Journal of Mathematical Sociology
1996, Vol. 21(1-2), pp. 57-76
Reprints available directly from the publisher
Photocopying permitted by license only

SOCIAL STRUCTURE, NETWORKS, AND E-STATE STRUCTURALISM MODELS

JOHN SKVORETZ and KATHERINE FAUST

University of South Carolina

THOMAS J. FARARO

University of Pittsburgh

The method of E-state structuralism provides dynamic models for the evolution and development of networks in small groups. Our interest lies in the kind of social networks that these models produce. We ask the question of whether such models produce "interesting" structure from a network point-of-view, in particular, from the perspective of Holland and Leinhardt who argue that any network that can be modeled adequately using only properties of nodes and dyads has no social structure. We show that E-state structuralism models are models of social structure in this technical sense because they assume a bystander mechanism in the creation of ties.

INTRODUCTION

Recently, a concern with theory has blossomed in social network research to complement the extensive methodological development of the past several decades. Wasserman and Faust's (1994) compendium demonstrates how methodologically sophisticated social network analysis has become. The concern with theory is a concern with processes that generate a social structure as a network of ties among a set of actors. This question is one of development and evolution of networks from *a priori* theoretical considerations rather than their *ex post facto* analysis. A common thread in recent theoretical models for this problem (Zeggelink, 1993; Snijders, 1994) is the assumption of methodological individualism and goal-directed action: "the driving force...is constituted by the actors' actions; each actor takes actions in order to further his own goals; these actions are in the domain of his own behavior or of the directed relationships from him to others" (Snijders, 1996).

We describe a research program, E-state structuralism, that focuses on the evolution of social structures as networks in small groups. The program, furthermore, antedates the recent concern with theoretical models in social network research. Our interest lies in the kinds of networks that E-state structuralist models produce. The basic models are stochastic process models, in particular, discrete time and discrete state Markov chain models, that describe a group's trajectory through a state space of networks. The models have absorbing states. Each group begins in an initial state in which no ties exist between any pairs and eventually settles into one of the absorbing states. The networks represented by these states and the distribution of groups over such states are the focus of our concern. In particular, we ask the question: do groups typically evolve into networks, represented by the absorbing states, that exhibit "interesting" structure in the precise sense defined in social network analysis? We first identify what it means to say that a network exhibits "interesting" structure. We then describe the assumptions of E-state structuralism models and their implications for the paths typically followed by networks that evolve under the assumed dynamics.

THE SOCIAL NETWORK MODEL

The minimal social network model consists of a set (or sets) of actors and one (or more) relations defined on pairs of actors. The actors may be any kind of social entity such as people, organizations, communities, etc. (Freeman, 1989; Wasserman and Faust, 1994). The relations are ties or linkages between pairs of actors or from one actor to another. For example one could record the number of communications directed from each actor to each other actor in a discussion group. A social network representation may also include attributes of actors (such as the sex or race of a person) and/or multiple relations. The inclusion of relational information is what makes the social network approach distinctive.

We denote the set of actors by $N = \{n_1, n_2, \ldots, n_g\}$ and the relation by $\mathbf{X}$. The relation $\mathbf{X}$ is defined on pairs of actors. Relations may be dichotomous (present or absent) or valued (taking on strengths); and may be directional (oriented from one actor to another) or non-directional (connecting actors without directed orientation). We denote the tie from actor i to actor j by x_{ij}, where x_{ij} may be either dichotomous (0 or 1) indicating the presence or absence of a tie from actor i to actor j, or valued indicating the strength (frequency, intensity) of the tie from actor i to actor j. For non-directional ties, we necessarily have $x_{ij} = x_{ji}$. Much of social network analysis is concerned with formalizing and calculating properties of networks and of network structure. These properties may refer to actor subsets of various types, such as cliques (subsets among whom ties (interactions) are relatively frequent or intense) or actors in structurally equivalent positions, or to the entire network. Properties of the latter sort include centralization, density, connectivity/connectedness, and hierarchy among others.

These network properties pertain to different *levels*: actors, pairs of actors, triads of actors,...,subset of actors, and the entire network. For example, in a friendship network, the number of friends that person i has is an actor-level property. Whether or not actors i and j have a mutual friendship is a pair-level or dyadic-level property. The number of such mutual ties in the network summarizes properties of pairs. In a communication network, whether a message that originated with any person in the network could travel to each and every other person in the network would be a network-level property. One would have to consider not only all pairs of actors but the entire configuration of ties in the network. An important point to note here is that because actors may vary along quite different kinds of attributes, we must emphasize here that the actor-level properties refer to relationally induced attributes definable from the network itself. They do not refer to actor attributes (such as, gender or motivation) that are extrinsic to the ties defining the network nor to the positions of actors in other networks or structures that are brought to the particular relational system under analysis.

We think of these levels as going from the "lowest" singleton actor level to the "highest" entire network level in that units at a higher-level are composed of sets of units at a lower-level. Some authors have referred to properties of networks that pertain to the lower-levels (actors and dyads) as *local-level* properties and properties that pertain to the entire network as *global-level* properties (Holland and Leinhardt, 1976). An important question about a network's structure is whether higher level properties can be derived from lower level properties (of actors and pairs of actors) or whether these higher-level properties are emergent, that is, more than the simple aggregation of lower-level properties.

Holland and Leinhardt (1979) argue that any network in which higher level properties can be modelled adequately using only properties of nodes (actors) and dyads (pairs of actors) has no social structure. In their words, there is nothing inherently *social* about the structure of such a network. In such a network, higher-level properties of the graph are simply a result of lower-level properties and are not unexpected or unusual given the lower-level (actor and dyad) properties. The specific sense of "adequately modelled" that they have in mind is a statistical: the values taken on by higher-level properties are within the range expected given chance variation as constrained by the lower-level properties.

Holland and Leinhardt's claim echoes a fundamental idea in the social network view of social structure. This fundamental idea is that of *interlock*, that is, the idea that social structure is composed of "relations among relations" in Nadel's (1957) terms. Nadel's point is that social structure not only brings actors into relationships with one another but creates regular associations between the relationships themselves. This interlock idea is at the core of Lorrain and White's (1971) early and influential paper in modern social network analysis. Holland and Leinhardt give this notion a statistical interpretation. Nevertheless, the same basic argument is advanced: the essence of social structure is interlock as a web of contingency between the ties actors have with one another.

