

Problem Set One: The Life-Cycle
Hypothesis/Permanent Income Hypothesis
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1. Consumption with Variable Labor Supply.

Consider the problem of a household that has to choose both consumption and labor supply in a stochastic dynamic environment. The household's problem is

$$\begin{aligned} V_0 &= \text{Max} \left[E_0 \sum_{t=0}^T \beta^t u(C_t, L_t) \right] \\ &\text{s.t.} \\ &A_0 \text{ given} \\ A_{t+1} &= (1+r)(A_t + w_t L_t + I_t - C_t) \\ A_{T+1} &= 0 \end{aligned}$$

where L_t is the household's hours worked at t , w_t is the wage rate, I_t is transfer income, and within-period preferences are given by

$$u(C_t, L_t) = \log(C_t) - \left(\frac{B}{\gamma}\right) L_t^\gamma,$$

where $B > 0$ (so that the household gets disutility from working) and $\gamma > 1$. The household regards L as a choice variable, while w and I are seen as exogenous. Assume that the household knows w_t and I_t at time t , but not at time $t-1$ (in another word, w and I are stochastic). For convenience, note that there is no risky asset and that the interest rate is constant; you may go further and assume that $\beta(1+r) = 1$.

- (a) Warm-up question: show that $\gamma > 1$ insures that the marginal disutility of work is increasing in hours worked. Does this assumption make sense?

- (b) Derive the following three conditions for optimal behavior by the household:
- i. a "static" first-order condition relating today's labor supply to today's wage and today's consumption;
 - ii. an Euler Equation relating today's consumption to expectations involving tomorrow's consumption;
 - iii. an Euler Equation relating today's labor supply to today's wage and expectations involving tomorrow's labor supply and wage.
- (c) Suppose we receive an unexpected increase in transfer income I at time t . Suppose this shock is expected to be permanent. Assuming the wage is unaffected, how will current labor supply be affected? What if the shock is expected to be transitory? (No closed-form solutions are required – or even possible, for that matter. Just use the FOCs and intuition).
- (d) Now suppose that there is no uncertainty in the model. Suppose we observe that people's wages rises steadily over their lifetime, but that hours worked are roughly constant over their life cycle. What does this tell us about the magnitude of γ ?

2. Empirical Testing of the LCH

Suppose you are given the following data for a group of single men, aged 60 in year 1980 and 70 in year 1990:

- (1) wealth, income (including labor earnings and asset income) and consumer spending in 1980;
- (2) wealth, income (including asset income and pension income) and spending in 1990;
- (3) expectations, as of 1980, of the annual flow of pension income after retirement, where for convenience assume that there is no Social Security or other public pension available to everyone, but that employers may provide pensions to their retired workers.

Assume that pension coverage varies within your sample: some workers expect to receive no pension after retirement, others expect a small pension, others expect a larger pension, and some expect their pension income to be even higher than their earnings while working.

Assume that all of your sample men are working in 1980 but retired by 1990, and that all of your sample men are still alive as of 1990.

- (a) Explain how you would use your data to test the permanent income hypothesis, and how if the permanent income hypothesis were rejected one could distinguish myopia from liquidity constraints.
- (b) Suppose someone argues that people have different discount rates and that people who discount the future heavily will tend to select into jobs that have high earnings while working but low or no pension coverage after retirement. Assuming that this argument is true, explain how it could lead you to reject the permanent income hypothesis in part (a) even if the permanent income hypothesis is true. (Hint: think about omitted variables bias. If discount rates differ across individuals but we impose the same constant term across individuals in our consumption growth regression, then differences in discount rates will be an omitted variable and part of the error term. What happens in an OLS regression if a right-hand-side variable is correlated with the error term?)
- (c) Now suppose you have data on income and spending for this group of single men in 1970. Explain intuitively how you could use this data to test the argument laid out in part (b), assuming that discount rates don't change for individuals *over time*. (Hint: can we use Δc from 1970 to 1980 to control for differences in discount rates? Why or why not?)