Spatial Hedonics and the Willingness to Pay for Residential Amenities

Kenneth A. Small and Seiji S.C. Steimetz

University of California at Irvine (ksmall@uci.edu)
and
California State University at Long Beach (ssteimet@csulb.edu)

October 6, 2009

Abstract

Housing rents may be influenced by characteristics of nearby properties, an effect captured by spatial autoregression in a hedonic rent equation. We investigate the implications of spatial autoregression for measuring the marginal welfare effects due to a change in a residential amenity such as air quality. We show that if spatial price interdependence arises from technological spillovers, such that utility depends directly on neighboring property values, then the welfare change is given by the reduced form of the autoregressive model, effectively applying a "spatial multiplier" to the relevant implicit prices. If instead spatial interdependence arises from merely pecuniary spillovers, as is commonly supposed in motivating spatial autoregression, then no spatial multiplier on implicit prices is called for in computing welfare; but it is then especially important to use the autoregressive model to measure those implicit prices.

JEL codes: Q51, R21

Keywords: spatial hedonics, spatial autoregression, spatial lag, willingness to pay, welfare

Acknowledgment: We are grateful to Luc Anselin, Jan Brueckner, Michael Greenstone, Wade Martin, Francisco Martinez, Luis Rizzi, Jeffrey Zabel, and seminar participants at the Stanford Environmental Economics Workshop, the Oslo Workshop on Valuation Methods in Transport Planning (Institute of Transport Economics, Oslo), and Universitat Autònoma de Barcelona for comments on earlier drafts.

I. Introduction

It is well known that, because housing prices depend on residential amenities and local public goods, hedonic price techniques may provide a way to measure households' willingness to pay for them.[1] It is more recently established that housing prices can be influenced by the characteristics of surrounding houses; various forms of such influence can be distinguished econometrically through the use of "spatial lag" specifications in which price of one property is influenced by one or more variables measured for other properties, including price itself. A common structural specification, in which only the price variable is spatially lagged, is sometimes called "the spatial lag model" or "spatial autoregression."[2] It is especially appropriate if neighboring prices are thought to influence the price of a given house through some causal mechanism not otherwise explicitly incorporated in the model specification. A frequently cited example is the use of "comps," *i.e.* the prices of similar ("comparable") nearby houses, in devising offering prices in real-estate transactions.[3]

Spatial lags complicate benefits measurement because an amenity improvement at a given property now affects the values of neighboring properties, which in turn exert an additional effect on the first property. In practice this has given rise to a "spatial-multiplier" approach to measuring benefits, where the direct effect of an improvement is magnified by the spillover effects among neighboring properties. (It could be diminished rather than magnified, but that is rare.) This multiplier is determined by the coefficient of a weighted transformation of spatially lagged prices.

We examine the economic interpretations of these direct and spillover effects. We find that the spatial-multiplier approach to benefit measurement is valid only under fairly strong

---

[1] Rosen (1974). The basic idea is that in competitive equilibrium households equate their marginal willingness to pay to the marginal price of the amenity that they face in the market. Pioneering empirical studies focused on air pollution, for example Ridker and Henning (1967) and Freeman (1974). See Freeman (1993), Smith and Huang (1995), and Chay and Greenstone (2005) for summaries of more recent evidence.

[2] See Case (1991), Can (1992), and Anselin (2002) for discussions of various spatial econometric models — especially those with spatial lags in the dependent variable, as here, and those with spatial lags only in the error terms — and how to distinguish them empirically. We mostly follow the terminology of Anselin (2003), and for simplicity treat "spatial lag" and "spatial autoregression" as synonymous.

[3] See, for example, Anas and Eum (1984), Pace and Gilley (1998), and Kim et al. (2003).

assumptions about the economic nature of spatial dependence. In particular, we draw a distinction between two types of spillovers that can lead to an autoregressive process: *pecuniary* spillovers, where spatial dependence is purely monetary, and *technological* spillovers, where the enjoyment of living at a given location is actually influenced by the values of neighboring locations. We show that only in the second case does the spatial-multiplier approach give a correct benefit measure. If instead spatial dependence is pecuniary, then welfare is correctly measured by the direct effect alone. In this latter case, incorporating the spatial multiplier for *prices* is still crucial for welfare measurement as otherwise the direct and indirect effects would be empirically confounded.

