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EQUILIBRIUM IN A SYSTEM OF COMMUNITIES WITH LOCAL PUBLIC GOODS
A Diagrammatic Exposition

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Equilibrium community configurations are analysed using a simple model where a community's public good output is chosen by majority voting and financed by a head tax. Examples which contradict the Tiebout hypothesis are presented.

In a seminal paper, Tiebout (1956) argued that pessimism about the ability of a decentralized economy to achieve an efficient allocation of resources to public good production may be unjustified. He conjectured that individuals have an incentive to segregate into homogeneous communities where public goods can be provided efficiently. Analytical treatment of Tiebout's hypothesis has appeared only in the last few years, with important contributions made by McGuire (1974), Stiglitz (1977), Westhoff (1977), and Wheaton (1975). Unfortunately, the verdict of these studies is not encouraging: the efficiency (and even the existence) of community equilibria with local public goods appears problematical. The present paper presents negative results which are similar to those derived by the other writers. The paper's contribution is the development of a simple diagrammatic analysis which starkly illustrates some of the difficulties of the Tiebout hypothesis. The analysis complements earlier work, in which important conclusions are often masked by technical details or expositional deficiencies.

In the following analysis, it is assumed that individuals consume a local public good and a private numeraire good. Each community produces the non-exportable private good according to the production function $G(n)$, where n is population. Equal productivity is assumed for all individuals. Part of the output of the private good, A, is used to produce the public good z according to the function $z = F(A)$, while the remainder $G(n) - A$ is consumed directly. For generality, it is assumed that the public good is subject to congestion in that per capita consumption x is given by $x = f(z, n)$, with $f_n \leqslant 0$. Increasing n while holding z fixed may reduce consumption of the public good. Inverting F and f gives $A = K(z)$ and $z = h(x, n)$. Substituting for z in K gives $C(x, n)$, the cost in terms of the private good of providing a per capita consumption level x of the public good in a community of size n:

$C(x, n) = K(h(x, n))$. This model is very similar to one analysed by Stiglitz.

It is assumed that each individual in a community receives a wage equal to $G'(n)$, his marginal product in the private good production process, and that the profits (or losses) from private good production, $G(n) - nG'(n)$, are divided equally among the residents. Together, these assumptions yield an income for each consumer equal to $G(n)/n$ units of the private good. A further assumption is that production of the public good is facilitated by a uniform head tax. Public good consumption of x requires that a head tax of $C(x, n)/n$ be levied on each resident. Letting g denote per capita private good consumption, each individual's budget constraint is therefore $g = (G(n) - C(x, n))/n$.

While individuals are identical in production, differing tastes will give rise to different desired consumption levels. The desired public good consumption in a community of size n for an individual with utility function $u^i(x, g)$, denoted $x_i(n)$, is given by the solution to $u^i_x/u^i_g = C_x/n$. The individual equates his MRS to his marginal cost for the public good, which is $1/n$ of the community's marginal cost. In a community composed entirely of type i individuals, public good consumption will be set at $x_i(n)$ by unanimous consent. In a community composed of individuals with different tastes, there will be disagreement over the appropriate public good level. Since public (and hence private) consumption must be the same for everyone, a public choice rule must decide the outcome. Majority voting determines public good consumption in the following analysis.

For simplicity, an economy with two types of individuals is considered. The following functions are central to the analysis:

$$W_i(n) \equiv u^i(x_i(n), [G(n) - C(x_i(n), n)]/n), \qquad i = 1, 2,$$

and

$$W_i^j(n) \equiv u^i(x_j(n), [G(n) - C(x_j(n), n)]/n), \qquad j \neq i.$$

$W_i(n)$ gives the maximized utility level of a type i resident in a community of size n and $W_i^j(n)$ gives the utility level of a type i resident in a community of size n where the public good level is chosen to satisfy type j tastes. Clearly, $W_i^j(n) \leqslant W_i(n)$. While analysis shows that the properties of the functions W_i and W_i^j depend on the nature of the functions u^i, u^j, and C, it is assumed for illustrative purposes that the W_i and W_i^j curves are single-peaked and have approximately the same shape. The value of n at the peak of W_i, which gives the optimal community population from the point of view of a type i individual, is denoted n_i^*.

