Problems of Commitment in Arming and War: How Insecurity and Destruction Matter[†]

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Current Version: August 14, 2018

Abstract: This paper analyzes a guns-versus-butter model in which two agents compete for control over an insecure portion of their combined output. They can resolve this dispute either peacefully through settlement or by military force through open conflict (war). Both types of conflict resolution depend on the agents' arming choices, but only war is destructive. We find that, insofar as entering into binding contracts on arms is not possible and agents must arm even under settlement to secure a bigger share of the contested output, the absence of long-term commitments need not be essential in understanding the outbreak of destructive war. Instead, the ability to make short-term commitments could induce war. More generally, our analysis highlights how the pattern of war's destructive effects, the degree of output insecurity and the initial distribution of resources matter for arming decisions and the choice between peace and war. We also explore the implications of transfers for peace.

JEL Classification: D30, D74, F51.
Keywords: contested property rights, settlement mechanisms, destructive conflict, negotiation, armed peace.

[†]We thank Todd Sandler, Kostas Serfes, Ricardo Serrano-Padial, and participants of the Eighth Conference on Political Violence and Policy (2018) at the University of Texas-Dallas, for useful comments.
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1 Introduction

An expansive literature in political science aims to understand why war breaks out, both between and within countries, when an alternative, nondestructive means of resolving disputes is possible—namely, bargaining. Building on the traditional view that war is part of a bargaining process and following Fearon's (1995) lead, much of the recent research has examined possible reasons, based on rational behavior, for negotiations to fall apart and, thus, for destructive wars to emerge. These include the indivisibility of what is being contested, incomplete information regarding the rival's strength or preferences, and commitment problems.[1] Of particular interest to us in this paper are commitment problems under complete information. As analyzed in the existing literature, such problems come in a variety of forms, but all boil down to the idea that a negotiated deal cannot be enforced in the future and that undermines the possibility of peace today.[2]

This paper offers a different perspective on commitment problems in war and peace, showing that one of the contending parties in a dispute could have a short-run incentive to commit to war. Our analysis builds on the familiar guns-versus-butter framework in a one-period setting, where two agents dispute the distribution of an insecure portion of their produced butter.[3] There are two modes of conflict resolution: open conflict (or war) modeled as a winner-take-all contest and peaceful settlement wherein the agents divide the contested output. Peace requires the contending parties to come to a mutually beneficial agreement, whereas war emerges when at least one party declares it. Both modes of interaction induce the contending parties to arm or produce guns to advance their respective positions, and that diverts resources away from the production of butter, thereby making the dispute, however resolved, costly and the contested prize endogenous. But, because only war is destructive, the two modes generate different incentives to arm.

A key assumption that distinguishes our analysis from much of the previous literature concerns the pattern of war's destructive effects. In particular, while war destroys some fraction of contested output, its effect on the contested output of the defeated side is greater.

[1] See Jackson and Morelli (2011) for a relatively recent survey of the literature. To the reasons suggested by Fearon (1995), they add (i) agency problems that arise when the leaders' preferences are not aligned with those of their citizenry and (ii) problems in multilateral bargaining where coalitions can form to block any possible negotiated agreement that would benefit all.

[2] See Powell (2006) for an extensive discussion of commitment problems, especially as they arise with expected shifts in power away from one party to another. Absent a means of enforcing current deals in the future, the party expecting to lose power in the future is compelled to engage in war today, despite a short-run preference for peace due to war's destructive effects (see also Fearon 1998; Acemoglu and Robinson 2001). Others have emphasized that negotiated agreements made today can settle only a current dispute, so that maintaining peace over time requires repeated negotiations in the future. Because that involves the diversion of additional resources from production (e.g., arming), one or both sides might prefer to end the dispute today (or at least to severely weaken the opponent) by declaring war, again despite war's destructive effects (Garfinkel and Skaperdas 2000; McBride and Skaperdas 2014).

[3] See Garfinkel and Skaperdas (2007) for a survey of conflict models and their applications in economics.

One interpretation of this differential effect is that some of the contested output is lost or deteriorates in transit as the victor forcibly takes possession of it. Alternatively, war's destruction could simply be more severe for the losing side. In any case, this assumption introduces a possible endogenous asymmetry in the two sides' valuation of the prize under war. Provided initial resource endowments are distributed unevenly, it implies an asymmetric equilibrium in guns under war and possibly very different preferences for the contending parties over settlement and peace.[4]

In such a setting, the timing of actions matters for the outcome of the agents' choices between war and peace. When the two agents arm in advance of their negotiations and thus in advance of their eventual decision of whether to settle peacefully or to fight, the direct destructive effects of war on payoffs alone render settlement more appealing. Now suppose the timing is reversed as in Beviá and Corchón (2010). In this case, where the agents can effectively commit to war/peace but not to arming, the less endowed party always prefers peace, while the richer party could prefer war. To be sure, the overall and differential destructive effects of war have direct negative payoff consequences that tend to decrease the relative appeal of war to both sides. However, the payoff effect of war's differential destruction (on the defeated side's insecure output) is not as strong for the richer agent, since the amount of output the poorer agent contributes to the contested pool is naturally smaller. What's more, such destruction induces less arming by the rival relative to settlement. If war's overall destruction is not too severe, the richer agent (in contrast to the poorer rival) could exhibit a strict *ex ante* preference for war, and an increase in the degree of output security amplifies that preference. In such cases, his equilibrium payoff when the war/peace choice is made before the adversaries choose their arming exceeds his equilibrium payoff when the timing is reversed. Consequently, if it were possible for the agents to commit to war/peace in advance of arming, war could emerge even in a one-period setting.[5]

[4]If there were no differential destruction under open conflict, the two sides' preferences over war and peace would be identical, with both sides always strictly preferring settlement to avoid war's overall destructive effects. To the best of our knowledge, the implications of differential destruction for the endogenous choice between war and peace have never before been explored. Powell (1993), for example, similarly assumes differential destruction, but does not explore its implications for arming choices that would naturally influence, in turn, preferences over peace and war. While Grossman and Kim (1995) explore the implications of this assumption for defensive and predatory arming choices, their focus is on how one agent can arm sufficiently for defensive purposes to deter his rival from subsequently arming for predatory purposes.

[5]Others have found similarly that war can emerge as an equilibrium outcome in a one-period setting, although the underlying mechanisms at play are different. Specifically, in Beviá and Corchón (2010) where war's only costs are the resources dedicated to fighting and peace is costless, one agent might declare war as it forces a redistribution of resources in the victor's favor, whereas peace maintains the status quo. Abstracting from arming choices but incorporating war's destructive effects, Jackson and Morelli (2007) emphasize the role of political biases within a country, where the decision maker of the country stands to gain relatively more from a victory, to induce a war. In both analyses, assuming a proportional conflict technology (as we do), it is the poorer agent who stands to gain (on net) more from a redistribution of resources and thus is

Of course, it might not be possible to commit to war before having armed. What our analysis suggests, however, is that the contending agent wanting to fight would do whatever he could, in advance, to sabotage negotiations with his rival. In such cases, even interventions by dominant outside parties aiming to forge a peaceful outcome would likely be ineffective. Consider, for example, the Israeli-Palestinian bargain associated with the Oslo Accords in 1993 that was subsequently followed by much greater conflict. By the same token, the recent United Nations–backed "unity" government in Libya continues to face opposition from the two rival governments and a number of militias, resulting in ongoing conflict in that country.

But, like Beviá and Corchón (2010), Jackson and Morelli (2007) and others, we find that transfers made in advance of the choice between war and peace might be able to induce a peaceful outcome even if such transfers do not involve a commitment to choose peace. Since it is the less affluent agent who prefers peace, only he would consider making a transfer. Despite the effect of such transfers to result in a greater degree of resource disparity, they can serve to better align the preferences of the two agents.[6] We find this pacifying effect to be more effective when output is less secure, which suggests that higher output security need not be conducive to peace, even when we allow for transfers.

In what follows, we present our basic framework that allows for the two modes of conflict resolution (peaceful settlement and open conflict) and the possibility of differential destruction as well as overall destruction under open conflict. The case of settlement is a special case of open conflict with no destruction. In Section 3, we identify and characterize the subgame perfect equilibrium conditional on the mode of conflict resolution and compare the resulting payoffs. Based on that analysis, we examine in Section 4 the equilibrium choice of peace vs. war, with and without transfers made in advance of that decision. Section 5 concludes with a discussion of possible extensions of the analysis. All technical details are relegated to an online appendix.

2 Resolving disputes over output

Consider an environment in which $\bar{R}$ units of a productive resource are distributed among two risk-neutral agents, $i = 1, 2$. Agent i is endowed with R^i units, and $R^1 + R^2 = \bar{R}$. Each agent i can use this resource to produce, on a one-to-one basis, G^i ($\leq R^i$) units of "guns" and X^i ($= R^i - G^i$) units of a consumption good, say "butter." A fraction $\kappa \in [0, 1)$ of

more inclined to declare war. In a one-period setting that, like ours, supposes settlement requires arming and is thus costly, Chang and Luo (2017) find that war can be preferred *ex ante* over settlement by both agents when war's destructive effects depend positively on arming choices, since in this case war tends to induce less arming than settlement. Yet, that analysis does not consider the importance of the distribution of resource endowments.

[6] In Beviá and Corchón (2010) and Jackson and Morelli (2007), by contrast, only the more affluent agent might be willing to make a transfer, resulting in a more even distribution of resources (see footnote 5).

each agent's butter X^i is secure. The remainder is insecure and subject to contestation. Accordingly, we view $1-\kappa$ as a measure of output insecurity.

Property rights over the contested portion of the output of butter can be established in one of two ways: through open conflict or "war" where the winner takes the entire prize or through a peacefully negotiated division of the contested output. Either way, the outcome depends on the agents' arming decisions. In the case of war, guns affect the probability of victory for each agent. In the case of peaceful settlement, guns affect the division of the insecure quantities of butter between the two agents.[7] In both cases, an agent's arming is costly as it detracts from the production of butter.

There are a variety of ways one can model how arming matters for settlement, based on well-known bargaining protocols, such as Nash bargaining and split-the-surplus.[8] However, we rely on a formulation that allows us to highlight the important trade-offs involved without complicating the analysis unnecessarily. More precisely, let the aggregate quantity of guns chosen by the two agents be denoted by $\bar{G} = G^1 + G^2$. Then, the influence of guns on the outcome either under war or settlement operates via a simple conflict technology (or CSF), first introduced by Tullock (1980):

$$\phi^i = \phi^i(G^i, G^j) = \begin{cases} G^i/\bar{G} & \text{if} \quad \bar{G} > 0 \\ 1/2 & \text{if} \quad \bar{G} = 0 \end{cases}, \quad i \neq j = 1, 2. \tag{1}$$

In the case of open conflict, $\phi^i(G^i, G^j)$ represents the probability that agent i emerges as the victor; in the case of settlement, $\phi^i(G^i, G^j)$ represents his share of the contested output. The specification in (1) assumes that this probability or share for agent i is increasing in his own guns ($\phi^i_{G^i} > 0$) and decreasing in the guns of his rival ($\phi^i_{G^j} < 0$). It also implies that $\phi^i(G^i, G^j)$ is symmetric (so that $G^i = G^j = G \geq 0$ implies $\phi^i = \phi^j = \frac{1}{2}$) and concave in G^i. Finally, it implies that $\phi^i_{G^iG^j} \gtreqless 0$ as $G^i \gtreqless G^j$ for $i \neq j = 1, 2$.[9]

While the two forms of conflict resolution similarly rely on arms, they differ in that war has a destructive effect on the agents' contestable butter while peace does not. Specifically, suppose agent i emerges as the winner of the war, which occurs with probability ϕ^i. Then, he controls $(1-\kappa)\beta\left(X^i + \gamma X^j\right) + \kappa X^i$ units of butter. The fraction $\beta \in [0, 1]$ represents the fraction of his contestable butter that remains intact and $\beta\gamma \in [0, 1)$ is the corresponding

[7] Many scholars in political science (e.g., Powell 1993; Fearon 1995), by contrast, view settlement as resulting in the status quo. They either abstract from arming or treat it as exogenously determined.

[8] See Anbarci et al. (2002) and Garfinkel and Syropoulos (2018) who study the efficiency properties of rules of division based on these and other protocols in different settings.

[9] See Skaperdas (1996) who axiomatizes a more general class of CSFs, $\phi^i(G^i, G^j) = f(G^i)/\sum_{h=1,2} f(G^h)$ for $i \neq j = 1, 2$, where $f(\cdot)$ is a non-negative and increasing function. Hirshleifer (1989) explores the properties of the ratio form, where $f(G) = G^a$ with $a \in (0, 1]$, and of the difference form, where $f(G) = e^{\alpha G}$ with $\alpha > 0$. For simplicity and tractability, we chose the ratio form with $a = 1$.

fraction of his rival's contestable butter that similarly survives the war.[10] Importantly, for our purposes, $\gamma \in [0,1)$ reflects the possible differential damage inflicted upon the losing side's contestable butter that goes to the victor, with a lower value of γ implying a greater differential. This parameterization captures the notion that, while the forces of war generally destroy some portion of all that is contestable ($\beta < 1$), the component of the victor's winnings that comes from the defeated agent suffers additional damage ($\gamma < 1$) in the violent process by which it is seized by the victor.[11] Alternatively, if agent i loses the war, which occurs with probability $1-\phi^i$ $(=\phi^j)$, he controls only his secure output: κX^i units of butter. One can verify now that agent i's expected payoff function U^i under open conflict is given by:

$$U^i = U^i\left(G^i, G^j\right) = \phi^i \beta \left(1-\kappa\right)\left(X^i + \gamma X^j\right) + \kappa X^i,\ i \neq j = 1,2. \tag{2}$$

Though non-destructive, settlement like open conflict requires the diversion of resources from the production of butter, as emphasized above. Agent i's payoff in this case is given simply by

$$V^i = V^i\left(G^i, G^j\right) = \phi^i \left(1-\kappa\right)\left(X^i + X^j\right) + \kappa X^i,\ i \neq j = 1,2. \tag{3}$$

Comparing (2) and (3) shows that the payoff functions under settlement and open conflict would be equivalent if there were no destruction at all under open conflict: $\beta = \gamma = 1$.

A central objective of our analysis is to explore how the overall and differential rates of destruction under open conflict, respectively $\beta \leq 1$ and $\gamma < 1$, along with the degree of output security κ and the initial distribution of the resource $\bar{R}$ affect the agents' arming decisions and their payoffs under open conflict and settlement. In turn, the analysis unveils the forces that shape the agents' decisions of whether to fight or settle peacefully. In this context, we also address the question of whether resource transfers prior to any other decisions could avert war.

As in Beviá and Corchón (2010), choices are made in the following sequence: First, one agent possibly transfers some of his resource R^i to the other agent. Second, agents declare war or peace, with peace arising if and only if both sides choose it. Finally, agents arm and produce butter from their residual resource endowments. Each agent consumes the secure portion of his butter, and the remaining butter is distributed among them depending on the mode of conflict resolution determined in the second stage. This sequence of actions is important in that it allows us to also explore the value of commitments in choosing between

[10] As discussed below, our central results to follow remain unchanged when both secure and insecure output are subject to overall destruction, $1-\beta$.

[11] In contrast, when the two sides settle peacefully, the transfer of output from one agent to the other involves no violent force and thus would not be subject to such additional damage.

war and peace, when no such commitments are possible for arming choices.

3 Subgame perfect equilibria under settlement and conflict

Having described the essential features of the model and, in particular, the two modes of conflict resolution, we now explore the distinct equilibria that each induces, and then compare them in terms of payoffs for each agent.