To illustrate, assume that property values are spatially interdependent through some autoregressive process and consider a neighborhood that enjoys a uniform improvement in air quality. The improvement raises property values through two mechanisms. First, cleaner air at a given location increases the utility from living there: this is the "direct effect" of the improvement, measured by the coefficient on air quality in a properly specified hedonic model. Second, that location's value is further raised by the mutually reinforcing price increases described above: this is the "indirect effect" caused by spillovers, related to the coefficient of the spatially lagged dependent variable. Together, the direct and indirect effects on prices are unambiguously captured by multiplying the air-quality coefficient by the spatial multiplier.

But price changes need not equal welfare changes. For welfare analysis, the salient question is: how exactly are residents benefiting from the spillover effect? If the answer is simply through higher rents (or sales values), then these spillovers are transfers from renters to landlords (or from future homeowners to current homeowners). In other words, the spillover effect represents a *pecuniary* externality which is welfare neutral; only the direct effect, measured by the coefficient on air quality, is relevant for benefit measurement. Now suppose instead that residents derive pleasure from their neighbors' higher property values, perhaps because those neighbors use some of their capital gains to beautify their houses. In that case, the spillover effect represents a *technological* externality because it further increases each resident's utility. The spatial multiplier is then part of a full welfare analysis because both the direct and spillover effects increase utility in a manner that doesn't cancel across economic actors.

In practice it may be difficult to know whether a spatial spillover is pecuniary or technological. Indeed, the example just given suggests that the relationship between my neighbors' house prices and my utility is a reduced-form relationship describing the results of some more elaborate model of the neighbors' behavior which, ideally, should be spelled out to know what kind of externality is involved. For example, if the posited mechanism is "comps," one needs a theory of real estate transactions under limited information that explains how such a mechanism can persist in equilibrium. If the posited mechanism is prestige, one needs a theory of status effects that predicts to what extent my status is raised at someone else's expense. For other mechanisms, one may need to consider altruism, maintenance externalities (as in our verbal example), misperceptions, or other phenomena.[4]

We do not attempt here a comprehensive treatment of the many possible theories that could justify an autoregressive empirical specification. Instead, we examine two reduced-form models to which such theories might lead. In one, a household's utility depends only on its own residential characteristics, even though in equilibrium its housing price depends on neighboring prices. In the other, the household's utility itself (perhaps a partially indirect utility incorporating certain neighborhood market mechanisms) can be written as depending directly on neighboring prices. We show that in these two cases, the benefits of an amenity change can be estimated using the coefficient of that amenity in a spatial autoregression, multiplied by a spatial multiplier in the second case but not in the first. We also show through some published examples that benefits estimates can differ greatly depending on which of these reduced-form models is assumed to be valid.

We proceed by first reviewing the conventional hedonic model, clarifying the cancellation that makes its welfare analysis especially simple (Section II). We then consider pecuniary and technological externalities as alternative theoretical sources for spatial lags, showing that the same cancellation occurs in the former but not the latter (Section III). We review empirical studies that have used spatial multipliers for welfare analysis (Section IV), and draw conclusions for applied research (Section V).

[4] Wendner and Goulder (2008) and Bergstrom (2006) discuss how welfare evaluation is affected by "status effects" and altruism, respectively. Portney (1992) and Viscusi (2000, pp. 867-8) discuss perception, uncovering a conundrum for welfare analysis: is it socially valuable to reduce a perceived health threat that is actually illusory?

II. Conventional Hedonic Welfare Analysis

We begin by reviewing hedonic welfare analysis when there is no spatial interdependence among housing prices, following Freeman (1974) and Small (1975). While this material is not new, we point out certain cancellations in aggregation that help clarify just where the analysis is changed by the presence of spatial autoregression We use air quality as our example of the residential amenity to be evaluated. For expositional simplicity, we refer to all households as renters of properties owned by landlords, understanding that our analysis applies equally to owner occupants by considering them as playing dual roles of renter and landlord.

Let $q_j(\theta)$ measure the air quality associated with location j, which is valued positively by all households and which depends on some scalar index of overall pollution-abatement measures, θ. We assume $dq_j/d\theta>0$. The rental price of the property at j is $r_j=r\,(x_j,q_j)$ where x_j is a vector of characteristics of that property and $r(\cdot)$ is a hedonic rent function. Landlords in the area receive rental revenues $R\equiv\sum_j r(x_j,q_j)$.

