Liquidity and International Trade

Antonio Rodriguez-Lopez*
Department of Economics
University of California, Irvine

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Abstract

This paper introduces a framework to study the links between the supply of liquid assets for the financial market and the international allocation of economic activity. Private assets' liquidity properties—their usefulness as collateral or media of exchange in financial transactions—affect assets' values and interest rates, with consequences on firm entry, production, aggregate productivity, and total market capitalization. In a closed economy, the liquidity market increases the size and productivity of the sector of the economy that generates liquid assets. In an open economy, however, cross-country differences in financial development—as measured by the degree of liquidity of a country's assets—generate an allocation of real economic activity that favors the country that supplies the most liquid assets. In such a setting, trade liberalization magnifies the gap in economic activity between the countries.

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1 Introduction

Private assets such as equity, commercial paper, and corporate bonds provide liquidity services to the financial system because they can be used as media of exchange or as collateral in financial transactions. For example, according to ISDA (2015), in 2014 equities and corporate bonds accounted for 19.5 percent of non-cash collateral in the non-cleared derivatives market, which is higher than the 15.9 percent accounted for by U.S. government securities (considered to be the most liquid non-cash assets in the world). The money role of private assets not only expands the size of the financial sector by allowing more and larger financial transactions, but also affects real economic activity in sectors where the assets are generated. In particular, values of private assets include a liquidity premium that reflects their degree of *moneyness* in financial-sector activities; these augmented values in turn affect issuing firms' production, entry and exit decisions, and aggregate-level outcomes such as aggregate prices and productivity. At an international level, cross-country differences in financial development—as measured by the degree of liquidity services provided by a country's assets—potentially influence the organization of economic activity across borders, with consequences on international trade relationships.

The goal of this paper is to elucidate the links between the market for liquid assets and the international allocation of economic activity. Toward this goal, I introduce a theoretical model that describes the effects of the liquidity market on the size and aggregate productivity of the real-economy sector generating liquid assets. At an international level, I look at how cross-country differences in asset liquidity affect the international allocation of economic activity, and study the effects of trade liberalization. As well, this paper analyzes the impact of a liquidity crisis (similar to the 2007-2008 financial crisis) on interest rates and the allocation of economic activity. The framework offers transparent mechanisms that increase our understanding of the benefits and costs of a financial system evolving through innovations meant to extract liquidity services—by using complex processes of securitization—to almost any type of asset.[1]

The model introduces a market for liquid assets into the standard Melitz (2003) model of trade with heterogeneous (in productivity) firms. The market for liquidity—which follows Rocheteau and Rodriguez-Lopez (2014)—determines a full structure of equilibrium interest rates for multiple liquid assets and the equilibrium amount of liquidity in the economy. The supply of liquidity is composed of claims on Melitz firms' profits (private liquidity) and government bonds (public liquidity), while the demand for liquidity is given by financiers who need liquid assets to be used as collateral in

[1] Gorton and Metrick (2012) define securitization as "the process by which loans, previously held to maturity on the balance sheets of financial intermediaries, are sold in capital markets".

their financial activities.

The market for liquid assets has positive spillovers on the real economy. To show this, I start by describing a closed economy with three types of agents: households, financiers, and heterogeneous firms. Financiers fund the entry of heterogeneous firms in exchange for claims on the firms' future profits from their sales of differentiated-good varieties to households. In addition, financiers have random opportunities to trade financial services in an over-the-counter (OTC) market; these transactions are backed by a collateral agreement, with claims on firms and government bonds playing the collateral role. The simplest version of the model assumes that government bonds and all claims on producing firms have identical liquidity properties, being all fully acceptable in OTC transactions.

The model shows that—up to the rate of time preference—the financiers' demand for liquid assets is increasing in the assets' interest rate: when the interest rate increases, the financiers' cost of holding assets declines and hence they will hold more of them. When the interest rate reaches the financiers' rate of time preference, the holding cost is exactly zero and their holdings of liquid assets become indeterminate—financiers' liquidity needs are satiated. On the other hand, there is an inverse relationship between the supply of private liquidity and the interest rate. When the interest rate is equal to the rate of time preference, firms are priced at their "fundamental value", which is the value that would prevail in the absence of liquidity services from private assets (*i.e.*, when claims on the firms' profits are *illiquid*). For a lower level of the assets' interest rate, the average value of firms increases, driving up the total market capitalization of firms; hence, the supplied amount of private liquidity rises. In equilibrium, the interest rate is below the rate of time preference, and total market capitalization (*i.e.*, the amount of private liquidity) and the average productivity of firms are larger than at the fundamental-value outcome. Thus, the liquidity of private assets increases the size and productivity of the real-economy sector that generates them.

Adding government bonds to the set of liquid assets shifts to the right the total supply of liquidity, increasing the equilibrium interest rate and the equilibrium amount of liquidity. Importantly, even though the total amount of liquidity increases, the amount of private liquidity falls: the rise in the interest rate causes a decline in the total market capitalization of firms. This crowding-out effect of private liquidity by public liquidity, which also appears in Rocheteau and Rodriguez-Lopez (2014) and Holmström and Tirole (2011), finds strong empirical support in Krishnamurthy and Vissing-Jorgensen (2015). As well, this paper highlights further effects of an increase in government bonds on the real economy, with the crowding-out of private liquidity being accompanied by a decline in aggregate productivity and an increase in the aggregate price.

Once the synergies between the market for liquidity and the real economy have been established, the model is expanded to a two-country setting with differences in liquidity properties across assets within and between countries. There are four categories of assets—Home and Foreign private assets, and Home and Foreign government bonds—with the liquidity of each asset being determined by its *acceptability* as collateral in OTC transactions in the world financial system. Across private assets, acceptability is directly related to firm-level productivity and responds endogenously to changes in the economic environment. The model jointly determines a full structure of interest rates, production in each country, and the amount of trade.

The most liquid assets yield lower rates of return and differences in asset liquidity across countries affect the international allocation of economic activity. Assuming that countries are identical but for the acceptability of their assets in OTC transactions, it is shown that as Foreign assets become less liquid (less acceptable), the Home production sector displaces the Foreign production sector, aggregate productivity increases at Home but declines at Foreign, and the aggregate price declines at Home but increases at Foreign. Moreover, although trade liberalization has conventional Melitz-type effects in both countries—average productivity increases, the aggregate price declines, and the least productive firms exit—the total capitalization of firms increases at Home but declines at Foreign; *i.e.*, trade liberalization widens the gap in economic activity between the countries.

The model is useful to study the effects of a liquidity crisis—a shock to the acceptability or pledgeability of a country's private assets—on multiple liquid assets and international trade. If Home private assets become less acceptable in OTC transactions, but Home government bonds are the most acceptable asset in the world financial system, rate-of-return differentials between private assets and government bonds may increase substantially. If instead the liquidity shock affects the fraction of each Home private asset that can be pledged as collateral, there is a flight-to-liquidity phenomenon by which the liquidity premium increases not only for Home government bonds, but also for the private assets with the highest underlying productivities. In the latter case, aggregate productivity and total market capitalization of Home firms may increase.

This paper relates financial development to *a country's capacity to generate liquid assets*. However, the traditional literature on financial markets and trade relates financial development to *a country's degree of credit-market imperfections*—so that less financially developed countries have more credit-market frictions. Following this tradition, I extend the model to allow for credit frictions in the search for funding for differentiated-good firms. Differences in credit-market frictions across countries also yield an allocation of economic activity that favors the country with less frictions, and also cause unequal effects of trade liberalization.

The paper is organized as follows. Section 2 describes the theoretical and empirical background for the model developed here. Sections 3 and 4 introduce the closed-economy version of the model, which highlights the novel mechanisms of this framework. Section 5 presents the model with trade and heterogeneity in the liquidity properties across assets within and between countries. Section 6 describes the effects of cross-country differences in financial development on the allocation of economic activity, and studies the impact of trade liberalization and the effects of a liquidity crisis. Section 7 presents the model's extension with credit frictions. Lastly, section 8 concludes.

2 Theoretical and Empirical Background

Liquidity is priced: the most liquid assets—those with high degree of moneyness (easily traded and highly acceptable as media of exchange)—have higher prices and lower rates of return. Abundant evidence on the liquidity premium appears in the cross-section and over time in equity markets (see, *e.g.*, Pastor and Stambaugh, 2003 and Liu, 2006) and corporate bond markets (see, *e.g.*, Lin, Wang, and Wu, 2011). In comparison with U.S. Treasury bonds, which are considered to be the most liquid financial assets in the world, Chen, Lesmond, and Wei (2007) and Bao, Pan, and Wang (2011) find that corporate bond yield spreads—the rate-of-return difference between corporate bonds and U.S. Treasuries—decline with corporate bond liquidity.

As in the model in this paper, the pricing of liquidity depends on the availability of both public and private instruments. Related to this, Krishnamurthy and Vissing-Jorgensen (2012) document a negative relationship between the supply of Treasuries and both the interest rate spread between corporate bonds and Treasuries, and the interest rate spread between corporate bonds of different safety ratings. These results not only show the liquidity and safety properties of U.S. Treasuries, but also highlight the effects of public liquidity on the structure of private interest rates. In a follow-up paper, Krishnamurthy and Vissing-Jorgensen (2015) find a strong inverse relationship between the supply of U.S. Treasuries and the amount of private assets—similar evidence is found by Gorton, Lewellen, and Metrick (2012)—which lends empirical support to the crowding-out mechanism of private liquidity by public liquidity in this paper.

The interaction between the supply and demand for liquid assets determines aggregate liquidity and the structure of interest rates in financial markets, but who supplies and who demands liquid assets? The IMF (2012) estimates that by 2011 the supply of safe and liquid assets was about $74.4 trillion and was composed of OECD-countries sovereign debt (56 percent), asset-backed securities (17 percent), corporate bonds (11 percent), gold (11 percent), and covered bonds (4 percent). Regarding their country of origin, the U.S. is the main supplier of liquid assets for the world financial system. According to estimations by the BIS (2013), in 2012 the U.S. accounted for about half of the supply of high-quality assets eligible as collateral in financial transactions (the U.S. is followed by Japan, the Euro area, and the U.K.). The importance of the U.S. as a world provider of liquidity is even higher in the production of more sophisticated financial instruments. For example, according to Cetorelli and Peristiani (2012), from 1983 to 2008 the U.S. accounted for 73.1 percent of the issuance of asset-backed securities (ABS).

On the other hand, based on holdings of sovereign debt, the IMF (2012) estimates that by the end of 2010 the demand for safe and liquid assets was coming from private banks (34 percent), central banks (21 percent), insurance companies (15 percent), pension funds (7 percent), sovereign wealth funds (1 percent), and other entities (22 percent). Hence, most of the demand for liquid assets arises from inside the financial sector.[2] Accordingly, the demand for liquid assets in this model stems from financiers that need liquid assets to be used as collateral in their financial transactions. The crucial role of liquid assets as collateral in financial markets as well as the growing demand for high-quality collateral—fueled by new regulations following the recent financial crisis—are documented, among others, by the IMF (2012) and the BIS (2013).

The closed-economy version of the model shows the positive spillovers of the market for liquidity on the size and productivity of the sector that generates liquid assets: due to the liquidity services they provide, the interest rate on liquid private claims is below the rate of time preference (*i.e.*, the interest rate on illiquid assets), which then expands real economic activity. This result—and the basic intuition behind the liquidity market—strongly resembles the Bewley model of Aiyagari (1994), in which households accumulate claims on capital to self-insure against idiosyncratic labor income shocks. In that model *(i)* households' precautionary savings are increasing in the interest rate—the holding cost of a claim on capital declines as its rate of return increases—up to the discount rate (at that point the holding cost of a claim on capital is zero and savings tend to infinity), and *(ii)* the amount of capital in the production sector is declining in the interest rate (a higher interest rate implies a higher marginal product of capital, which then implies a lower level of capital—as usual, the marginal product of capital is declining). In equilibrium, due to the role of capital as a self-insurance device, the interest rate is below the discount rate and the aggregate capital stock in the economy is above its certainty level. Hence, the model here can be interpreted as a tractable version of Aiyagari's model in which instead of holding assets for precautionary-saving motives due to idiosyncratic income shocks, agents hold assets due to the liquidity services they

[2] See also Gourinchas and Jeanne (2012), who find that the demand for safe assets by the U.S. private real sector has been very stable over time (and also for the U.K., France, and Germany, but not for Japan), and hence attribute most of the increase in the demand for safe assets to the financial system.

provide in their random opportunities to trade in the OTC financial market.

