Investigating the development of knowledge using computational methods

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Johns Hopkins University
A current major issue facing education

Many children who are left behind with respect to early childhood learning experiences are put at a disadvantage for the rest of their education.
Designing intervention programs

One way to help these children is to design effective pre-school intervention programs that can stimulate their learning.

But how do we know what specific aspects of their learning experience to target?
Modern solutions to designing effective interventions

Interdisciplinary partnership between cognitive scientists and educators to

(1) identify the components of the learning experience that lead to typical learning vs. delayed learning

(2) determine how to enrich the learning experience to specifically benefit these components
Insights from some children to help other children

If we understand how learning proceeds for typically developing children who have an enriched learning environment, we can figure out how to trigger that learning process for other children.
Learning as ongoing mental computation
Learning as ongoing mental computation

Learning = given the available input,

You can touch them

Input

Do you see five of them?
Learning as ongoing mental computation

Learning = given the available input, information processing done by human minds

You can touch them

**Input**

Do you see five of them?

Mental computation
Learning as ongoing mental computation

Learning = given the available input, information processing done by human minds to build a system of knowledge

You can touch them

Input

Do you see five of them?

Mental computation

abstraction & generalization

VERB

see touch

5

Learning as ongoing mental computation

Learning = given the available input, information processing done by human minds to build a system of knowledge

You can touch them

Input

Do you see five of them?

Mental computation

abstraction & generalization

VERB

see touch

5
Learning as ongoing mental computation

Learning = given the available input, information processing done by human minds to build a system of knowledge whose output we observe.

You can touch them. Do you see five of them?

Input

Mental computation

Output

I see five!
Investigating learning

Many different questions about this mental computation
Investigating learning

Many different questions about this mental computation

What learning strategies comprise it?
Investigating learning

Many different questions about this mental computation

What learning strategies comprise it?

What learning biases do children need to succeed at it?

(Language: Pearl & Mis in rev., Pearl & Sprouse forthcoming, Pearl & Sprouse 2013, Pearl & Mis 2011, Pearl & Lidz 2009, Pearl 2008, Pearl & Weinberg 2007)
Many different questions about this mental computation

What learning strategies comprise it?

What learning biases do children need to succeed at it?

What knowledge representations can be easily learned using it?

(Language: Pearl et al. in prep., Pearl 2011, Pearl 2009)
Investigating learning

Many different questions about this mental computation

What learning strategies comprise it?

What learning biases do children need to succeed at it?

What knowledge representations can be easily learned using it?

When do children learn different aspects of knowledge using it, and what data are available to them to do so?

(Language: Pearl & Braunwald in prep., Caponigro, Pearl et al. 2012, Caponigro, Pearl et al. 2011)
Methods of empirical investigation
Methods of empirical investigation

Theoretical methods:

What knowledge is (and what children have to learn)

LOOK at the Kitties

VERB see look at

5
Methods of empirical investigation

Experimental methods:
When knowledge is acquired, what the input looks like, & plausible capabilities underlying how learning works

\[ p(k\text{itty} \mid \text{ ) } \propto p( \mid \text{kitty}) p(\text{kitty}) \]
Methods of empirical investigation

Computational methods:
Strategies for how children acquire knowledge, sophisticated quantitative analysis of children’s input & output

could see
would look at

see look at

would see
could look at
Today’s Plan

Using **computational methods** to look at one question about children’s learning and discussing connections to education
Today’s Plan

Using computational methods to look at one question about children’s learning and discussing connections to education

When do children learn different aspects of knowledge?
Today’s Plan

Using computational methods to look at one question about children’s learning and discussing connections to education

When do children learn different aspects of knowledge?

Assessing individual abilities:
When does the child attain a certain level of knowledge?
Connections: literacy development
Today’s Plan

Using **computational methods** to look at one question about children’s learning and discussing connections to education

When do children learn different aspects of knowledge?

**Assessing individual abilities:**
When does the child attain a certain level of knowledge?
*Connections: literacy development*

**Identifying factors that underlie the observable output:**
What signifies atypical development, which may require intervention?
*Connections: math readiness*
Today’s Plan

Using computational methods to look at one question about children’s learning and discussing connections to education

When do children learn different aspects of knowledge?

Assessing individual abilities:
When does the child attain a certain level of knowledge?
Connexions: literacy development

Identifying factors that underlie the observable output:
What signifies atypical development, which may require intervention?
Connexions: math readiness
Language & Literacy

Language learning: Developing linguistic knowledge
  → phonemes, words, grammatical categories, phrases, ...