Households derive utility from a numeraire good z, housing characteristics, and air quality. Thus when deciding where to live, household i with income y_i maximizes utility $u_i(z,x,q)$ subject to budget constraint $z+r(x,q)=y_i$. Households need not have identical utility functions. Assuming that a continuum of values of x and q are available in the market, we can represent this decision process as:

$$\underset{z,x,q,\lambda}{Max}\ \Lambda_i \equiv u_i(z,x,q)+\lambda_i\cdot[y_i-z-r(x,q)]. \tag{1}$$

Assuming an interior solution, a first-order condition yields:

$$\frac{u_{qi}}{\lambda_i}=r_{qi} \tag{2}$$

where $u_{qi}\equiv\partial u_i/\partial q$ is the marginal utility of air quality for household i and $r_{qi}\equiv\partial r(x_i,q)/\partial q$ is the implicit price of air quality at the location chosen by that household; both derivatives are evaluated at the solution values which we denote (z_i,x_i,q_i,λ_i).[5] The properties of Lagrangian multipliers imply that λ_i is the marginal utility of income. Thus, equation (2) shows that residents will choose a location where the marginal amenity value of air quality just equals its implicit marginal price. Inserting the full solution to (1) into the utility function yields indirect utility V_i, which is a function of y_i, θ, and all the parameters of the rent function $r(\cdot)$.

Using (2) and recalling that q_i is a function of θ, the marginal willingness to pay by household i for abatement is:

$$M_{i\theta} \equiv -\left.\frac{dy_i}{d\theta}\right|_{dV_i=0} = \frac{dV_i/d\theta}{\partial V_i/\partial y_i} = \frac{u_{qi}}{\lambda_i}\frac{dq_i}{d\theta} - \frac{dr_i}{d\theta} \qquad (3)$$

where the total derivatives of V_i and r_i include general-equilibrium effects arising in the land market.[6] Aggregating over all households and applying (2), aggregate household willingness to pay for abatement is:

$$M_\theta \equiv \sum_i M_{i\theta} = \sum_i r_{qi}\frac{dq_i}{d\theta} - \sum_i \frac{dr_i}{d\theta} \quad . \qquad (4)$$

The first term on the right-hand side of equation (4) shows the increased amenity value of the improvement, which we may call a technological effect because it results from the direct effect of air quality on utility. The second term shows the rent increase induced by the improvement, a pecuniary effect because it shows up as a monetary benefit to landlords.

Aggregate rents increase with changes in θ by the marginal amount:

$$R_\theta \equiv \sum_i \frac{dr_i}{d\theta}. \qquad (5)$$

---

[5] For notational simplicity, we henceforth use the same index i for the household and its location in equilibrium. Hence $r_i \equiv r(x_i,q_i)$ is the rent of the location chosen by household i.

[6] But these derivatives do not include relocation effects. Since household i is optimizing, it is sufficient to consider what happens at the fixed location i.

Therefore the rent changes, $\Sigma_i dr_i/d\theta$, cancel when aggregating the benefits to all residents and landlords:[7]

$$B_\theta \equiv M_\theta + R_\theta = \sum_i r_{qi} \frac{dq_i}{d\theta} . \tag{6}$$

Equation (6) shows that the social benefit of the improvement depends only on the technological effects. Part or all of these benefits might be captured by landlords, but the total level of the benefit is unaffected by such pecuniary transfers.[8] A special case of (6) is when abatement is uniform across all locations and measured in the same units as air quality. We then have $dq_i/d\theta=1$ for every i, and

$$B_\theta = \sum_i r_{qi} . \tag{7}$$

An empirical hedonic specification for estimating these benefits can be written

$$\mathbf{r} = X\beta + \varepsilon \tag{8}$$

where $\mathbf{r}$ is an nx1 vector of equilibrium property prices (distinguished from the scalar function $r(\cdot)$ by bold font), X is an nxk data matrix including observations on both x and q (possibly including transformations and interactions), β is a kx1 parameter vector, and ε is an nx1 vector of mean-zero disturbances which may or may not be mutually correlated. In the special case where

[7] The use of aggregate willingness to pay as a welfare indicator has been debated at length in the literature. The rationale for doing so is perhaps best explained by Kaldor (1939), Hicks (1941), and Polinsky (1972).

[8] We abstract from any cost savings that the landlord may enjoy, such as less frequent painting or air-filter replacement. Equation (6) applies only to a *marginal* air-quality improvement, for which the effects of any induced relocations cancel out because of the envelope theorem applied to each household — so long as such relocations involve no unpriced externalities. Equation (6) corresponds to the "Stage 1" and "Stage 2" effects described in Bartik (1988, pp. 176-177); Bartik goes on to show that with non-marginal air-quality improvements, relocations can add further benefits (a "Stage 3" effect). Because we assume a fixed supply of housing, (6) applies even if landlords exercise market power; such power would affect the division of benefits between renters and landlords (via the terms $dr_i/d\theta$), but not the total.

air quality appears linearly in $X\beta$ with a single coefficient β_q, then $r_{qi}=\beta_q$ and (7) shows that the marginal aggregate willingness to pay for a uniform improvement in air quality is $n\beta_q$.

