AIRLINE ALLIANCES AND SERVICE QUALITY*

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August 2018

Abstract

Convenient scheduling, characterized by adequate flight frequency, is the main quality attribute for airline services. However, the effect of airline alliances on this important dimension of service quality has received almost no attention in the literature. This paper fills this gap by providing such an analysis in a model where flight frequency affects schedule delay and connecting layover time. While an alliance raises service quality when layover time has zero cost, the reverse occurs when layover time is costly. The source of this surprising result is that costly layovers eliminate the additive structure of the full trip price, which consists of the sum of the subfares plus the weighted sum of the reciprocal flight frequencies when layover cost is zero. The paper also shows that nonaligned carriers adjust frequencies to suit passenger preferences in business and leisure markets, while an alliance is less responsive to such preference differences. With hub-airport congestion, greater internalization by allied carriers tends to reduce frequency, but this force is not enough to overturn the positive alliance effect in the low-cost layover case.

Keywords: service quality; alliance; double marginalization; congestion

JEL Classification Numbers: D43; L13; L40; L93; R4

*We are grateful to the audience at the 2018 Hong Kong ITEA Conference for useful comments. Flores-Fillol acknowledge financial support from the Spanish Ministry of Science and Innovation (ECO2016-75410-P), Generalitat de Catalunya (2014SGR631), and Recercaixa.
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1 Introduction

Service quality is an important element in the air transportation industry. It includes many different dimensions on the ground (bag handling, gate location, connecting layover times) and in the air (flight schedules, in-flight services, legroom, seat characteristics). However, there is a clear consensus in the literature that convenient scheduling, characterized by adequate flight frequency, is the main quality attribute for airline services. A higher flight frequency on a route is important because it reduces schedule delay, defined as the gap between a passenger’s preferred and actual arrival times. However, a negative impact of higher frequencies can be greater congestion at busy hub airports, generating delays, missed connections and flight cancellations. Of course, these factors can reduce airline service quality.

Given the importance of flight frequency in determining airline service quality, the recent literature has added this dimension in studying issues such as airline network structure, schedule competition between carriers, intermodal competition, the emergence of regional jets, and the effects of mergers. However, the theoretical literature on airline alliances has mainly focused on fare impacts, without considering the effect of alliances on service quality. The now-standard result is that alliances eliminate double marginalization in the setting of interline fares, where travel spans the networks of both airline partners. The result is a lower fare (and a higher traffic volume) than would be charged for the same trip by nonaligned carriers (Brueckner, 2001).

Relative to nonaligned carriers, it is recognized that alliances raise service quality on connecting trips by offering gate proximity at international connecting points, a single-stop check in process, and reciprocity in frequent-flyer programs. However, the effect of alliances on the important frequency dimension of service quality has received almost no attention in the theoretical literature. The purpose of the present paper is to fill this gap by providing such an analysis. We consider the simplest possible network, which consists of two route segments linking different endpoints to a hub airport, with the segments operated by two different carriers, acting independently or as alliance partners. Travel demand exists only between the non-hub endpoints, requiring interline (connecting) service by the two carriers.

To capture service quality, the passenger demand function incorporates aversion to schedule delay and connecting layover time. This disutility depends on the flight frequencies of the two carriers. When the frequencies are equal ex ante, as in the case of an alliance (which sets common values), then layover time is zero and schedule delay is proportional
to the reciprocal of the common frequency, as in the previous models that ignore flight connections. But when the carriers are nonaligned and choose their frequencies independently, the values differ ex ante even though they end up equal in a symmetric equilibrium. The challenge, then, is to derive an expression for passenger disutility from schedule delay and layover time in the no-alliance case, where frequencies differ ex ante. This derivation is a major contribution of the paper.

We carry out the required derivation under two different polar-case assumptions: a zero-cost layover time and a layover cost high enough that passengers completely avoid layovers. The first case could apply to leisure passengers and second to business passengers. In both cases, the ex-post layover time cost is zero, either by assumption (first case) or by choice (second case), so that passenger disutility (denoted $g$) consists entirely of schedule-delay cost. With zero-cost layover time, we show that $g$ is proportional to the sum of the reciprocals of the individual carrier flight frequencies. This disutility expression is added to the sum of the subfares charged by the carriers for their portions of the interline trip, leading to the full trip price. With this additive structure, we might expect that the standard double marginalization result on fares would also extend to frequencies, with frequency rising and the overall fare falling in moving from the no-alliance to the alliance case.

With high-cost layover time, the passengers choose flights whose arrival and departure times at the hub coincide so as to avoid any layover. In this case, we show that $g$ is proportional to the reciprocal of the minimum of the two airline frequencies. With the additive structure eliminated, we would no longer expect double marginalization in the choices of frequencies by nonaligned carriers.

The results of the paper can be summarized as follows. With zero-cost layover time, an alliance raises flight frequency relative to the no-alliance case, in line with the predicted double marginalization story. Interestingly, however, the same conclusion need not apply to fares, with the overall fare being either higher or lower than in the non-alliance case. However, an alliance does beneficially reduce the full trip price, thus yielding the same increase in interline traffic as in the standard model.

With high-cost layover time, an alliance reduces the overall fare, as in the standard model. But since the high-cost case does not exhibit the double-marginalization structure of the low-cost case with respect to frequencies, the opposite frequency impact occurs, with an alliance leading to a reduction in flight frequency. Because of lower frequency, the full
trip price can either rise or fall, so that an alliance could lead to a reduction in traffic, in a surprising reversal of the standard result. The upshot is that, when a service-quality dimension involving flight frequencies is added to an alliance model, the conclusions it generates may be unfamiliar.

Our findings can be applied to analyze the effect of airline alliances on service-quality heterogeneity across business and leisure markets. The different values of time for these two passenger types affects their attitudes toward layover time and schedule delay. This difference generates a service-quality gap between business and leisure markets in the no-alliance case, with frequencies being higher in business markets. After an alliance is formed, the flight-frequency difference between the two types of markets narrows but does not disappear.

Finally, we add hub-airport congestion to the model. The literature on airport congestion shows that airlines internalize the congestion they impose on themselves, neglecting the congestion they impose on other airlines. Therefore, given that allied carriers will take into account the congestion imposed on their partners, alliances are expected to increase the extent of internalization. Since this internalization effect puts downward pressure on frequencies in the alliance case, it is conceivable that the positive alliance frequency effect in the absence of congestion could be reversed when congestion is present. The results of the analysis show, however, that this conjecture is not upheld, with an alliance still leading to higher frequency and service quality when layover cost is zero.