To assess statistically whether an observed amount of a higher-level property departs significantly from what would be expected by chance requires knowledge of the distribution of the property over an appropriately chosen population of random graphs (Holland and Leinhardt, 1979). The question is: is the observed value of the property significantly greater or less than expected relative to an appropriately constructed population of random graphs? To construct the population of random graphs, we think of drawing a sample of graphs but "conditioning" on certain properties of the observed graph. The specific conditional distribution provides a *baseline* against which properties of the observed graph are to be compared. For example, we might be interested in whether or not the observed amount of transitivity in a graph is greater than expected given the observed indegrees and outdegrees of nodes in the graph. The necessary conditional distribution for this particular question and others can be extremely complex. Often network researchers must resort to approximations such as those based on graph sampling described by Snijders (1991).

TABLE 1
Indegree and Outdegree Distribution in a Six Person Group

Actor	Indegree	Outdegree
A	0	5
B	1	4
C	2	3
D	3	2
E	4	1
F	5	0

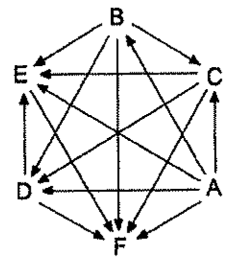
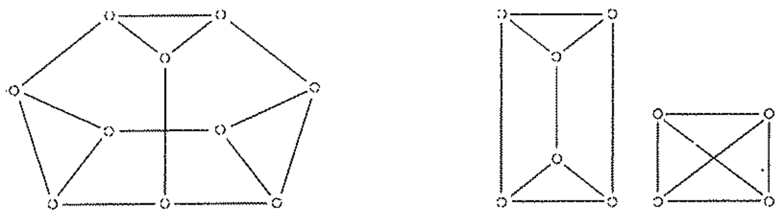


FIGURE 1. Graph for the indegree and outdegree distribution in Table 1.

We can illustrate the point with some simple examples. Consider the indegree and outdegree distribution for the six person group in Table 1. Such a distribution could result from a group in which the coded relation is "who dominates whom" and the group exhibits a linear hierarchy, that is, one actor dominates all others and is dominated by no one, a second actor is dominated only by the first actor and dominates the remaining actors and so on, until the actor at the bottom of the hierarchy dominates no actors and is dominated by all others. The graph for this group is given in Figure 1. Researchers are interested in the amount of transitivity exhibited in such graphs. One measure of this property is the number of triples of actors such that if i dominates j and j dominates k, then i dominates k. There are 20 unordered triples in a six person group. In the Figure 1 group, all 20 triples are transitive.

Is this degree of transitivity more or less than expected given the indegree and outdegree distribution of Table 1? In fact, it is exactly the level expected because there is only one graph consistent with Table 1's distribution of indegrees and outdegrees, the graph depicted in Figure 1. Thus, if the appropriate statistical baseline is all random graphs exhibiting the observed graph's distribution of indegrees and outdegrees, then the level of transitivity exhibited in Figure 1's graph is (trivially) not significantly different from chance expectations. In effect, we conclude that the level of transitivity, a property of triads and one of the higher-level properties, is completely determined by the lower-level properties of the actors. So against this baseline, we conclude that Figure 1 exhibits no social structure.

For a second illustration, consider the two graphs presented in Figure 2. Both graphs were generated using a biased net algorithm presented in Skvoretz (1990). Both graphs consist of ten nodes and exactly three undirected ties per node. One



Random: Transitivity bias $\sigma = 0.0$, degree = 3 for all nodes

Biased: Transitivity bias $\sigma = 0.5$, degree = 3 for all nodes

FIGURE 2. A random and a biased network.

graph, "random," is generated with no bias towards transitivity while the other graph, "biased," is generated with a fixed amount of bias towards transitivity. The random network is generated so that, subject to the condition that each node have only three ties, each pair of nodes has an equal and independent probability of having a tie present. Since ties are generated independently for all pairs of nodes, this graph, by design, has no social structure once we condition on its indegree and outdegree distribution. The biased graph is generated by setting the level of triad closure bias, σ, at 0.50. This means that in constructing the graph, the probability of adding a tie between i and j is increased if there exists a node (or nodes) k to whom both i and j have ties. Degree of closure is a triad level property and, by design, the biased graph exhibits a level of closure beyond that expected given the indegree and outdegree distributions. Therefore, relative to a population of random graphs with the same indegree and outdegree distribution, the biased graph exhibits social structure.

To demonstrate this statistically, we use a triad census program based on Holland and Leinhardt's (1970, 1976) work written by Walker and Wasserman (1988). This program counts the number of triads of various types defined by the ties connecting the three dyads composing a triad. In the general case, ties are directed arcs and a particular dyad is mutually choosing if two arcs are present, asymmetric if only one arc is present, or null if no arcs are present. Triad types are identified by a triple of numbers, the numbers of mutual (M), asymmetric (A), and null (N) dyads, and, when needed a letter. For instance, a type "102" triad contains one mutual dyad and two null dyads. Associated with the count of triads types is a summary statistic called τ. This statistic counts the number of transitive triples in the census, compares this number to the number expected under the random graph distribution, and standardizes the difference by the standard error of the expected number of transitive triples. The resulting value is usually assumed to have an approximate normal distribution with mean zero and variance unity.

Table 2 contains the triad counts for the graphs in Figure 2 along with an expected count based on a population of random graphs constrained to have the same numbers of M, A, and N dyads. The value of τ-statistic measuring transitivity is also given and may be interpreted as a z-score. Clearly, the biased graph (with $\tau = 1.37$)

TABLE 2
Triad Censuses for Figure 2 Graphs

Triad Type	Random	Biased	Expected
003	27	24	34.33
102	69	78	55.18
201	21	12	26.64
300	3	6	3.85
τ-statistic	−0.54	1.37	

departs much more from baseline expectations than does the random graph (with $\tau = -0.54$), and we would reject the null hypothesis for the biased graph at the 0.10 level. Thus we conclude that the biased graph exhibits social structure to a greater degree than does the random graph.

These illustrations demonstrate several points. First, there is a choice of baseline against which to assess whether a particular network exhibits social structure. Researchers using different baselines may draw different conclusions. Therefore, it is essential to determine which baseline is the appropriate one to use. Second, regardless of which baseline is used, the intuitive idea behind the assessment does not vary. The essential question is whether the values of lower-order properties of the graph can account for, within chance variation, the values of higher-order properties. Finally, the statistical detection of social structure in a network does not explain why that network exhibits social structure. That is, the statistical concern with the detection of social structure is a methodological rather than theoretical concern. In the next section, we describe a theoretical method that dynamically models the evolution of network structures in small groups and ask the question whether the networks that typically evolve exhibit social structure. Put another way, does the theoretical method we review constitute a theory of social structure?

