What Drives Offshoring Decisions? Selection and Escape-Competition Mechanisms

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Abstract

I present a model of offshoring decisions with heterogeneous firms, random adjustment costs, and endogenous markups. The model proposes a tractable probabilistic framework that goes beyond the conventional view of self-selection of more productive firms into offshoring. By characterizing the offshoring decision as a lumpy investment decision subject to heterogeneous adjustment costs, the model obtains an inverted-U relationship between firm-level productivity and the probability of offshoring. A tougher competitive environment (due, for example, to trade liberalization in final goods) has two opposing effects on firm-level offshoring likelihood: the conventional selection effect—accounting for the negative effect of competition on offshoring profits—and an escape-competition effect—accounting for the effect of competition on the opportunity cost of offshoring. In addition, the model highlights strong complementarities between offshoring and exporting decisions.

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1 Introduction

Recent models of offshoring with firm heterogeneity in productivity rely on self-selection mechanisms to explain firms’ offshoring decisions. For example, in the workhorse offshoring model of Antràs and Helpman (2004) heterogeneous firms sort into different offshoring modes based on their capacity to cover homogeneous fixed costs.\(^1\) Hence, as with the exporting decision in the Melitz (2003) model, there is a productivity-driven selection mechanism of offshoring: only the most productive firms offshore because only they are able to cover the fixed costs of offshoring. But given that by offshoring—which allows firms to take advantage of other countries’ lower factor prices—a firm aims to reduce its marginal cost, who has more incentives to offshore? A high-productivity firm or a low-productivity firm struggling for survival? It may well be the case that not-so-productive firms look for opportunities to offshore as a way to keep up with the pressure of more efficient firms or, using the terminology of Aghion \textit{et al.} (2005) in their competition and innovation analysis, as a means to “escape” competition. This paper develops a heterogeneous-firm model in which a firm’s decision to offshore is probabilistic and determined by the \textit{escape-competition mechanism}, as well as by the conventional selection mechanism.

The model starts from the observation that the offshoring decision is similar to an investment decision, and as such, it implies a production-process adjustment subject to disruption costs. Therefore, as with investment, the offshoring decision is lumpy in nature. In light of this observation, I characterize the offshoring decision as a lumpy investment decision subject to heterogeneous—across firms and over time—adjustment costs that are likely to be higher for larger firms. The model obtains an inverted-U relationship between firm-level productivity and offshoring probability, so that a more productive firm is not necessarily more likely to offshore than a less productive firm. As a consequence, the model predicts a high degree of coexistence between low-productivity offshoring firms and high-productivity non-offshoring firms. Furthermore, the model generates productivity distributions for offshoring and non-offshoring firms that closely resemble distributions found in empirical studies.

The paper then shows how the selection and escape-competition mechanisms interact in response to a tougher competitive environment. On the one hand, more competition decreases profits of offshoring firms, giving non-offshoring firms less incentives to alter their production processes. This

\(^1\)There are two organizational modes of offshoring: foreign outsourcing (arm’s-length trade) and vertical foreign direct investment (related-party or intra-firm trade). In the first type, the offshoring firm subcontracts a part of its production process with an independent foreign firm; in the second type, the offshoring firm owns a subsidiary in a foreign country (see Antràs and Rossi-Hansberg, 2009 for a review of the literature on production organization and trade). The model of Antràs and Helpman (2004) sorts firms into the different offshoring modes based on a homogeneous fixed cost for each mode. In that model, the least productive firms keep all their production activities inside the firm in a single location (these firms are not able to cover the fixed cost for any of the offshoring modes), a second set of more productive firms engage in foreign outsourcing (they are able to cover the outsourcing fixed cost), while the most productive firms vertically integrate their production process across international borders (they are able to cover the higher vertical FDI fixed cost).
is the selection effect of competition and causes a decline in the offshoring probability. On the other hand, although profits of both offshoring and non-offshoring firms might decline in a tougher environment, their difference—the incremental profits from offshoring—may increase, making offshoring more attractive relative to non-offshoring. This is the escape-competition effect and causes an increase in the offshoring probability. Due to the opposite forces of the selection and escape-competition effects, the offshoring probability declines for some firms but increases for others. I prove the existence of a productivity threshold that separates non-offshoring firms according to the dominant effect, with the selection effect dominating for the least productive firms.

The model’s main ingredients are firm heterogeneity in productivity, non-convex adjustment costs of offshoring, and endogenous markups. In this framework, more productive firms (with lower marginal costs) charge lower prices, have larger market shares, and have higher markups. Given the model’s Melitz-type structure, a productivity threshold determines the tradability of differentiated goods so that firms with productivity levels below the threshold do not produce. The price set by a firm with a productivity level identical to the threshold is equal to the marginal cost; that is, its markup is zero. A differentiated-good firm, however, can move a part of its production process to another country to take advantage of lower wages. If the firm starts to offshore, its marginal cost declines and its markup and profits increase.

Although offshoring implies lower marginal costs and higher profits, not all firms offshore because the offshoring decision is costly: it involves non-negligible relocation and reorganization costs. Following the model of Caballero and Engel (1999) on lumpy investment decisions, I introduce random adjustment costs of offshoring. Every period, each non-offshoring firm draws an offshoring adjustment cost from a probability distribution—adjustment costs vary through time and are not necessarily the same for two firms with identical productivity. If the adjustment cost draw is below an endogenously determined threshold, the firm adjusts its production process and begins offshoring. The adjustment cost has two components: one related to the firm’s size, and one independent of it. The first component, which is standard in the lumpy-investment literature, is proportional to the firm’s profits and thus, it is more important for more productive (and larger) firms—larger firms are likely to face higher reorganization costs. The second component, which is standard in models of trade with heterogeneous firms, is relatively more important for low-productivity (and smaller) firms—smaller firms have lower opportunity costs of reorganization because they have lower profits and hence, their offshoring constraints are mainly related to the new fixed costs they would have to incur.

Following Bergin and Feenstra (2000), the model assumes translog preferences to generate endogenous markups. The purpose of including an endogenous-markup structure is twofold: first, it adds a new and important dimension of reality because markups are indeed endogenous and vary from firm to firm (see, e.g., the recent empirical contributions of Amiti, Itskhoki and Konings,
second, it allows us to define in a natural and precise way what we mean by a “tougher competitive environment.” Regarding the latter, given the intimate link between firms’ markups and the level of competition in a market, we say that the competitive environment is tougher if every active firm that keeps the same production process is forced to reduce its markup. Importantly, the advantages of this variable markups’ approach come at no cost in tractability when compared to a version of the model with CES (constant elasticity of substitution) preferences and thus exogenous markups. Moreover, I show that the non-monotonic relationship between productivity and the probability of offshoring always emerges in this and other variable-markup settings, while it only arises as a special case in a CES version of the model.

I then extend the model to allow for trade in final goods. I show that trade liberalization in final goods toughens the competitive environment in the domestic and export markets, reducing markups in firms that do not alter their production process, and triggering selection and escape-competition effects in offshoring decisions. In contrast, a decline in offshoring costs (i.e., input trade liberalization) eases the competitive environment in the export market and may also soften competitive pressures for offshoring firms in the domestic market, and thus, this type of liberalization causes an increase in markups for some firms. Therefore, this model can reconcile the reduction in markups observed in empirical studies focusing in final-good trade liberalization, with the most recent findings of De Loecker et al. (2012), who find increasing markups for Indian firms after input trade liberalization. In spite of this difference, both types of trade liberalization increase the fraction of offshoring firms as well as the fraction of exporting firms, implying strong complementarities between exporting and offshoring decisions—this is ultimately reflected in the result that offshoring firms are likely to be also exporting firms.

The paper is organized as follows. Section 2 discusses the model’s theoretical and empirical background. Section 3 presents the model, with special emphasis in the description of the offshoring decision problem. Section 4 presents the model’s implications for changes in the competitive environment, including a discussion of the selection and escape-competition effects. Section 5 presents the extension with trade in final goods, and section 6 concludes. A separate Appendix includes the proofs of the lemmas and propositions, and additional supporting material.²

2 Theoretical and Empirical Background

Heterogeneous-firm models with homogeneous fixed costs of offshoring (e.g., the model of Antràs and Helpman, 2004 and its variations) imply strong truncations in the productivity distributions of offshoring and non-offshoring firms. Figure 1a illustrates this point: in theory, there is no overlap between the productivity distributions of offshoring and non-offshoring firms because they

are separated by a productivity threshold—the productivity distribution of non-offshoring firms is right-truncated and the productivity distribution of offshoring firms is left-truncated. If a small percentage of firms engage in offshoring activities, the truncation of the distributions should occur at a high productivity level. In the U.S., for example, Bernard et al. (2007) report that only 14% of manufacturing firms were involved in importing activities in 1997, which then implies that—if the data satisfies the homogeneous-fixed-cost assumption—evidence of truncation for U.S. manufacturing firms should appear in the last quintile of the productivity range.

Instead, evidence for Japan (from Tomiura, 2007) and Spain (from Antràs and Yeaple, 2014) shows productivity distributions of offshoring and non-offshoring firms that look like those in Figure 1b. That is, although the productivity distribution of offshoring firms is to the right of the distribution of non-offshoring firms—so that offshoring firms are on average more productive than non-offshoring firms—they exhibit substantial overlap: there is a remarkable coexistence of low-productivity offshoring firms and high-productivity non-offshoring firms. This overlap should not be surprising if one considers other dimensions of firm heterogeneity (e.g. firm heterogeneity in quality or managerial ability); the fact that the distribution of offshoring firms is to the right of the distribution of non-offshoring firms may still be evidence of selection in productivity. There may be, however, other factors driving the gap between the distributions.

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3See Figures 1, 2, and 3 in Tomiura (2007), and Figures 2.9 and 2.11 in Antràs and Yeaple (2014).

4 Despite the evidence that firms that participate in international markets—as exporters or importers—are on average larger and more productive than firms that only operate domestically (see Bernard, Jensen and Schott, 2009), a truncation of the type suggested in Figure 1a is also absent in the evidence for exporting and non-exporting firms. For example, for U.S. exporters and non-exporters, Bernard et al. (2003) show bell-shaped empirical productivity distributions with a substantial degree of overlap: though the exporters' distribution is to the right of the non-exporters distribution, there is a well-established coexistence between low-productivity exporters and high-productivity non-exporters (see their Figure 2B). Melitz and Trefler (2012) show similar distributions for Canada (see their Figure 5). As well, Hallak and Sivadasan (2013) present evidence of the coexistence of large non-exporting firms and small exporting firms in Chile, Colombia, India, and the United States.
In particular, there is strong theoretical and empirical support for the existence of an offshoring productivity effect—the decline in marginal cost due to offshoring—which would cause a shift to the right of the productivity distribution of offshoring firms.\(^5\) Hence, the ex-ante productivity distribution of offshoring firms may be identical (or even to the left) to the distribution of non-offshoring firms, but ex-post, the distributions may look as in Figure 1b due exclusively to the offshoring productivity effect. In that case, the average productivity of offshoring firms is higher precisely because they offshore, not the other way around. This is consistent with the results of Smeets and Warzynski (2013) for Danish manufacturing firms: although importing firms are on average more productive than non-trading firms, they find no evidence of selection into importing but—in accordance with the offshoring productivity effect—find long-run evidence of “learning by importing”. By adding an escape-competition mechanism in an otherwise conventional Melitz-type offshoring structure, the model in this paper can generate this type of outcome.

The model’s treatment of the offshoring decision as an investment decision is not trivial. Far more than a coincidence in name (recall that one of the forms of offshoring is FDI), the offshoring decision is no different to an investment decision because it always involves a production-process adjustment and as such, it is discrete, may involve large capital adjustments, and creates disruptions as the firm reorganizes.\(^6\) Therefore, I follow the extensive literature on lumpy investment and rely on non-convex adjustment costs to model offshoring decisions. Empirically, this type of costs has been shown to be crucial. In particular, studies using U.S. plant- and firm-level data show that non-convex adjustment costs are necessary to match the dynamics of plant-level investment (Cooper and Haltiwanger, 2006), and output fluctuations after uncertainty shocks (Bloom, 2009). As well, I follow Caballero and Engel (1999)—who also obtain large estimates for non-convex adjustment costs using U.S. data—and add a stochastic element to the offshoring adjustment cost. The stochastic approach not only makes the model highly tractable, but also adds a new dimension of reality, taking into account that offshoring opportunities present themselves at random, with offshoring adjustment costs differing across firms (even if they are equally productive) and varying over time.

Though this is a paper about offshoring decisions, my framework can be used more generally to model any type of firm-level decision involving a productivity-enhancing innovation, e.g. technology upgrading. This paper is then related to papers that study the effects of competition on innovation. In my model, any shock that alters the competitive environment affects the opportunity cost of the offshoring decision. Along the same lines, Holmes, Levine and Schmitz (2012) present a model

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\(^6\)For example, Smeets and Warzynski (2013) find that although there are long-run “learning by importing” productivity effects in Danish firms, they suffer a negative shock in the year they start to import. Consistent with the production-process adjustment story, they suggest that the observed temporary adverse shock in these firms “could be explained by a need to adapt their products or supply chain”.

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about the effects of increased competition on technology adoption when costs from switchover disruptions matter. In their model, a monopolist is unwilling to adopt a new technology because of the high opportunity cost of switchover disruptions. The price that the monopolist can charge is, however, limited by the marginal cost of potential rivals. Hence, if that marginal cost falls—in a shock interpreted as an increase in competition—the monopolist’s opportunity cost of switchover disruptions also falls, which then may drive the firm to adopt the new technology. Bloom et al. (2013) obtain a similar result in their trapped-factors model of innovation.\(^7\) The inverse relationship between competition and the opportunity cost of innovation obtained in these models is just another version of the escape-competition effect that appears in my model.\(^8\) Nevertheless, none of the previous models consider firm heterogeneity and hence cannot explain how firms’ offshoring (or innovation) incentives vary with firm-level productivity.

Likewise, Aghion et al. (2005) (from which I borrow the “escape competition” terminology) document an inverted-U relationship between competition and innovation at the industry level. To explain this fact, they propose a model in which the effects of competition on pre-innovation and post-innovation profits depend on whether an industry is leveled (composed of neck-and-neck firms) or unleveled (composed of leaders and followers). In neck-and-neck sectors the difference between pre- and post-innovation profits increases with competition, and hence firms innovate to escape competition. The opposite happens for laggard firms in unleveled sectors—the Schumpeterian (or selection) effect of competition is stronger—and hence innovation declines in these sectors. In the end, the industry-level inverted-U shape is generated by changes in the composition of leveled and unleveled sectors in the economy. The objective of Aghion et al. is to explain the competition-innovation relationship at the industry level and therefore—and in contrast to my model—they abstract from firm heterogeneity and variable markups’ considerations.

There is substantial empirical evidence on the interaction of selection and escape-competition effects after shocks that alter the competitive environment. In their survey of industry-specific and trade liberalization studies, Holmes, Levine and Schmitz (2012) observe two general facts: (i) competition reduces establishment and industry sizes (which is consistent with selection effects), and (ii) competition spurs establishment-level productivity (which is consistent with escape-competition effects). In a related survey, Syverson (2011) mentions similar evidence and highlights the positive impact on aggregate productivity of the selection and within-firm effects of competition.\(^9\) The

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\(^7\)In the model of Bloom et al. (2013), production factors are partially trapped in producing old goods because of good-specific sunk investments (e.g. learning by doing): if the firm redeploys workers to innovation activities, it loses the human capital related to the production of the old good—a switchover cost. An increase in competition from a low-wage country drives down the firm’s profit from the old good, causing a decline in the opportunity cost of innovation, and hence allowing the reallocation of the trapped factors to innovation activities.

\(^8\)The idea that firms’ production-process enhancements occur after reductions in opportunity costs due to lower profitability has also been used in the context of firms’ restructuring decisions during recessions (see, for example, Aghion and Saint-Paul, 1998 and Berger, 2012).

\(^9\)As an example, Bloom, Draca and Van Reenen (2012) find that import competition from China increases innovation and productivity in European firms through both between-firm (selection) and within-firm effects. Similar
within-firm effect refers to firms making productivity-enhancing decisions, and hence it is analogous to the escape-competition effect in this paper. In my model, and in line with the empirical evidence, a tougher competitive environment rises aggregate productivity through both the selection and escape-competition effects.

