to produce judgments using the logical forms of judgment. Because both intuitions and concepts are needed for any experience at all, some concepts must be available a priori. These are the pure concepts of the understanding, which include the notions of an individual object with properties and of the dependence of a consequent on its ground.²³ For these pure categories to be applied to the stuff of intuition, they must be 'schematized', that is, made compatible with spatiotemporal inputs. So, for example, the schematized category corresponding to the pure category of object with properties is that of a spatiotemporal object with properties; the schematized category corresponding to the pure category of ground and consequent is causality.

The key to Kant's account of a priori knowledge is that none of this is the sort of empirical analysis of actual human cognition that might have been undertaken by 'the celebrated Locke' (A ix) or a contemporary cognitive or neuro-psychologist. It is, somehow, transcendental. It tells us not how humans happen to cognize, but how intellects of our general type²⁴ *must* cognize. Once we have performed this transcendental analysis, we see that we can know, prior to any experience, that the world we experience will be spatiotemporal, and that it will involve spatiotemporal objects standing in causal relations.

At this point, our a priori knowledge is of the world of experience, as opposed to the world as it is in itself, but this is a transcendental distinction. If we drop back to the empirical level, the world we experience simply is the world, and our a priori knowledge of space, time and causation is knowledge of that world. Kant's position combines the two perspectives. Space and time, the forms of intuition, are transcendently ideal; that is, viewed transcendently, they are present in the world of experience only because of how we cognize, not because of how the world is in itself. But space and time are also empirically real; that is, viewed empirically, they are objective features

of the world. The same goes for principles like the law of cause and effect: transcendentally, it is true of the experience of any knower like us; empirically, it is objectively true.

To see how logic fits into this picture, we need to pause and ask what is meant by the phrase ‘knowers like us’. For Kant, we are, primarily, discursive intellects, which means that our knowledge of objects isn’t direct, but mediated by concepts. (This is another way of putting the previous point that our knowledge requires both intuitions and concepts, both sensibility and understanding.) We humans also have our two particular forms of intuition — space and time — but a discursive intellect could, in principle, have different forms of intuition.²⁵

Now Kant believes that every discursive intellect, just by reason of its discursivity, must judge using a certain limited range of logical forms; these are presented in the famous Table of Judgments at the beginning of the ‘Transcendental Analytic’, which includes subject/predicate and if/then constructions.²⁶ This Table of Judgments is connected with the Table of Categories via the so-called ‘Metaphysical deduction’, so the discursive knower must also judge using the pure categories. Object-with-properties and ground/consequent, in particular, correspond to the logical forms subject/predicate and if/then. What matters for our purposes is that all this is independent of our particular forms of intuition: any discursive intellect, regardless of its forms of intuition, must judge using the logical forms and the pure, unschematized categories. Finally, the judgments themselves are regarded as rules, and the valid inferences are mere applications of these rules.²⁷ In sum, any discursive intellect, whatever its forms of intuition, is required to judge by these forms, and thus is bound by the laws of logic as inevitable patterns of its mode of thought.²⁸

This means that logical truth is grounded in the pure understanding just as geometry is grounded in the forms of intuition and the law of causality is grounded in the forms of intuition and (the schematized concepts of) the pure understanding. Speaking transcendentally, this means that logic is also ideal; it is true of the world of experience because of the structure of the discursive understanding, not because of features of the world as it is in itself. But logical truth is not dependent on the forms of intuition. So it is transcendentally ideal, but less of our cognitive structure is involved; we might say that it is transcendentally ideal in a weaker sense than geometry and the law of causality.

Empirically, though, logical truth is as robustly objective as spatiotemporality and causality; logic is true of the world. How do we come to know these truths? According to Kant, we can’t help believing them, as they are determined by the most general forms of our capacity to judge: the logical forms of judgment and the pure (unschematized) categories. After our transcendental analysis, we see that these forms and pure concepts are necessary to us as discursive intellects, and thus, that the world of our experience will necessarily conform to them. Thus Kant’s combined transcendental/empirical analysis secures our a priori knowledge of these facts about the world.

IV. A FREGEAN UPDATE

If this can be taken as a fair, if irresponsibly brief, sketch of Kant's position, let me now propose an adjustment inspired by the subsequent replacement of the Aristotelian logic of Kant's time with our modern Fregean variety. We might see this as amending the Table of Judgments in light of Frege's more powerful analysis of logical form.²⁹ What I have in mind, obviously, is the profound and far-reaching shift from the Kantian form subject/predicate to the Fregean notion of argument and function. I'll have more to say in what follows about what I take to be the content of this new form of judgment (and its corresponding category); for now we should simply note that this single move provides the flexible system of complex relations that we know today, and opens the door to quantification. Frege himself was confident

I believe that the replacement of the concepts *subject* and *predicate* by *argument* and *function*, respectively, will stand the test of time. (Frege [1879], p. 7)

In this he was surely right.

As far as the form if/then is concerned, Frege remarks that his understanding of the conditional

has a close affinity with the important relation of ground and consequent. (Frege [1880/1], p. 37)

Our update will not touch this form or category. And finally, if we replace 'subject/predicate' in the Table of Judgments with 'argument/function', we must also replace 'object with properties' in the Table of Categories with 'objects in relations'. (Presumably this would schematize to 'spatiotemporal objects in relations'.) Call the result of this update the Kant/Frege forms and categories.

V. THE NATURALISTIC ALTERNATIVE

Imagine now that our naturalist is faced with this neo-Kantian proposal. The style of consideration Kant raises at the empirical level will be familiar and congenial, but what will she make of the transcendental additions and corrections to her scientific theorizing? Viewed as a scientific hypothesis, idealism will be unappealing — even Kant would agree to that — and his extra-scientific, transcendental reasoning will be unpersuasive when evaluated by ordinary scientific norms.³⁰

The plan, then, is to take Kant's transcendental account, with its Tables of Judgments and Pure Categories (as updated), and convert it into something palatable to the naturalist. The trick is to replace Kant's transcendental and empirical levels of investigation with one scientific inquiry. There are several ways this might be done. To get a sense of the possibilities, let's consider the two simplest options first: adopt the theory developed at one or the other of Kant's two levels, viewing it as straightforwardly scientific.

Suppose, then, that we try to retain the general thrust of the transcendental analysis, but treat it as describing the workings of the

empirical ego rather than the transcendental ego, in other words, as an ordinary psychological theory of cognition. Translating as directly as possible, we get the empirical claim: any cognizer whose knowledge is mediated by concepts must judge according to the logical forms of judgment and the pure categories (as modified), and thus, must be bound by the laws of logic. The roughest spot here, the one we didn't so much as hint at sorting out in the Kantian version, is the connection between the mere fact of thinking in terms of concepts and the detail of the Kant-Frege Tables of Judgements and Pure Categories.³¹ Indeed, it seems plausible to suppose that there could be cognizers who use concepts, but whose fundamental modes of judging and conceptualizing nevertheless differ sharply from ours. If we forgo this part of the neo-Kantian position, our empirical claim is simply that any cognizer who uses our forms of judgment — a logic of relations, with conditionals — and the corresponding pure concepts — objects in relations, ground/consequent — will be bound by the laws of logic. Alas, what we've come to here is a psychologistic account of logic — logic is grounded in the structure of human cognition — one of the options we had hoped to avoid.

