Offshoring and Jobs: The Myriad Channels of Influence

Appendix — For Online Publication

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A The Mass of Entrants and Market Shares

To obtain $N_E$, note first that we can write the price for the basket of differentiated goods as

$$P = \left[ N_n \bar{p}_n^{1-\sigma} + N_o \bar{p}_o^{1-\sigma} \right] \frac{1}{1-\sigma}, \quad (A-1)$$

where $\bar{p}_s \equiv p(\hat{\phi}_s)$ is the average price of firms with offshoring status $s$, for $s \in \{n,o\}$, with

$$\hat{\phi}_n = \left[ \int_{\hat{\phi}}^{\hat{\phi}_o} \varphi^{\sigma-1} g(\varphi \mid \hat{\phi} \leq \varphi < \hat{\phi}_o) d\varphi \right]^{\frac{1}{\sigma}}, \quad \hat{\phi}_o = \left[ \int_{\hat{\phi}_o}^{\infty} \varphi^{\sigma-1} g(\varphi \mid \varphi \geq \hat{\phi}_o) d\varphi \right]^{\frac{1}{\sigma}}. \quad (A-2)$$

Substituting $N_n = [G(\hat{\phi}_o) - G(\dot{\phi})] N_E$ and $N_o = [1 - G(\hat{\phi}_o)] N_E$ into (A-1), we solve for $N_E$ as

$$N_E = \frac{P^{1-\sigma}}{[G(\hat{\phi}_o) - G(\dot{\phi})] \bar{p}_n^{1-\sigma} + [1 - G(\hat{\phi}_o)] \bar{p}_o^{1-\sigma}}. \quad (A-3)$$

Lastly, we only have to use the zero-cutoff-profit condition in (21) to substitute for $P$ in equation (A-3).

We can also obtain convenient expressions for the market shares of non-offshoring and offshoring firms. From the end of section 3.1 we know that the share in total differentiated-good expenditure of a producing firm with productivity $\varphi$ is $\mu(\varphi) = \frac{p(\varphi)^{1-\sigma}}{P^{1-\sigma}}$. Aggregating the shares of all firms with the same offshoring status, it follows that the total market share of firms with offshoring status $s$ is

$$\mu_s = \frac{N_s \bar{p}_s^{1-\sigma}}{P^{1-\sigma}}, \quad (A-4)$$

for $s \in \{n,o\}$. Note that $\mu_n + \mu_o = 1$ is equivalent to equation (A-1).
B  Existence and Uniqueness of Equilibrium

We assume search friction parameters so that the equilibrium value for \( \hat{w}_Z \)—from (16)—is in the range \((\lambda k(0)w^*, \lambda k(1)w^*)\), so that an interior solution for \( \hat{a} \)—from (17)—exists. For the existence and uniqueness of the equilibrium cutoff levels, note that after plugging in equations (22) and (23) into \( \Pi \) and using Leibniz’s rule, we obtain

\[
\frac{d\Pi}{d\hat{\phi}} = -\left(\frac{(\sigma - 1)f[1 - G(\hat{\phi})]}{\hat{\phi}^{\sigma}}\right) \left[ \frac{N_n}{N} \varphi_n^{\sigma - 1} + N_o \left( \frac{\varphi_o}{c(\hat{\alpha})} \right)^{\sigma - 1} \right] < 0,
\]

with average productivities given by (A-2). As \( \Pi \) is strictly decreasing in \( \hat{\phi} \) and it converges to zero as \( \hat{\phi} \) increases, the equilibrium exists and is unique as long as \( \Pi \) is greater than \( f_E \) when \( \hat{\phi} \) approaches \( \varphi_{\min} \) from the right, where \( \varphi_{\min} \geq 0 \) is the lower bound of the support of the productivity distribution. It follows from (23) that the equilibrium value for \( \varphi_o \) also exists and is unique.

C  Proofs of Lemmas and Propositions

Proof of Lemma 1. If \( L \) and \( L^* \) are the amounts of domestic and foreign labor employed by a firm with productivity \( \varphi \) in the production of inputs, we must satisfy

\[
\begin{align*}
L &= \int_{\hat{\alpha}(\varphi)}^{1} \ell(\alpha) d\alpha, \quad (C-1) \\
L^* &= \int_{0}^{\hat{\alpha}(\varphi)} \ell^*(\alpha) d\alpha, \quad (C-2)
\end{align*}
\]

where \( \hat{\alpha}(\varphi) \) is this firm’s fraction of offshored inputs, and \( \ell(\alpha) \) and \( \ell^*(\alpha) \) denote units of domestic and foreign labor employed in the production of input \( \alpha \).

Given equation (10) and the input production functions \( y(\alpha) = \ell \) and \( y^*(\alpha) = \frac{\ell}{k(\alpha)} \), it follows that the optimal allocation of domestic and foreign labor across different inputs for a firm with productivity \( \varphi \) satisfies

\[
\ell(\alpha) = \ell(\alpha') \quad \text{for } \alpha, \alpha' \in [0, \hat{\alpha}(\varphi)] \quad (C-3)
\]

\[
\ell^*(\alpha) = \ell^*(\alpha') \left( \frac{k(\alpha)}{k(\alpha')} \right)^{1-\rho} \quad \text{for } \alpha, \alpha' \in (\hat{\alpha}(\varphi), 1]. \quad (C-4)
\]

Thus, for domestic labor it follows from equations (C-1) and (C-3) that

\[
\ell(\alpha) = \frac{L}{1 - \hat{\alpha}(\varphi)} \quad (C-5)
\]

for \( \alpha \in (\hat{\alpha}(\varphi), 1]. \) For the allocation of foreign labor, note from equation (C-4) that \( \ell^*(\alpha) = \ell^*(\hat{\alpha}(\varphi)) \left[ \frac{k(\alpha)}{k(\hat{\alpha}(\varphi))} \right]^{1-\rho} \) for \( \alpha \in [0, \hat{\alpha}(\varphi)] \) must hold. Plugging in the previous expression into equation (C-2), we solve for \( \ell^*(\hat{\alpha}(\varphi)) \) and substitute that expression back into \( \ell^*(\alpha) \) to obtain

\[
\ell^*(\alpha) = \frac{\ell^*(\hat{\alpha}(\varphi)) 1-\rho L^*}{\int_{0}^{\hat{\alpha}(\varphi)} k(\alpha')^{1-\rho} d\alpha'}. \quad (C-6)
\]
Therefore, substituting equation (C-5) into \( y(\alpha) \) and equation (C-6) into \( y^*(\alpha) \), and then plugging in the resulting expressions into equation (10), we get

\[
Y(\varphi) = \left\{ \frac{1-\rho}{\rho} \left[ \int_0^{\hat{\alpha}(\varphi)} k(\alpha)^{1-\rho} d\alpha \right] \frac{1}{\rho} L^* \frac{\rho-1}{\rho} + \left[ 1 - \hat{\alpha}(\varphi) \right] \frac{1}{\rho} L \frac{\rho-1}{\rho} \right\}^\frac{\rho}{\rho-1}.
\] (C-7)

We can then rewrite \( Y(\varphi) \) as

\[
Y(\varphi) = \left[ \kappa(\varphi)L^* \frac{\rho-1}{\rho} + \upsilon(\varphi)L \frac{\rho-1}{\rho} \right]^\frac{1}{\rho} = 0,
\] (C-8)

where \( \kappa(\varphi) \equiv \frac{1-\rho}{\rho} K[\hat{\alpha}(\varphi)]^{\frac{1}{\rho}} \), \( K[\hat{\alpha}(\varphi)] = \int_0^{\hat{\alpha}(\varphi)} k(\alpha)^{1-\rho} d\alpha \), and \( \upsilon(\varphi) \equiv \left[ 1 - \hat{\alpha}(\varphi) \right]^{\frac{1}{\rho}}. \)

**Proof of Lemma 2.** We find the unit cost of \( Y(\varphi) \) in two steps. First, we obtain the unit cost for a given \( \hat{\alpha}(\varphi) \), and then obtain the optimal \( \hat{\alpha}(\varphi) \). For a given \( \hat{\alpha}(\varphi) > 0 \), denote the domestic and foreign labor requirements to produce \( Y(\varphi) = 1 \) by \( L_{Y(\varphi)=1} \) and \( L^*_{Y(\varphi)=1} \), respectively. Next, we set the Lagrangean

\[
\Upsilon = w^* L^*_{Y(\varphi)=1} + \hat{w}_Z L_{Y(\varphi)=1} + \xi \left\{ 1 - \left[ \kappa(\varphi)L^*_{Y(\varphi)=1} + \upsilon(\varphi)L_{Y(\varphi)=1} \right]^\frac{1}{\rho} \right\},
\]

and obtain the following first order conditions

\[
w^* - \xi \left[ \kappa(\varphi)L^*_{Y(\varphi)=1} + \upsilon(\varphi)L_{Y(\varphi)=1} \right]^\frac{1}{\rho} \kappa(\varphi)L^*_{Y(\varphi)=1} = 0 \quad (C-9)
\]

\[
\hat{w}_Z - \xi \left[ \kappa(\varphi)L^*_{Y(\varphi)=1} + \upsilon(\varphi)L_{Y(\varphi)=1} \right]^\frac{1}{\rho} \upsilon(\varphi)L_{Y(\varphi)=1} = 0 \quad (C-10)
\]

\[
1 - \left[ \kappa(\varphi)L^*_{Y(\varphi)=1} + \upsilon(\varphi)L_{Y(\varphi)=1} \right]^\frac{1}{\rho} = 0. \quad (C-11)
\]

Combining equations (C-9) and (C-10) we obtain the optimal relationship between \( L_{Y(\varphi)=1} \) and \( L^*_{Y(\varphi)=1} \):

\[
L^*_{Y(\varphi)=1} = \left[ \frac{\kappa(\varphi)}{w^*} \right]^\rho L_{Y(\varphi)=1} \quad (C-12)
\]

Plugging in equation (C-12) into (C-11), we get

\[
L_{Y(\varphi)=1} = \left[ \frac{\upsilon(\varphi)^{\rho} \hat{w}_Z^{-\rho}}{\left[ \kappa(\varphi)^{\rho} w^{*1-\rho} + \upsilon(\varphi)^{\rho} \hat{w}_Z^{1-\rho} \right]^\frac{1}{\rho-1}} \right]^\frac{1}{\rho-1} \quad (C-13)
\]

and then substituting (C-13) into (C-12) we obtain

\[
L^*_{Y(\varphi)=1} = \left[ \frac{\kappa(\varphi)^{\rho} w^{*1-\rho}}{\left[ \kappa(\varphi)^{\rho} w^{*1-\rho} + \upsilon(\varphi)^{\rho} \hat{w}_Z^{1-\rho} \right]^\frac{1}{\rho-1}} \right]^\frac{1}{\rho-1} \quad (C-14)
\]
Thus, the unit cost of \( Y(\varphi) \), denoted by \( C[Y(\varphi)] \) is given by

\[
C[Y(\varphi)] \equiv w^*L_{Y(\varphi)-1} + \hat{w}_Z L_{Y(\varphi)-1} = \left[ \kappa(\varphi)^\rho w^{1-\rho} + \varphi(\rho)^\rho \hat{w}_Z^{1-\rho} \right]^{1/\rho}.
\]