The extension of the model with trade in final goods shows how liberalization alters offshoring incentives. Similarly, Van Long, Raff and Stähler (2011) study the effects of trade liberalization on innovation incentives in an oligopolistic setting with heterogeneous firms and endogenous markups. Given that the innovation decision is made before a firm knows its productivity, the level of innovation is the same for every firm. Moreover, when allowing for firm entry, the firm-level innovation decision becomes independent of trade costs. In contrast, the process innovation decision in this paper (i.e., the offshoring decision) crucially depends on the firm’s productivity and even with free entry, this decision will be affected by changes in trade costs. Impullitti and Licandro (2013) also study the effects of trade liberalization on cost-reducing innovation. As in my setting, trade liberalization creates a tougher competitive environment, which lowers firms’ markups and increases innovation. The difference is that in my model markups are monotonically increasing with productivity, while in their setting all exporters set the same markup, which is smaller than the markup of less productive non-exporters.

Kasahara and Lapham (2013) study the effects of trade liberalization in a heterogeneous-firm model that incorporates the offshoring productivity effect along with complementarities between offshoring and exporting. In contrast to this paper, the offshoring decision in that model is subject to homogeneous fixed costs. Nevertheless, they assume random iceberg costs of exporting and offshoring, which allows for coexistence of low-productivity trading firms and high-productivity non-trading firms, but in a setting with CES preferences and thus constant markups.

A key implication of my model is the inverted-U relationship between firm-level productivity and the probability of offshoring. In a heterogeneous-firm model with endogenous markups, Spearot (2012) obtains a closely related result for the relationship between productivity and firm-level investment. His model obtains that mid-productivity firms engage more in investment and provides empirical evidence from U.S. firms that support that result. In Spearot (2013), he extends the model to study foreign acquisition decisions. The inverted-U relationship between firm-level productivity and investment in Spearot’s model arises due to the endogenous-markup structure and the assumption of increasing marginal costs. My model also contains an endogenous-markup structure, but assumes instead constant marginal costs and, as in the lumpy-investment literature, relies on non-convex adjustment costs.

There is further evidence suggesting an inverted-U relationship between firm-level productivity

results are obtained by Iacovone, Rauch and Winters (2013) for the responses of Mexican firms to Chinese competition. In both studies, offshoring to China magnifies the effects.
and productivity-enhancing investments. For example, Lileeva and Trefler (2010) find that improved access to the U.S. market drove low- and mid-productivity Canadian firms to make the decision to invest and export. In addition, Bustos (2011) finds that for Argentine firms facing tariff reductions from Brazil, most technology-upgrading changes happen in the third quartile of the distribution of firm size. If firm size is positively related to productivity, Bustos’s finding implies an inverted-U relationship between firm-level productivity and technology-upgrading likelihood. Following this paper’s approach to model the technology-upgrading decision as a lumpy investment decision, we can provide an appealing explanation for the observed inverted-U relationship.\footnote{Other theoretical papers highlighting within-firm productivity growth in heterogeneous-firm settings include Costantini and Melitz (2008), Atkeson and Burstein (2010), and Burstein and Melitz (2011).}

3 The Model

This section presents a heterogeneous-firm model of offshoring decisions with endogenous markups and random adjustment costs of offshoring.

I assume a country inhabited by a continuum of households in the unit interval and with two production sectors: a homogeneous-good sector and a differentiated-good sector. Firms in the differentiated-good sector are heterogeneous in productivity. Each household provides a unit of labor at a fixed wage level to any of the sectors in the economy. However, wages differ between this country and the rest of the world. In particular, the wage abroad is below the domestic wage. This fundamental difference gives firms in the differentiated-good sector an incentive to split the production process between the domestic country and the rest of the world. Nevertheless, to begin offshoring, a firm must incur adjustment—or disruption—costs.

First, I specify preferences, obtain the demand, and discuss pricing and production decisions in the differentiated-good sector. Second, I describe the offshoring decision and obtain the key relationship between productivity and offshoring probability. Third, I show the distributions of offshoring and non-offshoring firms, and describe the free-entry condition that closes the model. The section concludes with a numerical example.

3.1 Model Setup

3.1.1 Preferences and Demand

Households define their preferences over a continuum of differentiated goods and a homogeneous good. In particular, the utility function of the representative household is given by

\[ U = q_h^{1-\psi} Q^\psi, \]

where \( q_h \) denotes consumption of the homogeneous good, \( Q \) is a consumption index of differentiated goods, and \( \psi \in (0, 1) \). Following Feenstra (2003), I assume that \( Q \) satisfies the symmetric translog
expenditure function

\[
\ln E = \ln Q + \frac{1}{2\gamma N} + \frac{1}{N} \int_{i \in \Delta} \ln p_i di + \frac{\gamma}{2N} \int_{i \in \Delta} \int_{j \in \Delta} \ln p_i (\ln p_j - \ln p_i) dj di,
\]

(2)

where \( E \) is the minimum expenditure required to obtain \( Q \), \( \Delta \) denotes the set of differentiated goods available for purchase, \( N \) is the measure of \( \Delta \), \( p_i \) denotes the price of differentiated good \( i \), and \( \gamma \) indicates the level of substitutability between the varieties (a higher \( \gamma \) implies a higher degree of substitution). Equation (2) implies variable markups in the differentiated-good sector.\(^{11}\)

The production of each unit of the homogeneous good requires one unit of labor. This good is the numéraire and is sold in a perfectly competitive market. Hence, the wage—in terms of the numéraire—is also 1. Given the Cobb-Douglas utility function in equation (1) and the equivalence of the wage and the price of the homogeneous good, the total expenditure in differentiated goods of the representative household is simply given by \( \psi \), where we must satisfy \( \psi < 1 \).

The demand of the representative household for differentiated good \( i \) is given by \( q_i = \sigma_i \frac{\psi}{\hat{p}} \), where \( \sigma_i \) is the share of variety \( i \) in the total household expenditure on differentiated goods. By Shephard’s lemma—the derivative of equation (2) with respect to \( \ln p_i \)—we obtain that \( \sigma_i = \gamma \ln \left( \frac{\hat{p}}{p_i} \right) \), where

\[
\hat{p} = \exp \left( \frac{1}{\gamma N} + \ln \bar{p} \right)
\]

denotes the maximum price that firms can set in the differentiated-good sector (note that \( \sigma_i = 0 \) if \( p_i = \hat{p} \)), and \( \ln \bar{p} = \frac{1}{N} \int_{j \in \Delta} \ln p_j dj \).

3.1.2 Pricing and Production of Differentiated Goods

Because households are located in the unit interval, the market demand for differentiated good \( i \) is identical to the demand of the representative household. A producer of good \( i \) with a constant marginal cost, \( c_i \), who takes \( \hat{p} \) as given, sets the price that maximizes \( \pi_i = (p_i - c_i)q_i \). This maximization problem yields \( p_i = \left[ 1 + \ln \left( \frac{\hat{p}}{p_i} \right) \right] c_i \), from which we can solve for \( p_i \) as

\[
p_i = (1 + \mu_i)c_i,
\]

(4)

\(^{11}\)Alternative preferences to model endogenous markups include, among others, the quadratic utility function of Ottaviano, Tabuchi and Thisse (2002)—and used by Melitz and Ottaviano (2008) in a heterogeneous-firm setup—and the exponential specification of Behrens et al. (2012). Translog preferences have been used recently in several topics of trade and open economy macroeconomics. Feenstra and Weinstein (2010) use them to estimate the gains from trade in the U.S. due to declining markups and increased product variety. Arkolakis, Costinot and Rodriguez-Clare (2010) show that their gains-from-trade results in Arkolakis, Costinot and Rodriguez-Clare (2012) hold for the case of translog preferences with a Pareto distribution of productivity. Based on the translog function, Novy (2013) obtains a gravity equation with endogenous elasticity of trade with respect to trade costs, which performs better than the CES gravity equation (which implies an exogenous trade elasticity) across several dimensions. In their business-cycle model with endogenous entry, Bilbiie, Ghironi and Melitz (2012) show the usefulness of translog preferences to solve the puzzle of countercyclical markups and procyclical profits. Bergin and Feenstra (2009) and Rodriguez-Lopez (2011) use translog preferences to study exchange rate pass-through with endogenous markups.
where $\mu_i$ is producer $i$'s proportional markup over the marginal cost, which is given by

$$\mu_i = \Omega \left( \frac{\hat{p}}{c_i} e \right) - 1. \quad (5)$$

The function $\Omega(\cdot)$ denotes the Lambert $W$ function, which is the inverse of $f(\Omega) = \Omega e^\Omega$; that is, in the equation $x = z e^z$, we solve for $z$ as $z = \Omega(x)$. Among its properties, we have that if $x \geq 0$ then $\Omega'(x) > 0$, $\Omega''(x) < 0$, $\Omega(0) = 0$, and $\Omega(e) = 1$.\footnote{See Corless et al. (1996) for an overview of the Lambert $W$ function. Other of its properties include $\Omega'(x) = \frac{\Omega(x)}{x[1+\Omega(x)]}$ for $x \neq 0$, and $\ln[\Omega(x)] = \ln x - \Omega(x)$ when $x > 0$.} Note that $\mu_i$ is zero if $c_i = \hat{p}$ (so that the price of good $i$ equals its marginal cost), and is greater than zero if $c_i < \hat{p}$. If $c_i > \hat{p}$, firm $i$ will not produce.

Another useful result arising from the properties of the Lambert $W$ function is that

$$\ln p_i = \ln \hat{p} - \mu_i, \quad (6)$$

which follows from taking the natural log of (4) and using the property $\ln[\Omega(x)] = \ln x - \Omega(x)$ for $x > 0$. Using equation (6) in the expression for $\sigma_i$ in section 3.1.1 yields

$$\sigma_i = \gamma \mu_i. \quad (7)$$

That is, the market share density of producer $i$ is directly proportional to its markup.

Firms are heterogeneous in productivity. Following Melitz (2003), I assume that a firm knows its productivity—drawn from a probability distribution—only after paying a sunk entry cost of $f_E$. Knowing its productivity, the firm can decide between using only domestic labor ($L$) or use also foreign labor ($L^*$). The foreign wage, $w^*$, is less than the domestic wage of 1. I assume that an offshoring firm splits its production process in two complementary parts, one of which stays at home while the other is moved abroad. Let $s \in \{n, o\}$ denote a firm’s offshoring status, with $n$ meaning “not offshoring” and $o$ meaning “offshoring”. Then, the production function of a producer with productivity $\varphi$ and offshoring status $s$ is given by $y_s(\varphi) = \varphi L_s$, where

$$L_s = \begin{cases} L & \text{if } s = n \\ \min \left\{ \frac{L}{1-\kappa}, \frac{L^*}{\kappa \lambda} \right\} & \text{if } s = o. \end{cases} \quad (8)$$

In $L_s$, $\kappa \in (0, 1)$ represents the fraction of the production process being offshored, and $\lambda$ accounts for the variable cost of making foreign labor compatible with the domestic production process.\footnote{We can also think of $\lambda$ as an iceberg offshoring cost: a producer must hire $\lambda \geq 1$ units of foreign labor to produce the same amount of output than a unit of domestic labor.} Denoting the price of $L_s$ with $w_s$, we obtain that $w_n = 1$ and $w_o = 1 - \kappa + \kappa \lambda w^*$. Hence, the marginal cost of a firm with productivity $\varphi$ and offshoring status $s$ is $\frac{w_s}{\varphi}$. Throughout the paper, I assume that $\lambda$ is small enough so that $w_o < w_n$ and therefore, a firm’s marginal cost is always lower when offshoring.
From equations (4) and (5), we write the price set by a producing firm with productivity $\varphi$ and offshoring status $s$ as

$$p_s(\varphi) = [1 + \mu_s(\varphi)] \frac{w_s}{\varphi},$$

for $s \in \{n, o\}$, where

$$\mu_s(\varphi) = \Omega \left( \frac{\varphi^s}{w_s} \right) - 1.$$

Then, this firm’s equilibrium output and profit functions are respectively given by

$$y_s(\varphi) = \left[ \frac{\mu_s(\varphi)}{1 + \mu_s(\varphi)} \right] \frac{\gamma \psi \varphi}{w_s} \quad \text{and} \quad \pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1 + \mu_s(\varphi)} \gamma \psi. \quad (9)$$

### 3.1.3 Cutoff Productivity Levels

As a Melitz-type model, cutoff levels determine the tradability of goods: a firm sells its differentiated good if and only if its productivity is no less than the cutoff productivity level for all the firms with the same offshoring status. The existence of the upper bound for the price that firms can set, $\hat{p}$, allows us to obtain the cutoff productivity levels without the need to assume fixed costs of production (which are necessary in the Melitz (2003) model with CES preferences). Using the markup function in the previous section, we define the cutoff productivity level for firms with offshoring status $s$ as

$$\varphi_s = \inf \{ \varphi : \mu_s(\varphi) > 0 \} = \frac{w_s}{\hat{p}}, \quad (10)$$

for $s \in \{n, o\}$. The model’s cutoff productivity levels are then $\varphi_n$ and $\varphi_o$.

Note that we can use the zero-cutoff-markup condition in equation (10) to replace $\hat{p}$ in the markup equation from the previous section. Hence, we rewrite the markup of a firm with productivity $\varphi$ and offshoring status $s$ as

$$\mu_s(\varphi) = \Omega \left( \frac{\varphi^s}{\varphi_s} \right) - 1, \quad (11)$$

for $\varphi \geq \varphi_s$, and $s \in \{n, o\}$. Given the properties of $\Omega(\cdot)$ from the previous section, $\mu_s(\varphi)$ is strictly increasing in $\varphi$; that is, given offshoring status $s$, more productive firms charge higher markups.

Moreover, combining the two expressions that stem from (10), we obtain one of the two equations we need to solve the model:

$$\varphi_o = w_o \varphi_n. \quad (12)$$

As $w_o < 1$, it is always true that $\varphi_o < \varphi_n$. Hence, a firm whose productivity is in the interval $[\varphi_o, \varphi_n)$ will only produce if it offshores.

### 3.2 The Offshoring Decision

Following the model of Caballero and Engel (1999) on lumpy investment decisions in a generalized $(S, s)$ framework, I model the offshoring decision on the basis of random adjustment costs. A firm
which decides to offshore incurs adjustment costs due to the disruption and reorganization of the production process. These costs, however, can vary over time and are not necessarily the same for firms with the same level of productivity.

From section 3.1.2, we know that the total profit obtained every period by a firm with productivity $\varphi$ and offshoring status $s$ is

$$
\pi_s(\varphi) = \begin{cases} 
0 & \text{if } \varphi < \varphi_s \\
\frac{\mu_s(\varphi)^2}{1 + \mu_s(\varphi)^2} & \text{if } \varphi \geq \varphi_s,
\end{cases}
$$

for $\mu_s(\varphi)$ given by equation (11) and $s \in \{n, o\}$. Since the marginal cost is lower when a firm offshores, it is always the case that $\pi_o(\varphi) \geq \pi_n(\varphi)$, with strict inequality if $\varphi > \varphi_o$. This implies that the offshoring decision is irreversible.

At the beginning of each period, every non-offshoring firm finds out its offshoring adjustment cost, which includes a component that is positively related to the firm’s size plus a component unrelated to the firm’s size. The firm then decides whether to offshore. If the firm decides to offshore, it will continue offshoring until it is hit by an exogenous death shock. If the firm does not offshore, it can die at the end of the period (after an exogenous death shock), or survive and receive a new adjustment cost at the beginning of the following period. The offshoring adjustment cost for a firm with productivity $\varphi$, denoted by $A(\varphi)$, is then given by

$$
A(\varphi) \equiv \eta [\rho \pi_n(\varphi) + f_o],
$$

where $\eta$ is a non-negative random variable with cumulative distribution function $F(\eta)$, $\rho \in (0,1]$, and $f_o > 0$.

The term $\eta \rho \pi_n(\varphi)$ accounts for adjustment costs related to the firm’s size; for a given $\eta$, and due to the positive relationship between firm-level size and productivity, these costs are increasing in productivity if $\varphi \geq \varphi_n$. Although we follow Caballero and Engel (1999) and use profits as a measure of size, any other measure of size—for example, firm-level output, $y_n(\varphi)$, or total sales, $p_n(\varphi)y_n(\varphi)$—would keep the insights from the model unaltered. On the other hand, $\eta f_o$ accounts for adjustment costs that are independent of $\varphi$. Note that $E[A(\varphi)] \equiv E(\eta) [\rho \pi_n(\varphi) + f_o]$, where $\frac{dE[A(\varphi)]}{d\varphi} > 0$ if $\varphi \geq \varphi_n$ (and zero otherwise). As a special case, if $\rho$ and the variance of $\eta$ approach zero, the adjustment costs get closer to the usual homogeneous fixed costs of offshoring, so that the productivity distributions of non-offshoring and offshoring firms look similar to those in Figure 1a.

Ideally, in addition to the intrinsic similarities between offshoring and investment/innovation decisions, we would like to observe data on offshoring costs and measures of firm-level productivity.