This paper shows that for two countries that differ only in their financial development—defined here as a country's ability to generate liquid assets—the allocation of real economic activity favors the most financially developed country, with trade liberalization further exacerbating the gap between them. The model's extension with credit-market imperfections shows that cross-country differences in credit frictions produce similar results. As mentioned in the Introduction, the extension is motivated by the traditional literature on the effects of financial markets on international trade, which associates financial development to a country's degree of credit-market imperfections. That literature—pioneered by the theoretical contribution of Kletzer and Bardhan (1987)—finds that the most financially developed countries (with less credit-market imperfections) have comparative advantage in sectors that rely more on external funding.[3] Empirically, these comparative-advantage patterns are confirmed, among others, by Beck (2002) and Manova (2013), who also show that weak credit conditions are associated with overall low trade volumes (see Foley and Manova (2015) for an extensive survey). Of course, both definitions of financial development are likely to be highly correlated: a country with a well-functioning credit market will likely be able to generate more liquid assets.[4]

My model is also related to recent models that try to explain global imbalances—which feature capital flows from emerging countries to rich countries (the so-called Lucas paradox)—as a result of cross-country differences in financial development. The OLG model of Caballero, Farhi, and Gourinchas (2008) has a definition of financial development that is similar to the one in this model, while the Bewley-type models of Angeletos and Panousi (2011) and Mendoza, Quadrini, and Rios-Rull (2009) relate financial development to financial contract enforceability in the insurance of idiosyncratic risks. In all these models the demand for financial assets is lower in financially developed countries (*e.g.*, in the last two Bewley-type models, agents have less insurance needs in financially developed economies because markets are more complete) and hence they have higher autarky interest rates; financial integration equalizes interest rates and thus drives capital flows toward the most financially developed countries. In contrast, in this paper the demand for liquid assets is set in a world financial market (independently of each country's financial development) and liquidity differences across assets are the main drivers of capital flows, with each asset yielding an equilibrium interest rate in accordance with its liquidity properties. As a consequence, and different

[3]Other theoretical contributions along the same lines include Matsuyama (2005) and Ju and Wei (2011).

[4]Indeed, the empirical literature on credit frictions and trade frequently uses measures of financial development that are closely related to country-level capacity to generate liquid assets. For example, Manova (2008) shows that equity-market openness is associated with higher exports.

to the previous papers, my model can explain phenomena like the 2007-2008 financial crisis, which featured worldwide flight-to-quality toward U.S. Treasuries in spite of the U.S. private sector being the source of the crisis.

3 The Environment

To describe the basic interactions between the market for liquid assets and the real economy, we describe first a closed economy. The model is in continuous time, $t \in \mathbb{R}_+$, and there are three categories of agents: a unit measure of households, a unit measure of financiers, and an endogenous measure of heterogeneous (in productivity) firms. There are three types of goods: a *homogeneous good* that is produced and consumed by households and financiers and that is taken as the numéraire, a *heterogeneous good* that is produced in many varieties by heterogenous firms and that is consumed by households only, and a *financial service* that is produced and consumed by financiers only.

3.1 Households

Households are risk-neutral and discount future consumption at rate $\rho > 0$, with lifetime utility given by

$$\int_0^\infty e^{-\rho t} C(t) dt,$$

where $C(t)$ is the household's consumption index described as

$$C(t) \equiv H(t)^{1-\eta} Q(t)^{\eta}, \tag{1}$$

where $H(t)$ denotes the consumption of the homogeneous good, $Q(t) = \left(\int_{\omega \in \Omega} q^c(\omega, t)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$ is the CES consumption aggregator of differentiated-good varieties, and $\eta \in (0, 1)$. In $Q(t)$, $q^c(\omega, t)$ denotes the consumption of variety ω, Ω is the set of varieties available for purchase, and $\sigma > 1$ is the elasticity of substitution between varieties.

Each household is endowed with a unit of labor per unit of time devoted either to produce one unit of the homogeneous good (which is produced under perfect competition without any other costs), or to produce in the differentiated-good sector as an employee of a differentiated-good firm. In the absence of any frictions in the labor market, the wage of each household is 1 (in terms of the homogeneous good).

Given (1) and the unit wage, the representative household's total expenditure on differentiated-good varieties is η, and its total expenditure on the homogeneous good is $1 - \eta$. It follows that each household's demand for differentiated-good variety ω is

$$q^c(\omega,t) = \left[\frac{p(\omega,t)^{-\sigma}}{P(t)^{1-\sigma}}\right]\eta, \tag{2}$$

where $p(\omega,t)$ is the price of variety ω at time t, and $P(t) \equiv \left[\int_{\omega\in\Omega} p(\omega,t)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ is the price of the CES aggregator $Q(t)$. Given that there is a unit mass of households, equation (2) also corresponds to the market demand for variety ω, and $P(t)Q(t) \equiv \eta$ is the country's total expenditure on differentiated-good varieties.

3.2 Financiers

Financiers define their preferences over the consumption of financial services—traded in an over-the-counter market (which involves bilateral matching and bargaining)—and the consumption of the homogeneous good. A financier discounts time at rate ρ and its lifetime expected utility is

$$\mathbb{E}\left\{\sum_{n=1}^{\infty} e^{-\rho T_n}\left\{F\left[y(T_n)\right] - x(T_n)\right\} + \int_0^{\infty} e^{-\rho t} H(t)dt\right\},$$

where the first term accounts for the utility from consumption of financial services, and the second term accounts for the utility from consumption of the homogeneous good.

In the first term, $\{T_n\}$ is a Poisson process with arrival rate $\nu > 0$ that indicates the times at which the financier is matched with another financier. After a match is formed, a financier is chosen at random to be either user or supplier of services. For a user, the utility from consuming y units of financial services is $F(y)$, where F is strictly concave, $F(0) = 0$, $F'(0) \to \infty$, and $F'(\infty) = 0$. For a supplier, the disutility from providing x units of financial services is x. For a given financier, either $y(T_n) > 0$ (with probability 0.5) or $x(T_n) > 0$ (with probability 0.5). For any match, feasibility requires that $y(T_n) \leq x(T_n)$—the consumption of the user must be no greater than the production of the supplier.

At all $t \notin \{T_n\}_{n=1}^{\infty}$ financiers can produce and consume the homogeneous good. The technology to produce/consume the homogeneous good is, however, not available at times $\{T_n\}$ when financiers are matched in the OTC market. This assumption implies that the buyer of financial services will finance its purchase with a loan to be repaid after the match is dissolved. Assuming lack of commitment and monitoring, financiers will rely on liquid assets (to be used as collateral) to secure their loans in the OTC market.

3.3 Firms

Producers of differentiated-good varieties are heterogeneous in productivity. Following Melitz (2003), after paying a sunk entry cost of f_E units of the homogeneous good, a firm draws its productivity from a probability distribution with support $[\varphi_{\min}, \infty)$, cumulative function $G(\varphi)$, and density function $g(\varphi)$. Firms' entry costs are paid for by financiers in exchange for the ownership in the future profits of the firm. Crucially, these claims on firms' profits belong to the set of liquid assets that financiers can use as collateral in OTC trades.

The production function of a firm with productivity φ is $q(\varphi, t) = \Phi\varphi L(t)$, where Φ is an aggregate productivity factor, and $L(t)$ denotes labor. The are also fixed costs of operation, with each producing firm paying f units of the homogeneous good per unit of time. In addition, all firms are subject to a random death shock, which arrives at Poisson rate $\delta > 0$.

Given CES preferences for differentiated-good varieties, the firm's profit maximization problem for a firm with productivity φ yields the usual pricing equation with a fixed markup over marginal cost: $p(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{\Phi\varphi}$. Note that $p'(\varphi) < 0$, so that more productive firms set lower prices. The firm's gross profit (before paying the fixed cost) is then $\pi(\varphi, t) = [p(\varphi)/P(t)]^{1-\sigma}\eta/\sigma$. A firm only produces if its gross profit is no less than the fixed cost of operation, f. Hence, there exists a cutoff productivity level, $\hat{\varphi}(t)$, that satisfies the Melitz's zero-cutoff-profit (ZCP) condition, $\pi[\hat{\varphi}(t), t] = f$, so that firms with productivities below $\hat{\varphi}(t)$ do not produce. The ZCP condition can be written as

$$P(t) = \left(\frac{\eta}{\sigma f}\right)^{\frac{1}{1-\sigma}} p[\hat{\varphi}(t)]. \tag{3}$$

Equation (3) can then be used to rewrite the gross profit function as

$$\pi(\varphi, t) = \left[\frac{\varphi}{\hat{\varphi}(t)}\right]^{\sigma-1} f, \tag{4}$$

which shows that firm-level profits are increasing in productivity and declining with the cutoff productivity level.

We can also obtain a convenient expression for the mass of producing firms, $N(t)$. Note first that the aggregate price of differentiated-good varieties, $P(t)$, can be calculated as

$$P(t) = \left[N(t)\int_{\hat{\varphi}(t)}^{\infty} p(\varphi)^{1-\sigma} g[\varphi|\varphi \geq \hat{\varphi}(t)]d\varphi\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

It then follows from (3) and (5) that

$$N(t) = \frac{\eta}{\sigma f}\left[\frac{\hat{\varphi}(t)}{\bar{\varphi}(t)}\right]^{\sigma-1}, \tag{6}$$

where

$$\bar{\varphi}(t) = \left[\int_{\hat{\varphi}(t)}^{\infty} \varphi^{\sigma-1} g[\varphi|\varphi \geq \hat{\varphi}(t)]d\varphi\right]^{\frac{1}{\sigma-1}} \tag{7}$$

is the average productivity of producing firms.

3.4 Government bonds

There is a supply B of pure-discount government bonds that pay one unit of the homogeneous good at the time of maturity. The terminal payment of bonds is financed through lump-sum taxation on financiers. Along with claims on firms' profits, government bonds can serve as collateral in the OTC market.

4 The Market for Liquidity

In the absence of perfect commitment, financiers need liquidity to secure their debt obligations from their OTC transactions. This section describes the supply of private liquidity arising from differentiated-good firms, the demand of liquidity by financiers, and the determination of the real interest rate to clear the market for liquid assets. We focus on steady-state equilibria—the cutoff productivity level, the mass of firms, and the interest rate are constant over time—and hence, we can suppress the time index, t, in some parts of this section.

4.1 Supply of Liquidity

All claims on producing firms' profits are part of the liquidity of the economy, and therefore, the amount of private liquidity available to financiers is equivalent to the aggregate capitalization of firms.[5] Here we determine the aggregate capitalization of firms as a function of the interest rate on liquid assets, r.

A producing firm with productivity φ generates a flow dividend, $\pi(\varphi) - f$, and dies at rate δ. The value of this firm is denoted by $V(\varphi)$, which solves $rV(\varphi) = \pi(\varphi) - f - \delta V(\varphi)$; that is,

$$V(\varphi) = \frac{\pi(\varphi) - f}{r + \delta}, \tag{8}$$

so that the value of the firm is the discounted sum of its instantaneous profits, $\pi(\varphi) - f$, with the effective discount rate given by the sum of the interest rate and the death rate. Therefore, the average value of producing firms is $\bar{V} = \int_{\hat{\varphi}}^{\infty} V(\varphi) g(\varphi | \varphi \geq \hat{\varphi}) d\varphi$, which from equations (4), (7), and (8) can be written as

$$\bar{V} = \frac{f}{r + \delta} \left[\left(\frac{\bar{\varphi}}{\hat{\varphi}} \right)^{\sigma - 1} - 1 \right]. \tag{9}$$

Financiers fund the entry of each firm before the realization of the firm's productivity. Thus, in equilibrium, the pre-entry expected value of a firm, $V_E = \int_{\hat{\varphi}}^{\infty} V(\varphi) g(\varphi) d\varphi$, is equal to the sunk

[5] In the following section we consider the case in which only a fraction of the total capitalization of firms is part of the liquidity available to financiers.

entry cost, f_E. Note that $V_E = [1 - G(\hat{\varphi})]\bar{V}$ and therefore, the free-entry condition is given by

$$\frac{f[1 - G(\hat{\varphi})]}{r + \delta} \left[\left(\frac{\bar{\varphi}}{\hat{\varphi}} \right)^{\sigma - 1} - 1 \right] = f_E. \tag{10}$$

Equation (10) determines a unique $\hat{\varphi}$ for each r. Moreover, it follows from (10) that $\frac{d\hat{\varphi}}{dr} = -\frac{f_E \hat{\varphi}[\hat{\varphi}/\bar{\varphi}]^{\sigma-1}}{(\sigma-1)f[1-G(\hat{\varphi})]} < 0$: an increase in r negatively affects the value of firms and hence the value of entry, so that a decline in $\hat{\varphi}$ (which rises firm-level profits) is needed to restore the free-entry condition. Note also that the average value of producing firms can be written more compactly as $\bar{V} = \frac{f_E}{1-G(\hat{\varphi})}$.