The present paper complements the closely related work of Czerny, van den Berg and Verhoef (2016), which explores the effects of carrier cooperation on service frequency in a general model that can be interpreted as applying to airlines. In addition to being simpler, our model differs by providing microfoundations that translate individual flight frequencies into overall service quality (as described above) rather than relying on an abstract general relationship.4

The paper is organized as follows. In section 2 we present the setup and derive the service quality expression in the cases of zero-cost and high-cost layover time. Section 3 derives the conditions for profit-maximization in the no-alliance and alliance cases and provides a general analysis of alliance effects. Section 4 derives results under the assumption that demand is linear. Section 5 considers the effects of congestion, and section 6 offers conclusions.
2 The setup and the form of the \( g \) function

The analysis focuses on the simple network shown in Figure 1. Airline 1 operates route segment XH, and airline 2 operates route segment YH, but travel demand only exists in market XY, where passengers rely on interline service by airlines 1 and 2.

Passenger demand, which is for round trips, depends on the full price of the service, which in turn depends on the overall XY fare and a measure of service quality:

\[
Q = D (\text{fare} + g[f_1, f_2]),
\]

where \( Q \) is passenger traffic (common to both carriers), \( f_1 \) and \( f_2 \) are flight frequencies, and \( D' < 0 \). The function \( g[\cdot] \) captures passenger disutility from schedule delay and layover time. The following subsections derive the precise functional form for \( g[\cdot] \) under different alternate assumptions.

As in Brueckner (2004) and Brueckner and Flores-Fillol (2007), we avoid the complexities of spatial models by using a framework where consumers ultimately care about overall flight frequencies rather than the departure times of individual flights. This approach relies on the assumption that a passenger’s preferred arrival time is unknown prior to the purchase of the airline ticket and is thus viewed as random, being uniformly distributed around the clock. Under this assumption, schedule delay (the difference between a passenger’s preferred and actual arrival times) depends on flight arrival times but is otherwise random, although its expected value (which matters to the consumer) can be derived conditional on flight times.

In addition to preferring less (expected) schedule delay, passengers also dislike layover time. Depending on the cost of layover time relative to the cost of schedule delay, different scenarios arise, which yield different functional forms for the function \( g[\cdot] \). Two polar cases are considered, with the cost of layover time either equal to zero or equal to a large enough value that minimization of layover time is the passenger’s main goal, with schedule delay being a secondary consideration.

Consider outbound flights from X to H (airline 1) and from H to Y (airline 2). We assume that the carriers evenly space their flights around the clock, represented by time circle. In addition, when the carriers operate the same numbers flights, with \( f_1 = f_2 \), the flights operate at the same time. When \( f_1 > f_2 \), the extra flights of airline 1 evenly fill the gaps between airline 2’s flights, as shown in Figure 2. Airline 1’s flights are given by the rule \( f_1 = 2^k f_2 \), where \( k = 0, 1, 2, \ldots \), so that airline 1’s flight density increases with
k. Note that under this rule, $f_1$ doubles with each successive increase in $k$, with the extra flights filling in the gaps between airline 1’s existing flights. The opposite pattern occurs when $f_1 < f_2$ with $f_2 = 2^k f_1$. Similar reasoning applies to inbound flights, whose volume exactly matches that of outbound flights for each airline. Note that the time gap between airline $i$’s flights is $T/f_i$, for $i = 1, 2$.

2.1 The form of $g$ with zero-cost layover time

In the first case, layover cost is zero, and the passenger considers only schedule delay in choosing flights, a case that could apply to leisure passengers. Focusing on outbound flights originating at X, the expected arrival schedule delay depends only on airline 2’s flight frequency, regardless of the relative values of $f_1$ and $f_2$, as will be demonstrated by considering Figure 3. In the figure, the arrows show the arrival times of airline 1’s flights at the hub H as well as the departure times of airline 2’s flights. With air travel assumed, without loss of generality, to be instantaneous, airline 2’s flight departure time from H and arrival time at Y are the same. The figure also shows the passenger’s preferred arrival time.

Suppose that $f_1 \geq f_2$, as shown in panel I of Figure 3. The passenger prefers flight B of airline 2, which arrives closest to her preferred time, but the passenger is indifferent between flights A and B of airline 1 since they both allow a connection with flight B of airline 2. The longer layover generated by flight A is immaterial since layover cost is zero. The upshot is that additional flights of airline 1 generate no benefit for the consumer, with expected schedule delay depending only on the gap between airline 2’s flights. Expected schedule delay is $1/4$ of this gap, or $T/4f_2$.

When $f_1 < f_2$ (panel II), the passenger takes flight A of airline 1 and waits costlessly at the hub airport to take flight B of airline 2, which minimizes her schedule delay. Again, expected schedule delay depends only on $f_2$, and the cost of this delay is given by $\delta T/4f_2$, where $\delta > 0$ is the schedule-delay cost parameter. Using $\gamma \equiv \delta T/4$, the previous expression can be rewritten as $\gamma/f_2$.

In the inbound direction (the return trip from Y to X), the roles of the airlines are switched, so that expected inbound schedule delay is $\gamma/f_1$. Average schedule-delay cost in both directions, which gives travel disutility $g$ given the absence of layover costs, is then

$$g[f_1, f_2] = \frac{1}{2} \left[ \frac{\gamma}{f_1} + \frac{\gamma}{f_2} \right].$$

(2)
2.2 The form of \( g \) with high-cost layover time

When layover time is costly (as with business travelers), the passenger’s main goal is to minimize it, with schedule delay a secondary consideration. In this situation, the relative values of \( f_1 \) and \( f_2 \) become crucial. Starting with the case where \( f_1 \geq f_2 \), it is easy to see that a passenger will always choose flights whose arrival and departure times coincide so as to avoid any layover, as shown in Figure 4.

Consider panel I and assume again that the passenger’s preferred arrival time lies between flights A and C of airline 2. Then, depending on the exact location of the preferred time, the passenger will choose either flight A or C of airline 1 along with the corresponding flight of airline 2 (the C flights will be chosen in the case shown in the figure), incurring a zero layover. In this case, flight B of airline 1 is irrelevant, as in Figure 3-I, and the expected schedule delay is again \( \gamma / f_2 \).

The difference in the costly layover case arises when \( f_1 < f_2 \), as shown in panel II of Figure 4. Despite the presence of flight B of airline 2, the passenger will make the same choice as in panel I, combining either the A flights or the C flights of the two airlines. When the preferred arrival is as shown, the C flights will be used, leading to a zero layover and the schedule delay shown in panel II. This choice is made even though a shorter schedule delay would be achieved by taking airline 1’s A flight and connecting to airline 2’s B flight. But since this option involves a layover equal to the gap between airline 1’s A flight and airline 2’s B flight, it will be shunned in favor of the option with no layover. In contrast to panel I, it is now airline 2’s B flight that is irrelevant, and the result is that expected schedule delay equals 1/4 of the gap between airline 1’s flights. Expected schedule-delay cost is then \( \gamma / f_1 \). Combining the results from the two panels of Figure 4, expected schedule-delay cost for the outbound trip is thus \( \gamma / \min \{f_1, f_2\} \).

