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LOGIT MODELS FOR AFFILIATION NETWORKS

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Once confined to networks in which dyads could be reasonably assumed to be independent, the statistical analysis of network data has blossomed in recent years. New modeling and estimation strategies have made it possible to propose and evaluate very complex structures of dependency between and among ties in social networks. These advances have focused exclusively on one-mode networks—that is, networks of direct ties between actors. We generalize these models to affiliation networks, networks in which actors are tied to each other only indirectly through belonging to some group or event. We formulate models that allow us to study the (log) odds of an actor's belonging to an event (or an event including an actor) as a function of properties of the two-mode network of actors' memberships in events. We also provide illustrative analysis of some classic data sets on affiliation networks.

1. INTRODUCTION

Affiliation networks represent actors' ties to events. The events may refer to well-defined collectivities like membership in country clubs or on corporate boards of directors or to more ephemeral collections like the guests at a party or spectators at a sporting event. Much of network analysis, including the statistical analysis of relational ties, focuses on one-mode

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networks—that is, networks in which the ties are from actors to actors or from collectivities to collectivities. In contrast, affiliation networks are two-mode networks because the ties link together different types of entities, actors, and collectivities. Affiliation networks have theoretical significance, despite the fact that they are not at the center of network analysis.

Social theorists have long recognized the importance of individuals' affiliations with groups, including both informal social encounters and more institutionalized memberships in organizations. Simmel (1950, 1955) forcefully contends that people are defined socially by the intersection of the various collectivities (family, occupation, neighborhood, voluntary organizations) to which they belong. Others have argued that participation in these collectivities heightens the likelihood of direct linkages emerging between pairs of individuals (Feld 1981, 1982; McPherson and Smith-Lovin 1982). Patterns of memberships not only define individual social identities and facilitate linkages between pairs, but overlapping memberships constrain individual action and provide the basis for social control (Breiger 1990).

From a different perspective, Homans (1951) argues that the identity of social groups emerges from the patterns of informal interactions among collections of people. Such groups can be located by examining patterns in people's participation in informal social activities. Variation in levels of participation and in comemberships among subsets of people indicates internal divisions defining important groups within a population (Breiger 1974; Davis, Gardner, and Gardner 1941; Doreian 1979; Freeman and White 1993; Homans 1951). From the perspective of the collectivities, individuals' overlapping memberships allow for flow of information between groups and for potential coordination of groups' activities. Common members who produce interlocks between organizations allow organizations to monitor one another's actions, to coopt potential competitors, or to coordinate multifaceted production activities by linking together different kinds of organizations.

Affiliation networks, consisting of a set of actors and a collection of "events" (or social occasions) with which subsets of actors are affiliated, have been used to investigate the empirical implications of these theoretical insights (Breiger 1974). They have been used in a wide variety of substantive studies, including the following: interlocking boards of directors (Allen 1982; Bearden and Mintz 1987; Levine 1972; Mariolis 1975; Mintz and Schwartz 1981a, b; Mizruchi 1982; Sonquist and Koenig 1975); voluntary organizations (Bonacich 1978; McPherson 1982); informal social gatherings (Bernard, Killworth, and Sailer 1980, 1982; Breiger 1974; Davis, Gardner, and Gardner 1941; Freeman and Romney 1987; Freeman, Romney, and Freeman 1987; Freeman, Freeman, and Michaelson 1989; Homans 1950); common political activities (Schweizer 1991, 1996); and ceremonial events (Foster and Seidman 1984; Schweizer, Klemm, and Schweizer 1993).

Affiliation networks—also called membership networks (Breiger 1974, 1990), hypernetworks (McPherson 1982), or dual networks (Berkowitz 1982)—differ in important ways from the usual social networks mapping linkages between pairs of actors. First, affiliation networks consist of two different kinds of entities: actors and events. Thus affiliation networks are two-mode networks. In addition, pairs of actors are not directly linked via dyadic ties; rather ties are recorded on subsets of actors (the members of the events or collectivities) and link these actors to the events or collectivities to which they belong. Because affiliation networks are two-mode, nondyadic networks, methods designed to study one-mode networks are not generally appropriate for studying affiliation networks. Furthermore, because of the duality in the relationship between actors and events, appropriate methods for affiliation networks permit one to study the linkages between people through shared memberships, the linkages between groups through common members, and the relationship between people and the groups to which they belong.

Although methodology for one-mode social networks has developed rapidly over the past several decades, there has not been similar development of methods for studying affiliation networks. Graphical displays using concept lattices have been proposed for studying the relationships between actors and events simultaneously (Freeman and White 1993; Schweizer 1991, 1996; Wasserman and Faust 1994). Centrality measures for affiliation networks have been explored (Bonacich 1991; Borgatti and Everett 1997; Faust 1997; Mizruchi, Mariolis, Schwartz, and Mintz 1986), as have methods for finding positions in two-mode networks (Borgatti and Everett 1992).

Despite the theoretical significance of affiliation networks, techniques for their statistical analysis have typically lagged behind those for the analysis of one-mode data. In an early generalization of models for one-mode networks, Snijders and Stokman (1987) extended Holland and Leinhardt's (1970, 1975) U|MAN model for triads to two-mode networks. One of the first statistical models for one-mode network data was Holland and Leinhardt's p_1 model (1981). It was an "independent dyad choice" model and it proposed that the probability of a tie from i to j depended on node level parameters measuring the expansiveness and attractiveness of nodes and on the tendency for choices to be reciprocated. It was well-known in the literature before similar models for two-mode network data were published by Galaskiewicz and Wasserman (1989), Iacobucci and Wasserman (1990), and Wasserman and Iacobucci (1991). These models, reviewed below, shared the p_1 model's assumption of dyad independence—that is, that the occurrence of a tie between i and j was independent of the occurrence of a tie between j and k, or i and k, or any other dyad.

