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COMPARISON OF METHODS FOR POSITIONAL ANALYSIS: STRUCTURAL AND GENERAL EQUIVALENCES *

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This paper explores the conceptualization and measurement of social position in relational data. It is argued that social positions are evidenced in the interactions among individuals, which are encoded in measured social relations. Given a set of measured relations the task is to reveal social positions which consist of groups of individuals with similar patterns of relations. Methods based on two alternative approaches are discussed. The first set of approaches is based on *structural equivalence*, and locates groups of similar individuals based on the extent to which they share identical ties with identical others. A second set of approaches, here called *general equivalences*, locates groups of similar individuals based on their sharing of "types" of ties with "types" of others. Procedures based on these different approaches are described and applied to actual data and to a constructed example. Results suggest that these different approaches identify different kinds of social groups. It is argued that structural equivalence is an unsuitable basis for analysis of relational data if the goal is detection of social positions.

1. Introduction

The related notions of social position, social role and social status provide a common motivation for a large body of social networks research. Methods based on these notions have been referred to as positional analysis techniques (Burt 1976), and are distinguished by the dual foci on the similarity of actors with respect to their relations, and of relations with respect to their occupants. In the past decade positional analysis has come to occupy a prominant spot in social networks

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research. This is evidenced by the use of position, role, or status as a general description for a body of analysis techniques (Burt 1980; Knoke and Kuklinski 1982), as a theoretical or explanatory construct (Burt 1978, 1983, 1987; Friedkin 1984), and as the theoretical motivation for the development of models and computational procedures (H. White *et al.* 1976; Breiger *et al.* 1975; Sailer 1978; Burt 1976; D. White and Reitz 1983, 1985; Winship and Mandel 1983; Breiger and Pattison 1986).

One outcome of the attention to position in social networks research is an explosion of methods to accomplish "positional analysis". The models, formal definitions, procedures and techniques that have been proposed have in common the notions of social position, social role, or social status as either their explicit or implicit theoretical foundation, and structural equivalence or some generalization of it as a point of formal grounding. This common motivation has led to two issues which require consideration: first is the assumption that methods in fact accomplish positional analysis (that is, the results correspond to sociological positions) and second is the implicit assumption that in empirical analysis the methods are in some sense substitutible.

As I will discuss below, while these methods do find common motivation in the notion of social position, they rely on quite different assumptions about how relational properties of actors or other social units suggest social positions. Since selection of an appropriate method for a given application depends on a match between the theoretical construct and its realization in the particular analytic procedure, specification of the formal basis for a procedure and the mapping between that definition and the theoretical construct is critical.

This paper examines methods based on two quite different approaches to positional analysis: *structural equivalence*, and what I refer to here as *general equivalence*. The goal is to present results of methods based on different approaches to allow comparison of the formal basis of the methods and of their outcomes. Several questions are addressed: What is meant by "position" in relational data? Are the results based on different approaches convergent enough to be considered substitutable? Do the results of these methods correspond to what is theoretically, or intuitively, meant by social position? What are the formal properties of the methods that lead to kinds of outcomes we see? How can the formal aspects of the methods be stated so as to facilitate choice of an appropriate method?

The next section of this paper discusses the application of social position to relational data. The third section reviews measures of structural equivalence and presents results using two of these methods. The fourth section discusses generalizations of structural equivalence (general equivalences) and looks in detail at regular equivalence, local role equivalence and ego algebras. In the fifth section we examine the question of defining position in relational data by looking at the results of different methods when applied to a constructed example with obvious positional structure. In the conclusion we return to the questions posed above, and discuss implications for interpretation of network analyses and directions for development of models of relational data.

2. Positions in relational data

The related concepts of social role, position and status have received considerable attention from sociologists, anthropologists and social psychologists. Among the most widely quoted definitions are those given by Linton, who defines a status as "the polar position in... patterns of reciprocal behavior". It is "a collection of rights and duties". When one "puts the rights and duties which constitute the status into effect he is performing a role" (1936: 113–114). Regardless of whether we focus on role, position or status, several important features of these concepts are apparent. First, at the core they refer to a social construct at a level of generality intermediate between the individual and the entire society. It is an aggregate class, category, or type of individual and their actual or expected behaviors or attributes. Second, this aggregate is defined on the basis of similarity in social activity, attributes, or social function of individuals in the category relative to the rest of the social system and members of other categories, rather than on the basis of geographical, social, cultural or interactional proximity. It is the patterns of, expectations for, and regularities in attributes and behaviors of individuals in a social category that provide the common core for these concepts.

Application of positional analysis to social network data rests on the assumption that the role structure of the group and the positions of individuals in the group are evidenced in measured social relations. As Lorrain and H. White observe, "the total role of an individual in a social system has often been described as consisting of sets of relations of various types linking this person as ego to sets of others" (1971: 50). In this sense role becomes identified with the manifest, measured relations. The goal of positional analysis is to provide an explicit means for inferring underlying roles and positions on the basis of measured relations among individuals. These relations are taken as indicators of the rights, duties, obligations, and expectations which obtain among positions. They are the observable indicators of the unobservable role and positional structure.

The task of positional analysis is inherently two-sided. Inferring positional structure from relational data requires both the aggregation of individuals based on similarity in their relations, and modelling similarities among relations based on their occupants. In social networks research *position* has come to refer primarily to an aggregate of individuals who are similar in their relations with others in a network (see Burt 1976) while *role* is more commonly used to describe a system of relations, either for an entire group or from an individual's perspective (see Breiger and Pattison 1986, or Winship and Mandel 1983). In this paper I will use *position* to refer to a category or group of individuals. The focus of this paper is on techniques which have as their primary objective the aggregation of individuals based on relational similarity. The product of such analysis is a statement about the structure of similarities among individuals or other social units in a given social system.

The distinctiveness of positional approaches to relational data rests on their reliance on *similarities* among actors based on their structural location in a social system. (We will discuss the measurement of this similarity in detail below.) Contrasting network models focus on *proximities* of actors: their geographic, physical or social closeness, their frequency or likelihood of interaction, or their reachability in a network. Such approaches have been referred to as relational, in contrast to positional (Burt 1978, 1980), or as based on cohesion rather than structural equivalence (Burt 1987; Friedkin 1984). Proximity based analyses aim to make statements about groups of actors who are closely connected to each other. Techniques based on proximity include clique detection, graph theoretic distance, and measures of density, cohesion and connectedness.