3.1 Settlement

We start with settlement as that provides a natural benchmark for our subsequent (and significantly more complex) analysis of open conflict. By the specification of the conflict technology in (1) under settlement, a positive quantity of guns produced by agent i always yields a positive share of the contested prize, and that share is increasing in G^i given the rival's choice G^j. But, each agent i's arming choice is subject to his resource constraint $G^i \in \left[0, R^i\right]$ for $i = 1, 2$; and, depending on the initial distribution of resources, this constraint could bind. Keeping that in mind, let agent i's best response to $G^j > 0$ ($j \neq i$) under settlement be denoted by $B_s^i\left(G^j; \cdot\right)$. Differentiating V^i in (3) with respect to G^i shows agent i's incentive to arm, given his rival's arming choice $G^j > 0$:

$$V_{G^i}^i = \phi_{G^i}^i \left(1 - \kappa\right) \left(\bar{R} - \bar{G}\right) - \left[\left(1 - \kappa\right) \phi^i + \kappa\right], \ \ i = 1, 2, \tag{4}$$

where $\bar{R} - \bar{G} = \sum_i X^i = \sum_i (R^i - G^i)$. The first term on the right hand side (RHS) of the expression above represents the marginal benefit to agent i from arming under settlement (MB_s^i). It reflects the expansionary effect of an increase in agent i's guns on his share of the contested pool of butter, which is itself endogenous. The second term is the marginal cost of arming under settlement (MC_s^i) that derives from the effect of arming to reduce the production of butter, part of which is not secure and part of which is secure. Higher output insecurity ($1 - \kappa \uparrow$) clearly increases MB_s^i; and, since $\phi^i < 1$, it also reduces MC_s^i. These two reinforcing effects imply higher output insecurity alone fuels agent i's arming incentives. Furthermore, inspection of the $\bar{R} - \bar{G}$ term in MB_s^i reveals that, when neither agent's arming decision is constrained by his resource endowment, an arbitrary transfer of that resource from agent i to agent j (where $dR^j = -dR^i$ so as to leave $\bar{R}$ unchanged) is inconsequential for arming incentives. Finally, observe that an increase in the rival's guns G^j influences each agent i's net benefit of arming. One can show this influence is positive (negative) when $G^i > G^j$ ($G^i < G^j$).[12]

Based on the above ideas using (4), one can see that an agent i's best-response function

[12]See the proof to Proposition 1 in the online appendix.

under settlement is given by

$$B_s^i\left(G^j;\kappa,R^i,\bar{R}\right)=\min\left\{R^i,\widetilde{B}_s^i(G^j)\right\},\ \ i\neq j=1,2 \tag{5a}$$

where

$$\widetilde{B}_s^i\left(G^j\right)\equiv -G^j+\sqrt{(1-\kappa)\,G^j\bar{R}} \tag{5b}$$

is agent i's unconstrained best-response function.[13] Next, we define the following:

$$R_L^s\equiv \tfrac{1}{4}\left(1-\kappa\right)\bar{R}\ \ \text{and}\ \ R_H^s\equiv\left[1-\tfrac{1}{4}\left(1-\kappa\right)\right]\bar{R}, \tag{6}$$

where "L" ("H") denotes the low (high) threshold level of the resource, conditional on the agents' settling their dispute peacefully ("s"), that together define the parameter space for which an agent is not resource constrained in his arming choice as detailed below. Clearly, $R_L^s+R_H^s=\bar{R}$, and the difference between these values, $R_H^s-R_L^s=\frac{1}{2}[1+\kappa]$, is increasing in the degree of output security κ.

Using (5) and (6), we establish the following:

Proposition 1 (Arming under Settlement) *Assume output is not perfectly secure ($\kappa<1$) and both agents choose to settle their dispute peacefully. Then, there exists a unique equilibrium in arming, with positive quantities of guns produced by both agents $G_s^i>0$, $i=1,2$. For any given $\bar{R}$ such that $R^i+R^j=\bar{R}$ ($i\neq j=1,2$), these quantities are characterized as follows:*

(a) *If $R^i\in[R_L^s,R_H^s]$ for $i=1,2$, then $G_s^i=R_L^s$, with $dG_s^i/d\kappa<0$.*

(b) *If $R^i\in(0,R_L^s)$ for $i\neq j=1$ or 2, then $G_s^i=R^i$ and $G_s^j=\widetilde{B}_s^j(R^i)>G_s^i$, with $dG_s^j/d\kappa<0$.*

This proposition establishes that an uneven distribution of $\bar{R}$ across the two agents (R^i,R^j) matters only insofar as that distribution implies one agent is constrained in his production of guns. Specifically, part (a) shows that when the distribution of resources across the two agents is sufficiently even such that neither agent's resource constraint binds, the equilibrium in arming is symmetric, and a transfer of the resource from one agent to the other has no effect on arming by either one provided the transfer leaves both agents' constraints non-binding. By contrast, as shown in part (b), when agent i's resource constraint binds, agent j is not resource constrained and the equilibrium becomes asymmetric with the unconstrained

[13] To avoid cluttering of notation, we suppress the dependence of $\widetilde{B}_s^i$ on κ, R^i and $\bar{R}$.

agent (j) naturally arming by more.[14] Whether the initial distribution of $\bar{R}$ is sufficiently even or uneven, the unconstrained agent's arming is decreasing in the degree of output security ($\kappa \uparrow$). Due to its effect on arming incentives, an increase in κ also expands the range of resource distributions for which neither agent is resource-constrained, $R^i \in [R_L^s, R_H^s]$. Fig. 1(a), which depicts agent $i = 1$'s equilibrium arming under settlement for various allocations of the resource and alternative values of κ, illustrates these findings.

Building on our characterization of equilibrium arming choices in Proposition 1 and using (3), we turn to the agents' payoffs.

Proposition 2 (Payoffs under Settlement) *Agent i's equilibrium payoff under settlement V_s^i $(i = 1, 2)$ depends on the distribution of resource ownership $R^i \in (0, \bar{R})$ and the degree of output security κ as follows:*

(a) *If $R^i \in [R_L^s, R_H^s]$ for $i \neq j = 1, 2$, then $V_s^i = \frac{1}{4}(1-\kappa)\bar{R} + \kappa R^i$, such that $dV_s^i/d\kappa \gtreqless 0$ as $R^i \gtreqless \frac{1}{4}\bar{R}$.*

(b) *If $R^i \in (0, R_L^s)$ for $i \neq j = 1$ or 2, then* (i) *$V_s^i < V_s^j$;* (ii) *$dV_s^i/dR^i > 0$ with $\lim_{R^i \to 0} V_s^i = 0$ and $dV_s^i/d\kappa < 0$; and,* (iii) *$dV_s^j/dR^j > 0$ with $\lim_{R^j \to \bar{R}} V_s^j = \bar{R}$, and $dV_s^j/d\kappa > 0$.*

Fig. 1(b), which shows that an agent's payoff is non-decreasing in his own endowment, illustrates these results. Part (a) establishes specifically that, even when the distribution of resource endowments is sufficiently symmetric to ensure equalization of arming, the two agents' payoffs will differ due to differences in their initial resource endowments, provided that there is some output security (i.e., $\kappa > 0$). Although an arbitrary transfer of resources from agent i to agent $j \neq i$ that leaves $\bar{R}$ unchanged has no effect on arming and thus generates no indirect payoff effects in this case (by Proposition 1(a)), it does imply a direct positive effect on the recipient's payoff and a direct negative effect on the donor's payoff (again provided that $\kappa > 0$). Part (b) shows that, when resource endowments are sufficiently uneven to constrain agent i's arming choice, the richer agent (j) enjoys a higher payoff; and, any transfer of the resource from the poorer agent to the richer one amplifies that difference, while a transfer from the richer agent to the poorer agent dampens it.

Perhaps the more interesting set of implications relate to improvements in output security ($\kappa \uparrow$) that tend to magnify the payoff effects of arbitrary resource transfers, resulting in a steeper payoff function over $R^i \in (0, \bar{R})$ as depicted in Fig. 1(b). In particular, when the distribution of resource endowments is sufficiently asymmetric to make one agent i constrained (so that $X^i = R^i - G^i = 0$), such improvements benefit the richer agent j while hurting the poorer agent i. The mechanisms at play here can be seen by considering the

[14] That only one agent i at most can be resource constrained follows since, by the definition of $\bar{R}$, $R^j = \bar{R} - R^i$ $(j \neq i = 1, 2)$, such that $R^i \in (0, R_L^s)$ implies $R^j \in [R_H^s, \bar{R})$.

direct and indirect payoff effects. Recall from Proposition 1 that an increase in κ reduces arming incentives for both agents. Less arming by the richer agent (j) implies a positive strategic payoff effect for the poorer agent (i). However, that effect is swamped by the negative direct effect for the poorer agent due to the fact that $X^i = 0$ (see equation (3)). Since the poorer agent cannot adjust his guns and the envelope condition holds for the richer agent (j), only the direct payoff effect matters for the richer agent, and that effect is positive since $\phi^j < 1$. What's more, even if neither agent is resource-constrained (i.e., $R^i \in [R_L^s, R_H^s]$), the poorer one would be worse off as κ rises, provided his endowment is sufficiently smaller than that of his rival: $R^i < \frac{1}{4}\bar{R}$.[15] However, if R^i and R^j are each greater than $\frac{1}{4}\bar{R}$, both agents benefit from improvements in output security. Nonetheless, the important point for our purposes is that, in the absence of commitments to restrain arming, exogenous improvements in security could be unappealing to the less affluent side even when both choose to resolve their dispute peacefully.

3.2 Open conflict

To explore the implications of open conflict for arming by each agent i, consider the marginal effect of an increase in G^i on agent i's expected payoff U^i in (2):

$$U^i_{G^i} = \phi^i_{G^i}\beta\left(1-\kappa\right)\left(X^i + \gamma X^j\right) - \left[\left(1-\kappa\right)\beta\phi^i + \kappa\right], \;\; i \neq j = 1, 2. \tag{7}$$

The first term in the RHS of (7) represents agent i's expected marginal benefit to arming (MB^i_c) that arises from the effect of an additional gun to increase agent i's probability of taking possession of the entire (endogenously determined) prize net of destruction. Since increases in the degree of output security (κ), the overall rate of destruction $(1-\beta)$, and the differential rate of destruction on the defeated agent's butter $(1-\gamma)$ reduce the value of the contested output, such changes reduce MB^i_c. The second term in the RHS of (7) represents agent i's opportunity cost of arming (MC^i_c) that (like MC^i_s) depends positively on output security κ. While independent of the differential destruction parameter, MC^i_c depends negatively on the overall rate of destruction $(1-\beta)$, and that tends to offset the effect of such destruction on the marginal benefit; however, as will become clear shortly, the effect on the marginal benefit dominates. Bringing these results together shows that the direct effect of an increase in κ, $1-\beta$, and/or $1-\gamma$ on arming incentives, all else the same, is negative. Furthermore, an increase in the guns produced by agent i's rival G^j under open conflict, like under settlement, influences the net marginal benefit of arming. However, the sign of that influence depends not only on the relative ranking of guns, but also on the

[15] Observe from (6) that the maximum value of R_L^s, which obtains when $\kappa = 0$, is precisely equal to this cutoff. Thus, for $\kappa > 0$, $R_L^s < \frac{1}{4}\bar{R}$.

differential destruction parameter γ.

Does the initial distribution of resource endowments across agents matter in this context? As in the case of settlement, an agent's resource endowment under open conflict could constrain his production of guns. What differs under open conflict derives from the presence of differential damage on the insecure component of the defeated agent's output, $\gamma \in (0,1)$. For given arming choices, consider an arbitrary transfer of the resource from agent i to agent j. In the case of settlement (where $\gamma = 1$), such a transfer reduces X^i and expands X^j, but leaves $\bar{X} = X^i + X^j$ unchanged. By contrast, in the case of open conflict with $\gamma \in (0,1)$, the $X^i + \gamma X^j$ component of the value of the prize from the donor's (i) perspective falls. Exactly the opposite holds for the value of the recipient's (j) prize (i.e., $X^j + \gamma X^i$ rises). Accordingly, this resource reallocation tends to reduce the donor's arming and to increase the recipient's arming. The adjustment in arms by the recipient, in particular, imparts an adverse strategic effect on the donor's expected payoff. Thus, consistent with the often voiced concern about providing aid or more generally making a concession to an adversary, such a transfer could hurt the donor beyond the direct loss of income. Below we study these effects more carefully and consider their implications for the agents' preferences over war and peace. For now, it is important to emphasize that, under open conflict (with $\gamma < 1$) in contrast to what happens under settlement, arming decisions always depend on the distribution of resource endowments.

Let $B_c^i\left(G^j;\cdot\right)$ denote agent i's best response to $G^j > 0$ ($j \neq i$) under conflict. The first-order condition (FOC) implied by (7) and the resource constraint on arming (which requires $G^i \in [0, R^i]$) give

$$B_c^i\left(G^j;\theta,\gamma,R^i,\bar{R}\right) = \min\left\{R^i, \widetilde{B}_c^i\right\}, \quad i \neq j = 1,2 \tag{8a}$$

where

$$\widetilde{B}_c^i\left(G^j\right) \equiv -G^j + \sqrt{\theta G^j\left[\bar{R} - (1-\gamma)\left(R^j - G^j\right)\right]} \tag{8b}$$

is agent i's unconstrained best-response function[16] and

$$\theta \equiv \frac{\beta(1-\kappa)}{\beta(1-\kappa)+\kappa} \in [0,1] \tag{8c}$$

is a measure of the relative importance of an agent's own *contested* output net of destruction to his total output net of destruction. Clearly, this relative importance parameter is increasing in β and decreasing in the degree of output security κ. As already suggested by our earlier discussion of how κ and β influence MB_c^i and MC_c^i, an increase in θ amplifies

[16] Once again, to avoid cluttering of notation, we suppress the dependence of $\widetilde{B}_c^i$ on θ, γ, R^i and $\bar{R}$.

agent i's incentive to arm.

Building on the ideas above, we now turn to the subgame perfect equilibrium under open conflict.

Proposition 3 (Equilibrium under Open Conflict) *Assume property is not perfectly secure ($\kappa < 1$) and at least one agent declares war, with $\beta \leq 1$ and $\gamma < 1$. Then, for any given $\bar{R}$ such that $R^i + R^j = \bar{R}$, there exists a unique equilibrium in arming, with positive quantities produced by both agents $G_c^i > 0$ for $i = 1, 2$ and unique thresholds R_L^c and R_H^c ($\equiv \bar{R} - R_L^c$) such that $R_L^c \leq \frac{\theta}{4}\bar{R} < \left(1 - \frac{\theta}{4}\right)\bar{R} \leq R_H^c$. These thresholds depend on θ and γ as follows:*

- (a) *$dR_L^c/d\theta > 0$ with $\lim_{\theta \to 0} R_L^c = 0$ and $\lim_{\theta \to 1} R_L^c = \gamma\bar{R}/(1 + \sqrt{\gamma})^2$.*
- (b) *$dR_L^c/d\gamma > 0$ with $\lim_{\gamma \to 0} R_L^c = 0$ and $\lim_{\gamma \to 1} R_L^c = \frac{\theta}{4}\bar{R}$.*

By the definition of R_H^c ($\equiv \bar{R} - R_L^c$) and the definition of θ in (8c), parts (a) and (b) imply that R_H^c is increasing in war's overall destructive effect ($\beta \downarrow$) and in war's differential destructive effect ($\gamma \downarrow$). Since $\beta = \gamma = 1$ holds under settlement, a noteworthy implication of these points is that $R_L^c < R_L^s$ and $R_H^s < R_H^c$ for all $\beta \in (0, 1)$ and $\gamma \in (0, 1)$.

Using Proposition 3 and equation (8), we now characterize equilibrium arming in the case of open conflict as it depends on the distribution of resources across the two agents as well as on the parameters of destruction and output security.

Proposition 4 (Arming under Open Conflict) *For any given $\bar{R}$ such that $R^i + R^j = \bar{R}$, $\beta \leq 1$ and $\gamma < 1$, the equilibrium quantities of guns under open conflict (G_c^i, G_c^j) have the following properties:*

- (a) *If $R^i \in [R_L^c, R_H^c]$ for $i \neq j = 1, 2$, then as $R^i \gtreqless R^j$, we have $G_c^i \gtreqless G_c^j$, with $dG_c^i/d\xi > 0$, $dG_c^j/d\xi > 0$ and $d\phi_c^i/d\xi \lesseqgtr 0$ for $\xi \in \{\theta, \gamma\}$.*
- (b) *If $R^i \in (0, R_L^c)$ for $i \neq j = 1$ or 2, then $G_c^i = R^i$ and $G_c^j = \widetilde{B}_c^j(R^i) > G_c^i$, with $dG_c^i/d\theta = 0$ while $dG_c^j/d\theta > 0$ and $dG_c^i/d\gamma = dG_c^j/d\gamma = 0$.*

Part (a) establishes that, in the case of open conflict, the distribution of resources between the two agents matters for equilibrium arming even if neither agent is resource constrained. Comparing the unconstrained best-response functions under settlement and open conflict, shown respectively in (5b) and (8b), reveals that the differential destruction parameter $\gamma < 1$ plays a pivotal role here. Specifically, if there were no differential destruction under open conflict (i.e., $\gamma = 1$), the maintained assumption that agent i is not resource constrained implies his equilibrium arming (given rival j's choice) would be independent of R^i. Thus, as is true under settlement, the result would be a symmetric equilibrium in arms for $R^i \in [R_L^c, R_H^c]$: $G_c^i = G_c$ for $i = 1, 2$. But, even if $\gamma = 1$ were to hold, arming under the two modes of conflict resolution need not be identical. Provided that war is destructive (i.e.,

$\beta < 1$) such that $\theta < 1$ for $\kappa > 0$, $G_c < G_s$ would hold, as expected since war's overall destructive effects detract from the value of the prize. However, given $\gamma < 1$ under open conflict, G_c^i does depend on R^i, so that the two agents arm identically only when they are identically endowed. More generally for distributions $R^i \in [R_L^c, R_H^c]$ and $\gamma < 1$, the richer agent (say i) arms by more, resulting in an asymmetric equilibrium where that agent is more powerful (i.e., $\phi_c^i > \frac{1}{2}$).[17] The last components of part (a) indicate increases in θ (due to either less overall destruction $\beta \uparrow$ or to less output security $\kappa \downarrow$) and decreases in differential destruction ($\gamma \uparrow$) fuel arming incentives for both agents. But, the richer agent responds by relatively less, thereby reducing his power advantage without eliminating it.