Now, the optimal choice of \( \hat{\alpha}(\varphi) \) is obtained by minimizing \( C[Y(\varphi)] \) with respect to \( \hat{\alpha}(\varphi) \):

\[
\frac{dC(Y(\varphi))}{d\hat{\alpha}(\varphi)} = \frac{\rho}{1-\rho} \left[ \kappa(\varphi)^\rho w^{1-\rho} + \varphi(\rho)^\rho \hat{w}_Z^{1-\rho} \right]^{\frac{\rho}{1-\rho}} \left[ w^{1-\rho} \lambda(\varphi)^{\rho-1} \frac{d\kappa(\varphi)}{d\hat{\alpha}(\varphi)} \right] + \hat{w}_Z^{1-\rho} \lambda(\varphi)^{\rho-1} \frac{d\varphi}{d\hat{\alpha}(\varphi)} = 0
\]

Using the definitions of \( \kappa(\varphi) \) and \( \varphi(\rho) \) from Lemma 1, we obtain \( \frac{d\kappa(\varphi)}{d\hat{\alpha}(\varphi)} \) and \( \frac{d\varphi(\rho)}{d\hat{\alpha}(\varphi)} \) and plug them into the previous expression. We get that \( \frac{dC[Y(\varphi)]}{d\hat{\alpha}(\varphi)} = 0 \) if and only if

\[
\lambda w^* k(\hat{\alpha}(\varphi)) = \hat{w}_Z.
\]

Solving for \( \hat{\alpha}(\varphi) \) we get \( \hat{\alpha}(\varphi) = k^{-1} \left( \frac{\hat{w}_Z}{\lambda w^*} \right) \). Note that the solution does not depend on \( \varphi \), and thus, all offshoring firms offshore the same fraction of inputs:

\[
\hat{\alpha} = k^{-1} \left( \frac{\hat{w}_Z}{\lambda w^*} \right).
\]

Using this result along with the definitions of \( \kappa(\varphi) \) and \( \varphi(\rho) \) from Lemma 1, we can rewrite \( C[Y(\varphi)] \) as \( c(\hat{\alpha}) \hat{w}_Z \), where

\[
c(\hat{\alpha}) = \left[ k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha} \right]^{\frac{1}{1-\rho}}.
\]

(C-15)

To show that \( c(\hat{\alpha}) \hat{w}_Z \) is decreasing in \( \hat{\alpha} \), we only need to show that \( c'(\hat{\alpha}) < 0 \). Taking the derivative of equation (C-15) and using that \( K'(\hat{\alpha}) = k(\hat{\alpha})^{1-\rho} \), we obtain

\[
c'(\hat{\alpha}) = - \left[ k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha} \right]^{\frac{\rho}{1-\rho}} k(\hat{\alpha})^{\rho-2} K(\hat{\alpha}) k'(\hat{\alpha}) < 0.
\]

The last inequality follows from the fact that \( k'(\hat{\alpha}) > 0 \).

Let us now prove that \( c(\hat{\alpha}) \hat{w}_Z \in (w^*, \hat{w}_Z) \) for \( \hat{\alpha} > 0 \). First, note that \( K(0) = 0 \) and hence \( c(0) = 1 \). Given that \( k(\hat{\alpha}) = \frac{\hat{w}_Z}{\lambda w^*} \) and the definition of \( K(\hat{\alpha}) \), we can write \( c(\hat{\alpha}) \) as

\[
c(\hat{\alpha}) = \left\{ \left( \frac{w^*}{\hat{w}_Z} \right)^{1-\rho} \int_0^{\hat{\alpha}} \left[ \lambda k(\alpha) \right]^{1-\rho} d\alpha + 1 - \hat{\alpha} \right\}^{\frac{1}{1-\rho}}.
\]

Thus, \( c(1) = \frac{w^*}{\hat{w}_Z} \left\{ \int_0^1 \left[ \lambda k(\alpha) \right]^{1-\rho} d\alpha \right\}^{\frac{1}{1-\rho}} \). As \( \lambda k(\alpha) > 1 \) for every \( \alpha \), it follows that

\[
\left\{ \int_0^1 \left[ \lambda k(\alpha) \right]^{1-\rho} d\alpha \right\}^{\frac{1}{1-\rho}} > 1
\]

for every \( \rho \), and therefore \( c(1) > \frac{w^*}{\hat{w}_Z} \). Given that \( c'(\hat{\alpha}) < 0 \), it must be the case that \( c(\hat{\alpha}) \in \left( \frac{w^*}{\hat{w}_Z}, 1 \right) \) for \( \hat{\alpha} > 0 \). Hence, \( c(\hat{\alpha}) \hat{w}_Z \in (w^*, \hat{w}_Z) \) for \( \hat{\alpha} > 0 \). \qed
Proof of Lemma 3. The equilibrium gross profit function for a firm with productivity $\varphi \geq \hat{\varphi}$ can be written as
\[
\pi(\varphi) = \left( p(\varphi) - \frac{c(\hat{\alpha})I_{\{\varphi \geq \hat{\varphi}_{o}\}} \hat{w}_{Z}}{\varphi} \right) z(\varphi).
\]
Substituting equations (19) and (22) in the previous expression, we can solve for the equilibrium output as
\[
z(\varphi) = \left( \frac{\varphi}{c(\hat{\alpha})I_{\{\varphi \geq \hat{\varphi}_{o}\}} \hat{\varphi}} \right)^{\sigma} f.
\]
Moreover, given that $z(\varphi) = \varphi Y(\varphi)$, we get
\[
Y(\varphi) = \left( \frac{\varphi}{c(\hat{\alpha})I_{\{\varphi \geq \hat{\varphi}_{o}\}} \hat{\varphi}} \right)^{\sigma-1} f.
\]
(C-16)

For a producing non-offshoring firm, so that $\varphi \in [\hat{\varphi}, \hat{\varphi}_{o})$, we know from equation (11) that $Y(\varphi) = L$. Thus, given equation (C-16) it follows that this firm’s demand for domestic labor, $L_n(\varphi)$, is given by
\[
L_n(\varphi) = \left( \frac{\varphi}{c(\hat{\alpha})I_{\{\varphi \geq \hat{\varphi}_{o}\}} \hat{\varphi}} \right)^{\sigma-1} f.
\]
(C-17)
The subscript in $L_n(\varphi)$ identifies the firm’s status: “not offshoring”.

For an offshoring firm, so that $\varphi \geq \hat{\varphi}_{o}$, we use equation (C-13) to write its demand for labor to meet variable-input requirements as
\[
L_o(\varphi) = \left[ \frac{(1 - \hat{\alpha})\hat{w}_{Z}^{-\rho}}{(\kappa(\varphi)^{\rho}w^{1-\rho})^{\rho-1}} \right] Y(\varphi)
\]
The subscript in $L_o(\varphi)$ identifies the firm’s status: “offshoring”. Using the definitions of $\kappa(\varphi)$ and $v(\varphi)$ from Lemma 1, along with $k(\hat{\alpha}) = \hat{w}_{Z}^{\rho}$, we get $L_o(\varphi) = (1 - \hat{\alpha})c(\hat{\alpha})^{\rho}Y(\varphi)$. Lastly, plugging in equation (C-16) into the previous expression we rewrite $L_o(\varphi)$ as
\[
L_o(\varphi) = \left( \frac{(1 - \hat{\alpha})\varphi}{c(\hat{\alpha})^{\rho}w^{1-\rho} \hat{w}_{Z}} \right)^{\sigma-1} f.
\]
(C-18)

Proof of Lemma 4. For $\zeta_{\hat{\alpha},\lambda} = \frac{d \ln \hat{\alpha}}{d \ln \lambda}$, note first that $\frac{d \ln k(\hat{\alpha})}{d \ln \lambda} = \frac{\hat{\alpha}k'(\hat{\alpha})}{k(\hat{\alpha})} \zeta_{\hat{\alpha},\lambda}$. Now, from the optimality condition for $\hat{\alpha}$, $\lambda k(\hat{\alpha}) w^{\ast} = \hat{w}_{Z}$, it must be the case that $\frac{d \ln (\lambda k(\hat{\alpha}) w^{\ast})}{d \ln \lambda} = 0$. Hence,
\[
\frac{d \ln k(\hat{\alpha})}{d \ln \lambda} = -1.
\]
(C-19)
It follows that $\frac{\hat{\alpha}k'(\hat{\alpha})}{k(\hat{\alpha})} \zeta_{\hat{\alpha},\lambda} = -1$, so we can solve for $\zeta_{\hat{\alpha},\lambda}$ as
\[
\zeta_{\hat{\alpha},\lambda} = \frac{d \ln \hat{\alpha}}{d \ln \lambda} = - \frac{k(\hat{\alpha})}{\hat{\alpha}k'(\hat{\alpha})}.
\]
(C-20)
Given that $k'(\hat{\alpha}) > 0$, it is the case that $\zeta_{\hat{\alpha}, \lambda} < 0$.

Let us now obtain $\zeta_{c(\hat{\alpha}), \lambda}$. Given that $k(\hat{\alpha}) = \frac{\hat{w}_x}{\hat{w}_z}$, we can rewrite equation (C-15) as $c(\hat{\alpha}) = \left[ \left( \frac{\hat{w}_x}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha} \right]^{\frac{1}{1-\rho}}$. Hence, it follows from the envelope theorem that

$$\frac{dc(\hat{\alpha})}{d\lambda} = \left[ \left( \frac{\hat{w}_x}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha} \right]^{\frac{1}{1-\rho}} \lambda^{-\rho} \left( \frac{\hat{w}_x}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha})$$

Therefore,

$$\zeta_{c(\hat{\alpha}), \lambda} = \frac{dc(\hat{\alpha})}{d\lambda} \frac{\lambda}{c(\hat{\alpha})} = \frac{(\lambda w^*/\hat{w}_z)^{1-\rho} K(\hat{\alpha})}{(\lambda w^*/\hat{w}_z)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha}}.$$  

Using $k(\hat{\alpha})^{\rho-1} = (\lambda w^*/\hat{w}_z)^{1-\rho}$ and the definition of $c(\hat{\alpha})$, we rewrite the previous expression as

$$\zeta_{c(\hat{\alpha}), \lambda} = \frac{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha})}{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha}} = 1 - (1 - \hat{\alpha})c(\hat{\alpha})^{\rho-1}. \quad (C-21)$$

For $\hat{\alpha} \in (0, 1)$, we have that $(1 - \hat{\alpha})c(\hat{\alpha})^{\rho-1} > 0$ because $c(\hat{\alpha}) > 0$. Also, $c(\hat{\alpha})^{1-\rho} = k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha} > 1 - \hat{\alpha}$ because $k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) > 0$, and hence $(1 - \hat{\alpha})c(\hat{\alpha})^{\rho-1} < 1$. It follows that $\zeta_{c(\hat{\alpha}), \lambda} \in (0, 1)$. Note that for $\rho = 1$, $\zeta_{c(\hat{\alpha}), \lambda}$ reduces to $\hat{\alpha}$.

