Ernst Cassirer on 'Substance-Concepts' and 'Function-Concepts'

Jeremy Heis
University of California, Irvine
Dept of Logic and Philosophy of Science

Berkeley
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ERNST CASSIRER'S *SUBSTANZBEGRIFF UND FUNKTIONSBEGRIFF*

- Published 1910. The high point of Marburg Neo-Kantian philosophy of science.
- Recent attention from philosophers of mathematics and science:
  - Seems to anticipate "structural realism": Gower 2000, French 2001, French and Ladyman 2003.
  - Gives a relativized theory of the a priori and a "dynamic theory" of the rationality of scientific theory change: Friedman 2001, 2005, 2010.
  - Defends a structuralist account of mathematical objects: Gower 2000, Heis 201?, Reck 201?, Yap 201?.

ERNST CASSIRER'S *SUBSTANZBEGRIFF UND FUNKTIONSBEGRIFF*

- Employs a “historical” style
  - Criticized by Reichenbach 1924, Schlick 1921
  - Benacerraf 1960: “Cassirer says things closely allied to the views here expressed …[For instance, Cassirer writes (p.39)]: "What is here expressed is just this: that there is a system of ideal objects whose content is exhausted in their mutual relations. The 'essence' of the numbers is completely expressed in their positions." One might wish that Cassirer were somewhat clearer than he is in his mode of expression (and possibly thought).”

WHAT IS THE BOOK ABOUT?

The investigations contained in this volume were first prompted by studies in the philosophy of mathematics. In the course of an attempt to comprehend the fundamental concepts of mathematics from the point of view of logic, it became necessary to analyse more closely the function of the concept itself and to trace it back to its presuppositions. Here, however, a peculiar difficulty arose: the traditional logic of the concept, in its well-known features, proved inadequate even to characterize completely the problems to which the theory of the principles of mathematics led. It became increasingly evident that exact science had here reached questions for which there existed no precise correlate in the formal language of traditional logic. The material content of mathematical knowledge pointed back to a fundamental form of the concept not clearly characterized and recognized within logic. ... [This analysis] led to a renewed analysis of the principles of concept formation itself.

CASSIRER ON RUSSELL'S NEW LOGIC

- It is, it appears to me, in fact a new and fruitful point of view, which is introduced by Russell in his treatment of formal logic. The entire "classical" logic has concerned itself with nothing but the subsumption of contents, with the super- and sub-ordination of the spheres of two concepts. […] Syllogistic appears overall as a particularly reactionary and inhibiting moment. Logic remains bound to the point of view of substance and thereby to the fundamental form of the judgment of predication, while the living scientific thought more clearly aims at the concept of function [Funktionsbegriff] as its own systematic middlepoint. One recognizes in this connection the value and necessity of the new foundation on which Russell is seeking to place logic. (Cassirer 1907, p.7)

- But Cassirer also defends Dedekind over Russell, and criticizes Russell's philosophy.

MYSTERIOUS FEATURES OF *SF*

- How did Cassirer begin with a criticism of the traditional logic – and so in agreement with Russell – and end with a view opposed to Russell's?

- *SF* is a work of "transcendental logic": it takes science as a fact and investigates the conditions of its possibility. But why does it seem contrary to Neo-Kantianism?

- How does Cassirer argue - against Russell - for a Dedekindian account of mathematical objects as "positions in structures"?

- Why focus on concept-formation [Begriffsbildung]?

- What *are* "substance-concepts" and "function-concepts"?

OVERVIEW OF TALK

- **Section 1.** Cassirer's choice of Begriffsbildung was an ingenious and dialectically subtle way to pull together many seemingly unrelated concerns of Cassirer's philosophy.

- **Section 2.** His central distinction between substance-concepts and function-concepts is multi-faceted and breaks up into a series of interrelated contrasts between logical, metaphysical, and epistemological theses.