The private provision of liquidity is defined as $A = N\bar{V}$. Using (6), (9), and (10), it follows that

$$A(r) = \frac{\eta f_E}{\sigma \left\{ f[1 - G[\hat{\varphi}(r)]] + f_E(r + \delta) \right\}}, \tag{11}$$

where $dA(r)/dr < 0$: as the real interest rate increases, the average value of producing firms, $\bar{V}$, declines and even though the mass of producing firms may increase or decrease (depending on the assumed productivity distribution), the private supply of liquidity shrinks. Moreover, from (10) we obtain that $\hat{\varphi}(-\delta) \to \infty$, so that $G[\hat{\varphi}(-\delta)] \to 1$ and thus $A(-\delta) \to \infty$; on the other hand, $A(\rho)$ is positive and finite.

The aggregate liquidity supply of the economy, $L_S(r)$, is given by the sum of the private provision of liquidity, $A(r)$, and the public provision of liquidity, B. As we will see below, due to the liquidity services provided by private and public assets, their equilibrium rate of return, r, will be smaller than the rate of time preference, ρ, which is the rate of return on illiquid assets.

4.2 Demand for Liquidity

Financiers demand liquid assets to be used as collateral in their OTC transactions. Here we obtain the relationship between the financiers' holdings of liquid assets and the interest rate. The relationship is straightforward: the higher the interest rate an asset yields, the lower the financier's cost of holding this asset, and hence the higher the financier's demand for this asset.

This section follows the OTC-market description of Rocheteau and Rodriguez-Lopez (2014), which is related to the OTC structures of Duffie, Garleanu, and Pedersen (2005) and Lagos and Rocheteau (2009). Importantly, this is not the only way to generate a positive relationship between the demand for liquidity and the interest rate: as long as financiers have a precautionary motive for holding some types of assets, a positive relationship between the demand for these assets and their interest rate will emerge even if financiers meet in a competitive market. I follow the OTC structure with bilateral matching and bargaining because of the predominance of OTC trades in financial transactions.

The financier's problem can be written as

$$\max_{a(t),h(t)} \left\{ \mathbb{E} \int_0^{T_1} e^{-\rho t} h(t) dt + e^{-\rho T_1} Z\left[a(T_1)\right] \right\} \tag{12}$$

subject to

$$\dot{a} = ra - h - \Upsilon \tag{13}$$

and $a(t) \geq 0$, with $a(0) > 0$. From (12), the financier chooses asset holdings, $a(t)$, and homogeneous-good consumption, $h(t)$, that maximize the discounted cumulative consumption up to T_1—the random time at which the financier is matched with another financier—plus the present continuation value of a trading opportunity in the OTC market at T_1 with $a(T_1)$ holdings of liquid assets, $Z\left[a(T_1)\right]$. The financier's budget constraint in (13) shows that the financier's change in asset holdings ($\dot{a}$) should equal the return on those assets (ra) plus the financier's production of the homogeneous good ($-h$) net of taxes (Υ).

Given the assumption that T_1 is exponentially distributed with arrival rate ν (waiting times of a Poisson process are exponentially distributed), the maximization problem in (12)-(13) can be rewritten as

$$\max_{a(t),h(t)} \int_0^{\infty} e^{-(\nu+\rho)t} \left\{ h(t) + \nu Z\left[a(t)\right] \right\} dt \quad \text{subject to} \quad \dot{a} = ra - h - \Upsilon.$$

The current-value Hamiltonian is then $\mathcal{H}(h, a, \xi) = h + \nu Z(a) + \xi\left(ra - h - \Upsilon\right)$, with state variable a, control variable h, and current-value costate variable ξ. From the first necessary condition $\mathcal{H}_h(h, a, \xi) = 0$, it follows that $\xi = 1$ for all t. From the second necessary condition, $\mathcal{H}_a(h, a, \xi) = (\nu + \rho)\xi - \dot{\xi}$, and given that $\xi = 1$ and $\dot{\xi} = 0$, it follows that the demand for liquid assets is determined by

$$Z'(a) = 1 + \frac{\rho - r}{\nu}. \tag{14}$$

In (14), $Z'(a)$ is the financier's benefit from an additional unit of liquid assets, which should be equal to the cost of purchasing the asset (which is 1 because liquid assets are in terms of the numéraire) plus the asset's expected holding cost until the next OTC match, $(\rho - r)/\nu$ (the average time until the next OTC match is $1/\nu$).

Let us now describe $Z(a)$ and $Z'(a)$ more precisely. When T_1 arrives, the financier has an equal chance of being a buyer or seller of financial services, and thus, $Z(a) = \left[Zb(a) + Z^s(a)\right]/2$, where Z^b is the value of being a buyer of financial services and Z^s is the value of being a seller of those services. Once the roles of the financiers are established, the buyer sets the terms of the OTC contract with a take-it-or-leave-it offer to the seller.

The OTC contract, (y, α), includes the buyer's consumption of financial services, y, and the transfer of liquid assets from the buyer to the seller, α. If the buyer holds a^b units of liquid assets, the buyer's problem is

$$\max_{y,\alpha} \{F(y) - \alpha\} \quad \text{subject to } \alpha \geq y \text{ and } \alpha \in \left[0, a^b\right].$$

Hence, the contract (y, α) maximizes the buyer's surplus from trade, $F(y) - \alpha$, subject to the participation constraint for the seller, $\alpha \geq y$, and the feasibility condition for the buyer, $\alpha \in \left[0, a^b\right]$. The solution is $y = \alpha = \hat{y}$, where $F'(\hat{y}) = 1$, if $a^b \geq \hat{y}$; otherwise, $y = \alpha = a^b$. Intuitively, the buyer's surplus-maximizing consumption of financial services is $\hat{y}$, but that outcome occurs only if the buyer has enough liquid assets to transfer to the seller (*i.e.*, if $a^b \geq \hat{y}$). If $a^b < \hat{y}$, the buyer is liquidity constrained and the best she can do is to transfer all of her liquid assets to the seller and get in exchange an equivalent amount of financial services.

The value function for the buyer is $Z^b(a) = \max_{y \leq a} \{F(y) - y\} + W(a)$, where the first term is the whole surplus of the match (which is equal to $F(\hat{y}) - \hat{y}$ if $a \geq \hat{y}$, and is equal to $F(a) - a$ if $a < \hat{y}$), and $W(a)$ is the financier's continuation value. The seller's surplus from the match is zero, and thus, $Z^s(a) = W(a)$. It follows that

$$Z(a) = \frac{1}{2} \max_{y \leq a} \{F(y) - y\} + W(a), \tag{15}$$

which indicates that with probability 1/2 the financier is a buyer, in which case she will transfer up to a units of liquid assets in exchange for y. Therefore, the financier's benefit from an additional unit of liquid assets at the time of the match (but before knowing her buyer or seller role) is

$$Z'(a) = \begin{cases} W'(a) & \text{if } \ a \geq \hat{y} \\ \frac{F'(a)-1}{2} + W'(a) & \text{if } \ a < \hat{y}. \end{cases} \tag{16}$$

Given that $f'(y) > 0$, $F''(y) < 0$, and $f'(\hat{y}) = 1$, it follows that $f'(a) - 1 > 0$ if $a < \hat{y}$, and is exactly zero if $a = \hat{y}$. Using these results along with the fact that $W'(a) = \xi = 1$, we can rewrite (16) as

$$Z'(a) = \frac{[F'(a) - 1]^+}{2} + 1, \tag{17}$$

where $[x]^+ = \max\{x, 0\}$.

From (14) and (17) we obtain $(\rho - r)/\theta = [F'(a) - 1]^+$, where $\theta = \nu/2$ is the rate at which a financier is matched as a buyer. It follows that the financier's consumption of financial services, $y = \min\{a, \hat{y}\}$, solves

$$F'(y) = 1 + \frac{\rho - r}{\theta} \tag{18}$$

for $r \leq \rho$. If $r < \rho$, so that $F'(y) > 1$ and $y = a < \hat{y}$, the financier's demand for liquid assets is $a^d = F'^{-1}[1 + (\rho - r)/\theta]$. If $r = \rho$, so that the cost of holding liquid assets is zero and $y = \hat{y}$, the financier's demand for liquid assets takes any value in the range $[\hat{y}, \infty)$.

There is a unit measure of financiers, which implies that the aggregate demand for liquid assets, $L_D(r)$, is identical to the financier's individual demand. Thus, we have that

$$L_D(r) = \begin{cases} F'^{-1}\left(1 + \frac{\rho - r}{\theta}\right) & \text{if } \; r < \rho \\ [\hat{y}, \infty) & \text{if } \; r = \rho. \end{cases} \tag{19}$$

If $r < \rho$, there is a positive relationship between $L_D(r)$ and r: an increase in the interest rate on liquid assets causes a decline in their holding cost, $(\rho - r)/\theta$, which drives financiers to hold more of them. When $r = \rho$, liquidity is costless to hold and hence financiers will hold any amount in the range $[\hat{y}, \infty)$.

4.3 Equilibrium

The equilibrium in the market for liquidity occurs at the intersection of supply and demand:

$$L_S(r) \equiv A(r) + B = L_D(r), \tag{20}$$

where $A(r)$ is given by (11) and $L_D(r)$ is given by (19). Figure 1 shows a graphical representation of the equilibrium in the market for liquid assets. The supply of private assets, $A(r)$, is downward sloping, with its lowest value being $A(\rho)$ and tending to infinity when r approaches $-\delta$ from the right. The aggregate liquidity supply, $L_S(r)$, adds B to $A(r)$, and hence it is simply a right-shifted version of $A(r)$. The demand for liquidity, $L_D(r)$, is upward sloping as long as $r < \rho$, and it becomes horizontal at $r = \rho$. The intersection of supply and demand gives a unique equilibrium, (L^e, r^e). The formal definition of a steady-state equilibrium follows.

Definition 1. *A steady-state equilibrium is a triple $(\hat{\varphi}, y, r)$ that solves (10), (18), and (20).*

The steady-state equilibrium is unique: there is unique r that clears the market for liquidity, $\hat{\varphi}$ is uniquely determined from (10), and y is uniquely determined from (18). We can now describe key relationships between the market for liquid assets and the real economy. In Figure 1, $A(\rho)$ denotes the market capitalization of firms that would prevail in the absence of liquidity services of private assets. We refer to $A(\rho)$ as the "fundamental-value" capitalization. Due to the liquidity services that private assets provide to the financial sector, the equilibrium total market capitalization of differentiated-good firms is $A^e > A(\rho)$. Moreover, $\hat{\varphi}(r^e) > \hat{\varphi}(\rho)$ (recall that $d\hat{\varphi}/dr < 0$), which implies from (5) and (7) that when compared to the fundamental-value outcome, the aggregate

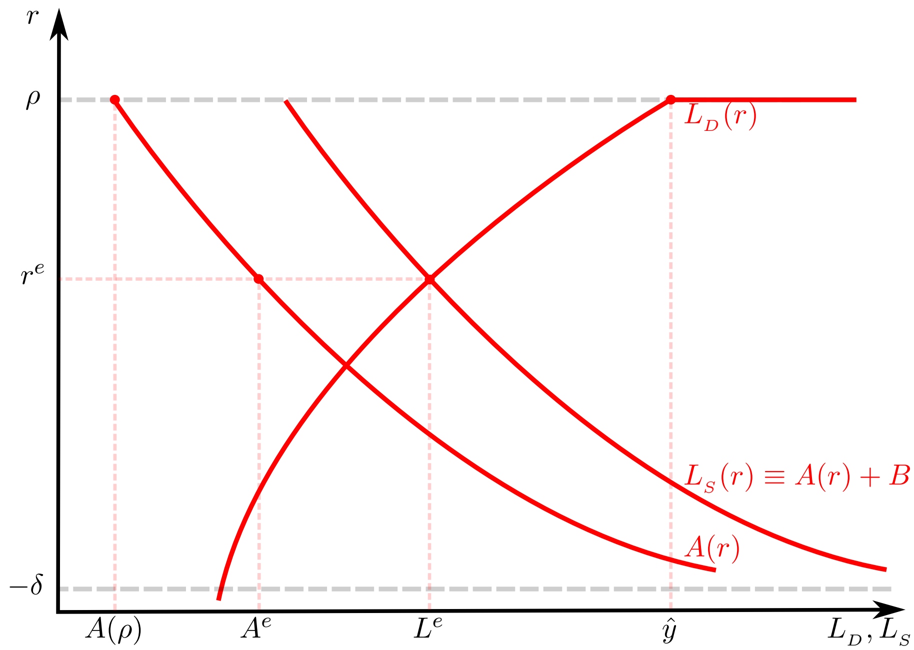


Figure 1: Equilibrium in the market for liquidity

price, P, is lower and the average productivity, $\bar{\varphi}$, is higher when private assets provide liquidity services.

Note that if $B = 0$, the equilibrium in the market for liquidity would be given by the intersection of $A(r)$ and $L_D(r)$, which implies a lower equilibrium interest rate and higher equilibrium level of private liquidity. As in Rocheteau and Rodriguez-Lopez (2014), this result highlights the crowding-out effect that public liquidity, B, has on private liquidity, A. Note that if the government is interested in maximizing the surplus in the financial sector by increasing the amount of public liquidity (so that $\hat{y}$ can be reached), it would push the differentiated-good sector toward the fundamental-value outcome.