For $\zeta_{\Gamma(\hat{\alpha}), \lambda}$, we use the expression for $\Gamma(\hat{\alpha})$ in equation (24) to get

$$\zeta_{\Gamma(\hat{\alpha}), \lambda} = \frac{d \ln \Gamma(\hat{\alpha})}{d \ln \lambda} = \left( 1 + \frac{c(\hat{\alpha})^{\sigma-1}}{1 - c(\hat{\alpha})^{\sigma-1}} \right) \zeta_{c(\hat{\alpha}), \lambda} = (1 + \Gamma(\hat{\alpha})^{\sigma-1}) \zeta_{c(\hat{\alpha}), \lambda} > \zeta_{c(\hat{\alpha}), \lambda}. \quad (C-22)$$

For $\zeta_{\varphi_o, \lambda}$, it follows from equation (23) that

$$\zeta_{\varphi_o, \lambda} = \zeta_{\Gamma(\hat{\alpha}), \lambda} + \zeta_{\varphi, \lambda}. \quad (C-23)$$

To obtain $\zeta_{\varphi, \lambda}$, note that given the free entry condition—equation (26)—it must be true that $\frac{d \Pi}{d x} = 0$. Using equation (22) to rewrite $\Pi$ as

$$\Pi = \int_{\bar{\varphi}}^{\varphi_o} \left( \frac{\varphi}{\bar{\varphi}} \right)^{\sigma-1} f(g) d\varphi + \int_{\bar{\varphi}}^{\varphi_o} \left( \frac{\varphi}{c(\hat{\alpha})} \right)^{\sigma-1} f(g) d\varphi - f(1 - G(\hat{\varphi})) - f_o(1 - G(\bar{\varphi}_o)), \quad (C-24)$$

we apply Leibniz’s rule along with $B^{\sigma-1} = \frac{\bar{\varphi}}{\bar{\varphi}_o}$, and equations (23) and (C-23) to obtain that $\frac{d \Pi}{d x} = 0$ is equivalent to

$$\zeta_{\varphi, \lambda} [1 - G(\hat{\varphi})] \bar{\varphi}^\sigma - 1 + \zeta_{c(\hat{\alpha}), \lambda} [1 - G(\bar{\varphi}_o)] \left( \frac{\bar{\varphi}_o}{c(\hat{\alpha})} \right)^{\sigma-1} = 0, \quad (C-25)$$

where

$$\bar{\varphi} = \left[ \frac{N_n}{N} \bar{\varphi}_n^{\sigma-1} + \frac{N_o}{N} \left( \frac{\bar{\varphi}_o}{c(\hat{\alpha})} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \quad (C-26)$$

is the average productivity of domestic active producers, with $\bar{\varphi}_n$ and $\bar{\varphi}_o$ defined as in (A-2). Given that $\frac{N_o}{N} = \frac{1 - G(\bar{\varphi}_o)}{1 - G(\bar{\varphi})}$, we can solve for $\zeta_{\varphi, \lambda}$ from (C-25) as

$$\zeta_{\varphi, \lambda} = -\frac{N_o}{N} \left( \frac{\bar{\varphi}_o}{c(\hat{\alpha})} \right)^{\sigma-1} \zeta_{c(\hat{\alpha}), \lambda}. \quad (C-27)$$
Lastly, from (19), (A-1), and (A-4) it follows that \( \mu_o = \frac{N_o}{N} \left( \frac{\hat{\phi}_o}{c(\hat{\alpha})\hat{\varphi}} \right)^{\sigma-1} \) and hence, we can rewrite (C-27) as

\[
\zeta_{\hat{\varphi},\lambda} = -\mu_o \zeta_{c(\hat{\alpha}),\lambda},
\]

where \( \mu_o \in (0, 1) \) is the market share of offshoring firms. It follows that \( \zeta_{\hat{\varphi},\lambda} \in (-\zeta_{c(\hat{\alpha}),\lambda}, 0) \). This result and equations (C-22) and (C-23) imply that \( \zeta_{\hat{\varphi},\lambda} > 0 \).

**Proof of Proposition 1.** For active firms that do not change their offshoring status, we substitute the results from the proof of Lemma 4 into equation (39) and obtain

\[
\zeta_{L_s(\varphi),\lambda} = \begin{cases} 
(\sigma - 1)\mu_o \zeta_{c(\hat{\alpha}),\lambda} > 0 & \text{if } s = n \\
\frac{(1 - \hat{\alpha})k(\hat{\varphi})}{(1 - \hat{\alpha})k'(\hat{\varphi})} \left( (\sigma - \rho) \zeta_{c(\hat{\alpha}),\lambda} + (\sigma - 1)\mu_o \zeta_{c(\hat{\alpha}),\lambda} \right) > 0 & \text{if } s = o.
\end{cases}
\]

Hence, after a decline in \( \lambda \), non-offshoring firms (with productivities between \( \hat{\varphi} \) and \( \hat{\varphi}_o \)) decrease their employment of domestic labor (\( \zeta_{L_n(\varphi),\lambda} > 0 \)). For offshoring firms (\( \varphi \geq \hat{\varphi}_o \)), the job-relocation (first term) and selection (third term) effects always cause destruction of domestic labor after a decline in \( \lambda \). The productivity effect (second term), however, causes job creation if \( \rho < \sigma \), job destruction if \( \rho > \sigma \), and has no impact if \( \rho = \sigma \). Thus, \( \zeta_{L_o(\varphi),\lambda} > 0 \) if \( \rho \geq \sigma \).

Let us now see the case when \( \rho < \sigma \). Note that the sum of the second and third terms is negative—so that after a decline in \( \lambda \) the job creation due to the productivity effect dominates the job destruction due to the selection effect—if and only if

\[
\rho < \sigma - (\sigma - 1)\mu_o = \bar{\rho},
\]

where \( \bar{\rho} \in (1, \sigma) \). Nevertheless, even in this case the final effect is ambiguous due to the job-relocation effect. Summarizing, for continuing offshoring firms \( \zeta_{L_o(\varphi),\lambda} > 0 \) if \( \rho \geq \bar{\rho} \) and its sign is ambiguous otherwise.

After a decline in \( \lambda \), \( \hat{\varphi} \) increases and \( \hat{\varphi}_o \) decreases. Given our assumption that \( \hat{\varphi} < \hat{\varphi}_o \), we get that the firms between the old and new \( \hat{\varphi} \) die (their employment drops to zero), while the firms between the new and old \( \hat{\varphi}_o \) change their offshoring status from non-offshoring to offshoring. For the last group of firms, note that we can rewrite equation (30) as

\[
L_s(\varphi) = \frac{1 - \hat{\alpha} \cdot \mathbb{I}\{s = o\} (\sigma - 1)}{c(\hat{\alpha})^{\mathbb{I}\{s = o\}}} \left( \frac{\varphi}{\hat{\varphi}} \right)^{\sigma - 1} f,
\]

where \( \mathbb{I}\{s = o\} \) is an indicator function taking the value of 1 if the firm is offshoring. Thus, we see that for firms that change their status to offshoring after a decline in \( \lambda \): i) \( \hat{\alpha} \cdot \mathbb{I}\{s = o\} \) changes...
from 0 to $\hat{\alpha}$, ii) $\hat{\varphi}$ increases, and iii) $c(\hat{\alpha})^\{s=0\}$ changes from 1 to $c(\hat{\alpha}) < 1$. i) and ii) account, respectively, for the job-relocation and selection effects and push for a decrease in domestic labor demand of new offshoring firms. On the other hand, iii) accounts for the productivity effect and moves in the opposite direction to the other two effects if $\rho < \sigma$, and in the same direction if $\rho > \sigma$.

Proof of Proposition 2. For the response of the mass of entrants, $N_E$, to a change in $\lambda$, note first from (21) that

$$\zeta_{P,\lambda} = -\left(\frac{\sigma - 1}{\sigma - \eta}\right) \zeta_{\hat{\varphi},\lambda}. \quad \text{(C-31)}$$

Using Leibniz’s rule to take the derivative of the log of $N_E$ in (A-3) with respect to the log of $\lambda$, and using (C-31) we obtain

$$\zeta_{N_{E,\lambda}} = \frac{\hat{\varphi}^{\sigma-1}}{(1 - G(\hat{\varphi}))^{\varphi^{\sigma-1}} \left( B^{\sigma-1} g(\hat{\varphi}) \hat{\varphi} \zeta_{\hat{\varphi},\lambda} + g(\hat{\varphi}) \hat{\varphi} \zeta_{\hat{\varphi},\lambda} \right) + \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} \zeta_{\hat{\varphi},\lambda}}. \quad \text{(C-32)}$$

Using the results in the proof of Lemma 4, equations (24), (C-26), the definition of $\hat{\varphi}_o$, and $N_o = \frac{1}{1 - G(\hat{\varphi}_o)}$, we get

$$\frac{\zeta_{\hat{\varphi}_o,\lambda}}{\zeta_{\hat{\varphi},\lambda}} = -\frac{1}{B^{\sigma-1}} \left[ \int_{\varphi_o}^{\infty} g(\varphi) \left( \frac{\varphi}{\hat{\varphi}_o} \right)^{\sigma-1} g(\varphi) d\varphi \right]. \quad \text{(C-33)}$$

Substituting equation (C-33) into (C-32) and rearranging terms we get

$$\zeta_{N_{E,\lambda}} = -\frac{g(\hat{\varphi}) \hat{\varphi} \zeta_{\hat{\varphi},\lambda}(J - 1)}{(1 - G(\hat{\varphi}))^{\varphi^{\sigma-1}}} + \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} \zeta_{\hat{\varphi},\lambda} \quad \text{(C-34)}$$

where

$$J = \frac{\int_{1}^{\infty} u^{\sigma-1} g(u) \left( \frac{1}{g(u)} \right) du}{\int_{1}^{\infty} u^{\sigma-1} g(u) \left( \frac{1}{g(u)} \right) du}. \quad \text{(C-35)}$$

As $\zeta_{\hat{\varphi},\lambda} < 0$, the first term of (C-34) is positive if $J > 1$, negative if $J < 1$, and equals zero if $J = 1$, while the second term is negative (because $\eta > 1$).