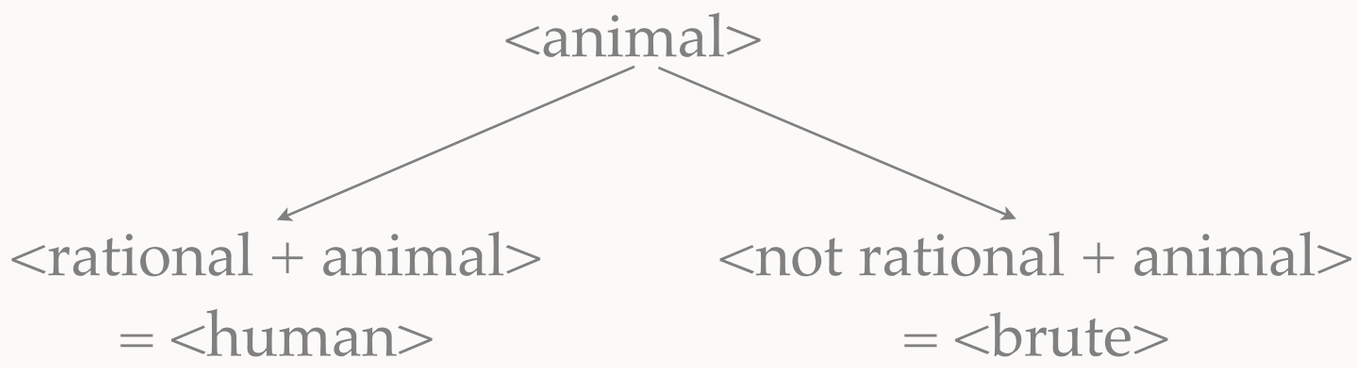
- **Section 3.** There is an argument that begins with considerations of concept formation and ends with a defense of mathematical structuralism.

- **Section 4**. Cassirer gives a two element theory of the a priori that explains the objectivity of theory change and is motivated by the failure of the abstractive theory of concept formation.

SECTION 1: WHY CONCEPT FORMATION?

- Late 19th century logic texts: how are concepts formed? what do scientists do and what justifies their practice?
- Cassirer's target is the traditional theory of concepts.
- The traditional theory has two parts:
  - *'Aristotelianism' about conceptual structure:* concepts are either simple or are composed of simple concepts by conjunction, addition, or exclusion.
  - *Abstractionism about concept formation*: concepts are formed by noticing similarities or differences among particulars and abstracting the concept, as the common element, from these similarities or differences.

CONCEPT FORMATION AND CONCEPTUAL STRUCTURE



All concepts are formed by specifying a genus; abstraction is the inverse of specification.

Thus, to attack the abstractionist theory of concept formation is to attack the traditional theory of conceptual structure.

CONCEPT FORMATION AND MARBURG NEO-KANTIANISM

- Natorp and Cohen: sensibility *does not* make a contribution to our knowledge that is independent of the understanding.
  - sensibility is passive (or "receptive");
  - understanding – the faculty of concepts – is active (or "spontaneous").
- But abstractionism requires that sensibility *does* make a contribution to our knowledge that is independent of the understanding.
- Cohen argued that there is not and cannot be something "given" [gegeben] in an experience (Cohen 1902, pp.24-5, 48-51).

SELLARS ON CONCEPT FORMATION AND THE "GIVEN"

- "[A]ll [forms taken by the myth of the given] have in common the idea that the awareness of certain sorts [...] is a primordial, non-problematic feature of immediate experience." ("Empiricism and the Philosophy of Mind," p. 157)

- "[T]he abstractive theory, as Kant saw, makes the mistake of supposing that the logical space of the concept simply transfers itself from the objects of direct perception to the intellectual order, or better, is transferred by the mind as Jack Horner transferred the plum." ("Phenomenalism," p.90)

CASSIRER READS KANT AS ATTACKING ABSTRACTIONISM

- “If, according to the traditional logical doctrine, the concept is merely the result of "abstraction" from a plurality of sensory data, so has it now been shown that "similar" impressions must be placed under a determinate rule of judging, before they – as is necessary for the process of "abstraction" – can be cognized as similar and be comprehended in a common genus. The unity of a genus presupposes the unity of an ideal norm, and the abstractive comparison presupposes a constructive connection. In its proper fundamental meaning, a concept is nothing other than the consciousness of this unity of synthesis.” (*EP* 2, 676; cf. 667)

- Abstractionism confuses *having similar impressions* with *recognizing that impressions are similar*.