III. Welfare Analysis with Spatial Lags

We now allow for the possibility that the rent at location j depends on rents at other locations. We do so by replacing $r(x,q)$ in (1) by a function $r(x,q,r_{-j})$, where r_{-j} is the vector of rents at all locations other than the one being considered; it is equal to the rent vector **r** with its j-th element set equal to zero. In other words, the market rent at location j follows the expanded hedonic rule:

$$r_j = r(x_j, q_j, r_{-j}) \ . \qquad (9)$$

We can express the empirical analogue to (9) as the spatial autoregressive form

$$\mathbf{r} = \rho W\mathbf{r} + X\beta + \varepsilon \qquad (10)$$

where W is an nxn spatial weighting matrix whose diagonal elements are zero, and ρ is a scalar spatial coefficient to be estimated. The error vector ε has mean zero but is otherwise unrestricted.[9] As is usual, we assume W to be normalized to sum to one by row: $W\boldsymbol{\iota}=\boldsymbol{\iota}$ where $\boldsymbol{\iota}$ is the nx1 unit vector.

It is sometimes convenient to write this model in reduced form by solving for the dependent-variable vector **r**:

$$\mathbf{r} = [I-\rho W]^{-1}X\beta + [I-\rho W]^{-1}\varepsilon \qquad (11)$$

[9] Kelejian and Prucha (1998) provide a general estimation procedure when both spatial autocorrelation and spatial lags are present, and Greenbaum (2002) provides an example where the two can be distinguished econometrically.

where I is the nxn identity matrix. The matrix $[I-\rho W]^{-1}$ is known as a "spatial-multiplier matrix" (Anselin 2003).[10]

We will make use of the fact that the row-normalization of W implies the following property of the spatial-multiplier matrix (Kim et al. 2003):

$$[I-\rho W]^{-1}\boldsymbol{\iota} = (1-\rho)^{-1}\boldsymbol{\iota} \ . \tag{12}$$

Thus post-multiplication by ι causes the spatial-multiplier matrix to simplify to a scalar $(1-\rho)^{-1}$, which is called the "spatial multiplier".

As before, we will be interested in the special case where data matrix X is linear in q: that is, where it contains air quality only in the form of a single column, q, with coefficient β_q. In that case, the total derivative matrix of rent vector **r** is:

$$\frac{d\mathbf{r}}{dq'} = [I-\rho W]^{-1}\beta_q \tag{13a}$$

which explains the term "multiplier".[11] We can express this rent gradient as the sum of direct and indirect (spatial-spillover) effects:

$$\frac{d\mathbf{r}}{dq'} = \overbrace{I\beta_q}^{direct} + \overbrace{\rho W[I-\rho W]^{-1}\beta_q}^{indirect} \tag{13b}$$

[10] Anselin (2003) refers to (10) and (11) as a model with "spatial externalities in both modeled and unmodeled effects" (p. 161) because its reduced form applies a spatial multiplier to both the independent variable and the errors. He also points out that it is constrained by postulating a single multiplier matrix for both.

[11] If q enters **r** non-linearly, (13a) is replaced by the more general expression

$$\frac{d\mathbf{r}}{dq'} = [I-\rho W]^{-1}\frac{d(X\beta)}{dq'} = [I-\rho W]^{-1} r_q^M ,$$

where r_q^M is an nxn diagonal matrix whose i-th diagonal element is the partial derivative r_{qi} defined in (14).

where we make use of the fact that $[I-\rho W]^{-1} = [I+\rho W+\rho^2 W^2+\rho^3 W^3+\ldots]$.[12] Consider in addition a policy that changes air quality uniformly at every location by an amount θ, so that $dq/d\theta = \iota$; then the vector of rents changes according to:

$$\frac{d\mathbf{r}}{d\theta} = \frac{d\mathbf{r}}{dq'}\boldsymbol{\iota} = \overbrace{\beta_q \boldsymbol{\iota}}^{direct} + \overbrace{\rho(1-\rho)^{-1}\beta_q \boldsymbol{\iota}}^{indirect} = (1-\rho)^{-1}\beta_q \boldsymbol{\iota} \ . \tag{13c}$$

The marginal change in rent at every location is now just the air-quality coefficient multiplied by the (scalar) spatial multiplier, and the aggregate rent change is $n(1-\rho)^{-1}\beta_q$.

Equations (13a) through (13c) make clear how spatial multipliers apply to *rent* changes. We now consider *welfare* changes by considering two types of theoretical models that can lead to specification (10).

