Schedule Competition Revisited

by

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Abstract

This paper proposes and analyzes a simple model of schedule competition, where transport providers choose service frequency and fares. The key assumptions are that passengers care about average schedule delay, a consequence of committing to travel before knowing their departure times, and that they exhibit brand loyalty to particular carriers. While the most general version of model is not amenable to analysis, familiar functional-form assumptions produce a tractable framework that generates a host of useful comparative-static results along with a clearcut efficiency evaluation. The model also sheds light on the phenomenon of excess transport capacity.

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1.0 Introduction

Transport providers compete on price, but competition also occurs on an important dimension of product quality: service frequency. In the airline industry, flight frequency is valued by passengers since higher frequencies give a broader range of choice in departure times. For the same reason, frequency matters in other, less visible, transport modes such as intercity bus service, where multiple providers often compete. Service frequency is also a key factor in freight transportation, and its importance has grown with the widespread adoption of just-in-time production processes, which rely on frequent deliveries of factory inputs.

Despite the central role of service frequency in transport, economists have devoted little effort to analyzing competition on this key product-quality dimension. The resulting scarcity of papers on "schedule competition" is partly due to the daunting nature of the problem when it is viewed in an explicitly spatial context. For example, in the spatial models of Schipper et al. (2003, 2007) and Lindsey and Tomaszewska (1999), consumers have a distribution of desired departure times, and service providers (airlines) time their departures taking account of this distribution, along with the schedules of competitors, while also setting fares. The resulting problem is one of spatial competition (d'Aspremont et al., 1979), but its difficulty is compounded by the fact that the *number* of spatial sites at which a firm operates (the number of flights) is endogenously chosen along with the locations of those sites (departure times) and prices.[1]

To circumvent the complexities of the spatial approach to schedule competition, the present paper analyzes a different type of model in which passengers care about overall service frequency without focusing on individual departure times.[2] The key assumption is that a consumer must make travel decisions before knowing his preferred departure time, which is drawn from a uniform distribution on the circle. Letting T denote the circle's time circumference, expected "schedule delay" (the difference between the preferred and actual departure times) then equals $T/4f$, where f is the service provider's number of evenly spaced departures. The cost of schedule delay can then be written γ/f, and the "full price" of a trip is $p + \gamma/f$, where p denotes the fare.

In a competitive context, the service provider ("carrier") offering the lowest full price would attract all the passengers in the market in the absence of some other friction. In the model, the required friction is supplied by consumer brand loyalty to particular carriers. Even when its full price is relatively high, brand loyalty allows a carrier to still attract its most loyal passengers. Along with idiosyncratic consumer preferences for particular carriers, the existence of loyalty-inducing mechanisms such as airline frequent-flier programs makes brand loyalty a plausible assumption.

In a previous paper, Brueckner and Flores-Fillol (BFF, 2007) analyzed a model based on the above assumptions, but their approach embodied a key limitation. Passengers were assumed to make a single trip, with demand for travel being perfectly inelastic. As a result, total travel volume could only vary through the decisions of some consumers not to travel at all.[3] In the present model, by contrast, individual trip volume is elastic, varying in response to a change in the full price. The no-travel option, which generates trip-volume changes in BFF, is absent since preferences take a form that makes travel essential. In addition to using this more-natural setup, the paper offers a further generalization by allowing an arbitrary number of competitors, which permits an analysis of entry (BFF assumed a duopoly).

It is important to note that the paper's travel-commitment assumption means that consumers must commit to a particular carrier before knowing their preferred departure time. This approach may not be fully accurate for some individual consumers, who may look at competing schedules with a departure time in mind when selecting a carrier. The approach better matches the choice setting of a corporate travel department, which must sign an exclusive contract with a particular carrier (in this case, an airline) for transporting its employees. The travel department cares about the average schedule delay for the company employees, while also seeking low fares. It signs an exclusive contract with the airline providing the best combination of these features (the lowest full price). Alternatively, the model could apply to individual business travelers, who cannot predict their travel times and thus purchase refundable full-fare tickets from the airline offering the lowest full price, tickets that allow them to board the next flight upon arrival at the airport. In both cases, the precise departure times of individual flights are not relevant, justifying the model's setup.

The plan of the paper is as follows. Section 2 formulates a general version of the model, using general specifications of traveler preferences, the distribution of brand loyalty and carrier costs (which give cost per departure as a function of seat capacity). Because analysis of the general model is impractical, specific function forms are then imposed, including Cobb-Douglas preferences, a uniform distribution of brand loyalty, and a linear carrier cost function. Under these assumptions, the model can be solved, and Section 3 derives the first-order conditions and presents comparative-static analysis. The analysis shows how service frequency, the fare, the full price (and thus the individual trip volume), and seat capacity depend on the model's parameters, both in the short and long run. Section 4 carries out an efficiency analysis, comparing the equilibrium outcome to the social optimum. The fare is shown to be too high and trip volume too low in equilibrium, as is frequency. The analysis also demonstrates that a second-best regime of fare regulation, like that imposed on airlines in the US prior to 1980, raises service frequency, matching the claims of contemporary critics of airline regulation. Section 5 analyzes the question of excess capacity (empty seats), which may be optimal when carriers face a minimum-seat-capacity constraint for each departure. In this situation, the carrier may wish to leave some seats unfilled to avoid sacrificing frequency, and the analysis identifies parameter conditions leading to this outcome. Section 6 presents conclusions. Overall, the paper offers a superior approach to analyzing schedule competition, providing a useful contribution to the literature of transport economics.

Before proceeding, it should be noted that the analysis can be viewed as an addition to the more-general literature on product-quality competition. The closest link is to models of vertical product differentiation, as analyzed by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) and summarized by Tirole (1988). In such models, consumers purchase one unit of a good, with utility depending inversely on $p - \lambda b$, where b is product quality, p is price, and λ captures the valuation of quality. The present full-price expression $p + \gamma/f$ differs only in that product quality (service frequency) appears in reciprocal form rather than as a negative linear term. More significantly, consumer heterogeneity in the vertical-differentiation model arises through a distribution on λ, which in turn leads firms to offer different product qualities in equilibrium, with better quality commanding a higher price p. Here, γ is the same for all consumers but heterogeneity arises through symmetric brand loyalty, which generates a symmetric equilibrium with identical qualities (frequencies). Such a symmetric, rather than differentiated, equilibrium is of interest since it matches the pattern in many transportation markets, where similar competing firms offer comparable service frequencies.

Note that the brand loyalty that underlies this symmetric outcome can be viewed as an exogenous, consumer-specific type of product differentiation, with consumers having different, idiosyncratic perceptions of the products of competing firms. Models with this brand-loyalty feature could be used to analyze quality competition in other contexts where the desired focus is on symmetric equilibria. Conversely, a vertical-differentiation variant of the present model could generate differentiated schedule-competition equilibria (with high and low-frequency carriers serving passengers with different schedule-delay costs) if the goal were to analyze such outcomes.