---

14 Caballero and Engel (1999) interpret adjustment costs that are proportional to the before-change profits as the amount of profits that a firm stops receiving during the adjustment (Bloom, 2009 and Cooper and Haltiwanger, 2006 assume similar costs). As mentioned before, in the models of Holmes, Levine and Schmitz (2012) and Bloom et al. (2013) the opportunity cost of innovation is also directly related to the before-change profits. My approach is also similar to Atkeson and Burstein (2010), who construct a heterogeneous-firm model of trade in which costs of process innovation are increasing in firm size.
to look for direct support of the adjustment-cost equation in (14). To my knowledge, however,
there is not any firm-level dataset with this type of information. Nevertheless, I argue that the
assumption of offshoring adjustment costs that are (in expectation) increasing in firm size is appro-
priate because: (i) the expression for adjustment costs is in absolute terms, and hence, if two firms
will offshore a fraction $\kappa$ of their production processes, but one is 100 times larger than the other, it
seems reasonable that the largest firm is more likely to face larger absolute costs of disruption and
reorganization (whether $\kappa$ approaches 0 or 1); (ii) in proportion to size, however, adjustment costs
are (in expectation) decreasing in productivity—note that $E[A(\phi)/\pi_n(\phi)] \equiv E(\eta)[\rho + f_o/\pi_n(\phi)]$
is decreasing in $\phi$—so that more productive and larger firms are effectively more efficient in handling
a reorganization or disruption due to offshoring; and (iii) as I show in section 3.5.1, assuming
$\rho = 0$ (so that absolute adjustment costs are homogeneous in expectation, i.e., $E[A(\phi)] = E(\eta)f_o$)
implies unrealistically high values for the fraction of offshoring firms and for the productivity gap
between offshoring and non-offshoring firms, while generating productivity distributions that bear
little resemblance to those found in empirical studies.

As in Melitz (2003), let $\delta$ be the probability of an exogenous death shock at the end of each
period. In steady state, the per-period profit of an offshoring firm with productivity $\phi$, $\pi_o(\phi)$, is
constant; thus, this firm’s expected lifetime profits are $\bar{\pi}_o(\phi)$. Hence, at the beginning of each period,
the Bellman equation for the value of a non-offshoring firm with productivity $\phi$ and adjustment
factor $\eta$ is

$$V(\phi, \eta) = \max \left\{ \pi_o(\phi) - \frac{\eta}{\delta} \left[ \rho \pi_n(\phi) + f_o \right], \pi_n(\phi) + (1 - \delta)E[V(\phi, \eta')] \right\}.$$  \hspace{1cm} (15) 

The first term in braces is the value of the firm if it decides to start offshoring, and is composed of
the expected lifetime offshoring profits net of the adjustment cost. The second term is the value
of the firm if it decides not to offshore; the firm receives $\pi_n(\phi)$ in the current period and survives
to the next period with probability $1 - \delta$, in which case it draws a new adjustment factor, $\eta'$. Let
$\hat{\eta}(\phi)$ be the value for $\eta$ that makes a non-offshoring firm with productivity $\phi$ indifferent between
offshoring or not. The following proposition describes the solution for $\hat{\eta}(\phi)$.

**Proposition 1. (The cutoff adjustment factor)**

*Given the Bellman equation (15) and a continuous $F(\eta)$, the cutoff adjustment factor for a
non-offshoring firm with productivity $\phi$, $\hat{\eta}(\phi)$, is the unique solution to

$$\hat{\eta}(\phi) = \frac{z(\phi)}{\delta} - \frac{1 - \delta}{\delta} \int_0^{\hat{\eta}(\phi)} F(\eta) d\eta,$$ \hspace{1cm} (16) 

where

$$z(\phi) \equiv \frac{\pi_o(\phi) - \pi_n(\phi)}{\rho \pi_n(\phi) + f_o} \geq 0$$ \hspace{1cm} (17) 

is an adjusted measure of the distance between the firm’s offshoring and non-offshoring profits.*
Therefore, at the beginning of each period, for the set of non-offshoring firms with productivity $\varphi$, those drawing an adjustment factor below $\hat{\eta}(\varphi)$ become offshoring firms. We can be more precise and pin down the probability that a non-offshoring firm with productivity $\varphi$ begins to offshore in a particular period. Denoting this probability with $\Lambda(\varphi)$, it follows that $\Lambda(\varphi) = F[\hat{\eta}(\varphi)]$. The following proposition describes the behavior of $\Lambda(\varphi)$.

**Proposition 2. (The probability of offshoring)**

There is an inverted-U relationship between firm-level productivity and the probability of offshoring: $\Lambda(\varphi) = 0$ for $\varphi \leq \varphi_o$, $\Lambda(\varphi) \to 0$ if $\varphi \to \infty$, and given $\varphi_n$ and $\varphi_o$, the level of productivity that maximizes $\Lambda(\varphi)$ approaches $\varphi_n$ from the right as $f_o$ declines or as $\rho$ increases.

Figure 2 presents a graphical description of Proposition 2. The offshoring probability is zero for a firm with productivity at or below $\varphi_o$, as this firm cannot make positive profits even if it offshores. For firms with productivities above $\varphi_o$, it is useful to refer to the adjusted measure of the incremental profits from offshoring, $z(\varphi)$, which is the most important determinant of the shape of $\Lambda(\varphi)$. The larger $z(\varphi)$ is, the higher the adjustment factor that a non-offshoring firm is willing to accept, and hence the higher the offshoring probability. Non-offshoring firms with productivities between $\varphi_o$ and $\varphi_n$ do not produce—have zero profits—and thus, their offshoring decision only depends on the comparison of offshoring profits and the component of adjustment costs unrelated to productivity, $\eta f_o$. These firms’ offshoring prospects increase with productivity, and hence $\Lambda(\varphi)$ is increasing in this range. For non-offshoring firms with productivities above $\varphi_n$ (so that they produce and have positive profits), their offshoring decision also considers the adjustment costs associated with their size, $\eta \rho \pi_n(\varphi)$. For those firms close to $\varphi_n$ (from the right), they are small enough so that the most important adjustment cost they face is $\eta f_o$. Thus, there exists a range of firms—starting at $\varphi_n$—for which the offshoring probability increases with productivity. As the adjustment cost related to the firm’s size becomes more important, there exists a point from which the offshoring probability starts to decline.

There are two key differences of this model compared to heterogeneous-firm models that only consider homogeneous fixed costs of offshoring. In those models, every firm with a productivity no less than a cutoff level will offshore: denoting that cutoff level by $\hat{\varphi}$, these models imply that $\Lambda(\varphi) = 0$ if $\varphi < \hat{\varphi}$, and $\Lambda(\varphi) = 1$ if $\varphi \geq \hat{\varphi}$. On the other hand, in this model (i) there is no cutoff level that separates non-offshoring and offshoring firms, and (ii) the most productive firms can have offshoring probabilities that are below the offshoring probabilities of much less productive firms.

Figure 2 shows $\Lambda(\varphi)$ for different levels of $f_o$ and $\rho$. These parameters determine the importance of adjustment costs related to firm’s size relative to adjustment costs independent of firm’s size. A lower $f_o$ or a higher $\rho$ imply a higher importance of the former, causing a more pronounced inverted-U relationship. Note, for example, that the range of a positive relationship between productivity and
offshoring probability is narrower for lower levels of $f_o$ and higher levels of $\rho$. This result gives us an insight into how the offshoring probability function, $\Lambda(\varphi)$, would look like for different industries. In those industries for which offshoring implies large disruptions in the production process—so that the adjustment cost related to size is relatively more important—we should expect to see a well-defined inverted-U shape in $\Lambda(\varphi)$. On the other hand, in those industries for which offshoring mostly implies adjustment costs unrelated to firm’s size, $\Lambda(\varphi)$ will show a weak inverted-U shape and hence, will give the general impression that more productive firms are more likely to offshore.

In section B.2 in the Appendix, I show that Proposition 2 holds if we use the quasilinear-quadratic preferences of Melitz and Ottaviano (2008), which also generate variable markups. On the other hand, in section B.1.2 in the Appendix I show that Proposition 2 does not hold with CES preferences (i.e., with exogenous markups). In the CES case, an inverted-U relationship between productivity and offshoring likelihood appears only if $f_o < \rho f$, where $f$ is a fixed cost of production; otherwise, there is a non-decreasing relationship between productivity and the offshoring probability. In contrast, in the translog and quasilinear-quadratic cases, the inverted-U shape emerges even with the assumption of zero fixed costs of production (assuming $f > 0$ would only reinforce the inverted-U relationship). This comparison across preferences shows that the results in Proposition 2 are not only a consequence of the assumed type of offshoring adjustment costs, but also depend crucially on the endogenous-markup structure.

### 3.3 Distribution and Composition of Firms

After entry, a firm draws its productivity from the interval $[\varphi_{\text{min}}, \infty)$ according to the cumulative distribution function $G(\varphi)$, with probability density function denoted by $g(\varphi)$. There are offshoring and non-offshoring firms. For each level of productivity, the determinants of the proportion of each
type of firm are the death probability, $\delta$, and the probability of offshoring, $\Lambda(\varphi)$. In particular, for productivity level $\varphi$ the steady-state proportion of offshoring firms is given by

$$\Gamma(\varphi) = \frac{\Lambda(\varphi)}{\delta + (1-\delta)\Lambda(\varphi)}. \quad (18)$$

Note that (i) $\Gamma(\varphi) = \Lambda(\varphi) = 0$ if $\varphi \leq \varphi_o$; and (ii) for $\varphi > \varphi_o$, $\Gamma(\varphi) \to \Lambda(\varphi)$ if $\delta \to 1$, and $\Gamma(\varphi) \to 1$ if $\delta \to 0$.\(^{15}\) It follows that for productivity level $\varphi$ the steady-state proportion of non-offshoring firms is $1 - \Gamma(\varphi)$.

Let $h_o(\varphi)$ and $H_o(\varphi)$ denote, respectively, the probability density function and the cumulative distribution function for the productivity of offshoring firms. Using $\Gamma(\varphi)$ and $g(\varphi)$ we obtain

$$h_o(\varphi) = \frac{\Gamma(\varphi)g(\varphi)}{\bar{\Gamma}}, \quad (19)$$

where $\bar{\Gamma} = \int_{\varphi_o}^{\infty} \Gamma(\varphi)g(\varphi)d\varphi$ is the steady-state proportion of offshoring firms. Analogously, let $h_n(\varphi)$ and $H_n(\varphi)$ denote the probability density function and the cumulative distribution function for the productivity of non-offshoring firms. We then have that

$$h_n(\varphi) = \frac{[1 - \Gamma(\varphi)]g(\varphi)}{1 - \bar{\Gamma}}. \quad (20)$$

As mentioned in section 3.1.1, $N$ denotes the measure of the set of goods that are available for purchase. As each firm produces a single good, the set of actual producers also has measure $N$. The set of actual producers comprises non-offshoring firms, with measure $N_n$, and offshoring firms, with measure $N_o$; that is, $N = N_n + N_o$. In steady state the firms that die due to the exogenous death shock are exactly replaced by successful entrants, and thus

$$\delta N_n = [1 - H_n(\varphi_n)](1 - \bar{\Gamma})N_E, \quad (21)$$
$$\delta N_o = \bar{\Gamma}N_E, \quad (22)$$

where $N_E$ denotes the mass of entrants. In equations (21) and (22), the left-hand side accounts for the firms that die, while the right-hand side accounts for the mass of successful entrants of each type of firm. For non-offshoring firms, we know that a fraction $1 - \bar{\Gamma}$ of entrants will not offshore and of these, a fraction $1 - H_n(\varphi_n)$ will have productivity levels no less than $\varphi_n$ and hence will produce. For offshoring firms, a fraction $\bar{\Gamma}$ of entrants will offshore and all of them have productivity levels no less than $\varphi_o$; that is, $H_o(\varphi_o) = 0$ and every offshoring firm produces. Adding (21) and (22) we can solve for $N$ as

$$N = \left[1 - (1 - \bar{\Gamma})H_n(\varphi_n)\right] \frac{N_E}{\delta}. \quad (23)$$

Hence, to obtain expressions for $N_n$, $N_o$, and $N$ in terms of the cutoff productivity levels, $\varphi_n$ and $\varphi_o$, we need to obtain first the steady-state mass of entrants, $N_E$.

\(^{15}\)If by the end of each period all the firms die (so that $\delta = 1$), it must be the case that $\Gamma(\varphi)$ is identical to $\Lambda(\varphi)$. On the other hand, if firms never die (so that $\delta = 0$) then every firm with productivity above $\varphi_o$ eventually becomes an offshoring firm and therefore, in steady state $\Gamma(\varphi) = 1$.\[^{16}\]
Lemma 1. The steady-state mass of entrants, $N_E$, is given by
\[
N_E = \frac{\delta \{ (1 - \Gamma) [1 - H_n(\varphi_n) \bar{\mu}_n + \Gamma \bar{\mu}_o] \}}{\gamma},
\]
where $\bar{\mu}_s = \int_\varphi^\infty \mu_s(\varphi) h_s(\varphi | \varphi \geq \varphi_s) d\varphi$ is the average markup of firms with offshoring status $s$, for $s \in \{n, o\}$.

We can also write expressions for average productivities, average prices, and market shares. For producing firms with offshoring status $s$, for $s \in \{n, o\}$, the average productivity and the average price are respectively given by
\[
\bar{\varphi}_s = \int_\varphi^\infty \varphi h_s(\varphi | \varphi \geq \varphi_s) d\varphi \quad \text{and} \quad \bar{p}_s = \int_\varphi^\infty p_s(\varphi) h_s(\varphi | \varphi \geq \varphi_s) d\varphi.
\]
Then, the overall average productivity and the overall average price can be written as $\bar{\varphi} = \frac{N_n}{N} \bar{\varphi}_n + \frac{N_o}{N} \bar{\varphi}_o$ and $\bar{p} = \frac{N_n}{N} \bar{p}_n + \frac{N_o}{N} \bar{p}_o$. For market shares, from equation (7) it follows that the market share density of a firm with productivity $\varphi$ and offshoring status $s$ is $\sigma_s(\varphi) = \gamma \mu_s(\varphi)$: given offshoring status $s$, more productive firms charge higher markups and have larger market shares. Integrating the previous expression over all firms with the same offshoring status, I obtain that the total market share of firms with offshoring status $s$ is $\sigma_s = \gamma N_s \bar{\mu}_s$, for $s \in \{n, o\}$, with $\sigma_n + \sigma_o = 1$.

3.4 Free-Entry Condition and Equilibrium

As in Melitz (2003), firms enter as long as the expected value of entry is no less than the sunk entry cost, $f_E$. A potential entrant knows that the expected profit of a firm with productivity $\varphi$ for its first period of existence is
\[
\tilde{\pi}(\varphi) = [1 - \Lambda(\varphi)] \pi_n(\varphi) + \Lambda(\varphi) \{ \pi_o(\varphi) - E[\eta | \eta \leq \hat{\eta}(\varphi)] [\rho \pi_n(\varphi) + f_o] \},
\]
which is a weighted average between the non-offshoring profits and the offshoring profits minus the expected adjustment cost, with the weights determined by the offshoring probability, $\Lambda(\varphi)$. Taking into account the exogenous death shock at the end of every period, the potential entrant also knows that the expected profit of a firm with productivity $\varphi$ for its $t^{th}$ period of existence is given by
\[
\tilde{\pi}^t(\varphi) = (1 - \delta)^{t-1} \left\{ [1 - \Lambda(\varphi)]^{t-1} \tilde{\pi}(\varphi) + [1 - (1 - \Lambda(\varphi))^{t-1}] \pi_o(\varphi) \right\},
\]
where the first term inside the braces accounts for the expected profit at time $t$ if the firm has not yet decided to offshore by $t - 1$, while the second term accounts for the profit the firm receives at $t$ if it is already offshoring by $t - 1$. Of course, $\tilde{\pi}^1(\varphi) = \tilde{\pi}(\varphi)$.

Given the productivity distribution of new firms—with probability density function $g(\varphi)$—a potential entrant’s expected value of entry is given by
\[
\tilde{\pi}_E = \int_{\varphi_o}^\infty \left[ \sum_{t=1}^\infty \tilde{\pi}^t(\varphi) \right] g(\varphi) d\varphi.
\]
Substituting the expressions for $\bar{\pi}(\varphi)$ and $\bar{\pi}(\varphi)$ into $\bar{\pi}E$, and using equations (18), (19), and (20), we can write the free-entry condition, $\bar{\pi}E = f_E$, as

$$(1 - \bar{\Gamma}) \int_{\varphi_n}^{\infty} \frac{\pi_n(\varphi)}{\delta} h_n(\varphi)d\varphi + \bar{\Gamma} \int_{\varphi_o}^{\infty} \left\{ \frac{\pi_o(\varphi)}{\delta} - E[\eta | \eta \leq \bar{\eta}(\varphi)] [\rho \pi_n(\varphi) + f_o] \right\} h_o(\varphi)d\varphi = f_E. \quad (27)$$

Note that the expression for $\bar{\pi}E$ in the free-entry condition presents the value of entry as a weighted average between a potential entrant’s lifetime expected profits if it never offshores, and the expected lifetime offshoring profits minus the one-time adjustment cost, with the weights determined by the steady-state proportion of offshoring firms, $\bar{\Gamma}$. We can now define the equilibrium of the model.