Pecuniary Spatial Effects

We say the dependence of r_j on r_{-j} in (9) is pecuniary if the household's utility function is as described in (1). The Lagrangian problem for household utility maximization is thus the same as before, and so is the first-order condition (2). The empirical meaning of this condition, however, is now rather different: the rent function to which it applies now contains argument r_{-i} , a vector of equilibrium rents for households and locations other than i, which is held constant in the partial differentiation. To be explicit, r_{qi} in (2) is the scalar quantity:

$$r_{qi} \equiv \left[\partial r(x_i, q, r_{-i})/\partial q\right]\big|_{q_i} \ . \tag{14}$$

where the vertical line indicates that the expression is evaluated at value $q{=}q_i$.

Furthermore, equations (3)-(7) describing welfare calculations still hold without modification. Again, the economic interpretation of them is somewhat different. In (3) and (4), the total derivative $dr_i/d\theta$, reflecting the entire process of adjustment of rents to pollution

[12] Therefore $\rho W \cdot [I-\rho W]^{-1} = [\rho W + \rho^2 W^2 + \rho^3 W^3] = [I + \rho W + \rho^2 W^2 + \rho^3 W^3] - I = [I-\rho W]^{-1} - I$.

abatement, now includes the indirect effects via pollution's influence on values of nearby locations. These indirect effects are captured by the spatial multipliers in (13a) through (13c), but that does not really matter because they cancel in (6) just as before. What does matter is that the rent function in (14) be estimated in full, including the spatial lags, so that its partial derivative can be taken in order to compute benefit measure (7).

The validity of (7) shows that in computing aggregate benefits from the fully specified hedonic model with spatial lags, only the partial effect of air quality, r_{qi}, is relevant for welfare analysis. This is because the indirect spatial effects enter M_θ and R_θ with equal and opposite magnitudes, and thus cancel — just as they did in the conventional analysis of Section 2. Thus, for example, if our hedonic specification is (10) then the aggregate benefit of a uniform improvement is $n\beta_q$, *i.e.* only the direct effect in (13c) is relevant. If we were to include the indirect effects, we would erroneously inflate these benefits by a factor $(1-\rho)^{-1}$.

What if in designing the estimation, we failed to recognize that a spatial autoregression is needed to reflect the underlying data generating process? If the true rent function is (10) but (8) is estimated instead, then the measured gradient r_{qi} would likely be overestimated given that least-squares is often found to overstate marginal effects of variables in the presence of spatial lags.[13] In that case, the usual benefit measure given by (7) would result in overstated benefits.

We should acknowledge that there is a potential conceptual problem with the pecuniary effects model as a justification for spatial autoregression. We have emphasized that the values of r_{-i} are held constant in taking the partial derivative (14). This is because the model assumes that a household can observe air quality q_j and neighborhood prices r_{-j} independently, and can choose the first conditional on the second. For this to be possible, r_{-i} must not be perfectly correlated with combinations of the other explanatory variables. But if the household does not care about those surrounding property values, then it would refuse to pay a premium to be near them, which would tend to reduce or eliminate the equilibrium responsiveness of **r** to r_{-i}. Yet there is ample empirical evidence that at least one type of pecuniary externality exists, namely the use of "comparables" in pricing houses, and its effects apparently have been observed through spatial autoregressions. Perhaps this occurs because potential homeowners have imperfect information about the housing market, consistent with findings that housing prices do not incorporate all

[13] For examples in labor economics and public finance, see Greenbaum (2002) and Case, Rosen and Hines (1993).

known information (Case and Shiller 1989). This discussion suggests the desirability of a more complete theory of the processes behind spatial autoregression than is typically provided to motivate empirical studies.

Technological Spatial Effects

Now suppose the reason for spatial-lag effects is that households derive enjoyment from unobservable characteristics that are associated with their neighbors' rent expenditures, so that utility at j depends on the vector of neighboring rents, r_{-j} — similar to what Brueckner (2002) calls a "spillover model" in the context of local governments.[14] The residential location problem is then

$$\underset{z,x,q,r_{-i},\lambda}{Max}\ u_i(z,x,q,r_{-j}) + \lambda_i \cdot [y_i - z - r(x,q,r_{-j})] \ . \tag{15}$$

where again r_{-j} denotes the set of rents at all locations other than the one under consideration. First-order condition (2) still applies. In addition, households choose the vector of neighboring property values, r_{-j}; for example, they may prefer a "more prestigious" location among otherwise identical houses. That yields an additional vector of first-order conditions that must apply for household i to be in equilibrium at location i:

$$\frac{1}{\lambda_i}\frac{\partial u_i}{\partial r_{-i}} = \frac{\partial r_i}{\partial r_{-i}} \tag{16}$$

where the derivatives with respect to r_{-i} are column vectors, with zero placed in the i-th component as a convention. This equation shows that the marginal amenity value obtained from each neighboring property's rent is balanced by the increment in own rent required to acquire it through location choice.[15]

[14] See the discussion of Brueckner's terminology in Anselin (2002, p. 249).