More recent advances in the statistical analysis of one-mode data discard the assumption of dyadic independence in favor of more complicated and hence more realistic structures of dependency between dyads. These models, termed p^* models by Wasserman and Pattison (1996), can be expressed in logit form and estimated approximately by logistic regression techniques, as demonstrated by the pioneering work of Strauss and Ikeda (1990). Frank and Strauss (1986) provided one early type of p^* model that they called "Markov" graphs. More recently, Wasserman and Pattison (1996), Pattison and Wasserman (1999), Robins, Pattison, and Wasserman (Forthcoming), and Anderson, Wasserman, and Crouch (Forthcoming) have given general form and characterization to these models. In all of this recent development, however, little attention has been paid to two-mode networks.

We generalize these logit models to the analysis of affiliation networks. These models allow us to study the (log) odds (or *logit)* of an actor's belonging to an event or an event including an actor as a function of properties of the two-mode network of actors' memberships in events. We begin with a review of the "independent dyad choice" models for affiliation data and then introduce the basics of logit models for one-mode data. We then generalize the approach to two-mode data. Finally, we analyze some classic examples of affiliation networks using the new modeling techniques and demonstrate how properties of affiliation networks can be incorporated into these models to yield useful insights into the structural features of these networks.

2. INDEPENDENT DYAD CHOICE MODELS FOR TWO-MODE NETWORKS

Following the notation of Wasserman and Faust (1994), we denote the set of actors by G and the set of events by H where g and h denote the number of actors and events, respectively. The matrix representing the affiliation network is denoted by $\mathbf{X}^{(G\ H)}$. Actors may belong to events at c different levels of intensity or participation $m = 0,1,\ldots,c-1$. We let $P(x_{ij} = m)$ denote the probability that actor i belongs to event j at level m. Following Iacobucci and Wasserman (1990), under the assumption that the dyads are independent, a simple dyad choice model has the following log-linear form

$$\log P(x_{ij} = m) = \lambda_{ij} + \theta_m + \alpha_{i(m)} + \beta_{j(m)} \tag{1}$$

for each m, subject to the constraints

$$\sum_{m=0}^{c-1} P(x_{ij} = m) = 1$$

$$\sum_i \alpha_{i(m)} = 0$$

$$\sum_j \beta_{j(m)} = 0\ . \tag{2}$$

The parameters are also constrained as follows: when $m = 0$, $\theta_m = 0$, $\alpha_{i(m)} = 0$, and $\beta_{j(m)} = 0$. The parameter $\alpha_{i(m)}$ measures the tendency for actor i to belong to events at level m, net of other factors—i.e., holding the other parameters constant, larger values of $\alpha_{i(m)}$ increase the probability that actor i belongs to event j at level m. The parameter $\beta_{j(m)}$ measures the tendency for event j to be belonged to by actors at level m, net of other factors. The $\{\theta_{\mathrm{m}}\}$ parameters measure general strength effects related to the overall frequency with which actors belong to events at level m. The $\{\lambda_{ij}\}$ parameters, finally, are technically required terms that ensure the first equality in equation set (2) is satisfied. This model is similar to the p_1 model in that it assumes the occurrence of a tie at level m between actor i and event j is independent of the occurrence of a tie at level m' between actor i' and event j'.

Certain special cases of equation (1) are immediately apparent. For instance, one could assume homogeneity (i.e., equality) of either the α parameters or the β parameters, or both, for a fixed level of participation at level m. An important simplification arises if actors or events or both can be "blocked" or partitioned into subsets within which equality of the relative parameters is assumed. Usually these subsets are defined *a priori* based on actor or event characteristics, as we illustrate below.[1] More importantly, though, all of these models are "independent dyad choice" models. This means that the joint probability distribution of the affiliation matrix is a product of the dyadic probabilities (recognizing that certain dyads—namely, all pairs of events and all pairs of actors—are constrained to take on value 0 with probability 1).

The assumption of dyadic independence is often regarded as suspect in analyses of one-mode networks. But, until the work of Frank and Strauss (1986) on Markov graphs, there was little choice but to make this assumption in order to have statistical models of network data. The assumption is equally dubious for affiliation networks, despite the much simpler structure of the basic independent dyad choice model. One can think of reasons why one actor's level of involvement in a particular event may not be independent of another actor's level of involvement in that event and vice-versa. One can also think of various reasons why an actor's level of involvement in one event may not be independent of his or her level of involvement in another event. These plausible but more complex dependency structures can be addressed within the framework of Markov graphs and logit models for network data. We now turn to a development of these ideas for affiliation networks.

3. LOGIT MODELS, MARKOV GRAPHS, AND PSEUDO-LIKELIHOOD ESTIMATION

Moving beyond "independent dyad choice" models required innovations in model building and in estimation. Both of these innovations are suggested, but not fully developed, in the work of Frank and Strauss (1986) on Markov graphs. Full development of the modeling approach is set out in Wasserman and Pattison's (1996) work on p^* logit models for social networks. Strauss and Ikeda (1990) provide the innovation in estimation, the use of pseudo-likelihood functions and logistic regression estimation procedures. We outline these innovations beginning with the p^* modeling framework.

The p^* modeling framework uses a log-linear model to express the probability of a graph G as a function of vector of parameters θ and an associated vector of graph statistics $x(G)$, and a normalizing constant $Z(\theta)$:

[1]Parameter estimates could also be used *a posteriori* to define subsets of stochastically equivalent actors (Wasserman and Anderson 1987).

$$P(G) = \frac{\exp(\theta' x(G))}{Z(\theta)} \quad . \tag{3}$$

The normalizing constant simply ensures that the probabilities sum to unity over all graphs. The θ parameters express how various "explanatory" properties of the graph affect the probability of its occurrence. These parameters must be estimated. However, estimation via maximum likelihood techniques is very difficult because of $Z(\theta)$ in the denominator of equation (3). We first describe some p^* models, and then return to the problem of estimation at the end of this section.