2.0 The Model

2.1. The general framework

The model focuses on a single transport market connecting two cities. Passengers in the market have mass M, and the market is served by n competing carriers. To develop the model, focus on carrier 1, and recall that a passenger using that carrier experiences an expected schedule delay of $T/4f_1$, where f_1 is the carrier's service frequency. Letting ν denote the cost per unit of schedule-delay time, $\nu T/4f_1 \equiv \gamma/f_1$ gives the cost of schedule delay on carrier 1, where $\gamma \equiv \nu T/4$. With p_1 denoting the carrier's fare, the full price of travel on carrier 1 is then $p_1 + \gamma/f_1$. Note that this full price depends on the fare, a price variable chosen by the carrier, as well as on a quantity variable, frequency, also chosen by the carrier.

Consumers value consumption, denoted x, and trips, denoted t, and both goods are essential, with infinite marginal utilities at zero consumption levels. Trip valuation depends on the carrier used to make the trip. If the consumer uses carrier 1, then utility is given by $U(x, a_1 t)$, where the a_1 factor measures the degree of brand loyalty to carrier 1. Brand loyalty is exogenous and varies across consumers, with each individual having particular values for the a_j, $j = 1, 2, \ldots, n$.[4]

Conditional on use of carrier 1, the consumer chooses x and t to maximize utility subject to the budget constraint

$$x \ + \ (p_1 + \gamma/f_1)t \quad = \quad y, \tag{1}$$

where y is the common level of income and where the full price appears as the price of a trip. Since second argument of the utility function can be replaced by $r \equiv a_1 t$ and expenditures rewritten as $x + [(p_1 + \gamma/f_1)/a_1]r$, it follows that the consumer's indirect utility function for travel on carrier 1 can be written as

$$V\left(\frac{p_1 + \gamma/f_1}{a_1},\ y\right). \tag{2}$$

The demand function for trips, however, depends separately on a_1 and the full price, being written[5]

$$t(p_1 + \gamma/f_1,\ a_1,\ y). \tag{3}$$

For the passenger to choose carrier 1, it must offer equal or higher utility than each of the competitors, which requires

$$V\left(\frac{p_1 + \gamma/f_1}{a_1},\ y\right) \quad \geq \quad V\left(\frac{p_j + \gamma/f_j}{a_j},\ y\right), \quad j \neq 1. \tag{4}$$

Since $V(\cdot)$ is decreasing in the price argument, it follows that (4) is equivalent to

$$a_j \quad \leq \quad W(p_j + \gamma/f_j,\ (p_1 + \gamma/f_1)/a_1,\ y), \quad j \neq 1. \tag{5}$$

for an appropriately chosen function W, indicating that brand loyalties to competing carriers must be sufficiently small.

The total number of travelers who prefer carrier 1 depends on the distribution of brand loyalty. Letting the density of consumers over the brand-loyalty space be denoted by $\Phi(a_1, a_2, \ldots, a_n)$, the number of travelers who prefer carrier 1 is given by the integral of Φ over the region specified in (5). To find carrier 1's passenger volume, denoted q_1, the density must be weighted by the trip volumes chosen by these travelers. The appropriate expression is then

$$
\begin{aligned}
q_1 \;=\; & \int_{a_1=0}^{\overline{a}_1} \int_{a_2=0}^{W(p_2+\gamma/f_2,\ (p_1+\gamma/f_1)/a_1,\ y)} \cdots \int_{a_n=0}^{W(p_n+\gamma/f_n,\ (p_1+\gamma/f_1)/a_1,\ y)} t(p_1+\gamma/f_1, a_1, y) \\
& \times \Phi(a_1, a_2, \ldots, a_n)\; da_1 da_2 \cdots da_n, \qquad (6)
\end{aligned}
$$

where the support of a_i is $[0, \overline{a}_i],\ j = 1, 2, \ldots, n$.[6] Using (6), carrier 1's revenue is then $p_1 q_1$.

On the cost side, carrier 1's cost per departure is given by $C(s_1)$, where s_1 equals seats per departure. This variable thus represents the seat capacity of the relevant transport conveyance (aircraft, bus, etc). The cost function $C(\cdot)$ is increasing and exhibits economies of seat capacity, with $C(s_1)/s_1$ decreasing in s_1. Cost per seat thus realistically falls as seat capacity rises. Carrier 1's total cost is equal to frequency f_1 times cost per departure, or $f_1 C(s_1)$.[7]

Total seats provided by the carrier must be sufficient to accommodate its passenger volume, requiring $f_1 s_1 \geq q_1$. As seen below, this constraint holds as an equality (implying that no seats are empty), as long as seat capacity s_1 is freely adjustable. This assumption is imposed until later in the analysis, where it is shown that the existence of a minimum seat capacity can make empty seats optimal.

Combining the above elements, carrier 1's profit-maximization problem can be stated. The carrier's goal is to choose frequency, the fare, and seat capacity per departure to maximize profit subject to above constraint, taking the choices of its competitors as given. Thus, the problem is

$$
maximize_{\{f_1, p_1, s_1\}}\ \pi_1 \;=\; p_1 q_1 \;-\; f_1 C(s_1) \;\; subject\ to \;\; f_1 s_1 \;=\; q_1, \qquad (7)
$$

where q_1 is given by (6).

2.2. Further assumptions

Although first-order conditions can be derived, nothing can be learned from this general formulation of the profit-maximization problem. To generate useful results, specific assumptions on functional forms must imposed, and the first such assumption is

$$A1. \quad \textit{Cobb-Douglas preferences: } U(x, a_i t) \quad \equiv \quad x^{1-\beta}(a_i t)^{\beta}$$

With this assumption, the demand for trips on carrier 1 is[8]

$$t(p_1 + \gamma/f_1,\ a_1,\ y) \quad \equiv \quad \frac{\beta y}{p_1 + \gamma/f_1} \tag{8}$$

and the indirect utility function is

$$V\left(\frac{p_1 + \gamma/f_1}{a_1},\ y\right) \quad \equiv \quad \delta y \left(\frac{p_1 + \gamma/f_1}{a_1}\right)^{-\beta}, \tag{9}$$

where $\delta = (1-\beta)^{1-\beta}\beta^{\beta}$. Substituting (9) into (4), the function W in (5) becomes

$$W(\cdot) \quad \equiv \quad \frac{a_1(p_j + \gamma/f_j)}{p_1 + \gamma/f_1}. \tag{10}$$

The second assumption is

$$A2. \quad \textit{A symmetric, uniform distribution of brand loyalty:}$$

$$\Phi(a_1, a_2, \ldots, a_n) \quad \equiv \quad \frac{M}{\alpha^n}, \quad \text{where } \overline{a}_i = \alpha,\ i = 1, 2, \ldots, n.$$

Recall that M is the mass of passengers, and note that the brand loyalty measure for each carrier thus ranges between 0 and α. Given $A1$ and $A2$, the integral in (6) reduces to

$$q_1 \quad = \quad \frac{\beta y}{p_1 + \gamma/f_1}\ \frac{M}{\alpha^n} \int_{a_1=0}^{\alpha} \int_{a_2=0}^{\frac{a_1(p_2+\gamma/f_2)}{p_1+\gamma/f_1}} \cdots \int_{a_n=0}^{\frac{a_1(p_n+\gamma/f_n)}{p_1+\gamma/f_1}} da_1 da_2 \cdots da_n \quad =$$

$$\frac{\beta y}{p_1 + \gamma/f_1}\ \frac{M}{\alpha^n} \int_{a_1=0}^{\alpha} \frac{a_1^{n-1}\Pi_{j\neq 1}(p_j + \gamma/f_j)}{(p_1 + \gamma/f_1)^{n-1}} da_1 \quad =$$

$$\frac{\beta y M\ \Pi_{j\neq 1}(p_j + \gamma/f_j)}{n(p_1 + \gamma/f_1)^n}. \tag{11}$$