Given that $u \geq 1$, a sufficient condition for $J \geq 1$ is

$$\frac{d}{d\varphi} \left[ \frac{d\ln g(\varphi)}{d\ln \varphi} \right] \leq 0 \quad \text{(C-36)}$$

for every $\varphi$. This condition is by no means restrictive, as it is satisfied by several commonly used distributions for non-negative random variables including the Pareto, lognormal, Weibull, $F$, Chi-squared, Exponential, Fréchet, and Gamma distributions.\(^1\) We assume that condition (C-36) holds,

\(^1\)To our knowledge, this list covers all the distributions that have been used in heterogeneous-firm models. The condition is satisfied with equality for the Pareto distribution.
and hence
\[
\zeta_{N,E,\lambda} = -\frac{g(\hat{\phi})\hat{\phi}^\sigma \zeta_{\hat{\phi},\lambda}(J-1)}{(1 - G(\hat{\phi}))\hat{\phi}^{\sigma-1}} + \frac{(\eta - 1)(\sigma - 1)\zeta_{\hat{\phi},\lambda}}{\sigma - \eta} + \left(\frac{\eta - 1}{\sigma - \eta}\right)\zeta_{\hat{\phi},\lambda} \tag{C-37}
\]

The sign of \(\zeta_{N,E,\lambda}\) is ambiguous, but it is more likely to become negative as \(\eta \to \sigma\).

For the mass of firms, \(N\), note from \(N = [1 - G(\hat{\phi})]N_E\) that \(\zeta_{N,\lambda} = \zeta_{N,E,\lambda} - \frac{g(\hat{\phi})\hat{\phi}^\sigma \zeta_{\hat{\phi},\lambda}}{1 - G(\hat{\phi})}\). Substituting (C-32) into the previous expression we get
\[
\zeta_{N,\lambda} = \frac{\hat{\phi}^{\sigma-1}}{(1 - G(\hat{\phi}))\hat{\phi}^{\sigma-1}}\left(B^{\sigma-1}g(\hat{\phi}_o)\hat{\phi}_o\zeta_{\hat{\phi},\lambda} - g(\hat{\phi})\hat{\phi}\zeta_{\hat{\phi},\lambda}\left[\left(\frac{\hat{\phi}}{\bar{\phi}}\right)^{\sigma-1} - 1\right]\right) + \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} \zeta_{\hat{\phi},\lambda}. \tag{C-38}
\]

The first term is positive because \(\zeta_{\hat{\phi},\lambda} > 0\), \(\zeta_{\hat{\phi},\lambda} < 0\), and \(\hat{\phi} > \bar{\phi}\) (recall that \(\bar{\phi}\) is the average productivity conditioning on \(\phi \geq \bar{\phi}\)). The second term moves in the opposite direction and therefore, the final effect on \(N\) is ambiguous.

For the mass of offshoring firms, \(N\), we know that \(N_o = [1 - G(\hat{\phi}_o)]N_E\) and therefore, \(\zeta_{N,\lambda} = \zeta_{N,E,\lambda} - \frac{g(\hat{\phi}_o)\hat{\phi}_o\zeta_{\hat{\phi},\lambda}}{1 - G(\hat{\phi}_o)}\). Plugging in (C-32) into the previous equation, we obtain
\[
\zeta_{N,\lambda} = \frac{\hat{\phi}^{\sigma-1}}{(1 - G(\hat{\phi}))\hat{\phi}^{\sigma-1}}\left(-\frac{g(\hat{\phi}_o)\hat{\phi}_o\zeta_{\hat{\phi},\lambda}}{\hat{\phi}^{\sigma-1}}\left[N\frac{\hat{\phi}^{\sigma-1} - B^{\sigma-1}\hat{\phi}^{\sigma-1}}{\hat{\phi}}\right] + g(\hat{\phi})\hat{\phi}\zeta_{\hat{\phi},\lambda}\right) + \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} \zeta_{\hat{\phi},\lambda}. \tag{C-39}
\]

The first term is negative because \(\zeta_{\hat{\phi},\lambda} > 0\), \(\zeta_{\hat{\phi},\lambda} < 0\), and \(\frac{N}{\hat{\phi}}\hat{\phi}^{\sigma-1} > B^{\sigma-1}\hat{\phi}^{\sigma-1}\)—the last result follows from (23), (24), and (C-26). Hence, \(\zeta_{N,\lambda} < 0\).

Lastly, for the mass of non-offshoring firms, \(N\), we know that \(N_n = [G(\hat{\phi}_o) - G(\hat{\phi})]N_E\) and therefore
\[
\zeta_{N,\lambda} = \left[\frac{g(\hat{\phi}_o)\hat{\phi}_o\zeta_{\hat{\phi},\lambda} - g(\hat{\phi})\hat{\phi}\zeta_{\hat{\phi},\lambda}}{G(\hat{\phi}_o) - G(\hat{\phi})}\right] + \zeta_{N,E,\lambda}. \tag{C-39}
\]

Although the first term is positive, the ambiguity in the sign of \(\zeta_{N,E,\lambda}\) carries over to \(\zeta_{N,\lambda}\). As with \(N_E\), \(N_n\) is more likely to increase after a decline in \(\lambda\) when \(\eta\) approaches \(\sigma\).

The case with \(\eta \to 1\): Note that the last term is (C-37) and (C-38) approaches zero as \(\eta \to 1\), and therefore, \(\zeta_{N,E,\lambda} \geq 0\) and \(\zeta_{N,\lambda} > 0\) as \(\eta \to 1\). From (C-39), it follows that \(\zeta_{N,\lambda} > 0\). Hence, when \(\eta \to 1\), a reduction in \(\lambda\) causes a decline in both \(N\) and \(N_n\), and \(N_E\) either declines or remains constant.

\[\square\]

**Proof of Proposition 3.** Let us work with each of the components of equation (40).

**Extensive margin.** In equation (40) we have that the net extensive margin (NEM\(_\lambda\)) has two components: the change in domestic employment due to non-offshoring firms that stop (or start)
producing due to the effect of \( \lambda \) on \( \hat{\varphi} \), \( \text{EM}_{1,\lambda} = \left[ -L_n(\hat{\varphi})g(\hat{\varphi}) \frac{d\hat{\varphi}}{d\lambda} \right] N_E \); and the change in domestic employment due to the effect of \( \lambda \) on the mass of entrants, \( \text{EM}_{2,\lambda} = \frac{L_n}{N_E} \frac{dN_E}{d\lambda} \).

For \( \text{EM}_{1,\lambda} \), we use equation (30) to rewrite it as

\[
\text{EM}_{1,\lambda} = \left[ -\frac{(\sigma - 1)f g(\hat{\varphi}) \hat{\varphi} \zeta_{\hat{\varphi},\lambda}}{\lambda \hat{w}_Z} \right] N_E > 0. \tag{C-40}
\]

\( \text{EM}_{1,\lambda} \) is positive because \( \zeta_{\hat{\varphi},\lambda} < 0 \).

For \( \text{EM}_{2,\lambda} \), note that we can rewrite it as \( \text{EM}_{2,\lambda} = \frac{L_n}{\lambda} \zeta_{N_E,\lambda} \). We know from Proposition 2 that the sign of \( \zeta_{N_E,\lambda} \) is ambiguous. It follows that \( \text{NEM}_{\lambda} = \text{EM}_{1,\lambda} + \text{EM}_{2,\lambda} \) is also ambiguous.

We will now rewrite \( \text{EM}_{2,\lambda} \) in a way that will allow us to make comparisons with the other margins. For \( L_Z \equiv N_n \bar{L}_n + N_o \bar{L}_o \), note that given the definitions of \( \bar{L}_n \) and \( \bar{L}_o \) in section 4 and given Lemma 3, it follows that

\[
\bar{L}_n = L_n(\hat{\varphi}_n) = \frac{(\sigma - 1)}{\hat{w}_Z} \left( \hat{\varphi}_n \hat{\varphi} \right)^{\sigma - 1} f \quad \text{and} \quad \bar{L}_o = L_o(\hat{\varphi}_o) = \frac{(1 - \hat{\alpha})(\sigma - 1)}{c(\hat{\alpha})^{\sigma - \rho} \hat{w}_Z} \left( \hat{\varphi}_o \hat{\varphi} \right)^{\sigma - 1} f.
\]

Using the expressions for \( N_n \) and \( N_o \), along with the equations for \( \zeta_{c(\hat{\alpha}),\lambda} \) and \( \zeta_{\hat{\varphi},\lambda} \) from Lemma 4, we obtain

\[
L_Z = \left[ (1 - G(\hat{\varphi})) (\sigma - 1) \frac{\hat{\varphi}^{\sigma - 1}}{\hat{w}_Z} \right] f (1 + \zeta_{\hat{\varphi},\lambda}) N_E. \tag{C-41}
\]

Lastly, from equations (C-40), (C-34), (C-41) we can write \( \text{EM}_{2,\lambda} = \frac{L_n}{\lambda} \zeta_{N_E,\lambda} \) as

\[
\text{EM}_{2,\lambda} = \text{EM}_{1,\lambda}(1 + \zeta_{\hat{\varphi},\lambda}) \left[ J - 1 - \frac{(\sigma - 1)(\eta - 1)(1 - G(\hat{\varphi}))}{(\sigma - \eta)\hat{\varphi}^{\sigma - 1}} \right]. \tag{C-42}
\]

**Intensive margin.** From equation (40) we know that the net intensive margin (NIM\(_\lambda\)) has three components: the change in domestic employment of firms that change their offshoring status, \( \text{IM}_{1,\lambda} = \left[ (L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o))g(\hat{\varphi}_o) \frac{d\hat{\varphi}_o}{d\lambda} \right] N_E \); the change in domestic employment of continuing non-offshoring firms, \( \text{IM}_{2,\lambda} = \left[ \int_{\hat{\varphi}_o}^{\hat{\varphi}_n} \frac{dL_n(\hat{\varphi})}{d\lambda} g(\hat{\varphi}) d\hat{\varphi} \right] N_E \); and the change in domestic employment of continuing offshoring firms, \( \text{IM}_{3,\lambda} = \left[ \int_{\hat{\varphi}_o}^{\hat{\varphi}_n} \frac{dL_o(\hat{\varphi})}{d\lambda} g(\hat{\varphi}) d\hat{\varphi} \right] N_E \).