The approach, proposed by Strauss and Ikeda (1990) and elaborated by Wasserman and Pattison (1996), first converts equation (3) into an expression for the log of the odds, or *logit*, a form that does not involve the normalizing constant. We use a mathematical identity that specifies the probability that $x_{ij} = 1$ given the rest of the adjacency matrix. We use G^{-ij} to denote this complement graph—that is, the graph including all adjacencies except the i, j th one. The graph G^+ is defined as the adjacency matrix plus $x_{ij} = 1$ while G^- is defined as the adjacency matrix plus $x_{ij} = 0$. Then with $P(G^+)$ the probability of G^+ and $P(G^-)$ the probability of G^-, the identity is

$$P(x_{ij} = 1 | G^{-ij}) = \frac{P(G^+)}{P(G^+) + P(G^-)} \quad . \tag{4}$$

Basically this equation expresses the probability that $x_{ij} = 1$ conditional on the rest of the graph. Note that it does not depend on the normalizing constant because upon rewriting we get

$$P(x_{ij} = 1 | G^{-ij}) = \frac{\exp(\theta' x(G^+))}{\exp(\theta' x(G^+)) + \exp(\theta' x(G^-))} \quad . \tag{5}$$

If we consider the odds of the presence of a tie from i to j to its absence, we get

$$\frac{P(x_{ij} = 1 | G^{-ij})}{P(x_{ij} = 0 | G^{-ij})} = \frac{\exp(\theta' x(G^+))}{\exp(\theta' x(G^-))} \quad . \tag{6}$$

From equation (6) we can then derive a simple form for the log of the odds or *logit* model:

$$\mathit{logit}\, P(x_{ij} = 1 | G^{-ij}) = \theta' [x(G^+) - x(G^-)] \quad . \tag{7}$$

The quantity in brackets on the right side is the vector of differences in the relevant graph statistics when x_{ij} changes from 1 to 0.

The specification of a p^* logit model requires a selection of network properties that are *a priori* assumed to affect the log odds of a tie being present to absent. A particularly simple case is the p_1 model expressed in logit form:

$$logit\ P(x_{ij} = 1|G^{-ij}) = \theta + \rho x_{ji} + \alpha_i + \beta_j \ . \tag{8}$$

The parameters of this model include expansiveness parameters α and attractiveness parameters β. The expansiveness parameters relate to an actor's tendency to initiate ties and the attractiveness parameters relate to an actor's tendency to receive ties. In addition, the model includes a reciprocity parameter ρ that expresses any tendency for a tie from j to i to be returned by a tie from i to j at greater (or lower) than chance levels. Following Wasserman and Pattison (1996), the vector of parameters and the associated vector of graph statistics for this model are

$$\theta = (\theta, \alpha_1, \ldots, \alpha_g, \beta_1, \ldots, \beta_g, \rho)'$$

$$x(G) = (L, x_{1+}, \ldots, x_{g+}, x_{+1}, \ldots, x_{+g}, M)' \ . \tag{9}$$

L is the number of edges in the digraph, M is the number of mutual dyads, and the remaining graph statistics are the set of outdegrees and the set of indegrees. It is easy to calculate the difference vector of graph statistics for this simple model.

The same logic works for "dependent dyad choice" models, such as those proposed in the Markov graph framework by Frank and Strauss (1986). A Markov graph is a random graph with a particular kind of dependency structure among its possible edges. The dependency structure obeys the following rule: If two dyads are node-disjoint (that is, they do not share a node), then they are conditionally independent (Frank and Strauss 1986:835). The idea is that the presence or absence of tie in one dyad is independent of the presence or absence of a tie in another dyad only when the dyads have no nodes in common. If they share a node, then the presence or absence of a tie in one may depend on the presence or absence of a tie in the other. In contrast to the basic assumption of independent dyad choice models, only some dyads are assumed to be independent in a Markov graph—namely, those that are node-disjoint. The Markov property generalizes in an obvious way to digraphs (Frank and Strauss 1986).

One of the simplest models proposed by Frank and Strauss is the $\rho\sigma\tau$ homogeneous Markov graph model, also called the triad model. Homogeneous models assume that nodes are *a priori* indistinguishable and so no node-specific parameters are necessary. The triad model is a further simplification of the basic homogeneous Markov model for graphs. In the basic model, the probability of a graph is given by a log-linear function of effects pertaining to various tie configurations in which different ties have nodes in common. In particular for a nondirectional relation, the relevant tie configurations are triangles and stars from degree 1 up to degree $g-1$. A triangle is a subset of three nodes where all three ties are present and a k-star is a subset of $k+1$ nodes where one node has a tie to the remaining k nodes. The basic homogeneous Markov model is

$$P(G) = \frac{\exp\left(\tau t + \sum_{k=1}^{g-1} \sigma_k s_k\right)}{Z(\tau, \sigma_1, \ldots, \sigma_k)} \quad , \tag{10}$$

where $Z(\tau, \sigma_1, \ldots, \sigma_k)$ is the normalizing constant, t is the count of triangles and s_k is the count of stars of degree k. The $\rho\sigma\tau$ model makes the simplifying assumption that stars of degree $k \geq 3$ have no effect on the probability of the graph beyond the effect of the $\binom{k}{2}$ 2-stars and the k 1-stars embedded in them. Specifically,

$$P(G) = \frac{\exp(\tau t + \rho r + \sigma s)}{Z(\tau, \rho, \sigma)} \quad , \tag{11}$$

where r is the number of edges in G—i.e., 1-stars—and s is the number of 2-stars. The quantities r, s, and t are the sufficient statistics for the model. Frank and Strauss (1986:836) note that an equivalent set of sufficient statistics is any three of the set of triad counts of G—that is, the number of subgraphs of size 3 having 0, 1, 2, or 3 ties.

