SLUGA, BURGE, AND BRANDOM SEE IN FREGE’S WRITINGS AN ECHO OF KANT’S DISCUSSION OF CONCEPTS AND JUDGMENTS

Whereas all intuitions, as sensible, rest on affections, concepts rest on functions. By 'function' I mean the unity of the act of bringing various representations under one common representation. Concepts are based on the spontaneity of thought, sensible intuitions on the receptivity of impressions. Now the understanding can make no other use of these concepts than that of judging by means of them. [...]

Concepts, as predicates of possible judgments, are related to some representation of a still undetermined object. The concept of body thus signifies something, eg, a metal, which can be cognized through that concept. It is therefore a concept only because other representations are contained under it by means of which it can be related to objects.

(A68/B93; A69/B94)
FREGE’S “PRIORITY PRINCIPLE”

In fact it is one of the most important differences between my mode of interpretation and the Boolean mode—and indeed I add the Aristotelian mode—that I do not proceed from concepts, but from judgements. (“On the Aim of the Begriffsschrift” 1882, p.94)

As opposed to [Boole], I start out with judgements and their contents, and not from concepts. … I only allow the formation of concepts to proceed from judgements. (“Boole’s Logical Calculus” 1880-1, p.16)

Now I do not think that concept formation can precede judgement because this would presuppose an independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content. (Letter to Marty, 1882)
VARIETIES OF PRIORITY

Ontological Priority: The being of a concept depends on the being of the judgment (as the being of a part in an organic whole depends on the being of the whole).

Weak Semantic Priority: A concept (or concept-word) has meaning because it is part of a judgment (or sentence) that has meaning.

Strong Semantic Priority: The meaning of a concept (or concept-word) consists in the contribution it makes to the meaning of a judgment (or sentence).

Epistemic Priority: A subject can come to know a concept only through coming to know a judgment containing that concept. (Or: a subject can form a concept only through forming a judgment containing it.)
Frege's (Epistemic) Priority Thesis: Judgments are prior to concepts because the formation of compound concepts requires first grasping a complete judgment and then replacing some of its components.

Kant’s (Semantic) Priority Thesis: Judgments are prior to concepts because a concept is a mediate representation of an object, and it is in virtue of being combined in judgments that concepts relate to objects.
Main Ideas of the Talk: Can We See Frege’s Views of Concepts and Judgments as Developments from Kant’s Views?

Part I: Kant’s and Frege’s Priority Principles.

- Kant's priority thesis is not identical to Frege's priority thesis.

- There is no argument using premises Kant would accept that would allow us to infer from Kant's priority thesis to Frege's priority thesis.

In this sense, then, the Fregean theses are not further developments of the Kantian theses.
Part 2: Kant’s Ambiguous Legacy: From the Nature of Concepts to the Structure of Concepts

- Kant’s priority thesis, as a thesis about object-directedness, came to be seen as a thesis with implications concerning the formation of concepts and the structure of concepts, judgments, and inferences

- I will present this historical development in 5 steps

- (I conclude with some remarks about Frege’s Context Principle).
Kant had argued against the theory of ideas ... that judgments are not formed out of previously given constituents, but that they possess an initial transcendental unity out of which we gain concepts by analysis. By the late nineteenth century the doctrine had become a standard argument in anti-naturalistic theories of knowledge. ... Through Lotze’s influence the doctrine also reached Frege. (Sluga, Frege)
In 1882 [Frege] wrote: "I do not think that the formation of concepts can precede judgment, for that would presuppose the independent existence of concepts; I rather imagine that the concept originates in the analysis of a judgeable content."