The distinction between similarity and proximity has proved useful in the development of network models. Not only does this distinction help to organize alternative analytic approaches, but in addition it parallels important theoretical explanations for behavior (role occupancy versus mutual contact, for example), and has provided contrasting explanations in empirical studies (Burt 1978, 1987; Friedkin 1984).

3. Structural equivalence

Formal leverage for drawing inferences about the dual positional-role structure from a set of measured relations is provided by the notion of structural equivalence. In their algebraic model of network structure Lorrain and H. White (1971) offer the most generally cited formal definition. "Objects a, b of a category **C** are structurally equivalent if, for any morphism M and any object x of **C**, aMx if and only if bMx, and xMa if and only if xMb. In other words, a is structurally equivalent to b if a relates to every object x of **C** in exactly the same ways as b does" (1971: 63). Restating this less formally, actors in a network are structurally equivalent if they have identical ties to and from all others in the network.

In the face of actual relational data, however, it is rarely the case that two actors will be perfectly structurally equivalent. The task then becomes one of measuring the degree to which actors approach structural equivalence, and perhaps grouping actors according to this measure. Viewed this way, the task can be seen as specific instance of the more general problem of measurement of similarity between observations, and secondarily of modelling these similarities.

One of the key distinctions between the two most widely used methods for positional analysis based on structural equivalence lies in different approaches to the measurement of similarity. In the following sections we examine these two methods. We look first at correlation as the basis for structural equivalence, as realized in the program CONCOR, and then turn to a discussion of Euclidean distance as an alternative measure, as found in the program STRUCTURE.

3.1. Correlation as an approach to structural equivalence

Correlation provides the basis for one of the most widely used approaches to positional analysis based on structural equivalence. The program CONCOR which uses correlation as the basis for measuring structural equivalence, grew out of work by H. White and his students on the "...application of algebraic concepts of structural equivalence to sociological theories of roles" (Breiger *et al.* 1975: 330). In a series of papers, a formal representation of data structure, called a blockmodel, was proposed as a way for presenting the information in a matrix (Arabie *et al.* 1978; Boorman and H. White 1976; Breiger *et al.* 1975; Heil and H. White 1976 and H. White *et al.* 1976). A blockmodel consists of a partition of a set of observations (for example, actors in a social network) into discrete sub-sets and the mapping of the original relations among individuals to an image matrix, which shows relationships among sub-sets (blocks) in this reduced matrix. The blocks are often interpreted as positions within the network (H. White *et al.* 1976). The program CONCOR was developed as a means for suggesting a blockmodel for a given set of data.

The usefulness of CONCOR for constructing blockmodels rests on the observation that computation of iterated Pearson product moment correlations among all pair of rows (or columns) in a data matrix (usually) converges to a matrix with values equal to $+1$ and -1. This matrix can be partitioned into two sub-matrices where correlations of $+1$ occur within sub-matrices and correlations of -1 occur between sub-matrices. This partition gives the groups for the blockmodel. Further iterations on the submatrices lead to finer partitions.

While the endpoint of CONCOR is a partition of the observations in a set of data, it is useful to think of the procedure as consisting of two parts: the computation of initial correlations among units in the original network data, and the construction of a partition based on correlations iterated on this matrix. The first correlation matrix may then be modelled using standard scaling or clustering techniques. This allows one to consider the first correlation as a measure of structural equivalence parallel to the distance measure employed by STRUCTURE (see below), and the partition produced by iterated correlations as a discrete model of the relations among subgroups.

CONCOR is readily available and widely used for positional analysis. Many of the classic data sets in social networks have been analyzed using CONCOR (Breiger *et al.* 1975; H. White *et al.* 1976; Arabie *et al.* 1978) and its relationship to other data analysis techniques including multidimensional scaling, cluster analysis and principal components analysis has been examined (Breiger *et al.* 1975; Schwartz 1977; Ennis

1982; Arabie and Boorman 1982; Arabie *et al.* 1978). In addition, CONCOR continues to be a common procedure for positional analysis of social networks, as evidenced by several recent applications (Arabie 1984; Friedkin 1984; Nemeth and Smith 1985; Carley 1986; Breiger and Pattison 1986). The method is applicable to a wide range of data types, since correlations may be computed across multiple relations, and with respect to nominations given or received (rows or columns of the sociomatrix), though the meaningfulness of such aggregation should be considered carefully.

We turn now to a discussion of distance as a measure of structural equivalence.

3.2. Distance as an approach to structural equivalence

The use of Euclidean distance as an alternative for measuring structural equivalence has been advocated by Burt (Burt 1976, 1978, 1980, 1983; Burt and Minor 1983) and is incorporated as part of the computer program STRUCTURE (Burt 1986b). The goal of this approach is to locate structurally non-equivalent statuses in a social network. Distances are computed using the familiar Euclidean distance (d_{ij})

$$d_{ij} = \left[\sum_{\substack{k=1 \\ k \neq i,\ k \neq j}}^{N} (x_{ik} - x_{jk})^2 + (x_{ki} - x_{kj})^2 \right]^{1/2} \tag{1}$$

where x_{ik} is the value of the relation between i and k, and N is the number of observations. As with the computation of correlation, distance can include multiple relations, and both row and column perspectives. As noted above, use of distance as a measure of structural equivalence is parallel to computing the first set of correlations in an analysis using CONCOR. (We discuss some of the implications of this choice below.) Once distances have been computed among pairs of actors they may be modelled using any of a range of standard techniques, such as hierarchical clustering or multidimensional scaling. This allows the question of representation of group structure to be answered separately from the measurement of structural equivalence. When a discrete model has been used to represent the distances the resulting groups have been referred to as statuses.

The Euclidean distance approach to positional analysis has been used extensively in social networks research, for example on Laumann and Pappi's community elites (Burt 1976), elites in sociological methodology (Burt 1983), Bernard, Killworth and Sailer's ham radio operators (Burt and Bittner 1981), and recently by Schott (1986) on systems of international exchange and Doreian and Fararo on journal-to-journal citations (Doreian and Fararo 1985).