Strauss and Ikeda (1990:206) give the logit form of this model as

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \rho + \sigma \Delta S + \tau \Delta T \quad , \tag{12}$$

where ΔS is the change in the number of 2-stars when x_{ij} changes from 1 to 0 and ΔT is the change in the number of triangles. For a directed graph, they note how this model can be made more complicated by including the expansiveness, attractiveness, and reciprocity parameters of the p_1 model

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \rho + \psi x_{ji} + \alpha_i + \beta_j + \sigma \Delta S + \tau \Delta T \ , \qquad (13)$$

where ΔS and ΔT change interpretation now that the underlying graph is directed.[2] Finally, they propose a blockmodel form of the basic triad model in which 2-stars and triangles are counted within blocks and b is the block indicator:

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \rho + \sigma^{(b)} \Delta S_{ij}^{(b)} + \tau^{(b)} \Delta T_{ij}^{(b)} \qquad (14)$$

Finally, Wasserman and Pattison (1996) propose entire family models, referred to as p^* models, with various structural aspects of networks as conditioning factors. Possible parameters for logit models of graphs include the triangles and stars already mentioned but, in addition, overall graph connectivity, various measures of graph centralization, and paths of varying length. In fact, any graph property is a candidate for inclusion. They note that some of these quantities assume a more complicated dependency structure than the simple Markovian one. For instance, a model with a parameter for k-paths assumes that all edges on paths of length k are conditionally dependent, even though pairs of these edges may have no node in common.

Estimation of the p^* models relies on pseudo-likelihood estimation, due to difficulty of maximum-likelihood estimation arising from $Z(\theta)$ in the denominator of equation (3). $Z(\theta)$ is a normalizing constant, given by the equation

$$Z(\theta) = \sum \exp^{\{\theta' x(G)\}} \ , \qquad (15)$$

where the summation is over all $2^{g(g-1)}$ graphs (Strauss and Ikeda 1990:205). For small values of g, $Z(\theta)$ can be calculated directly. However, as g increases (above about 6), direct calculation is all but impossible.

One possibility is to simulate a number of random graphs, each with the same number of nodes and lines as in the observed network, and estimate the θ parameters as a function of observed graph statistics, as sug-

[2]Note that Frank and Strauss's notation for the reciprocity effect and for the overall density effect differs from that of Wasserman and colleagues.

gested and illustrated by Frank and Strauss (1986) and Strauss (1986). However, given the computation intensity, this approach is not really practical (Frank and Strauss, 1986).

To estimate the p^* model, we use pseudo-likelihood estimation, a strategy hinted at by Frank and Strauss (1986) and elaborated by Strauss and Ikeda (1990) and Wasserman and Pattison (1996). The pseudo-likelihood function is defined as

$$PL(\theta) = \prod P(x_{ij} | G^{-ij}) \tag{16}$$

The idea is to maximize equation (16) with respect to the parameters, θ, where the maximum pseudo-likelihood estimator (MPE) is a value of θ that maximizes equation (16) (Strauss and Ikeda 1990:207). As Strauss and Ikeda note, "the pseudolikelihood function is simply the product of the probabilities of the $[x_{ij}]$ with each probability conditional on the rest of the data" (p. 204). This strategy is analogous to the procedure proposed by Besag (1974) in the context of spatial models and rectangular lattices, where pseudo-likelihood estimation is widely used in estimations that also involve difficult normalizing constants (Hjort and Omre 1994).

The estimation method proposed by Strauss and Ikeda (1990) forms a pseudo-likelihood function for the graph in terms of the conditional probabilities for x_{ij} as follows:

$$PL(\theta) = \prod_{i \neq j} P(x_{ij} = 1 | G^{-ij})^{x_{ij}} P(x_{ij} = 0 | G^{-ij})^{1-x_{ij}} \quad . \tag{17}$$

Strauss and Ikeda prove that equation (16) can be maximized using maximim likelihood estimation of the logistic regression, equation (7), assuming the x_{ij}'s are independent observations. Thus the p^* family of models can be estimated, albeit approximately, using logistic regression routines in standard statistical packages.[3] However, since the logits are not independent, the model is not a true logistic regression model and statistics from the estimation must be used with caution. Goodness-of-fit statistics are pseudo-likelihood ratio statistics, and it is questionable whether the

[3]The data array has $g \times (g-1)$ rows, one column for the dependent variable x_{ij} and the remaining columns express the change in the graph statistics that constitute the independent variables in a model. The extraordinary flexibility of p^* models means that care must be taken that the vector of independent variables does not unintentionally have a logically determinate relationship to the dependent variable.

usual chi-square distributions apply; in addition, standard errors have only "nominal" significance (see Crouch and Wasserman 1998).

In sum, much progress has been made, and made recently, toward statistical models for networks that abandon the restrictive assumption of dyadic independence. More complicated dependency structures can be formulated and estimated approximately by logistic regression techniques. However, all of this development takes place in the context of one-mode graphs or digraphs. In the next section, we extend these models to two-mode affiliation networks.

4. MODELS FOR AFFILIATION NETWORKS

Our models for affiliation networks focus on simply the presence or absence of a tie rather than its strength. Hence, in terms of earlier notation the number of levels of intensity, c, equals two.[4] The first model we consider adapts the basic triad model of Frank and Strauss (1986). However, because an affiliation network is a bipartite graph—the nodes can be partitioned into two subsets and all ties are between the two sets, so that all triples of nodes are constrained to have at most two ties—the adaptation produces a model whose entire structure depends simply on the degree sequences—that is, on the marginals of the adjacency matrix (see footnote 5 later). This model has some merit as a basic "baseline" model from which to address the question of whether a particular affiliation network displays any "interesting" structure. This question of "interesting structure" was first framed by Holland and Leinhardt (1979) who argued that any network in which higher-order properties could be adequately modeled using only the properties of nodes or dyads had no social structure. By "adequately modeled" they meant that the higher-order properties took on values within the range expected given chance variation as constrained by the lower-order properties (Skvoretz, Faust, and Fararo 1996). Thus if a particular affiliation network is fit well by the triad model, its higher-order properties are simply expected consequences of the lower-order degree sequences. We then propose additional models that use higher-order properties as "explanatory" variables in predicting the log odds on the presence of a tie.