Such an emphasis on the priority of judgments over concepts links Frege to the Kantian tradition in logic. Kant himself had considered it his greatest achievement in logic to have seen beyond the traditional view of judgments as mere composite concepts. Concepts, he had said, presuppose judgments since "the only use which the understanding can make of these concepts is to judge by means of them" (A68). And he had concluded that "concepts, as predicates of possible judgments, relate to some representation of a not yet determined object" (A69). [Sluga 1987, p. 87]
My concept-script commands a somewhat wider domain than Boole’s formula-language. This is a result of my having departed further from Aristotelian logic. For in Aristotle, as in Boole, the logically primitive activity is the formation of concepts by abstraction, and judgment and inferences enter in through an immediate or indirect comparison of concepts via their extensions. ... As opposed to this, I start out with judgements and their contents, and not from concepts ... I only allow the formation of concepts to proceed from judgements. (BLC, 15-6)
The x [in ‘$2^x = 16$’] indicates here the place to be occupied by the sign for the individual falling under the concept. We may also regard the 16 in $x^4 = 16$ as replaceable in its turn, which we may represent, say, by $x^4 = y$. In this way we arrive at the concept of a relation, namely the relation of a number to its 4th power. And so instead of putting a judgment together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of a possible judgment. (BLC 17)
THE TRADITIONAL DOCTRINE OF CONCEPTS

‘Aristotelianism’ about conceptual structure: concepts are either simple or are composed of simple concepts by conjunction, addition, or exclusion.

Abstractionism about conceptual formation: concepts are formed by noticing similarities or differences among particulars and abstracting the concept, as the common element, from these similarities or differences; compound concepts can arise from simpler concepts either by conjunction, disjunction, and exclusion, or by noticing similarities or differences among concepts and abstracting from these similarities or differences.
THE INTERCONNECTION OF BOOLEAN STRUCTURE AND CONCEPT FORMATION BY ABSTRACTION

[The Tree for the concept <animal>]

M (=<animal>)

M and N
=<$\text{rational and animal}$>
=<$\text{man}$>

M and non N
=<$\text{non-rational and animal}$>
=<$\text{brute}$>

M and N and O
=<$\text{learned and man}$>

M and N and non O
=<$\text{non-learned and man}$>
Kant obviously—as a result, no doubt, of defining them too narrowly—underestimated the value of analytic judgments [= a judgment provable using only general logical laws and definitions] ... He seems to think of concepts as defined by giving a simple list of characteristics in no special order; but of all the ways of forming concepts, that is one of the least fruitful. If we look through the definitions given in the course of this book, we shall scarcely find one that is of this description. The same is true of the really fruitful definitions in mathematics, such as those of the continuity of a function.
What we find in these is not a simple list of characteristics; every element in the definition is intimately, I might almost say organically, connected with the others. […] What we shall be able to infer from [a fruitful definition], cannot be inspected in advance; here, we are not simply taking out of the box again what we had put into it. The conclusions we draw from it extend our knowledge, and ought therefore, on Kant’s view, to be regarded as synthetic; and yet they are proved by purely logical means, and so are analytic. The truth is that they are contained in the definitions, but as plants are contained in their seeds, not as beams are contained in a house.
FREGE’S ARGUMENT FROM FA §88

1. Since in the begriffsschrift judgments are prior to concepts,

2. concepts expressible in begriffsschrift are not all formed by simple lists of common characteristics (that is, not by abstraction),

3. and so, some of the concepts expressible in begriffsschrift have a richer structure than Kant (or Boole and Aristotle) allow for,

4. and so, there are concepts expressible in begriffsschrift that are fruitful,

5. and so, the deductions represented in begriffsschrift can be epistemically ampliative, unlike the inferences Kant (or Boole and Aristotle) recognize.
PACE SLUGA, KANT’S PRIORITY PRINCIPLE IS NOT AN ALTERNATIVE TO THE VIEW THAT CONCEPTS ARE FORMED BY ABSTRACTION

To make concepts out of representations one must thus be able to compare, to reflect, and to abstract, for these three logical operations of the understanding are the essential and universal conditions for generation of every concept whatsoever. I see, e.g., a spruce, a willow, and a linden. By first comparing these objects with one another I note that they differ from one another in regard to the trunk, the branches, the leaves, etc.; but next I reflect on what they have in common among themselves, trunk, branches, and leaves themselves, and I abstract from the quantity, the figure, etc., of these; thus I acquire a concept of a tree. [Kant, Logic, §6]
1. One understands the structural nature of thoughts and components of thoughts ... by reflecting discursively on a large number of deductive inferences among sentences.

2. The key to understanding such structure lies in understanding the contribution of such components to determining truth conditions and to preserving truth in deductive inference.