- And so it overlooks the conceptual preconditions of perceptual knowledge.

SECTION 1: SUMMING UP

- Why concept formation?
- Because the topic of concept formation allows Cassirer to bring together three apparently unrelated philosophical concerns:

1. The methodology of the exact sciences,
2. The new logic of Russell and Frege,
3. Neo-Kantian epistemological doctrines -- the attack on the “given.”

SECTION 2: WHAT DOES CASSIRER MEAN BY 'SUBSTANZBEGRIFF' AND 'FUNKTIONSBEGRIFF'?

- Three uses of "Funktion" (besides the mathematical use):

  1. when alluding to Russell: "relation"

  2. when discussing scientific methodology: "role" or "purpose"

  3. when doing Kantian epistemology: a "rule-governed activity" of the mind.

- The third use is better expressed non-psychologistically: "epistemic preconditions."

USES OF "SUBSTANZBEGRIFF" AND "FUNKTIONSBEGRIFF"

- In some cases, the words distinguish two kinds of concepts. More often, they distinguish two opposing *philosophical views about concepts*.
- Core contrast:
  - The viewpoint of *Substanzbegriff*: a philosophical view that overlooks the epistemic preconditions of various kinds of knowledge,
  - The viewpoint of *Funktionsbegriff:* a philosophical view that recognizes the epistemic preconditions of various kinds of knowledge.
- The argument of the book begins with an attack on abstractionist theories of concept formation, and derives from that attack further kinds of epistemic preconditions.

USES OF "SUBSTANZBEGRIFF" AND "FUNKTIONSBEGRIFF"

- About concept formation:
  - *Substance-concept atomism*. It is possible to form a concept without possessing any other concepts or knowing any facts.
  - *Function-concept anatomism*. It is not possible to form a concept without possessing any other concepts or knowing any other facts.
- About the epistemic role of sensations:
  - *"Given" sensations*. There is a base level of sensations whose epistemic efficacy does not depend on having any concepts or knowing any facts.
  - *There are no "given" sensations.*

USES OF "SUBSTANZBEGRIFF" AND "FUNKTIONSBEGRIFF"