[15] Note that as in virtually all hedonic models, we assume a sufficient number of choices that the level of each "characteristic", here including the value of a nearby property, can be chosen from a continuum while holding

The marginal benefit to each household from a change in abatement parameter θ is now a more complicated version of (3):

$$M_{i\theta} \equiv \frac{dV_i / d\theta}{\lambda_i} = \frac{u_{qi}}{\lambda_i}\frac{dq_i}{d\theta} + \frac{1}{\lambda_i}\frac{\partial u_i}{\partial r'_{-i}}\frac{dr_{-i}}{dq'_{-i}}\frac{dq_{-i}}{d\theta} - \frac{dr_i}{d\theta} \tag{17}$$

where q_{-i} is the vector q with its i-th element replaced by zero. The first two terms on the right-hand side of (17) demonstrate two technological effects on utility: the direct amenity value of improved air quality, and the indirect amenity value of technological externalities reflected in an increase in neighboring property values.[16] (The second is technological because $\partial u_i/\partial r_{-i} \neq 0$.) These benefits are enjoyed at the expense of a pecuniary effect: namely, the total increase in rent resulting (through all channels) from the change in policy.

The aggregate benefit to households (as renters) is then given by the sum over locations of (17). Applying first-order conditions (2) and (16), we can convert the marginal utilities in (17) to marginal hedonic prices and write the households' aggregate benefit as:

$$M_\theta = \sum_i \left(r_{qi}\frac{dq_i}{d\theta} + \frac{\partial r_i}{\partial r'_{-i}}\frac{dr_{-i}}{dq'_{-i}}\frac{dq_{-i}}{d\theta} \right) - \sum_i \frac{dr_i}{d\theta} \ . \tag{18}$$

The aggregate benefit to landlords is still given by (5). When we add the benefits to households and landlords, the last term in (18) is canceled by (5) just as in the conventional analysis of Section II, leaving:

constant the characteristics of the house being purchased. The possible failure of this assumption is, of course, a limitation of hedonic analysis.

[16] The indirect effect is added across all the neighboring properties, as indicated by the two matrix multiplication operations in the indirect-effects term — recalling that for a given i, both $\partial u_i/\partial r_{-i}$ and $dq_{-i}/d\theta$ are vectors, the former with a zero value for component i.

$$B_\theta = M_\theta + R_\theta = \sum_i \left(\overbrace{r_{qi} \frac{dq_i}{d\theta}}^{\text{direct}} + \overbrace{\frac{\partial r_i}{\partial r'_{-i}} \frac{dr_{-i}}{dq'_{-i}} \frac{dq_{-i}}{d\theta}}^{\text{indirect}} \right). \tag{19}$$

Equation (19) now shows the aggregate benefit as a sum of direct *and* indirect effects to each household. The direct effect is that in the conventional result (6), whereas the indirect effect captures the end result of accounting for mutual price interactions. In the case considered before of a uniform improvement in pollution levels and an empirical specification in which **r** is linear in q, these direct and indirect effects are given by (13c), and their sum implies an aggregate benefit of $n(1-\rho)^{-1}\beta_q$. Thus if technological externalities are thought to generate spatially-autoregressive housing prices, the spatial-multiplier approach to benefits measurement correctly measures the welfare effect of a uniform improvement.

IV. Review of Spatial-Multiplier Evidence

Several recent papers employ the spatial-multiplier approach to estimate the benefits of residential amenity improvements. Here we briefly review some of their findings to numerically demonstrate the importance of distinguishing between pecuniary and technological spatial effects.

From the above analysis we see that if spatial dependence is pecuniary, but the spatial-multiplier approach is employed, then benefits will be incorrectly inflated by the magnitude of the spatial multiplier, $(1-\rho)^{-1}$. For example, Kim et al. (2003) estimate the benefits of a uniform reduction in air pollution and report a spatial-lag parameter estimate of ρ=0.55, implying a spatial multiplier of 2.22. Thus if the spatial dependence in their sample is pecuniary then the spatial-multiplier approach would overstate benefits by 122%. But if spatial dependence is technological then applying the spatial multiplier is appropriate, and omitting it would understate benefits by 55%.