Both CONCOR and the portion of the program STRUCTURE discussed here draw on structural equivalence as the formal basis for inferring social positions from relational data. They differ, however, in important ways, including how degree of structural equivalence is measured and the extent to which the representation of group positional structure is incorporated into the method rather than left as a separate question. With regard to the first point it should be noted that Euclidean distance is in a sense an unstandardized correlation coefficient. If relations are standardized to have equal means and equal variances across units (actors) then correlation and Euclidean distance are simply, and inversely, related. However when there are differences in means and variances of observations across units, the two methods may lead to quite different interpretations (Burt 1986; Burt and Bittner 1981; Faust and Romney 1985, 1986). Calculation of distance (or correlation) after some normalization of the original relations may give rise to a number of structural equivalence measures which focus on patterns of interaction (see Doreian and Fararo 1985, or Schott 1986 for examples). With regard to the second point, a blockmodel representation of social structure, as results from CONCOR, requires both a partition of social entities and a statement of the presence or absence of relational linkage between these groups. CONCOR, since it relies on successive bi-partitions, imposes a particular form on that structure: a binary tree. In a given application the researcher may or may not want to assume a particular a priori structure in the social relations of the group in question.

In the next section we present the results of these methods when applied to data from Sampson's classic study of monks in a monastery.

3.3. *Example using structural equivalence*

To illustrate results of positional analysis methods we use data from a study conducted by Sampson (1968) of 18 monks in a monastery. These data are important because of the extent of information gathered on the monks and the changes which occurred in the monastery during the course of Sampson's research. The monks in the sample include 18 lay and clerical novices in two cohorts. The first cohort is made up of six lay novices (Peter, Bonaventure, Berthold, Mark, Victor and Ambrose). The second cohort, who joined the monastery later, includes Romuald, John Bosco, Gregory, Basil, Louis, Winifrid, Amand, Hugh, Boniface, Albert, Elias and Simplicus. Prior to the arrival of the second class of novices, a number of changes were instituted to relax rules and procedures in the monastery. These changes, plus a number of other factors, including differences in previous education, led to a schism between the monks. This was evidenced in disagreements over rules and philosophy. Sampson noted that soon after the arrival of the second cohort John Bosco and Gregory emerged as leaders of the new group of monks, referred to by Sampson as the Young Turks. In addition to John Bosco and Gregory this group included Mark, Hugh, Winifrid, Boniface and Albert. The opposing group, the Loyal Opposition, was led by Peter and included Bonaventure, Berthold, Ambrose and Louis. Three monks, Victor, Romuald and Amand were intermediate between the other two groups and are labelled Interstitial (or Waverers). Elias, Simplicus and Basil were seen as relatively immature by Sampson and are called Outcasts. After a few months Elias, Simplicus and Basil (the Outcasts) along with Gregory were expelled from the monastery. Shortly thereafter John Bosco, Albert, Boniface, Hugh, Mark, Amand and Victor also left.

In addition to extensive questionnaires, Sampson collected sociometric information on four bi-polar relations. Each relation was presented a pair of scales, one generally positive and the other generally negative. The paired relations are: esteem-disesteem, like-dislike, influence-negative influence, and praise-blame. Each monk chose, in order, three others for each end of each of the four relations. Monks were questioned about each of five time periods. In the following analysis the esteem-disesteem relation from time four is used. In order to be appropriate for analysis by all of the methods discussed in this paper this relation was treated as two separate relations: esteem and disesteem. Each relation was then dichotomized by coding first, second, and third choices as 1, and coding 0 otherwise.

We turn now to analyses of the esteem and disesteem relations using methods based on structural equivalence. The UCINET package of network analysis programs was used for calculation of correlations, distances and CONCOR partitions (Freeman 1986). Figure 1 shows

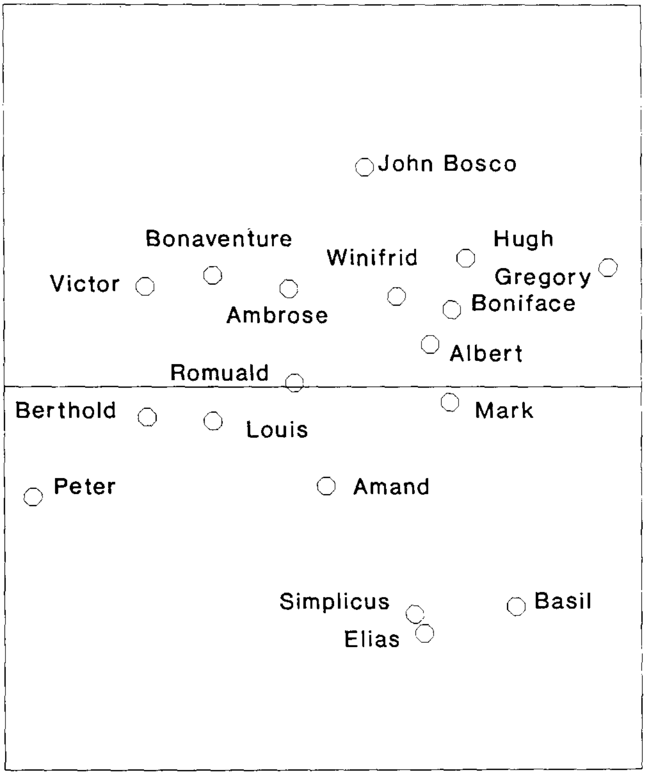

Fig. 1. Nonmetric multidimensional scaling of Euclidean distance as a measure of structural equivalence on esteem and disesteem from Sampson's monastery study.

results using distance as a measure of structural equivalence. Distances were computed across the dichotomized esteem and disesteem relations and their transposes, excluding diagonal entries. The two-dimensional nonmetric multidimensional scaling (Kruskal *et al.* 1973) is presented (stress formula 1 = .154).

Figure 2 presents a two-dimensional nonmetric multidimensional scaling of correlations on the dichotomized esteem and disesteem relations (stress formula 1 = .191). Pearson product moment correlations were computed across the esteem and disesteem relations and their transposes, excluding diagonal entries. A CONCOR analysis of these data to a four group partition gives the following sets: {Ambrose, Bonaventure, Albert, Mark}, {Winifrid, John Bosco, Boniface, Hugh, Gregory}, {Victor, Berthold, Romuald, Peter, Louis} and {Simplicus,