Applying the same reasoning for inbound flights, we get the same schedule-delay expressions: \( \gamma / f_2 \) for \( f_1 \geq f_2 \) and \( \gamma / f_1 \) for \( f_1 < f_2 \), expressed again as \( \gamma / \min \{f_1, f_2\} \). With the same outbound and inbound expressions, the average schedule-delay cost in both directions is given by this common expression. Since layover costs are zero given that no layovers are incurred, the \( g \) function again consists only of schedule-delay costs and is given by

\[
g[f_1, f_2] = \frac{\gamma}{\min \{f_1, f_2\}}.
\]

The proposition below summarizes the difference between the \( g \) functions in (2) and...
Proposition 1 With both zero-cost and high-cost layover time, the $g$ function consists only of schedule-delay costs. With zero-cost layover time, the $g$ function is proportional to the average of the reciprocals of the carriers’ flight frequencies, as in (2). With high-cost layover time, the $g$ function is proportional to the reciprocal of the minimum of the carriers’ flight frequencies, as in (3).

2.3 Airline costs

On a single route segment, airline $i$’s cost per flight is given by $\theta + \tau s_i$, where $s_i$ is carrier $i$’s aircraft size, as measured by the number of seats, $\theta$ is a fixed cost independent of aircraft size, and $\tau$ is the marginal cost per seat (as in Brueckner, 2004). Under this specification, cost per seat realistically falls with aircraft size, leading to economies of traffic density.

Assuming a 100% load factor, airline $i$’s flight frequency, aircraft size, and passenger traffic $Q$ are all related by the equation $s_i f_i = Q$, which says that aircraft size times the number of flights equals traffic (recall that passenger traffic is the same for both carriers). Note that $s_i$ is an airline choice variable, which is appropriate given that the demands of airlines ultimately determine the nature of aircraft supplied by manufacturers. While $s_i$ is thus endogenous, its value is determined residually once $Q$ and $f_i$ are known. Therefore, carrier $i$’s total cost is $f_i(\theta + \tau s_i)$ or, equivalently,

$$c_i = \theta f_i + \tau Q,$$

indicating that variable costs depend just on $Q$, being independent of the number of flights.

3 The general demand model

This section develops the profit-maximization conditions for the no-alliance and alliance cases under the general-demand specification in (1). In the alliance case, where $f_1 = f_2 = F$, the $g$ function reduces to $\gamma/F$ with both zero-cost and high-cost layover time, which means that the alliance conditions are the same in the two cases. The non-alliance profit-maximization conditions, however, differ between the cases. As a result, the no-alliance conditions are derived first for the two cases, with the common alliance conditions derived subsequently.
3.1 No-alliance conditions with zero-cost layover time

Taking into account the form of $g[\cdot]$ with zero-cost layover time (from (2)), demand in (1) becomes

$$Q = D \left( p_1 + p_2 + \frac{1}{2} \left[ \frac{\gamma}{f_1} + \frac{\gamma}{f_2} \right] \right), \quad (5)$$

where the overall fare is composed of two subfares ($p_1$ and $p_2$) that the carriers choose in noncooperative fashion, following previous alliance models.

Airline 1’s profit is therefore $\pi_1 = p_1 Q - c_1$, an expression that can be rewritten using (4) and (5) as

$$\pi_1 = (p_1 - \tau) D \left( p_1 + p_2 + \frac{1}{2} \left[ \frac{\gamma}{f_1} + \frac{\gamma}{f_2} \right] \right) - \theta f_1. \quad (6)$$

The corresponding expression for carrier 2 is given by interchanging subscripts.

Each carrier simultaneously chooses its subfare and flight frequency to maximize profit, treating the other carrier’s values as given. Focusing on the symmetric equilibrium, we set $p_1 = p_2 = P/2$ and $f_1 = f_2 = F$ after computing first-order conditions for the choice of $p_1$ and $f_1$, where $P$ is the overall fare for the XY trip and $F$ is the common frequency on both route segments. These conditions can be rewritten as

$$D + \left( \frac{P}{2} - \tau \right) D' = 0, \quad (7)$$

$$-\left( \frac{P}{2} - \tau \right) \frac{\gamma}{2F^2} D' - \theta = 0. \quad (8)$$

After rearrangement, the first condition says that the fare is set optimally when marginal revenue as a function of $P$ equals the marginal cost of a seat, $\tau$. The second condition says that $F$ is set optimally when fixed cost per flight ($\theta$) equals the revenue gain from an extra flight.

3.2 No-alliance conditions with high-cost layover time

With high-cost layover time, $g[\cdot]$ is given by (3), so that demand in (1) becomes

$$Q = D \left( p_1 + p_2 + \frac{\gamma}{\min \{f_1, f_2\}} \right), \quad (9)$$

and carrier 1’s total cost is again given by (4). Using these expressions, airline 1’s profit becomes

$$\pi_1 = (p_1 - \tau) D \left( p_1 + p_2 + \frac{\gamma}{\min \{f_1, f_2\}} \right) - \theta f_1, \quad (10)$$
while carrier 2’s profit expression comes from interchanging subscripts in (10).

Finding the Nash equilibrium in frequencies is less straightforward than before because of the presence of the min function. To derive carrier 1’s best frequency response function, suppose that the subfares in (10) are equal, as they will be in equilibrium, so that the frequency choices are symmetric for the two carriers. Let $f^*$ denote the $f_1$ value that maximizes (10) under the assumption that $f_2 > f_1$, in which case the frequency term in (10) equals $\gamma / f_1$. It is then easy to see that the optimal $f_1$ equals $f^*$ for $f_2 > f^*$, while the optimal value equals $f_2$ for $f_2 \leq f^*$. In the latter case, raising $f_1$ toward $f^*$ is desirable, but once $f_2$ is reached, further increases yield no benefit under the min function. The resulting best-response function for carrier 1 is shown as the solid curve in Figure 5. Carrier 2’s best-response function is the mirror image of 1’s function, shown as the dotted curve in the figure, which coincides with 1’s function over a portion of the 45-degree line. From the figure, it is clear that any $(f_1, f_2)$ pair lying where the best-response functions coincide is a Nash equilibrium in frequencies. We assume that, since the equilibrium at $(f^*, f^*)$ yields the highest profit for each carrier among these multiple equilibria, this is the equilibrium that will be selected.

The resulting optimality condition for $f_1$, which gives the common $f$ under symmetry, is then the same as the condition yielding $f^*$, which comes from replacing min $\{f_1, f_2\}$ by $f_1$ in (10) and differentiating. As before, this condition and the first-order condition for $p_1$ are evaluated under symmetry, with $p_1 = p_2 = P/2$ and $f_1 = f_2 = F$. The resulting conditions are

$$D + \left(\frac{P}{2} - \tau\right) D' = 0,$$  \hspace{1cm} (11)

$$- \left(\frac{P}{2} - \tau\right) \frac{\gamma}{F^2} D' - \theta = 0.$$  \hspace{1cm} (12)

Note that the the first condition is the same as the condition under zero-cost layovers (see (7)), whereas the second condition differs in the absence of a 2 factor multiplying $F^2$ (see (8)).