THE THEORETICAL METHOD OF E-STATE STRUCTURALISM

A key problem in the current state of social network analysis involves the development of ideas and methods for the formal study of processes conceived in relation to the network construct. From one analytical standpoint, a social network is given and its form constrains the process, as in theoretical models that explain exchange network outcomes (Skvoretz and Willer, 1993). From another analytical standpoint, the actor nodes are given and the process generates a network of ties, as in theoretical models that explain the emergence of dominance hierarchies in animal groups. From still another analytical standpoint, both nodes and ties are subject to birth and death processes.

Models of network evolution and transformation can take a variety of forms, depending upon the patterning of choices made among alternative conceptualizations. In particular, three conceptual questions help frame the choice set and illuminate the specific character of the E-state structuralism approach to the concept of social structure.

First, processes have transient states and settled states. A concept of social structure might or might not identify it with settled states. Following Homans (1950), we adopt the conceptual rule that "social structure" pertains to settled states of social networks. This means, in turn, that processes of transformation do not always produce social change. Social change occurs when one settled state (social structure) is transformed into another. Actually, for best coordination to sociological intuition, the change should involve shift to a social structure of a new type (Fararo, 1989: Ch.2). Given this conceptual decision, the expression "evolution of a social network" is the comprehensive term referring to all changes of states in a social network. Thus, the phrase covers, in our interpretation, not only "social change"—transformations carrying one social structure into another social structure of different type—but also any and all over-time transformations of a social network. In particular, the phrase covers a build-up of a social structure from initial conditions in which no structure is exhibited in the initial network state.

Second, one can think of the ties in social networks as observable relational data or as unobservable states of relational orientation. The method of E-state structuralism takes the second route, while not denying the legitimacy of models based on treating relations (e.g., institutionally defined ties) as objective features of the situation of action. Thus, for E-state structuralism, a social network is an unobservable complex of relational orientations. E-state structuralism is based on the idea of a "theoretical construct" (Berger, Cohen, Snell, and Zeldtich, 1962). This unobservable character of the approach raises the question of whether and how such models can have any explanatory function and how such models are coordinated to data, leading to the third choice.

Third, one can think of the relation between a theoretical construct model and observable data in two distinct ways. One way is to gather and interpret data as indicators of the constructs, "operationalizing" it. The other way regards the relevant observable data as functions of processes involving the unobservable constructs. E-state structuralism takes the second route of relating constructs and data. This last feature becomes clearer if we write down the canonical form of a state-determined discrete system or finite-state automaton in the form:

$$S' = f(S,i) \qquad o = g(S,i)$$

The first expression says that the next state S' of the system is a function of the current state and the input. The second expression says that the output of the system depends on its current state and the input. The i term can be thought of as "information" or perception. The o term can be thought of as observable behavior. In E-state structuralism, S is the state of the a network of relational orientations. Theoretical models are tested by what they say about the observed sequences of behavior, the o-term. Having discussed the logical patterning of choices among alternatives that have guided the specification of E-state structuralism as a theoretical method, we now turn to the details of the approach.

E-state structuralism proposes dynamic models by which dyadically based social psychological processes aggregate to produce stable power and prestige orders in groups of arbitrary size *via* the development of networks of ties among actors. It synthesizes concepts and ideas drawn from expectation states theory and from social network analysis. We first summarize the relevant ideas from expectation states theory. We then describe how these ideas are linked to the creation of ties among actors and the evolution of networks. We use basic E-state model axioms and example realizations to illustrate our remarks.

Expectation States Theory. Expectation states theory developed from a concern with the emergence of power and prestige orders in task-oriented groups of arbitrary size. These power and prestige orders were composed of stable distributions of opportunities to perform, of evaluations of contributions, of overall ratings of group members, and of relative influence of group members on final decisions of the group. Power and prestige orders emerged in studies of jury deliberations (Strodtbeck, James, and Hawkins, 1957; Strodtbeck and Mann 1956), bomber crews (Torrance, 1954), and professionals at a conference (Hurwitz, Zander, and Hymovitch, 1953). The seminal paper of Berger, Cohen, and Zelditch (1966) reviews these and other studies of power and prestige orders and how status external to the group affects a member's position in its internal power and prestige order.

Although expectation states theory was initially concerned with these processes as they occurred in groups of any size, large groups were, as a matter of fact, difficult to study experimentally. Furthermore, the social psychological processes postulated by expectation states theorists to account for the emergence of power and prestige orders—processes such as diffuse status activation, burden-of-proof process, and so on could be studied in a dyadic context, that is, a context in which a subject interacts with or is under the impression they are interacting with just one other person. Much of the advance in expectation states theory came from studying subjects in a dyadic context in which the expectations *vis-a-vis* an alter have been manipulated via the introduction of diffuse status differences or differences in specific performance characteristics (Webster and Foschi, 1988; Balkwell, 1991). However, this exclusive focus on behavior in dyads set aside the problem of how dyadic effects may or may not aggregate to yield coherent status effects in larger groups. E-state structuralism takes a more global view of the aggregation problem.

The basic theoretical construct of E-state structuralism is the concept of an "E-state." This idea is abstracted from the core assumptions of the expectation states research program (Berger, Wagner, and Zelditch, 1985). Formally, the E-state idea requires a distinction between the state of a system and its behavior with the accompanying idea that to explain the latter we must invoke the former. In terms of studying human behavior, what Weber called "observational understanding" yields a behavioral level of analysis: the actor does such-and-such an observationally recognized thing. However, for explanatory purposes, we often have to introduce "orientations." These entities are unobservable, but have observable consequences. They also change more slowly than the observationally understood behaviors. The orientations are connected to the observationally understood behaviors in two ways. First, they develop from the interplay of such behaviors and the situational responses to them. Second, they influence the production of such behaviors on later occasions of interaction. It is precisely this second point that ensures the unobservable orientations have observable consequences.

Orientations may be causally efficacious and thus postulated by an observer even if actors are unaware of holding such orientations. For instance, expectation states researchers recognize that the process of forming expectations based on status usually occurs without conscious thought. When gender acts as such a status dimension in, say, jury deliberations, it is not because women think to themselves "my partner is a man and men are generally more capable, therefore I will defer to his suggestions." Rather the orientation that creates the propensity to defer lies below conscious awareness and the appropriate behavior follows from this tacit adopted orientation.