If the supply of liquidity is abundant, so that the equilibrium occurs in the horizontal part of the demand for liquidity, the interest rate equals the discount rate and hence the price of liquidity is zero (*i.e.*, a liquidity premium does not exist). As previously discussed by Holmström and Tirole (1998) and Rocheteau (2011), liquidity premia only emerge if liquid assets are in scarce supply. Section 2 mentions evidence on the existence of liquidity premia in equity and corporate bond markets, which then indicates that the supply of liquid assets in financial markets is, indeed, scarce (*i.e.*, in the real world the equilibrium occurs in the upward-sloping part of the demand curve).

5 Liquid Assets and International Trade

The closed-economy model highlights the benefits of the market for liquidity on the real economy. But how do differences across countries in their abilities to generate liquid assets affect the international allocation of economic activity? This section extends the previous model to a two-country setting that allows for heterogeneity in liquidity properties across assets within and between countries.

There are two countries, Home and Foreign, and two production sectors in each country: a homogenous-good sector and a differentiated-good sector. The homogeneous good is traded costlessly and is produced under perfect competition, while each variety of the differentiated good is potentially tradable and is produced under monopolistic competition. Each country is inhabited by a unit measure of households, with each household providing a unit of labor per unit of time. We denote the variables for the Foreign country with a star (*).

There is an international OTC financial market in which Home and Foreign financiers trade financial services. There is a unit measure of financiers in the world.[6] To secure their transactions, financiers may use as collateral four categories of assets: Home and Foreign private assets, and Home and Foreign government bonds. However, there is heterogeneity in the liquidity properties across the four categories of assets, and across private assets within each country.

We start this section by describing the conventional Melitz's two-country structure, then we discuss the international market for liquid assets and define the equilibrium.

5.1 Preferences, Demand, and Production

The description of preferences and demand for Home is similar to section 3.1. Analogous expressions hold for Foreign. Hence, the total expenditure on differentiated goods in Foreign is η, and the Foreign's market demand for variety ω is $q^{*c}(\omega,t) = \left[\frac{p^*(\omega,t)^{-\sigma}}{P^*(t)^{1-\sigma}}\right]\eta$, where $p^*(\omega)$ is the Foreign price of variety ω, and $P^*(t) = \left[\int_{\omega\in\Omega^*} p^*(\omega,t)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$.

In both countries, producers in the differentiated-good sector are heterogeneous in productivity. Each Home and Foreign firm draws its productivity from the same cumulative distribution function, $G(\varphi)$. Each firm then decides whether or not to produce for the domestic and export markets. The decision to produce or not for a market is determined by the ability of the firm to cover the fixed cost of selling in that market. Although there can be imbalances from trading differentiated goods, costless trade in the homogeneous good ensures overall trade balance.

[6]Section 7 presents an extension with credit frictions in which there are endogenous measures of Home and Foreign financiers.

As before, the production function of a Home firm with productivity φ is given by $q(\varphi,t) = \Phi\varphi L(t)$, where Φ is an aggregate productivity factor for Home firms, and $L(t)$ denotes Home labor. Analogously, the production function of a Foreign firm with productivity φ is given by $q^*(\varphi,t) = \Phi^*\varphi L^*(t)$, where Φ^* is an aggregate productivity factor for Foreign firms, and $L^*(t)$ denotes Foreign labor.

The marginal cost of a Home firm with productivity φ for selling in the Home market is $\frac{1}{\Phi\varphi}$. If the Home firm decides to export its finished good, its marginal cost for selling in the Foreign market is $\frac{\tau}{\Phi\varphi}$, where $\tau > 1$ accounts for an iceberg exporting cost—the Home firm must ship τ units of the good for one unit to reach the Foreign market. Assuming market segmentation and given CES preferences, the prices that a Home firm with productivity φ sets in the domestic (D) and export (X) markets are given by $p_D(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{\Phi\varphi}$ and $p_X(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau}{\Phi\varphi}$, respectively. Using these pricing equations and the market demand functions, we obtain that this firm's gross profit functions—before deducting fixed costs—from selling in each market are

$$\pi_D(\varphi) = \frac{1}{\sigma}\left[\frac{P}{p_D(\varphi)}\right]^{\sigma-1}\eta \quad \text{and} \quad \pi_X(\varphi) = \frac{1}{\sigma}\left[\frac{P^*}{p_X(\varphi)}\right]^{\sigma-1}\eta,$$

which are increasing in productivity (*i.e.*, $\pi'_D(\varphi) > 0$ and $\pi'_X(\varphi) > 0$).

Similarly, the marginal cost for a Foreign firm with productivity φ is $\frac{1}{\Phi^*\varphi}$ from selling domestically, and $\frac{\tau}{\Phi^*\varphi}$ from selling in the Home market. Therefore, the prices set by a Foreign firm with productivity φ are $p^*_D(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{\Phi^*\varphi}$ in the domestic market, and $p^*_X(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau}{\Phi^*\varphi}$ in the export market. This firm's gross profit functions from selling in each market are

$$\pi^*_D(\varphi) = \frac{1}{\sigma}\left[\frac{P^*}{p^*_D(\varphi)}\right]^{\sigma-1}\eta \quad \text{and} \quad \pi^*_X(\varphi) = \frac{1}{\sigma}\left[\frac{P}{p^*_X(\varphi)}\right]^{\sigma-1}\eta.$$

5.2 Cutoff Productivity Levels

There are fixed costs of selling in each market. These fixed costs along with the CES demand system imply the existence of cutoff productivity levels that determine the tradability of each differentiated good in each market. For Home firms there are two cutoff productivity levels: one for selling in the domestic market, $\hat{\varphi}_D$, and one for selling in the export market, $\hat{\varphi}_X$. Then, for example, if a Home firm's productivity is between $\hat{\varphi}_D$ and $\hat{\varphi}_X$, the firm produces for the domestic market (as it will be able to cover the fixed costs of selling domestically), but not for the export market (as it will not be able to cover the fixed costs of exporting). Similarly, $\hat{\varphi}^*_D$ and $\hat{\varphi}^*_X$ denote the cutoff productivity levels for Foreign firms.

As before, we assume that all fixed costs are in terms of the homogeneous good. For $i \in \{D, X\}$, let f_i be the fixed cost of selling in market i for Home firms, and let f^*_i be the fixed cost of selling in market i for Foreign firms. Therefore, the cutoff productivity levels satisfy $\pi_i(\hat{\varphi}_i) = f_i$ and $\pi_i^*(\hat{\varphi}_i^*) = f_i^*$, for $i \in \{D, X\}$. Thus, using the gross profit functions from the previous section, we obtain the following zero-cutoff-profit conditions

$$\frac{1}{\sigma}\left[\frac{P}{p_D(\hat{\varphi}_D)}\right]^{\sigma-1}\eta = f_D, \tag{21}$$

$$\frac{1}{\sigma}\left[\frac{P^*}{p_X(\hat{\varphi}_X)}\right]^{\sigma-1}\eta = f_X, \tag{22}$$

$$\frac{1}{\sigma}\left[\frac{P^*}{p_D^*(\hat{\varphi}_D^*)}\right]^{\sigma-1}\eta = f_D^*, \tag{23}$$

$$\frac{1}{\sigma}\left[\frac{P}{p_X^*(\hat{\varphi}_X^*)}\right]^{\sigma-1}\eta = f_X^*. \tag{24}$$

Combining (21) and (24), and (22) and (23)—and using the pricing equations from the previous section—we obtain

$$\hat{\varphi}_X^* = \left(\frac{f_X^*}{f_D}\right)^{\frac{1}{\sigma-1}}\left(\frac{\Phi}{\Phi^*}\right)\tau\hat{\varphi}_D, \tag{25}$$

$$\hat{\varphi}_X = \left(\frac{f_X}{f_D^*}\right)^{\frac{1}{\sigma-1}}\left(\frac{\Phi^*}{\Phi}\right)\tau\hat{\varphi}_D^*. \tag{26}$$

These are two of the equations we need to solve for the equilibrium cutoff productivity levels and they indicate the link between the cutoff levels for firms selling in the same market. Moreover, using the zero-cutoff-profit conditions (21)-(24), we can substitute out P and P^* in the gross profit functions to rewrite them as

$$\pi_i(\varphi) = \left(\frac{\varphi}{\hat{\varphi}_i}\right)^{\sigma-1} f_i, \tag{27}$$

$$\pi_i^*(\varphi) = \left(\frac{\varphi}{\hat{\varphi}_i^*}\right)^{\sigma-1} f_i^*, \tag{28}$$

for $i \in \{D, X\}$.

5.3 Averages and the Composition of Firms

Let N and N^* denote, respectively, the masses of sellers of differentiated goods in Home and Foreign. In Home, N is composed of a mass of N_D Home firms and a mass of N_X^* Foreign firms, so that $N = N_D + N_X^*$. Similarly, $N^* = N_D^* + N_X$, where N_D^* is the mass of Foreign producers selling domestically, and N_X is the mass of Home exporters. As before, firms in each country are subject to a random death shock arriving at Poisson rate $\delta > 0$. In steady state, the firms that die are exactly replaced by successful entrants so that

$$\delta N_i = [1 - G(\hat{\varphi}_i)]\, N_E,$$

$$\delta N_i^* = [1 - G(\hat{\varphi}_i^*)]\, N_E^*,$$

where N_E and N_E^* denote the masses of Home and Foreign entrants per unit of time, $G(\varphi)$ is the cumulative distribution function from which Home and Foreign firms draw their productivities after entry, and $i \in \{D, X\}$. Thus, to obtain expressions for N_D, N_X, N_D^*, and N_X^* in terms of the cutoff productivity levels, we need to derive first the expressions for N_E and N_E^*.

To obtain N_E and N_E^*, note first that we can write the aggregate price equations in Home and Foreign as

$$P = \left[N_D \bar{p}_D^{1-\sigma} + N_X^* \bar{p}_X^{*1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{29}$$

$$P^* = \left[N_D^* \bar{p}_D^{*1-\sigma} + N_X \bar{p}_X^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{30}$$

where $\bar{p}_i = p_i(\bar{\varphi}_i)$ is the average price in market i of differentiated goods sold by Home firms, and $\bar{p}_i^* = p_i^*(\bar{\varphi}_i^*)$ is the average price in market i of Foreign firms' goods, with average productivities given by

$$\bar{\varphi}_i = \left[\int_{\hat{\varphi}_i}^{\infty} \varphi^{\sigma-1} g(\varphi|\varphi \geq \hat{\varphi}_i) d\varphi\right]^{\frac{1}{\sigma-1}} \quad \text{and} \quad \bar{\varphi}_i^* = \left[\int_{\hat{\varphi}_i^*}^{\infty} \varphi^{\sigma-1} g(\varphi|\varphi \geq \hat{\varphi}_i^*) d\varphi\right]^{\frac{1}{\sigma-1}},$$

for $i \in \{D, X\}$. Substituting the expressions for $\bar{p}_i$, $\bar{p}_i^*$, N_i, and N_i^*, for $i \in \{D, X\}$, into equations (29) and (30), and using the zero-cutoff-profit conditions to substitute for P and P^* along with equations (25)-(28), we obtain the system of equations that allows us to solve for N_E and N_E^* as

$$N_E = \frac{\delta\eta}{\sigma}\left[\frac{\Pi_D^* - \Pi_X^*}{\Pi_D \Pi_D^* - \Pi_X \Pi_X^*}\right], \tag{31}$$

$$N_E^* = \frac{\delta\eta}{\sigma}\left[\frac{\Pi_D - \Pi_X}{\Pi_D \Pi_D^* - \Pi_X \Pi_X^*}\right], \tag{32}$$

where

$$\Pi_i = \int_{\hat{\varphi}_i}^{\infty} \pi_i(\varphi) g(\varphi) d\varphi \quad \text{and} \quad \Pi_i^* = \int_{\hat{\varphi}_i^*}^{\infty} \pi_i^*(\varphi) g(\varphi) d\varphi$$

for $i \in \{D, X\}$. Notice that Π_i is the unconditional gross expected profit for a Home potential entrant from selling in market i, and Π_i^* is the unconditional gross expected profit for a Foreign potential entrant from selling in market i.

As is usual in Melitz-type heterogeneous-firm models, we assume that exporting costs are large enough so that $\hat{\varphi}_D < \hat{\varphi}_X$ and $\hat{\varphi}_D^* < \hat{\varphi}_X^*$: exporting firms always produce for the domestic market. This assumption implies that $\Pi_D \Pi_D^* > \Pi_X \Pi_X^*$ and therefore, the denominator in equations (31) and (32) is positive.[7] Hence, for an interior solution—so that N_E and N_E^* are positive—it must also hold that $\Pi_D > \Pi_X$ and $\Pi_D^* > \Pi_X^*$, which we assume to be the case.