These two insights “had already been enunciated by Kant, at least in germ [A65-9/B90-4]. Kant held that concepts are essentially predicates of possible judgments. He held that judgment aims at truth and is essentially propositional. Thus, he regarded components of thoughts as having their function and structure only in the context of propositional judgment and inferences among judgments.” [Burge 2005, p.14]
What is distinctive of my conception of logic is that I began by giving pride of place to the content of the word ‘true,’ and then immediately go on to introduce a thought as that to which the question ‘Is it true?’ is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgment; I come by the parts of a thought by analyzing the thought. (Notes for Ludwig Darmstaedter, July 1919, PW 253)
KANT ON JUDGMENT AND TRUTH

- A judgment is a relation of given representations that is *objectively valid* (B142)
- Kant equates objective validity with *truth* (or: *truth-directedness*) (A125)
- “[T]he problem as to how experience is possible by means of these categories, and only by means of them … can be solved almost by a single conclusion from the precisely determined definition of a judgment in general (an act by which given representations first become cognitions of an object).” (Metaphysical Foundations of Natural Science, Ak 4:475)
“WHAT DOES ONE MEAN, THEN, IF ONE SPEAKS OF AN OBJECT CORRESPONDING TO AND THEREFORE ALSO DISTINCT FROM THE COGNITION?”

• [O]ur thought of the relation of all cognition to its object carries something of necessity with it, since namely the latter is regarded as that which is opposed to our cognitions being determined at pleasure or arbitrarily rather than being determined a priori, since insofar as they relate to an object our cognitions must also necessarily agree with each other in relation to it, i.e., they must have that unity that constitutes the concept of an object. [A104-5]

• What is an object? That whose representation is a complex of a number of predicates appertaining to it. ... An object is that in the representation of which other representations can be thought as synthetically connected. (Refl 6350 (1790s))
THE KANTIAN PRIORITY THESIS

- Judgments are prior to concepts because a concept is a mediate representation of an object, and it is in virtue of being combined in judgments that concepts relate to objects.

- Cp. with Sluga’s different reading: “Kant had argued against the theory of ideas that judgments are not formed out of previously given constituents, but that they possess an initial transcendental unity out of which we gain concepts by analysis.”

- Cp. Burge: Kant “regarded components of thoughts as having their function and structure only in the context of propositional judgment and inferences among judgments.”
PART 1: REVIEW

- Kant’s priority thesis is not identical to Frege’s priority thesis.
- There is no argument using premises Kant would accept that would allow us to infer from Kant’s priority thesis to Frege’s priority thesis.
- Kant’s thesis concerns the function of concepts in cognition, not their structure.
- Kant’s thesis was not meant as an alternative to the theory of concept formation by abstraction.
PART 2: KANT’S AMBIGUOUS LEGACY: FROM THE NATURE OF CONCEPTS TO THE STRUCTURE OF CONCEPTS

- Kant’s priority thesis, as a thesis about object-directedness, came to be seen as a thesis with implications concerning the formation of concepts and the structure of concepts, judgments, and inferences.

- I will present this historical development in 5 steps.

- A surprising result is the role played by Hegel (and also Trendelenburg and Lotze).
THE HISTORICAL DEVELOPMENT FROM KANT’S PRIORITY THESIS TO FREGE’S: 5 STEPS

STEP #1 (Hegel): Kant’s priority thesis is inconsistent with the theory of concept formation by abstraction

STEP #2 (Hegel): The theory of concept formation by abstraction does not discriminate between essential and inessential marks

STEP #3 (Trendelenburg): Concepts structured in the traditional way as sums of marks will not be explanatory, and syllogisms containing these concepts will be valueless.

STEP #4 (Lotze): Concepts can be explanatory iff they are structured in a way that is more complicated than the traditional Aristotelian logic allows.