**Definition 1.** An equilibrium is a pair $(\varphi_o, \varphi_n)$ that solves equations (12) and (27).

In section C in the Appendix we show that an equilibrium exists under standard conditions, and that uniqueness is ensured with a sufficiently large $f_o$. The model does not assume any particular distributions for productivity, $G(\varphi)$, or the adjustment factor, $F(\eta)$, and thus, we rely on a condition on $f_o$ that is easy to satisfy and that allows us to keep the model’s generality.

### 3.5 Numerical Example

This section presents a numerical example that summarizes the model. Following Bergin and Feenstra (2000, 2001), I set the parameter of substitutability among varieties, $\gamma$, at 2. As mentioned in section 3.1.1, the domestic wage is 1. The parameter of preference for differentiated goods, $\psi$, is set at 0.5. As $\psi$ is equivalent to the total expenditure in differentiated goods, the assumed value implies that the representative household spends 50% of its income on differentiated goods. I assume $\kappa = 0.5$, which means that every offshoring firm ships abroad half of its production process.

The wage in the foreign country, $w^*$, is set at 0.5 and the iceberg offshoring cost, $\lambda$, is set to 1.3. The values for $\kappa$, $w^*$, and $\lambda$ imply that $w_o = 0.825$—the marginal cost for each firm is 17.5% lower if it offshores. The value of the sunk entry cost, $f_E$, is set at 1. For the death rate, $\delta$, we use the annual rate of job destruction in the U.S. estimated by Davis, Faberman and Haltiwanger (2006), which is 0.14.\(^{16}\)

The empirical productivity distributions described in the literature review in section 2 are bell-shaped and therefore, I assume that firm productivity is lognormally distributed with parameters $m$ and $v$; that is, the probability density function, $g(\varphi)$, is given by

$$g(\varphi) = \frac{1}{\varphi \sqrt{2\pi v}} e^{-\frac{(\ln \varphi - m)^2}{2v^2}},$$

and $G(\varphi)$ is the corresponding cumulative distribution function.\(^{17}\) I target the mean of the productivity distribution assumed by Ghironi and Melitz (2005) and set $m$ to 0.2 and $v$ to 0.16.

---

\(^{16}\)Job destruction is the result of both firms dying and firms contracting. Hence, the assumed value of $\delta$ implies a time frame that is longer than a year, which is appropriate for our steady-state model (the “periods” in the model should be sufficiently large to allow for firm exit and entry after changes in the competitive environment or, as we will see in section 5, after episodes of trade liberalization).

\(^{17}\)Combes et al. (2012) find that the productivity distribution of French firms is, by far, best approximated by a lognormal distribution than by a Pareto distribution. After fitting a mixture of the lognormal and Pareto distributions, they find that the empirical TFP distribution of French firms is 95% lognormal and only 5% Pareto.
Regarding the offshoring adjustment costs in (14), I assume that the adjustment factor, $\eta$, follows a log-logistic distribution; that is,

$$f(\eta) = \frac{\beta\alpha^\beta\eta^{\beta-1}}{(\alpha^\beta + \eta^\beta)^2} \quad \text{and} \quad F(\eta) = \frac{\eta^\beta}{\alpha^\beta + \eta^\beta}$$

with support $\eta \in [0, \infty)$, $\alpha > 0$, and $\beta > 0$. The median of $\eta$ is $\alpha$ and its variance is decreasing in $\beta$ (the variance approaches zero as $\beta \to \infty$, in which case the mean approaches $\alpha$).$^{18}$ I choose $\alpha = 2$ and $\beta = 6$ so that $E(\eta) = 2.09$ and $\text{var}(\eta) = 0.45$. Lastly, I set $\rho$ to 1 and $f_o$ to 0.184 and thus, the expected offshoring adjustment cost for a firm with productivity $\varphi$ is

$$E[A(\varphi)] = 2.09\pi_n(\varphi) + 0.38$$

(about twice its per-period non-offshoring profits plus costs unrelated to productivity that are about 40 percent of entry costs). The value of $f_o$ was chosen so that 30 percent of producing firms offshore in equilibrium.$^{19}$

Table 1 shows the solution for this numerical example. For comparison purposes, Table 1 also includes the autarky (no offshoring) solution, with autarky outcomes denoted with superscript "a".

### Table 1: Model’s numerical example

<table>
<thead>
<tr>
<th>Productivity</th>
<th>Prices</th>
<th>Markups</th>
<th>Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi_o$</td>
<td>0.486</td>
<td>$\hat{p}$</td>
<td>1.699</td>
</tr>
<tr>
<td>$\varphi_n$</td>
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<td>$\bar{p}$</td>
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<td>$\bar{p}_o$</td>
<td>0.998</td>
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<tr>
<td>$\bar{\varphi}_n$</td>
<td>1.355</td>
<td>$\bar{p}_n$</td>
<td>1.150</td>
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<tr>
<td>$\bar{\varphi}_E$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>1.352</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\varphi}_E$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi^n$</td>
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<td>$\bar{p}^n$</td>
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</tr>
<tr>
<td>$\bar{\varphi}^n$</td>
<td>1.350</td>
<td>$\bar{p}^n$</td>
<td>1.148</td>
</tr>
</tbody>
</table>

Note that the average productivity levels follow the same order as the cutoff levels: $\varphi_o < \varphi_n$ and $\bar{\varphi}_o < \bar{\varphi}_n$. Hence, contrary to a heterogeneous-firm model with homogeneous fixed costs of offshoring, in this model the average productivity of offshoring firms can be lower than the average productivity of non-offshoring firms. This result can be reversed (i.e., obtain $\varphi_o < \varphi_n$ and $\bar{\varphi}_o > \bar{\varphi}_n$)

---

$^{18}$The shape of the log-logistic distribution is very similar to the shape of the lognormal distribution. I use the log-logistic distribution for $\eta$ (instead of the lognormal) because it considerably reduces the computational burden when solving for the equilibrium (the conditional expectation for $\eta$ in equation (27) makes the problem computationally intensive).

$^{19}$According to Bernard et al. (2007), the proportion of U.S. manufacturing firms involved in importing activities in 1997 was 14%. On the other hand, Tomiura (2007) reports that in 1998 only 5% of manufacturing firms in Japan were involved in offshoring activities, while Kohler and Smolka (2012) report that 44.6% of Spanish firms (884 firms out of a sample of 1984 firms) engaged in offshoring during 2007. The 30 percent of offshoring firms targeted in this numerical example is the midpoint between the U.S. and Spanish numbers, while also considering that the share of U.S. offshoring firms has almost surely increased during the 2000s.
for larger levels of $f_o$, as the offshoring adjustment cost unrelated to the firm’s productivity level, $\eta f_o$, is the main driver in the offshoring decisions of low productivity firms.

There is another important point to make with respect to average productivities. The general finding in the empirical literature is that firms engaging in international activities are on average more productive than purely-domestic firms. For example, based on TFP measures (i) Bernard et al. (2007) find that U.S. manufacturing firms that import were on average 12% more productive than non-importing firms, while firms that do both export and import were on average 7% more productive; (ii) for Japan’s manufacturing firms, Tomiura (2007) finds that importing firms are on average 5.8% more productive, while the premium is 27% for importing/exporting firms; (iii) for Danish manufacturing firms, Smeets and Warzynski (2013) obtain premiums of 10% for importing firms, and 16% for importing/exporting firms; (iv) for Spain, Fariñas and Martín-Marcos (2010) report that importing firms are on average 7% more productive. In the previous paragraph we compared the average primitive (or ex-ante) productivities for offshoring and non-offshoring firms and obtained the opposite result; however, for offshoring firms, $\bar{\varphi}_o$ does not represent the average effective (or ex-post) productivity. As mentioned in section 2, the average effective productivity of offshoring firms incorporates the decline in marginal costs due to offshoring—the so-called productivity effect of offshoring.

Denoting by $\varphi^E$ the effective productivity of an offshoring firm with primitive productivity $\varphi$, it follows that $\varphi^E = \frac{\varphi}{w_o}$ (this is the inverse of the offshoring firm’s marginal cost—see section 3.1.2). Therefore, using $\bar{\varphi}_o^E$ to denote the average effective productivity of offshoring firms, it follows that $\bar{\varphi}_o^E = \frac{\varphi_o}{w_o}$. Comparing $\bar{\varphi}_o^E$ and $\bar{\varphi}_n$ we can see that the average effective productivity of offshoring firms is 20.4% higher than the average productivity of non-offshoring firms. Hence, in this example—and in contrast to the long-held (high-productivity) self-selection view—offshoring firms are on average (effectively) more productive than non-offshoring firms precisely because they offshore, not the other way around. As mentioned before, using Danish firm-level data, Smeets and Warzynski (2013) provide empirical evidence on this type of outcome.

To shed more light on this result, Figure 3 shows the productivity distribution for producing non-offshoring firms (solid line), along with the primitive (dotted line) and effective (dashed line) productivity distributions for offshoring firms. The probability density function for producing non-offshoring firms is $h_n(\varphi \mid \varphi \geq \varphi_n)$—recall that non-offshoring firms whose productivities are below $\varphi_n$ do not produce. For offshoring firms, $h_o(\varphi)$ is the density function for their primitive productivity, and we use $h_o^E(\varphi^E)$ to denote the density function for their effective productivity. To obtain $h_o^E(\varphi^E)$ we only need to apply a change of variable on $h_o(\varphi)$ in equation (19). Given that $\varphi^E = \frac{\varphi}{w_o}$, it follows that $h_o^E(\varphi^E) = w_o h_o(w_o \varphi^E)$.20

\[20\]
first-order stochastically dominates the productivity of producing non-offshoring firms (the pdf and cdf of the former are always located to the right of the latter’s pdf and cdf), with density functions that look very similar to the empirical distributions obtained by Tomiura (2007) (for Japan) and Antrás and Yeaple (2014) (for Spain) and summarized in Figure 1b. Ex-ante, however, the distributions have a very similar median, the density functions intersect a couple of times, and the cumulative functions are close to each other.

Comparing primitive and effective productivities of offshoring firms, note that the offshoring productivity effect shifts to the right and dilates the productivity distribution. The dilation observed in $h_o^E(\varphi)$ when compared to $h_o(\varphi)$ occurs because in this model, the offshoring productivity effect is larger (in absolute terms) for ex-ante more productive firms.

The average primitive productivity of all firms, $\bar{\varphi}$, is 1.352, while the average effective productivity of all firms, $\bar{\varphi}^E = \frac{N_n}{N} \bar{\varphi}_n + \frac{N_o}{N} \bar{\varphi}_o^E$, is 1.437 (6.3% higher). Compared to the autarky average productivity, $\bar{\varphi}^a = 1.35$, note that opening to offshoring yields almost no gains on the average primitive productivity; hence, almost all gains in the economy’s effective productivity when moving from autarky to offshoring are due to the offshoring productivity effect.

The rest of the benchmark results in Table 1 concern prices, markups, market shares, and number of firms. As a consequence of offshoring, $\hat{p}$ and $\bar{p}$ are below their autarky levels, $\hat{p}^a$ and $\bar{p}^a$. The average price of offshoring firms, $\bar{p}_o$, is smaller than the average price of non-offshoring firms.

21For example, with $w_o = 0.825$, the productivity jumps to 0.61 for a new offshoring firm with a primitive productivity level of 0.5, while it jumps to 2.42 for an offshoring firm with a primitive productivity level of 2. The study of Combes et al. (2012) on selection and agglomeration effects of large cities provides an illustrative guide for the interpretation of truncation, right-shifting, and dilation of firm-level productivity distributions.
Nevertheless, even with lower average prices, offshoring firms obtain higher average markups than non-offshoring firms: $\bar{\mu}_o > \bar{\mu}_n$. For the masses of firms, note that although offshoring firms ($N_o$) represent about 30% of the total mass of producing firms ($N$), their market share ($\sigma_o$) is 36%. That is, offshoring firms capture a larger part of the market through their lower prices, limiting the number of competitors—this is also reflected in the decline in $N$ when moving from autarky to offshoring.

Lastly, Figure 4 shows the offshoring probability for non-offshoring firms, $\Lambda(\varphi)$, along with the proportion of offshoring firms for each level of (primitive) productivity, $\Gamma(\varphi)$. Note that $\Gamma(\varphi)$ is a scaled function of $\Lambda(\varphi)$; indeed, taking the derivative of (18) we get $\text{sgn}[\Gamma'(\varphi)] = \text{sgn}[\Lambda'(\varphi)]$. In the plot, non-offshoring firms with a productivity of 1.18 have the greatest offshoring probability each period (about 8.42%), and among all the firms with that level of productivity, about 39.65% are offshoring. Importantly, in this example the firms with the highest offshoring incentives have a productivity level that is below the average productivity of non-offshoring firms, $\bar{\varphi}_n$.

### 3.5.1 Remarks on the Type of Offshoring Adjustment Costs

In this model, adjustment costs of offshoring are expected to be higher for larger (and more productive) firms. This assumption—along with the endogenous-markup structure—implies an inverted-U relationship between firm-level productivity and offshoring likelihood, generating productivity distributions for offshoring and non-offshoring firms that share strong similarities to those observed for some countries. It may be argued, however, that the observed empirical distributions may be simply the result of random fixed costs of offshoring that are independent of productivity; that is, the result of fixed costs of offshoring that are homogeneous in expectation. In our model this would
be equivalent to assuming that $\rho = 0$.

This section uses the numerical example to illustrate the differences of our framework with $\rho > 0$ against the case with $\rho = 0$. Note from (14) that with $\rho = 0$, the offshoring adjustment cost collapses to $A(\varphi) = A$ for every $\varphi$, where $A \equiv \eta f_o$, $E(A) = E(\eta)f_o$, and $\text{var}(A) = \text{var}(\eta)f_o^2$. When $\text{var}(\eta) \to 0$, the productivity distributions of offshoring and non-offshoring firms get very close to the truncated and separated distributions in Figure 1a. Moreover, with $\rho = 0$ the probability of offshoring, $\Lambda(\varphi)$, is strictly increasing in productivity for $\varphi \geq \varphi_o$.

Table 2 compares the $\rho = 0$ case with the benchmark example along two important outcomes: the fraction of offshoring firms, and the average productivity differences between offshoring and non-offshoring firms. In addition to solving the model for $\rho = 0$, we also explore the outcomes for different levels of $f_o$ and $\beta$—either a higher $f_o$ or a lower $\beta$ increases both $E(A)$ and $\text{var}(A)$. A bold parameter indicates a difference with respect to the benchmark value.

<table>
<thead>
<tr>
<th></th>
<th>Bench</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tr>
<td>$\rho$</td>
<td>1</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$f_o$</td>
<td>0.184</td>
<td>0.184</td>
<td>0.184</td>
<td>0.330</td>
<td>0.487</td>
</tr>
<tr>
<td>$\beta$</td>
<td>6.000</td>
<td>6.000</td>
<td>2.500</td>
<td>6.000</td>
<td>3.000</td>
</tr>
<tr>
<td>$E(A)$</td>
<td>0.384</td>
<td>0.485</td>
<td>0.691</td>
<td>1.178</td>
<td></td>
</tr>
<tr>
<td>$\text{var}(A)$</td>
<td>0.015</td>
<td>0.341</td>
<td>0.049</td>
<td>0.907</td>
<td></td>
</tr>
<tr>
<td>$N_o/N$</td>
<td>0.300</td>
<td>0.649</td>
<td>0.696</td>
<td>0.301</td>
<td>0.301</td>
</tr>
<tr>
<td>$\bar{\varphi}_o/\bar{\varphi}_n$</td>
<td>0.993</td>
<td>1.432</td>
<td>1.238</td>
<td>1.438</td>
<td>1.282</td>
</tr>
<tr>
<td>$\bar{\varphi}_{oE}/\bar{\varphi}_n$</td>
<td>1.204</td>
<td>1.736</td>
<td>1.500</td>
<td>1.743</td>
<td>1.554</td>
</tr>
</tbody>
</table>

In the benchmark case 30% of producing firms offshore, the average primitive productivity of offshoring firms is 0.7% lower than the average productivity of producing non-offshoring firms, but the effective average productivity of offshoring firms is 20.4% higher. To begin the comparison, column (1) shows the model’s outcome when $\rho = 0$, keeping all the other parameters identical. In that case the expected offshoring adjustment costs are lower for every firm with $\varphi \geq \varphi_n$ and hence, the fraction of offshoring firms increases to 64.9%. As more productive firms are more likely to offshore, the average primitive and effective productivities of offshoring firms are, respectively, 43.2% and 73.6% higher than the average productivity of non-offshoring firms. Given that the differences in average productivities are too high when compared to those found in previous studies for the U.S., Japan, Denmark, and Spain, in column (2) we assume a lower $\beta$ to increase $\text{var}(A)$ so that more low-productivity firms offshore. Although this drives down that average productivity gaps to 23.8% and 50% (which are still high), the fraction of offshoring firms rises to 69.6%.