Part (b) shows, when agent i is resource-constrained such that $G_c^i = R^i$, differential destruction $\gamma < 1$ is no longer relevant for either agent's arming. The constrained agent i simply cannot adjust his arming as γ changes; the unconstrained agent j can make adjustments but has no incentive to do so, since no part of his potential prize comes from the rival (i.e., $X^i = R^i - G^i = 0$). Furthermore, although the unconstrained agent behaves more aggressively under open conflict for larger values of θ, he remains less aggressive under open conflict with $\theta < 1$ than under settlement.

These results are illustrated in Fig. 2(a), which depicts country $i = 1$'s equilibrium arming as a function of the distribution of factor ownership for alternative values of γ with fixed values of κ $(= 0.1)$ and β $(= 1)$. The solid curve shows equilibrium arming under open conflict when $\beta = \gamma = 1$ (which coincides with that under settlement) and the remaining (dashed and dotted) curves show equilibrium arming under open conflict for various values of $\gamma < 1$. The dots along the horizontal axis point out the threshold values of R^i, R_L^c $(= R_L^s$ for $\gamma = \beta = 1)$ and R_H^c $(= R_H^s$ for $\gamma = \beta = 1)$, which change with γ as established in Proposition 3.

Using Proposition 4 along with (2), we now turn to equilibrium payoffs under open conflict.

Proposition 5 (Payoffs under Open Conflict) *An agent's equilibrium payoff under open conflict U_c^i $(i = 1, 2)$ depends on the distribution of resource ownership $R^i \in (0, \bar{R})$, the degree of output security $\kappa < 1$, the differential rate of destruction $1 - \gamma > 0$, and the overall rate of destruction $1 - \beta \geq 0$ as follows:*

(a) *If $R^i \in [R_L^c, R_H^c]$ for $i = 1, 2$, then* (i) *$dU_c^i/dR^i > 0$;* (ii) *$dU_c^i/d\kappa > 0$ for $R^i \in [\bar{R}/2, R_H^c]$;* (iii) *$dU_c^i/d\gamma > 0$ for $R^i \in \left[R_L^c, \bar{R}/2\right]$ whereas $\lim_{R^i \to R_H^c} dU_c^i/d\gamma < 0$; and* (iv) *$dU_c^i/d\beta > 0$.*

[17] As shown in the online appendix, each agent's arming choice can be, but need not be, monotonically related to his own endowment. In particular, we show there exists a $\gamma_0 \equiv \breve{\gamma}(\theta) \in (0, 1)$ with $\breve{\gamma}'(\theta) < 0$, such that for $\gamma \in [\gamma_0(\theta), 1)$, $dG_c^i/dR^i > 0$. Otherwise, we have $\lim_{R^i \to R_L^c} dG_c^i/dR^i > 0$, while $\lim_{R^i \to R_H^c} dG_c^i/dR^i < 0$. Nonetheless, the richer agent is more powerful in equilibrium.

(b) *If $R^i \in (0, R_L^c)$ for $i \neq j = 1$ or 2, then* (i) $dU_c^i/dR^i > 0$ *with* $\lim_{R^i \to 0} U_c^i = 0$, *whereas* $dU_c^j/dR^j > 0$ *with* $\lim_{R^j \to \bar{R}} U_c^j = [\beta(1-\kappa) + \kappa]\bar{R}$; (ii) $dU_c^i/d\kappa < 0$ *if* $(1-\beta)(1-\kappa) - \kappa < 0$, *while* $dU_c^j/d\kappa > 0$; (iii) $dU_c^i/d\gamma > 0$, *while* $dU_c^j/d\gamma = 0$; *and,* (iv) $dU_c^i/d\beta > 0$ *and* $dU_c^j/d\beta > 0$.

This proposition establishes the positive dependence of an agent's payoff on his initial resource holdings over the entire range of possible distributions $R^i \in (0, \bar{R})$. This dependence, driven by both direct effects and indirect effects through arming choices, implies that the richer agent always enjoys a higher payoff under open conflict than his poorer rival. As was the case under settlement, improvements in output security ($\kappa \uparrow$) tend to amplify the effects of endowment redistributions on payoffs. Furthermore, as expected, an increase in the overall destructive effect of war ($\beta \downarrow$) reduces both agents' payoffs for all $R^i \in (0, \bar{R})$.

By contrast, the payoff effects of greater differential destruction ($\gamma \downarrow$) depend on the initial resource holdings, as illustrated in Fig. 2(b) for agent $i = 1$. Following our convention in Fig. 2(a), the solid curve shows agent 1's payoff for $R^1 \in (0, \bar{R})$ under settlement $\gamma = \beta = 1$, while the remaining (dashed and dotted) curves show his payoffs under open conflict for different values of $\gamma < 1$ (and $\beta = 1$). As established in part (a) of the proposition and shown in the figure, a decrease in γ makes both agents strictly worse off provided that the distribution is sufficiently even—i.e., R^1 is sufficiently close to $\bar{R}/2$; however, as $R^1 \to R_H^c$, the payoff effects across the two agents start to diverge qualitatively, with the richer agent benefiting and the poorer one suffering. To see the underlying logic here, note first that, for all $R^1 \in [R_L^c, R_H^c]$, a decrease in γ reduces arming incentives, thereby generating a positive strategic effect for both agents. The divergence in total payoff effects derives from differences in the direct payoff effects. In particular, although a decrease in γ tends to reduce both agents' payoffs for given G^1 and G^2, that direct effect for the richer agent (1) vanishes as $R^1 \to R_H^c$, since that implies $R^2 \to R_L^c$ and, thus, $X^2 = R^2 - G^2 \to 0$. Meanwhile, the negative direct effect for the smaller agent (2) increases in magnitude since $X^1 = R^1 - G^1 (> 0)$ is increasing in R^1. Thus, the richer agent's (1) payoff rises with a decrease in γ as the positive strategic effect (eventually) dominates the negative direct effect. In contrast, as shown in the online appendix, the poorer agent's payoff falls with a decrease in γ because the direct (negative) effect dominates the strategic (positive) effect. For more uneven distributions where $R^1 \in (0, R_L^c)$, the richer agent (now 2) is unaffected by greater differential destruction (since $X^1 = 0$ for such distributions), whereas the poorer agent (1) continues to be adversely affected as R^1 falls and R^2 rises. These effects are depicted in Fig. 2(b), as a counterclockwise rotation of U_c^1 around a point $R^1 \in (\bar{R}/2, R_H^c)$.[18]

[18] Note that one can also visualize the effect of a decrease in β within Fig. 2(b) as a clockwise rotation of the payoff function at $R^1 = 0$.

3.3 Payoffs under settlement versus open conflict

With the help of Propositions 2 and 5, we now ask how the agents' payoffs compare under the two modes of conflict resolution. We take as our starting point the case where $\beta = \gamma = 1$, shown by the solid curve in Fig. 2(b), where $V_s^i = U_c^i$ for all possible $R^i \in (0, \bar{R})$. Of course, an increase in differential destruction ($\gamma \downarrow$) has no effect on V_s^i, but as we have just seen it induces a counterclockwise rotation of the U_c^1 function around a point to the right of $\bar{R}/2$. As illustrated in the figure, that rotation generates a range of distributions (to the right of $\bar{R}/2$ and the left of $\bar{R}$) under which player $i = 1$ consequently has an *ex ante* preference for open conflict over settlement.[19] We also know, however, that an increase in war's overall destruction ($\beta \downarrow$) leaves V_s^i unchanged, while it decreases U_c^i over the entire range of resource distributions $R^i \in (0, \bar{R})$.

To visualize this effect and its implications, consider Fig. 3, which shows agent 1's payoff function under settlement V_s^i (the solid curve) and his payoff functions under open conflict (the dashed and dotted curves) for given $\gamma < 1$ and various values of overall destruction ($\beta < 1$). Focusing on the highest (dashed) curve, notice that there is a range of resource allocations $R^1 \in (R^*, R^{**})$ for which $U_c^1 > V_s^1$. As the rate of war's overall destruction rises ($\beta \downarrow$), U_c^i falls, while V_s^i remains unchanged, such that the range (R^*, R^{**}) shrinks and eventually disappears once the rate of overall destruction becomes sufficiently severe. That critical value of β is depicted by the value β_0 associated with the payoff where $U_c^1(R_H^s) = V_s^1(R_H^s)$ in the figure; for all other resource allocations, $U_c^1(R^1) < V_s^1(R^1)$.

Building on these ideas with Propositions 2 and 5, we have the following result.

Proposition 6 (Comparison of Payoffs under Settlement and Open Conflict) *For any given degree of output security $\kappa \in [0, 1)$ and differential rate of destruction $\gamma \in [0, 1)$, there exists a unique threshold level of overall destruction, denoted by $1 - \beta_0$ where $\beta_0 \equiv \beta_0(\kappa, \gamma) \in (0, 1)$ with $\partial\beta_0/\partial\kappa < 0$ and $\partial\beta_0/\partial\gamma > 0$, that has the following implications for the comparison of payoffs under settlement and open conflict:*

(a) *If $\beta \leq \beta_0$, then $V_s^i \geq U_c^i$ for all $R^i \in \left(0, \bar{R}\right)$ and $i = 1, 2$.*

(b) *If $\beta > \beta_0$, then for each agent $i = 1, 2$ there exist threshold values, R^* and R^{**} where $\bar{R}/2 < R^* < R_H^s < R^{**} < R_H^c$, that imply* (i) *$U_c^i > V_s^i$ for all $R^i \in (R^*, R^{**})$ and* (ii) *$U_c^i < V_s^i$ for all $R^i \notin (R^*, R^{**})$.*

Part (a), which shows $U_c^i \leq V_s^i$ for all $\beta < \beta_0$, is quite intuitive, suggesting that, if destruction is sufficiently extensive, then both agents prefer peace over war. The dependence of β_0 on γ follows from the dependence of payoffs under conflict as detailed in Proposition 5 and just described in relation to Fig. 2(b). The dependence of β_0 on κ is a bit more complex

[19] As implied by Propositions 2(b) and 5(b) given $\beta = 1$, the figure shows $V_s^1(R^1) = U_c^1(R^1)$ for $R^1 \geq R_H^c$.

as an improvement in output security tends to increase the payoffs for the richer agent under both settlement as well as under open conflict. However, we can show that the effect on the payoff under open conflict dominates that under settlement.[20] Thus, when either output security improves or the rate of differential destruction increases, the minimum rate of overall destruction that gives not only the poorer agent but also the richer agent an *ex ante* preference for settlement must increase.

Part (b) is also obvious, suggesting that, when war is not very destructive (i.e., β is marginally below 1), the more affluent agent has an *ex ante* preference for war provided the distribution of factor ownership is not extremely uneven (i.e., for $R^i \in (R^*, R^{**})$). For all other distributions, both agents prefer peaceful settlement. The long-dashed curve in Fig. 3 relative to the solid curve illustrates these ideas. One can also show that, for given γ and β, greater output security implies a larger set of resource allocations for which the more affluent agent prefers war—i.e., the range (R^*, R^{**}) expands with increases in κ.

4 The extended game

While the comparison of payoffs across different resource distributions is interesting in its own right, our primary objective here is to understand the endogenous choice of war versus peace and the value of commitments in this context. In what follows, we study this choice first in the absence of transfers and second when transfers are possibly made in advance of the war/peace decision.

4.1 Without transfers

Much of the literature that studies the choice between war and peace in the context of settings with complete and perfect information assumes a particular sequence of actions that emphasizes war as a commitment problem in a dynamic setting. Consider, for example, the multi-period setting of Garfinkel and Skaperdas (2000). In each period, provided war was not previously declared and fought, arming decisions are made first, followed by negotiations and the decision of whether to declare war or not. In such a setting as in ours, this timing implies that, given any amount of guns brought to the negotiation table, both contenders have a short-run incentive to choose peace over war to avoid war's destructive effects. But, absent the ability to commit to future divisions of the resource, the dispute has to be settled again and again in the future, and that requires additional arming. If war today gives the victor a strategic advantage in future disputes (in the extreme case, eliminates the rival) whereby he can reduce costly arming for himself in the future, one or both agents could prefer war in the current period.

[20] See the online appendix.

Still, focusing on a single period alone and thus abstracting from that strategic advantage, war would be another Nash equilibrium in the setting of this paper if arming choices were made first. Given those choices and given that the emergence of peaceful settlement requires both agents to choose settlement, an agent would be indifferent between choosing peace or war if his rival declared war. Provided one agent declares war, that is the outcome of the second stage. While the equilibrium of war in this context is only one in weakly dominated strategies, its mere possibility underscores the importance of the assumption that arming choices are made first (which we view as reflecting the players' ability to commit to such choices). In particular, each agent's arming decision in the first stage will be based on what he expects the equilibrium to be in the second stage. Eliminating the weakly dominated strategy (war) would effectively fix those expectations such that settlement would emerge as the Nash equilibrium, even though it could result in a lower *ex ante* payoff to one agent, as suggested by Proposition 6.

To dig a little deeper, let us now assume the sequence of actions specified earlier: agents first choose between war and peace and then, contingent on the outcome of that first stage, make their arming decisions. In this setting, like that of Beviá and Corchón (2010), agents can commit to war or peace, but not their arming decisions. Even in this setting, war is always a possible equilibrium. That is, given one's rival chooses war, each agent would be indifferent between his two choices. Furthermore, war could be weakly dominated by settlement, in which case the two agents would want to engage in pre-play communication to coordinate on peace.

However, war need not be weakly dominated by peace in this case. Indeed, as indicated by Proposition 6, each agent's preferences over war and peace depend on his expectation of how both arm under each mode of conflict resolution and the associated payoffs. If peace induces arming decisions that prove to be payoff-reducing for one agent and war's overall destructive effects are not sufficiently severe given the degree of output security and differential destruction (i.e., $U_c^i(G_c^i, G_c^j) > V_s^i(G_s^i, G_s^j)$ for $i = 1$ or 2), peace would not be the weakly dominant strategy. Agent i would optimally declare war, and both agents would then arm accordingly as outlined in Proposition 4.

Consider Fig. 4, which depicts agent 1's payoff functions under open conflict (labeled $U_c^1 \equiv U_c^1(G_c^1, G_c^2)$ and shown as the short-dashed curve) and settlement (labeled $V_s^1 \equiv V_s^1(G_s^1, G_s^2)$ and shown as the long-dashed curve) over all $R^1 \in (0, \bar{R})$. The figure also shows the analogous payoff functions, without labels, for agent 2 initially having $R^2 = \bar{R} - R^1$ units of the resource that can be read from the right. In that figure, $\beta > \beta_0(\kappa, \gamma)$, such that there exists a range of resource distributions for each agent i where that agent prefers open conflict. Agent 1 prefers open conflict for $R^1 \in (R^*, R^{**})$, while agent 2 prefers open conflict for $R^2 \in (R^*, R^{**})$ or equivalently (as shown in the figure) for $R^1 \in (\bar{R} - R^{**}, \bar{R} - R^*)$.

Accounting for the fact that it takes just one agent to undermine peace, the solid curve with kinks and breaks (labeled W_N^1) shows the payoff to agent 1 in the extended game without transfers: $W_N^1 \equiv \max\{V_s^1, U_c^1\}$.

Of course, building arms requires time, and as such it would seem reasonable to suppose that agents arm first. In this case, choosing peace in the second stage is always a weakly dominant strategy for both players, because for any given guns the avoidance of destruction is payoff enhancing. But the important point, based on Proposition 6, is this: at the stage of arming, an agent's *ex ante* payoff when both agents arm in anticipation of peace could be lower than the payoff he could expect to obtain by committing to war prior to arming. If, in fact, $U_c^i(G_c^i, G_c^j) > V_s^i(G_s^i, G_s^j)$, agent i would certainly search for ways to commit to war. Such commitments could be made, for example, through outright declarations not to negotiate with the enemy, through the imposition of restraints on communication, or even through the strategic delegation of one's foreign policy to a "hawk." Alternatively, if $U_c^i(G_c^i, G_c^j) < V_s^i(G_s^i, G_s^j)$ for each agent, then both players would have an interest to engage in pre-play communication whereby they could ensure their coordination on peace and avoid war.