With (11) yielding a substantial simplification, the final assumption used to produce a tractable profit function is

$$A3. \quad \text{Linear costs: } C(s_1) \;\equiv\; \theta \,+\, \tau s_1.$$

Thus, cost per departure involves a fixed cost of θ and a marginal seat cost of τ, so that cost per seat declines with s_1. Using $A3$, carrier 1's cost is $f_1(\theta + \tau s_1) = \theta f_1 + \tau q_1$, given the constraint in (7). As a result, profit can be written as $(p_1 - \tau)q_1 - \theta f_1$ or, using (11),

$$\pi_1 \;=\; \frac{(p_1 - \tau)\mu\,\Pi_{j\neq 1}(p_j + \gamma/f_j)}{n(p_1 + \gamma/f_1)^n} \;-\; \theta f_1. \tag{12}$$

In (12), $\mu \equiv \beta y M$ is a "market size" parameter, which depends on the trip utility exponent, income, and the passenger mass, all of which affect the aggregate demand for trips. Note that, with linear costs, seat capacity s_1 disappears as an explicit choice variable in the profit function. This variable can be recovered, however, using the relationship $s_1 = q_1/f_1$ once carrier 1's frequency and fare have been determined. Note also that profit is independent of the degree of dispersion of brand loyalty, as captured by α.[9]

3.0 Profit Maximization and Comparative Statics

3.1. First-order conditions

Carrier 1 chooses f_1 and p_1 to maximize profit in (12), viewing f_j and p_j, $j \neq 1$, as given in Nash fashion. The first-order conditions are

$$\frac{\partial \pi_1}{\partial f_1} \;=\; \frac{(p_1 - \tau)\mu\,\Pi_{j\neq 1}(p_j + \gamma/f_j)}{(p_1 + \gamma/f_1)^{n+1}}\,\frac{\gamma}{f_1^2} \;-\; \theta \;=\; 0 \tag{13}$$

$$\frac{\partial \pi_1}{\partial p_1} \;=\; \frac{\mu\,\Pi_{j\neq 1}(p_j + \gamma/f_j)}{(p_1 + \gamma/f_1)^n}\left(\frac{1}{n} \;-\; \frac{p_1 - \tau}{p_1 + \gamma/f_1}\right) \;=\; 0. \tag{14}$$

With symmetric carriers, it is natural to focus on the symmetric Nash equilibrium, where p_i and f_i, $i = 1, 2, \ldots, n$ assume common values, denoted p and f.[10]

Making this substitution and canceling the multiplicative factor in (14), rearrangement yields a solution for p in terms of f:

$$p \;=\; \frac{\tau n}{n-1} \;+\; \frac{\gamma}{(n-1)f}. \tag{15}$$

Note that, regardless of f's value, p exceeds the marginal seat cost τ given $n/(n-1) > 1$. Next, after imposing symmetry in (13), the first ratio term reduces to $\mu(p-\tau)/(p+\tau/f)^2$. Eq. (15) is then used to eliminate p in this expression, and the resulting new version of (13) is rearranged to solve for f, yielding

$$\frac{\mu(n-1)}{\theta n^2} \;-\; f \;=\; \frac{\tau}{\gamma} f^2. \tag{16}$$

The Nash equilibrium is computed by using (16) to generate a solution for f in terms of parameters, with substitution in (15) then yielding a solution for p. Note that the f solution is given by the intersection of the downward-sloping line corresponding to the LHS of (16) and the parabola corresponding to the RHS, as shown in Figure 1.

Several observations regarding the equilibrium are in order. First, it is easily seen that the second-order conditions for the carrier's profit-maximization problem are satisfied.[11] Second, note that the f solution from (16) depends only on the ratios τ/γ and μ/θ, not the levels of the component parameters. However, since (16) involves τ and γ as well as f, it follows that p depends on the individual levels of γ, τ, μ and θ. Finally, observe that since full prices are equal across carriers at the symmetric equilibrium, the division of passengers is determined only by brand loyalty. In the duopoly case, where $n = 2$, passengers whose combination of brand loyalties lies below (above) the 45 degree line in the (a_1, a_2) plane choose carrier 1 (2), a pattern that generalizes for higher values of n.[12]

3.2. Short-run comparative statics

Comparative-static analysis is carried out using (15) and (16). Since the ensuing computations view the number of competitors n as parametric, they constitute a short-run analysis. The long-run case, where n is endogenous, is considered below.

The effect of γ, τ, θ, μ, and n on f can be inferred directly from Figure 1. Since an increase in γ or a decrease in τ shifts the parabola downward, the intersection point moves to the right and f rises. Thus,

$$\frac{\partial f}{\partial \gamma} \;>\; 0, \quad \frac{\partial f}{\partial \tau} \;<\; 0, \tag{17}$$

indicating that frequency rises with the cost of schedule delay and falls with the marginal seat cost, both natural conclusions.

Since an increase in θ, a decrease in μ, or an increase in n shifts the line's intercept downward in Figure 1, f falls. Thus,

$$\frac{\partial f}{\partial \theta} \;<\; 0, \quad \frac{\partial f}{\partial \mu} \;>\; 0, \quad \frac{\partial f}{\partial n} \;<\; 0, \tag{18}$$

indicating that frequency falls with the fixed cost, rises with market size, and falls with the number of competitors, again all natural conclusions.

To derive fare impacts, a parameter's effect on f along with any direct effect appearing in (15) must be taken into account. Since θ is absent from (15) and an increase lowers f, it follows that p rises, and the reverse conclusion applies when μ increases. Also, since the direct effect of τ is positive while f falls with τ, the combined impact on p is positive, so that

$$\frac{\partial p}{\partial \theta} \;>\; 0, \quad \frac{\partial p}{\partial \mu} \;<\; 0, \quad \frac{\partial p}{\partial \tau} \;>\; 0. \tag{18}$$

Naturally, an increase in either cost parameter raises the fare. However, an increase in market size reduces p, an interesting conclusion that is discussed further below.

The effects of γ and n on p require computation of the relevant f derivative from (16) and its use in (15) to capture indirect effects. The impact of γ on p equals[13]

$$\frac{\partial p}{\partial \gamma} \;=\; \frac{1}{n-1}\frac{\partial(\gamma/f)}{\partial \gamma} \;\simeq\; \frac{1}{f} \;-\; \frac{\gamma}{f^2}\frac{\partial f}{\partial \gamma} \;\simeq\; 1 - \frac{\tau f}{\gamma + 2\tau f} \;>\; 0, \tag{19}$$

where $\simeq$ means "same sign." The effect of n is given by

$$\frac{\partial p}{\partial n} \;\simeq\; -\left(1 - \frac{(n-2)\gamma/n}{\gamma + 2\tau f}\right) \;<\; 0 \tag{20}$$

(see the appendix for details). While the negative effect of n is natural, the positive impact of schedule-delay cost on the fare is not predictable a priori.