In columns (3) and (4) we target instead the fraction of offshoring firms to the benchmark level, 30%. Column (3) looks for the level of $f_o$ that reaches the target, while keeping $\beta$ at its benchmark
level. In this case the productivity gaps between offshoring and non-offshoring firms are the highest in Table 2: 43.8% (primitive) and 74.3% (effective). Column (4) alters both $f_o$ and $\beta$: a lower $\beta$ helps to drive down the productivity gaps but increases the fraction of offshoring firms, and thus, $f_o$ must increase to maintain the 30% target. The average productivity gaps remain very high at 28.2% (primitive) and 55.4% (effective), but also, the expected offshoring adjustment cost is higher than the entry cost and the variance of $A$ is 18.5 times higher than in column (3). Figure 5 shows the productivity distributions for the case in column (4). The primitive and effective distributions of offshoring firms show substantial dispersion (and hence a very low peak) when compared with the distribution of non-offshoring firms. In contrast, the density functions for offshoring and non-offshoring firms obtained by Antrás and Yeaple (2014) for Spain and Tomiura (2007) for Japan present similar degrees of dispersion.

To sum up, although assuming random adjustment costs of offshoring that are homogeneous in expectation can generate bell-shaped and overlapping distributions of offshoring and non-offshoring firms, it is very difficult to match simultaneously reasonable levels for (i) the fraction of offshoring firms and (ii) average productivity differences. Moreover, as we attempt to get closer to reasonable targets, the implied density functions show substantially different degrees of dispersion.

4 Competitive Environment and Offshoring Likelihood

This section discusses the model’s implications for the effects of changes in the competitive environment on firms’ offshoring decisions. I begin by defining what a tougher competitive environment is and then I identify some of the parameters whose changes create a tougher environment. I con-
tinue by discussing the effects of competition on offshoring probability: the selection effect and the escape-competition effect. Lastly, I continue with the numerical exercise from above.

4.1 A Tougher Competitive Environment

This paper takes advantage of the endogenous-markup structure of the model to define in a simple and intuitive way a tougher competitive environment.

Definition 2. A competitive environment is said to be tougher if every producing firm that keeps the same offshoring status is forced to reduce its markup. That is, if \( \mu_n(\varphi) \) declines for \( \varphi \geq \varphi_n \), and \( \mu_o(\varphi) \) declines for \( \varphi \geq \varphi_o \).

From equation (11) we know that the markup of a firm with offshoring status \( s \) and productivity \( \varphi \geq \varphi_s \) is given by \( \mu_s(\varphi) = \Omega \left( \frac{\varphi}{\varphi_s} \right) - 1 \), for \( s \in \{n,o\} \). From the properties of the Lambert \( W \) function mentioned in section 3.1.2, it follows that this firm’s markup declines if and only if \( \varphi_s \) increases. Hence, we write the following corollary to the definition above.

Corollary. A tougher competitive environment occurs if and only if both \( \varphi_o \) and \( \varphi_n \) increase.

We consider three parameters whose changes can cause a tougher environment: the parameter of substitutability between varieties, \( \gamma \), the total expenditure on differentiated goods, \( \psi \), and the parameter of adjustment costs unrelated to productivity, \( f_o \). Let \( \zeta_\gamma \) denote the elasticity of \( \varphi_o \) with respect to \( \gamma \); i.e., \( \zeta_\gamma = \frac{d\ln \varphi_o}{d\ln \gamma} \). As \( \varphi_o = w_o \varphi_n \) and \( w_o \) does not depend on \( \gamma \), it follows that \( \zeta_\gamma \) is also the elasticity of \( \varphi_n \) with respect to \( \gamma \). Similarly, let \( \zeta_\psi \) and \( \zeta_{f_o} \) denote, respectively, the elasticities of \( \varphi_o \) (and \( \varphi_n \)) with respect to \( \psi \) and \( f_o \). The following lemma describes how changes in these parameters alter the competitive environment.

Lemma 2. A tougher competitive environment occurs if either \( \gamma \) increases (\( \zeta_\gamma > 0 \)), or \( \psi \) increases (\( \zeta_\psi > 0 \)), or \( f_o \) declines (\( \zeta_{f_o} < 0 \)).

An increase in \( \gamma \) creates a tougher competitive environment because goods become less differentiated to the eyes of the consumers. Compared to a CES setting, an increase in \( \gamma \) is equivalent to an increase in the (CES) elasticity of substitution between varieties. An increase in \( \psi \) represents an increase in the size of the differentiated-good market. As first shown by Melitz and Ottaviano (2008) in a heterogeneous-firm model with endogenous markups, a larger market induces more entry and hence the environment becomes tougher. Lastly, a decline in \( f_o \) makes offshoring more attractive and increases the value of entry. As with the increase in \( \psi \), more entry creates a tougher environment and causes a downward pressure on markups.
4.2 The Selection and Escape-Competition Effects

The net effect of a change in the competitive environment on a firm’s offshoring likelihood is the result of two opposing forces. I refer to the negative effect of competition on offshoring probability as the selection effect, and to its positive effect as the escape-competition effect. The magnitude of these effects varies according to each firm’s productivity, and there exists a cutoff level that separates firms according to the dominant effect. The following proposition describes this result.

**Proposition 3. (The offshoring probability in a tougher competitive environment)**

For a tougher competitive environment (driven either by an increase in $\gamma$ or $\psi$, or by a decline in $f_o$) there is a unique productivity level, $\varphi^*$, such that the offshoring probability, $\Lambda(\varphi)$, declines if $\varphi < \varphi^*$, and increases if $\varphi > \varphi^*$. Hence, the selection effect dominates if $\varphi < \varphi^*$, and the escape-competition effect dominates if $\varphi > \varphi^*$.

To understand Proposition 3 and the intuition behind each of the effects, I analyze carefully the case of an increase in $\gamma$. The offshoring probability of a firm with productivity $\varphi$ is given by $\Lambda(\varphi) = F[\hat{\eta}(\varphi)]$, where $\hat{\eta}(\varphi)$ is given by the solution to equation (16). Hence, using Leibniz’s rule we get

$$\frac{d\Lambda(\varphi)}{d\gamma} = \left[ \frac{f[\hat{\eta}(\varphi)]}{\delta + (1-\delta)\Lambda(\varphi)} \right] \frac{dz(\varphi)}{d\gamma}. \hspace{1cm}(28)$$

Given that the term in brackets is positive, the negative or positive response of $\Lambda(\varphi)$ to a change in $\gamma$ is entirely determined by the response of $z(\varphi)$. Thus, we can understand the selection and escape-competition effects by looking only at $z(\varphi)$.

Recall from (17) that $z(\varphi) = \frac{\pi_s(\varphi)-\pi_o(\varphi)}{\rho\pi_o(\varphi)+f_o}$. Let us assume that an increase in $\gamma$ not only reduces $\mu_n(\varphi)$ and $\mu_o(\varphi)$, but also reduces $\pi_n(\varphi)$ and $\pi_o(\varphi)$ (below I discuss cases in which $\pi_s(\varphi)$ increases in spite of $\mu_s(\varphi)$ declining). The decline in $\pi_o(\varphi)$ drives the selection effect, reducing $z(\varphi)$ through the numerator. On the other hand, the decline in $\pi_n(\varphi)$ drives the escape-competition effect through two channels: (i) it increases the numerator, which directly counteracts the selection effect; and (ii) it also reduces the denominator because the offshoring adjustment cost is expected to fall with the decrease in $\pi_n(\varphi)$—this is similar to the decline in the opportunity cost of disruptions in the model of Holmes, Levine and Schmitz (2012).

The selection effect—also referred to as the Schumpeterian or Darwinian effect (see, e.g., Aghion *et al.*, 2005; Syverson, 2011)—refers to the cleansing effect of competition and affects offshoring incentives of non-offshoring firms by reducing their potential offshoring profits. On the other hand, the escape-competition effect refers to the impact of increased competition on offshoring incentives based on the response of a normalized measure of the incremental profits from offshoring; that is, although a tougher competitive environment may decrease both non-offshoring and offshoring profits, the gap between them—normalized by offshoring adjustment costs—may increase. Intuitively, this effect makes offshoring more attractive as a mean to “escape” competition. A firm’s offshoring
probability declines if the selection effect dominates, and increases if the escape-competition effect dominates.

Let us study the interaction between selection and escape-competition effects for each level of productivity $\varphi$. Using $\pi_s(\varphi)$ from (9) and letting $\hat{\varphi}_s$ denote the productivity level such that $\mu_s(\hat{\varphi}_s) = \zeta_\gamma$, for $s \in \{n, o\}$, we obtain that

$$\frac{d\pi_s(\varphi)}{d\gamma} = \left[ \frac{\mu_s(\varphi) - \zeta_\gamma}{\mu_s(\varphi)} \right] \frac{\pi_s(\varphi)}{\gamma} \geq 0 \text{ if } \varphi \geq \hat{\varphi}_s,$$

and is less than zero otherwise; thus, there are some firms for which, in spite of declining markups, profits increase after an increase in $\gamma$. With $\mu_s(\varphi)$ strictly increasing for $\varphi \geq \varphi_s$, and given that $\mu_o(\varphi) > \mu_n(\varphi)$ for every $\varphi > \varphi_o$, it follows that $\hat{\varphi}_o < \hat{\varphi}_n$ must hold. Moreover, and to cover all possible cases, I assume that $\varphi_n < \varphi_o$. Therefore, after an increase in $\gamma$, a non-offshoring firm with productivity $\varphi$ falls into one of the following five cases: (i) $\varphi < \varphi_o$; (ii) $\varphi \in [\varphi_o, \varphi_n)$; (iii) $\varphi \in [\varphi_n, \hat{\varphi}_o)$; (iv) $\varphi \in [\hat{\varphi}_o, \hat{\varphi}_n)$; and (v) $\varphi \geq \hat{\varphi}_n$.

In case (i) $\varphi < \varphi_o$, the firm does not make positive profits even if it offshores and hence, none of the effects is present. In case (ii) $\varphi \in [\varphi_o, \varphi_n)$, the non-offshoring firm does not produce—it begins to produce if and only if it becomes an offshoring firm—and therefore, only the selection effect is present ($z(\varphi) = \pi_o(\varphi)/f_o$ and $\pi_o(\varphi)$ decreases with an increase in $\gamma$). In case (iii) $\varphi \in [\varphi_n, \hat{\varphi}_o)$, the non-offshoring firm produces and both effects are present; both $\pi_n(\varphi)$ and $\pi_o(\varphi)$ decrease with an increase in $\gamma$. In case (iv) $\varphi \in [\hat{\varphi}_o, \hat{\varphi}_n)$, $\pi_o(\varphi)$ increases with $\gamma$ and hence there is no selection effect; moreover, $\pi_n(\varphi)$ continues to decline and thus, there is a reinforced escape-competition effect. Lastly, in case (v) $\varphi \geq \hat{\varphi}_n$, both $\pi_n(\varphi)$ and $\pi_o(\varphi)$ increase with $\gamma$; there is a weak escape-competition effect, as the increase in $\pi_n(\varphi)$ has a negative effect on the firm’s incentives to offshore.

Notice that the selection effect weakens as productivity increases and stops being relevant if $\varphi \geq \hat{\varphi}_o$. Given that the escape-competition effect is absent if $\varphi \in [\varphi_o, \varphi_n)$, $\Lambda(\varphi)$ declines for these firms. As the escape-competition effect emerges while the other weakens, the threshold $\hat{\varphi}$ after which the escape-competition effect dominates must be between $\varphi_n$ and $\hat{\varphi}_o$. Also, for $\varphi \geq \hat{\varphi}_n$ the escape-competition effect weakens as $\pi_n(\varphi)$ stops declining, and thus, although the most productive firms have an increase in their offshoring likelihood, the probability increase is larger for lower productivity firms. For our numerical exercise below, for example, Figure 6a shows that after $\gamma$ increases, $\hat{\varphi}$ occurs close to $\varphi_n$ and well below the average productivity of non-offshoring firms, $\bar{\varphi}_n$, with the largest increase in the offshoring probability happening for firms around the mean.

Profits increase for some firms in spite of reductions in markups because the substitutability parameter magnifies the market shares of firms with lower prices: as differentiated goods become more homogenous ($i.e.$, $\gamma$ increases), price differences matter more and consumers move towards cheaper varieties. Recall from section 3.3 that the market share density of a firm with productivity
\( \varphi \) and offshoring status \( s \) is given by \( \sigma_s(\varphi) = \gamma \mu_s(\varphi) \). We can then obtain that

\[
\frac{d \ln \sigma_s(\varphi)}{d \ln \gamma} = 1 - \left[ \frac{1 + \mu_s(\varphi)}{2 + \mu_s(\varphi)} \right] \frac{\zeta_\gamma}{\mu_s(\varphi)}.
\]

Given that the term in brackets is always less than 1, note that \( \sigma_s(\varphi) \) increases with an increase in \( \gamma \) not only if \( \mu_s(\varphi) \geq \zeta_\gamma \), but also for some firms with markups smaller than \( \zeta_\gamma \). Hence, the decrease in markups due to an increase in the substitutability parameter allows a range of firms to capture larger market shares, and a subset of these firms are even able to increase their profits.

The case of an increase in \( \psi \) is very similar to an increase in \( \gamma \). We also obtain cases of declining markups and increasing profits. For this case, however, market shares decline. In spite of declining market shares and markups, profits increase for some firms because the market itself is larger; some firms are able to sell more output than before, and profits increase for a subset of these firms.

For the case of a decline in \( f_o \), note first from equation (17) that there is a direct positive effect on \( z(\varphi) - f_o \) is in the denominator of \( z(\varphi) \). Hence, in contrast to an increase in \( \gamma \), the escape-competition effect is present for every firm with productivity no less than \( \varphi_o \) (even if \( \pi_n(\varphi) = 0 \)).

Also, we obtain that for a firm with offshoring status \( s \) and productivity \( \varphi \geq \varphi_s \), both the markup and profits fall; that is, for \( \varphi \geq \varphi_s \), not only \( \frac{d \mu_s(\varphi)}{d f_o} > 0 \) but also \( \frac{d \pi_s(\varphi)}{d f_o} > 0 \). Therefore, for a decline in \( f_o \), a non-offshoring firm with productivity \( \varphi \) may be in one of the following three cases: (i) \( \varphi < \varphi_o \) (no effects are present); (ii) \( \varphi \in [\varphi_o, \varphi_n) \) (both effects are present, with the escape-competition effect driven by the direct effect of \( f_o \) on \( z(\varphi) \)); and (iii) \( \varphi \geq \varphi_n \) (both effects are present, with the escape-competition effect driven by both the direct effect of \( f_o \) and the indirect effect through the decline in \( \pi_n(\varphi) \)). Due to the direct effect of the decline of \( f_o \) on \( z(\varphi) \), \( \varphi \) may occur in \( (\varphi_o, \varphi_n) \). In the exercise below, note from Figure 6b that \( \varphi \) is so close to the new \( \varphi_o \) that the range for which the selection effect dominates is not even noticeable; in this case, the larger increases in the offshoring probability occur for firms with productivities well below the mean.