One important part of this process:
Generalization = a more abstract unit encompasses a group of individual items.
Language & Literacy

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One important part of this process:
Generalization = a more abstract unit encompasses a group of individual items.

I love my ____.
Language & Literacy

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One important part of this process:
Generalization = a more abstract unit encompasses a group of individual items.

I love my ___.

Noun
Language & Literacy

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One important part of this process:
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I love my ___.

Noun

kitty    doggy    penguin
Language & Literacy

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Generalization = a more abstract unit encompasses a group of individual items.

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Noun

kitty
doggy
penguin

kʰlri
klri
klthᵊ
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One important part of this process:
Generalization = a more abstract unit encompasses a group of individual items.

I love my ___.

```
Noun
  _____________
  kitty       doggy   penguin
  kʰɪɾɪj     klri     klthi
  k^n

I love my kitty.
```
Language & Literacy

Spoken language knowledge: a prerequisite for literacy

Part of literacy: A mapping process

unconscious knowledge about spoken language

/kIrI/

Explicit symbolic representation of written language

kitty

“kitty” is a noun

Similar words: doggy, penguin
Language & Literacy

One part of literacy:
vocabulary development & reading comprehension

Current project with some ties to this process:
Investigating the development of grammatical category knowledge

\[
\text{kitty} = \text{noun}
\]
Grammatical categories

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know this: kitty = noun

You also know that all these are fine:
“**My kitty** is cute.”
“I love **that kitty**.”
“The little **tabby kitty** is particularly adorable.”
“I gave some fish **to the kitty**.”
Grammatical categories

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how a word behaves, you know what other words behave like it, which allows you to transfer knowledge across words.
Grammatical categories

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**Those two penguins** are eating fish.
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If you know how a word behaves, you know what other words behave like it, which allows you to transfer knowledge across words.

Those two penguins are eating fish.
Grammatical categories

How does this relate exactly?
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If you know how a word behaves, you know what other words behave like it, which allows you to transfer knowledge across words.

Those two penguins are eating fish.

Therefore:
Those two kittens are eating fish.
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how words behave, this can help you learn new words, based on how the new word combines with other words.

[Syntactic bootstrapping, in addition to semantic cues]
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
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If you know how words behave, this can help you learn new words, based on how the new word combines with other words.

[Syntactic bootstrapping, in addition to semantic cues]

That feline is playing with the dog again.
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how words behave, this can help you learn new words, based on how the new word combines with other words.
[Syntactic bootstrapping, in addition to semantic cues]

That *feline* is playing with the dog again.

That ____ is

↓

Noun

↓

*entity*
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how words behave, this can help you learn new words, based on how the new word combines with other words.

[Syntactic bootstrapping, in addition to semantic cues]

That feline is playing with the dog again.

That ____ is related to playing, dog

Noun

entity
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how words behave, this can help you learn new words, based on how the new word combines with other words.

[Syntactic bootstrapping, in addition to semantic cues]

That *feline* is playing with the dog again.

That *** is related to *playing, dog*

Noun

Guess:
Other animal that can be a pet?
Grammatical categories for vocabulary development & reading comprehension

How does this relate exactly?
Grammatical category = the way a word behaves in combination with other words

If you know how words behave, this can help you learn new words, based on how the new word combines with other words.

*[Syntactic bootstrapping, in addition to semantic cues]*

That *feline* is playing with the dog again.

That ____ is related to *playing, dog*

Noun

Guess:
Other animal that can be a pet?
Assessing knowledge of grammatical categories

Grammatical category = a group of words that behaves the same way

Very specific

*Share many syntactic & semantic properties*

Very general

*Share a few core syntactic & semantic properties*
Assessing knowledge of grammatical categories

Grammatical category = a group of words that behaves the same way

Very specific

*Share many syntactic & semantic properties*

Verbs of communication

*whisper, shout, say, mumble, tell, ...*

Very general

*Share a few core syntactic & semantic properties*

If you know “derb” is part of this category, you know this is probably okay:

Jack *derbed* the name to Lily.
Assessing knowledge of grammatical categories

Grammatical category = a group of words that behaves the same way

**Very specific**

- Share many syntactic & semantic properties

**Very general**

- Share a few core syntactic & semantic properties

**Verbs**

- whisper, hug, think, give, breathe, ...