The assumption of similar shapes for the W curves is made plausible by the following example: Suppose that type-one and type-two individuals have utility functions $g^{\alpha_1}x^{1-\alpha_1}$ and $g^{\alpha_2}x^{1-\alpha_2}$, respectively, that $F(A) \equiv A$, and that z is a pure public good so that $x \equiv z \equiv A$. Then it is easily established that

$$W_i(n) = \alpha_i^{\alpha_i}(1 - \alpha_i)^{1-\alpha_i}\, G(n)n^{-\alpha_i}, \qquad i = 1, 2,$$

and

$$W_i^j(n) = \alpha_j^{\alpha_i}(1 - \alpha_j)^{1-\alpha_i} G(n)n^{-\alpha_i}, \qquad i \neq j,$$

which means that $W_i^j(n)$ is proportional to $W_i(n)$. The derivatives of W_i and W_i^j are proportional to $n^{-\alpha_i}(G'(n) - \alpha_i G(n)/n)$, which changes sign from positive to negative as n increases if average product $G(n)/n$ is first increasing then decreasing in n. Under this assumption, W_i and W_i^j are single-peaked, and it is easily shown that the common value of n at their peaks is inversely related to α_i. This example is meant only to be illustrative; the single-peaked curves used in the following analysis are not inconsistent with public good congestion or a more complicated public good production function.

Fig. 1 illustrates the choice of the public good consumption level in a community of fixed population. The community transformation curve is shown and points of tangency between different types of indifference curves and the individual budget constraint $g = [G(n) - C(x, n)]/n$ are illustrated. The utility levels on the various indifference curves are indicated.

Using the model, a straightforward discussion of equilibrium in a system of communities is possible. Equilibrium requires (1) that the public good output in a community reflects the tastes of the majority in that community and (2) that no individual can reach a higher utility level by moving to another community. Migration between communities is assumed to be costless. In fig. 2, the upper panel shows the W_1 and W_1^2 curves while the lower panel shows the W_2 and W_2^1 curves. Let N_i,

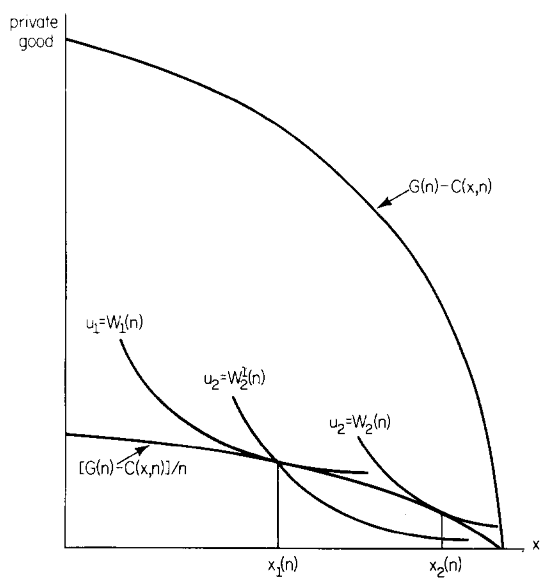

Fig. 1.

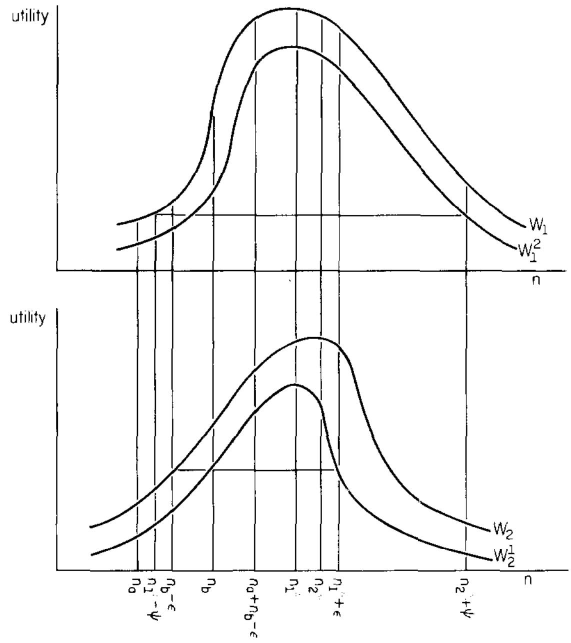

Fig. 2.