Also of interest are comparative-static effects on the full price, which in turn determine the impacts on the individual trip volume. Substituting (15), the full price is given by

$$p \; + \; \frac{\gamma}{f} \;\; = \;\; \frac{n}{n-1}\left(\tau \; + \; \frac{\gamma}{f}\right). \tag{21}$$

Using the above logic, it follows that

$$\frac{\partial(p+\gamma/f)}{\partial\theta} \; > \; 0, \quad \frac{\partial(p+\gamma/f)}{\partial\mu} \; < \; 0, \quad \frac{\partial(p+\gamma/f)}{\partial\tau} \; > \; 0, \quad \frac{\partial(p+\gamma/f)}{\partial\gamma} \; > \; 0, \tag{22}$$

although n has an ambiguous effect on the full price. Thus, the full price rises, reducing the individual trip volume, when fixed or variable cost or schedule-delay cost rises. The full price falls, raising the individual trip volume, when market size rises, again a noteworthy conclusion.

Finally, consider the comparative-static effects on seats per departure. Imposing symmetry in (11) to derive q (the symmetric passenger volume per carrier), it follows that seats per departure is given by

$$s \;\; = \;\; \frac{q}{f} \;\; = \;\; \frac{1}{f}\,\frac{\mu}{n(p+\gamma/f)} \;\; = \;\; \frac{\mu(n-1)}{n^2(\tau f+\gamma)}, \tag{23}$$

using (15). Noting the f impacts from (17) and (18),

$$\frac{\partial s}{\partial\gamma} \;\; < \;\; 0, \quad \frac{\partial s}{\partial\theta} \; > \; 0, \tag{24}$$

indicating that seats per departure fall with schedule-delay cost (reflecting greater frequency) while naturally rising with fixed cost. Since the appendix shows that τf increases with τ and that $\mu/(\tau f+\gamma)$ increases with μ,

$$\frac{\partial s}{\partial\tau} \; < \; 0, \quad \frac{\partial s}{\partial\mu} \; > \; 0. \qquad (25).$$

The first conclusion, which shows that a higher marginal cost reduces seats per departure, is natural. However, the second conclusion is not predictable a priori, since it shows that both seats per departure and frequency must increase to accommodate a larger market. Finally, it can be shown that

$$\frac{\partial s}{\partial n} \;<\; 0, \tag{26}$$

so that more competition leads to fewer seats per departure at the same time that it reduces frequency. Thus, as in the case of μ, seats and frequency move in the same direction when n increases (here, downward).

The foregoing results are summarized in Table 1 and in

Proposition 1. *The comparative-static properties of the model are as follows:*

(*i*) *An increase in schedule-delay cost* (γ) *raises frequency, the fare, and the full price while reducing seats per departure.*

(*ii*) *An increase in fixed cost* (θ) *reduces frequency and raises the fare, the full price and seats per departure.*

(*iii*) *An increase in market size* (μ) *raises frequency, reduces the fare and the full price, and raises seats per departure.*

(*iv*) *An increase in marginal seat cost* (τ) *reduces frequency, raises the fare and the full price, and reduces seats per departure.*

(*v*) *An increase in the number of competitors* (n) *reduces frequency, the fare and seats per departure but has an ambiguous effect on the full price.*

As indicated in the preceding discussion, the positive impacts of γ on the fare and full price (leading to a lower trip volume) are not at all predictable. But the various impacts of the fixed and marginal-cost parameters θ and γ are natural conclusions, as are the impacts of n, the number of competitors. By contrast, the negative impacts of market size on the fare and the full price (leading to a higher trip volume), may seem puzzling at first. However, these outcomes appear to reflect the increasing-returns property of the transportation technology, where cost per seat falls with seats per departure. Since service to a larger market ends up requiring more seats per departure, cost per passenger falls, and this effect is passed through in a lower fare. Moreover, since the larger market also requires higher frequency, the schedule-delay component of the full price also falls, leading to an overall reduction in that price and more individual trips.

3.2. Long-run analysis

In the long run, the number of competitors adjusts to yield zero profit. Evaluating the profit expression (12) at the symmetric equlibrium and equating it to zero yields

$$\pi \quad = \quad \frac{\mu(p-\tau)}{n(p+\gamma/f)} \; - \; \theta f \quad = \quad 0. \tag{27}$$

Using the earlier first-order condition (14), evaluated at the symmetric equilibrium, the ratio term in (27) equals μ/n^2. As a result, (27) can be rearranged to yield

$$f \quad = \quad \frac{\mu}{\theta n^2}. \tag{28}$$

Substituting in (16) then yields the following condition, which gives the long-run equilibrium value of n:

$$n^3 \; - \; 2n^2 \quad = \quad \frac{\tau\mu}{\gamma\theta} \tag{29}$$

With the LHS increasing in n, it follows that the equilibrium n decreases with γ and θ and increases with τ and μ. Thus, letting $\widehat{n}$ denote n's long-run equilibrium value,

$$\frac{\partial\widehat{n}}{\partial\gamma} \; < \; 0, \quad \frac{\partial\widehat{n}}{\partial\theta} \; < \; 0, \quad \frac{\partial\widehat{n}}{\partial\mu} \; > \; 0, \quad \frac{\partial\widehat{n}}{\partial\tau} \; > \; 0. \tag{30}$$

The positive effect of market size μ is natural, and the negative effect of γ may arise because higher schedule delay reduces total trip volume holding n fixed, allowing $\widehat{n}$ to fall. While the same argument could explain the negative effect of fixed cost, an intuitive explanation for marginal cost's positive impact on $\widehat{n}$ is not readily apparent.

Computation of the long-run comparative-static effects on frequency, the fare, the full price, and seats per departure takes into account direct impacts as well as indirect impacts operating through n. Impacts on frequency are computed making use of (28), noting the indirect impacts on n derived from (29). Effects on the fare are computed by using (28) to eliminate f in the fare equation (15), and by then evaluating both direct parameter effects and indirect impacts operating through n. Comparative-static effects on the full price and seats per departure are computed similarly. Lengthy and tedious analysis gives a simple conclusion: the long-run effects all have the same sign as the short-run effects. To show the nature of required analysis, the appendix provides selected calculations showing the impact of γ, and the other calculations are available on request. Summarizing yields

> **Proposition 2.** *In long-run equilibrium, the number of competitors* ($\widehat{n}$) *falls when the schedule-delay cost* (γ) *or fixed cost* (θ) *increases, while rising when market size* (μ) *or variable cost* (τ) *increases. The long-run comparative-static effects on the remaining variables have the same signs as the short-run effects.*

3.3. Representative applications of the model

The value of the model can be shown by using it to make some real world predictions. Consider, for example, the impact of the current escalation in fuel prices on the airline operations. The fuel price hike can be viewed as leading to an increase in both the fixed and marginal-seat-cost parameters. From Table 1, the model predicts that, in the long run, fuel price escalation will reduce flight frequencies, while raising both airline fares and the full price of travel. Individual trip volumes will decline in response. Since θ and τ have opposing effects on s, the impact of higher fuel prices on aircraft seat capacity is ambiguous. If the price hike mainly affects aircraft fixed cost, seat capacity will rise, but if the main effect is on marginal seat cost, capacity will fall. Note that a similar, but opposite, ambiguity applies to the effect of higher fuel prices on the number of competitors. Observe that these predictions implicitly hold aircraft fuel efficiency constant, and that a more sophisticated approach would make this factor endogenous (see below).