STEP #5 (Lotze): Concepts are structured in richer than Aristotelian ways only if they are formed, not by abstraction, but from compound judgments.
STEP #1 (Hegel): Kant’s priority thesis is inconsistent with the theory of concept formation by abstraction

- This original synthesis of apperception is one of the most profound principles for speculative development; it contains the beginning of a true apprehension of the nature of the Notion and is completely opposed to that empty identity or abstract universality which is not within itself a synthesis. (Hegel, Science of Logic, p.589)

- Hence we say that we cognize an object if we have effected synthetic unity in the manifold of intuition. But this is impossible if the intuition could not have been produced through a function of synthesis in accordance with a rule that makes the reproduction of the manifold necessary a priori and a concept in which this manifold is united possible. [A105]
STEP #2 (Hegel): The theory of concept formation by abstraction does not discriminate between essential and inessential marks

“Of course, if what is taken up into the Notion from the concrete phenomenon is to serve only as a mark or sign, it certainly may be any mere random sensuous particular determination of the object, selected from the others on the basis of any random external interest and of a similar kind and nature as the rest.” (Hegel, *Science of Logic*, p.588)

(Kant also argues that abstraction is not sufficient for isolating explanatory concepts. But he advocates supplementing the traditional theory, not replacing it. And his arguments are not derivable from the priority of judgment.)
STEP #3 (TRENDELENBURG): CONCEPTS STRUCTURED IN THE TRADITIONAL WAY AS SUMS OF MARKS WILL NOT BE EXPLANATORY, AND SYLLOGISMS CONTAINING THESE CONCEPTS WILL BE VALUELESS.

• Trendelenburg – *Logical Investigations* [1840/1870], “On Leibniz’s Universal Characteristic” [1867]: the structure of concepts in the traditional logic prevents it from expressing explanatory concepts.

• “In such a composition lies indeed an essential error. For the marks that we distinguish in a concept have a particular interconnection among themselves. This organic bond, through which the streaming life of the whole is designated, is destroyed in such a composition and turned into a mere sum of external marks. It is not sufficient in the [concept] human to add the mark animal to the mark rational. The essential relation of the two marks to one another is thereby neglected, namely how the animal life constitutes the foundation of the rational life.” (1870, p.21)
As a rule, the marks of a concept are not coordinated as all of equal value, but they stand to each other in the most various relative positions, offer to each other different points of attachment, and so mutually determine each other; ... an appropriate symbol for the structure of a concept is not the equation \( S = a + b + c + d \), etc, but such an expression as \( S = F( a, b, c, \text{ etc.}) \) indicating merely that, in order to give the value of \( S \), \( a, b, c, \text{ etc.} \) must be combined in a manner precisely definable in each particular case, but extremely variable when taken generally. (Lotze, *Logic*, §28)
• Mathematics requires non-syllogistic inferences and concepts whose structure is more complex than the traditional model allows for.

• Concepts are structured in richer than Aristotelian ways only if they are formed, not by abstraction, but from compound judgments (§8).

• We can isolate these various structures by reflecting on the various kinds of non-syllogistic inferences that appear in mathematical sciences.
LOTZE’S FUNCTIONAL THEORY OF THE CONCEPT AND FREGE’S FA §88

1. Since judgments are prior to concepts,
2. concepts are not all formed by abstraction,
3. and so, some concepts have a richer structure than Kant (or Boole and Aristotle) allow for,
4. and so, some concepts are fruitful,
5. and so, deductions can be epistemically ampliative, unlike the inferences Kant (or Boole and Aristotle) recognize.
FREGE’S CONTEXT PRINCIPLE AND KANT’S PRIORITY PRINCIPLE

The CP: “Only in the context of a sentence does a word have meaning.”

Three points:

1. The priority principle is not identical to, and does not imply the context principle.

2. Frege’s use of the CP in §60 is inextricably linked with his distinction between concepts and objects, and with his new 'logical' conception of objecthood. But in §89, Frege complains that Kant's concept/object distinction is confused, that Kant's notion of an object is different from his own, and that Kant therefore mistakenly denies that numbers are objects.
3. At §62, Frege writes:  “How, then, are numbers to be given to us, if we cannot have any ideas or intuitions of them? Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs.”

But at §89, he writes: “I must also protest against the generality of Kant's dictum [A51/B75]: without sensibility no object would be given to us. Zero and one are objects which cannot be given to us in sensation. […] Perhaps Kant used the word "object" in a rather different sense; but in that case he omits altogether to allow for zero and one [...] -- for these are not concepts either, and even of a concept Kant requires that we should attach its object to it in intuition.”