For \( \text{IM}_{1,\lambda} \), we use the expressions for \( L_n(\hat{\varphi}) \) and \( L_o(\hat{\varphi}) \) in equation (30), along with equations (23), (24), and Lemma 4 to obtain

\[
\text{IM}_{1,\lambda} = \text{EM}_{1,\lambda} \left[ -\frac{B^{\sigma - 1} \Gamma(\hat{\alpha})^{\sigma - 1} g(\hat{\varphi}_o) \hat{\varphi}_o \zeta_{\hat{\varphi}_o,\lambda}}{g(\hat{\varphi}) \hat{\varphi} \zeta_{\hat{\varphi},\lambda}} \right] \left[ 1 - \frac{1 - \hat{\alpha}}{c(\hat{\alpha})^{\sigma - \rho}} \right].
\]

Given the definition of \( J \) in (C-35), we rewrite equation (C-33) as \( \frac{\zeta_{\hat{\varphi}_o,\lambda}}{\zeta_{\hat{\varphi},\lambda}} = -\frac{g(\hat{\varphi}_o) \hat{\varphi}_o J}{B^{\sigma - 1} g(\hat{\varphi}_o) \hat{\varphi}_o} \). Thus, the expression for \( \text{IM}_{1,\lambda} \) simplifies to

\[
\text{IM}_{1,\lambda} = \text{EM}_{1,\lambda} J \Gamma(\hat{\alpha})^{\sigma - 1} \left[ 1 - \frac{1 - \hat{\alpha}}{c(\hat{\alpha})^{\sigma - \rho}} \right].
\]

The sign of \( \text{IM}_{1,\lambda} \) is ambiguous if \( \rho < \sigma \)—the term in brackets determines the sign. Note that \( \text{IM}_{1,\lambda} < 0 \), implying job creation by expansion—for new offshoring firms—after a decline in \( \lambda \), if
$c(\hat{\alpha})^{\sigma-\rho} < 1 - \hat{\alpha}$ (so that the productivity effect outweighs the job-relocation effect). If $\rho \geq \sigma$, it follows that $c(\hat{\alpha})^{\sigma-\rho} > 1 - \hat{\alpha}$ and thus, $\text{IM}_{1,\lambda} > 0$—job destruction by contraction for existing firms that begin to offshore after a decline in $\lambda$.

For $\text{IM}_{2,\lambda}$, note that we can rewrite it as

$$\text{IM}_{2,\lambda} = \frac{\zeta L_n(\varphi),\lambda}{\lambda} \left[ \int_{\hat{\varphi}}^\varphi L_n(\varphi) g(\varphi) d\varphi \right] N_E = \frac{\zeta L_n(\varphi),\lambda}{\lambda} N_n \bar{L}_n.$$  

From equation (C-29) in the proof of Proposition 1 we know that $\zeta L_n(\varphi),\lambda > 0$ and thus, $\text{IM}_{2,\lambda} > 0$.

Then, after a decline in $\lambda$, existing non-offshoring firms generate job destruction by contraction.

Following the same steps, for existing offshoring firms we obtain $\text{IM}_{3,\lambda} = \frac{\zeta L_o(\varphi),\lambda}{\lambda} N_o \bar{L}_o$. The sign for $\text{IM}_{3,\lambda}$ is then determined by the sign of $\zeta L_o(\varphi),\lambda$. Again, from equation (C-29) in the proof of Proposition 1, we get that $\text{IM}_{3,\lambda} > 0$ if $\rho \geq \sigma - (\sigma - 1) \mu_o$ and its sign is ambiguous if $\rho < \sigma - (\sigma - 1) \mu_o$. Therefore, after a decline in $\lambda$, existing offshoring firms generate job destruction by contraction if $\rho \geq \sigma - (\sigma - 1) \mu_o$ and the effect is ambiguous otherwise.

The sum of $\text{IM}_{2,\lambda}$ and $\text{IM}_{3,\lambda}$ is given by $\text{IM}_{2,\lambda} + \text{IM}_{3,\lambda} = \frac{\zeta L_n(\varphi),\lambda}{\lambda} N_n \bar{L}_n + \frac{\zeta L_o(\varphi),\lambda}{\lambda} N_o \bar{L}_o$. Using the expressions for $N_n$, $N_o$, $\bar{L}_n$, and $\bar{L}_o$ obtained above, along with equations (C-29) and (C-40) and Lemma 4, we rewrite the sum as

$$\text{IM}_{2,\lambda} + \text{IM}_{3,\lambda} = \frac{\zeta L_n(\varphi),\lambda}{\lambda} \left[ \int_{\hat{\varphi}}^\varphi L_n(\varphi) g(\varphi) d\varphi \right] N_E + \frac{\zeta L_o(\varphi),\lambda}{\lambda} N_o \bar{L}_o.$$  

Note that the last term inside the brackets is negative only when $\rho < 1$. More specifically, note that the sum of the second and third terms inside the brackets is only negative when

$$\rho < 1 - \frac{(\sigma - 1)(1 - \mu_o)\zeta c(\hat{\alpha}),\lambda}{1 - \zeta c(\hat{\alpha}),\lambda}.$$  

Thus, we have that $\text{IM}_{2,\lambda} + \text{IM}_{3,\lambda} > 0$ if $\rho \geq 1 - \frac{(\sigma - 1)(1 - \mu_o)\zeta c(\hat{\alpha}),\lambda}{1 - \zeta c(\hat{\alpha}),\lambda}$. Otherwise, the sign of $\text{IM}_{2,\lambda} + \text{IM}_{3,\lambda}$ is ambiguous.

The net intensive margin effect ($\text{NIM}_\lambda$) is given by the sum of $\text{IM}_{1,\lambda}$, $\text{IM}_{2,\lambda}$, and $\text{IM}_{3,\lambda}$. Even though $\text{IM}_{2,\lambda} + \text{IM}_{3,\lambda}$ is ambiguous only when $\rho < 1 - \frac{(\sigma - 1)(1 - \mu_o)\zeta c(\hat{\alpha}),\lambda}{1 - \zeta c(\hat{\alpha}),\lambda}$, the ambiguity of $\text{IM}_{1,\lambda}$ when $\rho < \sigma$ is carried out to the sign of $\text{NIM}_\lambda$. Hence, $\text{NIM}_\lambda > 0$ if $\rho \geq \sigma$ and its sign is ambiguous otherwise.

**Overall effect.** The net effect on employment, $\frac{dL}{d\lambda} = \text{NEM}_\lambda + \text{NIM}_\lambda$, is ambiguous even if $\rho \geq \sigma$. This overall ambiguity is driven by the ambiguous effect of $\lambda$ on the mass of entrants, $\text{EM}_{2,\lambda}$, due to the role of $\eta$. Note that as $\eta \to \sigma$, the negative term in (C-42) tends to $-\infty$. As $\eta$ declines, the negative term approaches 0 and overall job destruction after a decline in $\lambda$ is more likely.
The previous expression is positive because
\[
\eta_{\alpha} < 0, \quad \eta_{\alpha} \in (0, 1), \quad J \geq 1.
\]
In addition,
\[
NEM_{\lambda} + IM_{1, \lambda} \rightarrow EM_{1, \lambda} \left[ -\zeta_{\phi, \lambda} + J \left( 1 + \zeta_{\phi, \lambda} + \Gamma(\hat{\alpha})^{\sigma-1} - \frac{\Gamma(\hat{\alpha})^{\sigma-1}(1 - \hat{\alpha})}{c(\hat{\alpha})^{\sigma-\rho}} \right) \right].
\]
Using \( \zeta_{\phi, \lambda} = -(1 - \alpha)c(\hat{\alpha})^{\rho-1} = 1 - \zeta_{c(\hat{\alpha}), \lambda}, \) and \( \Gamma(\hat{\alpha})^{\sigma-1} = \frac{c(\hat{\alpha})^{\sigma-1}}{1-c(\hat{\alpha})^{\sigma-\rho}} \) into the previous expression, it simplifies to
\[
NEM_{\lambda} + IM_{1, \lambda} \rightarrow EM_{1, \lambda} \zeta_{c(\hat{\alpha}), \lambda} \left[ \mu_0 + J(1 - \mu_0 + \Gamma(\hat{\alpha})^{\sigma-1}) \right] > 0.
\]
The previous expression is positive because \( \mu_0 \in (0, 1), \) \( J \geq 1, \) \( \Gamma(\hat{\alpha}) > 0, \) and \( \zeta_{c(\hat{\alpha}), \lambda} > 0. \) Therefore, if \( \eta \rightarrow 1, \) any possible job creation due to \( IM_{1, \lambda} \) is dominated by the job destruction generated at the extensive margin.

It follows that with \( \eta \rightarrow 1, \) \( \frac{dL^Z}{d\alpha} > 0 \) if \( \rho \geq 1 - \frac{(\sigma-1)(1-\mu_0)\zeta_{c(\hat{\alpha}), \lambda}}{1-\zeta_{c(\hat{\alpha}), \lambda}}, \) where \( \frac{(\sigma-1)(1-\mu_0)\zeta_{c(\hat{\alpha}), \lambda}}{1-\zeta_{c(\hat{\alpha}), \lambda}} > 0, \) and the sign is ambiguous otherwise.