### 4.3 Numerical Example

We now continue with the numerical example from section 3.5 to review the model’s responses to a tougher competitive environment. Following Lemma 2, I consider an increase in the substitutability parameter, \( \gamma \), from 2 to 3, an increase in market size, \( \psi \), from 0.5 to 0.75, and a decline in the offshoring adjustment costs unrelated to productivity, \( f_o \), from 0.184 to 0.124. I choose the changes in \( \gamma \), \( \psi \), and \( f_o \) so that the fraction of offshoring firms, \( N_o/N \), increases to about 50 percent. The second, third, and fourth columns in Table 3 present the new steady-state solutions. Note that the second and third columns yield identical results, with the exception of the results for \( N \), \( N_o \), and \( N_n \). This is the case because \( \gamma \) and \( \psi \) enter the profit function as \( \gamma \psi \), and in our experiments \( \gamma \) and \( \psi \) change in the same proportion. Below we only refer to the effects of \( \psi \) when looking at the responses of the numbers of firms.
Table 3: Model’s responses to a tougher competitive environment

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Increase in $\gamma$ (to 3)</th>
<th>Increase in $\psi$ (to 0.75)</th>
<th>Decline in $f_o$ (to 0.124)</th>
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</thead>
<tbody>
<tr>
<td><strong>Productivity:</strong></td>
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<td></td>
<td></td>
<td></td>
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<tr>
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<td>0.993</td>
<td>1.026</td>
<td>1.026</td>
<td>0.928</td>
</tr>
<tr>
<td>$\bar{\varphi}_E/\bar{\varphi}_n$</td>
<td>1.204</td>
<td>1.244</td>
<td>1.244</td>
<td>1.125</td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>1.352</td>
<td>1.399</td>
<td>1.399</td>
<td>1.354</td>
</tr>
<tr>
<td>$\bar{\varphi}_E$</td>
<td>1.437</td>
<td>1.550</td>
<td>1.550</td>
<td>1.493</td>
</tr>
<tr>
<td><strong>Prices:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\varphi}$</td>
<td>1.699</td>
<td>1.382</td>
<td>1.382</td>
<td>1.676</td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>1.105</td>
<td>0.963</td>
<td>0.963</td>
<td>1.072</td>
</tr>
<tr>
<td>$\bar{\varphi}_o$</td>
<td>0.998</td>
<td>0.889</td>
<td>0.889</td>
<td>1.016</td>
</tr>
<tr>
<td>$\bar{\varphi}_n$</td>
<td>1.150</td>
<td>1.038</td>
<td>1.038</td>
<td>1.128</td>
</tr>
<tr>
<td><strong>Markups and shares:</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\bar{\mu}_o$</td>
<td>0.547</td>
<td>0.455</td>
<td>0.455</td>
<td>0.517</td>
</tr>
<tr>
<td>$\bar{\mu}_n$</td>
<td>0.417</td>
<td>0.310</td>
<td>0.310</td>
<td>0.426</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>0.360</td>
<td>0.599</td>
<td>0.599</td>
<td>0.552</td>
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<tr>
<td>$\sigma_n$</td>
<td>0.640</td>
<td>0.401</td>
<td>0.401</td>
<td>0.448</td>
</tr>
<tr>
<td><strong>Composition of firms:</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>1.097</td>
<td>0.871</td>
<td>1.306</td>
<td>1.060</td>
</tr>
<tr>
<td>$N_o$</td>
<td>0.329</td>
<td>0.439</td>
<td>0.659</td>
<td>0.534</td>
</tr>
<tr>
<td>$N_n$</td>
<td>0.768</td>
<td>0.432</td>
<td>0.647</td>
<td>0.526</td>
</tr>
<tr>
<td>$N_o/N$</td>
<td>0.300</td>
<td>0.505</td>
<td>0.505</td>
<td>0.504</td>
</tr>
<tr>
<td><strong>Dominant effect cutoff:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\varphi}$</td>
<td>—</td>
<td>0.853</td>
<td>0.853</td>
<td>0.530</td>
</tr>
</tbody>
</table>

From Lemma 2 we know that these changes cause a tougher competitive environment: $\varphi_o$ and $\varphi_n$ increase, which then implies lower markups for producing firms that do not change their offshoring status. For the change in $f_o$, however, the increases in the cutoff levels are minimal. In spite of this, Table 3 shows that changes in $f_o$ cause large steady-state changes in several outcomes.

For this example, the increase in $\gamma$ changes the ordering of average productivities to $\bar{\varphi}_n < \bar{\varphi}_o < \bar{\varphi}_E$. This is driven by increases in $\bar{\varphi}_o$ and $\bar{\varphi}_E$ that are proportionally larger than the increase in $\bar{\varphi}_n$. Conversely, for the decline in $f_o$, $\bar{\varphi}_o$ and $\bar{\varphi}_E$ fall, but $\bar{\varphi}_n$ increases. This happens because $\eta f_o$ is the main driver of offshoring decisions for low-productivity firms (and the only driver for those firms between $\varphi_o$ and $\varphi_n$); therefore, the decline in $f_o$ causes a large increase in the number of low-productivity firms that begin to offshore, causing a decline in the average productivity of
offshoring firms. Though the ordering of the benchmark case prevails, $\tilde{\varphi}_n$ is now 7.2% higher than $\tilde{\varphi}_o$, and $\tilde{\varphi}_o^E$ is now only 12.5% higher than $\varphi_n$.

The maximum price that firms can set, $\bar{p}$, the overall average price, $\bar{p}$, and the average price of non-offshoring firms, $\bar{p}_n$, decline in all cases. Meanwhile, the average price of offshoring firms, $\bar{p}_o$, falls for the increase in $\gamma$, but increases for the decline in $f_o$. The last result does not mean that offshoring firms are increasing their prices after $f_o$ declines, but is a consequence of the change in the composition of offshoring firms towards less productive firms. In all cases, $\bar{p}_o$ remains below $\bar{p}_n$.

In all cases, the average markup of offshoring firms, $\bar{\mu}_o$, declines. The average markup of non-offshoring firms, $\bar{\mu}_n$, declines for the increase in $\gamma$, but increases for the decline in $f_o$. For the last case, the increase in $\bar{\mu}_n$ in a tougher competitive environment may seem puzzling given that (i) the markup of each surviving firm that does not change its offshoring status declines, and (ii) the average price of non-offshoring firms also declines. As with the increase in $\bar{p}_o$, the increase in $\bar{\mu}_n$ is the result of a strong composition effect; in particular, the new $\bar{\mu}_n$ no longer includes low-productivity firms that begin to offshore (nor those that stop producing). Therefore, this model suggests that in the presence of composition effects, average markups may not be good indicators of the level of competition in a market.

The total mass of producing firms, $N$, declines after the increase in $\gamma$ and after the decline in $f_o$; hence, a tougher competitive environment is not necessarily associated with a larger mass of competitors. In contrast, $N$ increases with $\psi$, as more firms are needed to meet the needs of a larger market—recall that the effects of proportional changes in $\gamma$ and $\psi$ only differ with respect to the number of firms. For the components of $N$, note that in every case the mass of offshoring firms, $N_o$, increases while the mass of non-offshoring firms, $N_n$, declines, which then drives up the proportion of offshoring firms (along with their market share, $\sigma_o$).

For non-offshoring firms, we now look into the effects of a tougher competitive environment on the probability of offshoring, $\Lambda(\varphi)$. For each of the shocks considered in this numerical example, the last row in Table 3 shows $\tilde{\varphi}$, which is the productivity level that separates non-offshoring firms according to the dominant competition effect. From Proposition 3, we know that the offshoring probability declines for firms to the left of $\tilde{\varphi}$—where the selection effect dominates—and increases for firms to the right of $\tilde{\varphi}$—where the escape-competition effect dominates. For the increase in $\gamma$ (and $\psi$), the value for $\tilde{\varphi}$ in Table 3 is greater than the new steady-state value for $\varphi_n$. On the other hand, for the decline in $f_o$, $\tilde{\varphi}$ is between the new $\varphi_o$ and the benchmark steady-state level for $\varphi_n$. Hence, the range of $\varphi$ for which the escape-competition effect dominates is wider in the case of the decline in $f_o$.

As a graphical description of Proposition 3, Figure 6 presents the offshoring probability functions.

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22In other exercises we can also obtain more concentration even when the market size is bigger (i.e., a decline in $N$ after an increase in $\psi$), as new and existing offshoring firms replace and steal market share away from non-offshoring firms.
in the benchmark steady state, $\Lambda(\varphi)$, and in the new steady state, $\Lambda(\varphi)'$. Figure 6a shows the case of the increase in $\gamma$ or $\psi$, and Figure 6b shows the case of a decline in $f_o$. Note that the range of $\varphi$ for which the selection effect dominates in Figure 6b is not even noticeable, as opposed to the equivalent range in Figure 6a. Intuitively, and as seen in section 4.2, this difference arises because for the case of an increase in $\gamma$ (or $\psi$), the selection effect is the only effect present for firms with productivities below $\varphi_n$, while the escape-competition effect is present for the case of a decline in $f_o$ as long as the productivity of a firm is no less than the new $\varphi_o$.

Lastly, Figure 7 shows the effects of the increase in $\gamma$ (or $\psi$) and the decline in $f_o$ on the effective productivity distributions of offshoring and non-offshoring firms. The plots on the left are the probability density functions, $h_n(\varphi \mid \varphi \geq \varphi_n)$ and $h_o^E(\varphi^E)$, and the plots on the right are the cumulative distribution functions, $H_n(\varphi \mid \varphi \geq \varphi_n)$ and $H_o^E(\varphi^E)$. The solid lines are the functions for non-offshoring firms, and the dashed lines are functions for offshoring firms, with the thin light lines showing the benchmark functions from Figure 3.

For the increase in $\gamma$ or $\psi$, the distribution of offshoring firms shifts to the right as a result of both the selection and escape-competition effects: the increase in $\varphi_o$ automatically shifts the distribution to the right (selection), and in addition, note from Figure 6a that the firms that are more likely to offshore are more productive than before (the productivity level that maximizes $\Lambda(\varphi)$—and hence the fraction of offshoring firms, $\Gamma(\varphi)$—shifts to the right). On the other hand, the selection and escape-competition effects move in opposite directions for producing non-offshoring firms; the increase in $\varphi_n$ shifts the distribution to the right, but as more productive firms shift more towards offshoring, non-offshoring firms cluster towards low-productivity levels. In the end,
the cumulative functions in Figure 7a show a widening productivity gap between offshoring and non-offshoring firms along the distribution.

Meanwhile, the opposite happens after the decline in $f_o$. Given that the cutoff levels barely change, selection effects (which would shift the distributions to the right) are negligible. On the other hand, the escape-competition effect reduces the productivity gap between offshoring and non-offshoring firms along the entire distribution. This happens because the decline in $f_o$ has larger effects on lower productivity firms—note from Figure 6b that the productivity level that maximizes $\Lambda(\varphi)$ shifts to the left—which reduces the fraction of low-productivity non-offshoring firms and increases the fraction of low-productivity offshoring firms.
5 The Model with Trade in Final Goods

The model in the previous sections abstracts from trade in final goods in order to highlight the process of firms’ offshoring decisions, and how this process is affected by changes in the competitive environment. As mentioned in section 2, however, there is a growing theoretical and empirical literature on the effects of trade liberalization on productivity-enhancing innovation decisions. Along these lines, we extend the model to allow for trade in final goods to study the effects of final-good trade liberalization on (productivity-enhancing) offshoring decisions.

In addition, the model allows us to compare the effects of final-good trade liberalization with the effects of a decline in the variable cost of offshoring, \( \lambda \). The latter can be referred to as trade liberalization in inputs. We study how each type of trade liberalization affects the competitive environment in the domestic and export markets, and verify whether there are complementarities between exporting and offshoring decisions.

There are two countries, North and South, each of which is inhabited by a continuum of households in the unit interval. There are two sectors in each country: a homogeneous-good sector and a heterogeneous-good sector. The homogeneous good, which is the numéraire, is produced under perfect competition, while heterogeneous goods are produced under monopolistic competition. Labor is the only factor of production and is perfectly mobile across sectors (but not across countries), with each household providing one unit of labor each period. South variables are denoted with a star (*). Here we briefly describe the model, leaving most of the technical details for the Appendix.

5.1 Setup

The description of preferences for the North is identical to section 3.1.1. Similar preferences hold for the representative household in the South. There is, however, an important difference between the North and the South. As in the offshoring model with fixed wages of Antràs and Helpman (2004), we assume that South workers are less productive in the production of the homogeneous (and numéraire) good; in particular, one unit of South labor produces only \( w^* < 1 \) units of the homogeneous good. Therefore, assuming a positive production of the homogeneous good in both countries (ensured with large enough labor supplies), the wage in the South is \( w^* \).

Given Cobb-Douglas preferences between homogeneous- and heterogeneous-good consumption, \( U^* = q^h (1-\psi) Q^\psi w^* \), it follows that the representative South household spends a fraction \( \psi \) of its income on differentiated goods. Thus, its total expenditure on differentiated goods is given by \( \psi w^* \).

\(^{23}\)The fixed-wages assumption is innocuous as long as there are large and persistent wage differentials between the North and South. This is reasonable, for example, for the relationship between the U.S. (the North) and two of its three main trade partners, Mexico and China (the South). According to the International Labor Comparisons of the BLS, hourly compensation costs in manufacturing as a percent of costs in the U.S. were 15% in 1997 and 18% in 2012 for Mexico (see Table 1 in http://www.bls.gov/fls/ichcc.pdf), and ranged between 2% and 5% for China from 2002 to 2009 (see http://www.bls.gov/ilc/china.htm).
South household’s demand for differentiated good $i$ is then

$$q_i^s = \sigma_i^s \frac{\psi w^*}{p^*_i},$$

where $\sigma_i^s = \gamma \ln \left( \frac{\hat{p}^*}{p_i^*} \right)$ is the market share of variety $i$ in total South expenditure on differentiated goods, and $\hat{p}^*$ denotes the maximum price that differentiated-good firms can set in the South.

There is trade in both homogeneous and heterogeneous goods. As in the models of Chaney (2008), Helpman and Itskikhok (2010), and Helpman, Melitz and Yeaple (2004), international trade in the homogeneous good is costless, which implies balanced trade. Each differentiated-good variety is potentially tradable. After entry, each North and South differentiated-good firm draws its productivity from the common cumulative distribution function $G(\varphi)$. I also assume that firms in the differentiated-good sector are heterogeneous with respect to their exporting costs, with each of them drawing an iceberg cost parameter, $\tau \geq 1$, from a cumulative distribution function $M(\tau)$; an exporting firm must ship $\tau$ units of the good for one unit to reach the foreign market. The assumption of heterogeneous exporting costs accounts for the substantial coexistence of low-productivity exporters and high-productivity non-exporters documented by Bernard et al. (2003), Hallak and Sivadasan (2013), Melitz and Trefler (2012), among others.

Thus, on entry each differentiated-good firm in the North and South is defined by the pair $(\varphi, \tau)$, which remains constant over the firm’s lifetime.

North firms can offshore a part of their production process to take advantage of the South’s lower cost of labor. The production function of a North firm with productivity $\varphi$ and offshoring status $s$ is again given by $y_s(\varphi) = \varphi L_s$, where $L_s$ is defined as in (8) and $s \in \{n, o\}$. As before, the price of $L_s$ is $w_s$, with $w_n = 1$, $w_o = 1 - \kappa + \kappa \lambda w^*$, and $w_o < w_n$. Based on its offshoring status $s$ and the pair $(\varphi, \tau)$, a North firm decides whether or not to produce for the domestic $(D)$ and export $(X)$ markets. In particular, a North firm with the pair $(\varphi, \tau)$ and offshoring status $s$ produces for the domestic market if $\varphi \geq \varphi_{D,s}$, and produces for the export market if $\varphi \geq \tau \varphi_{X,s}$. The cutoff $\varphi_{D,s}$ has a similar interpretation to $\varphi_s$ in the model without final-good trade. On the other hand, $\varphi_{X,s}$ denotes the exporting cutoff level for a North firm with status $s$ and $\tau = 1$, so that no firm with status $s$ and productivity level below $\varphi_{X,s}$ exports. Equivalently, for the set of North firms with productivity $\varphi$ and status $s$, those with $\tau \leq \varphi / \varphi_{X,s}$ are exporters, and those with $\tau > \varphi / \varphi_{X,s}$ are non-exporters: the higher $\varphi$ is, the higher the $\tau$ that firms are willing to accept to become exporters.

South firms only employ South labor (they do not have incentives to offshore), with the production function of a South firm with productivity $\varphi$ given by $y^*(\varphi) = A^* \varphi L^*$, where $A^*$ is an

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24 While I assume heterogeneity in iceberg costs, other papers assume heterogeneity in fixed exporting costs to account for the overlapping distributions of exporters and non-exporters (see, e.g., Helpman et al., 2014). As in the model of section 3, one of the features of this model is that it does not need to assume fixed costs of accessing each market to pin down the productivity thresholds. Instead, the cutoff levels are naturally obtained from $\hat{p}$ and $\hat{p}^*$—the upper bounds for the prices that firms can set in the North and South. Thus, we abstract from fixed costs of exporting and rely on heterogeneity across iceberg costs. Adding fixed costs of production or exporting only complicates the model without providing further insights.
aggregate productivity factor for South firms. I assume that \( A^* < 1 \), so that South firms are less efficient than North firms in the production of heterogeneous goods. It follows that the marginal cost of a South firm with productivity \( \varphi \) is \( w^* / \varphi \). The cutoff productivity thresholds for South firms are \( \varphi_D^* \) and \( \varphi_X^* \), so that a South firm with the pair \((\varphi, \tau)\) produces for its domestic market if \( \varphi \geq \varphi_D^* \), and produces for the North market if \( \varphi \geq \tau \varphi_X^* \).