The second approach would be to settle for the empirical level unadorned: logic is true of the world; it is self-evident and universal. The trouble with this account is that it leaves us without answers to the very questions that inspired Kant's critical philosophy in the first place: how can we know about the world a priori? Without the transcendental part of the Kantian package, we have no epistemology.

Under the circumstances, it seems the most appealing way to naturalize our neo-Kantianism would be to somehow collapse the transcendental and the empirical levels into one without losing the attractions of either. As a first approximation, a position along these lines suggests itself: (1) psychologically, humans are so constructed that they conceptualize the world using the Kant/Frege forms of judgment and categories, and for this reason, their thinking is bound by the laws of logic; (2) objectively, the world has very general structural features that in fact correspond to the logical forms and (unschematized) categories — that is, the world consists of objects in relations, with ground/consequent dependencies between various of its aspects — and for this reason, the laws of logic are truths about the world; (3) humans believe the laws of logic because they are dictated by their fundamental conceptual machinery, but they come to know those laws are true by coming to know that the fundamental conceptualizations on which they are based are veridical, that is, by empirical investigation. On this view, logic would be true of the world, but independent of its spatiotemporal aspects (that is, those aspects corresponding to the forms of intuition); the laws of logic would be true in a non-spatiotemporal world, even an acausal world, as long as that world contained objects in relations and ground/consequent dependencies of some kind, but they need not be true in a world without those features.

This sketch immediately raises a series of pressing questions (perhaps in addition to a few eyebrows!): are we really so constituted

that we conceptualize the world in these ways? How does our use of these forms and concepts make the laws of logic correct prescriptions for our thought, or on the world's side, how do the corresponding features of the world make the laws of logic true? How do we know that the world has these features? Looking into these questions will lead us to a somewhat more subtle version of the view.

First, are the Kant/Frege forms and categories truly among our fundamental ways of conceptualizing the world? Observational and introspective evidence makes the thesis superficially plausible, and it is the sort of thing that can be and is being investigated empirically, by cognitive scientists. For the purposes of this paper, I won't attempt to mount a defense of this empirical claim.³² Its content should be clearer by the end, but its truth will remain a scientific conjecture.

The other two questions still loom, especially the second: how do these forms and categories underlie the laws of logic? We've seen that Kant's transcendental answer involved understanding judgments themselves as rules, but even if this idea could be transplanted into our empirical psychology, we'd lack the Kantian means to transform the logic so-generated into a series of truths about the world. So let's try a different approach.

Suppose we are in fact thinking in terms of objects arrayed in various relations to one another with various interdependencies. We've seen that the shift from Kant's object-with-properties to the more Fregean objects-in-relations is crucial, but I think we should regard the more modern conception as diverging from the older even in the area where they might seem to overlap, that is, in the case of a single object with a one-place relation or property. Kant's category here is 'subsistence and inherence', a descendent of the Aristotelian substance/attribute, carrying with it a complex and arcane metaphysics. Our modern (sub)category descends instead from the Boolean idea of an object as a member of a class of things, that is,

a collection of individuals, to each of which a particular name or description may be applied (Boole [1854], II.6, p. 28).

The underlying idea is quite simple: objects grouped together by their similarities; pairs of objects grouped together by their relational similarities; and so on. Theoretical elaborations eventually lead to our developed combinatorial notions of collection, function, etc.

Clearly, this thinking in terms of objects in relations immediately involves us in atomic claims of the familiar sort — $Rabc$. But notice that the very undertaking of seeing the world in these terms also seems to involve the understanding that some particular objects might enjoy more than one property or stand in more than one relation or stand in relation to more than one other object — Pa and Qa , $Rabc$ and $Rabd$, Sab and Sac ; that an object might enjoy one or another of several properties, or stand in relation to one or another of several other objects — Pa or Qa , Sab or Sac ; that some objects might fail to stand in some particular relations, that attributing a particular property to a particular object contains an implicit contrast with other properties this object

might lack, or other objects that might lack the particular property — not-Rabd, not-Pb.

In the first two cases, we have here minimal forms of conjunction and disjunction, holding only between atomic claims, counterparts of the intersections and unions that are so much a part of Boole's more elementary picture. In the third case, we have a sort of primitive opposition, which we represent using negation. Whether we count these as integral parts of the new category of objects-in-relations, or view them as additions to it, I think they are plausibly regarded as elements of our most rudimentary conceptual machinery. From here on, I will understand the naturalist as committed to this possibly more expansive notion of what's involved in the updated category objects-in-relations (plus a bit more, see below). In the end, the naturalist's burden will be to show that all these are present in our basic ways of cognizing and in the world.

We have here a picture of a property holding of some objects and not holding of others — an idea aptly represented by an open sentence,³³ where the language is rich enough³⁴ — and relations treated likewise. From this idea of running through a series of objects, identifying those that satisfy the open sentence and those that don't, all the more that's needed for quantification is the idea of a range of variation, a domain of quantification, and at the rudimentary level of conceptualization we're investigating now, this can be given by a property. (Again, if this seems an add-on to the bare category of objects-in-relations, let us add it.) And finally, our categorical thinking also involves situations conceptualized in terms of ground and consequent; these would naturally be expressed in terms of a conditional.

This analysis, so far, gives us rudimentary counterparts of all our modern primitive terms, but I think we are still some distance from our actual use of them. One important step is to use our various linguistic devices — 'and', 'or', 'not', 'all', 'some' — to extend the minimal versions to full connectives capable of applying to any description(s). But there is more to it than that.

First, consider: does thinking in terms of objects in relations and ground/consequent dependencies also commit us to granting, for example, that any given object must either have a given property or fail to have it? It seems to me that the answer must be no; we find it quite easy to think, for example, that it's neither true nor false that a given person is bald. Without bivalence, what we have at this stage is atomic sentences that can be true or false or lack truth value. Let me continue my conjecturing about our updated and naturalized category of objects-in-relations so far as to suggest that we understand conjunctions of such sentences to be true if both conjuncts are true, false if one or the other is false, and to lack truth value otherwise, and disjunctions to be true if one or the other disjunct is true, false if both are false, and to lack truth value otherwise.³⁵ (I'll abuse terminology here and call this conjunction and disjunction 'truth-functional', speaking as if 'lacks truth value' were another truth value.) In parallel,

a universal quantification of an open sentence over a range specified by a property is true if the open sentence is true of every object that enjoys the property, false if there is one object enjoying the property that fails to satisfy the open sentence, and without truth value otherwise. Similarly, the existential quantifier generalizes disjunction.