Table 1 provides the spatial-lag parameter estimates and corresponding spatial multipliers reported by several recent studies. The table implies that benefit estimates can differ substantially according to the nature of spatial dependence assumed to underlie the autoregression process.

Table 1: Spatial Parameter and Multiplier Estimates from Recently Published Studies

Study	Amenity Considered	Spatial Parameter ρ	Spatial Multiplier $(1-\rho)^{-1}$
Kim et al. (2003)	Air Quality	0.55	2.22
Anselin et al. (2008)[a]	Access to Potable Water	0.24	1.32
Cohen & Coughlin (2008)[b]	Airport Noise	0.54	2.17
Andersson et al. (2009)[a]	Road and Railway Noise	0.00 – 0.52	1.00 – 2.08
Anselin & Lozano-Gracia (2009)[a]	Air Quality	0.33	1.49
Kim & Goldsmith (2009)	Distance from Animal Feeding Operations	0.14 – 0.23	1.16 – 1.30

[a] These studies discuss the distinction made here between pecuniary and technological spatial externalities, citing an earlier draft of our paper.
[b] This study estimates airport noise-reduction benefits from a semi-logarithmic autoregressive specification in which noise levels are characterized by dummy variables. Applying a spatial multiplier to its relevant parameter estimates increases estimated benefits by 145% – see Steimetz (2009).

V. Conclusion

The fairly recent development of spatial-hedonic housing-price models introduces a spatial dimension to estimating the willingness to pay for residential amenity improvements. In particular, spatial-lag models have given rise to a spatial-multiplier approach, where both the direct and spatial-spillover effects of an improvement are included in benefits measurement. We demonstrate that this approach is only appropriate when spatial dependence among properties is technological as opposed to pecuniary. Moreover, we demonstrate that an incorrect assumption about the nature of spatial dependence can drastically misstate the benefits of an amenity improvement.

The intuition for our findings is straightforward. If, for example, reduced pollution increases my neighbors' property values, thereby increasing the value of my house, but does not further improve the amenity value of my house, then the spatial effect is pecuniary and, therefore, welfare-neutral. If, on the other hand, I derive increased utility from my neighbors' rise in property values, then the spatial effect is technological and is appropriately included in welfare analysis. In the former case, the direct coefficient on pollution in a spatial-lag specification produces the correct measure, whereas in the latter case the spatial-multiplier approach produces the correct measure.

We have not attempted to provide a means to determine whether spatial dependence is generated by technological or pecuniary externalities. Doing so empirically would presumably require data on the mechanism underlying spatial price interactions; for instance, temporal sales data might allow for identifying the nature of housing-price spillovers through sales-price comparisons, following Pace et al. (1998). In the absence of an empirical model that can identify this spillover mechanism, a practitioner must rely on theoretical reasoning to decide which interpretation of spatial dependence is more reasonable for a given situation.

More generally our analysis illustrates the importance of considering the spatial-lag model among competing specifications, for it is the only model capable of empirically disentangling the direct and spillover effects of an improvement (Anselin and Lozano-Gracia 2008). As such, it is the only model that allows for correctly measuring benefits under either assumption about the nature of spatial dependence.

References

Anas, Alex, and Sung Jick Eum, Hedonic analysis of a housing market in disequilibrium, *Journal of Urban Economics*, 15 (1984), 87-106.

Andersson, Henrik, Lina Jonsson, and Mikael Ogren, Property prices and exposure to multiple noise sources: hedonic regression with road and railway noise, *Environmental and Resource Economics*, Forthcoming (2009).

Anselin, Luc, Under the hood: issues in the specification and interpretation of spatial regression models, *Agricultural Economics* 27 (2002) 247-267.

Anselin, Luc, Spatial Externalities, Spatial Multipliers, and Spatial Econometrics, *International Regional Science Review*, 26 (2003), 153-166.

Anselin, Luc, and Nancy Lozano-Gracia, Errors in variables and spatial effects in hedonic house price models of ambient air quality, *Empirical Economics* 34 (2008) 5-34.

Anselin, Luc, and Nancy Lozano-Gracia, Spatial hedonic models, Working Paper 2008-02, GeoDa Center for Geospatial Analysis and Computation, Arizona State University (2008).

Bartik, Timothy J., Measuring the benefits of amenity improvements in hedonic price models, *Land Economics* 64 (1988) 172-183.

Bergstrom, Theodore C., Benefit-cost in a benevolent society, *American Economic Review*, 96 (2006), 339-351.

Brueckner, Jan K., Strategic interaction among governments: an overview of empirical studies, *International Regional Science Review*, 26 (April 2003) 175-188.