4.2 With transfers

Supposing that the agents make their war/peace decision first and then arm accordingly, we now ask if transfers of the initial resource endowments made in advance of the war/peace decision can induce peace. Following Beviá and Corchón (2010) and others, we assume an agent's receipt of a transfer does not commit him to peace. To proceed, let us return to Fig. 4 and focus on initial allocations $R^1 \in (\bar{R} - R^{**}, \bar{R} - R^*)$, where agent 1 prefers peace while agent 2 prefers war. Observe agent 2, by contrast, prefers peace to war where his allocation is just above R^{**} or equivalently at $R^1 = \bar{R} - (R^{**} + \epsilon)$ (for small $\epsilon > 0$). Indeed, he prefers this outcome to all outcomes when $R^1 \in (\bar{R} - R^{**}, \bar{R} - R^*)$. Hence, the smallest resource endowment that would induce agent 2 to choose peace equals $R^2 = R^{**} + \epsilon$, leaving agent 1 with at most $R^1 = \bar{R} - (R^{**} + \epsilon)$. Since $V_s^1(\bar{R} - (R^{**} + \epsilon)) > U_c^1(R^1)$ for all R^1 in the just noted range, agent 1 is willing to make such a transfer and peaceful settlement is sustainable for all $R^1 \in (0, \bar{R})$. The dotted line (labeled W_T^1) shows the adjusted payoff to agent 1 with the transfer from him when $R^1 \in (\bar{R} - R^{**}, \bar{R} - R^*)$ and to him when $R^1 \in (R^*, R^{**})$. Comparing those payoffs to the ones without transfers (i.e., the solid line, W_N^1) confirms that transfers which induce peace are mutually beneficial to both agents.

However, as shown in Fig. 5 that assumes a higher degree of output security, transfers from the poorer agent to the richer one need not support peace for all possible initial resource allocations $R^1 \in (\bar{R} - R^{**}, \bar{R} - R^*)$. Recall, greater output security ($\kappa \uparrow$) makes the payoff functions under both peace and war more sensitive to resource transfers. While expanding the range of initial resource allocations for which agent 2 prefers war $R^1 \in (\bar{R}-R^{**}, \bar{R}-R^*)$, a higher value of κ also implies that the difference in payoffs under peace and war for the less affluent agent $(V_s^1 - U_c^1)$ shrinks. To be sure, a transfer to the richer agent can induce that agent to choose peace over that entire range; however, in this case depending on the initial distribution of resources, the required transfer could be too large from the poorer agent's perspective, implying that war will emerge for some allocations. This possibility, which is shown in the figure for $R^1 \in (\bar{R} - \hat{R}, \bar{R} - R^*)$ where $V_s^1(\bar{R} - (R^{**} + \epsilon)) < U_c^1(R^1)$ while $V_s^2(R^{**} + \epsilon) > U_c^2(\bar{R} - R^1)$, obtains when κ exceeds a certain threshold (depending on β and γ) and thus suggests that higher output security need not be conducive to peace.

5 Concluding remarks

Without denying the relevance of the idea that the inability to make binding, long-term commitments today could foster conflict, our analysis based on a guns-versus-butter model suggests that the ability to make short-term commitments not to negotiate before arming choices could also foster open conflict, at least in a setting where decision makers have relatively short time horizons. Less extensive overall destruction and a greater degree of output security make war more likely, even when transfers can be offered in advance of the war/peace decision. These results can also be shown to follow from a rent-seeking model where the contested prize is independent of the two agents' arming choices.[21]

In our analysis, as in others that emphasize the importance of long-term commitments, the incentive to choose war is based on the notion that peaceful settlement, though not destructive, is costly as it requires the diversion of resources away from the production of goods for consumption. But, in our one-period setting, the inability to commit to arming plays a central role along with the differential destruction inflicted on the part of the prize taken from the defeated party. While both the direct and indirect payoff effects of differential destruction matter in shaping each side's preferences over war and peace, the influence of differential destruction to give the affluent side a preference for war can be attributed largely to the favorable strategic effect it imparts on that party (when the other party is not resource constrained). In particular, differential destruction reduces the less affluent side's incentive to arm under war relative to his incentive under peaceful settlement.

In view of the prominence of the strategic effects of destruction over its direct effects in shaping the more affluent agent's preferences for war versus peace, one might wonder if our

[21]Specifically, let T^i denote the value of agent i's resource that is contested and suppose it is independent of his endowment R^i. Then, $\beta(1-\kappa)(T^i + \gamma T^j)$ would be the value of agent i's $(i \neq j)$ prize under conflict and $(1-\kappa)(T^i + T^j)$ would the value of his prize under peace. The analysis goes through when we consider possible asymmetries in T^i and T^j for $\gamma < 1$. In this context, one could also consider asymmetries in the degree of security. While the functional dependence of guns on the distribution of resources and the various thresholds would change, the key insights on how destruction and the distribution of resources would affect the choice between peace and war would remain qualitatively intact.

logic remains intact when war destroys a fraction $(1-\beta)$ of both secure and insecure output. That is to ask, could the more affluent agent possibly prefer war in this case? The answer is yes. First, note that this modification would not influence arming under settlement (where $\beta = \gamma = 1$). Second, observe that this modification would not influence arming under war when there is no overall destruction ($\beta = 1$). Hence, in this case, the more affluent agent could prefer war provided there is differential destruction ($\gamma < 1$). What's more, by continuity, this preference ordering remains unchanged at least for β marginally below 1. Nonetheless, the threshold value of overall destruction $(1 - \beta_0)$ above which the more affluent agent could prefer war is smaller when both secure and insecure output are subject to overall destruction.

While our reliance on the conflict technology in (1) as a rule of division of the contested output under settlement is analytically convenient in allowing us to highlight the importance of differential destruction under war, that formulation implies arming plays a larger role in the negotiated division relative to that under Nash bargaining and split-the-surplus protocols and thus tends to inflate arming incentives relative to these other rules of division.[22] Thus, one important extension of the analysis, which we are currently pursuing, would consider such protocols. Since an agent's incentive to arm under settlement would depend not only on how arming influences his threat point (i.e., his payoff under war), but also on how his arming influences the surplus (i.e., the difference between the sum of their payoffs under settlement and sum under war), the effect of settlement on the agents' incentives to arm (relative to war) can be uncovered by studying the effect of their arming on the surplus. A related avenue for future research would be to consider a more general formulation of the conflict technology, either in its ratio or difference forms.

It would also be interesting to extend our framework to a multi-period setting. Assuming that agents arm first and then make their war/peace decisions, there would be a short-run incentive to settle peacefully. This extension could then explore how differential destruction interacts with the long-run benefits of conflict when resources are distributed unevenly.

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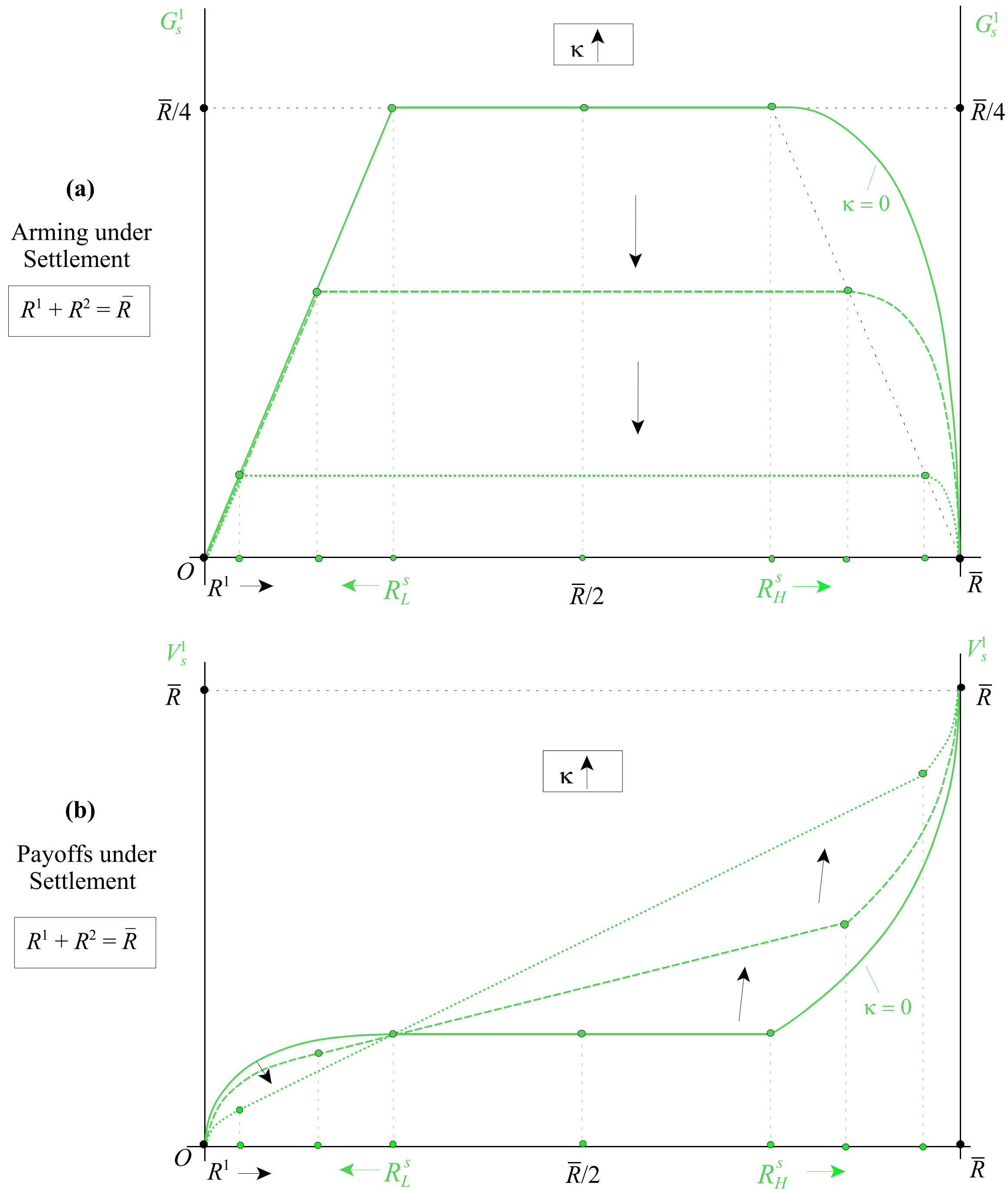

Fig. 1 The dependence of arming and payoffs under settlement on the distribution of resource endowments and insecurity

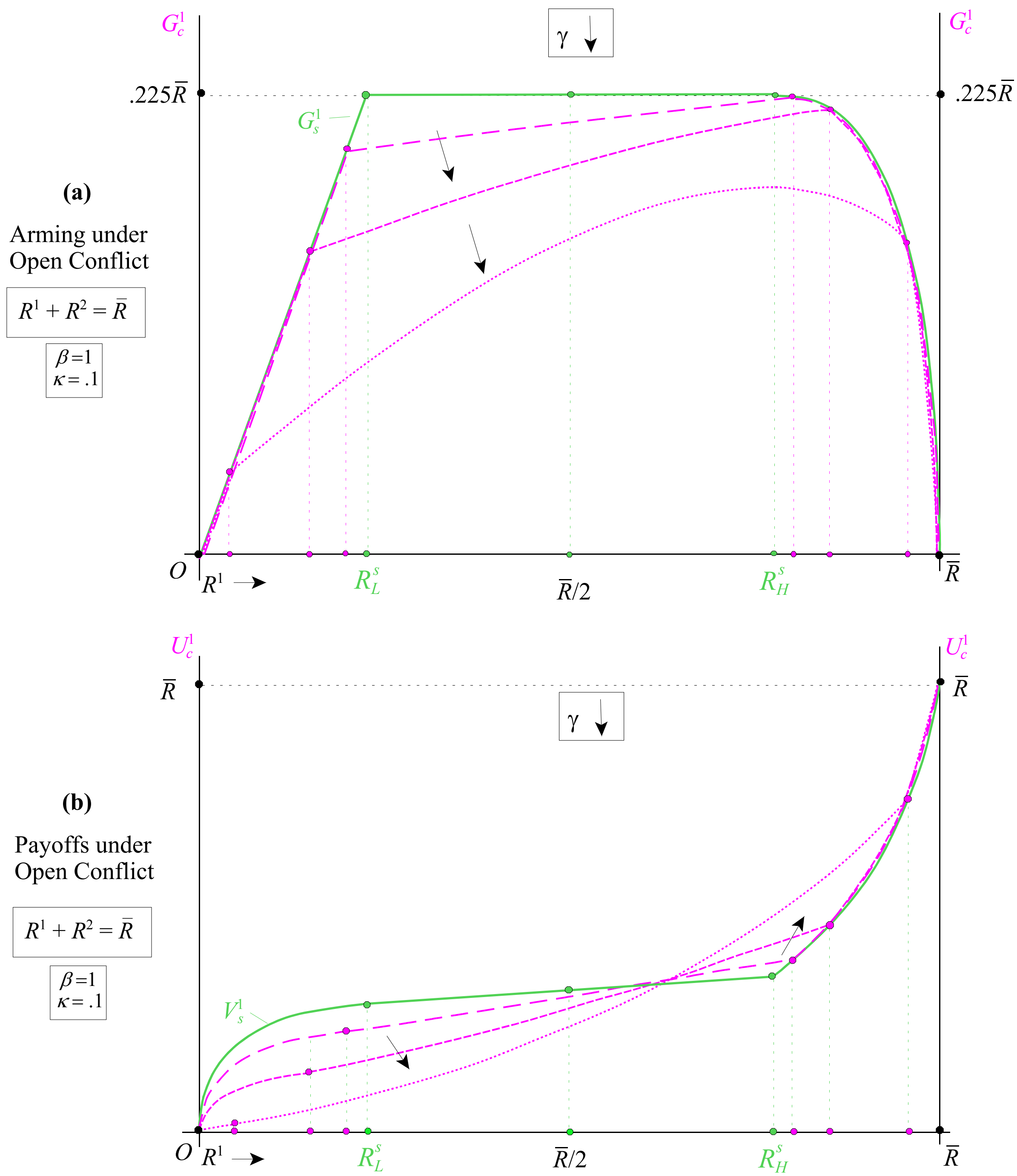

Fig. 2 Arming and payoffs under open conflict for alternative distributions of resource endowments and values of differential destruction γ

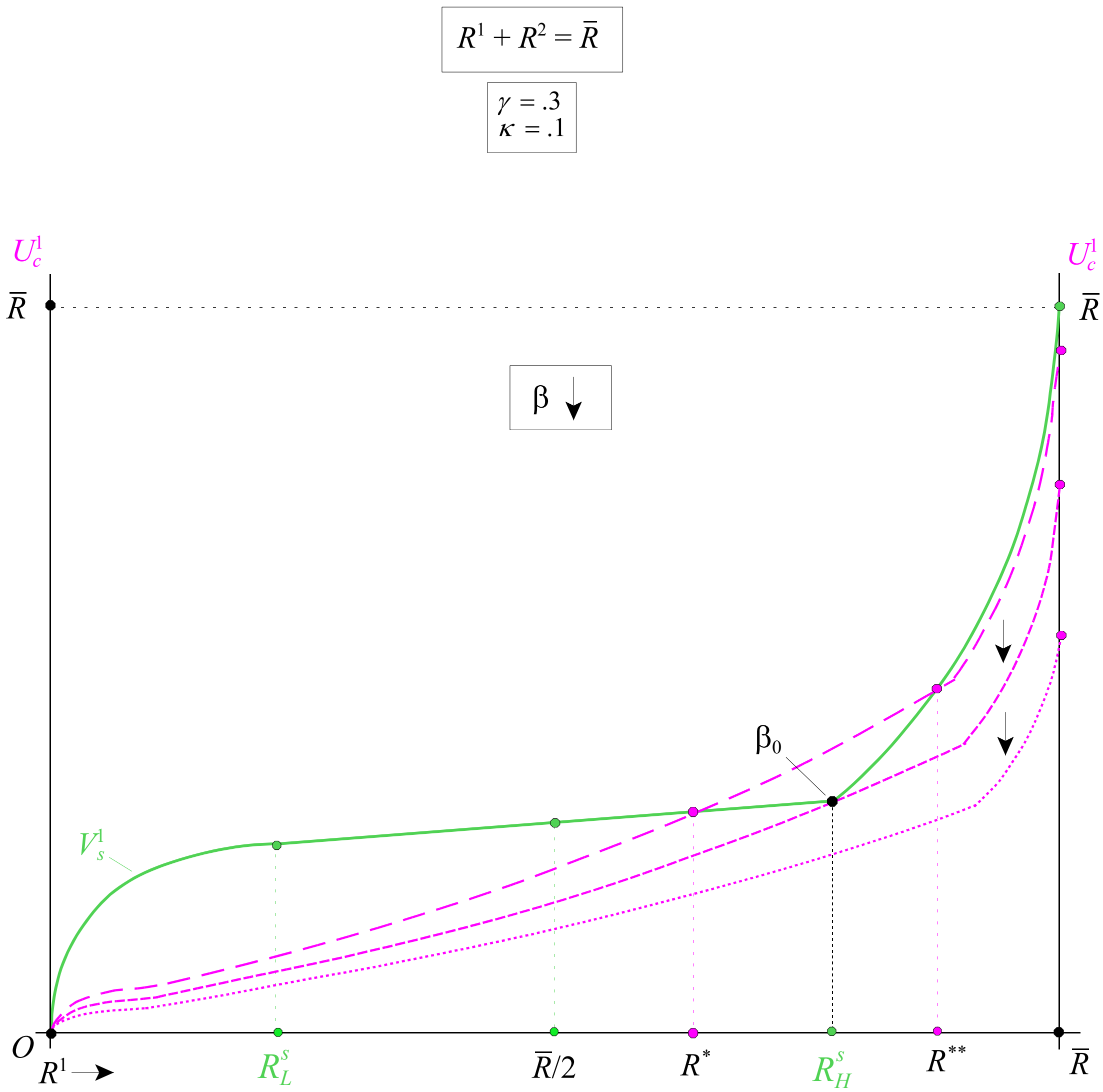

Fig. 3 The effect of changes in the rate of destruction β on payoffs and the determination of the critical rate β_0