Negation is more complex. The negation of a truth should be false, and the negation of a falsehood should be true, but what happens if what we negate lacks truth value to begin with? Perhaps the most natural idea is that if *p* lacks truth value, then so does not-*p*: if the claim that Joe is bald seems to lack truth value, so does the claim that he's not bald. But when pressed, we might be inclined to say, 'well, he's not bald, but then, he's not *not* bald, either!'³⁶ Here the unitalicized 'not's are understood to be true when applied to something that fails to be true, that is, something that is either false or lacking in truth value; so here what lacks truth value has a true negation. On the other hand, the italicized 'not' is understood to be true when applied to something false, and false otherwise; what lacks truth value has a false negation. So it seems we also have quite ordinary natural language uses of 'not' that regard the negation of something truth value-less to be true (the thing denied isn't true, after all), and others that regard such a thing as false (the thing denied isn't false, after all). As all three 'not's coexist in the non-bivalent context,³⁷ it seems prudent not to assume that we share some fundamental conceptual machinery that dictates a uniform value for the negation of what lacks truth value. All we know for sure about negation is that it expresses opposition: it sends true to false and false to true.

Alas, even less seems settled about our conditional. It expresses a kind of dependency, so perhaps we can insist on the falsity of any if/then with a true antecedent and a consequent false or lacking in truth value. Unfortunately, given the familiar vagaries of the category ground/consequent and the natural language if/then, it seems unlikely that the truth value of a conditional could be determined by the truth values of its components in any of the other combinations. Though modus ponens is preserved, even conditional proof is problematic in the non-bivalent context.³⁸

I've now sketched in more detail what my naturalist takes to be the logical content of our most fundamental modes of conceptualization (and the corresponding general structural features of the world). The question we're asking at this point is: what logical laws are generated on this slender basis? The first surprise is that it generates no logical truths whatsoever, because no classical tautology is always true in the presence of truth value gaps.³⁹ If we restrict our attention to conjunction and disjunction, we get some logical equivalences, in the sense that some pairs of statements will have the same truth value in every situation;⁴⁰ for example, the distributive pair — 'It's blue and it's either round or square' and 'Either it's blue and round or it's blue and square' — are equivalent in this sense. In addition, many classical inferences involving conjunction and disjunction will be valid — assuming we take validity to mean: if the

premises are true, then the conclusion is true — because, as Parsons puts it:

Assuming that the premises are true very often amounts to assuming that there are no gaps in them in positions that could make a classically valid conclusion lack truth value (Parsons [2000], p. 25).⁴¹

So, for example, we can conclude ‘Joe is bald’ from ‘Joe is bald and handsome,’ despite the fact that there is no corresponding logical truth, ‘if Joe is bald and handsome, then Joe is bald’.⁴²

This is without doubt a disappointingly meager logical stockpile, so perhaps it is not surprising that Frege insisted truth value gaps not be allowed for the purposes of logic:

This involves the requirement as regards concepts...that it shall be determinate, for any object, whether it falls under the concept or not. ...if this were not satisfied it would be impossible to set forth logical laws about them. (Frege [1891], p. 141)

In myth and fiction thoughts occur that are neither true nor false. Logic has nothing to do with these. In logic it holds good that every thought is either true or false, *tertium non datur*. (Frege [1906], p. 198)

Here bivalence in our classical logic is regarded as an idealization of sorts, or a restriction of our attention, for logical purposes, to cases in which it holds, rather than a universal claim that it holds in all cases.⁴³ This is the first major idealization required to cover the distance between the rudimentary logic of our fundamental conceptual machinery and the laws of modern logic. Removing all truth value gaps and restoring the standard truth-functional negation produces the full store of propositional tautologies involving conjunction, disjunction and negation.

But we are still a step away from our modern, fully truth-functional treatment of all propositional connectives and their interconnections (which must count as one of Frege’s great contributions to modern logic). To get there requires a second sort of idealization, this one in the definition of the conditional. Here again, we follow Frege, who explicitly notes the so-called paradoxes of material implication, pointing out that there need be no ‘causal connection’⁴⁴ (Frege [1879], p. 14) between the two components: ‘if the sun is shining, then $3 \times 7 = 21$ ’ is true, though the sun’s shining has nothing to do with the arithmetical fact. Here we make a clear and deliberate departure from the content of the underlying category, motivated by the needs of logical theorizing. Once it’s done, our bag of logical truths expands to contain all the classical tautologies, including the familiar interdefinability of the connectives.⁴⁵

Finally, what about quantification? In our rudimentary version, the range of variation is determined by a simple physical property, like ‘cats’ or ‘these blocks.’ Our modern use of the quantifiers requires instead a specified domain of quantification,⁴⁶ most often a set, which

must be non-empty and well-behaved in a sense that will become clear in the next section. So, to achieve the full stock of our classical validities, we need to add the linguistic extensions remarked earlier; both the idealizations already noted, bivalence and truth functional conditional, plus two more assumptions for every application of quantified logic: that the domain of quantification is non-empty and well-behaved enough to support the operation.

This account of the journey from the forms and pure categories to the laws of logic is obviously crude and incomplete, but the real point is clear: the answer to the question we began with — how do these concepts underlie the laws of logic? — is that they don't. The fundamental modes of conceptualization do provide us with rudimentary versions of the logical primitives, which means that the logical laws governing those primitives will be correct for our thought and that they will seem obvious to us. But several linguistic extensions and at least two substantial idealizations are needed before we can reach anything like the laws of logic as we now understand them. And we post-Fregeans require the added assumption, in each case, that the domain of quantification is non-empty and well-behaved. There's nothing shameful in any of this — idealizations are the life-blood of science — but we need to be aware of them just as we are elsewhere in science, so that we can assess their aptness. I hope this point will come clearer below.

Let's turn finally to our third question: how do we know that the world has features corresponding to the naturalist's forms and categories? Kant concocted his Copernican Revolution expressly to settle this sort of question, or perhaps we might say to erase it, but it remains pressing from our naturalistic perspective. And it seems to me that the results, though largely positive, are once again mixed.