Can, Ayse, Specification and estimation of hedonic housing price models, *Regional Science and Urban Economics* 22 (1992) 453-474.

Case, Anne C., Spatial patterns in household demand, *Econometrica*, 59(4) (July 1991) 953-965.

Case, Anne C., Harvey S. Rosen, and James R. Hines, Jr., Budget spillovers and fiscal policy interdependence, *Journal of Public Economics*, 52 (1993) 285-307.

Case, Karl E., and Robert J. Shiller, The efficiency of the market for single-family homes, *American Economic Review*, 79 (1989) 125-137.

Chay, Kenneth Y., and Michael Greenstone, Does air quality matter? Evidence from the housing market, *Journal of Political Economy* 113 (2005) 376-424.

Cohen, Jeffrey P., and Cletus C. Coughlin, Spatial hedonic models of airport noise, proximity, and housing prices, *Journal of Regional Science*, 48 (2008), 859-878.

Dubin, Robin A., Estimation of regression coefficients in the presence of spatially autocorrelated error terms, *Review of Economics and Statistics* 70 (1988) 466-474.

Freeman, A. Myrick III, Air pollution and property values: a methodological comment, *Review of Economics and Statistics* 53 (1972) 415-416.

Freeman, A. Myrick III, On estimating air pollution control benefits from land value studies, *Journal of Environmental Economics and Management* 1 (1974) 74-83.

Freeman, A. Myrick III, *The measurement of environmental and resource values: theory and methods*, Resources for the Future, 1993.

Greenbaum, Robert T., A spatial study of teachers' salaries in Pennsylvania school districts, *Journal of Labor Research*, 23 (2002), pp. 69-86.

Hicks, John R., The rehabilitation of consumers' surplus, *Review of Economic Studies*, 8 (Feb. 1941) 108-116.

Kaldor, Nicholas, Welfare propositions of economics and interpersonal comparisons of utility, *Economic Journal*, 49 (Sept. 1939) 549-552.

Kelejian, Harry H., and Ingmar R. Prucha, A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances, *Journal of Real Estate Finance and Economics*, 17 (1998), pp. 99-121.

Kim, Chong Won, Tim T. Phipps, and Luc Anselin, Measuring the benefits of air quality improvement: a spatial hedonic approach, *Journal of Environmental Economics and Management* 45 (2003) 24-39.

Kim, Jungik, and Peter Goldsmith, A spatial hedonic approach to assess the impact of swine production on residential property values, *Environmental and Resource Economics*, Forthcoming (2009).

Mills, Edwin S., and Bruce W. Hamilton, *Urban Economics*, HarperCollins, 1994.

Pace, R. Kelley, Ronald Barry, John M. Clapp, and Mauricio Rodriguez, Spatiotemporal autoregressive models of neighborhood effects, *Journal of Real Estate Finance and Economics*, 17 (1998), pp. 15-33.

Pace, R. Kelley, and Otis W. Gilley, Generalizing the OLS and Grid estimators, *Real Estate Economics* 26 (1998) 331-347.

Polinsky, A. Mitchell, Probabilistic compensation criteria, *Quarterly Journal of Economics*, 86 (Aug. 1972) 407-425.

Portney, Paul R., Trouble in Happyville, *Journal of Policy Analysis and Management*, 11 (1992), 131-132.

Ridker, Ronald G., and John A. Henning, The determinants of residential property values with special reference to air pollution, *Review of Economics and Statistics* 49 (1967) 246-257.

Rosen, Sherwin, Hedonic prices and implicit markets: product differentiation in pure competition, *Journal of Political Economy* 82 (1974) 34-55.

Small, Kenneth A., Air pollution and property values: further comment, *Review of Economics and Statistics* 57 (1975) 105-107.

Smith, V. Kerry, and Ju-Chin Huang, Can markets value air quality? A meta-analysis of hedonic property value models, *Journal of Political Economy* 103 (1995) 209-227.

Steimetz, Seiji S.C., Spatial multipliers in hedonic analysis: a comment on "Spatial hedonic models of airport noise, proximity, and housing prices", *Journal of Regional Science*, Forthcoming.

Tsutsumi, Morito, and Hajime Seya, Measuring the impact of large-scale transportation projects on land price using spatial statistical models, *Papers in Regional Science*, 87 (2008) 385-401.

Wendner, Ronald, and Lawrence H. Goulder, Status effects, public goods, provision, and excess burden, *Journal of Public Economics*, 92 (2008) 1968-1985.

Viscusi, W. Kip, Risk equity, *Journal of Legal Studies*, 29 (2000), 843-871.