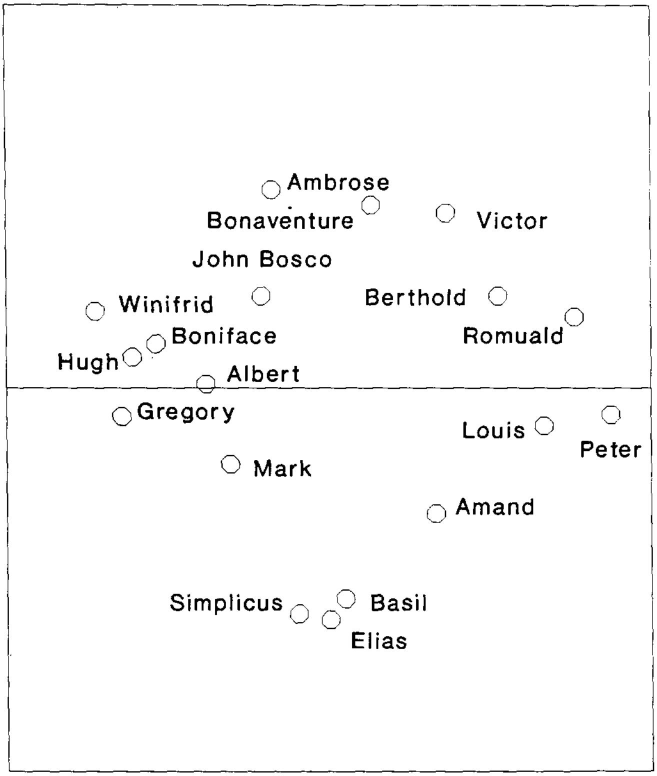

Fig. 2. Nonmetric multidimensional scaling of correlation as a measure of structural equivalence on esteem and disesteem from Sampson's monastery study.

Elias, Basil, Amand}. Referring to Figure 2 we can see that these groups are in close correspondence to regions of the multidimensional scaling solution. In addition, the first split from CONCOR (separating the first two groups from the third and fourth) occurs roughly along the vertical axis.

Several features of these two results are important. In both, the groups of monks identified by Sampson are in clear evidence. In Figure 2 (based on correlations) the Young Turks are to the left and the Loyal Opposition are to the right with the Outcasts toward the bottom. Figure 1 shows a similar pattern with the Young Turks in the upper right, the Loyal Opposition on the left, and the Outcasts at the bottom. A test of the correspondence between degree of structural equivalence and the four groups identified by Sampson is provided by a matrix

Table 1

Association among methods for structural and general equivalence on esteem and disesteem from Sampson's monastery study

Method	Method				
	Euclidean distance	Correlation	Regular equivalence	Ego algebra	Local role
Euclidean distance	–	−0.591	0.017	0.061	0.397
Correlation	0.605 (−10.330)	–	0.264	−0.228	−0.315
Regular equivalence	0.007 (0.676)	0.120 (4.706)	–	−0.392	−0.307
Ego algebra dist.	0.042 (−1.467)	0.037 (−2.729)	0.544 (−3.837)	–	−0.136
Local role equivalence	0.449 (7.294)	0.320 (−7.108)	0.092 (−2.866)	0.001 (−0.110)	–

Note: Wilson's e above diagonal, Pearson's r^2 and approximate Z from permutation test below the diagonal (in parentheses).

permutation test (Hubert and Schultz 1977). Table 2 presents approximate z scores and measures of association between a model consisting of a partition into the four groups and each measure of structural equivalence. Approximate z scores of 6.872 and −5.808 comparing the model with correlation and distance respectively indicate statistically significant relationships. The degree of association is also strong, as evidenced in the squared Pearson's product moment correlation coefficients of .310 and .224, and Wilson's e (a measure of strict monotone association, Wilson 1974) of .669 and −.601 between the model and distance and correlation respectively.

The fact that both distance and correlation based approaches to structural equivalence correspond to the groups labeled by Sampson is not surprising. Indeed, this correspondence was noted by H. White *et al.* (1976), though their results using CONCOR differ slightly from those presented here since they analyzed all eight relations. They note for a three block partition that "Sampson's Loyal Opposition is wholly contained in the first block; the Young Turks are exactly the men in the second block; the Outcasts are wholly contained in the third block. Sampson's Waverers 8 and 10 are in the Loyal Opposition block, whereas Waverer 13 is in the Outcast block" (1976: 753).

Not only are the two opposing "factions" revealed by structural

Table 2

Association between methods for structural and general equivalence and four groups for Sampson's monastery data

Measure	Method				
	Euclidean distance	Correlation	Regular equivalence	Ego algebra	Local role
Pearson's r^2	0.310	0.224	0.013	0.019	0.054
Z (permutation test)	(6.872)	(-5.808)	(1.381)	(-1.678)	(-2.857)
Wilson's e	0.669	-0.601	0.139	-0.109	-0.065

equivalence methods, but the leaders of the respective groups tend to be on extreme ends of the first dimension of the multidimensional scaling solution (compare Gregory and Peter). Furthermore the results using correlation and distance are quite similar. Table 1 presents the results of a matrix permutation test comparing the two methods. An approximate Z score of -10.330, Pearson's r^2 of .605 and Wilson's e of $-.591$ indicate very similar results. This is due to the fact that in these data there are only slight differences in mean and variance of observations across actors.

These results indicate that regardless of whether one uses correlation or distances as a measure of structural equivalence one does quite well in locating the factions described by Sampson. However, one might question whether these factions are in fact the sort of groups we hope to identify when we seek positions in relational data. Are the monks who are similar in these analyses ones we would consider as occupying the same *social position*? A negative response to this question leads us to seek alternative models for positional analysis.

4. General equivalences

Although structural equivalence has provided formal leverage for positional analysis of social networks, recently attention has turned to more general conceptualizations of equivalence. This more general focus arises in part from the insight that joint occupancy of a social position, or performance of a social role, derives not from the fact that occupants have identical social worlds, but rather from the fact that they have structurally similar social worlds (see Sailer 1978). This has led a number of people to explore alternative ways of thinking about structural location, and similarity of location, in relational data which more closely parallel intuitions about social position and social role. Procedures which take this more general approach include Winship and Mandel's (1983) local role equivalence, Breiger and Pattison's (1986) ego algebras, and D. White and Reitz's (1985) regular equivalence. I refer to these collectively as *general equivalences*.

Approaches using more general notions of equivalence depart from structural equivalence in two important ways. First, they employ a different way of thinking about structural location, and second, they require different ways of measuring similarity in location. Regular equivalence of D. White and Reitz, and related approaches such as Breiger and Pattison's ego algebras and Winship and Mandel's local role equivalence are all similar in that they focus on the *types* of relations in which actors participate and calculate similarity among actors with respect to these types. For example, if one were looking at an organizational hierarchy one would hope to distinguish actors who only give orders from actors who only receive orders, and to distinguish both of these groups from actors who both give and in turn receive orders regardless of the specific source or target of the order. Such groups would reflect the levels in the corporate hierarchy. Locating positions defined in this way requires a more general view of structural location and of similarity in location. However, as we shall see below, the problem of computing similarity in "type" is less straightforward than computing measures of structural equivalence, since widely familiar measures of dissimilarity or correlation may not be appropriate.