As shown in section D in the Appendix, from the zero-cutoff-markup conditions we obtain four of the six equations we need to solve for the equilibrium of this model:

\[
\varphi_X^* = w^* \varphi_D^*/A^*, \quad \varphi_X = A^* \varphi_D^*/w^*, \quad \varphi_D = w_0 \varphi_D^*, \quad \text{and} \quad \varphi_X = w_0 \varphi_X^*. \tag{29}
\]

The equations in (29) show the relationship between the cutoff productivity levels of firms competing in the same market; for example, all the firms competing in the North market are bounded by \( \hat{p} \) and hence, a link must exist between the productivity thresholds that determine who sells in that market \((\varphi_D, \varphi_D^*, \text{and} \varphi_X^*)\). As \( w_0 < 1 \), note that it is always the case that \( \varphi_D^* < \varphi_D \) and \( \varphi_X^* < \varphi_X \): there are always North firms that are able to reach the domestic or export markets only because they took the decision to offshore.

### 5.2 The Offshoring Decision

The description of the offshoring decision of North firms is similar to section 3.2. Let \( \pi_s(\varphi, \tau) \) be the total profit that a North firm with the pair \((\varphi, \tau)\) obtains every period under offshoring status \( s \), where \( s \in \{n, o\} \). That is, let

\[
\pi_s(\varphi, \tau) = \pi_{D,s}(\varphi) + \pi_{X,s}(\varphi, \tau), \tag{30}
\]

where

\[
\pi_{D,s}(\varphi) = \left[ \frac{\mu_{D,s}(\varphi)^2}{1 + \mu_{D,s}(\varphi)} \right] \gamma \psi \mathbb{1}\{\varphi \geq \varphi_{D,s}\}, \tag{31}
\]

\[
\pi_{X,s}(\varphi, \tau) = \left[ \frac{\mu_{X,s}(\varphi, \tau)^2}{1 + \mu_{X,s}(\varphi, \tau)} \right] \gamma \psi w^* \mathbb{1}\{\varphi \geq \tau \varphi_{X,s}\}, \tag{32}
\]

with \( \mu_{D,s}(\varphi) \) and \( \mu_{X,s}(\varphi, \tau) \) denoting the markups that the firm obtains in each market, and \( \mathbb{1}\{\cdot\} \) denoting an indicator function taking the value of 1 if the condition inside the brackets is met (and zero otherwise). As before, it is always the case that \( \pi_o(\varphi, \tau) \geq \pi_n(\varphi, \tau) \) and therefore, the offshoring decision is irreversible.

The rest follows closely section 3.2. To start offshoring a firm with productivity \( \varphi \) must pay an adjustment cost of \( A(\varphi, \tau) \), which is randomly drawn every period and given by

\[
A(\varphi, \tau) \equiv \eta \left[ \rho \pi_n(\varphi, \tau) + f_o \right], \tag{33}
\]

where \( \eta \) is a non-negative random variable with cdf \( F(\eta) \), \( \rho \in (0, 1] \), and \( f_o > 0 \). It follows that a version of Proposition 1 holds for the model with trade in final goods: \( \hat{n}(\varphi, \tau) \) denotes the
maximum adjustment factor that a non-offshoring firm with the pair \((\varphi, \tau)\) is willing to accept to begin offshoring, and is the unique solution to

\[
\hat{\eta}(\varphi, \tau) = \frac{z(\varphi, \tau)}{\delta} - \frac{1 - \delta}{\delta} \int_0^{\hat{\eta}(\varphi, \tau)} F(\eta) d\eta,
\]

where

\[
z(\varphi, \tau) = \frac{\pi_o(\varphi, \tau) - \pi_n(\varphi, \tau)}{\rho \pi_n(\varphi, \tau) + f_o} \geq 0.
\]

The offshoring probability of a North firm with the pair \((\varphi, \tau)\) is then \(\Lambda(\varphi, \tau) = F[\hat{\eta}(\varphi, \tau)]\).

With export opportunities, an inverted-U relationship between productivity and the offshoring probability is not the only possible outcome. Figure 8a illustrates this point by showing \(\Lambda(\varphi, \tau)\) for different levels of \(\tau\). For non-offshoring firms that draw small and medium levels of the iceberg cost, \(\tau\), the offshoring probability follows the same inverted-U behavior described in Proposition 2. On the other hand, non-offshoring firms drawing high levels of \(\tau\) exhibit a M-shape relationship between \(\varphi\) and \(\Lambda(\varphi, \tau)\). For these firms, their draw of \(\tau\) is so high that there exist a large range of low-productivity firms that will never be able to access the export market; the range is large enough that, as in the benchmark model without export opportunities, an inverted-U shape emerges for these firms. As the productivity increases, export opportunities are more likely in spite of a high \(\tau\) draw; thus, these high-productivity firms consider the access to the export market in their offshoring decision and another inverted-U shape emerges. In the end, there are two maximums in the offshoring probability: one for low-productivity firms (who only care about the domestic market), and another for high-productivity firms (who care also about the export market).

Let \(\bar{\Lambda}(\varphi)\) be the average offshoring probability of non-offshoring firms with productivity \(\varphi\), so that

\[
\bar{\Lambda}(\varphi) = \int_1^{\infty} \Lambda(\varphi, \tau)m(\tau) d\tau.
\]
Figure 8b shows $\bar{\Lambda}(\varphi)$ for different levels of $f_o$, which is very similar to Figure 2a for $\Lambda(\varphi)$ in the case without export opportunities. It is also possible, however, to obtain a M-shape relationship between $\varphi$ and $\bar{\Lambda}(\varphi)$ if the random variable $\tau$ has a high mean and low variance, so that $\Lambda(\varphi, \tau)$ has a M-shape for most values of $\tau$.

5.3 The Distribution and Composition of Firms

Given $\Lambda(\varphi, \tau)$ and the exogenous death shock with rate $\delta$, the fraction of North firms with $(\varphi, \tau)$ that engage in offshoring is

$$\Gamma(\varphi, \tau) = \frac{\Lambda(\varphi, \tau)}{\delta + (1-\delta)\Lambda(\varphi, \tau)}. \quad (37)$$

Therefore, the bivariate density functions for offshoring and non-offshoring North firms are given by

$$h_o(\varphi, \tau) = \frac{\Gamma(\varphi, \tau)g(\varphi)m(\tau)}{\bar{\Gamma}} \quad \text{and} \quad h_n(\varphi, \tau) = \frac{[1-\Gamma(\varphi, \tau)]g(\varphi)m(\tau)}{1-\bar{\Gamma}}, \quad (38)$$

where $g(\varphi)$ and $m(\tau)$ are the probability density functions for the distributions of productivity and iceberg costs at the time of entry, and $\bar{\Gamma} = \int_{\varphi_{min}}^{\infty} \int_{1}^{\infty} \Gamma(\varphi, \tau) d\tau d\varphi$. Hence, the marginal density function of $\varphi$ for North firms with status $s$ is given by

$$h_s(\varphi) = \int_{1}^{\infty} h_s(\varphi, \tau) d\tau. \quad (39)$$

On the other hand, South firms never offshore and therefore, their bivariate density function is simply $g(\varphi)m(\tau)$.

Firms enter in each country as long as their expected value of entry is no less than a sunk entry cost. The sunk entry costs are in terms of the homogeneous good, and are given by $f_E$ in the North, and by $f^*_E$ in the South. Section D.2 in the Appendix presents the North and South free-entry conditions, and defines the equilibrium of this model as the list $(\varphi_{D,o}, \varphi_{X,o}, \varphi_{D,n}, \varphi_{X,n}, \varphi^*_D, \varphi^*_X)$ that solves the two free-entry conditions and the four equations in (29). A nice feature of the solution to this model is that, in contrast to a conventional Melitz-type model with identical iceberg costs, we can obtain exporting productivity thresholds that are below domestic productivity thresholds (e.g., $\varphi_{X,s} < \varphi_{D,s}$ for $s \in \{n,o\}$), so that there may be firms that produce only for the export market. Once we obtain the equilibrium productivity thresholds, we can solve for different masses of firms, market shares, and average productivities, prices, and markups.

Let $N$ represent the mass of sellers in the North market, and let $N^*$ represent the mass of sellers in the South market. $N$ comprises non-offshoring North producers, $N_{D,n}$, offshoring North producers, $N_{D,o}$, and South exporters, $N^*_X$. On the other hand, $N^*$ comprises South producers, $N^*_D$, non-offshoring North exporters, $N_{X,n}$, and offshoring North exporters, $N_{X,o}$. That is, $N = N_{D,n} + N_{D,o} + N^*_X$ and $N^* = N^*_D + N_{X,n} + N_{X,o}$. Moreover, every period there is a measure $N_E$ of North entrants, and a measure $N^*_E$ of South entrants. In section D.3 in the Appendix we derive the solution for $N_E$ and $N^*_E$, which then allows us to obtain $N_{r,s}$, $N^*_r$, $N$, and $N^*$, for $r \in \{D, X\}$.
and $s \in \{n,o\}$. Note that the mass of North exporters is given by $N_X = N_{X,n} + N_{X,o}$. In addition, we denote the mass of North producers with $N_P$, and the mass of South producers with $N_P^*$. The mass $N_P$ comprises non-offshoring firms, $N_n$, and offshoring firms, $N_o$.\footnote{If $\varphi_{D,s} \leq \varphi_{X,s}$, so that North exporting firms are a subset of firms producing for the domestic market, it must be the case that $N_P = N_{D,n} + N_{D,o}$, $N_o = N_{D,n}$, and $N_n = N_{D,o}$. Otherwise, there will be a small fraction of firms that export but do not produce for the domestic market and $N_P > N_{D,n} + N_{D,o}$, $N_o > N_{D,n}$, and $N_n > N_{D,o}$. Analogously, for South firms $N_P^* = N_D^*$ if $\varphi_D \leq \varphi_X^*$, and $N_P^* > N_D^*$ otherwise.}

For North firms producing for market $r$ and status $s$, section D.4 in the Appendix describes expressions for their average markup ($\bar{\mu}_{r,s}$), average price ($\bar{p}_{r,s}$), average productivity ($\bar{\varphi}_{r,s}$), and market share ($\sigma_{r,s}$). As well, for South firms producing for market $r$ we have $\bar{\mu}_r^*$, $\bar{p}_r^*$, $\bar{\varphi}_r^*$, and $\sigma_r^*$. For market shares, it is the case that $\sigma_{D,o} + \sigma_{D,n} + \sigma_X^* = 1$ and $\sigma_D^* + \sigma_{X,o} + \sigma_{X,n} = 1$. The overall average prices in the North and South are given by

$$\bar{p} = (N_{D,n}/N)\bar{p}_{D,n} + (N_{D,o}/N)\bar{p}_{D,o} + (N_X^*/N)\bar{p}_{X,n}^*,$$

$$\bar{p}^* = (N_D^*/N^*)\bar{p}_D^* + (N_{X,n}/N^*)\bar{p}_{X,n}^* + (N_{X,o}/N^*)\bar{p}_{X,o}^*.$$

The average productivity of producing North firms with status $s$ is denoted by $\bar{\varphi}_s$, and hence, the overall average productivity of North firms is given by $\bar{\varphi} = (N_o/N_P)\bar{\varphi}_o + (N_n/N_P)\bar{\varphi}_n$—see section D.4 in the Appendix for the $\bar{\varphi}_o$ and $\bar{\varphi}_n$ equations. As in the model without final-good trade, the effective productivity of offshoring firms takes into account their lower marginal cost due to cheaper foreign labor; that is, $\bar{\varphi}_E^{r} = \bar{\varphi}_{r,o}/w_o$ for $r \in \{D,X\}$, and $\bar{\varphi}_E^D = \bar{\varphi}_o/w_o$. Therefore, the overall effective productivity of North firms is $\bar{\varphi}_E^D = (N_o/N_P)\bar{\varphi}_o + (N_n/N_P)\bar{\varphi}_n$. Lastly, the overall average productivity of South firms, derived in the Appendix, is denoted by $\bar{\varphi}^*$.\footnote{If $\varphi_{D,s} \leq \varphi_{X,s}$, so that North exporting firms are a subset of firms producing for the domestic market, it must be the case that $N_P = N_{D,n} + N_{D,o}$, $N_o = N_{D,n}$, and $N_n = N_{D,o}$. Otherwise, there will be a small fraction of firms that export but do not produce for the domestic market and $N_P > N_{D,n} + N_{D,o}$, $N_o > N_{D,n}$, and $N_n > N_{D,o}$. Analogously, for South firms $N_P^* = N_D^*$ if $\varphi_D \leq \varphi_X^*$, and $N_P^* > N_D^*$ otherwise.}

### 5.4 The Impact of Trade Liberalization

In this section we study the effects of trade liberalization on offshoring and exporting decisions, aggregate productivity, and the distribution of firms. We consider trade liberalization in final goods, as well as reductions in the variable cost of offshoring. To highlight this extension’s main insights, we focus on expanding the numerical example presented above.

The parameter values for the North are as described in section 3.5, as is the wage in the South, $w^*$. Both countries have the same preferences (identical $\psi$ and $\gamma$), and each differentiated-good firm draws its productivity from the same lognormal distribution. For South firms, I set $A^*$ to 0.53 and $f_E^*$ to 0.85. The value for $A^*$ implies that $w_o < w^*/A^* < w_n$, so that for each level of relative productivity $\varphi$, South firms are more efficient than non-offshoring North firms but less efficient than offshoring North firms.

In both countries, the distribution of the iceberg costs for differentiated-good firms is assumed to be uniform in the interval $[\tau, \tilde{\tau}]$, so that $m(\tau) = 1/(\tilde{\tau} - \tau)$ if $\tau \in [\tau, \tilde{\tau}]$ (and zero otherwise), and $M(\tau) = (\tau - \bar{\tau})/(\tilde{\tau} - \tau)$. I consider three cases for $\tau \sim U(\tau, \tilde{\tau})$: (i) $\tau \equiv \tau^\infty$, where $\tau \rightarrow \infty$
and $\tau \to \infty$, so that there is no trade in final goods; (ii) $\tau \equiv \tau^H \sim U(1,6)$, so that $E(\tau^H) = 3.5$; and (iii) $\tau \equiv \tau^L \sim U(1,2.5)$, so that $E(\tau^L) = 1.75$. In addition, we consider three cases for the variable cost of offshoring: $\lambda \equiv \lambda^\infty \to \infty$, so that there is no offshoring; $\lambda \equiv \lambda^H = 1.3$, which is the value assumed in sections 3.5 and 4.3; and $\lambda \equiv \lambda^L = 1.05$. In total we look at six cases: $(\tau^\infty, \lambda^\infty)$, $(\tau^\infty, \lambda^H)$, $(\tau^H, \lambda^H)$, $(\tau^H, \lambda^L)$, $(\tau^L, \lambda^H)$, and $(\tau^L, \lambda^L)$. The cases $(\tau^\infty, \lambda^\infty)$ and $(\tau^\infty, \lambda^H)$—autarky and offshoring without final-good trade—correspond to the cases in Table 1, while $(\tau^L, \lambda^L)$ corresponds to a case with substantial liberalization in both final-good trade and offshoring. Table 4 presents the solution for each case. Here we focus on the outcomes that are relevant for North firms and the North market, while the Appendix presents the relevant results for South firms and the South market.

The responses of the cutoff levels to changes in $\lambda$ or the distribution of $\tau$ indicate changes in the competitive environment faced by each type of firm. As before, North firms face a tougher competitive environment in market $r$ if both $\varphi_{D,o}$ and $\varphi_{D,n}$ increase, so that all firms that keep the same offshoring status and continue producing for market $r$ are forced to reduce their markups. As shown in Table 4, trade liberalization in final goods ($i.e.$, going from $\tau^\infty \to \tau^H \to \tau^L$, keeping $\lambda$ constant) increases $\varphi_{D,o}$, $\varphi_{D,n}$, $\varphi_{X,o}$, and $\varphi_{X,n}$, creating a tougher competitive environment for North firms in both the domestic and export markets. On the other hand, reductions in the variable cost of offshoring ($\lambda^\infty \to \lambda^H \to \lambda^L$, keeping the same distribution of $\tau$) create an easier competitive environment for North firms in the export market ($\varphi_{X,o}$ and $\varphi_{X,n}$ decline), but the effects on the domestic market are more complex. On the one hand, $\varphi_{D,n}$ rises as $\lambda$ declines, indicating that non-offshoring firms face a tougher domestic environment as offshoring becomes cheaper. On the other hand, the domestic environment for offshoring firms depends on the degree of final-good trade liberalization: if $\tau \equiv \tau^H$, $\varphi_{D,o}$ decreases as $\lambda$ declines (easing the domestic competition for offshoring firms), but the opposite happens if $\tau \equiv \tau^L$. As discussed below, the non-monotonic relationship between $\varphi_{D,o}$ and $\lambda$ is due to changes in the importance of the export market on North firms’ entry decisions.