The model can also be used to predict service and fare differences between airline routes traveled mainly by business passengers, who have a high schedule-delay cost γ, and routes traveled by leisure passengers, whose γ is lower. The model predicts that business routes will be served with greater frequency using smaller aircraft than leisure routes, and that fares will be higher. The first two predictions are confirmed in recent paper by Pai (2008), who investigates how endpoint characteristics affect aircraft size and flight frequency on nonstop US route segments. Pai shows that an increase in the managerial share of the workforces at the endpoints of a route leads airlines to use smaller aircraft and offer higher frequencies. His results also confirm the model's predictions regarding the effect of market size, with greater endpoint populations or incomes boosting aircraft size and flight frequency.[14]

3.4. Possible modifications of the model

Two modifications of the model could be considered. The first would introduce carrier asymmetry, with cost differences being a natural way of distinguishing the firms. Unfortunately, computation of the resulting asymmetric equilibrium is unworkable, even in the 2-carrier case, a barrier that BFF also encountered in their model.

A second modification would be to assume a two-stage choice of frequencies and fares, with frequencies realistically chosen first. However, while sequential choice yielded a different outcome than simultaneous choice in BFF's setup, the two approaches are equivalent in the present model. To see the reason, observe that, in a sequential setting, the first-order condition (14) determines carrier 1's fare conditional on the first-stage choice of frequencies. But since the first expression cancels, p_1 depends *only on the carrier's own frequency*, not on those of its competitors. Given this fact, it is easily shown that sequential and simultaneous choice are equivalent.[15]

4.0 The Social Optimum

4.1. The first-best optimum

The next step is to compare the equilibrium to the social optimum. Because utility is not transferable given Cobb-Douglas preferences, it is not possible to write down an explicit objective function for the planner to maximize, equal to the sum of consumer surplus and profit. Instead, a local approach to deriving the optimum is taken, as follows.

When full price increases by an amount dz, an individual consumer's utility falls at the rate $\partial U/\partial z = V_1/a$, where the subscript denotes the partial derivative of V with respect to the first argument (see (9)) and a denotes the experienced level of brand loyalty. Dividing by the marginal utility of income, given by $\partial U/\partial y = V_2$, the utility decline associated with the higher full price can be converted into a dollar value, with $(\partial U/\partial z)/(\partial U/\partial y) = \partial y/\partial z = (1/a)V_1/V_2$. Using (9), the resulting expression under Cobb-Douglas preferences is equal to

$$\frac{\partial y}{\partial z} \quad = \quad - \frac{(\beta\delta/a)[(p+\gamma/f)/a]^{-\beta-1}}{\delta[(p+\gamma/f)/a]^{-\beta}} \quad = \quad - \frac{\beta y}{p+\gamma/f}, \tag{31}$$

which equals $-t$ from (8). Multiplying by the consumer mass M then gives the aggregate dollar value of the utility loss from a marginal increase in the full price, equal to $-\mu/(p+\gamma/f)$.

In characterizing the social optimum, let the number of carriers be fixed at the long-run equilibrium value. At the optimum, the profit gain to these $\widehat{n}$ carriers from an increase in p should just cancel the dollar utility loss. Thus,

$$M\frac{\partial y}{\partial z}\frac{\partial z}{\partial p} \; + \; \frac{\partial(\widehat{n}\pi)}{\partial p} \quad = \quad 0 \tag{32}$$

must hold, where π is given by the expression in (27). Noting $\partial z/\partial p = 1$, substituting (31), and differentiating (27) to derive the second term in (32), the condition reduces to

$$- \frac{\mu}{(p+\gamma/f)} \; + \; \frac{\mu}{(p+\gamma/f)} \; - \; \frac{\mu(p-\tau)}{(p+\gamma/f)^2} \quad = \quad 0, \tag{33}$$

implying $p = \tau$. Thus, the socially optimal fare equals the marginal seat cost, a familiar requirement (the fare then lies below the equilibrium level).

The condition for the optimal frequency comes from replacing p by f in the derivatives in (32) and noting $\partial z/\partial f = -\gamma/f^2$. The resulting condition is

$$\frac{\mu}{(p+\gamma/f)}\,\frac{\gamma}{f^2} \; + \; \frac{\mu(p-\tau)}{(p+\gamma/f)^2}\frac{\gamma}{f^2} \; - \; n\theta \quad = \quad 0, \tag{34}$$

With $p = \tau$, (34) reduces to

$$\frac{\mu}{\theta n} \; - \; f \quad = \quad \frac{\tau}{\gamma}f^2, \tag{35}$$

a condition with the same form as (16). Given $(n-1)/n < 1$, it follows that the downward-sloping line on the LHS of (35) is higher than the corresponding line in (16), implying that the socially optimal f, denoted f^*, exceeds the equilibrium value. Frequency is thus higher at the optimum than in the equilibrium, although the two values converge as n increases.

The full price at the optimum equals $\tau + \gamma/f^*$, and this expression is less than the equilibrium full price from (21) given the smaller equilibrium f and $n/(n-1) > 1$. As a result, individual trip volumes are higher at the optimum than in the equilibrium, although convergence again occurs as n increases. Finally, the relationship between the equilibrium and socially optimal seat capacities is ambiguous. Summarizing yields

> **Proposition 3.** *Frequency at the social optimum is higher than in the equilibrium, and the optimal fare* (*which equals the marginal seat cost*) *is lower than the equilibrium fare. The full price is thus lower at the optimum than in the equilibrium, leading to higher individual trip volumes.*

Thus, the efficiency analysis yields the familiar conclusion that the equilibrium price is too high and quantities too low. The quantity comparison, however, applies to both trip volume and service frequency, distinguishing this efficiency verdict from a standard one.

4.2. The second-best optimum and regulatory outcomes

Given that the fare covers marginal but not fixed cost, the optimum imposes the usual losses on firms, and this outcome may make its attainment infeasible. The planner could instead pursue a second-best optimum, where consumer utility is maximized subject to a break-even constraint for the carriers. However, this second-best solution is complex (with f determined by a fifth-degree equation), so that a comparison to the equilibrium is not possible.

The second-best solution is nevertheless easily compared to the first-best optimum, recognizing that the second-best optimal fare (whatever its magnitude) must exceed τ in order for zero profit to be realized. When p is raised up from τ to the second-best level, the effect on frequency can be found using (34), which gives the optimal f conditional on p. Differentiation of (34) shows that f must fall as p rises above τ, implying that the second-best frequency is below the first-best level. Since the rise in p and the decline in f raise the full price, trip volume falls below the first-best level in moving to the second-best solution.[16]

Pursuit of the second-best optimum presumes that a government regulator can dictate carrier choices of both frequency and the fare. However, in the best-known example of transport regulation (US airline regulation prior to 1980), fares alone were regulated. Moreover, the decline in real airfares over the decades following deregulation indicates that, instead of reducing fares, the regulatory regime kept fares *above* the levels that would have emerged under competition (see Morrison and Winston, 1995). As for regulation's effect on service quality, many contemporaneous observers of the regulatory regime argued that, with price competition ruled out, competition on the remaining frequency dimension led to excessive flight frequencies. See, for example, Douglas and Miller (1974) and Morrison and Winston (1986).