In the following sections we will discuss general equivalences in details. First we will look at D. White and Reitz's regular multiplex equivalence. Following that we turn to the closely related methods of of Winship and Mandel, and Breiger and Pattison.

4.1. Regular multiplex equivalence

D. White and Reitz (1985) have proposed a number of measures of equivalence with the goal of relaxing the requirement of structural equivalence that equivalent actors be tied in identical ways to identical others. They take as a point of departure the insight that people who occupy a social position will relate in the same ways to occupants of other social positions. This leads to a series of measures of equivalence which differ primarily in the way similarity is defined across multiple and compound relations. The reader is referred to their paper for a detailed discussion of these measures. D. White and Reitz advocate the use of regular multiplex equivalence (referred to below as *regular equivalence*) as the most natural embodiment of the notion of social position or social role. Regularly equivalent actors are defined as being "identically linked by multiplex relations to equivalent others" (D. White and Reitz 1985). Equivalence must be satisfied simultaneously across all relations in a network, but the alters to whom equivalent actors are tied, are required to be *equivalent*, not necessarily identical.

Computing regular equivalence for a pair of actors requires that one consider the equivalences of alters to whom members of the pair are tied. This leads to an iterative procedure in which estimates of equivalence for pairs of actors are successively refined in light of the equivalence of the alters. The endpoint is a measure of the degree of regular equivalence between each pair of actors. The measure, denoted M_{ij}, is 1 if two actors (i and j) are perfectly regularly equivalent, and 0 if two actors are not at all equivalent.

The regular equivalence for two actors, i and j, M_{ij}, at iteration $t+1$ is given by the formula in D. White and Reitz (1985: 18).

$$M_{ij}^{t+1} = \frac{\sum_{k=1}^{N} \max_{m=1}^{N} \sum_{q=1}^{Q} M_{km}^{t} * \left[iqj \text{ Match}^{t}_{km} + jqi \text{ Match}^{t}_{km} \right]}{\sum_{k=1}^{N} \overset{*}{\max_{m}} \sum_{q=1}^{Q} \left[iqj \text{ Max}_{km} + jqi \text{ Max}_{km} \right]} \tag{2}$$

where

N is the number of actors;
Q is the number of relations;
$\overset{*}{\max_{m}}$ means choose the m as chosen for k in the numerator;
$iqj \text{ Match}_{km} = \min(x_{ikq}, x_{jmq}) + \min(x_{kiq}, x_{mjq})$;
$iqj \text{ Max}_{km} = \max(x_{ikq}, x_{jmq}) + \max(x_{kiq}, x_{mjq})$; and
x_{ijq} is the value of the tie from i to j on relation q.

The numerator of this measure is a weighted match of how well i's ties across the set of relations are matched by j's ties, and how well j's ties are matched by i's. A best matching counterpart for each alter (k) tied to i is found in the set of alters (m) tied to j (and vice versa). The goodness of this match depends not only on the number of shared relations, but also on the equivalence of the corresponding alters (M^t_{km}, the equivalence of k and m from the preceding iteration). The denominator is a scaling factor which insures that the value of M_{ij} lies between zero and one. It is equal to the degree of i plus the degree of j, which is the maximum value that the numerator could take if i and j were perfectly equivalent, that is, if their ties to others (equal in number to the sum of their degrees) were perfectly matched.

In the simplest case of a single relation the first iteration distinguishes (roughly) between those actors who have both in and out degree, those who have only in degree, those who have only out degree, and actors who are isolates. The second iteration distinguishes between actors who are tied to actors who themselves have other ties and actors who are tied only to actors who have no other ties. The third iteration takes the chain of connection one step further. The iterative process of computing equivalence is analogous to computing equivalence across compound relations. For example, one could consider whether two actors both have friends, or friends who themselves have friends, and friends whose friends have friends. Although composition of relations could be considered for strings (words) of any length, in practice strings longer three do not seem to provide additional stable information, though this remains an open question. In all analyses discussed here equivalences were calculated for three iterations.

Calculation of regular equivalence requires an iterative procedure in which "types" of relations between actors and partners arises implicitly in the iterations. In the next two sections we discuss methods for general equivalence which make the notion of "type" explicit. We look first at Winship and Mandel's local role equivalence and then discuss Breiger and Pattison's ego algebras.

4.2. *Relational profiles and types of ties*

In order to understand the approaches of Winship and Mandel and Breiger and Pattison it will be helpful to make more explicit what I have referred to above as a "type" of tie. We first consider social group with N members and a set of binary relations, R, defined on $N \times N$. R contains one or more simple relations. For example it may consist of the collection: {"friend of", "acquaintance of" and "colleague of"}.

We next define another set consisting of both simple and compound relations derived from R, which we denote R^*. In our example R^* might consist of {"friend of", "colleague of", "acquaintance of", "friend of a friend" and "colleague of an acquaintance"}. We let Q be the number of simple and compound relations in R^*. The set R^* is defined by the particular method, as we shall see below. Following Winship and Mandel (1983) we can present the relations in R^* among the N members of our group in an $N \times N \times Q$ array, called a *relation box*. The different layers and vectors from this array contain indicators of the types of relations from R^* which obtain for individuals, for pair, and for the group. For example one of the $N \times N$ layers of this array (there are Q such layers) is a sociomatrix containing a binary relation. Each of the $N \times Q$ layers presents the relations (from R^*) engaged in by a specific individual with each other individual (indexed by the N columns). Winship and Mandel call this array a *relation plane*. The collection of profiles of relations which hold between pairs of individuals in the group provides a summary of the types of role relations present among individuals as occupants of positions in the social system.

While both Winship and Mandel's local role equivalence approach and Breiger and Pattison's ego algebra approach employ similar perspectives in focussing on "types" of relations, they differ from each other in two fundamental ways: (1) the selection of the relevent set of relations (R^*), and (2) the calculation of similarity (or dissimilarity) among pairs of individuals based on their relational profiles. Each of these issues will be taken up in descriptions of the individual procedures below.