Begin with objects-in-relations. We've noted that our common sense strongly endorses the idea that the world comes divided into medium-sized physical objects and that these bear various properties (that is, one-place relations) and stand in various complex relations. Science for the most part agrees: where common sense sees an individuated physical object, scientific investigation reveals a microstructure with a real, if fuzzy, distinction between the make-up of the atoms in the space occupied by the object and the atoms in the surrounding space; these same macroscopic physical objects are found to have real properties largely determined by this microstructure, and to stand in identifiable relations to other such objects. The world cooperates admirably with our thinking in terms of this category,⁴⁷ at least until we reach the vagaries of quantum mechanics. There, alas, it becomes difficult to regard electrons, for example, as objects⁴⁸ with properties⁴⁹ in the familiar categorical sense. As the view under consideration would suggest, this in turn seems to undermine the most rudimentary of logical laws — those that flow from our fundamental forms and categories even without the benefit of idealizations and added assumptions — in this case, the distributive law equivalences.⁵⁰

Common sense views about ground/consequent interdependencies are also largely, but not universally, supported by much of our scientific theorizing. In acausal cases, like ‘if it’s red, then it’s colored,’ linguistics confirms that our common sense is supported by facts of language use.⁵¹ Causal influences are ubiquitous in our everyday thinking, and science gives more detailed and explanatory analyses of most of those same interconnections. But once again, there is a breakdown in quantum mechanics, where there seem to be dependencies, but linguistic facts are not at issue and our common sense and ordinary scientific notions of causation seem no longer to apply.⁵²

Finally, concerning the added assumption that the domain of quantification is well-behaved, both common sense and science identify unproblematic natural classifications, and we also seem well capable of defining other collections for quantifying over as need be (all registered Democrats, all even numbers, etc.). Though there seem to be no problems with quantifying over collections of physical objects, questions do arise in mathematics, as we’ll see in the next section.

So it seems that the answer to our third question — how do we know that the world has these features? — is once again that we don’t. In many of its aspects, the world seems fully amenable to being represented using the Kant-Frege categories, by which I mean that they seem to correspond to real worldly structures, but there are important exceptions, as noted. Still, given that our categories are veridical over a wide range of cases, we might hope to connect our answers to the first and third questions, that is, we might hope our empirical account of how we come to conceptualize the world in these ways will take the form of an account of how we have come to be sensitive to these very general features of the world, insofar as they are present.

Let’s pause and take stock. I think it is now clear that our first approximation to a naturalistic view of logic — the view embodied in the (1)-(3) a few pages back — falls short in several ways. To see this, begin, by recalling (1), the claim that humans are so constructed that they conceptualize using the Kant-Frege forms and categories, and for this reason, their thinking is bound by the laws of logic. In the intervening pages, I’ve spelled out in more detail what my naturalist takes those forms and categories to include; let’s continue to suppose that empirical psychology can confirm our common sense and introspective evidence that these modes of conceptualizing are fundamental to our thought. Even granting this supposition, (1) still fails, because, as we’ve seen, the logic implicit at this rudimentary conceptual level falls far short of the laws of modern logic. So we must substitute (1’): humans are so constructed that they conceptualize using the Kant-Frege forms and categories, and for this reason, their thinking is bound by a rudimentary logic, but the complete laws of modern logic result only after some significant idealizations and additional assumptions.

Moving on, recall (2): objectively, the world has very general structural features that in fact correspond to the forms and categories, and for this reason, the laws of logic are truths about the world. Here the same failure as in (1) reasserts itself, namely, the realization that only a rudimentary logic is implicit in the forms and categories. But there is a second failure in the case of (2); there are aspects of the world that don't seem to conform to the forms and categories. So for (2), we substitute: (2') to a large extent, the world has general structural features corresponding to the forms and categories, but there are exceptions, in which cases, even the rudimentary logic loses its foundation.

Finally, recall (3): humans believe the laws of logic because they are dictated by their fundamental conceptual machinery, but they come to know those laws are true by coming to know that the fundamental conceptualizations on which they are based are veridical. This account of the source of our belief now covers only the rudimentary logic. The justificatory process — verifying the veridicality of the forms and categories — will be the same, but we now recognize that it will fail in some cases. Finally, the extensions of logic past the rudimentary will be justified by their appropriateness and their fruits, like idealizations elsewhere in science. So let (3') read: humans believe the rudimentary parts of logic because they are dictated by their fundamental conceptual machinery, but they come to know those laws only to the extent that those fundamental conceptualizations can be shown to be veridical.⁵³

I'll come back to the significance of these breakdowns in the next section, but for now, let's try to solidify our understanding of this naturalistic alternative by testing it on a number of the standard themes in the philosophy of logic: necessity, a priority, analyticity, normativity. This exercise should bring out the contrasts with more familiar positions and illuminate the outlines of the proposal more starkly.

Metaphysically, on the proposed view, logical truth is contingent on various features of the world: that it is fairly described as consisting of stable objects, that these fall into identifiable groupings and relations, that there are systematic interdependencies between some of these situations and others. On the other hand, there is some lingering trace of necessity in that logical truth does not depend on the spatiotemporal structure of the world, that is, it is independent of the spatiotemporality of the objects in our world and the causal nature of their interdependencies. Under the circumstances, it's not surprising that we would imagine logic to be necessary; these forms and categories are so fundamental to our thinking that we overlook the possibility of a world without the corresponding objects and interconnections.

Notice, by the way, that to say the world can be described in terms of our categories is not to say it couldn't also be described in terms of other concepts. The world, in its inherent complexity, might well be amenable to various forms of description — others of which correspond to aspects we miss or which don't interest us — but it

isn't amenable to any old form (as our troubles applying our categories to quantum mechanics amply demonstrate). For our purposes, it's enough that our fundamental categories correspond to some aspects of reality. Where the world fails to cooperate, our logic no longer applies. If the world overall were less cooperative, logic would not apply at all.

But this dependence on the veridicality of the categories isn't the whole story of the contingency of logic. In addition, the reliability of logic depends on whether or not our idealizations and assumptions are benign in the particular context of a given application, that is, on whether or not they distort the underlying phenomena to the point of leading us astray. I think it is worth distinguishing these requirements from the categorical ones for reasons that should become clear in the next section.

Epistemologically, we believe the truths of logic at first because they appear obvious to us, insofar as they reflect some of our deepest modes of conceptualizing the world. Such belief doesn't count as knowledge until we verify that, in a given context, these modes of conceptualization are veridical and the various idealizations and assumptions are benign, so our knowledge is not a priori. Still, this is not direct empirical verification of logical truths, as imagined by previous versions of empiricism: it is not simple enumerative induction, it is not holistic inference to items playing a central role in our preferred web of belief,⁵⁴ and it does not apply to all logical truths equally in all contexts. Though logic is falsifiable, this is not always a simple matter of recalcitrant observations or failed experiments. It can be, as for example, when we discover that the predicate we're dealing with is vague, and conclude that we should not presuppose bivalence in this case. But such cases affect only the conscious idealizations and assumptions, not the fundamental categories.

To see how the more difficult cases work, consider Putnam's distinction between statements that can be disconfirmed by ordinary observations and experiments, and statements that can only be disconfirmed 'by thinking of a whole body of alternative theory as well' ([1995], p. 272). The idea is that a logical truth like an instance of modus ponens is not necessary, that it could be false, but that we couldn't become convinced of this by ordinary empirical considerations; we would need some further instruction on how to conceptualize its falsity.⁵⁵ The view proposed here implies something similar, if somewhat more specific: in order to falsify an instance of modus ponens, presumably you would have to modify our category of ground and consequent.⁵⁶ This might be done, but it would not be done easily. (Notice that the same could be said about basic physical beliefs based on our categories, for example, that there are physical objects.)