If you know “derb” is part of this category, you know these are probably okay:

- ...could *derb*...
- ...might *derb*...
- ...will *derb*...
Assessing knowledge of grammatical categories

When do children first develop knowledge of the abstract category of “verb”?

- Very specific: Share many syntactic & semantic properties
- Very general: Share a few core syntactic & semantic properties

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How can we tell?

One indicator:

Knowledge about how one word combines with other words is transferred within the category.

...could think...

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How can we tell?

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...could think...
Assessing knowledge of grammatical categories

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Pearl & Braunwald in prep.
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How can we tell?

One indicator:
Knowledge about how one word combines with other words is transferred within the category.

...could think...

...could Verb...

Verb

whisper

hug

think

give

breathe

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How can we tell?

One indicator:

Knowledge about how one word combines with other words is transferred within the category.

...could give...

...could Verb...

Verb

whisper

hug

think

give

breathe

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How can we tell?

One indicator:

This causes the child to combine words of the same category with similar words, so that there’s overlap in usage within a category.

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How can we tell?

One indicator:

This causes the child to combine words of the same category with similar words, so that there’s overlap in usage within a category.

This overlap (sometimes called “productivity”) is something we can quantitatively assess.

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

When does this kind of grammatical category knowledge typically develop?

Some evidence exists that it may already be in place around the age of two (Valian 1986), but some evidence also exists that it may appear significantly later than that (Tomasello 1992, Pine & Lieven 1997)

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

Focus: Development of the category Verb

Data:
Longitudinal data from a typically developing child (Laura) in naturalistic contexts

[Braunwald-Max Planck corpus from CHILDES database + hand-written diary data kept by Braunwald, 20- to 24-month subsection]

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

For example, should we expect every verb to combine with every auxiliary?

- could give...
- would give...
- might give...
- could hug...
- might hug...
- will hug...
- could think...
- would think...
- will think...

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

For example, should we expect every verb to combine with every auxiliary?

Probably not – remember, these are naturalistic outputs. We don’t say everything we know when we speak – we say things to communicate our intended meaning at the time.

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

In fact, it turns out naturalistic linguistic output shows power-law behavior (a Zipfian distribution)...

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<th>verb</th>
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Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

In fact, it turns out naturalistic linguistic output shows power-law behavior (a Zipfian distribution), where a few things are said very frequently...

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*Pearl & Braunwald in prep.*
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

In fact, it turns out naturalistic linguistic output shows power-law behavior (a Zipfian distribution), where a few things are said very frequently and most things are said very infrequently.

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Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

In fact, it turns out naturalistic linguistic output shows power-law behavior (a Zipfian distribution), where a few things are said very frequently and most things are said very infrequently.

\[
\log(\text{frequency}) \quad \quad \log(\text{rank})
\]

This shows up as a linear relationship in logarithmic space.

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

Laura’s verb production data during 20-24 months show this power-law distribution.

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

One implication: We can’t expect much overlap in combinatorial usage for verbs that only are used a few times (and certainly not for those that are only used once).

Laura’s verb usage 20-24 months

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

We need to somehow factor in that Laura may have known the combinatorial usage transferred to other verbs, but just didn’t choose to say those other verbs with other words.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

where this is equivalent to this

\[ O(r, N, D, S) = 1 + (D - 1)(1 - p_r)^S - \sum_{i=1}^{D} [(d_i p_r + 1 - p_r)^S] \]

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

\{hug, give, take, read, want, think, ...\} = ???

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

The probability that a particular verb with rank \( r \) will have overlap...

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Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[
O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S)
\]

...given the total number of verb vocabulary items N combining with the lexical items from the class of interest...

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

verb (used with auxiliary) total = 59
like
go...
cook...
make...

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

...and the number of combinatorial vocabulary items from the lexical class of interest \( D \)...

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

auxiliary total = 25
\[
\begin{align*}
& \text{don’t} \\
& \text{can’t} \\
& \text{...} \\
& \text{do} \\
& \text{...} \\
& \text{are} \\
& \text{...}
\end{align*}
\]

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

How much overlap do we expect to see if this child knows the category Verb?

A way to do this: quantitative methods from Yang 2010, 2011

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

...and the number of verb usages observed with the combinatorial vocabulary items of interest \( S \).