So described, these orientations formally render the construct "expectation states" as it has guided research for more than three decades. The initial use of the term "E-state" in Fararo and Skvoretz (1986) occurs in an effort to model the formation of dominance structures in animal groups and adapts this type of construct to animal interactions. Expectation states are generally framed in an explicitly Meadian symbolic interactionist way in terms of self-other concepts and evaluations. Our "E-state" construct generalizes the concept of an expectation state by removing its embeddedness in symbolic interaction. In the method of E-state structuralism, the extent to which explanations require the symbolic interaction framing remains an open question. The method clearly does not presume its importance.

Fararo and Skvoretz (1986) also take the novel step of deploying the E-state construct in a social network context. By postulation, each actor has a relational E-state toward others in the network. Behavior toward others then depends upon the E-state but is conceptually and formally distinct from it. The social network is a set of actors together with the configuration or pattern of relational E-states. By itself, such a network is not an observable. It is a state-space concept or construction. At any time, the observable behaviors or interactions depend upon it (in a way to be specified in a formal model) and, on the other hand, these states arise out of the consequences of interactions for the actors. This conception of social networks described in terms of relational E-states defines the general idea of "E-state structuralism." To construct a definite theoretical model instantiating this theoretical method involves setting out a set of axioms about a process of interaction conceived in terms of the interplay between these two levels, the level of relational E-states codified as ties in a network and the level of observable behaviors.

The *basic E-state model*, constructed by Fararo and Skvoretz (1986) deals with the classical problem of dominance structure formation in the barnyard and the fact that the structures tend to be highly transitive (Chase, 1974; Mazur, 1973; Freeman, Freeman, and Romney, 1992). Chase's (1982) experiments on this problem show how a triadic focus is necessary to explain the formation of highly transitive structures. Fararo and Skvoretz embed this idea in a formal model that has two mechanisms by which dominance ties are formed. Conceptually, dominance ties refer to pairs of complementary E-states in which one organism expects to dominate another and the second expects to defer to the first. Such a tie may develop between two organisms, firstly if one attacks the other. This is a "victim" effect. The victim effect by itself, however, does not ensure high degrees of transitivity. A second mechanism, the "bystander" effect, is required. By virtue of this mechanism, ties may form between bystanders to an agonistic encounter and its participants. Bystanders form such E-states by mirroring what they observe: the model postulates that in observing an attack a bystander may form a deference orientation to the attacker (and the attacker, a dominance orientation to the bystander) and may form a dominance orientation to the victim (and the victim, a deference orientation to the bystander). By virtue of this mechanism, high probabilities of transitivity follow logically from the axioms of the model.

It is worth presenting the formal assumptions of the basic model to see more clearly how networks of dominance ties evolve in groups. Let xNy indicate that no tie exists between x and y and let xDy indicate that a dominance tie from x to y exists. The axiom set of the basic model numbers five:

Axiom 1 At $t = 0$, every pair of actors is in state N.

Axiom 2 For any x and y, if xDy at t then xDy at $t + 1$.

Axiom 3 At any t, if a pair of actors is in state N and if x attacks y, then $\Pr(xDy) = \pi$.

Axiom 4 If x attacks y, then (a) $\Pr(xDz) = \theta$ for any bystander z and the attacker x such that xNz; and (b) $\Pr(zDy) = \theta$ such that for any bystander z and the victim y such that yNz.

Axiom 5 Let T be the number of ordered pairs at time t such that either xDy or xNy and let xAy denote the event that x attacks y at time t. Then if xDy, $\Pr(yAx) = 0$, otherwise, $\Pr(xAy) = 1/T$.

The first axiom states that there are no pre-established dominance ties in the group and the second, that dominance ties once formed are stable. The third axiom stipulates the victim effect with the probability of tie formation given by a parameter denoted π, $0 \leq \pi \leq 1$. The fourth axiom describes how attacks may affect the orientation of bystanders, z, to the attacker x and to the victim y in terms of a parameter of tie formation denoted θ, $0 \leq \theta \leq 1$. Here those bystanders who have not already formed ties with respect to attackers or victims may do so in each case with the probability given by θ. The final axiom describes how the evolving network of dominance ties affects the probability that attacks occur between particular pairs. Specifically, if xDy holds, then y never attacks x, but x may attack y with a probability that depends on the number of ordered pairs such that either xDy or xNy.

Figure 3 displays one example of the evolution of a network of dominance ties in a group of six organisms. For this illustration, π and θ are set at .50. The network evolves from a null state in which no ties exist between pairs of actors to a state in which 15 dominance ties have formed. After the first agonistic encounter in which 2 attacks 1, three ties form by way of bystander effects: 2 over 4, 2 over 6, and 3 over 1. Organism 1 then attacks organism 6 and four ties form via bystander effects: 1 over 2, 1 over 4, 3 over 6, and 5 over 6. The third attack, organism 4 on organism 5 results in further evolution of the network as three ties are added via bystander effects: 1 over 5, 2 over 5 and 3 over 5. The next three attacks, 2 to 5, 2 to 4, and 2 to 5, result in no change in the network, i.e., no new dominance ties are created. The seventh attack of 1 on 4 results in four ties added again via bystander effects: 3 over 4, 5 over 4, 1 over 6, and 6 over 4. No change results from the eighth attack of 3 on 1. The ninth attack of 3 on 2 evokes the final dominance tie via a victim effect, namely, 3 over 2. The network has evolved to an absorbing, equilibrium state in which further attacks may occur, but according to the axioms of the model, these attacks cannot alter the configuration of ties.

