1 Multiple Choice Questions

1. C
2. A

2 Calculations

Note: when you answer calculations and essays, please lay out all the necessary steps to illustrate your logical reasoning. The following answer key to question 3 shows you how.

3.

Step 1: the golden rule saving rate is the saving rate that maximizes the steady-state level of consumption per capita.

Step 2: find the per-capita production function. Divide both sides of the production function by \( L \), we get \( \frac{Y}{L} = \left( \frac{K}{L} \right)^{0.3} \). Let small letters to represent per-capita variables, it is \( y = k^{0.3} \). This is the per-capita production function.

Step 3: find what determines the steady-state consumption per capita.

In the Solow growth model with population growth but no technological progress, capital dynamics are given by \( \Delta k_t = sf(k_t) - (\delta + n)k_t \), where \( \delta \) is the capital depreciation rate and \( n \) is the population growth rate. At the steady state, \( \Delta k_t = 0 \), so that

\[
\begin{align*}
sf(k_{ss}) &= (\delta + n)k_{ss} \\
sk_{ss} &= 0.13k_{ss} \\
k_{ss} &= \left( \frac{s}{0.13} \right)^{1/\sigma}
\end{align*}
\]

The subscript “ss” represents the steady-state-level variables. Therefore, the steady-state consumption per capita, is

\[
c_{ss} = (1 - s)k_{ss}^{0.3} = (1 - s) \left( \frac{s}{0.13} \right)^{0.3} = \frac{(1 - s)s^{0.3}}{0.13^{0.3}} = 0.13^{-0.3} \left( s^{0.3} - s^{10} \right)
\]
Step 4: Maximize

$$Max_{s} \left[ 0.13^{-\frac{3}{7}} \left( s^{\frac{2}{7}} - s^{\frac{3}{7}} \right) \right]$$

Take the first order condition with respect to $s$, we get

$$0.13^{-\frac{3}{7}} \left( \frac{3}{7} s^{-\frac{4}{7}} - \frac{10}{7} s^{\frac{3}{7}} \right) = 0$$

$$3s^{-\frac{4}{7}} = 10s^{\frac{3}{7}}$$

$$s = 0.3$$

Therefore, the golden-rule saving rate is 0.3.

3 Essays