In the end, the model in consistent with the pro-competitive effects of trade identified since Krugman (1979): trade liberalization in final goods creates a tougher competitive environment and decreases markups. Empirical evidence of the decline of firm-level markups after final-good trade liberalization has been found by Badinger (2007), Feenstra and Weinstein (2010), Harrison (1994), and Krishna and Mitra (1998), among others. On the other hand, a decline in offshoring costs causes an increase in markups of North exporters, and may increase domestic markups of North offshoring firms. This result is consistent with empirical evidence from India by De Loecker

\[\text{\footnotesize{\textsuperscript{26}}In Krugman (1979) the elasticity of demand is endogenous and thus features variable markups. In spite of endogenous markups being in the central analysis of the seminal work of Krugman, Feenstra and Weinstein (2010) argue that the study of the pro-competitive effects of trade was left on the sidelines for several years due to the prominence of CES preferences (which imply exogenous markups) in trade models.}}\]
Table 4: The effects of trade liberalization

<table>
<thead>
<tr>
<th>Productivity:</th>
<th>Autarky</th>
<th>Offshoring</th>
<th>Final-good trade and offshoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_{D,o} )</td>
<td>—</td>
<td>0.486</td>
<td>0.512</td>
</tr>
<tr>
<td>( \varphi_{D,n} )</td>
<td>0.581</td>
<td>0.589</td>
<td>0.620</td>
</tr>
<tr>
<td>( \varphi_{X,o} )</td>
<td>—</td>
<td>—</td>
<td>0.406</td>
</tr>
<tr>
<td>( \varphi_{X,n} )</td>
<td>—</td>
<td>—</td>
<td>0.492</td>
</tr>
<tr>
<td>( \varphi_{D,o} )</td>
<td>—</td>
<td>1.345</td>
<td>1.409</td>
</tr>
<tr>
<td>( \varphi_{D,n} )</td>
<td>1.350</td>
<td>1.355</td>
<td>1.333</td>
</tr>
<tr>
<td>( \bar{\varphi}_{X,o} )</td>
<td>—</td>
<td>—</td>
<td>1.547</td>
</tr>
<tr>
<td>( \bar{\varphi}_{X,n} )</td>
<td>—</td>
<td>—</td>
<td>1.764</td>
</tr>
<tr>
<td>( \bar{\varphi}_{o} )</td>
<td>1.350</td>
<td>1.355</td>
<td>1.332</td>
</tr>
<tr>
<td>( \bar{\varphi}_{E} )</td>
<td>—</td>
<td>1.631</td>
<td>1.708</td>
</tr>
<tr>
<td>( \dot{\varphi}<em>{o}/\bar{\varphi}</em>{n} )</td>
<td>—</td>
<td>0.993</td>
<td>1.058</td>
</tr>
<tr>
<td>( \dot{\varphi}<em>{E}/\bar{\varphi}</em>{n} )</td>
<td>—</td>
<td>1.204</td>
<td>1.282</td>
</tr>
<tr>
<td>( \bar{\varphi} )</td>
<td>1.350</td>
<td>1.352</td>
<td>1.360</td>
</tr>
<tr>
<td>( \bar{\varphi}_{E} )</td>
<td>1.350</td>
<td>1.437</td>
<td>1.467</td>
</tr>
<tr>
<td>Prices:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>1.721</td>
<td>1.699</td>
<td>1.613</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>1.148</td>
<td>1.105</td>
<td>1.103</td>
</tr>
<tr>
<td>( \hat{p}_{D,o} )</td>
<td>—</td>
<td>0.998</td>
<td>0.954</td>
</tr>
<tr>
<td>( \hat{p}_{D,n} )</td>
<td>1.148</td>
<td>1.150</td>
<td>1.133</td>
</tr>
<tr>
<td>( \bar{p}_{X,o} )</td>
<td>—</td>
<td>—</td>
<td>1.534</td>
</tr>
<tr>
<td>( \bar{p}_{X,n} )</td>
<td>—</td>
<td>—</td>
<td>1.615</td>
</tr>
<tr>
<td>Markups and shares:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{\mu}_{D,o} )</td>
<td>—</td>
<td>0.547</td>
<td>0.542</td>
</tr>
<tr>
<td>( \bar{\mu}_{D,n} )</td>
<td>0.428</td>
<td>0.417</td>
<td>0.378</td>
</tr>
<tr>
<td>( \bar{\mu}_{X,o} )</td>
<td>—</td>
<td>—</td>
<td>0.303</td>
</tr>
<tr>
<td>( \bar{\mu}_{X,n} )</td>
<td>—</td>
<td>—</td>
<td>0.253</td>
</tr>
<tr>
<td>( \sigma_{D,o} )</td>
<td>—</td>
<td>0.360</td>
<td>0.411</td>
</tr>
<tr>
<td>( \sigma_{D,n} )</td>
<td>1.000</td>
<td>0.640</td>
<td>0.509</td>
</tr>
<tr>
<td>( \sigma_{X,o} )</td>
<td>—</td>
<td>—</td>
<td>0.131</td>
</tr>
<tr>
<td>( \sigma_{X,n} )</td>
<td>—</td>
<td>—</td>
<td>0.098</td>
</tr>
<tr>
<td>Composition of firms:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N )</td>
<td>1.169</td>
<td>1.097</td>
<td>1.231</td>
</tr>
<tr>
<td>( N_{P} )</td>
<td>1.169</td>
<td>1.097</td>
<td>1.054</td>
</tr>
<tr>
<td>( N_{o}/N_{P} )</td>
<td>—</td>
<td>0.300</td>
<td>0.360</td>
</tr>
<tr>
<td>( N_{X}/N_{P} )</td>
<td>—</td>
<td>—</td>
<td>0.388</td>
</tr>
<tr>
<td>( N_{X,o}/N_{o} )</td>
<td>—</td>
<td>—</td>
<td>0.569</td>
</tr>
<tr>
<td>( N_{X,n}/N_{n} )</td>
<td>—</td>
<td>—</td>
<td>0.287</td>
</tr>
</tbody>
</table>
et al. (2012), who find lower marginal costs and higher markups after trade liberalization in inputs. Thus, this model provides a theoretical framework that reconciles the opposite empirical results found for final-good and input trade liberalizations.

In all cases with trade in final goods in Table 4 we obtain $\varphi_{X,s} < \varphi_{D,s}$ and hence, there is small fraction of purely exporting North firms (those with $\varphi \in [\varphi_{X,s}, \varphi_{D,s})$ and $\tau \leq \varphi/\varphi_{X,s}$, for $s \in \{n, o\}$). The average productivity of exporters, however, is higher than the average productivity of firms selling for the domestic market ($\bar{\varphi}_{X,s} > \bar{\varphi}_{D,s}$), but their gap declines as trade liberalization deepens; from $(\tau^H, \lambda^H)$ to $(\tau^L, \lambda^L)$, $\varphi_{X,o}/\varphi_{D,o}$ declines from 1.097 to 1.006, and $\varphi_{X,n}/\varphi_{D,n}$ declines from 1.323 to 1.150. Note also that trade liberalization causes an increase in the average productivity of offshoring firms, $\bar{\varphi}_o$, and a decline in the average productivity of non-offshoring firms, $\bar{\varphi}_n$, which then raises the ratio $\bar{\varphi}_o/\bar{\varphi}_n$ from 0.993 in $(\tau^H, \lambda^H)$ to 1.134 in $(\tau^L, \lambda^L)$. Importantly, the overall average productivity of North firms, $\bar{\varphi}$, barely changes after trade liberalization: in autarky $\bar{\varphi}$ equals 1.35, but it only rises to 1.393 (a 3.2% increase) in the $(\tau^L, \lambda^L)$ case, even with 71% of North producers engaged in offshoring and 88.2% engaged in exporting. On the other hand, the overall average effective productivity of North firms, $\bar{\varphi}^E$, rises 26.8% from autarky to $(\tau^L, \lambda^L)$, with the response being larger for reductions in $\lambda$.

Both types of liberalization drive down the average price in the North, $\bar{p}$, the maximum price that sellers can set, $\hat{p}$, and the average domestic price of offshoring firms, $\bar{p}_{D,o}$. There is a slight increase in $\bar{p}_{D,n}$ when moving from autarky to $(\tau^\infty, \lambda^H)$, but that does not mean that non-offshoring firms are increasing their prices (to the contrary, $\bar{p}_{D,n}$ rises, which implies lower prices and markups for each active non-offshoring firm), but is simply a composition effect due to changes in the distribution of non-offshoring firms. In all other cases, both types of liberalization cause a decrease in $\bar{p}_{D,n}$. Lastly, the average prices of North firms in the export market, $\bar{p}_{X,o}$ and $\bar{p}_{X,n}$ decline with final-good trade liberalization, but increase with the decline in $\lambda$.

Average markups of North exporters, $\bar{\mu}_{X,o}$ and $\bar{\mu}_{X,n}$, increase after any type of liberalization. Given that the competitive environment becomes tougher in the export market after final-good trade liberalization, so that every continuing exporter keeping the same status $s$ reduces its markups, increasing average markups of exporters is the result of changes in their productivity distribution due to selection and escape-competition effects. As in the case without final-good trade, this result suggests that average markups may not be a good indicator of the level of competition in a market, as they are subject to composition effects. The average markups of North firms selling domestically, $\bar{\mu}_{D,o}$ and $\bar{\mu}_{D,n}$, decline for both types of liberalization, with the exception of the slight increase in $\bar{\mu}_{D,o}$ when $\lambda$ declines and $\tau \equiv \tau^H$. Markets shares move in the expected direction, with the shares of offshoring firms, $\sigma_{D,o}$ and $\sigma_{X,o}$, increasing after any type of liberalization, the share of non-offshoring firms in the domestic market, $\sigma_{D,n}$, contracting after any type of liberalization, and the market share of non-offshoring firms in the export market, $\sigma_{X,n}$, increasing after final-good
Figure 9: The average probability of offshoring and trade liberalization

trade liberalization, but decreasing after λ declines.

The number of varieties consumed in the North market, N, decreases after λ declines (as new and old offshoring firms wipe out non-offshoring firms and South firms), but increases after final-good trade liberalization. The behavior of the mass of producing North firms, N_P, which is directly related to entry, depends on the degree of trade liberalization: there is an increase in N_P when moving from (τ_H, λ_L) or (τ_L, λ_H) to (τ_L, λ_L), but a decrease otherwise. This happens because as one type of trade liberalization deepens (τ ≡ τ_L or λ ≡ λ_L), further liberalization of the other type has similar effects to an increase in market size because it raises North firms’ potential profits from exporting.

Both the fraction of North producers that offshore, N_o/N_P, and the fraction of North producers that export, N_X/N_P, rise with either final-good trade liberalization or a decline in λ. These results highlight strong complementarities between offshoring and exporting decisions: lower final-good exporting costs drive more firms to offshore, and vice versa. This complementarity is also observed in the large fraction of offshoring firms that also engage in exporting, N_{X,o}/N_o; in the (τ_H, λ_H) case, for example, 56.9% of firms that offshore also export (in contrast, 28.7% of non-offshoring firms are exporters) and this proportion rises to more than 95% in the (τ_L, λ_H) case.\footnote{Bernard et al. (2007) find that 79% of U.S. firms that import also export. The model in this paper can account for this high correlation without requiring the strong selection effects implied by models with homogeneous fixed costs of exporting and importing.}

Figure 9 shows the evolution of the average probability of offshoring, \bar{Λ}(ϕ), when final-good trade costs change (keeping λ constant), and when the variable offshoring cost changes (keeping the same distribution of τ). Comparing Figures 9a and 6a, we find that trade liberalization in final goods has similar effects to a toughening of the competitive environment driven by an increase in γ or ψ: there are selection and escape-competition effects, with the selection effect dominating.
(a) Final-good trade liberalization: \((\tau^\infty, \lambda^H) \rightarrow (\tau^L, \lambda^H)\)

(b) Decline in offshoring costs: \((\tau^L, \lambda^H) \rightarrow (\tau^L, \lambda^E)\)

Figure 10: Effective probability density functions: producing non-offshoring firms \((h_n(\varphi|\text{active}), \text{solid})\) and offshoring firms \((h_o^E(\varphi^E), \text{dashed})\)

for the least productive firms, and the escape-competition effect dominating for the rest of the firms. Note that in the cases showed in Figure 9a, the productivity levels at which the average escape-competition effect starts to dominate the average selection effect are below 1, which is well below the average productivity of North firms. This result is remarkably consistent with the empirical finding of Lileeva and Trefler (2010), who find that better access to the U.S. market drives productivity-enhancing investments mainly in low- and mid-productivity Canadian firms, who make simultaneous decisions to invest and export. Figure 9b shows the response of \(\bar{\Lambda}(\varphi)\) to a decline in \(\lambda\), which is very similar to Figure 6b for a decline in \(f_o\). As offshoring becomes cheaper, the probability of offshoring for every non-offshoring firm with \(\varphi \geq \min\{\varphi_{D,o}, \varphi_{X,o}\}\) increases, and thus, \(\bar{\Lambda}(\varphi)\) shifts up.

Figure 10 shows the evolution of the effective productivity distributions for offshoring and active non-offshoring North firms. As before, \(\varphi^E = \varphi\) for a non-offshoring firm, and \(\varphi^E = \varphi/w_o\) for an offshoring firm. Figure 10a looks at the effect of trade liberalization in final goods (from \((\tau^\infty, \lambda^H)\) to \((\tau^L, \lambda^H)\)), and Figure 10b looks at the effect of a decline in the variable costs of offshoring (from \((\tau^L, \lambda^H)\) to \((\tau^L, \lambda^L)\)). Similar to an increase in \(\gamma\) or \(\eta\), both types of liberalization cause the distribution of active non-offshoring firms to cluster towards \(\varphi_{D,n}\): on the one hand, \(\varphi_{D,n}\) rises with any type of liberalization, so that selection effects shift \(h_n(\varphi|\text{active})\) to the right; on the other hand, with the increase in the offshoring likelihood for most of the firms with \(\varphi > \varphi_{D,n}\) due to escape-competition effects (where \(\varphi_{D,n} = \varphi_{D,o}^E \equiv \varphi_{D,o}/w_o\)), there is a decline in the of mass of non-offshoring firms with low-mid and higher productivity levels. The effective distribution of offshoring firms, \(h_o^E(\varphi^E)\), shifts to the right after trade liberalization in final goods (which reflects
both selection and escape-competition effects), and becomes flatter (with more mass in both tails) after a decline in offshoring costs.

6 Conclusion

The model in this paper started from the following premise: A firm’s offshoring decision is a productivity-enhancing innovation decision that implies adjustment costs due to the reorganization (and/or disruption) of the firm’s production process. As such, I assumed that the decision is lumpy and subject to non-convex and random adjustment costs that are, in absolute terms, expected to increase with firm size (e.g., the adjustment costs due to offshoring should be, in absolute terms, greater for a one-billion-dollar firm than for a one-million-dollar firm). This assumption and a demand structure with endogenous markups imply an inverted-U relationship between firm-level productivity and the probability of offshoring, generating overlapping productivity distributions of offshoring and non-offshoring firms that look similar to those in empirical studies.

The model delivers a strong message with respect to the relevance of selection in productivity for offshoring decisions. Although the long-held view is that a high productivity is necessary to offshore, this model shows that the higher average productivity of offshoring firms observed in empirical studies may only be due to the firms’ productivity gains driven by offshoring decisions—removing the productivity effect of offshoring, the average productivity of offshoring firms can even be below the average productivity of non-offshoring firms.

When analyzing the responses of offshoring decisions to a tougher competitive environment (driven, for example, by an increase in the substitutability between goods, an increase in market size, or by trade liberalization in final goods), it was showed the existence of the conventional selection effect—the cleansing effect of competition—and of an escape-competition effect. The latter effect—which has been studied extensively in the innovation literature, but is largely absent in the trade literature—accounts for the impact of competition on the opportunity cost of offshoring: even if offshoring and non-offshoring profits decline with a tougher environment, their difference (relative to adjustment costs) may increase, and hence offshoring incentives may increase.

This paper does not discuss implications regarding the different organizational modes of offshoring. This a central topic in offshoring models with heterogeneous firms in the tradition of Antrás and Helpman (2004). Most of these models assume a rigid fixed-cost structure for each offshoring mode, and hence rely on pure selection in productivity. This paper abstracts from offshoring modes because its objective is to show that, at its most elemental level, the offshoring decision not only depends on selection mechanisms, but is also affected by its opportunity cost (which is not necessarily lower for the most productive firms). A topic for future research is then to expand the current model to analyze its implications for offshoring-mode choices.
References