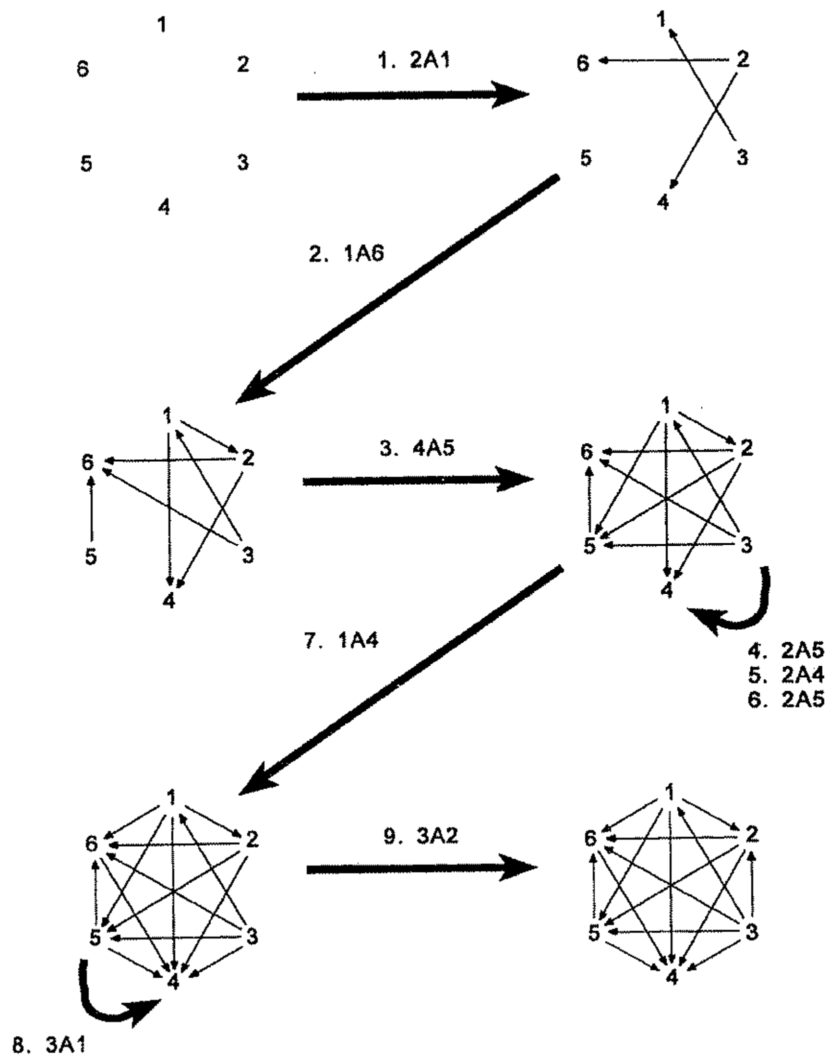


FIGURE 3. Evolution of a network of dominance ties.

The example has several interesting points. First, the end result is a completely transitive hierarchy of dominance relations with organism 3 on the top and organism 4 on the bottom. Second, the network evolves comparatively rapidly, due to the relatively substantial values of the key parameters that control tie formations. Third, 14 of the 15 ties occur via bystander effects and only the last one, via the victim. When π and θ are of similar magnitude, the preponderance of bystander effects is typical: each participation can create only one tie via the victim effect, but a maximum of 8 ties via bystander effects (there are four bystanders to each attack and, as set out in Axiom 4, a tie may form for each bystander with respect to the attacker and to the victim). Finally, the illustration clearly shows an important consequence of the bystander assumption, namely, that events occurring in different dyads—attacks and the formations of ties—are not necessarily independent: attacks in one dyad can affect the outcome of tie formations in other dyads.

More recent work on dominance structure formation has dropped some of the simplifying assumptions of the basic model (Fararo, Skvoretz and Kosaka, 1994) and has extended the domain of the first model to task oriented discussion groups of humans (Skvoretz and Fararo forthcoming). In the first extension, it is no longer assumed that complementary E-states form instantaneously. Thus, actor A may form a dominance orientation to actor B and so may B toward A. Such a relation is taken to be one of conflict. On the other hand, A and B may form deferential orientations to each other, a type of relation anticipated by Goffman's (1967) analysis of the typical mutual deference found in ritual interactions. This extension may be termed a *contingent complementarity E-state model.*

One variant of the contingent complementarity model addresses another problem with the basic E-state model, namely, its scope restriction to relatively small groups. If the group is even moderately large, it is no longer reasonable to assume that only one attack can occur in a given time interval. Instead, attacks may overlap in time. Following recent usage in other disciplines, this complication is termed a "parallelism" element, wherein numerous instances of processes of the same generic type may be going on at the same time in various subsets of the overall group. Moreover, this feature makes the bystander mechanism more complex. First, there may be any number of bystanders, not just one or two. Second, bystanders may shift roles, attacking each other. Third, and this was already possible in the simplest cases, a bystander may join in an ongoing attack to "gang up" on the victim or, on the contrary, the bystander may come to the aid of the victim. These and various other such interactive events introduce complexities that require that their properties be studied by simulation rather than formal analytical derivations, even when the models are set out in axiomatic terms. This variant yields a *contingent complementarity model with parallelism.*

In the second extension to task oriented discussion groups of humans, the tie formation axioms of the basic model are modified. The modifications reinterpret the links between actors as relations of "status precedence" rather than dominance and incorporate differential standing on exogenous status characteristics into the tie formation process. The interactive events that drive the tie formation process are participations in task discussions that one actor directs towards another. Bystander mechanisms play a key role in the formation of precedence orders as actors react to the participations others direct to one another. The aim of the model is to account for participation differentials in task group discussions and their relationship to actors' ranks on exogenous status characteristics as a function of the placement of actors in the internal "status precedence" order of the group. This extension may be termed an *E-state precedence model.*

The dynamics of the basic model and its extensions ensure that networks evolve from the null state of no ties to some absorbing state in which all pairs are linked by some form of relationship. Figure 3 illustrates for the basic model. In these absorbing states, the process of network evolution is in equilibrium: actors continue to emit behavior relevant to the formation of ties but the dynamics ensure that no

TABLE 3
Number of Tournaments for Unlabeled Graphs of Size g

g	Number of Tournaments
3	2
4	4
5	12
6	56
7	456
8	6880

events occur to add or subtract any existing ties. Our concern with E-state structuralism as a theory of social structure is a concern directed to these equilibrium networks: under what conditions do the equilibrium networks that evolve under E-state assumptions exhibit social structure?

NETWORKS EVOLVED BY E-STATE STRUCTURALISM MODELS

We consider first the equilibrium networks that evolve in the basic E-state model and the E-state precedence model and then examine networks that evolve from the contingent complementarity model (without parallelism). The networks that evolve under the first two types of models are simpler because E-states are assumed to be complementary. Therefore, each (i, j) dyad can take on only one of two relational forms at equilibrium: either i dominates or has precedence over j or j dominates or has precedence over i. In a contingent complementarity model, a dyad can be in any of four relational states and this complicates the analysis. In the simpler models, despite the different terms used to describe the nature of the ties linking actors, the equilibrium networks that evolve are *tournaments*, directed graphs in which "every pair of points are joined by exactly one arc" (Harary and Palmer, 1973: 5). Table 3, adapted from Harary and Palmer (1973: 245), lists the number of tournaments for group sizes $g = 3$ to $g = 8$.