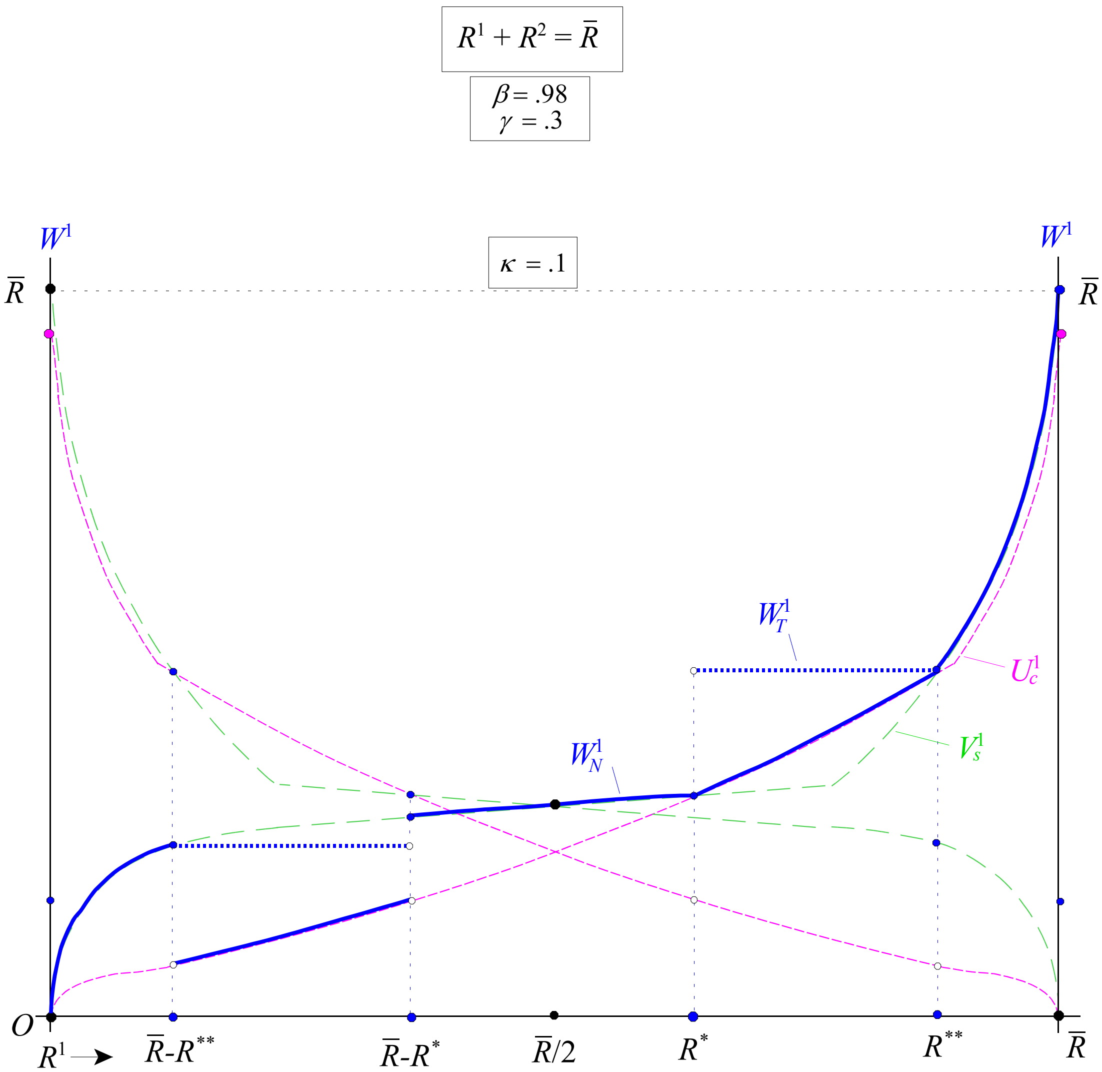

Fig. 4 Equilibrium payoffs in the extended game with and without resource transfers

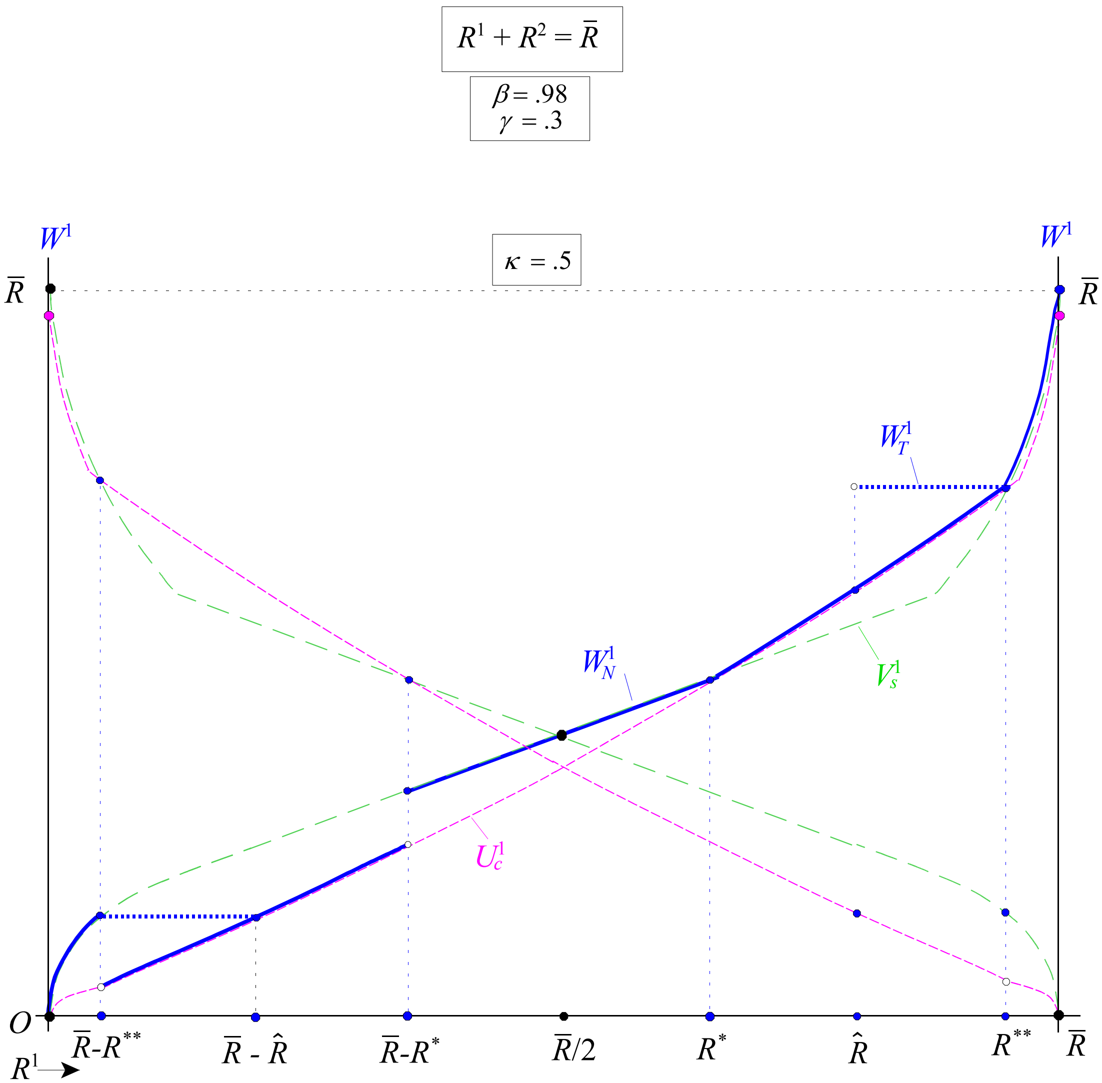

Fig. 5 Equilibrium payoffs in the extended game with and without resource transfers when output is more secure

A Online appendix

Proof of Propositions 1 and 3. Since settlement can be viewed as conflict with no destruction (i.e., $\beta = \gamma = 1$) to save on space we first prove Proposition 3 which deals with conflict, and then point out how the analysis carries over to settlement covered by Proposition 1.

Existence and uniqueness of the unconstrained equilibrium. We start by focusing on the case where neither agent is resource-constrained in his arming choice. Thus, agent i's FOC, which requires $U^i_{G^i}$ shown in (7) to be equal to zero, implicitly defines his (unconstrained) best-response function $\widetilde{B}^i_c(G^j)$ shown in (8b) for $i \neq j = 1, 2$.[23] Differentiation of $U^i_{G^i}$ with respect to G^i yields

$$\widetilde{U}^i_{G^iG^i} = \phi^i_{G^i}\beta\left(1-\kappa\right)\left[-2 + \left(X^i + \gamma X^j\right)\frac{\phi^i_{G^iG^i}}{\phi^i_{G^i}}\right] < 0, \tag{A.1}$$

where the negative sign follows from the properties of ϕ^i in (1) that $\phi^i_{G^i} > 0$ and $\phi^i_{G^iG^i} < 0$. Since U^i is concave in $G^i \in (0, R^i]$ for $i = 1, 2$ regardless of the values of the various parameters, we have established that an equilibrium always exists both under conflict and under settlement.

To prove uniqueness, we evaluate the expression in (A.1) at an optimum where $U^i_{G^i} = 0$ using (7) and the properties of ϕ^i, to obtain:

$$\widetilde{U}^i_{G^iG^i} = -2\left[\beta\left(1-\kappa\right) + \kappa\right]\phi^i/G^i < 0. \tag{A.2}$$

Next, differentiate $U^i_{G^i}$ with respect to G^j to find

$$\widetilde{U}^i_{G^iG^j} = \beta\left(1-\kappa\right)\phi^i_{G^j}\left[-1 - \frac{\phi^i_{G^i}}{\phi^i_{G^j}}\gamma + \left(X^i + \gamma X^j\right)\frac{\phi^i_{G^iG^j}}{\phi^i_{G^j}}\right], \tag{A.3}$$

which, when evaluated at i's optimum, using the properties of ϕ^i and the definition of θ in (8c), simplifies to

$$\widetilde{U}^i_{G^iG^j} = \left[\beta\left(1-\kappa\right) + \kappa\right]\left[\phi^i - \phi^j + (1-\gamma)\theta\left(\phi^j\right)^2\right]/G^j. \tag{A.4}$$

Since $G^i/G^j = \phi^i/\phi^j$, the slope of agent i's unconstrained best-response function can be

[23] Below we establish conditions on the initial resource allocation that ensure the existence of an unconstrained equilibrium. Equilibria in which at least one agent is resource constrained can be addressed in a straightforward manner.

written, using (A.2) and (A.4), as

$$\frac{d\widetilde{B}_c^i\left(G^j\right)}{dG^j} = -\frac{\widetilde{U}_{G^iG^j}^i}{\widetilde{U}_{G^iG^i}^i} = \frac{\phi^i - \phi^j + (1-\gamma)\,\theta\left(\phi^j\right)^2}{2\phi^j}. \tag{A.5}$$

Suppose $\gamma = 1$, as is the case under settlement. Inspection of (A.5) readily reveals that, in this case, $\widetilde{V}_{G^iG^j}^i \gtreqless 0$ (so $d\widetilde{B}_s^i\left(G^j\right)/dG^j \gtreqless 0$) if $G^i \gtreqless G^j$. Thus, under settlement, each agent i's arming is a strategic complement (substitute) for its rival's arming when $G^i > G^j$ ($G^i < G^j$). By contrast, under open conflict where $\gamma \in [0,1)$, an agent i's arming can be a strategic complement for its rival j's arming even if $G^i < G^j$.

To prove uniqueness of the unconstrained equilibrium, it suffices to show that $\widetilde{U}_{G^iG^i}^i\widetilde{U}_{G^jG^j}^j - \widetilde{U}_{G^iG^j}^i\widetilde{U}_{G^jG^i}^j > 0$ or, equivalently, that

$$\Delta \equiv \widetilde{U}_{G^iG^i}^i\widetilde{U}_{G^jG^j}^j\left[1 - \frac{d\widetilde{B}_c^i\left(G^j\right)}{dG^j}\frac{d\widetilde{B}_c^j\left(G^i\right)}{dG^i}\right] > 0.$$

Since $\widetilde{U}^i$ is concave in G^i for $i = 1,2$, we need only to show that the expression inside the square brackets is positive. Using (A.5) for $i \neq j = 1,2$ gives

$$1 - \frac{d\widetilde{B}_c^i\left(G^j\right)}{dG^j}\frac{d\widetilde{B}_c^j\left(G^i\right)}{dG^i} = \frac{1 - (1-\gamma)\,\theta\left[\left(\phi^i - \phi^j\right)^2 + (1-\gamma)\,\theta\left(\phi^i\phi^j\right)^2\right]}{4\phi^i\phi^j},$$

which is, in fact, positive.[24] As such, we have established uniqueness of unconstrained equilibrium for all $\gamma \in [0,1]$, $\beta \in (0,1]$ and $\kappa \in [0,1)$, thus covering both conflict and settlement. For future purposes, we also note that

$$\Delta = \frac{\left[(1-\kappa)\,\beta + \kappa\right]^2}{G^iG^j}D > 0, \tag{A.6a}$$

where

$$D \equiv 1 - (1-\gamma)\,\theta\left(\phi^i - \phi^j\right)^2 - (1-\gamma)^2\,\theta^2\left(\phi^i\phi^j\right)^2 > 0. \tag{A.6b}$$

Equilibrium under settlement (*Proposition 1*). Focusing on settlement, suppose $\beta = \gamma = 1$ (which from (8c) implies $\theta = 1 - \kappa$). The FOCs associated with $\widetilde{V}_{G^i}^i = 0$ for $i \neq j = 1,2$, using (4), together imply that $G_s^i = \frac{1}{4}(1-\kappa)\,\bar{R}$ for $i = 1,2$, which requires $G_s^i \leq R^i$. It follows then that the threshold levels of the resource are given by $R_L^s \equiv \frac{1}{4}(1-\kappa)\,\bar{R}$ and $R_H^s \equiv \left[1 - \frac{1}{4}(1-\kappa)\right]\bar{R}$, as shown in (6). From the expression for R_L^s, it follows immediately

[24] To confirm this claim, observe that this expression is least likely to be positive when $\gamma = 0$ and $\theta = 1$. With those parameter values, the expression simplifies to $(4 - \phi^i\phi^j)/4$ which is positive.

that $dR^s_L/d\kappa < 0$, as required in part (a). Clearly, only one player, say i, can be constrained by his endowment as covered in part (b) of the proposition. In this case, $G^i_s = R^i$ while $G^j_s = \widetilde{B}^j_s\left(R^i\right)$, where $\widetilde{B}^j_s(\cdot)$ is shown in (5b). In turn, differentiating $\widetilde{B}^j_s(\cdot)$ with respect to κ shows once again that the unconstrained agent's arming decreases as κ increases, thereby completing the proof of this proposition.

