**Question 1.**
Assuming that Raul leaves no bequest when he dies, his total consumption is equal to his total income, which is $4,000. His average consumption is his total consumption divided by the total number of years, that is $4,000/80 = $50. His consumption in the year of his 34th birthday is $50 because of consumption smoothing in the LCH model.

**Question 2.**

a) 

b) According to the LCH model with perfect smoothing, Raul smoothes consumption during all his life, which means that in the periods were he earns income he does not consume all of it. He saves a part that he spends when he retires.

**Question 3.** According to the permanent income model if Raul has an unexpected income of 240 he would smooth out this extra income during all his life. Remember that under this model,

\[ TC = Y_p + Y_t \]

Consumption in the remaining periods = \( (Y_p + Y_t)/T = (100*33 + 240)/53 = $66.80 \) per period

In the short run Raul average propensity to consume decreases (before the lucky night his APC was 50/100=.5) in the period that he gets the extra income, his APC = 66.80/340=.19.

In the long run, e.g. in period 60 his APC=.66.
In words: the effect of the lucky night is an increase in consumption over the remaining periods but is a relatively small increase in every single period. Raul smoothes consumption. The average propensity to consume decreases in the short run but stabilizes in the long run. In other words, people save most of their transitory income.

**Question 4.**

Keynes model: Myopic, consumption in a given period depends on that period’s income. An increase in \( Y_t \) leads to an increase in \( C_t \). This concords with short-time data but does not with long-time ones.

LCH model (Modigliani): Consumption does not depend on current income alone, as in the Keynesian model. It depends on lifetime income. Consumption is stable, people smooth consumption over their lives.

Friedman: Is also forward looking as LCH model but introduces uncertainty. People experience non-anticipated changes in their income (transitory income) but they save most of this unexpected income so we observe some increase in consumption after a increase in current period’s income but the increase is small because most of the extra income is saved to smooth consumption in the remaining periods.

**Question 5**

\[
\begin{align*}
\text{Max } & \quad U(C_1) + \beta U(C_2) \\
\text{s.t. } & \quad C_1 + \frac{C_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \\
& \quad C_1 \leq Y_1
\end{align*}
\]

a) The first constraint is the intertemporal budget constraint. It says that in this model people’s total consumption over their life is equal to their total income.

b) The second constraint is the borrowing constraint. It says that you can’t borrow money so that you cannot spend more money in period 1 than you earn.

c) \[
L = U(C_1) + \beta U(C_2) - \lambda_1[C_1 + \frac{C_2}{1 + r} - Y_1 + \frac{Y_2}{1 + r}] - \lambda_2[C_1 - Y_1]
\]

d) F.O.C

\[
C_1 : U'(C_1) - \lambda_1 - \lambda_2 = 0 \\
C_2 : \beta U'(C_2) - \frac{\lambda_1}{1 + r} = 0
\]

e) Euler’s equation:

\[
U'(C_1) = U'(C_2) + \lambda_2
\]

From the Euler equation is easy to see that \( U'(C_1) \) is greater in this case than \( U'(C_2) \). Given that \( U'' < 0 \) and that \( \lambda_2 \) is non-negative, this implies that \( C_1 \leq C_2 \), which
means also that the new $C_1$ is also smaller than in the case with no borrowing constraint.

f) Binding constraint:

Not binding constraint
Question 6.

a)

\[ \text{Max } \ln C_1 + 0.8 \ln C_2 \]
\[ \text{s.t. } C_1 + \frac{C_2}{1.1} = 200 \]

b) Lagragian:

\[ L = \ln C_1 + 0.8 \ln C_2 - \lambda \left( C_1 + \frac{C_2}{1.1} - 200 \right) \]

F.O.C.

\[ C_1 : \frac{1}{C_1} - \lambda = 0 \]
\[ C_2 : \frac{0.8}{C_2} - \frac{\lambda}{1.1} = 0 \]
\[ \lambda : C_1 + \frac{C_2}{1.1} = 200 \]

c) From the first two F.O.C the Euler’s equation is:

\[ \frac{1}{C_1} = 0.8 \cdot \frac{1.1}{C_2} \]

d) Solving for \( C_2 \) in the budget constraint and substituting its value in the Euler’s equation we obtain:

\[ C_1 = \frac{(200 - C_1) \cdot 1.1}{0.8 \cdot 1.1} \]
\[ C_1 = 111.11 \]
Substituting in the budget constraint we obtain:

\[ C_2 = 97.779 \]

e) The new set up of the problem is:
Max \( \ln C_1 + .8 \ln C_2 \)

s.t. \( C_1 + \frac{C_2}{1.1} = 200 \)
\( C_1 \leq 100 \)

The lagrangian is now:

\[ L = \ln C_1 + .8 \ln C_2 - \lambda_1 \left[ C_1 + \frac{C_2}{1.1} - 200 \right] - \lambda_2 [C_1 - 100] \]

The F.O.C are:

\( C_1 : \frac{1}{C_1} \lambda_1 - \lambda_2 = 0 \)

\( C_2 : \frac{.8}{C_2} \lambda_1 = 0 \)

\( \lambda_1 : C_1 + \frac{C_2}{1.1} = 200 \)
\( \lambda_2 : C_1 = 100 \) (because the constraint is binding so that \( \lambda_2 > 0 \))

Euler's equation (from the first two F.O.C.):

\( \frac{1}{C_1} = .8 \cdot \frac{1.1}{C_2} + \lambda_2 \) (note that is different from the Euler's equation in c), here \( C_1 \) is smaller

f) solving the sistem of equations above (you only need the two budget constraints) yields:

\( C_1 = 100 \) (smaller than before)
\( C_2 = 110 \) (bigger than before)