**Proof of Lemma 5.** Note from the condition for the optimal choice of \( \hat{\alpha} \) that
\[
\lambda k(\hat{\alpha})w^* = \hat{w}_z.
\]
Therefore, \( \frac{d\ln k(\hat{\alpha})}{d\ln \hat{w}_z} = 1 \) or \( \frac{\hat{\alpha} k'(\hat{\alpha})}{k(\hat{\alpha})} \frac{d\ln \hat{\alpha}}{d\ln \hat{w}_z} = 1 \) or \( \zeta_{\alpha, \hat{w}_z} = \frac{k(\hat{\alpha})}{\hat{\alpha} k'(\hat{\alpha})} > 0. \)

Let us now obtain \( \zeta_{c(\hat{\alpha}), \hat{w}_z}. \) Given that \( k(\hat{\alpha}) = \frac{\hat{w}_z}{\lambda w^*}, \) we can rewrite equation (C-15) as \( c(\hat{\alpha}) = \left[ \left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha} \right]^{-1/(1-\rho)} \). Hence, it follows from the envelope theorem that
\[
\frac{dc(\hat{\alpha})}{d\hat{w}_z} = -\left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha} \left[ \left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) \right]^{-1/(1-\rho)} \frac{1}{\hat{w}_z} \left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha})
\]
Therefore,
\[
\zeta_{c(\hat{\alpha}), \hat{w}_z} = \frac{dc(\hat{\alpha})}{d\hat{w}_z} \frac{\hat{w}_z}{c(\hat{\alpha})} = -\frac{\left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha})}{\left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} K(\hat{\alpha}) + 1 - \hat{\alpha}}.
\]
Using \( k(\hat{\alpha})^{\rho-1} = \left( \frac{\lambda w^*}{\hat{w}_z} \right)^{1-\rho} \) and the definition of \( c(\hat{\alpha}) \), we rewrite the previous expression as
\[
\zeta_{c(\hat{\alpha}), \hat{w}_z} = -\frac{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha})}{k(\hat{\alpha})^{\rho-1} K(\hat{\alpha}) + 1 - \hat{\alpha}} = (1 - \hat{\alpha})c(\hat{\alpha})^{\rho-1} - 1 = -\zeta_{c(\hat{\alpha}), \lambda} \in (-1, 0).
\]
For \( \zeta_{\Gamma(\hat{\alpha}), \hat{w}_z} \), we use the expression for \( \Gamma(\hat{\alpha}) \) in equation (24) to get
\[
\zeta_{\Gamma(\hat{\alpha}), \hat{w}_z} = \left( 1 + \frac{c(\hat{\alpha})^{\sigma-1}}{1 - c(\hat{\alpha})^{\sigma-1}} \right) \zeta_{c(\hat{\alpha}), \hat{w}_z} = \left( 1 + \Gamma(\hat{\alpha})^{\sigma-1} \right) \zeta_{c(\hat{\alpha}), \hat{w}_z} < \zeta_{c(\hat{\alpha}), \hat{w}_z} < 0.
\]
To obtain \( \zeta \hat{\phi}, \hat{w}_Z \) we follow the same steps as in the proof for \( \zeta \hat{\phi}, \lambda \) in Lemma 4 to obtain that

\[
\zeta \hat{\phi}, \hat{w}_Z = -\mu_o \zeta c(\hat{\alpha}), \hat{w}_Z \in (0, -\zeta c(\hat{\alpha}), \hat{w}_Z) > 0.
\]

Given that \( \zeta \hat{\phi}_o, \hat{w}_Z = \zeta \Gamma(\hat{\alpha}), \hat{w}_Z + \zeta \hat{\phi}, \hat{w}_Z \), it follows that

\[
\zeta \hat{\phi}_o, \hat{w}_Z = (1 - \mu_o + \Gamma(\hat{\alpha})\sigma^{-1}) \zeta c(\hat{\alpha}), \hat{w}_Z < 0.
\]

\[
\begin{array}{l}
\zeta L_o(\varphi), \hat{w}_Z = \left\{ \begin{array}{ll}
\frac{(\sigma - 1)\mu_o \zeta c(\hat{\alpha}), \hat{w}_Z - 1}{\zeta c(\hat{\alpha}), \hat{w}_Z} < 0 & \text{if } s = n \\
\frac{\hat{\alpha}}{1 - \alpha} \zeta c(\hat{\alpha}), \hat{w}_Z - (\sigma - \rho) \zeta c(\hat{\alpha}), \hat{w}_Z + (\sigma - 1)\mu_o \zeta c(\hat{\alpha}), \hat{w}_Z - 1 & > 0 \text{ if } \rho < \sigma \\
< 0 & \text{if } \rho \geq \sigma
\end{array} \right.
\end{array}
\]

Hence, after a decline in \( \hat{w}_Z \), non-offshoring firms increase their employment of domestic labor \( (\zeta L_o(\varphi), \hat{w}_Z < 0) \). For offshoring firms, the job-relocation (first term), selection (third term), and direct marginal cost (fourth term, \(-1\)) effects always cause creation of domestic labor after a decline in \( \hat{w}_Z \). The productivity effect (second term), however, causes job destruction if \( \rho < \sigma \), job creation if \( \rho > \sigma \), and has no impact if \( \rho = \sigma \). Thus, \( \zeta L_o(\varphi), \hat{w}_Z < 0 \) if \( \rho \geq \sigma \).

For the \( \rho < \sigma \) case, the sum of the second, third, and fourth terms is positive if and only if

\[
\rho < \frac{1 + \zeta c(\hat{\alpha}), \hat{w}_Z}{\zeta c(\hat{\alpha}), \hat{w}_Z} + (\sigma - 1)(1 - \mu_o) \equiv \tilde{\rho},
\]

where \( \tilde{\rho} < \sigma - 1 \). Then, for continuing offshoring firms \( \zeta L_o(\varphi), \hat{w}_Z < 0 \) if \( \rho \geq \tilde{\rho} \) and its sign is ambiguous otherwise.

After a decline in \( \hat{w}_Z \), \( \hat{\phi} \) declines and \( \hat{\phi}_o \) increases. Then, firms between the new and old \( \hat{\phi} \) are being born, while the firms between the old and new \( \hat{\phi}_o \) change their offshoring status from offshoring to non-offshoring. Note from (C-30) that for firms that stop offshoring after a decline in \( \hat{w}_Z \), their demand for domestic labor increases due to: i) \( \hat{\alpha} \cdot \mathbb{1}\{s = o\} \) changes from \( \hat{\alpha} \) to 0 (job relocation), ii) \( \hat{\phi} \) declines (selection effect), iii) the direct effect of the decline in \( \hat{w}_Z \). Also, \( c(\hat{\alpha})^{(s=o)} \) changes from \( c(\hat{\alpha}) \) to 1 (productivity effect) and causes a decline in the demand for domestic labor if \( \rho \leq \sigma \), and an increase if \( \rho > \sigma \). Hence, the effect is ambiguous for these firms if \( \rho < \sigma \), but they expand labor if \( \rho \geq \sigma \).
For the response of the mass of entrants, \( N_E \), to a change in \( \hat{w}_Z \), note first from (21) that

\[
\zeta_{P,\hat{w}_Z} = \left( \frac{\sigma - 1}{\sigma - \eta} \right) (1 - \zeta_{\phi,\hat{w}_Z}). 
\]  

Using Leibniz’s rule to take the derivative of the log of equation (A-3) with respect to the log of \( \hat{w}_Z \), and using (C-43) we obtain

\[
\zeta_{N_E, \hat{w}_Z} = \frac{\hat{\phi}^{\sigma-1}}{(1 - G(\hat{\phi}))^{\bar{\phi}^{\sigma-1}}} \left( B^{\sigma-1} g(\hat{\phi}_o) \hat{\phi}_o \zeta_{\phi_o, \hat{w}_Z} + g(\hat{\phi}) \hat{\phi} \zeta_{\phi, \hat{w}_Z} \right) - \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} (1 - \zeta_{\phi, \hat{w}_Z}). 
\]  

(A-3)

A useful result that spans from the proofs of Lemmas 4 and 5 is that \( \zeta_{\hat{a}, \hat{w}_Z} = -\zeta_{\hat{a}, \lambda}, \zeta_{\hat{c}(\hat{a}), \hat{w}_Z} = -\zeta_{\hat{c}(\hat{a}), \lambda}, \zeta_{\hat{\phi}, \hat{w}_Z} = -\zeta_{\hat{\phi}, \lambda}, \zeta_{\hat{\phi}_o, \hat{w}_Z} = -\zeta_{\hat{\phi}_o, \lambda} \). Therefore, from the proof of Proposition 2 we can rewrite (C-44) as

\[
\zeta_{N_E, \hat{w}_Z} = \frac{g(\hat{\phi})\hat{\phi}^{\sigma}(J - 1)}{(1 - G(\hat{\phi}))^{\bar{\phi}^{\sigma-1}}} - \frac{(\eta - 1)(\sigma - 1)}{\sigma - \eta} (1 + \zeta_{\phi, \lambda}) < 0, 
\]  

which because \( J \geq 1 \) (condition (C-36) holds) and \( \zeta_{\phi, \lambda} \in (-1, 0) \).

For the mass of non-offshoring firms, \( N_n \), we use equation (30) to rewrite it as

\[
\zeta_{N_n, \hat{w}_Z} = \left[ \frac{g(\hat{\phi}_o)\hat{\phi}_o \zeta_{\phi_o, \hat{w}_Z} - g(\hat{\phi}) \hat{\phi} \zeta_{\phi, \hat{w}_Z}}{G(\hat{\phi}_o) - G(\hat{\phi})} \right] + \zeta_{N_E, \hat{w}_Z} < 0
\]

because first term is negative (\( \zeta_{\hat{\phi}_o, \hat{w}_Z} < 0, \zeta_{\hat{\phi}, \hat{w}_Z} > 0 \), and \( G(\hat{\phi}_o) > G(\hat{\phi}) \)) and \( \zeta_{N_E, \hat{w}_Z} < 0 \). For the mass of offshoring firms, \( N_o \), we know that \( N_o = (1 - G(\hat{\phi}_o))N_E \) and therefore, \( \zeta_{N_o, \hat{w}_Z} = \zeta_{N_E, \hat{w}_Z} - \frac{g(\hat{\phi}) \hat{\phi} \zeta_{\phi, \hat{w}_Z}}{1 - G(\hat{\phi}_o)} \). The second term is positive, and thus, the sign of \( \zeta_{N_o, \hat{w}_Z} \) is ambiguous.

For the last part of the proposition, note that

\[
\frac{dL_z}{d\hat{w}_Z} = \left[ -L_n(\hat{\phi})g(\hat{\phi}) \frac{d\hat{\phi}}{d\hat{w}_Z} \right] N_E + \frac{L_z dN_E}{N_E d\hat{w}_Z} + \left( \begin{array}{c} \frac{\hat{\phi}_o d\phi}{d\hat{w}_Z} \left( L_n(\hat{\phi}_o) - L_o(\hat{\phi}_o) \right) + \int_{\hat{\phi}_o}^{\hat{\phi}} \frac{dL_n(\phi)}{d\hat{w}_Z} g(\phi) d\phi + \int_{\hat{\phi}_o}^{\infty} \frac{dL_o(\phi)}{d\hat{w}_Z} g(\phi) d\phi \end{array} \right] N_E. 
\]  

(C-46)

Let us work with each of the components of equation (C-46).

**Extensive margin.** In equation (C-46) we have that the net extensive margin (NEM\(_{\hat{w}_Z}\)) has two components: the change in domestic employment due to non-offshoring firms that stop (or start) producing due to the effect of \( \hat{w}_Z \) on \( \hat{\phi} \), \( \text{EM}_1, \hat{w}_Z = \left[ -L_n(\hat{\phi})g(\hat{\phi}) \frac{d\hat{\phi}}{d\hat{w}_Z} \right] N_E \); and the change in domestic employment due to the effect of \( \hat{w}_Z \) on the mass of entrants, \( \text{EM}_2, \hat{w}_Z = \frac{L_z dN_E}{N_E \hat{w}_Z} \).