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

auxiliary + verb usage total = 220

don’t close the door
can’t reach it
...
do not come
...
are you cooking that?
...

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

Once we know this expected overlap, we can look at the overlap we actually observe in the empirical data and see if they match.

Expected overlap

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

Estimate of empirical overlap:

Pearl & Braunwald in prep.
Assessing knowledge of grammatical categories

Once we know this **expected overlap**, we can look at the **overlap** we actually observe in the empirical data and see if they match.

**Expected overlap**

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

**Estimate of empirical overlap**:

If verb is used with more than one lexical item within the lexical class (ex: auxiliaries), overlap for that verb = 1.

...could give...
...would give...
...could give...
...could give...

Overlap for *give* = 1

*Pearl & Braunwald in prep.*
Assessing knowledge of grammatical categories

Once we know this expected overlap, we can look at the overlap we actually observe in the empirical data and see if they match.

**Expected overlap**

\[ O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S) \]

The proportion of observed verbs that should have some overlap: 0.0 - 1.0

**Estimate of empirical overlap:**
If verb is used with more than one lexical item within the lexical class (ex: auxiliaries), overlap for that verb = 1. Otherwise, overlap = 0.

...could give...
...could give...
...could give...
...could give...
overlap for **give** = 0

*Pearl & Braunwald in prep.*
Assessing knowledge of grammatical categories

Once we know this expected overlap, we can look at the overlap we actually observe in the empirical data and see if they match.

**Expected overlap**

\[
O(D, N, S) = \frac{1}{N} \sum_{r=1}^{N} O(r, N, D, S)
\]

**Estimate of empirical overlap:**

If verb is used with more than one lexical item within the lexical class (ex: auxiliaries), overlap for that verb = 1. Otherwise, overlap = 0.

If expected and empirical overlap match, this child’s output is compatible with knowing the grammatical category Verb.

*Pearl & Braunwald in prep.*
Grammatical category of Verb at 20-24 months?

Investigate a number of different lexical classes that can combine with verbs – each one is an individual assessment of productivity, which can be viewed collectively to assess whether the category Verb is known.

<table>
<thead>
<tr>
<th></th>
<th>Expected overlap</th>
<th>Empirical overlap</th>
<th>Exp - Emp</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>0.70</td>
<td>0.63</td>
<td>0.45</td>
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<td></td>
<td>0.75</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0.57</td>
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Pearl & Braunwald in prep.
Grammatical category of Verb at 20-24 months?

Comparing expected to empirical overlap for 7 classes of lexical items, it doesn’t seem like verbs combine freely with words from different lexical classes.

Knowledge is not transferring across different verbs.

<table>
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<th></th>
<th>subj</th>
<th>obj</th>
<th>non-obj</th>
<th>neg</th>
<th>aux</th>
<th>wh</th>
<th>emb cla</th>
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Pearl & Braunwald in prep.
Implication:
Despite the number of verbs the child is producing (260 verb vocabulary items), development of grammatical category knowledge (Verb) does not occur until after this age.
Assessing the development of knowledge about language

One example of using computational methods to assess the development of language knowledge, given naturalistic data.

When does this knowledge typically develop?

Very specific

*Share many syntactic & semantic properties*

Very general

*Share a few core syntactic & semantic properties*

This takes some time to develop, even when children already have a significant vocabulary.

*Pearl & Braunwald in prep.*
Assessing the development of knowledge about language

When does more specific knowledge about abstract categories in language typically develop?

- **Very specific**
  - Share many syntactic & semantic properties

- **Verbs of communication**
  - whisper, shout, say, mumble, tell, ...

- **Very general**
  - Share a few core syntactic & semantic properties
Assessing the development of knowledge about language

More generally: Determine the typical age of development for different kinds of abstract knowledge that can aid vocabulary acquisition

That **feline** is playing with the dog again.

That ____ is playing related to **dog**

Animate Noun

**animate entity**

Guess:

Other animal that can be a pet?
Assessing the development of knowledge about language

Assess whether children are typical or atypical in their categorical knowledge, given their naturalistic output, such as writing samples. (Can avoid test anxiety.)

Does this child group together verbs of communication? How often are these used with animate subjects, language objects (ex: “word”, “name”), and indirect objects?

whisper, shout, say, mumble, tell, ...
Helping children learn

Delayed learners: Help design individualized intervention methods based on a delayed individual’s specific vocabulary category deficits.

