Problem Set Two: More on Consumption
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Min Ouyang

1. The Life-cycle Hypothesis and Durable Goods

   So far this semester we have looked at models in which consumption goods yield flow utility only in the period in which they are bought. This assumption may be appropriate for nondurable goods and services, but is clearly not appropriate for durable goods such as cars and houses, since a durable good purchased at t will typically yield flow utility at t and for several periods thereafter.

   Assume that a consumer's utility depends on the flow of services from a stock of durable goods. The consumer's problem is

$$Max \quad E_0 \left\{ \sum_{t=0}^{T} \left[\beta^t u\left(K_t\right) \right] \right\} \quad \text{s.t.}$$

$$K_t = (1-\delta)K_{t-1} + C_t \tag{1}$$

$$A_{t+1} = (1+r)(A_t + Y_t - C_t) \tag{2}$$

$$A_{T+1} = 0, \quad A_0, K_0 \text{ given,}$$

   where K_t is the consumer's stock of durable goods, C_t is purchases of durables, Y_t is labor income, A_t is financial wealth, and $0 < \delta < 1$ is the depreciation rate of durables. Assume that Y_t is random as of $t-1$, that the riskless interest rate r is constant, and that there is no risky asset.

   We can restate the consumer's problem in Dynamic Programming form as follows (note that K_{t-1} is one of the state variables, not K_t)

$$V_t(A_t, K_{t-1}) = \underset{C_t}{Max} \left\{ u\left(K_t\right) + \beta E_t \left[V_{t+1}\left(A_{t+1}, K_t\right) \right] \right\} \tag{3}$$

   (a) What additional assumptions would we have to make in order for the value-function $V_t(A_t, K_{t-1})$ to be time-invariant? Write down the time-invariant value function.

   (b) Use the Bellman's equation (3) to establish the following Euler Equation for the stock of durable goods:

$$u'\left(K_t\right) = \beta(1+r)E_t\left[u'\left(K_{t+1}\right)\right] \tag{4}$$

Hints: the first-order condition for C will involve $u'(K)$ and and the expected derivatives of V w.r.t. both A and K. Evaluate the derivatives of V using the envelope theorem. Show that $V_K = (1-\delta)V_A$, and use this to eliminate the term involving V_K from the first-order condition for C. Then show that $V_A = \frac{(1+r)}{(r+\delta)}u'(K)$.

(c) Now assume that $u(K)$ is quadratic and that $\beta(1+r) = 1$.Show that the stock of durables follows a random walk:

$$K_{t+1} = K_t + \varepsilon_{t+1} \tag{5}$$

where $E_t(\varepsilon_{t+1}) = 0$. Show that durables purchases obey

$$C_{t+1} = C_t + \varepsilon_{t+1} - (1-\delta)\varepsilon_t \tag{6}$$

(d) Mankiw(1982) finds that aggregate durables purchases (C_t) in the postwar U.S. follow a random walk; given variables such as$C_{t-1}, (C_t - C_{t-1}), K_t$ and $(K_t - K_{t-1})$ do not help predict C_{t+1}. Is his result consistent with (5) and (6)? (Note, a reasonable value for δ for consumer durables is 0.3)

2. Precautionary Savings

Definition 1 *if X is lognormal, then the mean of X satisfies $E(X) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$, where $\left(\mu, \sigma^2\right)$ are the mean and the variance of $log(X)$.*

Now the problem. Assume that an individual faces the following two-period problem, with zero discount and interest rates:

$$\begin{aligned} & Max\ \{u(C_1) + E_1[u(C_2)]\} \\ & s.t. \\ C_2 &= (W_1 - C_1) + W_2 \end{aligned}$$

Assume that W_1 is know at time 1, while W_2 is normally distributed with mean μ_W and variance σ_W^2. Assume that utility is

$$U(C) = -\frac{1}{\alpha}\exp(-\alpha C)$$

Derive a closed-form solution for first-period consumption and first-period precautionary saving. How does the amount of precautionary saving vary with the variance of W_2?