The contrast with Quine should now be clear. First, notice that my naturalist's justification of our logical beliefs begins from the scientific observation that the world consists (largely) of objects in relations with ground/consequent dependencies, and proceeds

through a story of how we humans have come to see it in this way. In contrast, Quine's justification is holistic; it doesn't isolate the particular aspects of the world to which logic corresponds. Second, my naturalist traces the difficulty of revising our rudimentary logic to its grounding in our most basic modes of conceptualizing the world. In contrast, Quine cites the ubiquity of logic in our theory of the world, and the principle of minimum mutilation.

Let me pause here for a word on phraseology. In contexts where logic seems to break down, I've sometimes said that the laws of logic fail to apply and sometimes that they are false. In fact, I don't think much, if anything, hangs on this difference of expression. We often say things like: Euclidean geometry wasn't falsified by general relativity, it was just shown not to apply. On the other hand, we don't tend to say: Phlogiston theory wasn't falsified, it was just shown not to apply. In the case of logic, if it is falsified in the easier ways, by the failure of some idealization or assumption in the given context, we might well say it isn't false, it just doesn't apply here. Indeed, given that the world is as it is, the deeper failures come at the more esoteric reaches of science, so here again, we might say that we have a contextual failure, not a falsification. Driven to extremes, someone thinking along these lines might want to say that even in a world that doesn't conform to the categories at all, logic wouldn't be *false*, it just wouldn't happen to be applicable.

It seems to me that the real difference being marked by these locutions, the real difference between the cases in which we tend to say the theory is falsified and those in which we deny this, rests on the likelihood that the falsified theory will continue to be useful, either in other contexts or as an approximation in the original context. The trouble with phlogiston is that it didn't work in the one context where it had a chance. The immunity of Euclidean geometry comes from its continuing extreme usefulness as a workable approximation to the structure of actual space. Similarly, though logic may fail locally in some actual contexts and globally in some (half-) imagined worlds, it continues to be extremely useful in the vast majority of cases. This is surely worth noting, even if saying that it isn't falsified, that it's just not applicable here or there, isn't the most straightforward way of doing so. In any case, I invite those unhappy with the language of falsity to read such claims in their preferred idiom, substituting failure of applicability for failure of truth.

So, logical truth is contingent (or 'the applicability of logic is contingent'), and at the same time necessary for any world that conforms to the unschematized categories, some of which may be non-spatiotemporal. Logic is not a priori, but in its fundamentals at least, it also isn't simply empirical: to disconfirm it would take more than observation and experiment; it would take a revision of our most basic ways of thinking. Finally, it isn't analytic in the usual sense of being true by virtue of something purely linguistic.⁵⁷ Except for its independence of the spatiotemporal features of the world, logical truth differs from fundamental physical truths only in matters of degree, not kind.

Finally, what becomes of the purported normativity of logic: if we admit that it's either red or green, and it's not red, why *must* we admit that it is green? I've mentioned that, for Kant, judgments according to the logical forms are in some sense rules, so the force of the logical 'must' is the force of rule-following, but there is nothing of this idea left in our naturalized proposal. Here I think the naturalist will agree with Frege when he sorts out the sense in which logical laws are 'laws of thought': they are not laws of psychology, natural laws about how we happen to think.

Only in [this] sense can the laws of logic be called 'laws of thought': so far as they stipulate the way in which we ought to think. Any law asserting what is, can be conceived as prescribing what one ought to think in conformity with it, and is thus in that sense a law of thought. This holds for the laws of geometry and physics no less than for the laws of logic. (Frege [1893], p. 12)

The laws of logic, geometry and physics are all

prescriptions to which our judgements must conform in a different domain if they are to remain in agreement with the truth. (Frege [1897], pp. 145-6)

So the logical 'must', like the 'must' of physics or the 'must' of geometry, simply means that we must admit these things if we are to stay on the right side of the truth.

Still, there is a phenomenological sense in which we want to insist that the 'must' for our logical examples is especially compelling, more compelling than any physical 'must'. Because of their grounding in our most fundamental ways of thinking, at least some laws of logic strike us as obvious, as undeniable, as self-evident. As we've noted, these firm beliefs will not count as knowledge until the veridicality of our categories is confirmed, but the psychological conviction can hardly be increased by any such effort.⁵⁸

This, then, is the proposed naturalistic alternative. Nothing in it should strain against the naturalist's scientific world view, yet it differs from the familiar options of psychologism, simple empiricism, classical Quinean empiricism, and conventionalism. And it is surely more robustly realistic than has been common in recent years.

VI. DEVIANT LOGICS

Confronted by the suggestion that classical first-order logic rests on our most basic modes of conceptualization (which correspond, in turn, to structural features of the world), one can hardly help wondering how there can then be so many deviant logics, so let me close with a few observations on this topic. From the point of view proposed here, these departures from classical logic come in two varieties: those that reject one or another of the idealizations or assumptions, and those that reject one or another of the underlying categories. Those falling in the first group include: intuitionistic logic (which rejects the idealization of bivalence), relevance logic (which rejects the idealization of the truth functional conditional), and

predicativism (which rejects the assumption of a well-behaved domain of quantification).⁵⁹ In the smaller, second group, we find: quantum logic.⁶⁰ The issues raised by the likes of quantum logic run considerably deeper, so let me begin with a look at the examples in the first group.

Intuitionistic logic rejects bivalence, and hence the law of the excluded middle. As noted above, nearly everyone admits that bivalence does sometimes fail — for example, in the case of vague predicates, or in talk of fictional entities — and generally we refrain from applications of classical logic that would lead us astray in such circumstances. (For example, we don't conclude that a certain hair tonic is useless because those who aren't bald don't need it and those who are bald can't be helped by it; or that some definite proportion of the characters in *The Last Chronicle of Barset* are right-handed.) Intuitionists understand mathematical entities as mental constructions — somewhat akin to fictions in that they have (or lack) only the properties we prove them to have (or lack) — and conclude that the idealization of bivalence is inappropriate in this case, as well.⁶¹ We may disagree on the advisability of this approach to mathematics, but logically speaking, it is simply a case of avoiding full classical logic in a context where one of its idealizations is purportedly not appropriate.⁶²

As we've seen, in the absence of bivalence, negation also becomes problematic. There are three possibilities for the truth value of not- p when p lacks truth value, one of which would have to be chosen if the connective is to be truth-functional. The intuitionists do not choose: when p lacks truth value, not- p might also lack truth value (e.g., Goldbach's conjecture and its negation), but it might also be outright false (e.g., not- p can be false, and hence, not-not- p true, without p being true⁶³). The criterion is simply that not- p is true if and only if there is a proof of $1=0$ from p .