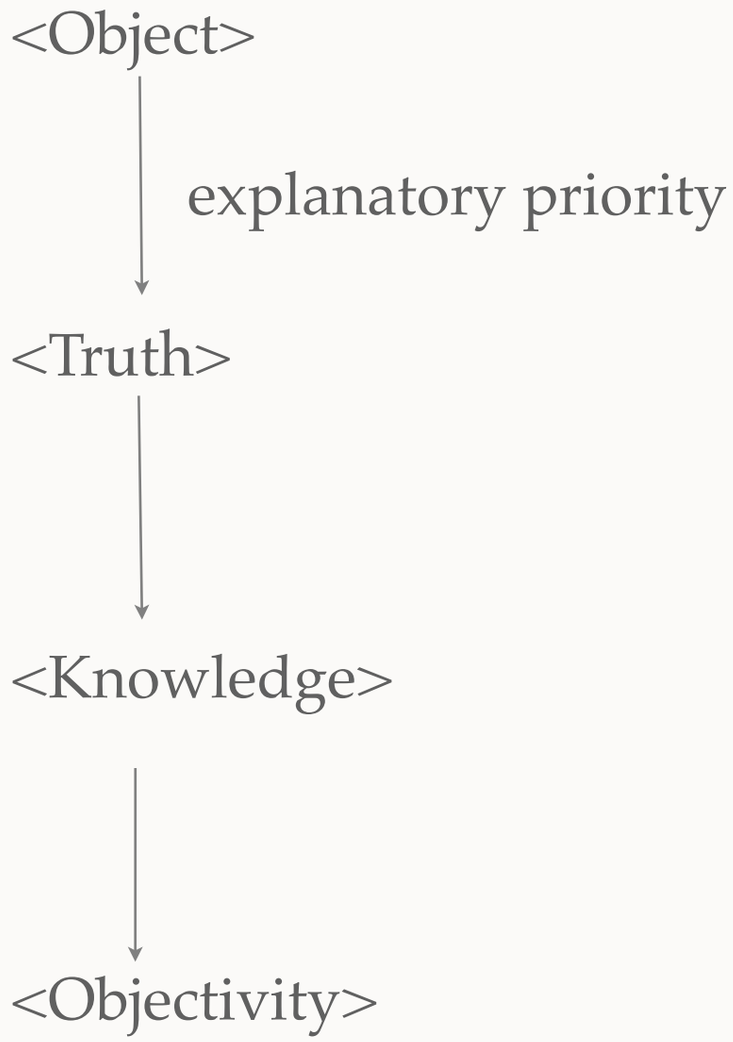
- About measurement:
  - The most basic results of scientific experimentation are "given."
  - The most basic results of scientific experimentation presuppose not only concepts and laws of pure mathematics, but also laws of nature and natural scientific concepts. (C follows Poincaré and Duhem.)

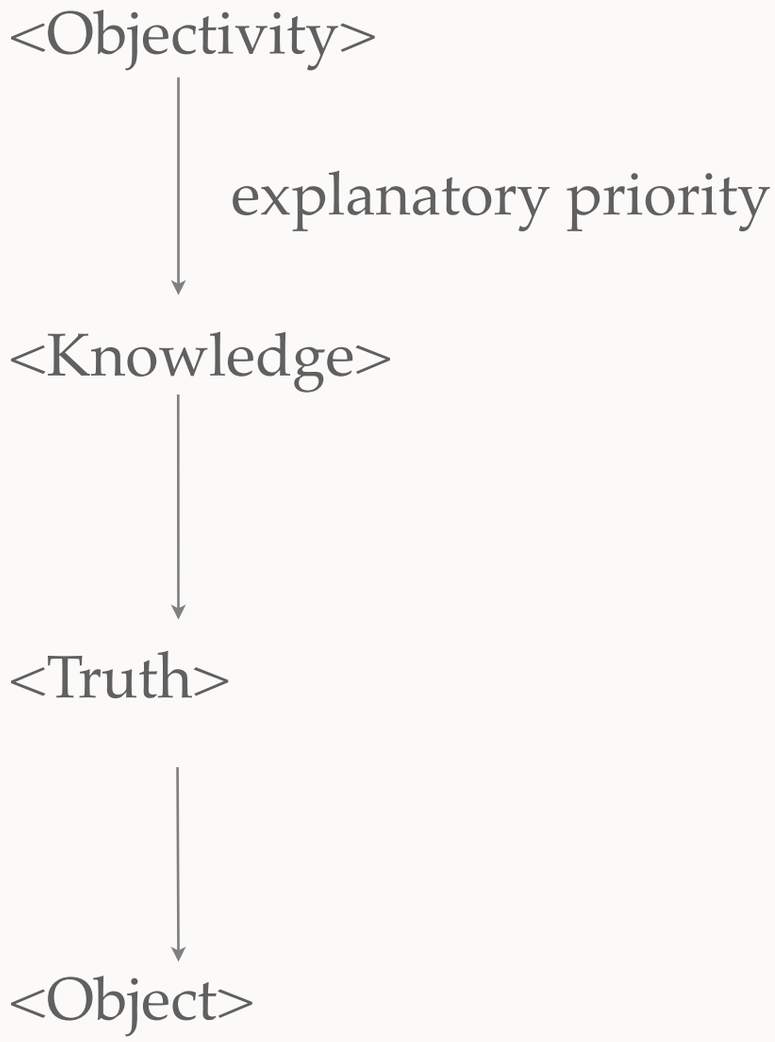
"Measurement presupposes certain theoretical principles and in the latter certain universal functions of connection, of shaping and coordination. We never measure mere sensations, and we never measure with mere sensations, but in general to gain any sort of relations of measurement we must transcend the "given" of perception and replace it by a conceptual symbol, which possesses no copy in what is immediately sensed."