The present model can be used to investigate the effect of regulation on service frequency. Letting $\overline{p}$ denote the regulated fare, carriers then choose f to maximize profit conditional on $\overline{p}$. Using (13), the symmetric equilibrium is characterized by

$$\frac{\mu(\overline{p}-\tau)}{(\overline{p}+\gamma/f)^2}\,\frac{\gamma}{f^2}\;-\;\theta\;=\;0. \tag{36}$$

To decide whether the resulting frequency level (denoted $\overline{f}$) is excessive, a benchmark is needed. One possible benchmark is simply the equilibrium frequency level itself, which satisfies (36) with $\overline{p}$ replaced by either the short-run or long-run equilibrium fare. To find the change in f in moving to the regulated equilibrium (where the fare is higher), (36) is differentiated with respect to $\overline{p}$. Calculation shows that the resulting derivative is positive provided that $\overline{p} < 2\tau + \gamma/\overline{f}$. It is easily seen that, when evaluated at the equilibrium frequency, the RHS expression in this inequality exceeds the equilibrium fare from (15) as long as $n \geq 2$. As a result, when $\overline{p}$ equals the equilibrium fare, the above inequality holds and $\partial\overline{f}/\partial\overline{p}$ is positive. Thus, when starting at the equilibrium fare level, a marginal, regulatory-induced fare increase raises frequency. Frequency continues to rise with $\overline{p}$ as long as the above inequality remains satisfied. Summarizing yields

> **Proposition 4.** *When fare regulation pushes p marginally above the equilibrium level, frequency rises, a conclusion that matches the claims of commentators regarding the impact of the US airline regulatory regime.*

A different benchmark for judging whether frequency is excessive is the f value that is socially optimal *conditional on the regulated fare.* This frequency level, denoted $\widetilde{f}$, satisfies (34) with p set equal to $\overline{p}$. It can be shown that the comparison between $\overline{f}$ and $\widetilde{f}$ is ambiguous in general, although $\overline{f} < \widetilde{f}$ holds (indicating that frequency is low relative to this alternate benchmark) in the duopoly case, where $n = 2$.

5.0 Excess Capacity

Empty seats are widespread in the provision of transportation services, and the model may provide some insight into this phenomenon. While stochastic travel decisions (failure to show up for a flight) are one cause of empty seats, the model highlights a different mechanism: carriers' desire to maintain frequency in the face of a minimum-seat-capacity constraint. To understand the mechanism, note that if seats per departure are fully flexible, then excess capacity can always be eliminated through a seat reduction. But if s must exceed some minimum $\overline{s}$ and the constraint binds, then empty seats can only be eliminated through a reduction in frequency, which may be undesirable. Such a minimum-capacity constraint would be realistic, for example, on transcontinental or international airline routes, where aircraft with the required range are necessarily large.

When seats are empty, the relationship $q_1 = f_1 s_1$ used in setting up the carrier's profit function no longer applies, with an inequality holding instead. As result, carrier 1's profit-maximization problem must be reformulated by the addition of two constraints: $f_1 s_1 \geq q_1$ and $s_1 \geq \overline{s}$. The Lagrangean expression for this new problem is

$$p_1 q_1 \; - \; f_1(\theta + \tau s_1) \; + \; \lambda(s_1 - \overline{s}) \; + \; \phi(f_1 s_1 - q_1), \tag{38}$$

where q_1 is again given by (11).

The first-order conditions, evaluated at a symmetric equilibrium, are

$$f: \qquad \frac{\mu(p-\phi)}{(p+\gamma/f)^2}\,\frac{\gamma}{f^2} \; - \; (\theta + \tau s) \; + \; \phi s \;\; = \;\; 0 \tag{39}$$

$$p: \qquad \frac{1}{n} \; - \; \frac{\mu(p-\phi)}{p+\tau/f} \;\; = \;\; 0 \tag{40}$$

$$s: \qquad -\tau f \ + \ \phi f \ + \ \lambda \ = \ 0 \tag{41}$$

along with the complementary slackness conditions. To begin analyzing the solution, note first that if $\lambda = 0$, indicating that the minimum-seat constraint is not binding, then (41) implies $\phi = \tau > 0$, indicating that the constraint $f_1 s_1 \geq q_1$ binds and that all seats are filled. Thus, empty seats cannot exist when the carrier is still able to reduce seat capacity. When τ is substituted in place of ϕ in (39) and (40), these conditions naturally reduce to the previous equilibrium conditions. By contrast, when the minimum-seat constraint binds (so that $\lambda > 0$), then (41) shows that $\phi < \tau$.

The strategy for analyzing the conditions leading to excess capacity is as follows. First, suppose that $\lambda > 0$ and $\phi > 0$ hold at the solution to the above problem, so that the minimum-seat constraint is binding and all seats are filled. Then, after solving for the resulting f, p and ϕ values, the parameter changes that lead to a decline in ϕ can be identified. Large enough parameter changes in the required direction will then drive ϕ to zero, at which point empty seats emerge.

To carry out this strategy, s in (39) is replaced with $\overline{s}$, and (39), (40) and the additional condition $f\overline{s} = \mu/n(p + \gamma/f)$ (which equals q) are then solved for f, p and ϕ. Normalizing μ at 1 to facilitate a solution, the resulting ϕ value is

$$\phi \ = \ \tau \ - \ \left(\frac{\gamma n^2(\theta + \tau\overline{s})}{n-1} \ - \ \frac{\theta}{\overline{s}} \right). \tag{42}$$

For some seats to be empty, contradicting the assumption that $f\overline{s} = q$, the ϕ solution in (42) should be negative instead of assuming the required positive value, an outcome that will obtain for particular parameter values. In particular, since large values of $\overline{s}$, n or γ lead to small values of ϕ in (42), large enough values will drive the ϕ solution below zero, leading to empty seats. The effects of τ and θ on the magnitude of (42) are, however, ambiguous. The first set of conclusions is mostly intuitive: a large minimum capacity induces empty seats, as does a high cost of schedule delay, which reduces seats per departure (as it raises frequency) in the absence of any constraint. Also, a higher n puts downward pressure on both frequency and

s in the unconstrained case, eventually leading to empty seats in the presence of the constraint. Summarizing yields

> **Proposition 5.** *A minimum-seat-capacity constraint can lead carriers to operate with excess capacity, leaving some seats empty. This outcome is encouraged by a large minimum capacity* ($\overline{s}$)*, a large schedule-delay cost* (γ)*, or a large number of competitors* (n).

It should be noted that, with the exception of τ, the parameter conditions that tend to make ϕ in (42) negative are the same conditions that lead to $\phi < \tau$ and thus $\lambda > 0$, implying a binding minimum-seat constraint. The requirements are large values of $\overline{s}$, γ, or n. But since $\phi < \tau$ only requires a positive value for the expression in parentheses in (42), the previous ambiguity regarding τ disappears. In particular, since a large τ will make the expression in (42) positive, a sufficiently large marginal seat cost will cause the minimum-seat constraint to bind (a natural conclusion), even though it has an ambiguous impact on the emergence of empty seats.