Frank and Strauss's triad model has as sufficient statistics the triad census of the graph. For an undirected graph, there are four triad equiva-

[4] We note that p^* models for one-mode networks have been extended to valued relations in Robins, Pattison, and Wasserman (forthcoming).

lence classes: the nonisomorphic three-subgraphs with zero, one, two, or three edges. But for an affiliation network (and bipartite graphs in general), the census has three, rather than four, equivalence classes since there can be no triads with three edges because there are no ties between actors or between events (Snijders and Stokman 1987). Furthermore, the triads in an affiliation network can be further distinguished by the number of actor and event nodes in the triad. All triads with three actors or three events are empty since ties cannot be present between nodes within the same set. Triads in which ties may be present must contain either two actors and one event or two events and one actor, and each triad may have zero, one, or two ties. Thus, once we distinguish between actors and events in an affiliation network, there are six equivalence classes of triads.

Our extension of the homogeneous triad model for an undirected graph (Frank and Strauss 1986) estimates separate parameters for the "two actor one event" and the "two event one actor" triads. As noted by Frank and Strauss (1986:836), for a given network, the sum of the counts of triads with zero, one, two, and three edges is a constant, the total number of triads in the network; therefore only three of four counts are needed as sufficient statistics. For an affiliation network, further dependencies among these counts mean that only a single count within each of the two sets is needed: We select the configurations with two edges depicted in Figure 1—2-stars with actors at their centers (actor 2-stars) and 2-stars with events at their centers (event 2-stars). Our model is homogeneous within each of the two sets because actors are interchangeable in one set and events in the other.

The direct generalization of the homogeneous triad model has the logit form

$$\mathit{logit}\, P(x_{ij} = 1 | G^{-ij}) = \theta + \sigma_a \Delta S_a + \sigma_e \Delta S_e \ , \tag{18}$$

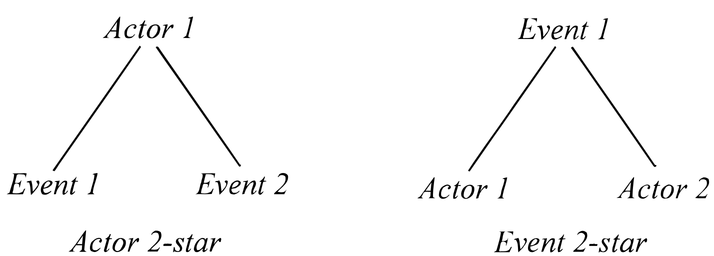

FIGURE 1. Actor 2-stars and event 2-stars.

where θ is an overall density effect and the two σ parameters refer to the impacts that actor 2-stars and event 2-stars have on the logit.[5] A positive σ_{a} effect means that the log odds of x_{ij} being present are increased if the absence of the tie disrupts links between event j and other events that are created through actors' participation in events. A positive σ_{e} effect means that the log odds of x_{ij} being present are increased if the absence of the tie disrupts links between actors that are created through an event's inclusion of multiple actors. These parameters are responsive to the ideas that an actor's involvement in a particular event may depend on other actors' involvement in that event (captured by the event 2-star count) and that an actor's involvement in one event may depend on his or her involvement in another event (captured by the actor 2-star count).

As Wasserman and Pattison (1996) have noted, a wide range of network structural effects can be incorporated into p^* models. Even parameters in the relatively simple Markov random graph models embody important structural properties. Consider the frequent observation that in an affiliation network actors are linked to one another through joint membership in events, and events are linked through joint participation of actors (Breiger 1974). Joint membership for actors is captured in the event 2-stars (equivalently in the count of triads with two lines and two actor nodes). The parallel effect for event overlap is captured in the actor 2-stars (equivalently in the count of triads with two lines and two event nodes).

The event 2-star effect parametrizes how multiple shared memberships for actors affect the likelihood of a single actor-event tie. If actor i belongs to many events with other actors, we might hypothesize that these multiple memberships influence the probability of actor i's membership in

[5]As a reviewer pointed out, the number of actor 2-stars and the number of event 2-stars are simple functions of sums of degrees and sums of degree squares. It is easy to verify that

$$S_a = \sum_i \frac{x_{i+}^2}{2} - \sum_i \frac{x_{i+}}{2}$$

$$S_e = \sum_j \frac{x_{+j}^2}{2} - \sum_j \frac{x_{+j}}{2} \quad ,$$

and since θ is a function of the number of edges in the graph, x_{++}, the probability distribution depends only on the average degree and the variance of actor degrees and the event degrees. Therefore the triad model for two-mode networks is a model about dispersion of degrees rather than about structure defined as pattern within the adjacency matrix conditional on the marginals.

the events shared by these other actors. As a general tendency for actors, this effect is captured in the parameter for event 2-stars. Similarly, we could consider the extent to which multiple overlapping members among a set of events would affect the probability of an event-actor tie. This is captured in the parameter for the actor 2-stars. All of these structural effects of actor comemberships and event overlaps are incorporated in the Markov graph model for affiliation networks with nonhomogeneous 2-star effects, or equivalently with nonhomogeneous triad parameters distinguishing between actor 2-stars and event 2-stars.

Some obvious extensions to the basic Markov model include the following ideas. First, we can investigate the effect of higher-order subgraphs on the presence/absence of tie. That is, we can add parameters that express the effects of various 3-stars, 4-stars, etc., following the full homogeneous Markov graph model proposed by Frank and Strauss (1986). But as Frank and Strauss note, since lower-order stars are embedded in higher-order stars, interpretation of the parameters is problematic.

Second, we can relax the homogeneity assumption to allow for specific actor and event effects related to the overall number of events an actor participates in and to the overall number of actors an event attracts. There are three possibilities:

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \theta + \alpha_i + \sigma_a \Delta S_a \tag{19}$$

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \theta + \beta_j + \sigma_e \Delta S_e \tag{20}$$

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \theta + \alpha_i + \beta_j + \sigma_a \Delta S_a + \sigma_e \Delta S_e \tag{21}$$

where α_{i} parametrizes actor i's expansiveness and β_{j} parametrizes event j's attractiveness. However, these models are not well-formed and cannot be estimated since the logits will assume the value $+\infty$ for ties that are actually present and $-\infty$ for ties that are actually absent.[6]