The idealization of the truth-functional conditional is also rejected; $p \rightarrow q$ means 'a proof of p can be converted into a proof of q ', a notion like that of inference from premises in beginning from the assumption that the premises are true. As a result, various of the equivalences noted above as holding in the presence of truth gaps, which generated legitimate inferences in that context, generate outright logical truths in the intuitionistic setting: e.g., if it's blue and it's round or square, then it's either blue and round or blue and square. Adding intuitionistic negation, some DeMorgan laws result, e.g., if it's not red or green, then it's not red and it's not green. From this it follows that not-not- $(p$ or not- $p)$, unlike the law of the excluded middle, is an intuitionistic logical truth, and so on. Here we have a developed alternative to classical logic that eschews its two main idealizations, but still agrees on the rudimentary logic based on the categories alone.

Relevance logic takes a less familiar turn, rejecting the idealization of the truth-functional conditional because of the so-called paradoxes of material implication.⁶⁴ Its proponents hope to get closer to the underlying concept of ground and consequent by insisting that the antecedent of a true conditional be 'relevant' to the conclusion. At

this point, it's debatable whether relevance logic actually provides a deeper analysis of ground/consequent:

It is a natural thought that for a conditional to be true there must be some connection between its antecedent and consequent. It was precisely this idea that led to the development of relevance logic. A sensible notion of connection is not so easy to spell out, however... (Priest [2001], 9.7.11)

But for our limited purposes, it is enough to note that the motivation, once again, is to forgo one of the central idealizations of classical logic.

Predicativism is a more esoteric deviation, motivated by concerns in the foundations of mathematics. The underlying idea is that

all mathematical objects beyond the natural numbers are to be introduced by explicit definitions. (Feferman [1987], p. 52)

This doctrine casts immediate doubt on the practice of defining a mathematical entity by reference to a collection of which the entity defined is to be a member. Predicativists agree with classicists that this method — impredicative definition — is legitimate for defining a natural number, because all that's happening is that one of the numbers is being picked out from the rest. But in the case of a real number, an impredicative definition would attempt to 'introduce' the new entity by presupposing a collection to which it belongs, that is, by presupposing (among other things) the very entity being 'introduced'!

To avoid such vicious circles, the predicativists rule out impredicative definitions, that is, for example, attempts to define a real number using a quantifier that ranges over all real numbers. By their lights, when it comes to reals, the domain of quantification is not well-behaved, the neo-Fregean assumption fails:

The totality of natural numbers is granted as clear and definite and each of its members can be singled out...[but] there is no justification in the assumption of a totality of [real numbers] independent of how these may be defined. (Feferman [1987], p. 54)

A clear distinction is drawn: the predicativist

accept[s] classical quantifier logic when applied to any established system of objects... (op. cit.)

including, for example, systems of reals that have been properly defined. But quantification over all reals is illegitimate and forbidden. Sticking to this restriction is difficult, as impredicative definitions occur essentially at various key junctures in the development of the classical theory of real numbers — e.g., in the proof that every non-empty set of reals with an upper bound has a least upper bound — but predicativists have devised ingenious systems in their effort to reproduce analysis.

The case of quantum logic differs from all of these; what breaks down here isn't an idealization or an added assumption, but the fundamental categories themselves, both objects-in-relations and ground/consequent, and with them, the distributive equivalences of our most rudimentary logic. Faced with this profound collapse, it's not surprising that some would suggest we need a new logic; what might be surprising is that the philosopher at the forefront here is once again Putnam. In this case, Putnam maintains that the distributive law is not the sort of logical law that cannot be disconfirmed without a body of new theory; he holds that quantum logic *could* be adopted on straightforward empirical grounds.⁶⁵ But the view of logic proposed here doesn't endorse this classification of distributivity. By our lights, a new logic for quantum mechanics, a new logic without the distributive equivalences, cannot rest on the Kant-Frege categories of objects-in-relations and ground/consequent; it will need a new categorical underpinning to found it. Logical revisions this deep may be possible, but they are far more difficult than suspending a conscious idealization or an explicit assumption.

As it happens, this is not what contemporary quantum logic puts on offer. The subject is a large one, but some basic points seem clear: the philosophical goal of the project is to clear the way for an interpretation of quantum mechanics that ratifies the *old* categories⁶⁶ rather than proposing new ones; quantum logic is viewed as a theoretical alternative to classical logic to be preferred on purely empirical grounds, because it makes possible a more palatable account of quantum mechanics;⁶⁷ and finally, quantum logic doesn't deliver on this promise — it doesn't succeed in removing the quantum quandaries and it doesn't ratify the old categories, either.⁶⁸ As Hughes observes, 'the package we have bought seems markedly less attractive than the product which was advertised' (Hughes [1989], p. 212).

But how, then, do we manage to develop quantum mechanics if classical logic doesn't apply and we have no appropriate alternative? The answer is that attention is shifted from particles — and their properties, relations, and interdependencies — to state vectors, that is, to mathematical items residing in a mathematical Hilbert space. This formalism issues forth predications of astounding accuracy, and — lucky coincidence! — it can be understood in the familiar terms of our Kant-Frege forms and categories: the objects of classical mathematics stand in relations and the certain conditions of some mathematical things often stand in ground/consequent (indeed, logical) relations with other conditions of these and other mathematical things. In this way, we can pursue quantum mechanics while hewing to our familiar logic.

But this move is just the beginning of the philosophy of quantum mechanics. As Hughes remarks:

The theory uses the mathematical models provided by Hilbert spaces, but it's not clear what categorial elements we can hope to find represented within them, nor, when we find them, to what extent the quiddities of these representations will impel us to

modify the categorial framework within which these elements are organized. (Hughes [1989], p. 176)

The term ‘categorial framework’, due to Körner (1969), descends from Kant by a somewhat different route from that of our naturalist’s categories, but the sentiment here is similar. What we don’t understand is how our mathematical formalism corresponds to the world, and clarifying this may require new logical forms and categories, a development that may or may not be possible. If such new forms and categories matched up well with the mathematical formalism, it’s even possible that classical logic would continue to apply. But none of this is understood.

In sum, then, most deviant logics deviate in ways easily understood on the proposed account of logical truth: they remove certain idealizations and assumptions in contexts where they are (purportedly) inappropriate. Doing so necessarily complicates the logic, as those idealizations and assumptions were first introduced to smooth the way; the quality of the motivation and the effectiveness of the result must be judged separately in each case. But attempts at deeper deviations, involving departures from the rudimentary logic of the categories themselves, are much less common, and their success to date is doubtful.

I have tried, as promised, to sketch an account of logical truth that is open to the naturalist but which differs from the familiar options of psychologism, simple empiricism, classical Quinean empiricism, and conventionalism. The view derives, somewhat circuitously, from Kant, with an update from Frege, making modifications necessary to remove all transcendental, and therefore unnaturalistic machinery. The view is realistic — the core of our logic reflects structural features of the world — and fallibilist.