6.0 Conclusion

This paper has proposed and analyzed a simple model of schedule competition, where transport providers choose service frequency and fares. Key assumptions are that passengers care about average schedule delay, a consequence of committing to travel before knowing their departure times, and that they exhibit brand loyalty to particular carriers. While the most general version of model is not amenable to analysis, familiar functional-form assumptions produce a tractable framework that generates a host of useful comparative-static results along with a clearcut efficiency evaluation. The model also sheds light on the phenomenon of excess transport capacity (empty seats).

Given its tractability, the model could be adapted to study the impacts of various transport policy measures. Brueckner and Girvin (2008) used BFF's related model to study the effect of airport noise regulation on flight frequencies and fares, treating the aircraft noise level as an endogenous choice variable, and a similar application is possible here. While effect of airline carbon taxes and general fuel price escalation could be studied using the simple approach mentioned above, a more ambitious treatment would view aircraft fuel efficiency as a decision variable, like aircraft noise, in an expanded version of the model.[17] In addition, the impact of airport congestion pricing on fares and frequencies as well aircraft sizes could be explored using the model, following the lead of Flores-Fillol (2008).

Although service frequency is an important consideration in freight transportation, as noted in the introduction, the present model pertains to the tranport of passengers, not freight. A somewhat different setup, which would nevertheless retain many of the current features, would be required to analyze schedule competition in freight transport. Development of such a model could be a task for future work. Transport economists have lacked a simple framework for the analysis of schedule competition, and this list of possible applications and extensions shows that the present model can open many doors.

Appendix

Derivation of (20)

Differentiation of (16) yields

$$\frac{\partial f}{\partial n} \quad = \quad \frac{\gamma\mu(2-n)/\theta n^3}{\gamma + 2\tau f}, \tag{$a1$}$$

and differentiation of (15) then yields

$$\begin{aligned}
\frac{\partial p}{\partial n} \quad &= \quad -\frac{\tau}{(n-1)^2} \;-\; \frac{\gamma}{(n-1)^2 f} \;-\; \frac{\gamma}{(n-1)f^2}\,\frac{\partial f}{\partial n} \\
&= \quad \frac{1}{(n-1)^2 f^2}\left(-\tau f^2 \;-\; \gamma f \;+\; \frac{\gamma^2\mu(n-2)(n-1)/\theta n^3}{\gamma + 2\tau f}\right) \\
&\simeq \quad -\left(1 \;-\; \frac{(n-2)\gamma/n}{\gamma + 2\tau f}\right) \;<\; 0. \qquad (a2),
\end{aligned}$$

where $\tau f^2 + \gamma f$ in the second line is eliminated using (16).

Derivation of (25)

From (23), τ's effect on s is the reverse of its effect on τf. Using $\partial f/\partial\tau = -f^2/(\gamma + 2\tau f)$, the latter derivative is proportional to the positive expression in (19), yielding $\partial s/\partial\tau < 0$.

The effect of μ on s is proportional to its effect on $\mu/(\tau f + \gamma)$, which has the same sign as

$$1 \;-\; \frac{\mu\tau}{\tau f + \gamma}\,\frac{\partial f}{\partial\mu}. \tag{$a3$}$$

Using $\partial f/\partial\mu = (\gamma\mu(n-1)/\theta n^2)/(\gamma + 2\tau f)$, ($a3$) reduces to the positive expression in (19), yielding $\partial s/\partial\mu > 0$.

Derivation of (26)

The impact of n on s is the reverse of its impact on $(n^2/(n-1))(\tau f+\gamma)$. Using $(a2)$, that derivative is equal to

$$\begin{aligned}
&\frac{n(n-2)}{(n-1)^2}(\tau f+\gamma) \;+\; \frac{\tau n^2}{n-1}\frac{\partial f}{\partial n} \;=\\
&\frac{n-2}{n-1}\left(\frac{n}{n-1}(\tau f+\gamma) \;-\; \frac{\gamma\tau\mu/\theta n}{\gamma+2\tau f}\right) \;\simeq\\
&\frac{\gamma\mu}{\theta n f} \;-\; \frac{\gamma\tau\mu/\theta}{n(\gamma+2\tau f)} \;\simeq\; n \;+\; \tau f \;>\; 0,
\end{aligned} \qquad (a4)$$

yielding $\partial s/\partial n < 0$ (the second equality relies on substitution from (16)).

Representative long-run calculations

Consider the long-run effects of an increase in γ. Differentiation of (29) shows that

$$\frac{\partial n}{\partial \gamma} \;=\; -\,\frac{\tau\mu/\gamma^2\theta}{3n^2-4n} \;<\; 0, \qquad (a5)$$

and $(a5)$ along with (28) yields $\partial f/\partial\gamma > 0$.

Using (16) together with $(a5)$ yields

$$\begin{aligned}
\frac{\partial p}{\partial \gamma} \;&=\; \left(\frac{\gamma\theta n/\mu}{n-1} \;-\; \frac{\tau+\gamma\theta n/\mu}{(n-1)^2}\right)\frac{\partial n}{\partial \gamma} \;+\; \frac{\theta n^2/\mu}{(n-1)}\\
&\simeq\; -\frac{\tau}{n}\frac{\partial n}{\partial \gamma} \;+\; \frac{\theta n^2}{\mu} \;>\; 0,
\end{aligned} \qquad (a6)$$

where the second equality relies on substitution from (29).

To compute the effect of γ on the full price, (28) is substituted in (21), so that the full price becomes $(\tau+\theta\gamma n^2/\mu)n/(n-1)$. Differentiation with respect to γ then yields (after repeated substitutions using (29))

$$\begin{aligned}
\frac{\partial(p+\gamma/f)}{\partial \gamma} \;&=\; \frac{\gamma\theta n^2/\mu}{n-1}\frac{\partial n}{\partial \gamma} \;+\; \frac{\theta n^3/\mu}{n-1}\\
&\simeq\; \frac{2(n^2-n)}{3n-4} \;>\; 0.
\end{aligned} \qquad (a7)$$

To compute γ's effect on seats per departure, (28) is substituted into (23), yielding $s = (n-1)/(\tau\mu/\theta n^2 + \gamma)$. Differentiation and substitution from (29) yields

$$\frac{\partial s}{\partial \gamma} \quad \simeq \quad \gamma(n-2)\frac{\partial n}{\partial \gamma} \; - \; n \quad < \quad 0. \tag{a8}$$