$i = 1, 2$, be the total number of type i individuals in the economy, and suppose first that $N_1 = n_1^*$ and $N_2 = n_2^*$. This means that one optimal size homogeneous community can be formed for each type of individual. It is easy to see that this community configuration is an equilibrium. This follows because $W_1(n_1^*) > W_1^2(n_2^*)$ and $W_2(n_2^*) > W_2^1(n_1^*)$ (see fig. 2); equilibrium condition (1) is trivially satisfied and no individual has an incentive to leave his homogeneous community to locate in the other group's community, where he would be a minority of one. Similarly, if $N_i = k_i n_i^*$, $i = 1, 2$, where the k_i are positive integers, then an equilibrium configuration with k_1 homogeneous type-one communities and k_2 homogeneous type-two communities may be formed. It is also easy to show that as $N_i \to \infty$, $i = 1, 2$, an equilibrium configuration of homogeneous communities can always be constructed. To see this, note first that $N_i = k_i n_i^* + \delta_i$, $i = 1, 2$, where k_i is some non-negative integer and $0 \leqslant \delta_i < n_i^*$. Then note that for each i, there exists a θ_i such that if $n_i^* \leqslant n \leqslant n_i^* + \theta_i$, then $W_i(n) \geqslant W_i^j(n')$ for arbitrary n'.[1] This means that if a homogeneous community can be formed with a population between n_i^* and $n_i^* + \theta_i$, then it will never be attractive for a resident to abandon the community and migrate to any community where he

[1] To find θ_i, draw a horizontal line tangent to W_i at its peak and find the line's intersection with the downward sloping part of W_i. The value of n at the intersection is $n_i^* + \theta_i$.

would be in the minority. Now, ignoring the fact that community populations must be integer valued, the type i population may be divided to form k_i communities of population $n_i^* + \delta_i/k_i$. Since δ_i is bounded between 0 and n_i^* while k_i increases in steps as N_i rises, δ_i/k_i approaches zero as $N_i \to \infty$. Thus, by choosing N_i large enough, identical homogeneous type i communities with populations between n_i^* and $n_i^* + \theta_i$ can be formed. Since migration out of these communities is unattractive, the community configuration is an equilibrium.

The community configuration with optimal size homogeneous communities, where the utility of each type of individual reaches its highest possible level, is what Tiebout envisioned in his path breaking paper (1956). Although it has been shown that an equilibrium which is arbitrarily close to this 'Tiebout equilibrium' may be constructed when the group populations are sufficiently large,[2] the following examples are designed to show that this is no basis for optimism about the economy's ability to generate efficient community configurations.

Returning to the case where $N_1 = n_1^*$ and $N_2 = n_2^*$, it is easy to see that other equilibria exist aside from the one with two homogeneous communities. In fig. 2, the configuration with one homogeneous type-one community with population $n_1^* - \psi$ and one mixed community with population $n_2^* + \psi$ is an equilibrium $[W_1(n_1^* - \psi) = W_1^2(n_2^* + \psi)$ and $W_2(n_2^* + \psi) > W_2^1(n_1^* - \psi)]$. In addition, any configuration with two mixed communities of population, $(n_1^* + n_2^*)/2$, with type-two's in the majority in both communities is an equilibrium, since members of a given group are equally well-off in the two communities. Also, the configuration with two homogeneous type-one communities of population, $n_1^*/2$, and two homogeneous type-two communities of population, $n_2^*/2$, is an equilibrium [although these populations are not shown in fig. 2, inspection shows that $W_1(n_1^*/2) > W_1^2(n_2^*/2)$ and $W_2(n_2^*/2) > (W_2^1(n_1^*/2)$]. Although each of these three community configurations satisfies equilibrium requirements (1) and (2) above, only the first configuration is stable. To see this, consider what would happen in the third configuration if a type-one individual were moved from one type-one community to the other. Since the relevant part of the W_1 curve is upward sloping, the utility level in the community which loses a resident falls while utility in the community which gains a resident rises. Therefore, an incentive arises for further migration out of the smaller community, and the equilibrium disintegrates. The same thing happens when the configuration with two mixed communities is perturbed; the community which loses a resident shrinks further as individuals are attracted to the higher utility level in the larger community. Note that these community configurations would have been stable had the relevant parts of the W_i and W_i^j curves been downward sloping. The stability of the first equilibrium follows from the fact that the W_1 curve at $n_1^* - \psi$ is steeper than the W_1^2 curve at $n_2^* + \psi$. To see this, note that if a type-one resident is moved from the larger to the smaller community, utility rises by more in the

[2] Note that this result holds as long as the W_i curves have a global maximum, and that the argument may be extended to include an arbitrary number of types of individuals.

larger community than in the smaller, causing the migrant to return to the larger community.