For \( \text{EM}_1, \hat{w}_Z \), we use equation (30) to rewrite it as

\[
\text{EM}_1, \hat{w}_Z = \left[ -\frac{(\sigma - 1)fg(\hat{\phi})\hat{\phi} \zeta_{\phi, \hat{w}_Z}}{\hat{w}_Z^2} \right] N_E < 0. 
\]  

(C-47)
EM$_1$,$\tilde{w}_z$ is negative because $\zeta_{\hat{\phi},\tilde{w}_z} > 0$.

For EM$_2$,$\tilde{w}_z$, note that we can rewrite it as $EM_2$,$\tilde{w}_z = \frac{L_{\hat{\phi}}}{w_z} \zeta_{N_E,\tilde{w}_z}$. We know from (C-45) that $\zeta_{N_E,\tilde{w}_z} < 0$ and hence EM$_2$,$\tilde{w}_z < 0$. It follows that NEM$_{\tilde{w}_z} = EM_1$,$\tilde{w}_z + EM_2$,$\tilde{w}_z < 0$. Thus, a decline in $\tilde{w}_z$ causes net job creation at the extensive margin.

**Intensive margin.** From equation (C-46) we know that the net intensive margin (NIM$_{\tilde{w}_z}$) has three components: the change in domestic employment of firms that change their offshoring status, IM$_1$,$\tilde{w}_z = \left[\left(\frac{E_{\hat{\phi}}}{w_z} - \frac{E_\phi}{\hat{\phi}}\right)\frac{\partial L_{\hat{\phi}}}{\partial \hat{\phi}}\right] N_E$; the change in domestic employment of continuing non-offshoring firms, IM$_2$,$\tilde{w}_z = \left[\int_{\hat{\phi}}^{\phi} \frac{\partial L_{\phi}}{\partial \phi} g(\phi) d\phi\right] N_E$; and the change in domestic employment of continuing offshoring firms, IM$_3$,$\tilde{w}_z = \left[\int_{\hat{\phi}}^{\phi} \frac{\partial L_{\phi}}{\partial \phi} g(\phi) d\phi\right] N_E$.

For IM$_1$,$\tilde{w}_z$, we use the expressions for $L_n(\phi)$ and $L_\phi(\phi)$ in equation (30), along with equations (23), (24), (C-33), and the results in the proof of Lemma 5 to obtain

$$IM_1$,$\tilde{w}_z = EM_1$,$\tilde{w}_z J\Gamma(\hat{\phi})\sigma^{-1} \left[1 - \frac{1 - \hat{\phi}}{c(\hat{\phi})^{\sigma-\rho}}\right].$$

The sign of IM$_1$,$\tilde{w}_z$ is ambiguous if $\rho < \sigma$—the term in brackets determines the sign. If $\rho \geq \sigma$, it follows that $c(\hat{\phi})^{\sigma-\rho} > 1 - \hat{\phi}$ and thus, IM$_1$,$\tilde{w}_z < 0$—job creation by expansion for existing firms that begin to offshore after a decline in $\tilde{w}_z$.

For IM$_2$,$\tilde{w}_z$, note that we can rewrite it as

$$IM_2$,$\tilde{w}_z = \frac{\zeta_{L_n(\phi),\tilde{w}_z}}{\tilde{w}_z} \left[\int_{\hat{\phi}}^{\phi} L_n(\phi) g(\phi) d\phi\right] N_E = \frac{\zeta_{L_n(\phi),\tilde{w}_z}}{\tilde{w}_z} N_n \hat{\phi}.$$

From above we know that $\zeta_{L_n(\phi),\tilde{w}_z} < 0$ and thus, IM$_2$,$\tilde{w}_z < 0$. Then, after a decline in $\tilde{w}_z$, existing non-offshoring firms create jobs by expansion.

Following the same steps, for existing offshoring firms we obtain IM$_3$,$\tilde{w}_z = \frac{\zeta_{L_\phi(\phi),\tilde{w}_z}}{\tilde{w}_z} N_\phi \hat{\phi}$. The sign for IM$_3$,$\tilde{w}_z$ is then determined by the sign of $\zeta_{L_\phi(\phi),\tilde{w}_z}$. Above we got that $\zeta_{L_\phi(\phi),\tilde{w}_z} < 0$ if $\rho \geq \hat{\phi}$, and then, IM$_3$,$\tilde{w}_z < 0$ if $\rho \geq \hat{\phi}$ and its sign is ambiguous if $\rho < \hat{\phi}$. Therefore, after a decline in $\tilde{w}_z$, existing offshoring firms create jobs by expansion if $\rho \geq \hat{\phi}$ and the effect is ambiguous otherwise.

The net intensive margin effect (NIM$_{\tilde{w}_z}$) is given by the sum of IM$_1$,$\tilde{w}_z$, IM$_2$,$\tilde{w}_z$, and IM$_3$,$\tilde{w}_z$. The ambiguity of IM$_1$,$\tilde{w}_z$ when $\rho < \sigma$ is carried out to the sign of NIM$_{\tilde{w}_z}$. Hence, NIM$_{\tilde{w}_z} < 0$ if $\rho \geq \sigma$ and its sign is ambiguous otherwise.

**Overall effect.** The net effect on employment, $\frac{dL_z}{d\tilde{w}_z} = NEM_{\tilde{w}_z} + NIM_{\tilde{w}_z}$, is ambiguous if $\rho < \sigma$. Otherwise, $\frac{dL_z}{d\tilde{w}_z} < 0$. Note that as $\eta \to \sigma$, NEM$_{\tilde{w}_z}$ tends to $-\infty$. Note also that NIM$_{\tilde{w}_z}$ is more likely to be negative for higher $\rho$. Thus, overall net job creation is more likely with higher $\eta$ and higher $\rho$. 

\[\square\]
D Fixed Costs of Offshoring and Employment

The channels through which a decline in the fixed cost of offshoring, \( f_o \), affect employment are the same as those described for a decline in \( \lambda \). The following proposition summarizes the effects of a decline in \( f_o \) on employment.

**Proposition D.1. (The fixed cost of offshoring and employment)**

A decline in \( f_o \) causes: (i) the death of the least productive non-offshoring firms, who then destroy all their jobs; (ii) job destruction (by contraction) at surviving firms that do not change their offshoring status; (iii) an ambiguous domestic labor response for existing firms that begin to offshore if \( \rho < \sigma \) (and job destruction otherwise); (iv) and increase in \( N_o \), but ambiguous responses of \( N_E \), \( N \), and \( N_n \); (v) an ambiguous domestic labor response at the extensive margin; (vi) an ambiguous response at the intensive margin if \( \rho < \sigma \) (and job destruction otherwise); (vii) an overall ambiguous effect, but net job creation is more likely for higher \( \eta \) and lower \( \rho \).

**Proof.** Parts (i), (ii), and (iii) of Proposition D.1 are the equivalent to Proposition 1 for a change in \( f_o \). In this case, \( \hat{\alpha} \) and \( c(\hat{\alpha}) \) are not affected by a change in \( f_o \); that is, the elasticities \( \zeta_{\hat{\alpha}, f_o} \) and \( \zeta_{c(\hat{\alpha}), f_o} \) are zero. Hence, from equation (30) we obtain that for active firms that do not change their offshoring status

\[
\zeta_{L_s(\varphi), f_o} = -(\sigma - 1) \frac{\hat{\varphi} f_o}{\bar{\varphi} f_o} \tag{D-1}
\]

for \( s \in \{n, o\} \). Following similar steps as those followed in the proof of Lemma 4 to obtain \( \zeta_{\hat{\varphi}, \lambda} \), we get that

\[
\zeta_{\hat{\varphi}, f_o} = -\frac{\mu_o(1 - c(\hat{\alpha})\sigma^{-1})}{\sigma - 1} \left( \frac{\hat{\varphi}}{\bar{\varphi}} \right)^{\sigma^{-1}} < 0. \tag{D-2}
\]

Given that \( \mu_o \in (0, 1) \), \( c(\hat{\alpha}) \in (w^*/w^2, 1) \), \( \hat{\varphi} < \bar{\varphi} \), and \( \sigma > 1 \), it follows that \( \zeta_{\hat{\varphi}, f_o} \in (-\frac{1}{\sigma - 1}, 0) \). Therefore, plugging in equation (D-2) into (D-1), we get

\[
\zeta_{L_n(\varphi), f_o} = \zeta_{L_o(\varphi), f_o} = \mu_o(1 - c(\hat{\alpha})\sigma^{-1}) \left( \frac{\hat{\varphi}}{\bar{\varphi}} \right)^{\sigma^{-1}} \in (0, 1) \tag{D-3}
\]

for active firms (with \( \varphi \geq \hat{\varphi} \)) that do not change their offshoring status. For these firms, only the selection effect is present when \( f_o \) changes. Thus, after a decline in \( f_o \), continuing firms that do not change their offshoring status destroy labor by contraction.

As with a change in \( \lambda \), there will be firms that change their offshoring status after a change in \( f_o \). From equation (23), note that \( \zeta_{\varphi, f_o} = \zeta_{B, f_o} + \zeta_{\hat{\varphi}, f_o} \) (\( \Gamma(\hat{\alpha}) \) does not depend on \( f_o \)). Given that \( \zeta_{B, f_o} = \frac{1}{\sigma - 1} \) and equation (D-2), we obtain

\[
\zeta_{\varphi, f_o} = \frac{1}{\sigma - 1} \left[ 1 - \mu_o(1 - c(\hat{\alpha})\sigma^{-1}) \left( \frac{\hat{\varphi}}{\bar{\varphi}} \right)^{\sigma^{-1}} \right] \in \left( 0, \frac{1}{\sigma - 1} \right). \tag{D-4}
\]
This implies that, as with a decline in $\lambda$, $\hat{\varphi}$ increases and $\hat{\varphi}_o$ decreases with a decline in $f_o$. Hence, for new offshoring firms, the last paragraph of the proof of Proposition 1 applies identically.

Part (iv) of Proposition D.1 is the equivalent to Proposition 2 for a change in $f_o$. For the response of the mass of entrants, $N_E$, to a change in $f_o$, note first from (21) that

$$\zeta_{P,f_o} = -\left(\frac{\sigma - 1}{\sigma - \eta}\right)\zeta_{\hat{\varphi},f_o}. \tag{D-5}$$

Using Leibniz’s rule to take the derivative of the log of equation (A-3) with respect to the log of $f_o$, and using (D-5) we obtain

$$\zeta_{N_E,f_o} = \frac{\zeta_{\hat{\varphi},f_o}}{<0} \left[\frac{(\sigma - 1)^2}{\sigma - \eta} \frac{g(\hat{\varphi})\hat{\varphi}^\sigma}{1 - G(\hat{\varphi})} + \frac{(\sigma - 1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1 - G(\hat{\varphi}_o)} \right], \tag{D-6}$$

which has an ambiguous sign.