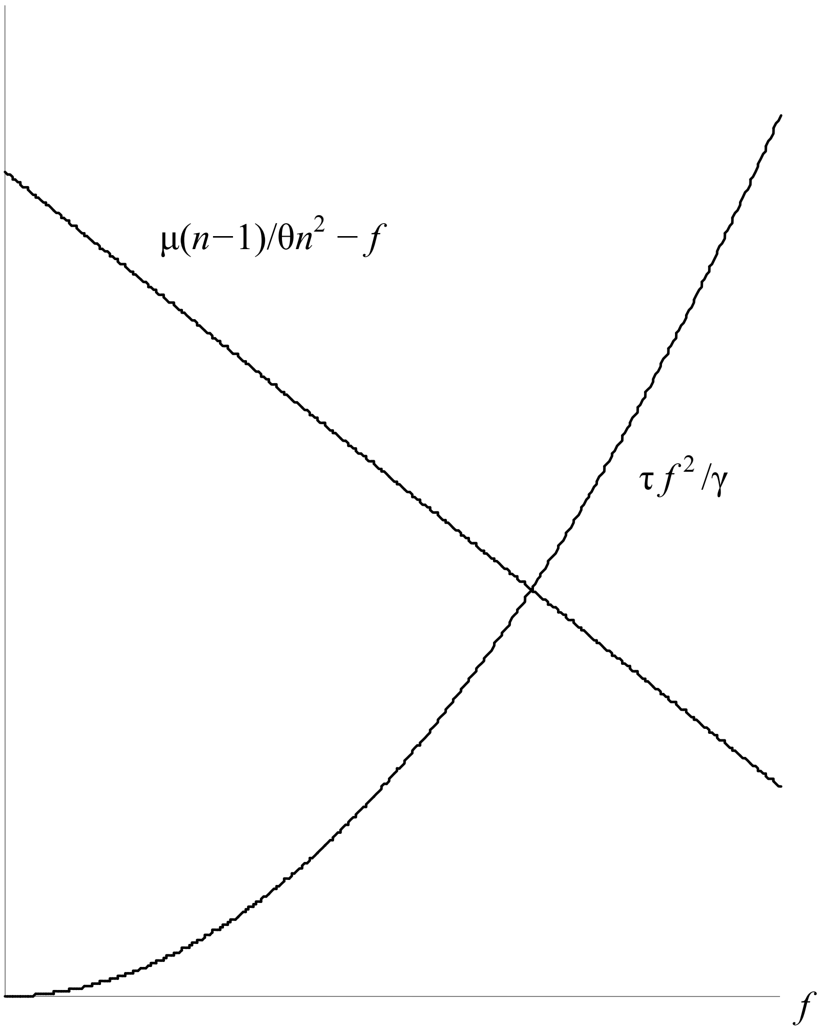

Figure 1: Determination of f

Table 1.

Comparative-Static Effects

PARAMETER: VARIABLE:	*schedule-delay cost* (γ)	*fixed cost* (θ)	*market size* (μ)	*marginal seat cost* (τ)	*competitors* (n)
frequency (f)	+	−	+	−	−
fare (p)	+	+	−	+	−
full price ($p + \gamma/f$)	+	+	−	+	?
seats per departure (s)	−	+	+	−	−

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Footnotes

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[1] Using a spatial approach, Brueckner and Zhang (2001) analyzed a scheduling model for a monopoly airline, with the goal of understanding how network structure affects flight frequencies. By contrast, Panzar (1979) used a spatial model to analyze frequency equilibria when each airline operates a single flight. Since free entry along with a zero-profit condition determines equilibrium flight frequency, his model does not depict schedule competition. Salop (1979) presented a similar but more-general model of monopolistic competition in a circular spatial market. Firm locations are interpreted as product brands, but they could also be viewed as flight departure times in the case where each airline offers a single flight. In another analysis of scheduling, Encaoua et al. (1996) analyzed the strategic choice of departure times for two airlines when one relies on connecting traffic from the other.

[2] The source of this approach is Brueckner (2004), who analyzed a monopoly scheduling model.

[3] This outcome was generated by assuming two groups of passengers, with high and low travel benefits. High-benefit passengers always travel, so that their margin of choice is between the two carriers. Only those low-benefit passengers with strong brand loyalty to a particular carrier travel, and their margin of choice is between that carrier and not traveling. Hymer and Shy (2006) independently developed a very similar model, which they used in the analysis of airline alliances. Brueckner and Flores-Fillol's paper was also written for the airline context, although the model has general applicability.

[4] In a richer model, brand loyalty might be endogenous, being affected by firm efforts to build loyalty (for example, by the generosity of airline frequent-flier programs) or by the level of service quality itself.

[5] Note that, while the demand for r depends only on the ratio $(p_1 + \gamma/f_1)/a_1$, that demand must be divided by a_1 to get the demand for t.

[6] Note that if $W(p_j + \gamma/f_j,\ (p_1 + \gamma/f_1)/a_1,\ y) \geq \overline{a}_j$ holds, then all travelers will prefer carrier 1 to carrier j, leaving that carrier with no passengers. Since this outcome will not be observed in equilibrium, the reverse inequality can be assumed to hold, justifying the limits of integration in (6).

[7] To better grasp the logic of this cost formulation, note that each aircraft, bus, or other

conveyance will be used for fixed number of trips per unit time (say, 4 or 5 trips per day for an aircraft). The carrier incurs a cost per unit time for the conveyance (a lease rate, for example), and dividing this cost by the number of trips per unit time yields the cost per departure, $C(s_1)$.

[8] Note that, given the form of preferences, the strength of demand is not affected by brand loyalty. Under different assumptions, such a linkage could exist, and carriers could conceivably exploit it, assuming that the extent of each passenger's loyalty is observable. For example, if price discrimination were feasible and loyalty observable, the carrier could then charge higher fares to more-loyal passengers.

[9] This independence was not present in BBF's model, where profit depends on α.

[10] Asymmetric equilibria would emerge when the cost parameters differ across firms, as discussed further below.

[11] Computations show that $\partial^2\pi_1/\partial f^2$ and $\partial^2\pi_1/\partial p^2$ are negative and that $\partial^2\pi_1/\partial f\partial p = 0$ holds at the solution to (15) and (16), ensuring positivity of the profit function's Hessian determinant.

[12] Recall that each a value also has an upper limit of α.

[13] Differentiation of (16) shows that

$$\frac{\partial f}{\partial \gamma} = \frac{(n-1)/\theta n^2 - f}{\gamma + 2\tau f}.$$

[14] Givoni and Rietveld (2006) carry out a similar exercise focusing only on aircraft size but use a less revealing set of covariates. Among other things, their results show that an increase in market size (measured by total seats offered by all airlines in the market) leads to the use of larger aircraft.

[15] Letting the second-stage fare choice be given by $p_1 = g(f_1)$ and focusing on the duopoly case, carrier 1's profit can be written $\pi_1(g(f_1),\ g(f_2),\ f_1,\ f_2)$. The optimality condition for the first-stage choice of f_1 is then $g'(f_1)\partial\pi_1/\partial p_1 + \partial\pi_1/\partial f_1 = 0$. But since $\partial\pi_1/\partial p_1 = 0$, this condition reduces to $\partial\pi_1/\partial f_1 = 0$, the optimality condition under simultaneous choice. Suppose, however, that each carrier's second-stage fare choice depended on both f_1 and f_2 (as in BFF's model), so that $p_1 = h(f_1, f_2)$ and $p_2 = h(f_2, f_1)$ (given the model's symmetry, h is a symmetric function). After again zeroing out the initial term, the first-order condition

for f_1 in the first stage would be

$$h_2(f_2, f_1)\partial\pi_1/\partial p_2 \ + \ \partial\pi_1/\partial f_1 = 0,$$

where h_2 denotes h's partial derivative with respect to its second argument. This condition differs from the first-order condition for f_1 under simultaneous choice.

[16] Note that these comparisons are qualitatively the same as the equilibrium/optimum comparisons already stated in Proposition 3, which is natural given that $p > \tau$ also holds in the equilibrium.

[17] Although these additions are tractable in BFF's framework, the greater nonlinearity of the present model may force the use of numerical methods in the analysis.