### 3.3 Alliance conditions

The alliance maximizes joint profit, which is split equally between both partners. As in the standard model, the allied entity chooses an overall fare $P$. Since the alliance also sets a common flight frequency $F$ on both route segments, while operating inbound and
outbound flights at the same time, layover cost is identically zero, not being affected by
the cost of layover time. The upshot is that profit-maximization conditions for an alliance
are independent of layover cost, being the same in the zero-cost and high cost cases. With
common route frequencies, (average) schedule delay is $\gamma/F$, so that demand is given by

$$Q = D \left( P + \frac{\gamma}{F} \right), \quad (13)$$

Joint profit is then written

$$\Pi = (P - 2\tau) D \left( P + \frac{\gamma}{F} \right) - 2\theta F, \quad (14)$$

where the 2 factors are needed because the alliance operates two route segments. Note that,
while the same full price $(P + \gamma/F)$ emerges in the symmetric no-alliance equilibrium, here
it appears in the original optimization problem.

After dividing by 2, the first-order conditions for the choices of $P$ and $F$ are

$$\frac{D}{2} + \left( \frac{P}{2} - \tau \right) D' = 0, \quad (15)$$
$$- \left( \frac{P}{2} - \tau \right) \frac{\gamma}{F^2} D' - \theta = 0. \quad (16)$$

### 3.4 Unified statement of the profit-maximization conditions

To analyze the effect of alliances on fares and frequencies, we first gather the previous
results in two unified statements of the profit-maximization conditions, one for the case
of zero-cost layover time and the other for the high-cost case. Then, we can carry out a
comparative-static analysis of the equilibrium conditions for the two cases to determine
the fare and service-quality effects of alliances.

#### 3.4.1 Zero-cost case

Combining (7), (8) and (15), (16), the unified statement of the profit-maximization conditions for the case of zero-cost layover time is

$$\alpha D + \left( \frac{P}{2} - \tau \right) D' = 0, \quad (17)$$
$$- \left( \frac{P}{2} - \tau \right) \frac{\gamma}{2\alpha F^2} D' - \theta = 0. \quad (18)$$
where the parameter $\alpha$ takes the value $1/2$ in the alliance case and $1$ when carriers are nonaligned.

To get a sense of the different frequency incentives of an alliance and nonaligned carriers, consider the frequency choices in the two cases implied by (18) when $P$ is held fixed at a common value. In the no-alliance case, $\alpha = 1$ makes the expression $\gamma/2\alpha F^2$ equal to $\gamma/2F^2$ and thus smaller than its value of $\gamma/F^2$ in the alliance case (where $\alpha = 1/2$). This difference leads to a smaller no-alliance $F$, holding $P$ fixed. That outcome reflects the fact that, in the no-alliance case, a change in a carrier’s frequency affects only half of the frequency portion of the full price rather than the entire frequency portion, which accounts for the $2$ in the denominator of the previous no-alliance expression and its absence in the alliance expression. This difference is the essence of double marginalization, where noncooperative behavior leads carriers to ignore beneficial spillovers from their choices on the other carrier, thus setting their decision variable at too low a value. While this discussion holds $P$ fixed, the ultimate frequency difference between the no-alliance and alliance cases must account for differences in $P$.

The $\alpha$ factor in (17) reflects double marginalization by nonaligned carriers in the choice of fares. In considering a higher subfare $p_1$, airline 1 ignores the effect on airline 2’s revenue, thus choosing a value higher than the one that would maximize joint profit (the alliance’s goal). Therefore, relative to the alliance case, the LHS of (17) is larger in the absence of an alliance (with $\alpha = 1$ instead of $1/2$) and thus reaches zero at a higher value of $P$.

### 3.4.2 High-cost case

Combining (11), (12) and (15), (16), the unified statement of the profit-maximization conditions for the case of high-cost layover time is

$$\alpha D + \left(\frac{P}{2} - \tau\right) D' = 0,$$  \hspace{1cm} (19)

$$- \left(\frac{P}{2} - \tau\right) \frac{\gamma}{F^2} D' - \theta = 0,$$  \hspace{1cm} (20)

where the parameter $\alpha$ again takes the value $1/2$ in the alliance case and $1$ when carriers are nonaligned.

Note the differences relative to the zero-cost case: although the first condition remains the same, the $2$ multiplying $F^2$ is absent in the second condition and $\alpha$ does not appear, with $\alpha$ present only in the first condition. The absence of $\alpha$ in (20) affects the previous
argument regarding frequency incentives. In contrast to the previous discussion for the zero-cost layover case, an alliance and nonaligned carriers will now choose the same $F$ value when $P$ is held fixed at a common value. This outcome reflects the fact, explained above, that the Nash equilibrium choice of $F$ has carrier 1 ignoring the min function and choosing $f_1$ in isolation. But since this is the same choice made by the alliance in choosing a common $F$ on the two route segments, the optimality conditions are then the same. As a result, the difference between the alliance and no-alliance values of $F$ arises only through differences in $P$, which affect the $F$ solution in (20). Thus, in contrast to the zero-cost layover case, double marginalization in the no-alliance frequency choices is absent, although it is still present in the choice of subfares.

### 3.5 The effects of alliances on the fare and frequency

In the general model developed so far, comparative-static analysis of the equilibrium conditions for the zero-cost and high-cost layover cases can be carried out to determine the fare and service-quality effects of alliances in the two cases. The derivatives $\partial P/\partial \alpha$ and $\partial F/\partial \alpha$ can be computed to gauge the effect of a change in $\alpha$ on the equilibrium fare and frequency in moving from the alliance to the no-alliance case. In doing the calculations, it is important to recognize the equilibrium conditions for the alliance case are the direct derivatives of alliance profit function with respect to $P$ and $F$, which means that the alliance second-order conditions can be used for the comparative-static analysis, which increases $\alpha$, starting at the alliance value of 1/2 and moving toward the no-alliance value of 1. By contrast, because symmetry has been imposed in generating the no-alliance equilibrium conditions, the second-order conditions for that case cannot be derived simply by differentiating these conditions with respect to $P$ and $F$. A return to the underlying profit-maximization problem is required.

The computation requires total differentiation of the two sets of alliance equilibrium conditions (17)-(18) or (19)-(20), with respect to $P$, $F$ and $\alpha$, evaluating the results at $\alpha = 1/2$. Denoting the expressions in the alliance equilibrium conditions by $\Pi_P$ and $\Pi_F$ (using the same notation for the zero-cost and the high-cost cases), their derivatives are $\Pi_{PP}$, $\Pi_{PF}$, $\Pi_{FP}$, and $\Pi_{FF}$, where the double subscript denotes second derivative. Then, for $\alpha = 1/2$, the following second-order conditions are assumed to hold:

$$\Pi_{PP}, \Pi_{FF} < 0 \text{ and } \Lambda = \Pi_{PP}\Pi_{FF} - \Pi_{PF}\Pi_{FP} > 0.$$  

(21)
To proceed, we adopt

**Assumption 1** $P$ and $F$ are strategic complements ($\Pi_{PF} = \Pi_{FP} > 0$), which is true under a linear-demand specification.