4.3. Local role equivalence

In the *local role equivalence* approach of Winship and Mandel (1983) the set of relations, R^*, consists of all simple and compound relations formed from an initial (generator) set, R, up to a given length. Distance between a pair of individuals is calculated by locating best matching alters from the perspective of each member of the pair. This strategy is similar to that employed by regular equivalence, as described above. Specifically, the distance between individuals i and j is found by locating for each k tied to i a best matching counterpart in the set of m's tied to j. The goodness (badness) of each counterpart is computed as:

$$d(R_{ik}, R_{jm}) = \sum_{q=1}^{Q} |x_{ikq} - x_{jmq}|$$

where x_{ikq} is the value of the tie from i and k on relation q, N is the size of the group, and Q is the number of relations in R^*. Thus $d(R_{ik}, R_{jm})$ counts the numbers of failures to match presence (absence) of relations between i with alter k and j with alter m, across all relations in R^*. The measure or local role equivalence, denoted D_{ij}, is calculated as:

$$D_{ij} = \sum_{k=1}^{N} \min_{m} \left[d(R_{ik}, R_{jm}) \right] + \sum_{m=1}^{N} \min_{k} \left[d(R_{jm}, R_{ik}) \right]. \tag{3}$$

D_{ij} locates the best matching counterparts for all alters (k) tied to i in alters (m) tied to j (and vice versa). Notice that local role equivalence is a dissimilarity, with an upper bound which depends on the number of relations in R^* and the size of the group.

4.4. Ego algebras

Recently Breiger and Pattison (1986) have presented a comprehensive scheme for modelling individual roles and group social structure simultaneously. The *ego algebra* approach discussed in this section is a subset of their more extensive conceptualization of the simultaneous duality of individual role systems and group role structure. In keeping with our current emphasis on positional techniques, this section will focus on that portion of Breiger and Pattison's procedure which allows one to make statements about similarities among individuals. The reader is referred to their paper for a discussion of the more general scheme. In a manner similar to that of Winship and Mandel's procedure, the ego algebra approach expresses the relations of each individual in the group as a collection of relations from R^* which holds between that individual as ego and each alter. Breiger and Pattison call this array an *ego algebra*. For Breiger and Pattison the relevant set of relations, R^*, is the set of all *unique* simple and compound relations which can be formed from the generator relations in R. This set is also called the semigroup of relations.

Once each actor is characterized in terms of their ego algebra, the dissimilarity of two ego algebras is used as a measure of role dissimilarity for the pair. The strategy follows that described by Boorman and H. White (1976) for comparing two semigroups. The distance between two ego algebras is defined as a function of the distance of each from the joint homomorphic reduction of the two. In somewhat simplified terms, each ego algebra is a semigroup of relations from ego's perspective. This semigroup can be viewed as a partition of the set of all possible unique simple and compound relations, by equation of relations. For example, if for some individual all of their friends' friends were also their own friends, the relation "friend of" would be identical to the compound relation "friend of a friend" for that individual. The joint homomorphic reduction of two ego algebras is defined as their union. Since the union establishes more inclusive subsets, the joint homomorphic reduction partitions each of the two ego algebras, and will (usually) be a coarser partition than that produced by either of the two ego algebras. A measure of how coarse the partition (homomorphic reduction) is, in comparison to each individual partition (ego algebra), provides the basis for the calculation of distance between the two ego algebras. Following Boorman and H. White (1976), a measure of the coarseness of a partition of set **S** is:

$$h(P) = \frac{\sum_{i=1}^{m} \binom{|c_i|}{2}}{\binom{N}{2}}.$$

P is a partition into m subsets ($P = \{c_1, c_2, \ldots, c_m\}$), $|c_i|$ is the size of c_i, and N is the number of elements in S.

We let P_i and P_j be the partitions of each of two ego algebras (for individuals i and j respectively) determined by their joint homomorphic reduction, and let $h(P_i)$ and $h(P_j)$ be the distance of each from their joint homomorphic reduction. The distance between the two ego algebras is defined as:

$$\delta_{ij} = h(P_i) + h(P_j). \tag{4}$$

This measure is symmetric, and ranges from 0 to 2. Distances computed among all pairs of ego algebras provide a summary of pairwise role dissimilarity for the group. These may then be modelled using standard scaling or clustering techniques (see Breiger and Pattison 1986 for an example).

4.5. Example using general equivalences

In this section we return to the data on esteem and disesteem from Sampson's monastery study and compare the results of analyses using the three methods for general equivalences. All analyses use the same two binary matrices used to illustrate structural equivalence methods.

Figure 3 presents a two-dimensional nonmetric multidimensional scaling of pairwise regular equivalences (stress formula 1 = .113). Results of two dimensional nonmetric multidimensional scaling using local role equivalence are presented in Figure 4 (stress formula 1 = .181). The program ROLE (Breiger 1986) was used, and simple and compound relations up to length two were included in the calculation. Distances among ego algebras were calculated using the RELE portion of the program ROLE (Breiger 1986). Initial scaling of distances among ego algebras revealed that Romuald and Winifrid were quite different from the remaining 16 monks, leading to a degenerate scaling solution in which differentiation among the remaining monks was not apparent. To better represent the bulk of the group structure, Romuald and Winifrid were omitted, and the data were re-scaled. However, calculation of ego algebras and all comparisons discussed below include the full set of 18 monks. The two dimensional non-metric multidimensional scaling of the 16×16 matrix of distances among ego algebras is presented in Figure 5 (stress formula 1 = .185).

Several features of these results, in contrast to the results using structural equivalence are important. First, the groups labeled by Sampson do not occupy tight clusters in Figures 3, 4 and 5 as they did in Figures 1 and 2. Permutation tests show either no significant relationship, or only a marginal relationship, between a partition into four groups and the degree of equivalence for any of the three general equivalence methods. Table 2 shows the results of these comparisons. The Pearson's r^2's are .013, .054 and .019 with corresponding approximate Z scores of 1.381, -2.857, and -1.678, for regular equivalence, local role distance and distances among ego algebras, respec-