[6]Echoing the caution in footnote 3, we note that in each case, there is a logically determinate relationship between the observed value of x_{ij} and the corresponding vector of graph statistic differences used as independent variables. For instance, consider equation (20). The vector of difference statistics is of length $h + 2$. If $x_{ij} = 1$, then the vector equals $(1,0,\ldots,1,\ldots,0,x_{+j} - 1)$: the first position equals 1, the change in the overall number of ties as x_{ij} goes from present to absent; there are 0s in all but the j^{th} location in the next h positions (corresponding to the fact that only the degree of the j^{th} event changes—and by 1—as x_{ij} goes from present to absent), and the last position equals the degree of event j minus 1, the number of changes in event 2-stars as x_{ij} goes from present to absent. If $x_{ij} = 0$, then only the last entry changes. The last position now equals just the degree of event j, since if x_{ij} were to be present, x_{+j} additional event

Third, we may consider subgroup effects within the basic Markov model for affiliation networks. A specific model we estimate in the next section blocks only on events. This block diagonal model includes effects for actor 2-stars when the events are in the same block:

$$logit\, P(x_{ij} = 1 | G^{-ij}) = \theta + \sum_{b} \sigma_a^{(b)} \Delta S_a^{(b)} + \sigma_e \Delta S_e \tag{22}$$

Of course, one could add parameters of the off-diagonal blocks or make the assumption that the effect parameters for the various blocks are equal or build an analogous model for blocking on actors. The parameters capture the idea that it is the extent to which multiple overlapping members among a block of events, rather than the entire set of events, affects the probability of an event-actor tie.

Finally, we illustrate models that condition on higher-order properties of an affiliation network. We examine two properties of interest, both of which have been argued to be theoretically important features of affiliation networks. The first property is called "subgroup overlap," or in our context either "actor overlap" or "event overlap." We use a measure proposed by Bonacich (1972).[7] Events overlap to a greater degree when more actors participate in both of them. Actors overlap to a greater degree when they both jointly participate in many events. Bonacich's measure of overlap is logically independent of the size of the events or the number of events attended by the actors. Event overlap varies from 0 if no actors jointly participate in the two events, to 1 if all actors attending one event attend the other (and vice versa). Actor overlap varies from 0 if the two

2-stars would be created. We can select values for the parameters $\theta, \beta_1, \ldots, \beta_h$, and σ_e such that x_{ij} becomes a determinate linear function of the difference vector. For instance, if we let $\beta_j = \theta - x_{+j}$ and let $\sigma_e = -1$, it is easy to see that

$$x_{ij} = \theta + (\theta - x_{+j})(1) + \sum_{k \neq j} (\theta - x_{+k})(0) + (-1)(x_{+j} - x_{ij}).$$

[7]Bonacich's measure r is defined as

$$r = \begin{cases} \dfrac{n_{11} n_{22} - \sqrt{n_{11} n_{22} n_{12} n_{21}}}{n_{11} n_{22} - n_{12} n_{21}} & \text{if} \quad n_{11} n_{22} \neq n_{12} n_{21} \\ = 0.5 & \text{otherwise,} \end{cases}$$

where n_{11} is the number of actors belonging to both groups or events, n_{22} is the number belonging to neither group/event, n_{12} is the number belonging to the first group/event but not the second, and n_{21} is the number belonging to the second but not the first.

actors attend no events together, to 1 if the two attend exactly the same set of events. We explore models that condition the occurrence of a tie on the *average* amount of overlap between events and the *average* amount of overlap between actors. In logit form, the log odds of the presence of a tie is modeled as a function of the change in the average amount of overlap between events (or between actors) when the tie goes from 1 to 0.

The second property of interest also takes two forms depending on whether we consider paths from events to events or paths from actors to actors. We consider the path length between actors and between events as measured by the *average* number of events on the shortest path between two actors and by the *average* number of actors on the shortest path between two events. Prevalence of short paths between pairs of actors or pairs of events is indicative of system-level integration, whereas prevalence of long paths can indicate a tendency for segregation into subgroups with little connection between them (Granovetter 1973). In logit form, the log odds of the presence of a tie is modeled as a function of the change in the average path length when the tie goes from 1 to 0. Note that the change will always be zero or negative—that is, removing a tie will either leave the average path length unchanged or increase it. This is not true for the first property—removing a tie can increase the average amount of measured overlap.

One reason to examine these properties is that they are not determined by the degree sequences—i.e., by the marginals of the affiliation matrix. It is possible to construct two affiliation networks with the same degree sequences but with different values for average event or actor overlaps and for the average path lengths. Furthermore, both of these properties imply that the underlying graph is not Markovian: Models based on these properties postulate dependencies between dyads that do not share a node. However, such effects can be easily parametrized and approximately estimated by a logit model of the p^* family. Thus these properties give us an opportunity to illustrate the application of p^* models to affiliation networks.

5. TWO ILLUSTRATIONS

We illustrate various models on two data sets. The first is Davis, Gardner, and Gardner's (1941) classic affiliation network of the participation of 18 Southern women in 14 social events (see also Homans 1951 and Breiger 1974). The second is Galaskiewicz's (1985) data on the board and club

TABLE 1
Logit Models of Data for Davis, Gardner, and Gardner (1941)

Model	Number of Parameters	Pseudo-Likelihood Ratio Statistic
1. Choice	1	327.292
2. Choice + 2-stars	2	305.328
3. Choice + event 2-stars	2	308.273
4. Choice + actor 2-stars	2	325.618
5. Choice + event 2-stars + actor 2-stars	3	304.784

memberships of corporate executive officers (CEOs) in Minneapolis–St. Paul. We use the subset of data on 26 CEOs and 15 boards/clubs reprinted in Wasserman and Faust (1994). This second example includes a four-category subgrouping variable for the type of board or club: country club, metropolitan club, board of FORTUNE 500 firms or FORTUNE 50 banks, and board of cultural or religious organizations.

Table 1 presents the pseudo-likelihood ratio statistics for the fits of models to Davis, Gardner, and Gardner's Southern women data. The simplest interesting model has a homogeneous effect for 2-stars (model 2). Models 3 and 4 consider, separately, effects of event 2-stars and actor 2-stars (respectively) ignoring the other type of 2-star. Model 5 includes nonhomogeneous effects for type of 2-star. Interestingly, this model is not an improvement over the homogeneous effect of 2-stars (model 2). Thus for these data there is no advantage in distinguishing between actor-centered and event-centered 2-stars.