It is obvious in fig. 2 that the stable equilibrium with one mixed and one homogeneous type-one community is Pareto-inferior to the equilibrium with two optimal size homogeneous communities; everyone is better off in the latter configuration. The stability of the inefficient configuration clearly contradicts Tiebout's conjecture that the economy tends to generate an efficient set of homogeneous communities. Although everyone would benefit from a major community reorganization, the economy fails to generate the superior configuration because individual migration looks unattractive to each community member. Similar negative conclusions have been reached by the writers cited above.

The following example shows that when a group population is small, a stable equilibrium configuration of homogeneous communities may not even exist. If $N_1 = n_1^*$ and $N_2 = n_b < n_2^*$ in fig. 2, then the configuration with two homogeneous communities is not an equilibrium, since $W_2(n_b) < W_2^1(n_1^*)$. Furthermore, the above discussion of stability shows that even if an equilibrium could be constructed by dividing each group into many identical homogeneous communities with less than optimal populations, the equilibrium would be unstable because the relevant parts of the W_i curves are upward sloping. Hence, a stable equilibrium with homogeneous communities cannot be constructed in this example. However, the configuration with one homogeneous type-two community of population, $n_b - \epsilon$, and one mixed community of population, $n_1^* + \epsilon$, is a stable equilibrium [in fig. 2, $W_2(n_b - \epsilon) = W_2^1(n_1^* + \epsilon)$, $W_1(n_1^* + \epsilon) > W_1^2(n_b - \epsilon)$ and $|\mathrm{d}W_2(n_b - \epsilon)/\mathrm{d}n| < |\mathrm{d}W_2^1(n_1^* + \epsilon)/\mathrm{d}n|$], but everyone is worse off than in the configuration with two homogeneous communities, ($W_2(n_b - \epsilon) < W_2(n_b)$ and $W_1(n_1^* + \epsilon) < W_1(n_1^*)$. The domination of the mixed equilibrium by a homogeneous configuration which is not itself an equilibrium suggests a conclusion even gloomier than the previous example's. Not only can equilibria be inefficient, but desirable community configurations may not be supportable in a decentralized economy.

A final example shows that a stable equilibrium with homogeneous communities may be Pareto-inferior to an equilibrium with one mixed community. If $N_1 = n_b - \epsilon$ and $N_2 = n_a$, the configuration with two homogeneous communities is an equilibrium [in fig. 2, $W_1(n_b - \epsilon) > W_1^2(n_a)$ and $W_2(n_a) > W_2^1(n_b - \epsilon)$], but everyone would be better off in one mixed community of population $n_a + n_b - \epsilon$, where type-ones are in the majority [$W_1(n_a + n_b - \epsilon) > W_1(n_b - \epsilon)$ and $W_2^1(n_a + n_b - \epsilon) > W_2(n_a)$]. [3] This example shows that an equilibrium configuration of homogeneous communities may be inefficient, a result which looks curious from the perspective of the Tiebout model.

[3] Note that a configuration with one mixed community composed of all the individuals in the economy is always an equilibrium under the natural assumption $G(0) = 0$. This assumption implies $W_1(0) = W_2(0) = 0$, which means that no individual would find it attractive to abandon any existing community to form a community of one.

While the reader will no doubt be able to construct other illuminating examples using the diagrammatic approach developed in this paper, the above discussion is sufficient to suggest several observations. First, general analysis of equilibrium in a system of communities appears impossible. The number and properties of equilibrium community configurations appear to depend crucially on the sizes of groups and on the shapes of the W_i and W_i^j curves. Second, in spite of the strong appeal of Tiebout's belief that the economy tends to evolve toward an efficient community configuration, this optimistic view of the operation of the economy appears untenable in light of the examples presented in this paper.

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