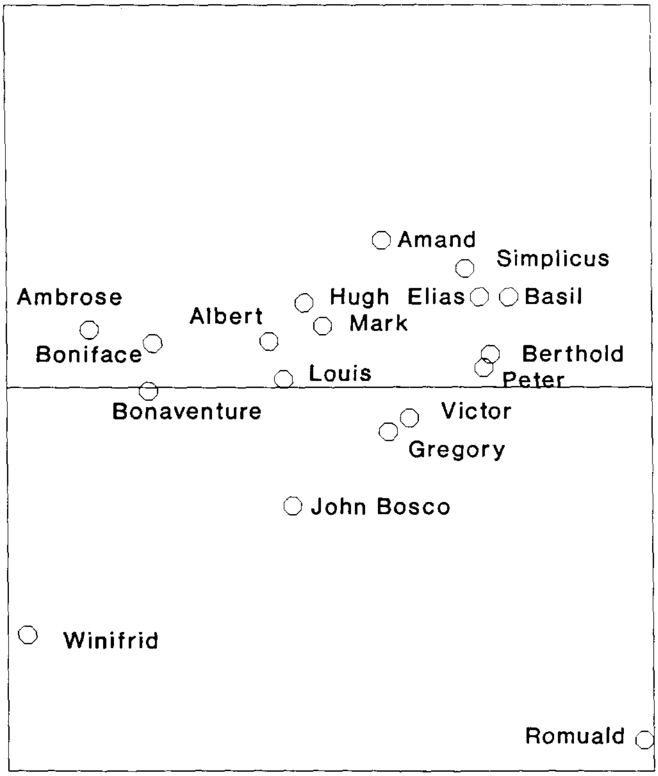

Fig. 3. Nonmetric multidimensional scaling of regular equivalence as a measure of general equivalence on esteem and disesteem from Sampson's monastery study.

tively. These can be contrasted with Z scores of 6.872 and -5.808 for Euclidean distance and correlation compared to the four group partition.

A second feature of results using general equivalences is their sensitivity to actors with distinctive relational patterns. For example, Winifrid, who made no disesteem nominations, and Romuald, who gave neither esteem nor disesteem nominations were so different from the remaining 16 monks in the ego algebra analysis that they had to be excluded in order to represent the structure of the entire group. These two monks also appear to be distinctive in the analysis using regular equivalences. Both are located at the bottom of the multidimensional scaling plot in Figure 3. Another group of monks with distinctive relational patterns includes Ambrose, Boniface, Bonaventure and

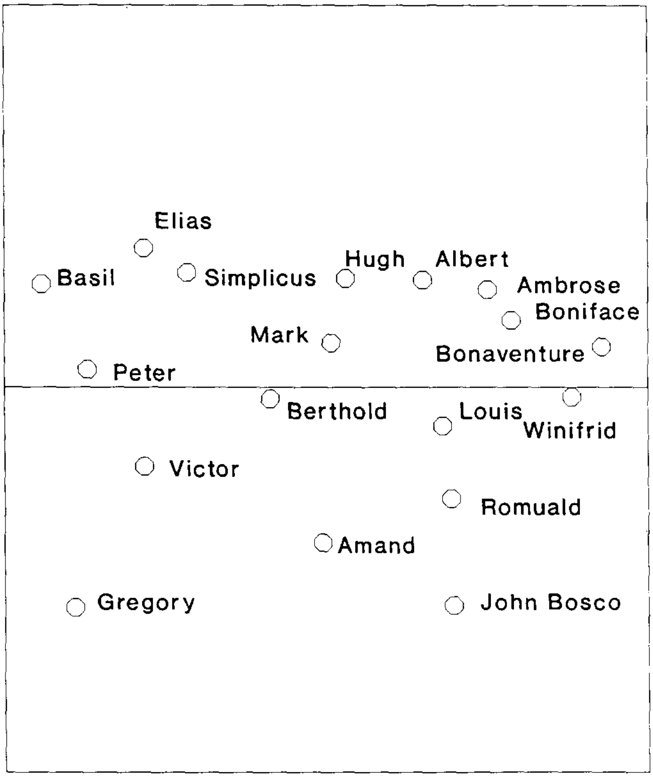

Fig. 4. Nonmetric multidimensional scaling of local role equivalence as a measure of general equivalence on esteem and disesteem from Sampson's monastery study.

Winifrid, all of whom received no disesteem nominations. These monks are also similar to each other in regular equivalences (where they all appear to the left of the figure) and in local role equivalence (where they are all in the upper right corner of the figure). Although suggestive at this point, these results indicate that general equivalences may be quite sensitive to distinctive relational configurations, such as the absence of a particular generator relation.

Comparing the results based on structural and general equivalence we note several points of contrast. Both methods based on structural equivalence are more similar to each other than to the results using general equivalences, though local role equivalence is more similar to the structural equivalence methods than to the other two general equivalences. Structural equivalence methods do well at locating the

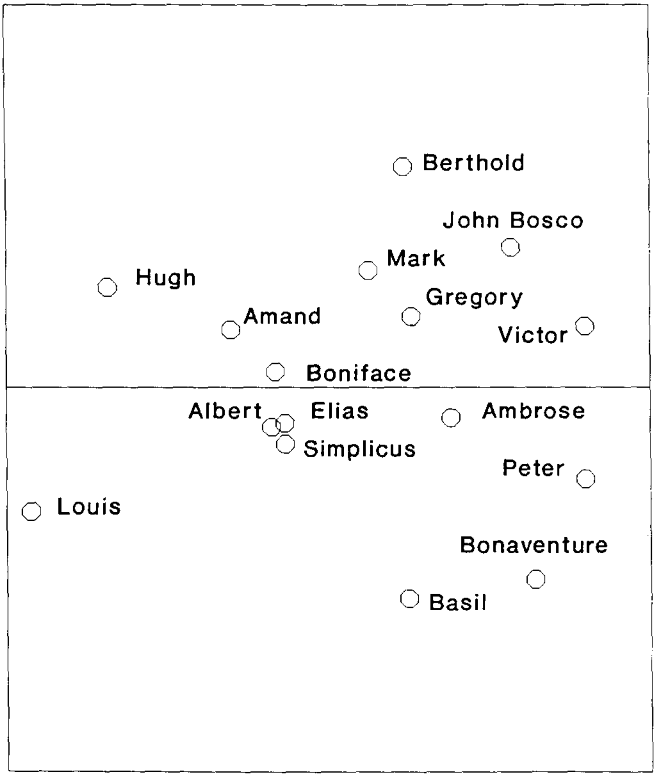

Fig. 5. Nonmetric multidimensional scaling of distance between ego algebras as a measure of general equivalence on esteem and disesteem from Sampson's monastery study.

four groups described by Sampson. General equivalences do not locate these factions.

5. Detecting positions?

One might ask whether these contrasting results are due to the particular example selected, or whether they are characteristic of the methods in general. This section addresses this point by looking at a constructed example. Following the strategy employed by Sim and Schwartz (1979) we look at a relational system in which the "positional" structure is apparent in the relations. The question we ask is: do the results of the methods correspond to what we would consider to be "social positions"?