[7] To prove that $\Pi_D \Pi_D^* > \Pi_X \Pi_X^*$ when $\hat{\varphi}_D < \hat{\varphi}_X$ and $\hat{\varphi}_D^* < \hat{\varphi}_X^*$, we use the following results: *(i)* $\int_a^\infty \varphi^{\sigma-1} g(\varphi) d\varphi > \int_b^\infty \varphi^{\sigma-1} g(\varphi) d\varphi$ if a and b are positive and $a < b$; *(ii)* $\tau^{2(\sigma-1)} = \left(\frac{\hat{\varphi}_X \hat{\varphi}_X^*}{\hat{\varphi}_D \hat{\varphi}_D^*}\right)^{\sigma-1} \frac{f_D f_D^*}{f_X f_X^*} > 1$, which follows from the product of equations (25) and (26).

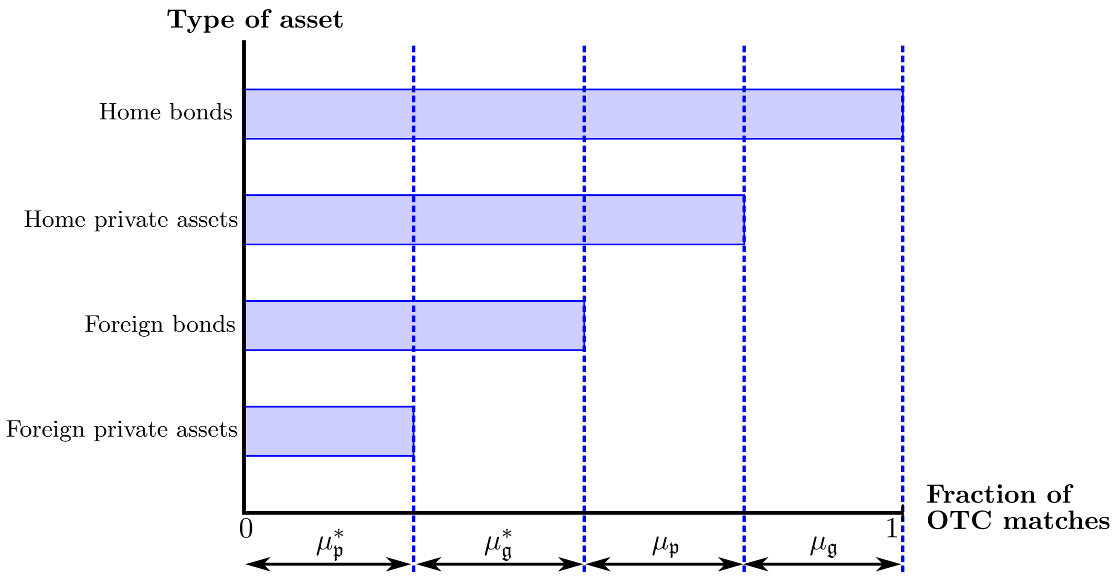


Figure 2: Acceptability of Home and Foreign assets in OTC matches

5.4 The International Market for Liquidity

This section presents the market for liquidity in our two-country setting with multiple Home and Foreign assets. I start by describing the liquidity properties of the different categories of assets, then we look at the demand for liquidity, and lastly we describe the supply of liquid assets and define the steady-state equilibrium.

5.4.1 Differences in Liquidity Properties

I now introduce liquidity differences across assets by assuming that the different categories of assets have different acceptability properties in OTC matches. In particular, I assume that *(i)* Home assets are acceptable as collateral in a larger fraction of OTC matches than Foreign assets, *(ii)* for each country's assets, public liquidity is acceptable in a larger fraction of matches than private liquidity, and *(iii)* there is heterogeneity in acceptability across private assets, with firm-level productivity being positively correlated with collateral fitness.

Figure 2 presents a description of assumptions *(i)* and *(ii)*. In a fraction $\mu_{\mathfrak{g}}$ of OTC matches only Home government bonds are acceptable as collateral, in a fraction $\mu_{\mathfrak{p}}$ of matches both public and private Home assets are acceptable, in a fraction $\mu_{\mathfrak{g}}^*$ of matches Home assets and Foreign government bonds are acceptable, and in the remaining $\mu_{\mathfrak{p}}^*$ fraction of matches all categories of assets are acceptable. Analogously, Foreign private assets are acceptable in a fraction $\mu_{\mathfrak{p}}^*$ of OTC matches, Foreign bonds are acceptable in a fraction $\mu_{\mathfrak{p}}^* + \mu_{\mathfrak{g}}^*$ of matches, Home private assets in a fraction $\mu_{\mathfrak{p}}^*+\mu_{\mathfrak{g}}^*+\mu_{\mathfrak{p}}$ of matches, and Home bonds are acceptable in all matches ($\mu_{\mathfrak{p}}^*+\mu_{\mathfrak{g}}^*+\mu_{\mathfrak{p}}+\mu_{\mathfrak{g}} = 1$).

Regarding *(iii)*, to each producing Home firm (with $\varphi \geq \hat{\varphi}_D$) we associate a loan-to-value ratio, $\lambda(\varphi) \in [0,1)$, that specifies the fraction of the asset value that can be pledged as collateral in an OTC transaction: a financier can obtain a loan of size $\lambda(\varphi)\, a(\varphi)$ if she commits $a(\varphi)$ assets of type φ as collateral. The function $\lambda(\varphi)$ satisfies $\lambda'(\varphi) > 0$ for all $\varphi \geq \hat{\varphi}_D$, $\lambda(\hat{\varphi}_D) = 0$, $\lambda(\infty) \to 1$, and $d\lambda(\varphi)/d\hat{\varphi}_D < 0$. Hence, firm-level productivity is positively related to collateral fitness, which captures the idea that low-productivity firms are seen by financiers as more volatile and sensitive to shocks than more productive firms and thus they get lower loan-to-value ratios. Note that a firm at the cutoff $\hat{\varphi}_D$ is illiquid and hence must yield a return of ρ—financiers know that this firm will die for any minimal shock causing an increase in $\hat{\varphi}_D$, so they are unwilling to accept assets of type $\hat{\varphi}_D$ in OTC transactions. Analogous properties hold for loan-to-value ratios of Foreign private assets, which are described by the function $\lambda^*(\varphi)$.

Although the analysis below only requires $\lambda(\varphi)$ and $\lambda^*(\varphi)$ to meet the properties described above, I assume a useful functional form that depends on a single parameter:

$$\lambda(\varphi) = 1 - \left(\frac{\hat{\varphi}_D}{\varphi}\right)^{\beta} \quad \text{and} \quad \lambda^*(\varphi) = 1 - \left(\frac{\hat{\varphi}^*_D}{\varphi}\right)^{\beta^*}$$

where $\varphi \geq \varphi_D$ for Home firms, $\varphi \geq \varphi^*_D$ for Foreign firms, $\beta > 0$, and $\beta^* > 0$. If $\beta \to \infty$, then $\lambda(\varphi) \to 1$ for all $\varphi > \hat{\varphi}_D$, which approximates the special case in which all claims on producing firms are equally liquid. Note also that $d\lambda(\varphi)/d\beta > 0$ for all $\varphi > \hat{\varphi}_D$, so that a decline in β is useful to analyze the effects of a liquidity crisis affecting loan-to-value ratios of Home private assets.

Furthermore, I assume that for a Home or Foreign private asset to be part of the available liquidity to financiers, the asset must be certified by a rating agency that makes public the asset's underlying productivity. The certification process involves sunk costs of f_A for Home private assets, and f^*_A for Foreign private assets (these costs are in terms of the homogeneous good). These costs imply the existence of two more cutoff productivity levels, $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$, that separate assets into "non-certified" and "certified" categories. Non-certified assets have underlying productivities in the range $[\hat{\varphi}_D, \hat{\varphi}_A)$, they are illiquid and hence pay the illiquid rate of return, ρ. Certified assets have underlying productivities in the range $[\hat{\varphi}_A, \infty)$, they are liquid and hence pay a return below ρ.

Let $r(\varphi)$ denote the rate of return of Home private assets with underlying productivity φ, so that $r(\varphi) = \rho$ if $\varphi \in [\hat{\varphi}_D, \hat{\varphi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\varphi}_A, \infty)$. Similarly, let $r^*(\varphi)$ denote the rate of return of Foreign assets with underlying productivity φ. To pin down $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$, note that an asset with underlying productivity φ will be certified if and only if the value of the firm when certified minus the sunk certification cost, is no less than the value of the firm when not certified;

this condition holds with equality for a firm at the cutoff. Thus, $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$ solve

$$[[\pi_D(\hat{\varphi}_A) - f_D]\,\mathbb{1}\{\hat{\varphi}_A \geq \hat{\varphi}_D\} + [\pi_X(\hat{\varphi}_A) - f_X]\,\mathbb{1}\{\hat{\varphi}_A \geq \hat{\varphi}_X\}]\left[\frac{1}{r(\hat{\varphi}_A)+\delta} - \frac{1}{\rho+\delta}\right] = f_A \tag{33}$$

$$\left[\left[\pi^*_D(\hat{\varphi}^*_A) - f^*_D\right]\mathbb{1}\{\hat{\varphi}^*_A \geq \hat{\varphi}^*_D\} + \left[\pi^*_X(\hat{\varphi}^*_A) - f^*_X\right]\mathbb{1}\{\hat{\varphi}^*_A \geq \hat{\varphi}^*_X\}\right]\left[\frac{1}{r(\hat{\varphi}^*_A)+\delta} - \frac{1}{\rho+\delta}\right] = f^*_A, \tag{34}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function taking the value of 1 if the condition inside the brackets is satisfied (and is zero otherwise). The left-hand side in (33) and (34) shows the difference between the discounted sum of instantaneous profits when certified (with an effective discount rate of $r(\hat{\varphi}_A)+\delta$) and the discounted sum of instantaneous profits when not certified (with an effective discount rate of $\rho+\delta$). The right-hand side in (33) and (34) shows the sunk certification cost in each country.

5.4.2 Supply of Private Liquid Assets

Financiers fund the entry of differentiated-good firms in both countries in exchange for claims on firms' profits. As mentioned before, financiers may use these claims as private liquidity (*i.e.*, as collateral in their financial transactions). In contrast to the closed-economy case, however, the total market capitalization of firms is no longer equivalent to the amount of private liquidity available. In particular, in the presence of loan-to-value ratios below 1 and certification costs that give rise to the cutoffs $\hat{\varphi}_A$ and $\hat{\varphi}^*_A$, the total supply of Home private liquidity, A, and the total supply of Foreign private liquidity, A^*, are now a fraction of the total market capitalization of firms in each country.

At Home, the value of a firm with productivity φ is defined as

$$V(\varphi) = \frac{[\pi_D(\varphi) - f_D]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_D\} + [\pi_X(\varphi) - f_X]\,\mathbb{1}\{\varphi \geq \hat{\varphi}_X\}}{r(\varphi)+\delta} \tag{35}$$

where $r(\varphi) = \rho$ if $\varphi \in [\hat{\varphi}_D, \hat{\varphi}_A)$ and $r(\varphi) < \rho$ if $\varphi \in [\hat{\varphi}_A, \infty)$, and $\mathbb{1}\{\cdot\}$ is the indicator function. As a firm knows its productivity only after entry, the pre-entry expected value of a firm for Home potential entrants is $V_E = \int_{\hat{\varphi}_D}^{\infty} V(\varphi)g(\varphi)d\varphi$. With similar expressions for Foreign firms, and assuming entry costs of f_E for Home entrants, and f^*_E for Foreign entrants, the free-entry conditions for differentiated-good firms at Home and Foreign are

$$\int_{\hat{\varphi}_D}^{\infty}\left[\frac{\pi_D(\varphi) - f_D}{r(\varphi)+\delta}\right]g(\varphi)d\varphi + \int_{\hat{\varphi}_X}^{\infty}\left[\frac{\pi_X(\varphi) - f_X}{r(\varphi)+\delta}\right]g(\varphi)d\varphi = f_E + [1 - G(\hat{\varphi}_A)]f_A, \tag{36}$$

$$\int_{\hat{\varphi}^*_D}^{\infty}\left[\frac{\pi^*_D(\varphi) - f^*_D}{r^*(\varphi)+\delta}\right]g(\varphi)d\varphi + \int_{\hat{\varphi}^*_X}^{\infty}\left[\frac{\pi^*_X(\varphi) - f^*_X}{r^*(\varphi)+\delta}\right]g(\varphi)d\varphi = f^*_E + [1 - G(\hat{\varphi}^*_A)]f^*_A. \tag{37}$$

In (36), the left-hand side is V_E, with the first term showing the expected discounted profits from selling domestically, and the second term showing the expected discounted profits from exporting;

the right-hand side shows the sunk entry cost plus the expected certification cost (which is only paid if the entrant's productivity draw is $\hat{\varphi}_A$ or higher). Equation (37) has an analogous interpretation for Foreign potential entrants.