Note that $\Pi_{PF}$ has the sign of $- [\alpha D' + (P/2 - \tau) D'']$, evaluated at $\alpha = 1/2$. Therefore, Assumption 1 seems reasonable because one would expect the negative sign of $D'$ to dominate in determining the sign of this expression, in which case it would be positive.

In the zero-cost layover case, $\Pi_{P\alpha} = D > 0$ and $\Pi_{F\alpha} = -\theta < 0$. Therefore, using Cramer’s rule, the impacts of a change in $\alpha$ on $P$ and $F$ are given by

\[
\begin{align*}
\frac{\partial P}{\partial \alpha} &= \frac{-\Pi_{P\alpha} \Pi_{FF} + \Pi_{PF} \Pi_{F\alpha}}{\Lambda}, \quad (22) \\
\frac{\partial F}{\partial \alpha} &= \frac{-\Pi_{PP} \Pi_{F\alpha} + \Pi_{P\alpha} \Pi_{FP}}{\Lambda}. \quad (23)
\end{align*}
\]

As can be seen from inspection of (22) and (23), these derivatives are ambiguous in sign.

Moving to the high-cost layover case, $\Pi_{P\alpha} = D > 0$ (as before) but $\Pi_{F\alpha} = 0$. Hence, the impacts of a change in $\alpha$ are now simpler:

\[
\begin{align*}
\frac{\partial P}{\partial \alpha} &= \frac{-\Pi_{P\alpha} \Pi_{FF}}{\Lambda} > 0, \quad (24) \\
\frac{\partial F}{\partial \alpha} &= \frac{\Pi_{P\alpha} \Pi_{FP}}{\Lambda} > 0. \quad (25)
\end{align*}
\]

These derivatives show a clear effect of an alliance in the high-cost layover case, where an increase in $\alpha$ (movement away from the alliance case) raises $P$ and $F$, with an alliance thus yielding a lower fare and a lower frequency. Summarizing these results yields

**Proposition 2** Under Assumption 1, formation of an alliance leads to a lower fare and lower frequency in the high-cost layover case, while having ambiguous effects in the zero-cost layover case.

Thus, the only general result on the service-quality effect of an alliance is unexpected: flight frequency falls in the high-cost layover case when an alliance is formed. To gain further insight, we now put more structure on the model to generate determinate results in
the zero-cost layover case, and to provide some additional results in the high-cost layover case. We use a linear demand specification, which is widely applied in theoretical work on airline economics (see, e.g., Brueckner, 2004; and Brueckner and Flores-Fillol, 2007). The advantage is that clear analytical solutions can be derived directly from first principles. Additionally, the linear-demand set-up can show the effect of an alliance on the full price \( P + \gamma/F \) and the traffic volume \( Q \).

4 A linear demand model

In the material that follows, we assume that the demand function in (1) takes the form

\[
Q = a - b (\text{fare} + g [f_1, f_2]),
\]

where \( a, b > 0 \). Microfoundations yielding this linear demand function have been developed in previous papers (see, for example, Brueckner, 2004).

4.1 Zero-cost layover time

With zero-cost layover time, (26) becomes

\[
Q = a - b \left( p_1 + p_2 + \frac{1}{2} \left[ \frac{\gamma}{f_1} + \frac{\gamma}{f_2} \right] \right).
\]

With \( D' = b \), the zero-cost layover conditions in (17) and (18) reduce to

\[
\alpha \left[ a - b \left( P + \frac{\gamma}{F} \right) \right] - \left( \frac{P}{2} - \tau \right) b = 0,
\]

\[
\frac{P}{2} - \tau = \frac{2\alpha\theta}{b\gamma} F^2.
\]

Plugging \( P \) from (29) into (28), the following condition for \( F \) emerges:

\[
\frac{(a - 2b\tau)}{L(F)} F - b\gamma = \frac{2\theta}{\gamma} (2\alpha + 1) F^3.
\]

Using (30), the equilibrium frequency is found graphically, as seen in Figure 6. The \( F^* \) solution occurs at the intersection between the linear function \( L(F) \) on the RHS and the cubic expression \( C(F) \) on the LHS. It is easy to verify that \( a - 2b\tau > 0 \) must hold, so that the linear function is upward sloping. There are three possible solutions to (30), two of
them positive, but compliance with the second-order conditions makes the second positive solution relevant (details are in Appendix A).

Some comparative-static results that apply to both the alliance and no-alliance cases can be obtained by simple inspection of (30) along with Figure 6. Higher demand (an increase in $a$), rotates the line in Figure 6 counterclockwise and thus raises frequency, a natural conclusion. An increase in the demand slope $b$ (which leads to the reverse line rotation as well as a downward shift), has the opposite effect. Higher fixed or variable costs also have natural effects on $F$: a higher marginal seat cost $\tau$ rotates the line downward, reducing $F$, while a higher flight fixed cost $\theta$ has the same effect since it raises the level of the cubic curve. An increase in the disutility of schedule delay $\gamma$ seems to produce an ambiguous effect on $F$ (since it shifts both the line and the cubic curve downward), but a more thorough analysis shows that $F$ increases with $\gamma$. Therefore, frequency rises in response to a higher passenger sensitivity to schedule delay.\(^6\)

With the comparative-static effects of $a$, $b$, $\tau$, $\theta$, and $\gamma$ understood, attention now shifts to the effects of alliances. The impact of an alliance on $F$ can be easily ascertained graphically by plotting (30) for the no-alliance ($\alpha = 1$) and the alliance ($\alpha = 1/2$) cases, as shown in panel I of Figure 7. The cubic function in the alliance case $C(F)|_a$ is lower than $C(F)|_{na}$, so that the equilibrium $F$ is larger: $F^*_a > F^*_na$, where subscripts $a$ and $na$ denote alliance and no-alliance equilibrium values.

It is impossible, however, to determine the direction of the alliance fare effect, as can be seen by referring to (29). Since reducing $\alpha$ from 1 to 1/2 raises $F$, the change in the magnitude of the RHS expression is ambiguous, making the change of $P$ on the LHS also ambiguous. However, the alliance’s effect of the full price of travel can be derived. Eliminating $P/2 - \tau$ in (28) using (29) yields

$$a - b \frac{P + \gamma}{F} = \frac{2\theta}{\gamma} F^2. \quad (31)$$

Since the alliance raises $F$, the LHS of (31) rises as well, implying a higher traffic level $Q$ and lower full price $P + \gamma/F$. Therefore, even though the alliance may not lead to a lower fare, the full price of travel falls and traffic rises, just as in the standard alliance model where service quality is not a concern of passengers. Summarizing yields the following proposition.\(^7\)
Proposition 3  Under zero-cost layover time, an alliance leads to higher service quality (a higher $F$) than in the no-alliance case, although the fare $P$ may rise or fall. However, the full price of travel falls with an alliance, raising traffic $Q$.

Note that, while the general ambiguity of the alliance fare impact is not resolved in the linear case, a determinate frequency impact now emerges along with a conclusion regarding the impact on the full price and traffic.