Table 2 gives the parameter estimates for models 2 and 5 for the Davis, Gardner and Gardner data. The parameter estimate for the effect of

TABLE 2
Parameter Estimates for Models 2 and 5 from Table 1

	Parameter Estimate	
Effect	Model 2	Model 5
Choice	−2.503	−2.374
2-stars	0.175	
Event 2-stars		0.186
Actor 2-stars		0.131

TABLE 3
Logit Models for CEOs and Boards/Clubs Network (Galaskiewicz 1985)

Model	Number of Parameters	Pseudo-Likelihood Ratio Statistic
1. Choice	1	439.717
2. Choice + 2-stars	2	400.878
3. Choice + event 2-stars	2	391.940
4. Choice + actor 2-stars	2	429.746
5. Choice + event 2-stars + actor 2-stars	3	387.013
6. Choice + actor 2-stars within blocks	5	403.777
7. Choice + actor 2-stars between blocks	2	437.480
8. Choice + actor 2-stars within and between blocks	6	401.136
9. Choice + actor 2-stars within and between blocks + event 2-stars	7	369.700

2-stars is positive, indicating that the greater the number of 2-stars disrupted by the absence of a particular actor-event tie, the greater the log odds that the tie is present versus absent. Clearly the type of 2-stars that have this enhancing effect are event 2-stars. This indicates that it is coattendance at events over pairs of actors that is primarily responsible for the positive 2-star effect.

Table 3 presents fits of models to Galaskiewicz's CEOs and boards/clubs network. In this example we fit the same models as for the Davis, Gardner and Gardner data but also include models with a blocking of the events. This blocking operates on the actor 2-stars and captures whether or not the two events in the actor 2-star are in the same block (for blocks 1 through 4) or whether they are in different blocks (regardless of the specific blocks).

First consider the models without event blocking. In contrast to the results for Davis, Gardner, and Gardner's data, the addition of nonhomogeneous effects distinguishing actor 2-stars and event 2-stars provides an improvement of fit for Galaskiewicz's CEOs and boards/clubs network (compare models 2 and 5 in Table 3). Table 4 gives the parameter estimates for model 5 for these data. Actor 2-stars and event 2-stars have contrasting effects on the likelihood of a tie; actor 2-stars decrease whereas event 2-stars increase this probability. Comparing models 3 and 4 with model 1 suggests that event 2-stars provide more leverage than do actor 2-stars.

Models 6 through 9 in Table 3 add event blocking to the actor 2-stars. These event blockings are added separately for within block (model 6) and

TABLE 4
Parameter Estimates for Models 5 and 9 from Table 3

	Parameter Estimate	
	Model 5	Model 9
Choice	−1.338	−1.284
Event 2-stars	0.154	0.144
Actor 2-stars	−0.238	
Blocked actor 2-stars:		
Block 1		−7.210
Block 2		−0.512
Block 3		−1.102
Block 4		−0.031
Between blocks		−0.142

between block (model 7) actor 2-star effects and then for both within and between block actor 2-star effects (model 8). Finally event blocking of actor 2-stars is considered in combination with event 2-stars (model 9). Comparing model 8 with model 4, and model 5 with model 9 shows the additional effect that the event blocking has on the actor 2-stars. In both cases event blocking improves the fit of the model. Parameter estimates for model 9 are in Table 4.

Recalling that block 1 is composed of two country clubs, the large negative effect of actor 2-stars for this block of events means that membership in the clubs tends strongly to being mutually exclusive—that is, actors belong either to one club or to the other, but not both. The next largest effect is in block 3, which is composed of boards of Fortune 500 firms or Fortune 50 banks. Again the tendency here, although it is not as strong, is for memberships on some of these boards to depress the likelihood of membership on others. The negative effect means that the greater the number of actor 2-stars that would be created by a tie from an actor to a board, the lower is the probability of the tie being present. So there is a “ceiling effect” on total number of memberships on boards within a block—the more boards of a given type to which an actor belongs, the lower is the likelihood that he belongs to one more. That the effect of event 2-stars is positive indicates the tie connecting an actor to clubs or boards with relatively many members is a “stronger” tie than one connecting the actor to relatively small clubs or boards. The first tie creates relatively many event 2-stars and so, as compared with the condition in which it is absent, the log odds of it being present are increased substantially. The second tie creates relatively few event 2-stars and so the "force" of its presence (as measured by the log odds) is not as strong.

The next set of estimated models condition on the average overlap between events and between actors and/or the average minimum path length between actors and between events. Table 5 presents zero-order correlations between relevant variables for the two data sets. The cases here are the $(g \times h)$ dyads, and the variables are the dyad change scores corresponding to the independent variables in our models. In both data sets, there are substantial positive correlations between the change in actor 2-stars and the change in event overlap and between the change in event 2-stars and actor overlap. There is also a substantial negative correlation between the change in the average distance between actors and the change in actor overlap. The latter, however, is not paralleled by the correlation between the change in the average distance between events and the change in event overlap. While these correlations are substantial, they are not perfect, indicating that the overlap and distance measures are not simple linear functions of the 2-stars and the underlying degree sequences. Finally, we note that in both data sets there are moderate positive correlations between the presence of a tie (the dependent variable) and event 2-stars and actor overlap. In the Davis, Gardner, and Gardner data, there is a moderate positive correlation between the dependent variable and the average distance between events while in the Galaskiewicz data, this correlation is essentially zero.

The p^* models we estimate for these data sets are presented in Tables 6 and 7. We begin with the basic model that includes both actor and event 2-stars and then add the overlap and distance measures. The best fitting model for the Davis, Gardner, and Gardner data includes effects for event overlap and for the distance between events as measured by the number of actors on the shortest path between them. The best fitting model for the Galaskiewicz data only includes an effect for the distance between events. Parameter estimates corresponding to the basic model and the best-fitting models are present in Tables 8 and 9. Interpretation of these effects can be made either in terms of how the change in the corresponding independent variable affects the log odds on the presence of a tie or, preferably, in terms of the underlying p^* models in which the independent variable impacts the probability of a tie being present.