Let $A(\varphi)$ denote the supply of liquidity stemming from Home firms with productivity φ, and let $A^*(\varphi)$ denote the supply of liquidity stemming from Foreign firms with productivity φ. The aggregate value of Home firms with productivity φ is given by $N_A V(\varphi) g(\varphi|\varphi \geq \hat{\varphi}_A)$, where $N_A = [1 - G(\hat{\varphi}_A)] N_E/\delta$ denotes the measure of certified Home firms. Analogously, $N_A^* V^*(\varphi) g(\varphi|\varphi \geq \hat{\varphi}_A^*)$ is the aggregate value of Foreign firms with productivity φ for all $\varphi \geq \hat{\varphi}_A^*$, where $N_A^* = [1 - G(\hat{\varphi}_A^*)] N_E^*/\delta$ is the measure of certified Foreign firms. Given that only fractions $\lambda(\varphi)$ and $\lambda^*(\varphi)$ of the value of these firms can serve as collateral in the OTC market, it follows that

$$A(\varphi) = \lambda(\varphi) N_A V(\varphi) g(\varphi|\varphi \geq \hat{\varphi}_A)$$
$$A^*(\varphi) = \lambda^*(\varphi) N_A^* V^*(\varphi) g(\varphi|\varphi \geq \hat{\varphi}_A^*)$$

for all $\varphi \geq \hat{\varphi}_A$ at Home and $\varphi \geq \hat{\varphi}_A^*$ at Foreign. The supplies of Home private liquidity, $A = \int_{\hat{\varphi}_A}^{\infty} A(\varphi) d\varphi$, and Foreign private liquidity, $A^* = \int_{\hat{\varphi}_A^*}^{\infty} A^*(\varphi) d\varphi$, can then be written as

$$A = N_A \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi) \left[\frac{[\pi_D(\varphi) - f_D] + [\pi_X(\varphi) - f_X] \mathbb{1}\{\varphi \geq \hat{\varphi}_X\}}{r(\varphi) + \delta} \right] g(\varphi|\varphi \geq \hat{\varphi}_A) d\varphi, \tag{38}$$

$$A^* = N_A^* \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi) \left[\frac{\left[\pi_D^*(\varphi) - f_D^*\right] + \left[\pi_X^*(\varphi) - f_X^*\right] \mathbb{1}\{\varphi \geq \hat{\varphi}_X^*\}}{r^*(\varphi) + \delta} \right] g(\varphi|\varphi \geq \hat{\varphi}_A^*) d\varphi. \tag{39}$$

5.4.3 Demand for Multiple Liquid Assets

Let $a(\varphi)$ and $a^*(\varphi)$ denote the financier's holdings of Home and Foreign assets of type φ, and let b and b^* denote the financier's holdings of Home and Foreign government bonds. In addition to Home and Foreign private assets' returns, $r(\varphi)$ and $r^*(\varphi)$, the rates of return of Home and Foreign government bonds are r_b and r_b^*. Hence, the budget constraint of a Home financier is

$$\int_{\hat{\varphi}_D}^{\infty} \dot{a}(\varphi) d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} \dot{a}^*(\varphi) d\varphi + \dot{b} + \dot{b}^* = \int_{\hat{\varphi}_D}^{\infty} r(\varphi) a(\varphi) d\varphi + \int_{\hat{\varphi}_D^*}^{\infty} r^*(\varphi) a^*(\varphi) d\varphi + r_b b + r_b^* b^* - h - \Upsilon.$$

The left-hand side presents the change in the financier's wealth, which is given by the financier's total investment in private assets and government bonds from both Home and Foreign. The right-hand side shows the interest payments on the financier's portfolio net of homogeneous-good consumption (h) and taxes (Υ). In contrast to the closed-economy model, the financier's portfolio is now composed of assets with different liquidity properties and rates of return. The budget constraint of a Foreign financier is identical to Home financier's budget constraint, with the exception of the last term, which changes to Υ^* (taxes in Foreign).

The total amounts of Home private liquidity, a, and Foreign private liquidity, a^*, held by a financier are given by

$$a = \int_{\hat{\varphi}_A}^{\infty} \lambda(\varphi) a(\varphi) d\varphi \quad \text{and} \quad a^* = \int_{\hat{\varphi}_A^*}^{\infty} \lambda^*(\varphi) a^*(\varphi) d\varphi,$$

which weight the holdings of each asset by its loan-to-value ratio, and take into account that liquid assets have underlying productivities no less than $\hat{\varphi}_A$ and $\hat{\varphi}_A^*$.

Following similar steps to those in section 4.2 to obtain (15), we find that the continuation value of a financier upon being matched (but before realizing its buyer or seller role), $Z(a, a^*, b, b^*)$, is given by

$$\begin{aligned} Z(a, a^*, b, b^*) =& \frac{\mu_{\mathfrak{p}}^*}{2} \max_{y_{\mathfrak{p}}^* \le a+a^*+b+b^*} \left\{F(y_{\mathfrak{p}}^*) - y_{\mathfrak{p}}^*\right\} + \frac{\mu_{\mathfrak{g}}^*}{2} \max_{y_{\mathfrak{g}}^* \le a+b+b^*} \left\{F(y_{\mathfrak{g}}^*) - y_{\mathfrak{g}}^*\right\} \\ &+ \frac{\mu_{\mathfrak{p}}}{2} \max_{y_{\mathfrak{p}} \le a+b} \left\{F(y_{\mathfrak{p}}) - y_{\mathfrak{p}}\right\} + \frac{\mu_{\mathfrak{g}}}{2} \max_{y_{\mathfrak{g}} \le b} \left\{F(y_{\mathfrak{g}}) - y_{\mathfrak{g}}\right\} + W(a, a^*, b, b^*). \end{aligned}$$

This expression shows that with probability 1/2 the financier is the buyer in the OTC match, in which case she can make a take-it-or-leave-it offer to the seller in order to maximize her surplus, $F(y) - y$. With probability $\mu_{\mathfrak{p}}^*$, the financier is in a match in which private and public assets from both Home and Foreign are acceptable as collateral and thus, she can transfer up to $a + a^* + b + b^*$ in exchange for $y_{\mathfrak{p}}^*$. With probability $\mu_{\mathfrak{g}}^*$ Home assets and Foreign government bonds are acceptable, so that the financier can transfer up to $a + b + b^*$ to purchase $y_{\mathfrak{g}}^*$. With probability $\mu_{\mathfrak{p}}$ only Home assets are acceptable, so that the financier can transfer up to $a + b$ to purchase $y_{\mathfrak{p}}$. Lastly, with probability $\mu_{\mathfrak{g}}$ only Home government bonds are acceptable, so that the financier can transfer up to b to purchase $y_{\mathfrak{g}}$.

Similar to the derivation of equation (18) in the closed-economy model, the financier's optimal portfolio solves

$$\frac{\rho - r^*(\varphi)}{\theta} = \mu_{\mathfrak{p}}^* \lambda^*(\varphi) \left[F'(y_{\mathfrak{p}}^*) - 1\right] \tag{40}$$

$$\frac{\rho - r_b^*}{\theta} = \mu_{\mathfrak{p}}^* \left[F'(y_{\mathfrak{p}}^*) - 1\right] + \mu_{\mathfrak{g}}^* \left[F'(y_{\mathfrak{g}}^*) - 1\right] \tag{41}$$

$$\frac{\rho - r(\varphi)}{\theta} = \mu_{\mathfrak{p}}^* \lambda(\varphi) \left[F'(y_{\mathfrak{p}}^*) - 1\right] + \mu_{\mathfrak{g}}^* \lambda(\varphi) \left[F'(y_{\mathfrak{g}}^*) - 1\right] + \mu_{\mathfrak{p}} \lambda(\varphi) \left[F'(y_{\mathfrak{p}}) - 1\right] \tag{42}$$

$$\frac{\rho - r_b}{\theta} = \mu_{\mathfrak{p}}^* \left[F'(y_{\mathfrak{p}}^*) - 1\right] + \mu_{\mathfrak{g}}^* \left[F'(y_{\mathfrak{g}}^*) - 1\right] + \mu_{\mathfrak{p}} \left[F'(y_{\mathfrak{p}}) - 1\right] + \mu_{\mathfrak{g}} \left[F'(y_{\mathfrak{g}}) - 1\right]. \tag{43}$$

Equations (40)-(43) define the optimal choice of each type of asset.[8] On one extreme, the left-hand side of equation (40) is the holding cost of Foreign private asset of type φ, while the right-hand

[8] Note that the closed-economy equation (18) can be obtained from (42) and (43) by assuming that $\mu_{\mathfrak{p}}^* = \mu_{\mathfrak{g}}^* = \mu_{\mathfrak{g}} = 0$, $\mu_{\mathfrak{p}} = 1$, and $\beta \to \infty$ so that $\lambda(\varphi) \to 1$ for all $\varphi \ge \hat{\varphi}_D$.

side indicates the expected marginal surplus from holding an additional unit of that asset. That Foreign asset can only be used in a fraction $\mu_{\mathfrak{p}}^*$ of all matches with a pledgeability ratio of $\lambda^*(\varphi)$, in which case the marginal surplus of the financier is $F'(y_{\mathfrak{p}}^*) - 1$. On the other extreme, the left-hand side of (43) shows the holding cost of a Home government bond, while the right-hand side is its marginal surplus from its use in all matches in the financial market.

Similar to the closed-economy case, the quantity of financial services traded in an OTC match is the minimum between the value of the buyer's liquidity in that match and the surplus-maximizing quantity, $\hat{y}$. The difference is that in the closed-economy case only domestic private and public assets are used, and they are all fully acceptable in all matches. Given that financiers hold identical portfolios and that there is a unit measure of financiers in the world, market clearing implies that $a = A$, $a^* = A^*$, $b = B$, and $b = B^*$. Thus, we get

$$y_{\mathfrak{p}}^* = \min\left\{A + A^* + B + B^*, \hat{y}\right\}, \tag{44}$$

$$y_{\mathfrak{g}}^* = \min\left\{A + B + B^*, \hat{y}\right\}, \tag{45}$$

$$y_{\mathfrak{p}} = \min\left\{A + B, \hat{y}\right\}, \tag{46}$$

$$y_{\mathfrak{g}} = \min\left\{B, \hat{y}\right\}. \tag{47}$$

Note from (40)-(43) and (44)-(47) that we can have several scenarios. Suppose, for example, that liquidity is scarce in matches that only accept Home assets, but is abundant in matches that also accept Foreign assets; *i.e.*, $A + B < \hat{y}$ but $A + B + B^* > \hat{y}$. This implies that $F'(y_{\mathfrak{p}}^*) - 1 = F'(y_{\mathfrak{g}}^*) - 1 = 0$, but $F'(y_{\mathfrak{g}}) - 1 > F'(y_{\mathfrak{p}}) - 1 > 0$. Therefore, from (40)-(43) we obtain that all Foreign assets pay the maximum rate of return, $r_b^* = r^*(\varphi) = \rho$, while Home liquid assets give returns $r_b < r(\varphi) < \rho$.