Consider a simple example of two stylized corporate hierarchies. Each has three levels: the CEO, the managers and the employees. Two relations are used to generate this system. These might be thought of as "oversees the work of" and "cooperates on the job with". This is illustrated in Figure 6a. Before analyzing this example let's briefly describe what we would expect if we hoped to infer positions from the relations. Given the theoretical definition one would hope to identify three social positions in this example: the "Chief Executive Officer", the "Manager" and the "Employee". One would therefore expect a positional analysis method to reveal three equivalence classes, with a high degree of equivalence within and an intermediate or low degree of equivalence between classes. These classes would correspond to the levels in the corporate hierarchy, regardless of company affiliation.

The results using methods based on structural equivalence are in fact quite different from these expectations. CONCOR makes its first division by separating the two companies. The second division within each company puts the CEO and the two managers together, in contrast to the employees (see Figure 6c). Using distance in combination with complete link clustering produces a first cluster containing the CEO's and then groups them with all employees and finally adds

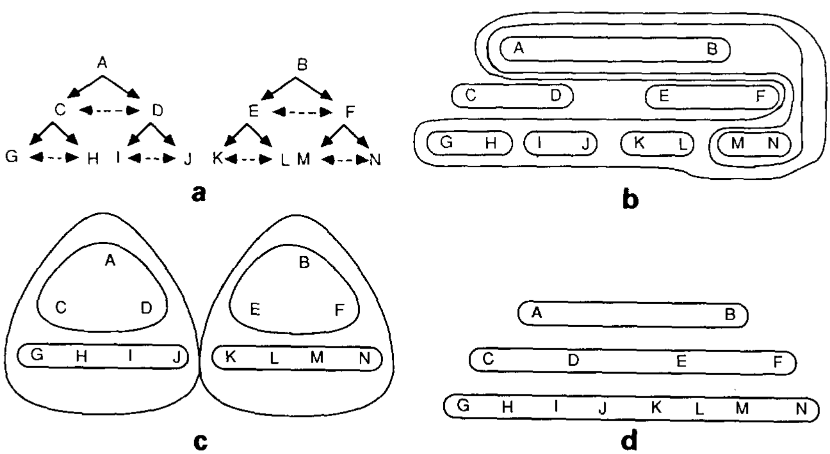

Fig. 6. Example of two hierarchies constructed from two relations: (a) The two hierarchies; (b) Structural equivalence groups using Euclidean distance and complete link clustering; (c) Structural equivalence groups using CONCOR; (d) General equivalence groups using regular equivalence, local role distance, or distances between ego algebras.

the managers for the most encompasing cluster (see Figure 6b). In contrast, all three methods based on general equivalences produce three groups corresponding exactly to the three levels in the corporate hierarchy. Equivalence is perfect within each level, and intermediate between levels, as shown in Figure 6d.

In the next section we discuss the implications of these results for the questions posed in the introduction.

6. Discussion

This paper has outlined the conceptualization and measurement of social position in relational data. It was argued that social positions are evidenced in the interactions among individuals as occupants of positions and performers of roles. Given a set of measured relations reflecting these interactions, the task is to locate social positions which consist of groups of individuals with similar patterns of relations. Two alternative approaches were discussed. The first, *structural equivalence*, locate groups of similar individuals based on the extent to which they share identical ties with identical others. A second set of approaches, here called *general equivalences*, locate groups of similar individuals based on their sharing of "types" of ties with "types" of others. Methods based on these approaches where applied to actual data and to a constructed example. The results suggest that these approaches identify different kinds of social groups. Groups which emerge from structural equivalence procedures tend to include individuals who are closely connected to one another or to the same other(s). In the data from Sampson's monastery study the groups identified by structural equivalence procedures closely mirror the "factions" labeled by Sampson. In a constructed example of two hierarchically organized corporations structural equivalence as measured by CONCOR groups together individuals belonging to the same "corporation". These results suggest that structural equivalence is an unsuitable basis for analysis if the goal is detection of social positions. On the other hand, groups which emerge from procedures based on general equivalences, whether regular equivalence, local role equivalence, or ego algebras, exactly reflect social types (positions) in an example constructed with clear positional structure, and locate groups of individuals with distinctive relational configurations when applied to the data from Sampson's monastery study.

In the introduction we posed several questions about positional analysis of relational data. As a way of conclusion we return to those questions, taking up each in turn.

First, what do we mean by position in relational data? It was argued that a social position is an aggregate of individuals who are similar in social activity relative to members of other positions. An assumption underlying positional analysis of social network data is that the measured relations serve as indicators of the rights, duties, obligations and expectations which obtain among positions. A position in relational data is therefore an aggregate of individuals who share similar "types" or profiles of relations with individuals in other positions. Similarity in "type" is satisfied between representatives of positions, without regard for the identity of the alters.

Second, are the results similar enough to be considered substitutable? Clearly the results presented above indicate that they are not. Methods based on structural equivalence and methods based on general equivalences reveal quite different groups of actors.

Third, do the results of the methods locate positions as defined for relational data? With respect to a relational system constructed with clear positional structure, methods based on structural equivalence do not recover sociological positions. All of the methods based on general equivalences (regular equivalence, local role equivalence and ego algebras) locate positions.

What are the formal properties of the methods that lead to kinds of outcomes we see? One of the key points which distinguishes methods based on structural equivalence from methods based on general equivalences is the focus on types of relations. Methods based on general equivalence attempt, either through iterative refinement or through direct calcuation, to describe and to compare actors in terms of a configuration of relational types. Comparison among actors is done with respect to these (abstract) types, rather than with respect to identical others. This (perhaps) contributes to results which are sensitive to distinct relational configurations, for example, the absence of a specific generating relation.

How can the formal aspects of the methods be stated so as to facilitate choice of the appropriate method? Appropriate choice of a method requires specification of the appropriate form or nature of structural similarity required in a substantive application. Doreian (1986) has recently presented the view that there are a range of equivalence definitions, from which one selects the appropriate one in a given situation. This is an area for further investigation and clarification.

As a final comment, this paper has examined positional analysis as a question of measurement. Clearly the usefulness of any of the methods explored here rests on their integration in substantive and theoretical problems. Structural equivalence and general equivalences capture different social structural effects. Making clear the nature of these effects, and investigating situations where they provide contrasting theoretical predictions is an important task for the future.

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