4.2 High-cost layover time

With high-cost layover time, the linear demand in (26) becomes

$$ Q = a - b \left( p_1 + p_2 + \frac{\gamma}{\min\{f_1, f_2\}} \right), $$

and the high-cost layover conditions in (19) and (20) are now

$$ \alpha \left[ a - b \left( P + \frac{\gamma}{F} \right) \right] - \left( \frac{P}{2} - \tau \right) b = 0, $$

$$ \frac{P}{2} - \tau = \frac{\theta}{b\gamma} F^2. $$

Eliminating $P$ in (33) using (34), we obtain the following condition for $F$:

$$ (a - 2b\tau) F - b\gamma = \frac{\theta}{\gamma} \left( \frac{1}{\alpha} + 2 \right) F^3. $$

The comparative-static properties of $a$, $b$, $\tau$, $\theta$, and $\gamma$ are as in the zero-cost layover case. As for the effect of alliances, panel II of Figure 7 shows the graphical representation of this equilibrium condition for $\alpha = 1/2$ and $\alpha = 1$. The relationship between the curves is opposite to the one in the zero-cost layover case, with $C(F)|_a$ now higher than $C(F)|_{na}$, implying $F^*_a < F^*_{na}$, so that the alliance leads to a reduction in $F$ rather than an increase.

Since, in contrast to case of zero-cost layover time, (34) does not involve $\alpha$, the RHS of this condition falls in moving to an alliance, implying that $P$ must fall as well. A reduction in $P$ is, of course, the usual effect of an alliance, but it is now accompanied by a reduction, rather than an increase, in service quality. Furthermore, in contrast to the case of zero-cost layover time, the alliance has an ambiguous effect on the full price of travel and thus on $Q$. Eliminating $P/2 - \tau$ in (33) using (34) yields an analog to (31), where the RHS is replaced by $\theta F^2/\alpha\gamma$. Since $F$ falls as $\alpha$ drops from 1 to 1/2, the change in the RHS is
ambiguous, implying an ambiguous alliance effect on the LHS. Therefore, in contrast to usual beneficial effect of alliances on traffic, which also emerges in the case of zero-cost layover time, $Q$ may fall with high-cost layover time. Summarizing yields

**Proposition 4** *With high-cost layover time, an alliance leads to a lower fare $P$ as well as lower service equality (a lower $F$). The full price of travel may either rise or fall, implying that traffic $Q$ may decrease.*

This proposition confirms the result obtained in the general model and provides further insights concerning the full price and traffic volume.

Recalling the earlier discussion of frequency incentives, the source of the lower alliance frequency is clear in the high-cost layover case. As explained earlier, the common form of (34) for the alliance and no-alliance cases meant that differences in $F$ arise solely through differences in $P$ between the cases. Thus, the smaller $P$ in the alliance case is the source of the lower frequency.

The analysis with linear-demand thus shows that, in a model where flight frequencies capture service quality, the cost of layover time is crucial in determining the effects of airline alliances. When layover cost is zero, the usual double marginalization phenomenon in the no-alliance case suggests that an alliance should reduce the fare and raise frequency. While the predicted negative fare effect does not emerge, frequency does rise and its increase is sufficient to reduce the full price, benefiting passengers and raising traffic volume. When layover cost is costly, the additive reciprocal frequencies are replaced by the reciprocal of their minimum value, thus eliminating the double marginalization structure. Therefore, an alliance reduces rather than raises frequency, leading to a possible increase in the full price and a reduction in traffic.

Another way to understand these results is by considering the effect of frequency on passengers’ willingness-to-pay for travel, as reflected in the position of the demand curve. In the zero-cost layover case, the increase in service quality (the higher $F$) under an alliance raises willingness-to-pay by shifting the demand curve outward. While this shift would tend to raise the fare, the elimination of double marginalization creates downward fare pressure, and these opposing effects make the direction of the fare change ambiguous. By contrast, the lower alliance frequency in the high-cost layover case lowers willingness-to-pay and tends to reduce the fare. Combined with the double-marginalization effect, the result is an unambiguous fare reduction in moving to an alliance.
4.3 Comparison

Since the alliance frequency is the same in both the zero-cost and high-cost layover cases, while the no-alliance frequency is lower (higher) than the alliance frequency in the zero-cost (high-cost) layover cases, we can state

**Corollary 1** The no-alliance frequency is lower in the low-cost layover case than in the high-cost case, with the alliance frequency lying between the two no-alliance values.

The corollary is illustrated in Figure 8, where superscript 0 refers to zero-layover cost and $H$ to high-layover cost.

4.4 The effect of different passenger types

Airline pricing and scheduling decisions often differentiate between business and leisure passengers, who are distinguished by different values of time. Within the model, this difference in time valuations can affect passenger attitudes toward layover time and schedule delay. To see the effects, consider two types of city-pair markets where business and leisure travelers are, respectively, dominant. Of course, although some city-pair markets are easy to classify into one of these two broad categories, there are others where the passenger mix makes this taxonomy inappropriate. In what follows, we apply our findings to analyze the effect of airline alliances on the heterogeneity between business and leisure markets.

**Layover time.** It seems reasonable to assume that waiting for connecting flights at a hub airport is more costly for business passengers than for leisure passengers, implying a higher cost of layover time. Since no-alliance service quality is higher with high-cost layoffs (Corollary 1), it follows that business city-pair markets will have higher flight frequencies than leisure markets in the absence of an alliance. But after an alliance is formed, the new entity coordinates flight schedules by setting a common $F$ that eliminates layover time. As a consequence, flight frequency increases in leisure markets and decreases in business markets (as reported in Propositions 3 and 4). Thus, alliances eliminate airline responses to layover-cost heterogeneity across markets.

**Schedule delay.** Because of their high valuation of time, business passengers are expected to be more sensitive to schedule delay, so that a gap between preferred and actual arrival times is more inconvenient for this type of passenger than for a leisure passenger. In the model, this heterogeneity translates into differences in the $\gamma$ parameter, which is larger
in business than in leisure city-pair markets. Since an increase in \( \gamma \) raises frequency regardless of layover cost or alliance status (see footnote 7), it follows that the larger \( \gamma \) in business markets will further widen the no-alliance frequency gap between business and leisure markets that already exists due to differences in layover cost.

Therefore, different passenger time valuations, operating through both layover and schedule-delay costs, generate a service-quality gap between business and leisure markets in the no-alliance case. However, in contrast to layover-cost differences, differences in \( \gamma \) continue to have an effect once an alliance is formed, with business markets served by an alliance having higher frequency than leisure markets. Consequently, partial convergence between business and leisure city-pair markets can be expected as a consequence of airline alliances.

