ECON 105 C
Problem Set 4

Question 1. Raul Gonzalez was born in 1900. He lives 80 years, leaving nothing to his descendants when he dies at the end of 1980. He earns nothing during his first 20 years of life. He gets a job as a postman when he is 20 and earns a constant income of $100 per year during the next forty years. He retires at the age of 60 and earns nothing from then on. According to the Life-Cycle-Hypothesis, what is his total consumption? What is his average consumption? What is his consumption in the year of his 34th birthday?

Question 2. The following graph refers to the situation described in question 1:

a) Fill in the question marks with a letter or a word that represent what they are and give numerical values for each of them. (HINT: The question marks inside squares refer to the area of the square).

b) Explain the graph

Question 3. Assume now the permanent income model proposed by Friedman and our consumer of previous questions. Suppose that Raul has an unexpected income of 240 in 1927 due to a lucky night in Las Vegas. Would this have any effect on his short-term consumption? And on his long-term consumption? Explain why.
**Question 4.** Consider Keynes consumption model, Modigliani’s LCH model and Friedman’s Permanent Income model:

a) What are the differences in the assumptions in each model?
b) What are the differences in the predictions of each model?

**Question 5.** Consider the following model:

\[
\begin{align*}
\text{Max} & \quad U(C_1) + \beta U(C_2) \\
\text{s.t.} & \quad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \\
& \quad C_1 \leq Y_1
\end{align*}
\]

a) Explain the two constraints.
b) Set up the Lagrangian.
c) What is the interpretation of the Lagrangian multiplier in this model?
d) Write the first order conditions with respect to \( C_1 \) and \( C_2 \)
e) Find the Euler’s equation
f) Suppose \( \beta(1+r) = 1 \). Is \( C_1 \) in this case bigger or smaller than in the model without borrowing constraint? (HINT: \( U''(.) < 0 \))
g) Show in two different graphs the solution to the model when the constraint is binding and when the constraint is not binding.

**Question 6.** Suppose that Raul Gonzalez is a student in period 1 and works as a salesman in period two, after which he dies. He has income of $100 from a fellowship in period 1 and 110 from his wage as salesman in period 2. He has no bequest at the beginning of the period and the interest rate is 10%. Raul’s preferences are defined by the following utility function:

\[
U(C_t) = \ln(C_t)
\]

a) Set up Raul’s utility maximization problem supposing that \( \beta = 0.8 \)
b) Write the first order conditions
c) Find the Euler’s equation
d) Solve for consumption. Is Raul a borrower or a saver?
e) Suppose that due to the financial markets crisis, the banks are not able to offer Raul any loan during period 1. Find the Euler’s equation in this case (HINT: the constraint is binding)
f) Is Raul’s consumption in period 1 bigger or smaller than before? What about consumption in period 2?
g) Show your results in a graph.