For such networks, two higher level properties of interest can be defined. The first, hierarchy, is a global property exhibited by those tournaments in which for all x, y, and z, if xDy and yDz, then xDz. In all other tournaments, there is some triple x, y, and z for which xDy and yDz but zDx. For each group size, there is only one tournament that is a perfect hierarchy, all others have one or more intransitive triples. The second property pertains to triples of actors and refers to the distribution of these triples over types of triads. In the case of tournaments we need to consider only two types of triads: the transitive triad and the cyclical triad. We use Holland and Leinhardt's (1977) notation of 030T and 030C to denote these types. In their MAN notational scheme the first number is the number of pairs in the triad exhibiting mutual ties (links in both directions), the second is the number of pairs exhibiting an asymmetric pattern (a link in only one direction), and the third is the number of pairs not tied at all ("null" pairs). Letters are used to differentiate between non-isomorphic tie patterns with the same counts. In tournaments, only asymmetric ties occur, hence only the 030T and the 030C triads are possible.

We use these higher order properties to assess whether the basic E-state model evolves networks that exhibit social structure. To complete the assessment we need to select a baseline random graph distribution for the comparison. That is, we need to examine whether the probability of a complete hierarchy and the distribution of triad types in the evolved networks depart significantly from expectations derived from the baseline random graph distribution. The key question now becomes: what is the appropriate baseline distribution? In particular, is it a distribution that conditions on the indegree and outdegree distributions of the nodes or is it a distribution that conditions on some property of dyads?

For an answer, we must inspect the assumptions of the model. If these assumptions constrain the indegree or the outdegree distribution of ties in any way, then the appropriate baseline random graph distribution should be conditioned on those constraints. But the basic E-state model does not begin with an assumption that specifies, for each organism, how many others it will dominate and how many others will dominate it. Rather the basic assumption is merely that a dominance tie will form in each pair of organisms given a sufficient length of time for interaction. Thus the proper baseline is not one that conditions on a particular indegree and outdegree distribution, but one that assumes that ties in dyads will take on a particular distribution, namely, asymmetric with probability 1.0. That is, the appropriate baseline conditions on the numbers of M, A, and N dyads where M = N = 0, and A = $\binom{g}{2}$. In short, the appropriate baseline distribution is one in which for each pair of actors x and y, the tie is as likely to be from x to y as from y to x. For this baseline, the chance probability of a complete hierarchy and the distribution of triad types are easily calculated. In particular, the chance probability of a hierarchy in a group of size g is:

$$\Pr(\text{Hierarchy}) = \frac{g!}{2^{\binom{g}{2}}}.$$

At all group sizes, furthermore, 75% of all triples are 030T triads while 25% are 030C triads. Thus the baseline chance expectation for triad types does not differ by group size.

Simulation studies summarized in Table 4 show how the probability of hierarchy and the distribution of triad types vary with the E-state parameters π and θ in groups of size 6. The statistics are based on 100 runs at each combination of parameter values. The main result is quite obvious: the basic E-state model evolves highly transitive networks with substantial probabilities of hierarchy only if the bystander effect is non-zero. The same finding holds for the E-state precedence model (see Skvoretz and Fararo, forthcoming.). In the basic E-state model, bystander effects contingently create ties between bystanders and attackers and between bystanders and victims that mirror the direction of an attack. In the E-state status precedence model, bystander effects operate in essentially the same way but the process is more complex because directed participations are interpreted in terms of any existing ties between actors. Nevertheless, the same conclusion holds: without bystander effects, transitivity only occurs at chance levels. Therefore, the theoretical method of E-state structuralism is a theory of social structure precisely because it incorporates a mechanism that creates dependence between dyads with respect to the occurrence of ties.

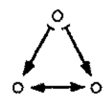
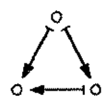
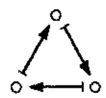
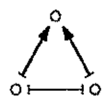
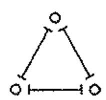
TABLE 4
Distribution of Triads and Probability of Hierarchy in Basic Model Outcome Networks for $g = 6$

Parameters		Triad Types		Hierarchy
π	θ	030T	030C	Prob
1.00	1.00	1.0000	0.0000	1.000
0.75	1.00	0.9935	0.0065	0.950
0.50	1.00	0.9865	0.0135	0.880
0.25	1.00	0.9780	0.0220	0.780
0.00	1.00	0.9830	0.0170	0.780
1.00	0.75	0.9705	0.0295	0.660
0.75	0.75	0.9615	0.0385	0.630
0.50	0.75	0.9540	0.0460	0.500
0.25	0.75	0.9450	0.0550	0.480
0.00	0.75	0.9210	0.0790	0.410
1.00	0.50	0.9385	0.0615	0.410
0.75	0.50	0.9270	0.0730	0.370
0.50	0.50	0.9080	0.0920	0.320
0.25	0.50	0.8930	0.1070	0.260
0.00	0.50	0.9015	0.0985	0.290
1.00	0.25	0.8855	0.1145	0.250
0.75	0.25	0.8900	0.1100	0.190
0.50	0.25	0.8585	0.1415	0.140
0.25	0.25	0.8595	0.1405	0.200
0.00	0.25	0.8530	0.1470	0.180
1.00	0.00	0.7555	0.2445	0.020
0.75	0.00	0.7440	0.2560	0.020
0.50	0.00	0.7655	0.2345	0.020
0.25	0.00	0.7625	0.2375	0.020
Baseline		0.7500	0.2500	0.022

Assessing whether the more complex E-state model, the contingent complementarity model, evolves networks with social structure is more involved. At equilibrium dyads can take on more than just an asymmetric state. This means the simple random graph population conditioned on the numbers of M, A, and N dyads cannot be used for the referent baseline distribution. We need to generalize this distribution to reflect the greater variety of dyadic outcomes.

In the contingent complementarity E-state model, a particular (i, j) dyad can be connected in any one of four ways at equilibrium. First, both actors may have dominance orientations to one another. Fararo, Skvoretz, and Kosaka (1994) call this a "conflict" tie. Second, both actors may have deference orientations to one another. Fararo, Skvoretz, and Kosaka (1994) label this a "mutual" tie. The third and fourth types of connection are asymmetric dominance relations distinguished only by direction. In these dyads, one actor holds a dominance orientation towards the other

TABLE 5
Stable Triads in Complex E-State Models*

Type	Graph	Number of Labeled Isomorphic Triples
0201D		3
0300T		6
0300C.		2
1200U		3
3000		1

* $\leftrightarrow$ is a C tie; $\vdash\!\!\dashv$ is an M tie; and $\longrightarrow$ is an A tie.

who holds a deference orientation to the first. Thus because each dyad can be in one of four states at equilibrium, an (i, j, k) triple can be in one of $4^3 = 64$ different states at equilibrium. In contrast, for the basic E-state model, each triple can be in only one of $2^3 = 8$ different states because each dyad has only two possible asymmetric states. These 8 states, in turn, reduce to two non-isomorphic structures: 030T and 030C. The first type has 6 different realizations and the second, two different realizations.

