Problem Set Three: Productivity Residual
Econ 210F, Spring 2009

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1. The Cobb-Douglas Production Function (walm-up question)

   First, consider the Cobb-Douglas production function with capital (K), labor (L, the number of workers), and technology (A).

   $$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

   (a) With output price normalized as one, capital (rental) price as r and wage rate (payment per worker) as w, find the profit-to-revenue ratio for a profit-maximizing firm that does not accumulate capital or inventory in a perfectly competitive market.

   (b) Let $g(\cdot)$ to represent the growth rate. Show that (1) suggests

   $$g(Y) = \frac{rK}{rk + wL}g(K) + \frac{wL}{rk + wL}g(L) + g(A)$$

2. Returns to Scale

   (a) Now, let's examine a more general production function with materials input (M):

   $$Y = AF(K, L, M), \tag{2}$$

   Suppose that $F(\cdot)$ is homogeneous of degree γ in capital, in labor and in materials. That is, it satisfies

   $$x^{\gamma}AF(K, L, M) = AF(xK, xL, xM), \ \forall x. \tag{3}$$

   (b) What is the profit-to-revenue ratio for a profit-maximizing firm with (2)?(Hints: differentiate (3) with respect to x and evaluate when $x = 1$).

(c) Let p be the material price, show that (2) suggests

$$\begin{aligned} g(Y) &= \gamma\left[s_k g(K) + s_l g(L) + s_m g(M)\right] + g(A) \\ s_k &= \frac{rK}{rk + wL + pM}; \\ s_l &= \frac{wL}{rk + wL + pM}; \\ s_m &= \frac{pM}{rk + wL + pM}. \end{aligned}$$

3. Factor Utilization

Now, suppose that, in additional to choosing L and K, firms may vary the intensity of input usage, so that

$$Y = AF\left(ZK, EHL, M\right)$$

where Z is the capital utilization rate, H is hours worked per employee, E is the effort of each worker. But varying utilization will put additional cost on labor so that:

$$w = \widetilde{w} g(H, E) v(Z), g_H > 0, g_E > 0, v_Z > 0$$

(a) Please show that

$$\begin{aligned} g(Y) &= \gamma\left[g(X) + g(U)\right] + g(A) \qquad (4) \\ \text{where } g(X) &= s_k g(K) + s_l\left[g(L) + g(H)\right] + s_m g(M) \\ g(U) &= s_k g(Z) + s_l g(E) \end{aligned}$$

where $\widetilde{w}$ is a constant, s_k, s_l and s_m are defined as in question two.

(b) Note that U is non-observable, since we cannot directly measure Z and E. Use the first order conditions to show that both E and Z are proportional to the observable H. (Hints: first show that E is proportional to H, then show that Z would be proportional to E).

(c) Show how you would estimate $g(A)$ in (4) using your answer to (b),

4. NBER-CES Manufacturing Productivity (MP) Database.

Visit "http://www.nber.org/nberces/nbprod96.htm". Download the MP Stata file in 1987 Standard Industry Classification (SIC) as well as the technical appendix (NBER Working Paper No. 205). Given the size of this data file, you would have to expand the memory size using command “set mem”.

(a) Print out the variable summaries and variable descriptions (use commands "sum" and "desc").

(b) How many industries are included in this file? How many years? Accordingly, what is the sample size?

(c) To estimate (4), how would you measure $g(Y)$?

(d) To estimate (4), how would you measure $g(X)$?

(e) To estimate (4), how would you measure $g(U)$? (using your answer to question 3, part (c)?

(f) Describe how you would use Instrumental Variable approach to get a better estimate for γ, and therefore a better estimate for $g(A)$.