Typical & delayed learners: Help design educational methods for introducing new vocabulary & aiding in reading comprehension.

Hermione whispered the name to Harry.
Harry shouted the name back to her.
Dumbledore announced a warning for them both.

Linguistic features associated with verbs of communication: animate subject, language object, indirect object.
Today’s Plan

Using computational methods to look at one question about children’s learning and discussing connections to education.

When do children learn different aspects of knowledge?

Assessing individual abilities:
When does the child attain a certain level of knowledge?
Connections: literacy development

Identifying factors that underlie the observable output:
What signifies atypical development, which may require intervention?
Connections: math readiness
Language & Math

Spoken language knowledge: a prerequisite for math

Language knowledge is required to create new concepts, such as exact numbers.

1, 2, 3, 4, 5, 6....

*counting list*
Numerical cognition

Knowledge of cardinal numbers is a fundamental building block for more sophisticated math concepts

• precursor for successful acquisition of mathematical knowledge: a signal of “math readiness”
• knowledge at kindergarten can predict later math achievement through at least the 5th grade (Ginsburg & Russell 1981, Duncan et al. 2007)

5 + 6 = 11
6 * 5 = 30
...
There’s wide variation in cardinal number knowledge in kindergarteners.

It’s unclear what factors contribute to this observable variation (nature vs. nurture) and how much each factor contributes.
Development of cardinal number knowledge

It takes a while for children to realize what the counting list maps to.

Sophisticated numerical knowledge = **Cardinal Principle**
The last number reached when counting the items in a set represents the entire set.

1, 2, 3, 4, 5, 6....there are 6!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Pre-number knowers haven’t mapped any of the counting list.

1….there are 4!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

One-knowers have only mapped 1.

1....there’s 1!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

One-knowers have only mapped 1.

1, 2....there’s 5!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Two-knowers have only mapped 1 and 2.

1, 2….there’s 2!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Two-knowers have only mapped 1 and 2.

1, 2, 3….there’s 4!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Three-knowers have only mapped 1, 2, and 3.

1, 2, 3....there’s 3!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Three-knowers have only mapped 1, 2, and 3.

1, 2, 3, 4....there's 6!
Development of cardinal number knowledge

Children progress through different levels of knowledge on their way to discovering the Cardinal Principle (Wynn 1992, Sarnecka & Carey 2008, Sarnecka & Lee 2009).

Cardinal Principle (CP) knowers realize the mapping between numerosity and the counting list.

1, 2, 3, 4, 5….there are 5!

1, 2, 3, 4, 5, 6….
Development of cardinal number knowledge

We can gauge which stage children are at using experimental methods.

Give-N Task (Wynn 1992):
“The way we play this game is: I will tell you what to put on the plate, and you put it there and sli-i-i-de it over to Kitty, like this [demonstrating]. OK, can you give one fish to Kitty?”
Development of cardinal number knowledge

We can then get a sense of when children typically pass through the different number-knower stages.

![Box plot showing age in months for different stages of number-knowledge: Pre-number knowers, Subset number knowers, and Cardinal Principle knowers.](Slusser 2010: Monolingual, High SES children)
Development of cardinal number knowledge

Training study in progress (Barbara Sarnecka): Collecting longitudinal data on children from different language (monolingual vs. bilingual) and SES (high vs. low) backgrounds to determine typical “time to cardinality”, as measured by the amount of number word input they receive.

For each child, we can get something like this:

![Graph showing the development of cardinal number knowledge]

Amount of number word input

Pearl & Sarnecka
Understanding the development of cardinal number knowledge

We can then investigate the factors that underlie this progression.

- One known external factor: **input quantity** (Levine et al. 2010, Gunderson & Levine 2011)
- Other potential internal factors: **creativity, conceptual capacity, conservativity**

For each child, we can get something like this:

```
CP-knower
three-knower
two-knower
one-knower
pre-knower
```

Amount of number word input

---

*Pearl & Sarnecka*
Understanding the development of cardinal number knowledge

We can create a cognitive process model (Yang 2002, Pearl 2011) that can reproduce this trajectory, and that depends on all these factors.

**Input quantity (I):** how much data the child has encountered (observed, external variable)

![Graph showing the development of cardinal number knowledge with different stages: pre-knower, one-knower, two-knower, three-knower, CP-knower.](image)
Understanding the development of cardinal number knowledge

We can create a cognitive process model (Yang 2002, Pearl 2011) that can reproduce this trajectory, and that depends on all these factors.