A similar reduction applies to the networks generated by the contingent complementarity model. To see this, we follow Fararo, Skvoretz, and Kosaka (1994) and extend the MAN notation to an MANC notation, in which C refers to the number of "conflict" dyads, M to the number of "mutual" (deference) dyads, A to the number of asymmetric dyads and N to the number of null ties in a triad. (The value of N is necessarily 0 for equilibrium networks). Letters distinguish non-isomorphic triads with the same count. There are 16 non-isomorphic triads composed of these ties. Table 5 labels and diagrams five of these 16 types and lists how many of the 64 different triple states exhibit each type. For instance, only one of the 64 potential configurations is of the all mutual deference type 3000. However, there are three different ways that type 0201D can be realized, depending on which node dominates the other two. Overall 15 of the possible 64 outcomes are one of the five types listed in Table 5.

These types are singled out because they constitute the only absorbing states of a triad under the contingent complementarity E-state model. That is, for a non-trivial model (one in which all key parameters are not zero) the dynamics of the process by which ties are formed ensures that only the five types in Table 5 are stable. Stability here does not mean stasis. Rather it means dynamically stable in that the structure of ties constrains the behaviors emitted by actors to those that cannot disturb the structure, that is, change actors' orientations to one another.

In the simple random graph distribution conditioned on the numbers of M, A, and N dyads, when all dyads are of type A, the random distribution of triads is identical for groups of all sizes. In particular, the expected frequencies are proportional to the number of ways of realizing the 030T and the 030C triads. Thus we expect the triad census for the random graph distribution to yield 75% transitive triads and 25% cyclical triads for any size group. For instance, the baseline expectation for groups of size $g = 6$ groups with 20 triads is 15 transitive triads and 5 cyclical triads.

We apply a similar logic to the 16 non-isomorphic states of a triad under the contingent complementarity model. First, the distribution of triads for the appropriate random graph distribution (conditioned now on the numbers of M—mutual deference, A, N, and C ties) will be identical for groups of any size g. Second, the expected frequency of each triad will be proportional to the number of particular states that realize each of the 16 triad types. It is this generalized triad census against which we can assess whether networks evolved under the contingent complementarity model have social structure.

Simulation studies of groups of size 6 reported in Table 6 present the probability of hierarchy and the distribution of triads found in networks that evolve under different parameter values for this more complex E-state model. The last line of the table summarizes the baseline expectations. Clearly, the networks that evolve under this model have triad distributions that depart dramatically from the baseline distribution. Moreover, the departures differ among each other depending on the parameter values. This means that the contingent complementarity E-state model not only evolves networks that exhibit social structure, but under different parametric conditions, evolves different types of social structure. Put in other terms, not only does the model evolve relational interlock, it evolves different kinds of relational interlock.

CONCLUSION

E-state structuralism relies on social-psychological principles to model the evolution of a network of ties in a group of initially unconnected individuals. In contradistinction to recent modeling trends that emphasize a purposive actor framework for dynamic network models (Snijders, 1996), E-state models propose that some networks evolve as organisms adopt expectational orientations towards one another that may not be consciously held or recognized. The E-state models and the purposive actor models are not inconsistent. For instance, Fararo (1989: Section 3.3.2)

TABLE 6
Triad Distribution and Probability of Hierarchy in Complex Model Outcome Networks for $g = 6$*

Parameters				Triad Type						H
π	θ	ρ	γ	0201D	0300T	0300C	1200U	3000	Others	Prob
1.00	1.00	0.75	0.50	0.000	0.985	0.015	0.000	0.000	0.000	0.78
1.00	1.00	0.75	0.00	0.000	0.609	0.000	0.351	0.040	0.000	0.05
1.00	0.50	0.75	0.50	0.000	0.954	0.046	0.000	0.000	0.000	0.53
1.00	0.50	0.75	0.00	0.000	0.853	0.046	0.100	0.002	0.000	0.16
1.00	0.00	0.75	0.50	0.000	0.737	0.263	0.000	0.000	0.000	0.03
1.00	0.00	0.75	0.00	0.000	0.746	0.254	0.000	0.000	0.000	0.03
0.50	1.00	0.75	0.50	0.000	0.968	0.032	0.000	0.000	0.000	0.66
0.50	1.00	0.75	0.00	0.000	0.664	0.004	0.306	0.026	0.000	0.04
0.50	0.50	0.75	0.50	0.000	0.927	0.073	0.000	0.000	0.000	0.44
0.50	0.50	0.75	0.00	0.000	0.832	0.034	0.129	0.005	0.000	0.18
0.50	0.00	0.75	0.50	0.000	0.746	0.254	0.000	0.000	0.000	0.02
0.50	0.00	0.75	0.00	0.000	0.764	0.236	0.000	0.000	0.000	0.02
0.00	1.00	0.75	0.50	0.000	0.951	0.049	0.000	0.000	0.000	0.56
0.00	1.00	0.75	0.00	0.098	0.603	0.005	0.270	0.024	0.000	0.06
0.00	0.50	0.75	0.50	0.000	0.928	0.072	0.000	0.000	0.000	0.42
0.00	0.50	0.75	0.00	0.046	0.776	0.028	0.145	0.005	0.000	0.14
Baseline				0.047	0.094	0.031	0.047	0.015	0.766	0.02

*The parameters π and θ govern the formation of orientations, the first for attackers and victims and the second for bystanders *vis-a-vis* attackers and victims. The ρ parameter governs the formation of complimentary orientations while the parameter γ governs whether the participants to an attack are aware of bystanders and thus at risk of forming orientation E-states *vis-a-vis* the bystanders.

considers a form of integration in which expectation states "control" the formation of goal states. But integration is possible in the other direction as well with goal states influencing expectational orientations. However, the models we explore do not have this "goal states" substructure, yet the networks that evolve display socially structured tie patterns. The nonpurposive orientations affect behavior and the recursive process cycling between behavior and structure produces networks that display "social structure" via the creation of dependencies between dyads. In this way, the models avoid "the assumption of independence between dyads [which] excludes a priori almost all sociologically interesting interactions" (Snijders, 1996: 150).