In the Davis, Gardner, and Gardner data, we find that event overlap has a negative effect on the presence of a tie. The size of the coefficient reflects the scale of the independent variable—in this data set the change score varies from -0.011 to 0.028. The negative effect means that as be-

TABLE 5
Zero-order Correlations for p^* models (Davis, Gardner, and Gardner [1941] above diagonal, Galaskiewicz [1985] below diagonal)

	Actor 2-stars	Event 2-stars	Actor Overlap	Event Overlap	*e*-on-*a* Path	*a*-on-*e* Path	Tie Present
Actor 2-stars	—	−0.11	0.12	0.81	0.36	0.19	0.08
Event 2-stars	−0.15	—	0.79	0.05	−0.55	0.15	0.28
Actor overlap	−0.08	0.86	—	0.21	−0.74	0.10	0.25
Event overlap	0.74	−0.20	−0.06	—	0.19	0.17	−0.06
e-on-*a* path	0.26	−0.77	−0.95	0.17	—	0.09	−0.12
a-on-*e* path	0.00	0.27	0.23	−0.01	−0.22	—	0.28
Tie present	−0.16	0.37	0.31	−0.16	−0.32	−0.03	—

TABLE 6
p^* Models for Davis, Gardner, and Gardner Data

Model	Number of Parameters	Pseudo-Likelihood Ratio Statistic
1. Choice + event 2-stars + actor 2-stars	3	304.784
2. 1 + actor overlap	4	304.698
3. 1 + event overlap	4	279.092
4. 1 + actor overlap + event overlap	5	279.035
5. 1 + *a*-on-*e* path	4	286.170
6. 1 + *e*-on-*a* path	4	304.783
7. 1 + *a*-on-*e* path + *e*-on-*a* path	5	285.835
8. 1 + event overlap + *a*-on-*e* path	5	263.979

tween two ties—say, x_{ij} and x_{kl}—the one that has the higher probability of occurrence is associated with a lower amount of event overlap. Net of other considerations, this effect has the consequence that an actor would be less likely to add a tie to an event if the actors already tied to that event are ones to whom the focal actor is not already tied via coparticipation in other events. Adding such a tie would increase overlap between events more than adding a tie to an event attended mostly by other actors to whom the focal actor is already tied via common participation in other events. The positive effect of average distance between events means that as between two ties, x_{ij} and x_{kl}, the one that has the higher probability of occurrence is associated with a longer average path length between events. In effect, these data exhibit an "anti-bridging" tendency—a tie from an actor to an event that would create shorter paths is less likely to occur than a tie

TABLE 7
p^* Models for Galaskiewicz Data

Model	Number of Parameters	Pseudo-Likelihood Ratio Statistic
1. Choice + event 2-stars + actor 2 stars	3	387.013
2. 1 + actor overlap	4	386.804
3. 1 + event overlap	4	386.711
4. 1 + *a*-on-*e* path	4	378.833
5. 1 + *e*-on-*a* path	4	386.989

TABLE 8
Parameter Estimates for Models 1 and 8 from Table 6

	Parameter Estimate	
Effect	Model 1	Model 8
Choice	−2.374	−4.238
Event 2-stars	0.186	0.232
Actor 2-stars	0.131	0.612
Event overlap		−158.177
a-on-*e* path		2.376

that would create longer paths. In the Galaskiewicz data, the effect of the average distance between events is negative, indicating that these data exhibit a "bridging" tendency. That is, in these data, as between two ties, the one with a higher probability of occurrence is associated with a shorter average path length between events.

We can explore this effect inspecting the event overlap matrices. In the Davis, Gardner, and Gardner data, there are 91 pairs of events; 25 have zero overlap—that is, there are no actors who attend both events—7 pairs overlap at just one actor, and the rest overlap from 2 to 9 actors. The zero and one overlap pairs are critical since a change in the value of a single tie could increase or decrease the distance between events. Since the zero cases are much more numerous than the one cases, there are more occasions where adding a tie would decrease path distance than there are occasions where deleting a tie would increase path distance. In addition, the zero cases predominately fall between two blocks of events. The fact that such critical ties are *not* present is the "anti-bridging" tendency, a tendency

TABLE 9
Parameter Estimates for Models 1 and 4 from Table 7

	Parameter Estimate	
Effect	Model 1	Model 4
Choice	−1.338	−1.837
Event 2-stars	0.154	0.180
Actor 2-stars	−0.238	−0.245
a-on-*e* path		−0.797

consistent with the frequently observed "clique" structure in these data (Homans 1951; Breiger 1974). In the Galaskiewicz data, there are 105 pairs of events; 35 have zero overlap, but 33 overlap at one actor; the zero cases do not appear to fall between entire blocks of events. Consequently, these data display a different tendency with respect to the probability of bridging ties, one where ties tend to create short paths or bridges between events.

6. CONCLUSION

We show that recent advances in the statistical analysis of one-mode network data can be extended to two-mode data from affiliation networks. The models we have proposed and evaluated do not exhaust the possible model structures. Our models begin with the basic idea of Markov graphs by postulating dependencies between dyads only if they share a node. Because of their nature, certain simple homogeneous Markov graph models simplify further when applied to affiliation networks. However, they become more complex in one respect—there is a natural heterogeneity between types of triads depending on whether they contain two actors and one event or one actor and two events. That these configurations can have empirically different effects is documented in our illustrative analyses.

Finally, by using measures for average path length and for actor and event overlap, we show how non-Markovian models can be proposed and estimated via the p^* framework. These models uncover both "bridging" and "anti-bridging" tendencies in the formation of affiliation networks. In the Davis, Gardner, and Gardner data set of Southern women, actors' ties to events appear to differentiate them and push them apart, whereas in the Galaskiewicz data set of CEOs, events appear to integrate actors and pull them closer together.

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