Furthermore, combining (40)-(43) and (44)-(47) we can write the full structure of interest rates as follows:

$$r^*(\varphi) = \rho - \mu_{\mathfrak{p}}^*\theta\lambda^*(\varphi)\left[F'(A + A^* + B + B^*) - 1\right]^+, \tag{48}$$

$$r_b^* = \rho - \mu_{\mathfrak{p}}^*\theta\left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^*\theta\left[F'(A + B + B^*) - 1\right]^+, \tag{49}$$

$$\begin{aligned} r(\varphi) = &\rho - \mu_{\mathfrak{p}}^*\theta\lambda(\varphi)\left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^*\theta\lambda(\varphi)\left[F'(A + B + B^*) - 1\right]^+ \\ &- \mu_{\mathfrak{p}}\theta\lambda(\varphi)\left[F'(A + B) - 1\right]^+, \end{aligned} \tag{50}$$

$$\begin{aligned} r_b = &\rho - \mu_{\mathfrak{p}}^*\theta\left[F'(A + A^* + B + B^*) - 1\right]^+ - \mu_{\mathfrak{g}}^*\theta\left[F'(A + B + B^*) - 1\right]^+ \\ &- \mu_{\mathfrak{p}}\theta\left[F'(A + B) - 1\right]^+ - \mu_{\mathfrak{g}}\theta\left[F'(B) - 1\right]^+, \end{aligned} \tag{51}$$

where $[x]^+ = \max\{x, 0\}$. From (48), note that Foreign private asset of type $\varphi \geq \hat{\varphi}_A^*$ dominates private asset of type $\tilde{\varphi} > \varphi$ in their rate of return, $r^*(\varphi) > r^*(\tilde{\varphi})$, provided that $\mu_{\mathfrak{p}}^* > 0$ and

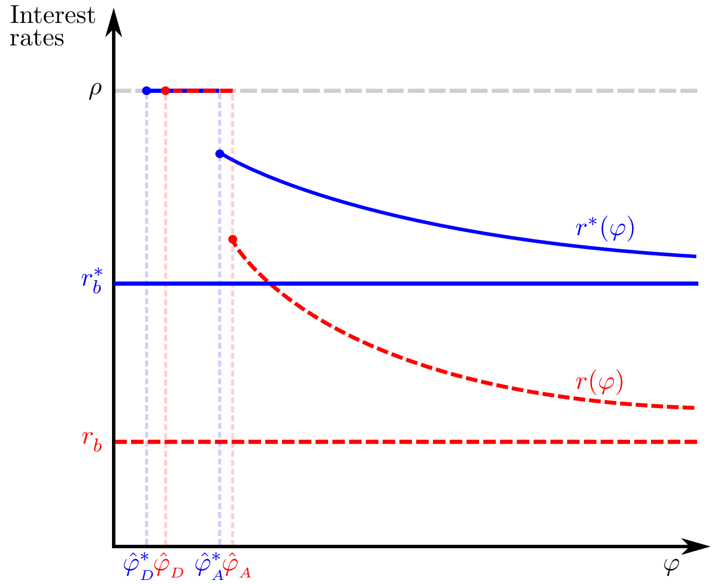


Figure 3: The structure of interest rates when liquidity is scarce in all OTC matches

$A + A^* + B + B^* < \hat{y}$. Similarly, the Foreign private asset of type $\tilde{\varphi}$ dominates Foreign government bonds in their rate of return, $r^*(\tilde{\varphi}) > r_b^*$, if either $\mu_{\mathfrak{p}}^* > 0$ and $A + A^* + B + B^* < \hat{y}$, or $\mu_{\mathfrak{g}}^* > 0$ and $A + A^* + B < \hat{y}$, or both. Similar rate-of-return comparisons across multiple assets can be obtained from (48)-(51).[9] Figure 3 shows the full structure of interest rates when liquidity is scarce in every match in the financial market; *i.e.*, when $A + A^* + B + B^* < \hat{y}$.

The definition of a steady-state equilibrium in the two-country model follows.

Definition 2. *A steady-state equilibrium in the two-country model is a list*

$$\left\langle \hat{\varphi}_D, \hat{\varphi}_X, \hat{\varphi}_D^*, \hat{\varphi}_X^*, \hat{\varphi}_A, \hat{\varphi}_A^*, A, A^*, y_{\mathfrak{p}}^*, y_{\mathfrak{g}}^*, y_{\mathfrak{p}}, y_{\mathfrak{g}}, r^*(\varphi), r_b^*, r(\varphi), r_b \right\rangle$$

that solves (25), (26), (33), (34), (36)-(39), *and* (40)-(47).

The steady-state equilibrium solves for the cutoff productivity levels that indicate the tradability of Home and Foreign goods in each market $(\hat{\varphi}_D, \hat{\varphi}_X, \hat{\varphi}_D^*, \hat{\varphi}_X^*)$, the cutoff productivity levels that separate certified and non-certified firms in each country $(\hat{\varphi}_A, \hat{\varphi}_A^*)$, the amounts of Home and Foreign private liquidity (A, A^*), the amount of financial services traded in each type of match $(y_{\mathfrak{p}}^*, y_{\mathfrak{g}}^*, y_{\mathfrak{p}}, y_{\mathfrak{g}})$, and the structure of interest rates $(r^*(\varphi), r_b^*, r(\varphi), r_b)$.

[9] As a by-product, note that this framework is useful to help explain the equity-premium puzzle, which refers to the observation that rates of return on equities are much higher than rates of return on government bonds. Lagos (2010) explores this venue in a related setting.

6 Financial Development, Trade Liberalization, and Liquidity Crises

We can now investigate the effects of cross-country differences in financial development on the allocation of economic activity, and study the model's implications for the effects of trade liberalization. This section also studies the effects of a liquidity crisis on trade and the structure of interest rates.

6.1 Financial Development and Trade

This paper defines financial development as a country's ability to generate assets that are acceptable as collateral or means of payment in financial transactions. In the current framework, financial development is captured by the acceptability parameters $\mu_{\mathfrak{p}}^*$, $\mu_{\mathfrak{g}}^*$, $\mu_{\mathfrak{p}}$, and $\mu_{\mathfrak{g}}$, by the loan-to-value ratio parameters β and β^*, and by the asset certification costs f_A and f_A^*. The analysis in this section focuses exclusively on $\mu_{\mathfrak{p}}^*$ and $\mu_{\mathfrak{p}}$, and hence we assume *(i)* $f_A = f_A^* = 0$, *(ii)* $\beta = \beta^* \to \infty$, and *(iii)* $\mu_{\mathfrak{g}}^* = \mu_{\mathfrak{g}} = 0$.

The first assumption implies that $\hat{\varphi}_D = \hat{\varphi}_A$ and $\hat{\varphi}_D^* = \hat{\varphi}_A^*$, while the second assumption implies that the loan-to-value ratio of every producing firm is 1. Under these two assumptions—and similar to the closed-economy model—the total market capitalization of firms in each country is identical to the country's total amount of private liquidity, A for Home and A^* for Foreign. The third assumption implies that all Home assets (public and private) are acceptable in all OTC matches ($\mu_{\mathfrak{p}}^* + \mu_{\mathfrak{p}} = 1$), while Foreign assets are only acceptable in a fraction $\mu_{\mathfrak{p}}^*$ of matches—analogously, in a fraction $\mu_{\mathfrak{p}}$ of OTC matches only Home assets are acceptable, while in the remaining $\mu_{\mathfrak{p}}^* = 1 - \mu_{\mathfrak{p}}$ fraction of matches all assets (from Home and Foreign) are acceptable. All together, the three assumptions yield the same interest rate for all Home assets, r, and the same interest rate for all Foreign assets, r^*.

We assume that Home and Foreign have identical production structures: $\Phi = \Phi^*$, $f_E = f_E^*$, $f_D = f_D^* = f$, and $f_X = f_X^* = f$. To simplify notation let $\mu \equiv \mu_{\mathfrak{p}}$, so that $\mu_{\mathfrak{p}}^* \equiv 1 - \mu$. Thus, the differences between Home and Foreign will only span from the value that μ takes. Focusing only on μ allows us to clearly elucidate the strong effects that liquidity differences across countries can have on the international allocation of economic activity. There are two extreme cases: on the one hand, if $\mu = 0$ all Home and Foreign assets are acceptable in all OTC matches and thus there are no liquidity differences across countries; on the other hand, if $\mu = 1$ Home assets are acceptable in all OTC matches, but Foreign assets are totally illiquid (they are never accepted in OTC transactions). Thus, as μ rises the relative liquidity differences between Home and Foreign assets become larger in favor of Home.

To gain tractability and obtain clear analytical results, I further assume a Pareto distribution of productivity so that $G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k$ and $g(\varphi) = \frac{k\varphi_{\min}^k}{\varphi^{k+1}}$, where $k > \sigma - 1$ is a parameter of productivity dispersion (higher k means lower heterogeneity). This version of the model can be conveniently reduced to a system of four equations and four unknowns,

$$A = \frac{(\sigma-1)\eta}{\sigma k}\left[\frac{\tau^k}{\tau^k(r+\delta)-(r^*+\delta)} - \frac{1}{\tau^k(r^*+\delta)-(r+\delta)}\right], \tag{52}$$

$$A^* = \frac{(\sigma-1)\eta}{\sigma k}\left[\frac{\tau^k}{\tau^k(r^*+\delta)-(r+\delta)} - \frac{1}{\tau^k(r+\delta)-(r^*+\delta)}\right], \tag{53}$$

$$r^* = \rho - (1-\mu)\theta\left[F'(A+A^*+B+B^*)-1\right]^+, \tag{54}$$

$$r = \rho - (1-\mu)\theta\left[F'(A+A^*+B+B^*)-1\right]^+ - \mu\theta\left[F'(A+B)-1\right]^+, \tag{55}$$

where an interior solution requires $\frac{2\tau^k}{\tau^{2k}+1} < \frac{r^*+\delta}{r+\delta} < \frac{\tau^{2k}+1}{2\tau^k}$, which we assume always holds. The endogenous variables are the total amounts of Home and Foreign private liquidity, A and A^*, and the interest rates, r and r^*, while our exogenous variables are μ, τ, B, and B^*.

6.1.1 Abundant Liquidity

Consider the case of abundant liquidity in every match in the OTC financial market (*i.e.*, $A+B \geq \hat{y}$) so that liquidity is not valued: $[F'(A+B)-1]^+ = [F'(A+A^*+B+B^*)-1]^+ = 0$ and thus $r = r^* = \rho$. It follows from (52) and (53) that the total capitalization of firms is the same in both countries, $A = A^* = \frac{(\sigma-1)\eta}{\sigma k(\rho+\delta)} = \underline{A}$, which is independent of μ and τ. This is the "fundamental-value" outcome of the conventional Melitz model with two identical countries, a Pareto distribution of productivity, and no liquidity services of private assets. Trade liberalization (a decline in τ) does not affect the total capitalization of firms nor entry ($N_E = N_E^* = \frac{\delta \underline{A}}{f_E}$, *i.e.*, entry and capitalization are directly proportional), but it induces reallocation of market shares toward more productive firms, which translates to higher average productivities and lower aggregate prices in both countries.

6.1.2 Scarce Liquidity

Let us now consider the case with scarce liquidity in all OTC matches (*i.e.*, $A+A^*+B+B^* < \hat{y}$) and $\mu \in [0,1)$, so that there are liquidity premiums for Home and Foreign assets: $[F'(A+B)-1]^+ > [F'(A+A^*+B+B^*)-1]^+ > 0$ and thus $r \leq r^* < \rho$.

If $\mu = 0$, so that Home and Foreign assets are equally liquid, the right-hand sides of (54) and (55) are identical and thus interest rates are the same in both countries, $r = r^* = r_0 < \rho$. From (52) and (53) it follows that $A = A^* = \frac{(\sigma-1)\eta}{\sigma k(r_0+\delta)} = A_0$, which is independent of τ. Hence, as in the abundant liquidity case, trade liberalization does not affect the total capitalization of firms but it has conventional Melitz's effects on aggregate productivity and prices. It is the case, however, that $A_0 > \underline{A}$ and in particular

$$\frac{A_0}{\underline{A}} = \frac{\rho + \delta}{r_0 + \delta} = 1 + \frac{\theta\left[F'(2A_0 + B + B^*) - 1\right]}{r_0 + \delta} > 1.$$

Therefore, as in the closed-economy case, the liquidity role of private assets in the financial market causes an expansion in the differentiated-good sector in each country, which translates to higher entry, higher average productivity, and lower aggregate prices. Note also that an increase in the supply of government bonds in either country (an increase in B or B^*) crowds out economic activity in both countries: $F'(2A_0 + B + B^*)$ declines toward 1, r_0 rises toward ρ, and A_0 gets closer to the abundant-liquidity outcome, $\underline{A}$.

If $\mu \in (0, 1)$, so that Home assets are more liquid than Foreign assets, we can see from (54) and (55) that $r < r^* < \rho$. Taking the ratio of (52) and (53) we get

$$\frac{A}{A^*} = 1 + \frac{(\tau^k + 1)^2 (r^* - r)}{(\tau^{2k} + 1)(r + \delta) - 2\tau^k (r^* + \delta)}.$$

In an interior solution the denominator in the second term is always positive and therefore, $A > A^*$ when $r < r^*$. Thus, even though Home and Foreign have identical production structures, the allocation of economic activity favors Home because of its lower interest rate due to its better ability to generate liquid assets for the financial market. This is also reflected in higher average productivity and a lower aggregate price at Home. In the presence of liquidity differences across countries's assets, Table 1 shows the equilibrium responses of A, A^*, r, and r^* to changes in μ, τ, B, and B^*.

exogenous → / endogenous ↓	μ	τ	B	B^*
A	+	−	−	−
A^*	−	+	+/−	+/−
r	+/−	−	+	+
r^*	+	+/−	+	+

Table 1: Comparative statics when $\mu \in (0, 1)$— Each cell indicates the sign of the derivative of the endogenous variable in the row with respect to the exogenous variable in the column

The first column of Table 1 shows that as μ rises—so that Foreign assets become less liquid—the total capitalization of Home firms increases, while the total capitalization of Foreign firms declines. To understand this result, let us start with a state of commercial autarky or "no trade" ($\tau \to \infty$). In that case, $\frac{dr}{d\mu}$ is negative, and (52)-(53) collapse to $A = \frac{(\sigma-1)\eta}{\sigma k(r+\delta)}$ and $A^* = \frac{(\sigma-1)\eta}{\sigma k(r^*+\delta)}$, so that there is a straightforward inverse relationship between a country's interest rate and total capitalization of firms. A rise in μ increases the financiers' demand for Home assets, causing a decline in their rate of return (the liquidity premium of Home assets rises), which in turn drives up entry of Home firms and their total capitalization. Along the way average productivity rises and the aggregate price declines at Home. The opposite happens at Foreign, with r^* increasing toward ρ as μ increases—A^* declines from A_0 toward the fundamental-value outcome, $\underline{A}$, as μ rises in the interval $(0, 1)$.