USES OF "SUBSTANZBEGRIFF" AND "FUNKTIONSBEGRIFF"

- About empirical confirmation:
  - *Confirmation Atomism*. Concepts are applied to experience independently of confirming the judgments that contain them. Judgments can be empirically confirmed in isolation from their fellows.
  - *Confirmation anatomism:* no single empirical statement of natural science can be confirmed atomistically.
- About existence claims:
  - Existence can be determined "directly," by acquaintance.
  - Existence claims are possible only within a system of concepts and laws.

THE "SUBSTANCE" THEORY V. THE "FUNCTIONAL" THEORY OF KNOWLEDGE





THE "FUNCTIONAL" THEORY OF KNOWLEDGE

- *The functional theory of objectivity:* A total theory is objective if its concepts and judgments have a systematic form.
  - Systematicity requires: logical coherence, general laws, and rules for theory change.

- *The functional theory of knowledge*: A representation is knowledge if it plays the sort of role within a system of representations that would make objectivity possible.

- *The functional theory of the object:* An object is that which is represented by fully objective knowledge.

§3. FROM THE POLEMICS AGAINST ABSTRACTIONISM TO DEDEKIND'S FOUNDATIONS OF ARITHMETIC

- *SF* begins with an attack on the traditional logic in Ch.1 – and thus with Russell – and then quickly moves (in Ch.2) to a defense of a Dedekindian – and so anti-Russellian – view that the "'essence' of the numbers is completely expressed in their positions" (*SF*, 39).

- How is this argument supposed to work?

- [W]hat was at stake [in the debate between Dedekind and Frege and Russell] was … the universal question of how knowledge is actually related to "objects" and what conditions it must fulfill in order to acquire "objective meaning." [...] The question is "What is meant by mathematical 'existence,' and how can there be any meaningful question about the proof of such existence?" (*Problem of Knowledge*, p.63)

A FALSE START

> The attempt to present the entirety of cognition in a systematic unity ends in final *Form-concepts* that bring to expression the possible kinds of relation between contents in general. In these fundamental relations are given the final invariants to which cognition is able to advance; therefore also the "objective" standing [Bestand] of being is grounded in them. For objectivity is – according to the critical analysis and meaning of this concept – itself only another designation for the validity of determinate combinatory connections that are to be separately discovered and are to be investigated in their structure. The task of *Erkenntniskritik* consists in this, to go back from the unity of the general concept of the object to the manifold of necessary and sufficient conditions that constitute it. In this sense the thing that cognition calls its 'object' is resolved into a web of relations that are themselves held together through the highest rules and principles. (1913, p. 13)

CASSIRER'S DEFENSE OF DEDEKIND, AND RUSSELL'S ATTACK

- Cassirer's evidence for the view that numbers are simply "terms of relations" is derived from "the development of scientific arithmetic in the last decades" (*SF*, p.35).

- Russell's objection to Dedekind, on the other hand, is purely metaphysical: "It is impossible that the ordinals should be, as Dedekind suggests, nothing but the terms of such relations as constitute a progression. If they are to be anything at all, they must be intrinsically something; they must differ from other entities as points from instants, or colors from sounds." (Russell *POM*, §242; cf. *SF*, p.39)

- When Russell objects to the procedure of mathematicians based on some prior metaphysical convictions, he is violating the "Functional Theory of Knowledge."