There is much more to be said, beginning with the job of filling in the disagreeably patchy places in the view as presented here. It also seems to me worth inquiring into the relations of this position with Wittgenstein’s, early and late, and I hope for some illuminating interconnections with views on the nature of simple arithmetic truths and the extent to which mathematized science can be regarded as literal. But I leave these for other occasions. For now, I’ll be fully satisfied if this preliminary discussion succeeds in stimulating the slightest interest in this position from anyone other than myself!⁶⁹

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ENDNOTES

1. See, e.g., Quine [1951]. Of course, Quine has expressed other views in other places, e.g. 'when [the deviant logician] tries to deny the doctrine he only changes the subject' ([1970], p. 81).
2. For purposes of this discussion, 'logic' is our contemporary first-order quantification theory and some of its precursors. (I confess to thinking that a logic should have a complete proof procedure, and thus to resisting the classification of higher order logics as logic, but I won't address these issues here.)
3. For a more extended discussion, see [2001]. There I use Richard Feynman as an example of a (more or less) natural-born naturalist.
4. For more discussion of this case, see [1997], II.6, and [2001].
5. Though we may be unable to give necessary and sufficient conditions for a method to be 'scientific', this doesn't mean that we have nothing to say about science (as suggested above, it begins from observation and experiment, proceeds through theory formation, etc.) or that we have no good arguments against such practices as astrology. See [2001] for more on this point.
6. For more on Quinean naturalism, see [1997], III.3.
7. Quine might also differ on the initial point, by holding that naturalism is an explicit doctrine, and/or that science can be demarcated. I won't pursue these exegetical matters.

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8. For a fuller explanation of these points, see [1997], II.6 and III.3.
 9. Quine's famous indispensability argument for our commitment to mathematical entities depends on these controversial aspects of his account of scientific practice. For discussion, see [1997], II.2 and II.6.
 10. Notice here that it is the opponent, not the naturalist, who seems to presuppose a criterion of demarcation between science and non-science.
 11. For more on this school of thought, see Sluga [1980], chapter 1, and especially, Gregory [1977].
 12. This quotation is from Moleschott, one of the three leading scientific materialists. See Gregory [1977], p. 146.
 13. See Gregory [1977], pp. 151-152.
 14. I think a persuasive case can be made that Mill regards simple logical truths as 'merely verbal'. See Jackson [1941] or my [FKTB], IV.
 15. For a naturalist-friendly interpretation, see [FKTB], III.
 16. Also Carnap, of course, in his famous *Principle of Tolerance*: 'In logic there are no morals' ([1934], p. 52). I avoid Carnap as my example here because his related distinction between external and internal questions seems to me to offend against naturalism. (See [2000] for discussion.)
 17. See Quine [1936] and [1954].
 18. See Lewis [1969].
 19. See Skyrms [2000]. This much is clear: on Skyrms' account, we don't acquire logic truths solely by adopting the conventional meanings; we need further evolutionary training.
 20. I don't bother to oppose psychologism as most everyone nowadays seems to agree with Frege that logic isn't a matter of how we think, but of how we ought to think.
 21. Putnam would deny that these are logical truths, preferring the likes of, 'for all classes S, M, P: if all S are M and all M are P, then all S are P' ([1971], pp. 334-5). In this matter, I'm sympathetic with Quine's opinion that the latter is actually a truth of elementary mathematics, that when a logician asserts 'if all S are M and all M are P, then all S are P', he's actually asserting that all the substitution instances of this schema are logical truths. In any case, I'm working from the simple idea that a logical truth is one that's true by virtue of its form, and my examples would seem to satisfy that condition.
 22. For more detail, see [1999].
 23. Kant's example of a ground/consequence dependence is: the existence of perfect justice is the ground of the punishment of the obstinately evil (A73/B98). Notice that this is not a causal dependence. (See Allison [1983], pp. 121-2.)
 24. That is, discursive intellects with our forms of intuition. See below.
 25. I think a discursive intellect must have some forms of intuition. See [1999], III.
 26. The connection between discursivity and the table of judgments is one of the murkier areas of Kantian exegesis. Reich [1932] attempts an explication of the connection between discursivity and the forms of judgment, but my impression is that the critical community has not been persuaded. (See [1999] for some discussion.) See Longuenesse [1998] for a more recent discussion.
 27. See [1999], pp. 103.
 28. This is not to say that we humans can't make mistakes; we can be led astray by a variety of empirical factors. See [1999], p. 104.
 29. I'm not suggesting that Frege himself was or wasn't a Kantian.
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30. For more discussion of naturalism and transcendental idealism, see [2000] and [2001].
31. The connection comes in two parts: the development of the Table of Judgments (see footnote 26) and the Metaphysical Deduction from the Table of Judgments to the Table of Categories.
32. I hope to take up the matter of empirical evidence in future work; see 'Toward a naturalistic philosophy of mathematics I: logic and arithmetic', in progress.
33. Speaking of mathematical functions, of which his functions are a generalization, Frege explains that 'it is precisely by the notation that uses "x" to indicate [a number] indefinitely that we are led to the right conception' ([1891], p. 133). By setting expressions with x's on either side of the equals sign, 'we thus express ... generality' ([1891], p. 136).
34. We shouldn't assume that properties are essentially linguistic in any way. See Wilson [1985].
35. These are the so-called Lukasiewicz or Strong Kleene connectives. The Weak Kleene connectives, under which both conjunctions and disjunctions lack truth value if either component does, seem better suited to interpretations of neither true nor false as 'non-sense' rather than our 'lacking in truth value'. Also, as is well-known, truth value gaps can be avoided by using supervaluations, but I doubt this sort of meta-linguistic mode of evaluation will turn up at the rudimentary conceptual level we're attempting to describe.
36. See Parsons [2000], pp. 19-21, for discussion.
37. Not to mention the intuitionistic non-truth functional 'not' discussed in the next section!
38. See Parsons [2000], pp. 87-91, for discussion.
39. This is true even if negation and conditional are truth functional.
40. By this formulation, I mean to include the requirement that the two statements lack truth value in the same situations.
41. Parsons takes negation in the first truth-functional sense — the negation of what lacks truth value also lacks truth value — so he gets a few inferences that we do not here, e.g., from not-not-p to p.
42. The key here is the truth-functional understanding of 'and' (coupled with the definition of validity): if 'Bj&Hj' is T, then 'Bj' is T and 'Hj' is T, so 'Hj' is T. If we prefer to carry out this argument in a meta-language as rudimentary as the object language, we must do without the conditional, say by redefining 'validity' to mean something like 'there is (in the ML) a good argument from the truth of the premises to the truth of the conclusion'. Then the step from 'Bj&Hj' is T, to 'Bj' is T and 'Hj' is T, is justified by the truth functional account of '&' in the object language, and the step from 'Bj' is T and 'Hj' is T to 'Hj' is T is justified by the same thing in the ML. Notice that a similar attempt to justify the inference from 'not-not-p' to 'p' fails because the rudimentary logic includes no truth functional account of 'not'.
43. Frege's rejection of referent-less names is similar: 'a logically perfect language [e.g., the language of [1893]] should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a *Bedeutung*' ([1892a], p. 163). See footnote 59.
44. By our lights, he should say that there need be no ground/consequent connection between the two, given that what's at issue is the pure categories, not the schematized categories. But I don't mean to suggest that Frege saw this move in our terms, as a departure from the underlying category. The only hint is his perhaps off-hand remark, cited above, about a 'close affinity'
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between his conditional and ‘the important relation of ground and consequent’ ([1880/1], p. 37).