Equilibrium under open conflict (*Proposition 3*). Turning to open conflict, we now establish the existence of unique threshold values of R^c_L and R^c_H. In the process, we also highlight an alternative way of deriving and characterizing the equilibrium under conflict, which is especially useful in the case of asymmetries of the sort studied here. In particular, given the agents' endowments, the rates of destruction and the degree of security in output, there is an interior equilibrium defined by a system of two equations (i.e., the two FOCs associated with $\widetilde{U}^i_{G^i} = 0$ in (7)) in two unknowns (G^i, G^j). Using these FOCs with the conflict technology (1) and the resource constraints $G^i \leq R^i$ for $i = 1, 2$, we transform that system into one of four equations in four unknowns. This transformation allows us to solve for the equilibrium value of ϕ^i (and thus $\phi^j = 1 - \phi^i$), from which we can back out equilibrium arming; in turn, it allows us to identify and characterize the range of resource allocations for which one agent is resource constrained ($G^i = R^i$).

To start, we focus on outcomes where neither agent is resource constrained. Note that (1) implies $\phi^i_{G^i} = \phi^i\phi^j/G^i$. Then, recalling that $X^i = R^i - G^i$ for $i = 1, 2$, and using the definition of θ in (8c), we rewrite $U^i_{G^i} = 0$ $(i = 1, 2)$ from (7) as

$$g^i = -\left[1 - \theta\left(\phi^j\right)^2\right]G^i - \theta\phi^i\phi^j\gamma G^j + \theta\left[R^i + \gamma R^j\right]\phi^i\phi^j = 0,\ \ i \neq j = 1, 2.$$

Next, we use $g^i = 0$ $(i = 1, 2)$ to solve for G^i:

$$G^i = \frac{\theta\phi^i\phi^j\left\{R^i\left[1 - \theta\phi^i\left(\phi^i + \gamma^2\phi^j\right)\right] + R^j\gamma\left(1 - \theta\phi^i\right)\right\}}{\left[1 - \theta\left(\phi^i\right)^2\right]\left[1 - \theta\left(\phi^j\right)^2\right] - \left(\gamma\theta\phi^i\phi^j\right)^2},\ i \neq j = 1, 2. \tag{A.7}$$

With (A.7), we use the implication of (1) that $G^i/G^j = \phi^i/\phi^j$ and solve for R^i/R, keeping in mind that $R^j = \bar{R} - R^i$. Doing so, after tedious algebra, gives the following condition that must hold when resource constraints on arming are not binding:

$$\rho^i\left(\phi^i; \gamma, \theta\right) = r^i, \tag{A.8}$$

where $\phi^i + \phi^j = 1$ for $i \neq j = 1, 2$, and

$$\rho^i\left(\phi^i; \gamma, \theta\right) \;\; \equiv \;\; \frac{1}{1-\gamma}\left\{1 - \frac{(1+\gamma)\left[1 - (1-\gamma)\,\theta\left(\phi^i\right)^2\right]\phi^j}{1 - (1-\gamma)\,\theta\phi^i\phi^j}\right\} \tag{A.9a}$$

$$r^i \equiv R^i/\bar{R}, \quad i \neq j = 1,2. \tag{A.9b}$$

Differentiating ρ^i with respect to ϕ^i, θ, and γ gives:

$$\rho^i_{\phi^i} = \frac{(1+\gamma)\left[1-(1-\gamma)\theta\left(\phi^i-\phi^j\right)^2-(1-\gamma)^2\theta^2\left(\phi^i\phi^j\right)^2\right]}{(1-\gamma)\left[1-(1-\gamma)\theta\phi^i\phi^j\right]^2} > 0 \tag{A.10a}$$

$$\rho^i_{\theta} = \frac{(1+\gamma)\phi^i\phi^j\left(\phi^i-\phi^j\right)}{\left[1-(1-\gamma)\theta\phi^i\phi^j\right]^2} \gtreqless 0 \text{ as } \phi^i \gtreqless \phi^j \tag{A.10b}$$

$$\rho^i_{\gamma} = \frac{\left(\phi^i-\phi^j\right)\left[1-\left(1-\gamma^2\right)\theta\phi^i\phi^j-(1-\gamma)^2\theta^2\left(\phi^i\phi^j\right)^2\right]}{(1-\gamma)^2\left[1-(1-\gamma)\theta\phi^i\phi^j\right]^2} \gtreqless 0 \text{ as } \phi^i \gtreqless \phi^j, \tag{A.10c}$$

for $i \neq j = 1,2$. Since $\rho^i_{\phi^i} > 0$ and $\rho^i\left(\phi^i;\cdot\right) \in (-\gamma/(1-\gamma), 1/(1-\gamma))$, it follows that, absent binding resource constraints, there exists a unique value of ϕ^i, denoted by $\tilde{\phi}^i_c$, that solves (A.8).[25] Note especially, $\tilde{\phi}^i_c = \frac{1}{2}$ if $r^i = \frac{1}{2}$.

Applying the implicit function theorem to (A.8) using (A.10) gives

$$d\tilde{\phi}^i_c/dr^i = 1/\rho^i_{\phi^i} > 0 \tag{A.11a}$$

$$d\tilde{\phi}^i_c/d\theta = -\rho^i_{\theta}/\rho^i_{\phi^i} \lesseqgtr 0 \text{ as } \phi^i \gtreqless \phi^j \tag{A.11b}$$

$$d\tilde{\phi}^i_c/d\gamma = -\rho^i_{\gamma}/\rho^i_{\phi^i} \lesseqgtr 0 \text{ as } \phi^i \gtreqless \phi^j. \tag{A.11c}$$

The first expression implies $\tilde{\phi}^i_c \gtreqless \frac{1}{2}$ as $r^i \gtreqless \frac{1}{2}$. The second and third expressions indicate that increases in θ (due to an increase in β or a decrease in κ) and/or γ tends to dampen differences in power between the two agents.

Given $\tilde{\phi}^i_c$, we recover $\widetilde{G}^i_c$ by combining $r^i \equiv R^i/\bar{R} = \rho^i$ in (A.11a) and G^i in (A.7), keeping in mind that $\phi^j = 1-\phi^i$:[26]

$$\widetilde{G}^i_c = \frac{(1+\gamma)\theta\left(\phi^i\right)^2\phi^j}{1-(1-\gamma)\theta\phi^i\phi^j}\bar{R}, \quad i \neq j = 1,2. \tag{A.12}$$

First, observe that $r^i = \frac{1}{2}$, which implies $\tilde{\phi}^i_c = \frac{1}{2}$, gives

$$\widetilde{G}^i_c\big|_{\phi^i=\frac{1}{2}} = \frac{(1+\gamma)\theta}{4-(1-\gamma)\theta}\left(\bar{R}/2\right) < \bar{R}/2,$$

[25] Because the domain of ρ^i is limited to only those values of ϕ^i that ensure both agents' resource constraints are non-binding, not all values of ρ^i in $(-\gamma/(1-\gamma), 1/(1-\gamma))$ are relevant. In any case, with more tedious algebra one can find that (A.8) is cubic in ϕ^i. As such, there is an explicit solution. However, that solution is long and cumbersome to manipulate algebraically. Nonetheless, we can characterize the equilibrium without deriving the explicit solution.

[26] We omit "∼" and subscript "c" from ϕ^i and ϕ^j in the expressions below to avoid cluttering.

which implies that $\widetilde{G}_c^i/\bar{R} < \frac{1}{2}$. Furthermore, we have

$$\left.\frac{d\widetilde{G}_c^i}{d\theta}\right|_{\phi^i=\frac{1}{2}} = \frac{2\left(1+\gamma\right)\bar{R}}{\left[4-\left(1-\gamma\right)\theta\right]^2} > 0 \quad \text{and} \quad \left.\frac{d\widetilde{G}_c^i}{d\gamma}\right|_{\phi^i=\frac{1}{2}} = \frac{\left(2-\theta\right)\theta\bar{R}}{\left[4-\left(1-\gamma\right)\theta\right]^2} > 0.$$

Thus, in the symmetric equilibrium which arises when the two agents are equally endowed, reductions in γ or in θ (due to a fall in β or an increase in κ) induce less arming.

Differentiation of $\widetilde{G}_c^i$ and $\widetilde{G}_c^j$ with respect to ϕ^i gives

$$\frac{d\widetilde{G}_c^i/d\phi^i}{\widetilde{G}_c^i} = \frac{\phi^j - \phi^i + \phi^j\left[1-\left(1-\gamma\right)\theta\phi^i\phi^j\right]}{\phi^i\phi^j\left[1-\left(1-\gamma\right)\theta\phi^i\phi^j\right]} \tag{A.13a}$$

$$\frac{d\widetilde{G}_c^j/d\phi^i}{\widetilde{G}_c^j} = -\frac{\phi^i - \phi^j + \phi^i\left[1-\left(1-\gamma\right)\theta\phi^i\phi^j\right]}{\phi^i\phi^j\left[1-\left(1-\gamma\right)\theta\phi^i\phi^j\right]}, \;\; i \neq j = 1,2. \tag{A.13b}$$

Now suppose that $r^i \geq \frac{1}{2}$. Inspection of (A.13a) suggests the presence of ambiguity in the sign of $d\widetilde{G}_c^i/d\phi^i$. While this sign is positive for r^i close to $\frac{1}{2}$ (as that generates a value of ϕ^i close to $\frac{1}{2}$), this sign could be negative, as we show more formally below in the proof of Proposition 4. By contrast, equation (A.13b) shows that, for $\phi^i > \phi^j$ and thus $r^i \geq \frac{1}{2}$, $d\widetilde{G}_c^j/d\phi^i < 0$ holds. Thus, while $\widetilde{G}_c^j/\bar{R} < r^j$ for $r^i = \frac{1}{2}$ initially, resource reallocations from agent j to agent i ($\neq j$) reduce both r^j and $\widetilde{G}_c^j/\bar{R}$. But, inspection of (A.13b) shows that ρ^j ($= r^j$) falls faster than $\widetilde{G}_c^j/\bar{R}$, such that the resource constraint for j eventually becomes binding—i.e., when R^j falls below a certain threshold level R_L^c (or, equivalently, when R^i rises above a threshold $R_H^c = \bar{R} - R_L^c$).

To pin down this threshold where $G_c^j = R^j$, first set $\widetilde{G}_c^j/\bar{R} = \rho^j$ ($= r^j$) (where $\widetilde{G}_c^j$ satisfies (A.12) and ρ^j satisfies (A.8)). From that equality, we obtain the following quadratic equation for ϕ^i:

$$\left(1-\gamma\right)\theta\left(\phi^i\right)^2 - \left[1+\gamma+\left(1-\gamma\right)\theta\right]\phi^i + 1 = 0\ . \tag{A.14}$$

The solution to this equation for $\gamma < 1$, which we denote by ϕ_H^i, identifies the value of ϕ^i when $G^j = R_L^c$ and $G^i = \widetilde{B}_c^i(R_L^c)$:

$$\phi_H^i \equiv \frac{\left[1+\gamma+\left(1-\gamma\right)\theta\right]}{2\left(1-\gamma\right)\theta}\left(1-\sqrt{1-\frac{4\left(1-\gamma\right)\theta}{\left[1+\gamma+\left(1-\gamma\right)\theta\right]^2}}\right). \tag{A.15}$$

An application of the implicit function theorem to (A.14) shows

$$\frac{d\phi_H^i}{d\theta} = -\frac{\left(1-\gamma\right)\phi^i\phi^j}{1+\gamma-\left(1-\gamma\right)\theta\left(\phi^i-\phi^j\right)} < 0 \tag{A.16a}$$

$$\frac{d\phi_H^i}{d\gamma} = -\frac{\left(1-\theta\phi^j\right)\phi^i}{1+\gamma-(1-\gamma)\,\theta\left(\phi^i-\phi^j\right)} < 0. \tag{A.16b}$$

To obtain the value of R_L^c, plug ϕ_H^i shown in (A.15) back into $\widetilde{G}_c^j$ $(= R_L^c)$ shown in (A.12). One can then identify the effects of changes θ and γ on R_L^c by appropriately differentiating $\widetilde{G}_c^j$ $(= R_L^c)$:

$$\frac{dR_L^c}{d\theta} = \frac{1+\gamma-(1-\gamma)\,\theta\phi^i\left[1-(1-\gamma)\,\theta\phi^i\phi^j\right]}{\theta\left[1+\gamma-(1-\gamma)\,\theta\left(\phi^i-\phi^j\right)\right]\left[1-(1-\gamma)\,\theta\phi^i\phi^j\right]}\bar{R} > 0 \tag{A.17a}$$

$$\frac{dR_L^c}{d\gamma} = \frac{\frac{1-2\theta\phi^i\phi^j}{1+\gamma}+\frac{\left(1-\theta\phi^j\right)\left\{\phi^i-\phi^j+\phi^i\left[1-(1-\gamma)\theta\phi^i\phi^j\right]\right\}}{\phi^j\left[1+\gamma-(1-\gamma)\theta(\phi^i-\phi^j)\right]}}{1-(1-\gamma)\,\theta\phi^i\phi^j}\bar{R} > 0. \tag{A.17b}$$

Based on (A.14), we have $\lim_{\theta\to 0}\phi_H^i = 1/(1+\gamma)$ and $\lim_{\theta\to 1}\phi_H^i = 1/(1+\sqrt{\gamma})$. Using these results in (A.12) for agent j, we find $\lim_{\theta\to 0} R_L^c = 0$ and $\lim_{\theta\to 1} R_L^c = \bar{R}\gamma/(1+\sqrt{\gamma})^2$. Furthermore, $\lim_{\gamma\to 0}\phi_H^i = 1$ and $\lim_{\gamma\to 1}\phi_H^i = 1/2$ which, again, can be used in (A.12) to confirm that $\lim_{\gamma\to 0} R_L^c = 0$ and $\lim_{\gamma\to 1} R_L^c = \theta\bar{R}/4$. ||

Before considering the payoff implications of settlement (detailed in Proposition 2), we complete our characterization of arming under open conflict building on the analysis above.

Proof of Proposition 4.

Part (a): We already know from the analysis in the proof to Proposition 3 that, when $R^i \in [R_L^c, R_H^c]$, $\tilde{\phi}_c^i \gtreqless \tilde{\phi}_c^j$ as $R^i \gtreqless R^j$. Then, from (A.12), we have $\widetilde{G}_c^i \gtreqless \widetilde{G}_c^j$ as $R^i \gtreqless R^j$. To fix ideas, suppose $R^i > R^j$ (or $r^i > r^j$), which implies $\tilde{\phi}_c^i > \tilde{\phi}_c^j$ and $\widetilde{G}_c^i > \widetilde{G}_c^j$. Furthermore, from (A.11), we have $d\tilde{\phi}_c^i/dR^i > 0$ whereas $d\tilde{\phi}_c^i/d\theta < 0$ and $d\tilde{\phi}_c^i/d\gamma < 0$.

To demonstrate the remaining components of this part of the proposition, we turn to standard methods.[27] Let $\xi \in \{R^i, \theta, \gamma\}$ be the parameter of interest and for now we suppress "$\sim$" when dealing with unconstrained values to avoid cluttering. Then, total differentiation of the FOCs (7) gives

$$dU_{G^i}^i = U_{G^iG^i}^i dG^i + U_{G^iG^j}^i dG^j + U_{G^i\xi}^i d\xi = 0 \tag{A.18a}$$

$$dU_{G^j}^j = U_{G^jG^i}^j dG^i + U_{G^jG^j}^j dG^j + U_{G^j\xi}^j d\xi = 0, \tag{A.18b}$$

which can be solved to obtain

$$\begin{pmatrix} dG^i \\ dG^j \end{pmatrix} = \frac{d\xi}{\Delta}\begin{pmatrix} -U_{G^jG^j}^j U_{G^i\xi}^i + U_{G^iG^j}^i U_{G^j\xi}^j \\ -U_{G^iG^i}^i U_{G^j\xi}^j + U_{G^jG^i}^j U_{G^i\xi}^i \end{pmatrix}, \tag{A.19}$$

where $\Delta > 0$ was previously defined in (A.6). Once we rewrite $U_{G^i}^i = 0$ using θ shown in

[27] Taking this approach is useful for the proof to Proposition 5.

