ECON 105C. Problem Set 5.

Question 1.

Consider the following definitions related to firm’s investment decisions over time:

\( \bar{K} \): Industry wide capital

\( \pi(\bar{K}) \): profits per unit of capital

\( K_t \) = Firm specific capital in period \( t \)

\( I_t \) = Firm specific investment in period \( t \)

\( P_t \) = Price of purchasing capital in period \( t \)

\( C(I_t) \) = Firms cost of adjusting its capital stock

a) Write up the firms profit maximization problem taking into account that

\( K_{t+1} = \bar{K}_t + I_t \)

b) Set up the Lagrangian (Hint: remember that there are \( t \) constraints)

c) What is the interpretation of the Lagrangian multiplier in this model?

d) Write up the Lagrangian expressing the multipliers in time \( t \) value.

(Hint: \( q_t = (1+r)^t \lambda_t \))

e) What is the interpretation of \( q_t \)?

f) Write the F.O.C. w.r.t \( I_t \) and \( K_t \). (Hint: for the F.O.C. w.r.t. \( K_t \) remember that \( K_t \) appears in two constraints)

g) Focus on the F.O.C. w.r.t. \( I_t \). Interpret this condition. (Hint: before interpreting multiply both sides by \( (1+r)^t \) and solve for \( q_t \))

h) Consider the case when \( C'(0)=0 \) (e.g. there is no investment in the period). What happens to capital? Does it increase, or decrease? Plot the condition derived in g) in the \( (\bar{K}_t, q_t/P_t) \) space (graph relating \( q_t/P_t \) to \( \bar{K}_t \))

i) Explain the dynamics of \( K_t \) in the graph. In order to do it consider what is the sign of \( C'(I_t) \) and \( I_t \) when \( q_t > 1 \) and when \( q_t < 1 \).

j) Now focus on the F.O.C w.r.t. \( K_t \). Multiply both sides of this equation by \( (1+r)^t \) and rearrange so that \( \pi(\bar{K}) \) appears as a function of \( q_t \) and \( \Delta q_t \). Interpret this condition.

k) Assume now that \( \Delta q_t = 0 \). Draw this condition in a graph relating \( q_t \) to \( \bar{K}_t \).

l) Explain the dynamics of \( q_t \) in the same graph.

m) Combine the graphs derived in h) and k). Explain the dynamics of \( K_t \) and \( q_t \) starting from each of the four quadrants determined by the graph.

n) Use the graph in m) to explain the effects (and the dynamics) of a permanent tax credit. What happens if the tax credit is temporary?

o) Use the graph in m) to explain the effects (and the dynamics) of a permanent upward shift in \( r \). What happens if the shift is temporary?