5 Congestion

In this section, we extend the analysis to account for congestion at the hub airport. The literature on airport congestion shows that airlines internalize the congestion they impose on themselves while neglecting the congestion they impose on other airlines. Therefore, alliances are expected to increase the extent to which congestion is internalized, given that allied carriers will take into account the congestion imposed on their partners. As a consequence, in a framework where service quality is not a factor, flight frequency is expected to decrease as compared to the case without congestion (as shown in Flores-Fillol (2010)). In the present model, where service quality matters, we would expect this effect to put downward pressure on frequency.

Given that an alliance yields a higher frequency under zero-cost layover time (Proposition 3) but lower frequency under high-cost layover time (Proposition 4), the analysis of the latter case is straightforward because the post-alliance internalization of congestion will reinforce the already reported decrease in frequency. However, the analysis of the case with zero-cost layover time is more interesting given that an alliance yields two conflicting effects on flight frequency: a positive one from the service quality effect and a negative one due to greater internalization of congestion. Consequently, the analysis that follows concentrates on low-cost layover case, where the existence of these opposing effects suggests that the previous positive flight-frequency effect could be reversed.

Congestion at the hub airport depends on the number of aircraft movements, given
by \( f_1 + f_2 \). Therefore, the per-flight congestion cost for each of the carriers is a function \( H (f_1 + f_2) \) with \( H' > 0 \) and \( H'' \geq 0 \), as in Brueckner and Van Dender (2008). Taking into account hub congestion, the cost function in (4) now becomes

\[
c_i = \theta f_i + \tau Q + f_i H (f_1 + f_2),
\]

for \( i = 1, 2 \). Hence, carrier 1’s profit function in the no-alliance case is given by (6) with the congestion-cost term \( f_1 H (f_1 + f_2) \) subtracted. In a similar way, alliance profit is given by (14) with the term \( 2FH (2F) \) subtracted.

Moving to the linear-demand case to assess the effect of alliances on fares and service quality, the \( H \) function is also assumed to be linear, so that \( H = \eta (f_1 + f_2) \) in the no-alliance case and \( H = 2\eta F \) in the alliance case, with \( \eta > 0 \) being the congestion-damage parameter. Analysis of the first-order conditions yields the following equilibrium condition for \( F \):

\[
\left( a - 2b \tau \right) F - b \gamma = \frac{2 (2\alpha + 1) F^3}{\gamma} \left[ \theta + \left( \frac{1}{\alpha} + 2 \right) \eta F \right],
\]

where the LHS is not affected by congestion, being the same linear function \( L (F) \) as in (30). By contrast, the RHS is now a quartic function \( D (F) \) due to the presence of the additional congestion term (see Figure 9). There are two possible positive solutions to (37), but compliance with the second-order conditions makes the second solution relevant.

It easy to check that \( D (F)|_a \) is lower than \( D (F)|_{na} \) in the positive quadrant, so that the equilibrium frequency is larger with an alliance, with \( F^*_a > F^*_na \). Therefore, the previous result, a higher frequency under an alliance, is preserved despite the offsetting effect from internalization of congestion.

As in the absence of congestion, the direction of the alliance fare effect cannot be determined, but the expression for the total traffic, equal to

\[
\underbrace{a - b \left( P + \frac{\gamma}{F} \right)}_{Q} = \frac{2F^2}{\gamma} \left[ \theta + \left( \frac{1}{\alpha} + 2 \right) \eta F \right],
\]

reveals that the full price \( P + \gamma/F \) falls and traffic \( Q \) rises with an alliance. This conclusion follows because an alliance leads to a smaller value for the RHS since it reduces \( \alpha \) and also increases \( F \). Consequently, the LHS rises as well, implying a lower full price and higher traffic.
It is also important to note that the presence of congestion reduces both $F_a^*$ and $F_{na}^*$ relative to their values in the absence of congestion, reflecting full or partial internalization of congestion. This outcome is due to the upward shift of the quartic functions for both the alliance and no-alliance cases when $\eta > 0$ relative to their positions when $\eta = 0$, shifts that move both intersections to the left. Note, however, that the magnitude of $(\frac{1}{\alpha} + 2) \eta$ captures the extent of the shift in the curves and thus the magnitude of the reductions in $F$ associated with congestion. The shift is larger with an alliance ($\alpha = 1/2$) than in the no-alliance case ($\alpha = 1$).

All of these results are summarized as follows:

**Proposition 5** Under zero-cost layover time,

i) Alliances yield two conflicting effects on flight frequency: a positive one derived from the double marginalization effect and a negative one due to greater internalization of congestion. The first effect dominates, so that an alliance leads to higher service quality (a higher $F$), a lower full price of travel and higher traffic, just as in the absence of congestion.

ii) The presence of congestion reduces frequency in both the alliance and no-alliance cases relative to the no-congestion case, but the alliance reduction is larger, reflecting greater internalization of congestion.

## 6 Conclusion

This paper has explored a topic that has received little attention in the literature: the service-quality impact of airline alliances. We focus on the most important dimension of service quality, flight frequency, ignoring the various other ways which alliances are thought to improve quality. To compare service qualities with and without an alliance, we need to distill the frequencies of two nonaligned carriers, which differ ex ante, into a service-quality measure for the no-alliance case. While past papers have recognized that the reciprocal of flight frequency (which captures schedule delay) is the proper measure for a nonstop route, no study has explored the case of a connecting passenger who travels on route segments exhibiting different flight frequencies. In this situation, the mismatched frequencies generate both layover time and schedule delay in a potentially complex fashion.

To make the analysis manageable, we focus on the polar cases of zero-cost and high-cost layover time. Then, the service-quality measure is proportional to either the sum of the reciprocal flight frequencies on the two route segments (zero-cost case) or to the reciprocal
of their minimum (high-cost case). With zero layover cost, choice of frequencies in the no-alliance case then has the same double-marginalization structure as the choice of subfares, leading to a higher frequency in the alliance case as double marginalization is eliminated, along with a lower full trip price. The surprising result of the paper arises with high-cost layover time, where double marginalization in frequencies is absent and where an alliance reduces service quality via a lower frequency, with the full price potentially rising.

Therefore, the paper offers a decidedly mixed message on the service-quality effects of alliances. Although the zero-cost layover case offers a welcome confirmation of existing results establishing the benefits of alliances, the possibility of an adverse effect remains. This possibility could be discounted by arguing that the high-cost layover case is unrealistic, but grounds for doing so are not clear. Perhaps further research looking at intermediate layover-cost cases could clarify the picture.

The paper also shows that nonaligned carriers adjust frequencies to suit passenger preferences in business and leisure markets, while an alliance is less responsive to such preference differences. This finding could suggest that alliances may have greater incentives for other kinds of business/leisure segmentation in interline markets, as compared to nonaligned carriers.