Demonstration of this claim, however, makes it clear how important the choice of a baseline is against which to assess significant departures in higher-order network properties. Just because a particular property is a lower-order network property, as the indegree and outdegree distribution is relative to triad transitivity, does not mean that the appropriate baseline random graph distribution must necessarily be conditioned on it. Theoretical considerations in relation to the models under evaluation must determine the selection of the appropriate baseline distribution. For E-state models, the appropriate baseline conditions on the dyad census and assesses whether the observed distribution of triads departs significantly from that expected by chance given the dyad distribution.

Finally, it is important to note that E-state structuralism models produce networks that exhibit social structure in this technical and statistical sense, but only when nonzero bystander effects are present. If bystanders do not attend to interactive events involving others or are not influenced in their relations to others by these events, the networks that emerge from interaction do not exhibit social structure. Thus, bystander attention and reaction are crucial to the emergence of social structure. In closing we suggest this conclusion applies to other theoretical models for the network evolution and development. That is, such models will produce networks that exhibit social structure only if they too deploy some form of a bystander mechanism.

REFERENCES

Balkwell, J. W. (1991) Status characteristics and social interaction: Assessment of theoretical variants. In E. J. Lawler, B. Markovsky, C. Ridgeway and H. A. Walker (Eds.), *Advances in Group Processes* **8:** 135–176, Greenwich, CT: JAI Press.

Berger, J., Cohen, B. P., Snell, J. L., and Zelditch, Jr., M. (1962) *Types of Formalization in Small Group Research*, Boston: Houghton-Mifflin.

Berger, J., Cohen, B. P., and Zelditch, Jr., M. (1966) Status characteristics and expectation states. In J. Berger, M. Zelditch, Jr., and B. Anderson (Eds.), *Sociological Theories in Progress* **1:** 29–46, Boston: Houghton-Mifflin.

Berger, J., Wagner, D. G., and Zelditch, Jr., M. (1985) Introduction: Expectation states theory: Review and assessment. In J. Berger and M. Zelditch, Jr. (Eds.), *Status, Rewards, and Influence*, San Francisco: Jossey-Bass.

Chase, I. (1982) Dynamics of hierarchy formation: The sequential development of dominance relationships. *Behavior* **80:** 218–240.

——— (1974) Models of hierarchy formation in animal societies. *Behavioral Science* **19:** 374–382.

Fararo, T. J. (1989) *The Meaning of General Theoretical Sociology*, New York: Cambridge University Press.

Fararo, T. J., and Skvoretz, J. (1986) E-state structuralism: A theoretical method. *American Sociological Review* **51:** 591–602.

Fararo, T. J., Skvoretz, J., and Kosaka, K. (1994) Advances in E-state structuralism: Further studies in dominance structure formation. *Social Networks* **16:** 233–265.

Freeman, L. C. (1989) Social networks and the structure experiment. In L. C. Freeman, D. R. White and A. K. Romney (Eds.), *Research Methods in Social Network Analysis*, Fairfax, VA: George Mason, 11–40.

Freeman, L. C., Freeman, S. C., and Romney, A. K. (1992) The implication of social structure for dominance hierarchies in red deer, *Cervus elaphus L. Animal Behaviour* **44:** 239–245.

Goffman, E. (1967) *Interaction Ritual*, New York: Doubleday.

Harary, F., and Palmer, E. M. (1973) *Graphical Enumeration*, New York: Academic Press.

Holland, P. W., and Leinhardt, S. (1970) A method of detecting structure in sociometric data. *American Journal of Sociology* **76:** 492–513.

——— (1976) Local structure in social networks. In D. R. Heise (Ed.), *Sociological Methodology—1976*, San Francisco: Jossey-Bass, 1–45.

——— (1979) Structural sociometry. In P. W. Holland and S. Leinhardt (Eds.), *Perspectives on Social Network Research*, New York: Academic Press, 63–83.

Homans, G. C. (1950) *The Human Group*, New York: Harcourt, Brace & World.

Hurwitz, J. I., Zander, A. F., and Hymovitch, B. (1953) Some effects of power on the relations among group members. In D. Cartwright and A. F. Zander (Eds.), *Group Dynamics: Research and Theory*, Evanston, IL: Row, Peterson and Co., 483–492.

Lorrain, F., and White, H. C. (1971) Structural equivalence of individuals in social networks. *Journal of Mathematical Sociology* **1:** 49–80.

Mazur, A. (1973) A cross-species comparison of status in small established groups. *American Sociological Review* **38:** 513–530.

Nadel, S. F. (1957) *The Theory of Social Structure*, London: Cohen and West Ltd.

Skvoretz, J. (1990) Biased net theory: Simulations, approximations and observations. *Social Networks* **12:** 217–238.

Skvoretz, J., and Fararo, T. J. (forthcoming) Status and participation in task groups: A dynamic model. *American Journal of Sociology* (forthcoming).

Skvoretz, J., and Willer, D. (1993) Exclusion and power: A test of four theories of power in exchange networks. *American Sociological Review* **58:** 801–818.

Snijders, T. A. B. (1991) Enumeration and simulation methods for 0–1 matrices with given marginals. *Psychometrika* **56:** 397–417.

——— (1996) Stochastic actor-oriented dynamic network models. *Journal of Mathematical Sociology* **21:** 149–172.

Strodtbeck, F. L., James, R. M., and Hawkins, C. (1957) Social status in jury deliberations. *American Sociological Review* **22:** 713–719.

Strodtbeck, F. L., and Mann, R. D. (1956) Sex role differentiation in jury deliberations. *Sociometry* **19:** 3–11.

Torrance, E. P. (1954) Some consequences of power differences on decision making in permanent and temporary three-man groups. *Research Studies*, State College of Washington **22:** 130–140.

Walker, M. E., and Wasserman, S. S. (1988) *Triads PC Network Analysis Package for Assessing Local Structure: User Manual*, University of Illinois.

Wasserman, S. S., and Faust, K. (1994) *Social Network Analysis: Methods and Applications*, New York: Cambridge University Press.

Webster, Jr., M., and Foschi, M. (Eds.) (1988) *Status Generalization: New Theory and Research*, Stanford, CA: Stanford University Press.

Zeggelink, E. (1993) *Strangers into Friends: The Evolution of Friendship Networks Using an Individual Oriented Modeling Approach*, Amsterdam: Thesis Publishers.