CASSIRER'S DEFENSE OF DEDEKIND, AND RUSSELL'S ATTACK

- Scientific arithmetic – and scientific geometry, number theory, and analysis – are paradigm objective disciplines that progress historically by employing identifiable and clear standards.

- So, according to the *functional theory of objectivity*, mathematics is objective.

- So, according to the *functional theories of knowledge and objecthood,* this objectivity allows us to say that mathematics provides genuine knowledge – true claims about independently existing objects.

- Moreover, Russell's need for intrinsic properties seems to be motivated by something like the abstractive theory itself.

CASSIRER'S ARGUMENT FOR STRUCTURALISM

- The functional theory of knowledge does not support robust metaphysical reflection; it *deflates* it.

- With metaphysical considerations deflated, we adopt the view of mathematical objects that gives the exact sciences all they need, and nothing more than what they need. This view is *structuralism*.

- Compare with Benacerraf's argument in "What Numbers Could Not Be."

SUBSTANZBEGRIFF UND FUNKTIONSBEGRIFF: TYING TOGETHER THE STRANDS

- §1. The question of concept formation brings together three historically distinct lines of research: Russell and Frege's new logic, investigations into the details of mathematical methodology by late nineteenth century German logicians, and (Neo-)Kantian epistemological reflections.

- §2. "Funktionsbegriff" refers fundamentally to Kantian epistemological reflections on concept use and objectivity.

- §3. These reflections provide the frame in which facts about mathematical methodology can come to the foreground. These methodological facts motivate mathematical structuralism, which is only coherently formulable with the new logic.

§4. THE THEORY OF ABSTRACTION AND THE A PRIORI

- Cassirer 1921 argues that Riemannian differential geometry and Einstein's principle of general covariance are conditions of the possibility of general relativity.

- But these principles are not certain, self-evident, or unrevisable.

- "That we in [science] find only a *relative* stopping point, that we therefore have to treat the *categories*, under which we consider the historical process itself, themselves as variable and capable of change, is obviously correct: but this kind of relativity does not indicate the limits, but rather the particular life of cognition." (Cassirer 1906, I:16)

- Richardson 1998 and Ryckman 2005 see Cassirer as anticipating the relativized a priori later defended in Reichenbach 1920.

CASSIRER'S TWO ELEMENT ACCOUNT OF THE A PRIORI: *RELATIVIZED* AND *INVARIANT* A PRIORI

> "The goal of critical analysis would be reached, if we succeeded in isolating in this way the ultimate common element of all possible forms of scientific experience; *i.e.*, if we succeeded in conceptually defining these moments, which persist in the advance from theory to theory because they are the conditions of any theory. At no given stage of knowledge can this goal be perfectly achieved; nevertheless, it remains as a demand, and prescribes a fixed direction to the continuous unfolding and evolution of the systems of experience. From this point of view, the strictly limited meaning of the "*a priori*" is clearly evident. Only those ultimate logical invariants can be called *a priori*, which lie at the basis of any determination of a connection according to natural law. A cognition is called *a priori* not in any sense as if it were *prior* to experience, but because and in so far as it is contained as a necessary premise in every valid judgment concerning facts." (Cassirer 1910, p.269)

THE INVARIANT THEORY

- Cassirer introduced the theory of the invariant a priori to address concerns about the rationality and objectivity of theory changes.
- These concerns would not arise if the epistemological atomism that forms the basis of the abstractive theory of concept formation were true.
- After rejecting abstractionism: the fact that two subjects are situated in the same world and are receiving impressions from the same physical objects is not sufficient to provide them with a common intersubjective basis.
- A Quinean holism without a priori elements cannot explain the objectivity and rationality of scientific theory change.
- So the rejection of abstractionism leads to this new theory of the a priori.