(8c) as

$$U^i_{G^i} = [(1-\kappa)\beta + \kappa]\left[\phi^i_{G^i}\theta\left(X^i + \gamma X^j\right) - \left(\theta\phi^i + 1 - \theta\right)\right] = 0, \;\; i = 1, 2,$$

it is straightforward to derive the direct effects of the parameters of interest on arming:

$$U^i_{G^i R^i} = \phi^i_{G^i}\theta(1-\gamma) > 0 \;\text{ and }\; U^j_{G^j R^i} = -\phi^j_{G^j}\theta(1-\gamma) < 0 \tag{A.20a}$$

$$U^i_{G^i\theta} = \phi^i_{G^i}\left(X^i + \gamma X^j\right) + \phi^j = \frac{1}{\theta} > 0 \tag{A.20b}$$

$$U^i_{G^i\gamma} = \phi^i_{G^i}\theta X^j > 0 \quad\text{and}\quad U^j_{G^j\gamma} = \phi^j_{G^j}\theta X^i > 0, \tag{A.20c}$$

for $i \neq j = 1, 2$. Equation (A.20a) shows that the direct effect of an increase in R^i on G^i is positive, as one would expect since an increase in agent's own resource that matches a decrease in the rival's resource means that less of the prize is subject to differential destruction ($\gamma < 1$). Exactly the opposite is true for G^j. But the equilibrium effects of R^i on $\widetilde{G}^i_c$ and $\widetilde{G}^j_c$ depend on the indirect effects of changes in the rival's (j) arming as well. And, as can be seen from (A.19), the indirect effect hinges on whether guns are strategic complements or strategic substitutes.

Working through the math, one can find that the total effects of R^i on guns are as follows:

$$\frac{d\widetilde{G}^i_c}{dR^i} = \frac{(1-\gamma)\theta\phi^i}{D}\left(\phi^j - \phi^i + \phi^j\left[1 - (1-\gamma)\theta\phi^i\phi^j\right]\right) \tag{A.21a}$$

$$\frac{d\widetilde{G}^j_c}{dR^i} = -\frac{(1-\gamma)\theta\phi^j}{D}\left(\phi^i - \phi^j + \phi^i\left[1 - (1-\gamma)\theta\phi^i\phi^j\right]\right), \tag{A.21b}$$

where $D > 0$ is shown in (A.6b). Clearly, from (A.21), $d\widetilde{G}^i_c/dR^i = d\widetilde{G}^j_c/dR^i = 0$ in the special case where $\gamma = 1$ so that there is no differential destruction. Returning to the assumption that $\gamma < 1$, as noted earlier and confirmed by (A.21a), $d\widetilde{G}^i_c/dR^i > 0$ if R^i sufficiently close to $\bar{R}/2$ (or equivalently if ϕ^i is sufficiently close to $\frac{1}{2}$). For completeness, we now demonstrate that $d\widetilde{G}^i_c/dR^i < 0$ can hold as $R^i \to R^c_H$, depending on the values of θ and γ. To this end, we use (A.14) that helps us pin down R^c_H and (A.21a) to show, after some tedious algebra, there exists a function $\breve{\theta}(\gamma)$, given by

$$\breve{\theta}(\gamma) \equiv 1 - \frac{(1+\gamma)^2}{2(1-\gamma)} + \frac{1}{2}\sqrt{4 + (1+\gamma)^2}, \tag{A.22}$$

that satisfies $\breve{\theta}'(\gamma) < 0$. Since $\breve{\theta}(\gamma)$ is monotonic in γ, we take its inverse, $\breve{\gamma}(\theta) = \breve{\theta}^{-1}(\gamma)$. One can then confirm, by taking the appropriate limits, that $\lim_{R^i \to R^c_H}(d\widetilde{G}^i_c/dR^i) \gtreqless 0$ as $\gamma \gtreqless \breve{\gamma}(\theta)$ for any $\theta \in (0, 1)$. In words, $\widetilde{G}^i_c$ is increasing in R^i for all $R^i \in [R^c_L, R^c_H]$ if γ is large enough, but decreasing in R^i in the neighborhood of R^c_H if γ is sufficiently small.

Going back to (A.20b), we see the direct effect of an increase in θ on guns is positive because it increases the value of each player's contested prize. Once again, however, the nature of the indirect effect depends on the initial values of guns and the parameters. Nonetheless, when we do the math we find

$$\frac{d\widetilde{G}_c^i}{d\theta} = \frac{\widetilde{G}_c^i/\theta}{D}\left[1+(1-\gamma)\,\theta\left(\phi^j\right)^2\right] > 0 \tag{A.23a}$$

$$\frac{d\widetilde{G}_c^j}{d\theta} = \frac{\widetilde{G}_c^j/\theta}{D}\left[1+(1-\gamma)\,\theta\left(\phi^i\right)^2\right] > 0. \tag{A.23b}$$

Lastly from (A.20c), the direct effect of a decrease in γ on both agents' guns is positive, which once again makes sense since an increase in γ represents a decrease in differential destruction. The total effects of γ on guns are given by

$$\frac{d\widetilde{G}_c^i}{d\gamma} = \frac{\theta\phi^i}{D}\left(2X^j\left(\phi^j\right)^2 + X^i\phi^i\left[\phi^i-\phi^j+(1-\gamma)\,\theta\left(\phi^j\right)^2\right]\right) > 0 \tag{A.24a}$$

$$\frac{d\widetilde{G}_c^j}{d\gamma} = \frac{\theta\phi^j}{D}\left(2X^i\left(\phi^i\right)^2 + X^j\phi^j\left[\phi^j-\phi^i+(1-\gamma)\,\theta\left(\phi^i\right)^2\right]\right) > 0. \tag{A.24b}$$

To confirm the sign of $d\widetilde{G}_c^i/d\gamma$ in (A.24a), we find an expression for X^i, using the definition of $\widetilde{G}_c^i$ from (A.12) and (A.8) with (A.9):

$$\tilde{X}_c^i = \frac{\bar{R}\left[\phi^i-\gamma\phi^j-(1-\gamma)\,\theta\phi^i\phi^j\right]}{(1-\gamma)\left[1-(1-\gamma)\,\theta\phi^i\phi^j\right]} \quad \text{for} \quad i\neq j=1,2, \tag{A.25}$$

which in turn implies

$$X^j - X^i = \frac{\bar{R}\,(1+\gamma)\left(\phi^j-\phi^i\right)}{(1-\gamma)\left[1-(1-\gamma)\,\theta\phi^i\phi^j\right]} \Rightarrow X^j \gtreqqless X^i \text{ if } \phi^j \gtreqqless \phi^i.$$

Clearly, if $R^i > R^j$, then $\phi^i - \phi^j > 0$ and $d\widetilde{G}_c^i/d\gamma > 0$. Suppose now that $R^i < R^j$ which implies $\phi^i - \phi^j < 0$. Still, it is easy to see that, once again, $d\widetilde{G}_c^i/d\gamma > 0$.

Part (b): If $R^i \in (0, R_L^c)$, then $G_c^i = R^i$ and $G_c^j = \widetilde{B}_c^j\left(R^i;\theta,\gamma\right) = -R^i + \sqrt{\theta R^i\bar{R}}$, so $\phi_c^i = R^i/\sqrt{\theta R^i\bar{R}} = \sqrt{R^i/\theta\bar{R}}$. The various comparative statics reported in this part can now be readily obtained by differentiating the relevant expressions appropriately. ||

Proof of Proposition 2.

Part (a): Assuming $R^i \in [R_L^s, R_H^s]$ where $R_L^s \equiv \frac{1}{4}(1-\kappa)\,\bar{R}$, we have from Proposition 1(a), $G_s^i = R_L^s$ for $i=1,2$, which implies (from (1)) $\phi^i = \frac{1}{2}$, $X^i = R^i - \frac{1}{2}(1-\kappa)\bar{R}$, and $\bar{X}_s = \frac{1}{2}(1+\kappa)\bar{R}$ for $i=1,2$. Substituting these values into (3) shows $V_s^i = \frac{1}{4}(1-\kappa)\,\bar{R}+\kappa R^i$ for $i=1,2$, which is clearly increasing in agent i's own resource R^i (given $\bar{R}$ and $\kappa > 0$) and increasing (decreasing) in κ for $R^i > \frac{1}{4}\bar{R}$ $(R^i < \frac{1}{4}\bar{R})$.

Part (b): If $R^i \in (0, R^s_L)$, then from Proposition 1(b), $G^i_s = R^i$ and from (5b) $G^j_s = -R^i + \sqrt{(1-\kappa)R^i\bar{R}}$, which from (1) imply $\phi^i_s = R^i/\sqrt{(1-\kappa)R^i\bar{R}}$. Furthermore, we have $\bar{X}_s = X^j_s = \bar{R} - \sqrt{(1-\kappa)R^i\bar{R}}$. Substituting these values into (3) and simplifying the resulting expression, we find the payoff function for the constrained agent i:

$$V^i_s = \bar{R}\sqrt{\frac{(1-\kappa)R^i}{\bar{R}}}\left(1 - \sqrt{\frac{(1-\kappa)R^i}{\bar{R}}}\right). \tag{A.26}$$

Clearly, $\lim_{R^i \to 0} V^i_s = 0$. Keeping in mind that $R^s_L/R^i > 1$, differentiation of V^i_s in (A.26) with respect to R^i and κ shows

$$\frac{dV^i_s}{dR^i} = \sqrt{R^s_L/R^i} - (1-\kappa) > 0 \tag{A.27a}$$

$$\frac{dV^i_s}{d\kappa} = R^i\left(1 - \frac{1}{1-\kappa}\sqrt{R^s_L/R^i}\right) < 0. \tag{A.27b}$$

Thus V^i_s is increasing in $R^i \in (0, R^s_L]$ and decreasing κ, as claimed in part (b). Using the expressions above, one can also verify $d^2V^i_s/(dR^i)^2 < 0$ and $d^2V^i_s/d\kappa^2 < 0$.

Applying the solutions above again in (3) but this time for the unconstrained agent (j), one can verify that his payoff is given by

$$V^j_s = \bar{R}\left(1 - \sqrt{\frac{(1-\kappa)R^i}{\bar{R}}}\right)^2, \; j \neq i. \tag{A.28}$$

Since $R^i = R - R^j$, it follows from (A.28) that (i) $\lim_{R^j \to \bar{R}} V^j_s = \bar{R}$ and (ii) $V^j_s/d\xi > 0$ for $\xi \in \{R^j, \kappa\}$. It is also straightforward to confirm that $d^2V^j_s/d\xi^2 > 0$ for $\xi \in \{R^j, \kappa\}$. ||

Proof of Proposition 5.

Part (a): To prove this part, which assumes $R^i \in [R^c_L, R^c_H]$, we can rely on the envelope theorem and study only the direct of changes in the parameters on an agent's payoff and strategic effects that operate through their influence on the rival's arming. In particular, for any $\xi \in \{R^i, \kappa, \gamma, \beta\}$ we need to study

$$dU^i_c/d\xi = U^i_\xi + U^i_{G^j}\left(d\widetilde{G}^j_c/d\xi\right). \tag{A.29}$$

Given the results in Proposition 4(a), we start by identifying the importance of the initial distribution of resources, R^i. Define $A \equiv (1-\kappa)\beta + \kappa$ (> 0) and recall the definition of θ in (8c). Then, partial differentiation of U^i in (2) with respect to R^i (keeping in mind that $R^j = \bar{R} - R^i$) and G^i gives

$$U^i_{R^i} = A\left[1 - \theta + \theta(1-\gamma)\phi^i\right] > 0$$

$$U^i_{G^j} \;\; = \;\; -A\theta\left(\phi^i/\phi^j\right)\left[1-\theta+\theta\phi^i+\gamma\phi^j\right]<0,$$

where in the second expression we use $\widetilde{U}^i_{G^i}=0$ with (7) and the properties of ϕ^i in (1) that imply $\phi^i_{G^j}/\phi^i_{G^i}=-\phi^i/\phi^j$. The direct effect of increasing R^i on U^i is positive because it expands both the value of agent i's contestable prize and the value of its secure output. In contrast, an increase in G^j reduces U^i because of its negative effect both on i's contested prize and the probability that i will prevail in conflict. But, from our earlier discussion in relation to (A.21), an increase in R^i (or equivalently a decrease in R^j) increases G^j only when R^i is sufficiently smaller than R^j and γ is sufficiently small. To proceed, we substitute the expressions for the direct effects above and (A.21b), using the definition of $D>0$ (A.6b), into (A.29) to find

$$\frac{d\widetilde{U}^i_c}{dR^i}=A\left[1-\theta+\frac{\theta\left(1-\gamma\right)\phi^i}{D}\Big(1+\left(\phi^i-\phi^j\right)\phi^j+\left(\phi^i\right)^2\left[2-\left(1-\gamma\right)\theta\left(1+\phi^j\right)\right]\Big)\right].$$

Since $2>\left(1-\gamma\right)\theta\left(1+\phi^j\right)$ holds, whenever $R^i>R^j$ (which implies $\phi^i>\phi^j$) the above expression is positive. Even when $R^i<R^j$ (so that $\phi^i<\phi^j$), we have $1>(\phi^j-\phi^j)\phi^j$, such that the positive direct effect dominates the negative strategic effect.

Next, we turn to the effects of an increase in κ. Differentiation of $\widetilde{U}^i_c$ gives

$$d\widetilde{U}^i_c/d\kappa=\tilde{X}^i_c-\beta\phi^i\left(\tilde{X}^i_c+\gamma\tilde{X}^j_c\right)+\overset{(-)}{U^i_{G^j}}\left(\overset{(-)}{d\tilde{G}^j_c}/d\kappa\right).$$

As shown above, the strategic effect of an increase in κ (represented by the third term in the RHS of the above expression) is positive. However, since a marginal increase in security increases U^i by the quantity of i's secure output and reduces U^i by the value of the agent's contested prize, the sign of the direct effect (represented by the sum of the first two terms) would appear to be ambiguous.

Given the positive sign of the strategic effect, we explore the direct effect $\partial U^i/\partial\kappa$ in finer detail. To proceed, observe the following: (i) As $R^i\to R^c_L$, $\tilde{X}^i_c\to 0$ and $\tilde{X}^j_c>0$, and thus $\partial U^i/\partial\kappa<0$. (ii) However, when evaluated at $R^i=\bar{R}/2$ (so that $\phi^i=\phi^j=1/2$ and $\tilde{X}^j_c=\tilde{X}^i_c>0$), $\partial U^i/\partial\kappa=[1-\frac{1}{2}\beta(1+\gamma)]\tilde{X}^i_c>0$ for $\beta<1$ and/or $\gamma<1$. (iii) As $R^i\to R^c_H$, $\tilde{X}^j_c\to 0$ and $\tilde{X}^i_c>0$, such that $\partial U^i/\partial\kappa=\left(1-\beta\phi^i\right)\tilde{X}^i_c>0$. Observations (ii) and (iii) raise the question of whether $\partial U^i/\partial\kappa$ is positive for all $R^i\in[\bar{R}/2,R^c_H]$. We now demonstrate that the answer is yes.