Creativity ($\alpha$): how likely the child is to consider a new hypothesis about the meaning of number words

![Diagram showing the development of number knowledge, with stages labeled as CP-knower, three-knower, two-knower, one-knower, and pre-knower. The x-axis represents the amount of number word input.](image-url)
Understanding the development of cardinal number knowledge

We can create a cognitive process model (Yang 2002, Pearl 2011) that can reproduce this trajectory, and that depends on all these factors.

Conceptual capacity (β): how difficult it is for the child to imagine rules like the Cardinal Principle

Amount of number word input

Pearl & Sarnecka
Understanding the development of cardinal number knowledge

We can create a cognitive process model (Yang 2002, Pearl 2011) that can reproduce this trajectory, and that depends on all these factors.

Conservativity ($\gamma$): how much a child’s belief in what number words mean shifts on the basis of a single data point

![Diagram showing the development of cardinal number knowledge with different stages: pre-knower, one-knower, two-knower, three-knower, CP-knower, and their corresponding levels of conservativity. The diagram illustrates the amount of number word input in relation to the stages of development.](image-url)
Understanding the development of cardinal number knowledge

Once we have these values for a large sample of children, we can determine what the range for typical development is.
Understanding the development of cardinal number knowledge

We can identify the values outside the norm associated with delayed learners, and for any given child, which factors seem to be causing the delay.

Pearl & Sarnecka
Helping children learn

This can then help identify children who are at risk for later math difficulty, and help design targeted interventions for them.

Pearl & Sarnecka
Recap

Using computational methods to identify when children learn different types of knowledge

Assessing individual abilities:
When is knowledge attained?

One finding: Basic grammatical category information develops later than two in a typically developing child.

Extensions: When do other linguistic categories develop that are useful for vocabulary acquisition and reading comprehension?

Connections: literacy development
Recap

Using computational methods to identify when children learn different types of knowledge

Identifying factors that underlie the observable output:
What causes typical and atypical development?

One application: Identifying internal and external factors underlying the development of cardinal number knowledge, which predicts later math achievement.

Connections: math education
Big picture: Understanding learning using computational methods

Computational methods are part of an arsenal of empirical investigation methods that we can use to help us understand the process of learning. This includes the learning strategies children use, the learning biases children have, the knowledge representations that are easily learnable, and the time course & causes of knowledge development.
Cognitive Science + Education: If we understand how learning proceeds in typically developing children who have enriched learning environments, we can design targeted, effective early childhood interventions that can recreate this learning experience for other children who are delayed in their knowledge development.
Thank you!

Sue Braunwald
Joseph Nunn

Barbara Sarnecka
Tricia Ignacio
Extra material
Grammatical category of Verb at 20-24 months?

Investigate a number of different lexical classes that can combine with verbs – each one is an individual assessment of productivity, which can be viewed collectively to assess whether the category Verb is known.

<table>
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Pearl & Braunwald in prep.
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as subjects?

I, you, he, Laura, Daddy, ...

Rule ≈ Subject + Verb

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It doesn’t seem like it.

Pearl & Braunwald in prep.
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as direct objects? *me, you, him, Laura, Daddy, ...*

Rule ≈ Verb + Object

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*Pearl & Braunwald in prep.*
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as indirect objects, locatives, preposition phrases, or predicate adjectives?

Rule \( \approx \) Verb + Non-Object

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Pearl & Braunwald in prep.
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as negations?

`not, n’t, no, ...`

Rule ≈ Negation + Verb

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`It doesn’t seem like it.`

*Pearl & Braunwald in prep.*
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as auxiliaries? *could, should, do, can, ....*

Rule ≈ Auxiliary + Verb

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Maybe.
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical items used as *wh*-words in questions? *who, where, why, ...*

Rule $\approx$ *wh*-word .... + Verb

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It doesn’t seem like it.

*Pearl & Braunwald in prep.*
Grammatical category of Verb at 20-24 months?

Can verbs combine freely with lexical item sequences used as embedded clauses?

...Eugene doing it, ...Susie coming, ...

Rule $\approx$ Verb + Embedded Clause

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It doesn’t seem like it.

Pearl & Braunwald in prep.
Grammatical category of Verb at 20-24 months?

In general?

Comparing expected to empirical overlap for 7 classes of lexical items, it doesn’t seem like it.

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