A caveat is that the model’s simplified route structure focuses on interline markets where air services are complementary, whereas actual alliances also involve interhub markets (also called gateway-to-gateway markets) where the airlines’ networks overlap (see Brueckner, 2001). Therefore, our analysis cannot be used to predict frequency effects on interhub routes, which is the subject of some recent empirical work (see Alderighi and Gaggero, 2014, and Bilotkach and Hüschelrath, 2012, who find positive effects).13

Overall, this paper opens new avenues for research on airline alliances. Further study of the service-quality impacts of alliances, both theoretical and empirical, can increase our understanding of the impacts of these important airline linkages and perhaps better inform the actions of the regulators who oversee them.
References


Notes

1Using a monopoly model, Brueckner (2004) studies the effect of network structure (hub and spoke vs. fully connected) on flight frequency. Brueckner and Flores-Fillol (2007) and Brueckner (2010) include scheduling decisions in a model of airline competition. de Palma, Criado and Randrianarisoa (2018) model frequency competition between carriers serving nearby airports using a Hotelling model. An empirical study by Pai (2010) shows effect of route distance on flight frequency. Bilotkach, Fageda and Flores-Fillol (2010) analyze how intermodal competition (cars vs. air services) affects the relationship between route distance and flight frequency. Brueckner and Pai (2009) and Fageda and Flores-Fillol (2012a and 2012b) study the possible emergence of new point-to-point routes served by regional jets (offering higher frequencies) or by low-cost carriers (offering poorer frequencies). Bilotkach, Fageda and Flores-Fillol (2013) examine the effects of the merger between Delta and Northwest on flight frequencies at the hubs dominated by the two airlines before the merger. Brueckner and Luo (2014) estimate reaction functions for carriers engaged in frequency competition.


3See Luttmann (2017) for an estimation of the value of layover time in the US airline industry.

4In a related study that does not focus on flight frequencies, Bilotkach (2007) shows how allied carriers alter their schedules to reduce connecting times while also lowering the interline fare.

5It can be shown that the first-order condition for \( f_1 \) found by ignoring the rule \( f_1 = 2^k f_2 \) and just differentiating with respect to \( f_1 \) is the same as the condition that emerges when the rule is used and differentiation occurs with respect to \( k \). In the latter case, however, \( k \) is treated as continuous, not integer valued, so that the resulting condition should be viewed as an approximation.

6The proof of \( \partial F/\partial \gamma > 0 \) is as follows (a similar proof can be found in Brueckner, 2004). Equating (30) to 0, we can compute \( \partial F/\partial \gamma = \frac{2\theta (2\alpha + 1) F^3 / \gamma^2 + \beta}{\Psi} \), where \( \Psi = 6\theta (2\alpha + 1) F^2 / \gamma - (a - 2b\gamma) \). Taking into account that compliance with the second-order condition requires the cubic function to be steeper than the linear one in (30), we conclude that \( \Psi > 0 \) (see Appendix A for the details). Finally, using again the equilibrium condition in (30) to rewrite the previous derivative, we obtain \( \partial F/\partial \gamma = (a - 2b\gamma) F / \gamma \Psi \), which is always positive.

7The effect of the alliance on the optimal aircraft size is unclear. With both \( Q \) and \( F \) rising, the change in \( s = Q/F \) is ambiguous.

8Brueckner and Pai (2009) and Fageda and Flores-Fillol (2012a) study the emergence of new point-to-point routes from the use of regional jets that allow higher frequencies in business city-pair markets.

9The same assumption motivates the analysis in Brueckner and Pai (2009) and Fageda and Flores-Fillol (2012a and 2012b).

10Some relevant contributions on the phenomenon of congestion internalization are Brueckner (2002), Mayer and Sinai (2003), Brueckner and Van Dender (2008), and Flores-Fillol (2010).

11For the sake of simplicity, we assume no passenger congestion costs. See Flores-Fillol (2010) for an
analysis including both airline and passenger congestion costs.

12The proof showing that only the second positive solution is relevant follows exactly the procedure described in Appendix A. Details are available from the authors on request.

13The positive effect of alliances on frequencies on these routes may actually have no link to incentives for better service quality, instead occurring because of the need to accommodate higher volumes of connecting traffic passing through the hubs, which is generated by removal of double marginalization for connecting trips.
Figures

Figure 1: The network.

Figure 2: Illustration of the case $f_1 \geq f_2$. 
Figure 3: Expected schedule delay under zero-cost layover time.

Figure 4: Expected schedule delay under costly layover time.
Figure 5: Airlines’ best-response functions.

Figure 6: The $F$ solution.
Figure 7: The effect of alliances on $F$.

Figure 8: Comparing the equilibrium frequencies.
Figure 9: The effect of alliances on $F$ under zero-cost layover time and congestion.
A Appendix: Second-order conditions and optimal $F$

in the linear-demand case

An analysis of the second-order conditions for the linear-demand case reveals that the second possible solution in Figures 6-8 is always the optimum.

A.1 No alliance

A.1.1 Zero-cost layover time

As mentioned above, the equilibrium frequency is found at the intersection between a linear and a cubic expression (see Figure 6). At the second positive solution in the no-alliance case (i.e., $\alpha = 1$), the cubic function is steeper than the linear one, implying $\frac{18\theta}{\gamma} F^2 > a - 2b\tau$, which can be rewritten as $\frac{18\theta}{\gamma} F^3 > (a - 2b\tau) F$. Using the condition in (30), this inequality becomes

$$b < \frac{12\theta}{\gamma^2} F^3.$$  \hspace{1cm} (A1)

Positivity of the Hessian determinant requires $\gamma < 8F \left(\frac{\gamma}{2} - \tau\right)$, which can be rewritten using (29) as

$$b < \frac{16\theta}{\gamma^2} F^3.$$  \hspace{1cm} (A2)

Therefore, satisfaction of (A1) implies satisfaction of the second-order condition in (A2). At the first positive solution in Figure 6, where the linear function is steeper than the cubic function ($b > \frac{12\theta}{\gamma} F^3$) the second-order condition may or may not be violated. If we rule out situations with two local maxima (which are possible if the second-order condition holds at both intersections), the second solution is the relevant one in the zero-cost no-alliance case.

A.1.2 High-cost layover time

At the second positive solution in the no-alliance case (i.e., $\alpha = 1$), the cubic function is again steeper than the linear one, implying $\frac{9\theta}{\gamma} F^2 > a - 2b\tau$ or $\frac{9\theta}{\gamma} F^3 > (a - 2b\tau) F$. Using (35), this inequality becomes

$$b < \frac{6\theta}{\gamma^2} F^3.$$  \hspace{1cm} (A3)
Positivity of the Hessian determinant requires $\gamma < 4F \left( \frac{P}{\gamma} - \tau \right)$, which can be rewritten using (34) as

$$b < \frac{4\theta}{\gamma^2} F^3$$

(A4)

Therefore, the second-order condition can only be satisfied at the second solution in the high-cost no-alliance case (this outcome, which is not guaranteed, is assumed).

A.2 Alliance

Repeating these steps for the alliance case (i.e., $\alpha = 1/2$), where the zero-cost and the high-cost layover cases converge, the cubic function is steeper than the linear one at the second positive solution, implying

$$b < \frac{8\theta}{\gamma^2} F^3.$$  

(A5)

Since positivity of the Hessian determinant requires exactly the same condition, the second-order condition holds only at the second intersection in the alliance case.