Noting from (A.8) and (A.11a) that r^i $(=\rho^i)$ and ϕ^i are positively related, define $h(\phi^i)\equiv\partial U^i\left(\phi^i\right)/\partial\kappa$. To establish our claim that the direct payoff effect of an increase in output security is positive in view of observations (ii) and (iii) above, it suffices to show that $h(\phi^i)$ is strictly quasi-concave in ϕ^i. Let $\phi^i_{\max}\in[1/2,\phi^i_H]$ denote a potential maximizer of

$h(\phi^i)$ for $R^i \in [\bar{R}/2, R_H^c]$. Differentiation of $h(\phi^i)$ with respect to ϕ^i using (A.13) shows

$$h'\left(\phi^i\right) = \frac{\bar{R}\left(1+\gamma\right)}{\left(1-\gamma\right)\left[1-\left(1-\gamma\right)\theta\phi^i\phi^j\right]^2}\left[\Psi^i - \Omega^i\right] \tag{A.30}$$

where

$$\begin{aligned}
\Psi^i &= \Psi\left(\phi^j\right) \equiv 1-\left(1-\gamma\right)\theta\left(\phi^j\right)^2 > 0 \\
\Omega^i &= \Omega\left(\phi^i,\phi^j\right) \equiv \beta\left(1-\gamma\right)\phi^i\left[2\left(1-\theta\right)+\left(2+\gamma\right)\theta\phi^i+\left(1-\gamma\right)\theta^2\phi^i\left(\phi^j\right)^2\right] > 0.
\end{aligned}$$

One can then evaluate (A.30) at $\phi^i = \frac{1}{2}$ to find

$$h'\left(\phi^i = \tfrac{1}{2}\right) = \frac{\bar{R}\left(1+\gamma\right)\left[1-\beta\left(1-\gamma\right)\left(1-\frac{\theta}{4}\right)\right]}{\left(1-\gamma\right)\left[1-\left(1-\gamma\right)\frac{\theta}{4}\right]} > 0, \tag{A.31}$$

which implies $\phi^i_{\max} > 1/2$. Thus, we must either have $h'\left(\cdot\right) > 0$ over the entire range—which would imply $\phi^i_{\max} = \phi^i_H$ and that would complete the proof—or we must have $h'\left(\phi^i_{\max}\right) = 0$ at some $\phi^i_{\max} \in (1/2, \phi^i_H)$ where $h''\left(\phi^i_{\max}\right) < 0$. (The latter inequality would also imply $\phi^i_{\max}$ is unique). Clearly $h'\left(\phi^i_{\max}\right) = 0$ requires $\Psi - \Omega = 0$. Additionally,

$$\text{sign}\left\{h''\left(\phi^i_{\max}\right)\right\} = \text{sign}\left\{-\phi^i\Psi^i_{\phi^j} + \phi^i\Omega^i_{\phi^j} - \phi^i\Omega^i_{\phi^i}\right\},$$

where every term inside the brackets is evaluated at $\phi^i = \phi^i_{\max}$, and

$$\begin{aligned}
\phi^i\Psi^i_{\phi^j} &= -2\left(1-\gamma\right)\theta\phi^i\phi^j < 0 \\
\phi^i\Omega^i_{\phi^j} &= 2\beta\left(1-\gamma\right)^2\theta^2\left(\phi^i\right)^3\phi^j > 0 \\
\phi^i\Omega^i_{\phi^i} &= \Omega + \beta\left(1-\gamma\right)\left(\phi^i\right)^2\left[\left(2+\gamma\right)\theta+\left(1-\gamma\right)\theta^2\left(\phi^j\right)^2\right] > 0 \\
&= 1-\left(1-\gamma\right)\theta\left(\phi^j\right)^2 + \beta\left(1-\gamma\right)\left(\phi^i\right)^2\left[\left(2+\gamma\right)\theta+\left(1-\gamma\right)\theta^2\left(\phi^j\right)^2\right] > 0.
\end{aligned}$$

We pre-multiply the expressions inside the brackets by $\phi^i > 0$ to obtain Ω^i in the RHS of $\phi^i\Omega^i_{\phi^i}$.[28] In turn, to obtain the second expression for $\phi^i\Omega^i_{\phi^i}$, we used the fact that at $\phi^i_{\max}$, $\Omega^i = \Psi^i$. From the expressions above we find, after simplifying,

$$\begin{aligned}
-\phi^i\Psi^i_{\phi^j} + \phi^i\Omega^i_{\phi^j} - \phi^i\Omega^i_{\phi^i} &= -\left[1-(1-\gamma)\theta\phi^j\left(2\phi^i+\phi^j\right)\right] \\
&\quad -\beta\left(1-\gamma\right)\theta\left(\phi^i\right)^2\left[2+\gamma+\left(1-\gamma\right)\theta\phi^j\left(2\phi^i-\phi^j\right)\right] < 0.
\end{aligned}$$

It is easy to verify that the sum of the terms inside the first set of brackets is positive. Therefore, $h''\left(\phi^i_{\max}\right) < 0$, thereby completing the proof that $\partial U^i/\partial\kappa > 0$ for $R^i \in [\bar{R}/2, R_H^c]$.

[28] Of course, doing so is legitimate because it does not change the relevant signs.

To identify the total payoff effect of a change in γ, we differentiate $\widetilde{U}_c^i$ appropriately. Using the definition $A \equiv (1-\kappa)\beta + \kappa$ and simplifying, we have

$$\frac{d\widetilde{U}_c^i}{d\gamma} = A\theta\phi^i \left[\tilde{X}_c^j - \left(\gamma + \frac{1-\theta+\theta\phi^i}{\theta\phi^j} \right) \overset{(+)}{\left(d\widetilde{G}_c^j / d\gamma \right)} \right]. \tag{A.33}$$

Clearly, the direct effect of an increase in γ is positive (because it increases the value of the contested prize) whereas the indirect effect is negative. But, $\lim_{R^i \to R_H^c} \tilde{X}_c^j = 0$. Therefore, $\lim_{R^i \to R_H^c} d\widetilde{U}_c^i / d\gamma < 0$.

To demonstrate that $d\widetilde{U}_c^i / d\gamma > 0$ holds for $R^i \in [R_L^c, \bar{R}/2]$, first note that we can rewrite $d\widetilde{G}_c^j / d\gamma$ in (A.24b) as

$$\frac{d\widetilde{G}_c^j}{d\gamma} = \frac{\theta\phi^j \tilde{X}_c^j}{D} \Upsilon^i.$$

where $\Upsilon^i \equiv \phi^i \left[2\phi^i (\tilde{X}_c^i / \tilde{X}_c^j) \right] + \phi^j \left[\phi^j - \phi^i + (1-\gamma)\theta(\phi^i)^2 \right]$. Now observe two points. First, since we are considering allocations $R^i \in [R_L^c, \bar{R}/2]$ that imply $\phi^i \leq \frac{1}{2} \leq \phi^j$ and $\tilde{X}_c^i \leq \tilde{X}_c^j$ (with both satisfied as an equality for $R^i = \bar{R}/2$), each of the two terms inside the square brackets in Υ^i is less than 1. Second, since $\phi^i + \phi^j = 1$, Υ^i is a weighted sum. Therefore, $\Upsilon^i \in (0, 1)$. Using the above, we now rewrite (A.33) as

$$\frac{d\widetilde{U}_c^i}{d\gamma} = A\theta\phi^i \tilde{X}_c^j \left[1 - \frac{1-(1-\gamma)\,\theta\phi^j}{D} \Upsilon^i \right].$$

But, while recalling $\phi^j \geq \frac{1}{2}$, inspection of the definition of D in (A.6b) reveals that $\left[1-(1-\gamma)\,\theta\phi^j\right]/D \in (0,1)$, which implies the expression inside the brackets is strictly positive. Accordingly, the direct effect of γ dominates its indirect effect: $d\widetilde{U}_c^i / d\gamma > 0$ for all $R^i \in [R_L^c, \bar{R}/2]$ and, by continuity, for points to the right of $\bar{R}/2$ that are sufficiently close to it. Panel (b) of Fig. 2, which depicts the effect of a reduction in γ on $\widetilde{U}_c^i$, illustrates how this payoff effect depends on the initial distribution of factor ownership.

Finally, we turn to β. We partially differentiate U^i in (2) with respect to β, using the property of ϕ^i that implies $\phi_{G^i}^i = \phi^i \phi^j / G^i$ and the expression for $\widetilde{G}_c^i$ in (A.12). When evaluated at the optimizing arming choice, the result is

$$U_\beta^i = C\left(\theta\phi^i + 1 - \theta\right), \text{ where } C \equiv \frac{\bar{R}\,(1-\kappa)\,(1+\gamma)\left(\phi^i\right)^2}{1-(1-\gamma)\,\theta\phi^i\phi^j} > 0.$$

Furthermore, using the expression for $U_{G^j}^i$ obtained earlier, the value of $\widetilde{G}_c^j$ from (A.12) and

$d\widetilde{G}_c^j/d\beta$ ($= (d\widetilde{G}_c^j/d\theta)(d\theta/d\beta > 0)$ from (A.23b) with $D > 0$ in (A.6b) yields

$$\frac{d\widetilde{U}_c^i}{d\beta} = C\phi^i \left(\theta + (1-\theta)\frac{\left[1 + (1-\gamma)\,\theta f\left(\phi^i\right)\right]}{D}\right) > 0,$$

where $f(\phi^i) \equiv 2\phi^j - \phi^i - \phi^i\phi^j$. The inequality holds for all $\gamma \in [0,1)$ and $\theta \in [0,1)$, since $f' < 0$ for all $\phi^i \in (0,1)$ and $f(1) = -1$.

Part (b): If $R^i \in (0, R_L^c)$, then $G_c^i = R^i$, $G_c^j = -R^i + \sqrt{\theta R^i \bar{R}}$, $X_c^i = 0$, $X_c^j = \bar{R} - \sqrt{\theta R^i \bar{R}}$ and $\phi_c^i = R^i/\sqrt{\theta R^i \bar{R}}$. Substituting these values into (2) for the constrained agent i shows

$$U_c^i = (1-\kappa)\,\beta\gamma R^i \left(\sqrt{\frac{\bar{R}}{\theta R^i}} - 1\right), \tag{A.34}$$

which clearly approaches 0 as $R^i \to 0$. Differentiation of U_c^i in (A.34) with respect to R^i, κ, γ and β gives

$$\frac{dU_c^i}{dR^i} = (1-\kappa)\,\beta\gamma\left(\sqrt{\frac{\bar{R}}{4\theta R^i}} - 1\right) \geq 0, \quad \frac{d^2U_c^i}{(dR^i)^2} < 0 \tag{A.35a}$$

$$\frac{dU_c^i}{d\kappa} = \frac{\theta\gamma\sqrt{R^i\bar{R}\theta}}{2\,(1-\kappa)}\left[1 - 2\left[\beta\,(1-\kappa) + \kappa\right]\left(1 - \sqrt{\frac{R^i\theta}{\bar{R}}}\right)\right], \quad \frac{d^2U_c^i}{d\kappa^2} < 0. \tag{A.35b}$$

$$\frac{dU_c^i}{d\gamma} = U_c^i/\gamma > 0, \quad \frac{d^2U_c^i}{d\gamma} = 0 \tag{A.35c}$$

$$\frac{dU_c^i}{d\beta} = \frac{1}{2}\,(1-\kappa)\,\gamma\sqrt{\frac{\bar{R}R^i}{\theta}}\left[1 + \theta\left(1 - \sqrt{\frac{4R^i}{\theta\bar{R}}}\right)\right] > 0, \quad \frac{d^2U_c^i}{d\beta^2} < 0. \tag{A.35d}$$

Since $R^i \leq R_L^c$ and (from Proposition 3) $R_L^c \leq \frac{1}{4}\theta\bar{R}$, we have $\bar{R}/4\theta R^i \geq 1$, which confirms the sign of (A.35a). Turning to (A.35b), note that $R^i\theta/\bar{R} \leq \theta^2/4$ or, equivalently, $\sqrt{R^i\theta/\bar{R}} \leq \theta/2$; therefore,

$$1 - 2\left[\beta\,(1-\kappa) + \kappa\right]\left(1 - \sqrt{\frac{R^i\theta}{\bar{R}}}\right) < (1-\beta)\,(1-\kappa) - \kappa.$$

Clearly, the sign of the RHS of the above inequality depends on the values of β and κ. The higher is the degree of security ($\kappa \uparrow$) and the lower is the rate of destruction ($\beta \uparrow$), the more likely it is that improvements in security will reduce the constrained agent's payoff.[29] Finally, from (A.35d), the sign of $dU_c^i/d\beta$ is positive because $R^i \leq \frac{1}{4}\theta\bar{R}$.

Turning to the unconstrained agent (j), we again substitute in the solutions above into

[29] Consistent with our findings under settlement, in the special case where $\beta = 1$ as under settlement, $dU_c^i/d\kappa < 0$.

(2), to find

$$U_c^j = \bar{R}[(1-\kappa)\beta + \kappa]\left(1 - \sqrt{\frac{R^i\theta}{\bar{R}}}\right)^2, \;\; j \neq i. \tag{A.36}$$

We identify the effects of changes in the various parameters $\xi \in \{R^i, \kappa, \gamma, \beta\}$ on agent j's payoff by invoking the envelope theorem and noting that

$$dU_c^j/d\xi = U_\xi^j + U_{G^i}^j \left(\partial B_c^i/\partial\xi\right), \quad j \neq i.$$

Focusing on $\xi = R^j$, it is clear that $U_{R^j}^j = U_{X^j}^j > 0$ while $U_{G^i}^j < 0$. Since $B_c^i = R^i$ and $dR^j = -dR^i$, it follows that $U_{G^i}^j(\partial B_c^i/\partial R^j) > 0$. In short, $dU_c^j/dR^j > 0$ due to positive direct and indirect effects. The convexity of U_c^j in R^j is obvious from (A.36). Since remaining parameters $\xi \in \{\kappa, \gamma, \beta\}$ have no influence on the rival's arming ($B_c^i = R^i$), only their respective direct effects will matter. The sign of these effects can be readily established upon inspection of U^j in (2). ||

Proof of Proposition 6. To start, assume $\beta = \gamma = 1$ and $\kappa \in [0, 1)$, so that conflict and settlement are indistinguishable. In the context of Fig. 2(b), this assumption implies that agent i's payoffs under the two regimes coincide with the solid curve where $R_J^c = R_J^s$ for $J \in \{L, H\}$. While maintaining the assumption that $\beta = 1$, now suppose γ falls to some level below 1. As demonstrated in Proposition 5 and shown in the figure, U_c^i rotates counter-clockwise around (a shifting pivot at $R^i > \bar{R}/2$) while V_s^i remains intact. The decrease in γ also implies that R_L^c falls while R_H^c rises so that $R_L^c < R_L^s$ and $R_H^c > R_H^s$. Furthermore, and as long as $\beta = 1$, $U_c^i = V_s^i$ for $R^i \in [R_H^c, \bar{R})$. The reason for the just described rotation of U_c^i is that, while the higher rate of differential destruction ($\gamma \downarrow$) generates an adverse direct effect on agent i's payoff for larger values of R^i (due to a reduction in the size of his prize), that effect is dominated by the favorable strategic effect of his rival j's arming reduction, when the distribution of factor ownership is sufficiently—but not very—uneven (i.e., R^i is close to R_H^s). Exactly the opposite is true for sufficiently lower values of R^i. Thus, a decrease in γ gives rise to a set of threshold values R^* and R^{**} where $\bar{R}/2 < R^* < R^{**}$ ($= R_H^c$ for $\beta = 1$), such that $U_c^i \geq V_s^i$ for all $R^i \in [R^*, R^{**}]$ (with equality at the endpoints of the interval), while $U_c^i < V_s^i$ for $R^i < R^*$ and $U_c^i = V_s^i$ for $R^i > R^{**}$.

Keeping κ and γ fixed at levels below 1, consider now the effect of increasing the indiscriminate rate of destruction ($\beta \downarrow$). As established in Proposition 5 and shown in Fig. 3, U_c^i will pivot clockwise at $R^i = 0$ due to an adverse direct effect that dominates the potentially favorable strategic effect. From the definition of θ in (8c), we know that, as $\beta \to 0$, $\theta \to 0$, and from Proposition 3, we know that $\lim_{\theta\to 0} R_L^c = 0$ and thus

$\lim_{\theta \to 0} R_H^c = \bar{R}$. What's more, from Proposition 4, one can infer that $\beta \to 0$ implies $G_c^i \to 0$ and $G_c^j \to 0$. These results taken together imply $\lim_{\beta \to 0} U_c^i|_{R^i = R_H^c} = \kappa \bar{R}$. Noting that $V_s^i|_{R^i = R_H^s} = [\kappa + \frac{1}{4}(1-\kappa)^2]\bar{R} = \frac{1}{4}(1+\kappa)^2 \bar{R}$, it follows that

$$\lim_{\beta \to 0} U_c^i\big|_{R^i = R_H^c} < V_s^i\big|_{R^i = R_H^s}.$$

On the basis of the above and the continuity of U_c^i in β we conclude that, for $R^i = R_H^s(\kappa) = \left[1 - \frac{1}{4}(1-\kappa)\right]\bar{R}$, there exists a unique $\beta_0 = \beta_0(\kappa, \gamma) \in (0,1)$ such that

$$U_c^i\big|_{R^i = R_H^s(\kappa)}(\beta_0, \gamma, \kappa) = V_s^i\big|_{R^i = R_H^s(\kappa)}(\kappa).$$

Let us now examine the dependence of β_0 on its arguments. Since γ enters as an argument in agent i's payoff under conflict U_c^i but not in V_s^i and $dU_c^i/d\gamma < 0$ while $dU_c^i/d\beta > 0$, it follows (by the implicit function theorem) that $\partial\beta_0/\partial\gamma > 0$. Next, note that the effect of κ on β_0 is transmitted both through V_s^i and U_c^i. But, while the effect of an increase in κ on V_s^i evaluated at $R_H^s > \frac{1}{4}\bar{R}$ is clearly positive (by Proposition 2), the effect on U_c^i is more involved. In particular, there are three channels of transmission of κ on U_c^i: (i) a direct channel (such that $U_\kappa^i > 0$ since $R^i > \bar{R}/2$); (ii) an effect channeled through the endowment $R^i = R_H^s(\kappa)$ (where $dR_H^s/d\kappa > 0$ and $U_{R^i}^i > 0$); and (iii) a strategic channel whereby G_c^j changes due to changes in κ (which tends to reduce G_c^j) and an indirect effect through the just noted change in $R^i = R_H^s$ (which normally tends to increase G_c^j). The ambiguity of effect (iii) complicates matters. Moreover, the sign of the effect of κ on β_0 is complicated by the fact that it depends on the sign of $d(V_s^i - U_c^i)/d\kappa$, evaluated at $V_s^i - U_c^i = 0$. Bypassing these issues, we report the $\partial\beta_0/\partial\kappa < 0$ effect based on numerical analysis of the model.

Parts (a) and (b) follow in a straightforward way from the above discussion. ||