45. If we undertake to settle on a truth-functional negation and conditional before introducing bivalence, there are several possibilities, perhaps the most appealing of which is the one Parsons adopts in his [2000]: the Lukasiewicz connectives plus a unary ‘determinately’ connective that takes truth to truth and everything else to falsehood. (See his pp. 22-28 for the fairly straightforward logic that corresponds to this semantics.) Here my goal is to catalog what bridges the gap between the most rudimentary logic and full classical logic, and I think the essential moves come out more clearly in the order presented in the text.

46. For Frege, first order quantifiers ranged over all objects whatsoever. See Goldfarb [1979] for discussion of how this modification of Frege developed.

47. For the record, Boole thinks there is a ‘manifest fitness between the intellectual procedure...and the conditions of that system of things by which we are surrounded, — such conditions, I mean, as the existence of species connected by general resemblances’, but he insists that the existence of ‘this correspondence between the forms of thought and the actual constitution of Nature...would in no sense lead to the consequence that one system is the mere product of the other’ ([1854], XXII.9, p. 420, and XXII.8, p. 418). In opposing Kant here — nature being partly constituted by our thought — he unfortunately also rejects the converse notion that the structure of our thought might be influenced by nature!

48. Suppose electrons A and B have all the same properties except for location. Now imagine switching the positions of A and B. On our usual conception of object, these are two distinct situations, but apparently they aren’t in fact. See Teller [1995], pp. 10-11, and chapter 2, for discussion of ‘what must clearly be left out [of our familiar notion of object] to get a concept that fits the facts of the quantum world’ (p. 10). See also van Fraassen [1991], chapter 11.

49. Thinking in our familiar categories, it seems that an electron has x-spin up or down and y-spin up or down, but that it cannot be said to have x-spin up and y-spin up or x-spin up and y-spin down or x-spin down and y-spin up or x-spin down and y-spin down. (See Albert [1992], pp. 1-11, for discussion of the experimental evidence that leads to these bizarre conclusions.)

50. As in the previous footnote.

51. Recall that Kant’s example of a ground/consequent dependency (footnote 23) was acausal. It isn’t clearly linguistic either; in fact, I suspect that it, like logical truths, bears a particular sort of intermediate status in Kant’s system. See [1999].

52. As Albert colorfully puts it: ‘Let’s say...precisely what the locality assumption amounts to: the assumption says that I can’t punch you in the nose unless my fist gets to where your nose is’ ([1992], p. 64, footnote 4). Quantum mechanics seems incorrigibly non-local.

53. By a very different route, Wilson arrives at what seems to me a closely related position. Speaking of the justification of logic, he writes: ‘we should look at the soundness proofs of logic in [a] provisional spirit: *if* we are guaranteed that the sets and other ingredients needed to supply truth-conditions are well defined within the setting at hand, *then* first order reasoning over this language can be trusted’ ([2000], p. 392).

54. I’ll spell the contrasts with Quine in a moment.

55. Putnam admits to being ‘torn’ on such questions ([1978], pp. 111-112), so my straightforward characterization in the text doesn’t include or even square with much of what he says in the relevant papers ([1962], [1978], and [1995]). But this simple idea is one thread in his thinking, and it’s the one that interests me here.

56. Putnam's example is the law of non-contradiction, which would fall in the same class as modus ponens on the view proposed here. The conviction that a statement can't be both true and false is part of the underlying conception of objects standing in relations, and thus revising it would also involve revising part of our fundamental categorical underpinning.

57. It might be analytic in a Kantian sense of not being synthetic. See [1999].

58. I'm grateful to Kyle Stanford for highlighting this sense of the logical 'must'.

59. In addition to the non-bivalent logic discussed above (see footnote 45), another in this group would be free logic, which rejects the idealization that all names have referents. (See footnote 43.) I leave this aside because I've been passing over names altogether.

60. Another candidate for the second group is paraconsistent logic, which denies that a contradiction implies everything. The most radical version of this view, dialetheism, insists that there are literally true contradictions, sentences that are both true and false (see Priest [1998]). This would violate the most rudimentary principles of conjunction and negation, and thus undermine the categorical foundations of our logic. I stick to the example of quantum logic in the text.

61. See Heyting [1971]: 'The idea that for the description of some kinds of objects another logic may be more adequate than the customary one has sometimes been discussed. But it was Brouwer who first discovered an object which actually requires a different form of logic, namely the mental mathematical construction' (p. 1).

62. I'm speaking here of the early intuitionism of Brouwer and Heyting, which focused on the nature of mathematical entities. Dummett's contemporary intuitionism springs from a number of other sources; one version (Dummett [1994]) rejects the well-behaved domain of quantification and derives the need for intuitionistic logic from that.

63. Not-(p or not-p) is an example, as we'll see in the next paragraph of the text.

64. E.g., see Priest [2001], 1.7-1.10, 4.8.

65. The classic source for Putnam on quantum logic is [1968]. For his continued support of a straightforwardly empirical quantum logic, and hence, for the distinction between the law of non-contradiction and the distributive law, see his [1995], footnote 10.

66. In the sense that particles have definite values for all relevant properties at all times. See footnote 49.

67. Thus the view suffers from the limitations of the classical Quinean empiricism noted above. Furthermore, as Gibbons observes, quantum logic is to be used in a 'rather limited and "unphysical" spirit...[the goal] is not to get the *right* answer for the two-slit pattern, which is the physicist's problem...[but] to *stop getting the wrong answer*. Avoidance of paradox is, I am afraid, typically the philosopher's but not the physicist's strategy' ([1987], p. 148).

68. See Gibbins [1987], chapter 10. What quantum logic does accomplish, according to Gibbins, is to provide 'an inexpressive language ... [that] does not *resolve* the paradox, it prevents its being formulated' (pp. 160-161).

69. I'm grateful to Aldo Antonelli, Mark Balaguer, Patricia and Paul Churchland, Mark Colyvan, Sean Duggan, Hartry Field, David Malament, Patricia Marino, Adina Roskies, Kyle Stanford, Jamie Tappenden, Eric Updike, David Velleman, Mark Wilson, helpful audiences at MIT, UCSD and the 2001 Romanell Lecture, and several anonymous referees for comments on earlier drafts.
