Defending the Axioms

Winter 2009

This course is concerned with the question of how set theoretic axioms are properly defended, of what counts as a good reason to regard a given statement as a fundamental assumption of set theory, and by extension, of contemporary classical mathematics. This question inevitably raises underlying issues of set-theoretic metaphysics and epistemology. The interaction between methodological matters -- which axioms and on what grounds? -- and metaphysical/epistemological matters -- what are sets and how do we know about them? -- will be the specific focus.

(I offer no apology for the topic, but I do apologize for proposing to spend so much time on my own approach to it. My selfish plan is to try out material from a book-in-progress, also called *Defending the Axioms*, in the second half of the course, and, perhaps unsurprisingly, reading material from my earlier work -- especially *Naturalism in Mathematics* -- strikes me as the best way to set the stage. I promise a more responsible reading list for next quarter's seminar on 'The New Wave in Philosophy of Mathematics': articles from Ferreirós and Grey, eds., *The Architecture of Modern Mathematics* (2006) and Mancosu, ed., *The Philosophy of Mathematical Practice* (2008), along with Charles Parsons's new book, *Mathematical Thought and its Objects* (2008), which isn't strictly 'new wave', but is indisputably new.)

The default requirement for those taking the course for a grade (other than S/U) is three short papers (750-1250 words) due at the beginning of class in the 4th week, 7th week, and 10th week. Each paper should isolate one localized point in the readings and offer some analysis and/or critique. Those preferring to write one longer paper (2500-3750 words) should let me know and consult on potential topics.

I assume everyone has access to copies of *Naturalism in Mathematics* and Benacerraf and Putnam's anthology. For helpful background, see

Dauben, *Georg Cantor.*

Ebbinghaus, *Ernst Zermelo.*

Ferreirós, *Labyrinth of Thought*.

Fraenkel, Bar-Hillel and Levy, *Foundations of Set Theory*.

Hallett, *Cantorian Set Theory and Limitation of Size*.

Moore, *Zermelo's Axiom of Choice*.

Jean van Heijenoort, ed., *From Frege to Gödel*.

Copies of other assigned readings will be available for photocopying in a box outside my office door. Please come to the first meeting prepared to discuss the readings in Topic #1.

Topics:

1. The problem

Naturalism in Mathematics, pp. 1-85.

Extra reading:

Boolos, ‘The iterative conception’, ‘Iteration again’ and ‘Must we believe set theory?’

Parsons, ‘What is the iterative conception of set?’

Russell, ‘The regressive method of discovering the premises of mathematics’.

Shapiro, ‘Foundations: structures, sets, and categories’.

Zermelo, ‘A new proof of the possibility of a well-ordering’ and ‘Investigations in the foundations of set theory’.

2. Some data

‘Believing the axioms’, pp. 490-508, 736-748.

Koellner, ‘On the question of absolute undecidability’, pp. 167-186.

Rapalino, notes.

Extra reading:

Dehornoy, ‘Recent progress on the Continuum Hypothesis (after Woodin).

Martin, ‘Mathematical evidence’.

3. Robust Realism

Moore, ‘Introduction to Gödel’s “What is Cantor’s continuum problem?”’.

Gödel, ‘What is Cantor’s continuum problem?’

Naturalism in Mathematics, pp. 87-129.

Extra reading:

Parsons, ‘Introduction to Gödel’s “Russell’s mathematical logic”’.

Gödel, Russell’s mathematical logic’.

Boolos, ‘Introduction to Gödel’s Gibbs lecture’.

Gödel, the Gibbs lecture.

Solovay, ‘Introduction to Gödel [1970abc]’.

Gödel, three little pieces on CH ([1970abc]).

4. Troubles for Robust Realism

Benacerraf, ‘Mathematical truth’.

Steiner, *Mathematical Knowledge*, pp. 109-116.

Hart, Review of Steiner, pp. 123-126.

Field, *Realism, Mathematics and Modality*, pp. 25-30.

Naturalism in Mathematics, pp. 130-160.

Colyvan, *The Indispensability of Mathematics*, chapter 5.

Extra reading:

Realism in Mathematics, pp. 36-50.

Second Philosophy, pp. 404-407.

Papers in Leng et al, eds., *Mathematical Knowledge*.

Leng, ‘Mathematical explanation’.

5. Naturalism

Naturalism in Mathematics, pp. 161-234.

Löwe, ‘A first glance at non-restrictiveness’ and ‘A second glance at non-restrictiveness’.

6. Second Philosophy

Second Philosophy, pp. 9-46, 83-96, 305-313.

Exchange with Stroud.

Extra Reading:

Second Philosophy, pp. 47-82, 97-111, 392-411.

‘Naturalism, transcendentalism and therapy’.

7. Mathematics, applied and pure

Second Philosophy, pp. 314-360.

‘How applied mathematics became pure’.

8. Thin Realism

Second Philosophy, pp. 361-377.

‘Thin realism’.

Extra reading:

Liston, ‘Thin- and full-blooded platonism’ and ‘Review of *Second Philosophy*’.

Tait, ‘Truth and proof’ and ‘Beyond the axioms’.

9. Arealism

Second Philosophy, pp. 377-391.

‘Arealism’.

Extra reading:

Liston, ‘Thin- and full-blooded platonism’ and ‘Review of *Second Philosophy*’.

10. Morals

‘A plea for extrinsic justifications’.

‘Robust realism revisited’.

Extra reading:

Burge, ‘On knowing the foundation’.

Martin, ‘Mathematical evidence’.

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[1989] ‘Iteration again’, reprinted in his [1998b], pp. 88-104.

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[1998a] ‘Must we believe set theory?’, in his [1998b], pp. 120-132.

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