For the mass of firms, $N$, note from $N = [1 - G(\hat{\varphi})]N_E$ that $\zeta_{N,f_o} = \zeta_{N_E,f_o} - \frac{g(\hat{\varphi})\hat{\varphi}\zeta_{\hat{\varphi},f_o}}{1 - G(\hat{\varphi})}$. Substituting (D-6) into the previous expression we get

$$\zeta_{N,f_o} = \frac{\zeta_{\hat{\varphi},f_o}}{<0} \left[\frac{(\sigma - 1)^2}{\sigma - \eta} \frac{g(\hat{\varphi})\hat{\varphi}}{1 - G(\hat{\varphi})} - \frac{(\sigma - 1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1 - G(\hat{\varphi}_o)} \right], \tag{D-7}$$

Without further assumptions on the distribution of $\varphi$, the sign of $\zeta_{N,f_o}$ is ambiguous. For the mass of offshoring firms, $N_o$, we know that $N_o = [1 - G(\hat{\varphi}_o)]N_E$ and therefore $\zeta_{N_o,f_o} = \zeta_{N_E,f_o} - \frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1 - G(\hat{\varphi}_o)}$.

Plugging in (D-6) into the previous equation, we obtain

$$\zeta_{N_o,f_o} = -\frac{(\sigma - 1)g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{1 - G(\hat{\varphi}_o)} + \frac{\zeta_{\hat{\varphi}_o,f_o}}{<0} \left[\frac{(\sigma - 1)^2}{\sigma - \eta} \frac{g(\hat{\varphi})\hat{\varphi}^\sigma}{1 - G(\hat{\varphi})} + \frac{g(\hat{\varphi})\hat{\varphi}^\sigma}{1 - G(\hat{\varphi})} \right] < 0.$$

Then, a decline in $f_o$ causes an increase in the mass of offshoring firms. For the mass of non-offshoring firms, $N_n$, we know that $N_n = [G(\hat{\varphi}_o) - G(\hat{\varphi})]N_E$ and therefore

$$\zeta_{N_n,f_o} = \left[\frac{g(\hat{\varphi}_o)\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o} - g(\hat{\varphi})\hat{\varphi}_o\zeta_{\hat{\varphi}_o,f_o}}{G(\hat{\varphi}_o) - G(\hat{\varphi})}\right] + \zeta_{N_E,f_o}.$$

As in the proof of Proposition 2 for a change in $\lambda$, the first term is positive but the ambiguity in the sign of $\zeta_{N_E,\lambda}$ carries over to $\zeta_{N_n,\lambda}$.

Parts (v), (vi), and (vii) of Proposition D.1 are the equivalent to Proposition 3 for a change in $f_o$. Using Leibniz’s rule to take the derivative of equation (32) with respect to $f_o$, we decompose the effect of $f_o$ on $L_Z$ into its extensive- and intensive-margin components as
\[
\frac{dL_Z}{df_o} = \left[ -L_n(\hat{\varphi})g(\hat{\varphi}) \frac{d\hat{\varphi}}{df_o} \right] N_E + \frac{L_Z}{N_E} \frac{dN_E}{df_o} + \\
\left[ (L_o(\hat{\varphi}_o) - L_o(\hat{\varphi}_o)) g(\hat{\varphi}_o) \frac{d\hat{\varphi}_o}{df_o} + \int_{\varphi_o}^{\hat{\varphi}_o} \frac{dL_n(\varphi)}{df_o} g(\varphi) d\varphi + \int_{\varphi_o}^{\hat{\varphi}_o} \frac{dL_o(\varphi)}{df_o} g(\varphi) d\varphi \right] N_E.
\tag{D-8}
\]

**Extensive margin.** The net extensive margin (NEM\(_{fo}\)) in equation (D-8) has two components: the change in domestic employment due to non-offshoring firms that stop (or start) producing due to the effect of \(f_o\) on \(\hat{\varphi}\), EM\(_{1,fo}\) = \[-L_n(\hat{\varphi})g(\hat{\varphi}) \frac{d\hat{\varphi}}{df_o}\] \(N_E\); and the change in domestic employment due to the effect of \(f_o\) on the mass of entrants, EM\(_{2,fo}\) = \[\frac{L_Z}{N_E} \frac{dN_E}{df_o}\).

For EM\(_{1,fo}\), we use equation (30) to rewrite it as

\[
EM_{1,fo} = \left[ -\frac{(\sigma - 1)f g(\hat{\varphi})\hat{\varphi} \zeta_{\hat{\varphi},fo}}{f_o \hat{w}_Z} \right] N_E > 0.
\tag{D-9}
\]

EM\(_{1,fo}\) is positive because \(\zeta_{\hat{\varphi},fo} < 0\).

For EM\(_{2,fo}\), note that we can rewrite it as EM\(_{2,fo}\) = \[\frac{L_Z}{f_o} \zeta_{N_E,fo}\). Substituting equations (C-41) and (D-6) into EM\(_{2,fo}\)—and using equations (23), (24), and (D-2)—we obtain

\[
EM_{2,fo} = \frac{(\sigma - 1)(1 + \zeta_{\hat{\varphi},\lambda})}{\hat{w}_Z} \begin{cases} 
\frac{[1 - G(\hat{\varphi}_o)](\sigma - 1)}{\sigma - \eta} < 0 & \\
\frac{f g(\hat{\varphi})\hat{\varphi} \zeta_{\hat{\varphi},fo}}{f_o} & \text{for } \eta \leq \rho < \sigma; \\
\frac{g(\hat{\varphi}_o)\hat{\varphi} \zeta_{\hat{\varphi},fo}}{\hat{w}_Z} & \text{for } \rho \leq \sigma.
\end{cases}
\tag{D-10}
\]

Given (D-6), the sign of EM\(_{2,fo}\) is also ambiguous. This ambiguity carries over to NEM\(_{fo}\) = EM\(_{1,fo}\) + EM\(_{2,fo}\). Note that EM\(_{2,fo}\)—and hence NEM\(_{fo}\)—is more likely to cause job creation after a decline in \(f_o\) if \(\eta\) approaches \(\sigma\).

**Intensive margin.** From equation (D-8) we know that the net intensive margin (NIM\(_{fo}\)) has three components: the change in domestic employment of firms that change their offshoring status, IM\(_{1,fo}\) = \[(L_n(\hat{\varphi}_o) - L_o(\hat{\varphi}_o)) g(\hat{\varphi}_o) \frac{d\hat{\varphi}_o}{df_o}\] \(N_E\); the change in domestic employment of continuing non-offshoring firms, IM\(_{2,fo}\) = \[\int_{\varphi_o}^{\hat{\varphi}_o} \frac{dL_n(\varphi)}{df_o} g(\varphi) d\varphi\] \(N_E\); and the change in domestic employment of continuing offshoring firms, IM\(_{3,fo}\) = \[\int_{\varphi_o}^{\hat{\varphi}_o} \frac{dL_o(\varphi)}{df_o} g(\varphi) d\varphi\] \(N_E\).

For IM\(_{1,fo}\), we use the expressions for \(L_n(\varphi)\) and \(L_o(\varphi)\) in equation (30), along with equation (23) and \(B^{\sigma-1} = \frac{L_Z}{\hat{w}_Z}\), to obtain

\[
IM_{1,fo} = g(\hat{\varphi}_o)\hat{\varphi}_o \zeta_{\hat{\varphi},fo} \left[ \frac{(\sigma - 1)\Gamma(\hat{\alpha})^{\sigma-1}}{\hat{w}_Z} \left( 1 - \frac{1 - \hat{\alpha}}{c(\hat{\alpha})^{\sigma-\rho}} \right) \right] N_E.
\]

As with IM\(_{1,\lambda}\), the sign of IM\(_{1,fo}\) is ambiguous if \(\rho < \sigma\). Note that if \(c(\hat{\alpha})^{\sigma-\rho} \leq (1 - \hat{\alpha})\)—which can only happen if \(\rho < \sigma\)—new offshoring firms create jobs by expansion after a decline in \(f_o\) (IM\(_{1,fo} < 0\)). The sign of IM\(_{1,fo}\) is positive if \(\rho \geq \sigma\).
For $IM_{2,f_o}$, we rewrite it as

$$IM_{2,f_o} = \frac{\zeta L_n(\phi)_{f_o}}{f_o} \left[ \int_{\hat{\phi}}^{\hat{\phi}_o} L_n(\phi) g(\phi) d\phi \right] N_E = \frac{\zeta L_n(\phi)_{f_o}}{f_o} N_n \bar{L}_n,$$

where we use that $N_n = (G(\hat{\phi}_o) - G(\hat{\phi})) N_E$. From equation (D-1) we know that $\zeta L_n(\phi)_{f_o} > 0$ and thus, $IM_{2,f_o} > 0$. Similarly, for existing offshoring firms we obtain

$$IM_{3,f_o} = \frac{\zeta L_o(\phi)_{f_o}}{f_o} N_o \bar{L}_o,$$

where we use that $N_o = (1 - G(\hat{\phi}_o)) N_E$. Again, from equation (D-1) we know that $\zeta L_o(\phi)_{f_o} > 0$ and therefore, $IM_{3,f_o} > 0$. Then, after a decline in $f_o$, existing firms that do not change their offshoring status destroy jobs by contraction.

The net intensive margin effect ($NIM_{f_o}$) is given by the sum of $IM_{1,f_o}$, $IM_{2,f_o}$, and $IM_{3,f_o}$. Even though $IM_{2,f_o} + IM_{3,f_o} > 0$, the ambiguity of $IM_{1,f_o}$ is carried out to the sign of $NIM_{f_o}$.

**Overall effect.** The net effect on employment, $\frac{dL_Z}{df_o} = NEM_{f_o} + NIM_{f_o}$, is ambiguous. Even though $EM_{1,f_o} + IM_{2,f_o} + IM_{3,f_o} > 0$, the ambiguity of $EM_{2,f_o}$ carries over to the overall effect. Note that a decline in $f_o$ is more likely to cause net job creation for higher $\eta$ (through its impact on $EM_{2,f_o}$) and lower $\rho$.

Compared to the effects of a decline in $\lambda$, the key difference in Proposition D.1 comes from the fact that there are no productivity and job-relocation effects for continuing offshoring firms—note that $\hat{\alpha}$ and $c(\hat{\alpha})$ do not depend on $f_o$. As with the employment of continuing non-offshoring firms, the only effect for continuing offshoring firms is the selection effect and hence, their employment decreases after a decline in $f_o$. The productivity and job-relocation effects are, however, present for firms that switch from no offshoring to offshoring, and the impact of a decline in $f_o$ on their domestic employment is ambiguous.