With trade in differentiated goods, the impact of cross-country liquidity differences on the allocation of economic activity are magnified. An increase in μ increases entry at Home more than in commercial autarky because Home firms now have access to the Foreign market. On the other hand, entry at Foreign declines more than under commercial autarky because in addition to higher interest rates, Foreign firms are subject to tougher competition from new Home entrants. The sign of $\frac{dr}{d\mu}$ becomes ambiguous because the increase in A implies that more liquidity is available in OTC financial transactions that only accept Home assets ($F'(A+B)$ declines), and with the magnifying effects of trade the rise in A can be sufficiently large that r could even go up for high levels of μ.

For a given $\mu \in (0, 1)$, the second column in Table 1 shows the unequal effects of trade liberalization. A decline in τ increases A and reduces A^*, exacerbating the gap in the allocation of economic activity across countries. Given the difference in the liquidity properties between Home and Foreign assets, we start from a state in which entry is lower at Foreign and

$$\hat{\varphi}_D^* < \hat{\varphi}_D < \hat{\varphi}_X < \hat{\varphi}_X^*,$$

where $\hat{\varphi}_X = \tau \hat{\varphi}_D^*$ and $\hat{\varphi}_X^* = \tau \hat{\varphi}_D$. Hence, average productivity is higher at Home ($\bar{\varphi}_D > \bar{\varphi}_D^*$), but Foreign exporters are on average more productive than Home exporters ($\bar{\varphi}_X^* > \bar{\varphi}_X$). Entry incentives at Foreign are mostly driven by profit opportunities from selling domestically, as the Home market is more competitive and thus more difficult to access. On the other hand, the smaller gap between $\hat{\varphi}_D$ and $\hat{\varphi}_X$ shows that entry incentives at Home are more balanced between domestic sales and export opportunities. Trade liberalization has conventional Melitz-type effects in both countries: $\hat{\varphi}_D^*$ and $\hat{\varphi}_D$ rise while $\hat{\varphi}_X$ and $\hat{\varphi}_X^*$ decline. Therefore, in each country profit opportunities from selling domestically decline but profit opportunities from exporting increase, the least productive firms are wiped out, average productivity increases, and the aggregate price declines. However, the decline in τ disproportionately affects Foreign firms' profit opportunities from selling domestically, who now face tougher competition from Home. Although now it is easier to export to the competitive Home market, the Foreign entry incentives from exporting are weak compared to the decreased entry incentives due to tougher Home competition. In contrast, Home firms thrive as they access the less competitive Foreign market. In the end, trade liberalization causes even more entry at Home and less entry at Foreign, which translates to a higher A and a lower A^*.

A decline in τ has an ambiguous effect on the interest rate of Foreign assets, r^*. Given $\mu \in (0,1)$, from (54) note that the response of r^* to τ depends on world liquidity, $A + A^*$. If the contraction at Foreign dominates the expansion at Home so that $A + A^*$ declines, liquidity is more scarce in OTC financial matches that accept Foreign assets, $F'(A + A^* + B + B^*)$ increases, and thus r^* declines (the liquidity premium on Foreign assets rises). The opposite happens if $A + A^*$ increases. On the other hand, given $\mu \in (0,1)$, trade liberalization always increases the interest rate on Home assets, r; *i.e.*, the liquidity premium on Home assets declines. From (55), this results implies that the increase in liquidity in matches that only accept Home assets is sufficiently large (so that the decline in $F'(A + B)$ is sufficiently strong) to offset the downward pressure on r from a possible reduction in liquidity in OTC matches that also accept Foreign assets.

The last two columns in Table 1 show the impact of changes in the supply of Home and Foreign public liquidity. The results are relatively similar to those obtained in the closed-economy case: a rise in B or B^* increases the amount of available liquidity in financial matches, driving an increase in interest rates of both Home and Foreign assets, and crowding out Home private liquidity (A declines). However, the response of Foreign private liquidity, A^*, is ambiguous.

Without trade in differentiated goods ($\tau \to \infty$), $\frac{dA^*}{dB}$ and $\frac{dA^*}{dB^*}$ are negative and therefore Foreign private liquidity is also crowded out by an increase in B or B^*. With a low enough τ so that trade matters, an increase in B has a positive effect on A^* if μ is sufficiently high. To see this, note that if μ is very close to 1 (so that Foreign assets are almost never accepted as collateral in the financial market), then an increase in B has a negligible impact on r^* because the latter is already very close to its upper bound ρ. In contrast, given that Home assets are basically the only source of liquidity for the financial market, the increase in B causes a large increase in r, which in consequence crowds out an important amount of Home private liquidity, A. With entry and average productivity declining at Home, Foreign firms export incentives increase and therefore entry, average productivity, and total capitalization rise at Foreign. An increase in Foreign government bonds, B^*, may also increase A^* but the conditions for that to happen are much harder to satisfy: τ should be low, and μ cannot be too low (with a higher μ changes in public liquidity mostly affect Home firms) nor too high (if $\mu \to 1$ Foreign public and private assets are nearly irrelevant in the liquidity market).

Lastly, we consider the case with scarce liquidity and $\mu = 1$, so that Foreign assets are totally illiquid—never accepted as collateral in the financial market—and hence $r < r^* = \rho$.[10] After

[10] This is equivalent to a case with $\mu \in (0,1)$ in which liquidity is abundant in OTC matches that accept Foreign assets (*i.e.*, $A + A^* + B + B^* > \hat{y}$), but scarce in matches that only accept Home assets (*i.e.*, $A + B < \hat{y}$).

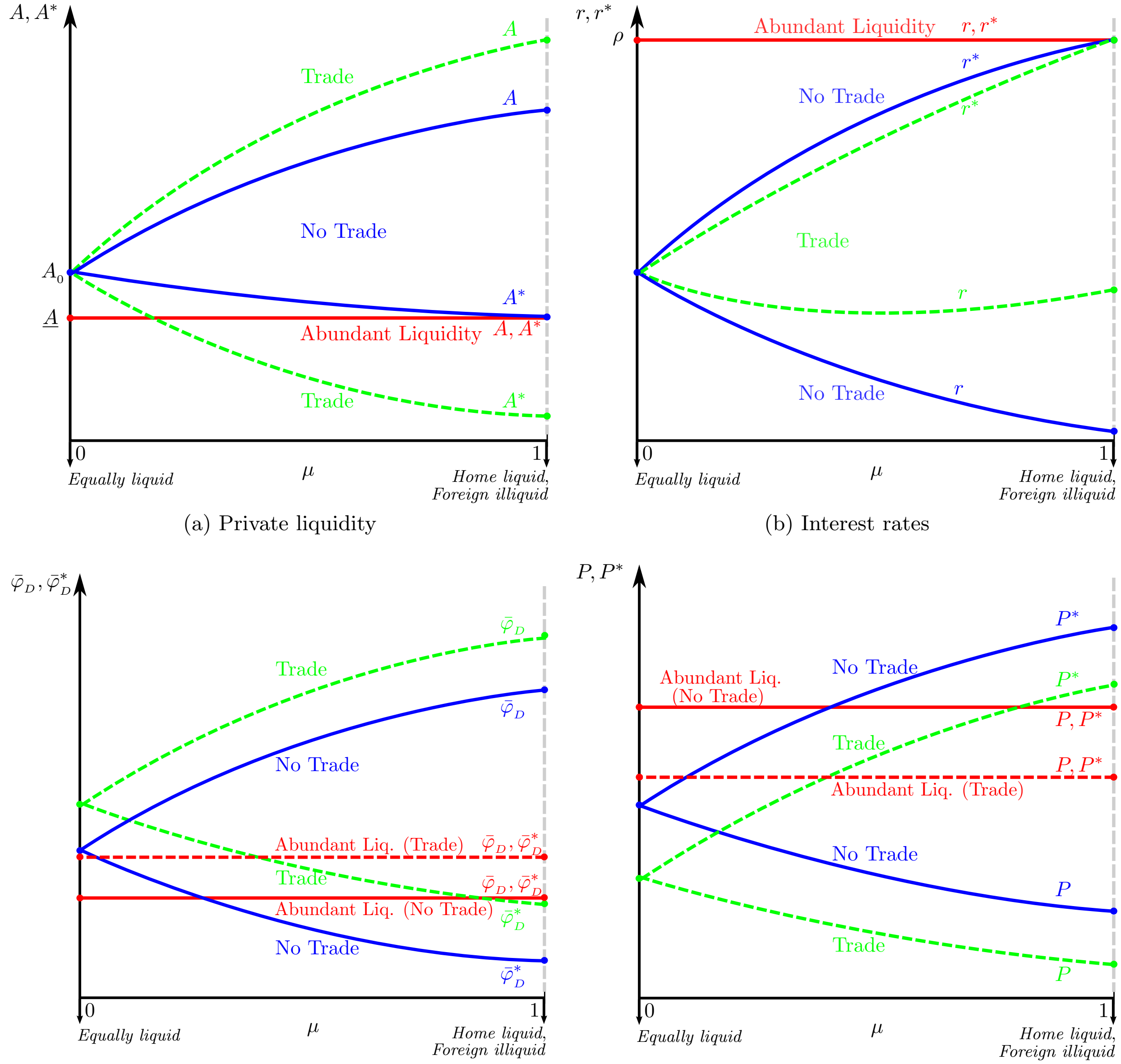
replacing r^* with ρ and μ with 1, the solution of the model simplifies to solving for A and r from (52) and (55). The effects of a decline in τ are similar to the case with $\mu \in (0,1)$: there are conventional Melitz-type effects on on average productivity and aggregate prices in both countries, but there are unequal effects on real economic activity (A increases and A^* declines), and r rises because liquidity, $A+B$, is more abundant in OTC matches. A rise in B increases r and partially crowds out Home private liquidity, A. As Home becomes less competitive, export opportunities for Foreign firms improve, driving up entry and total capitalization, A^*.

6.1.3 Summary

Figure 4 presents a graphical summary of our main results on the relevance of financial development and the unequal effects of trade. It shows the responses of several variables to changes in μ under three scenarios: *(1)* "Abundant Liquidity" corresponds to the case when there is a large enough supply of liquid assets so that OTC matches always reach the surplus-maximizing consumption of financial services, $\hat{y}$; *(2)* "No Trade" corresponds to a case with scarce liquidity and no international trade in differentiated goods ($\tau \to \infty$); and *(3)* "Trade" corresponds to the case with scarce liquidity and τ sufficiently small to allow for international trade flows in differentiated goods.

Starting from the "equally liquid" case ($\mu = 0$), note in Figure 4a that the liquidity role of private assets expands the market capitalization of firms in both countries whether there is differentiated-good trade or not ($A_0 > \underline{A}$). With no trade and scarce liquidity, the level of A^* declines toward its fundamental value as Foreign assets become less liquid, while the level of A increases. When there is trade, however, the differentiated-good sector in Foreign gets wiped out by Home firms as μ rises. Further trade liberalization would continue to increase the gap between A and A^* until the Foreign differentiated-good sector is totally depleted. Hence, liquidity differences across countries influence the allocation of economic activity in favor of the country that is better in generating liquid assets for the financial system. In such as setting, trade liberalization magnifies the gap between the countries.

Figure 4b shows that when Home and Foreign assets are equally liquid, their interest rates are identical and well below ρ (the rate of return on illiquid assets). As μ rises the return on Foreign assets, r^*, increases toward ρ and the return on Home assets, r, initially declines but if τ is sufficiently low it may increase for higher levels of μ. This happens because the rise in A increases the amount of liquidity in matches that only accept Home assets, and with the magnifying effect of trade—which is more powerful the higher μ is—the supply of liquidity effect (driven by A and pushing for higher r) can dominate the effect of the increase in demand for Home liquidity (driven



(a) Private liquidity (b) Interest rates

(c) Average productivity (d) Aggregate prices

Figure 4: The impact of differences in financial development and the unequal effects of trade

by μ and pushing for lower r).

Figure 4c shows the conventional Melitz's result of the positive effect of trade liberalization on aggregate productivity: for every level of μ, aggregate productivity in both countries is higher under trade than under commercial autarky. It also shows, however, that as Foreign assets becomes less liquid, aggregate productivity increases at Home but declines at Foreign. Hence, differences in financial development not only shrink the size of Foreign's real economy, but they also make it less productive. As a mirror to aggregate productivity changes, we see in Figure 4d that for each level of μ aggregate prices are lower under trade, but the relationship between μ and the aggregate price is again adverse for the Foreign country, but it benefits the Home country.

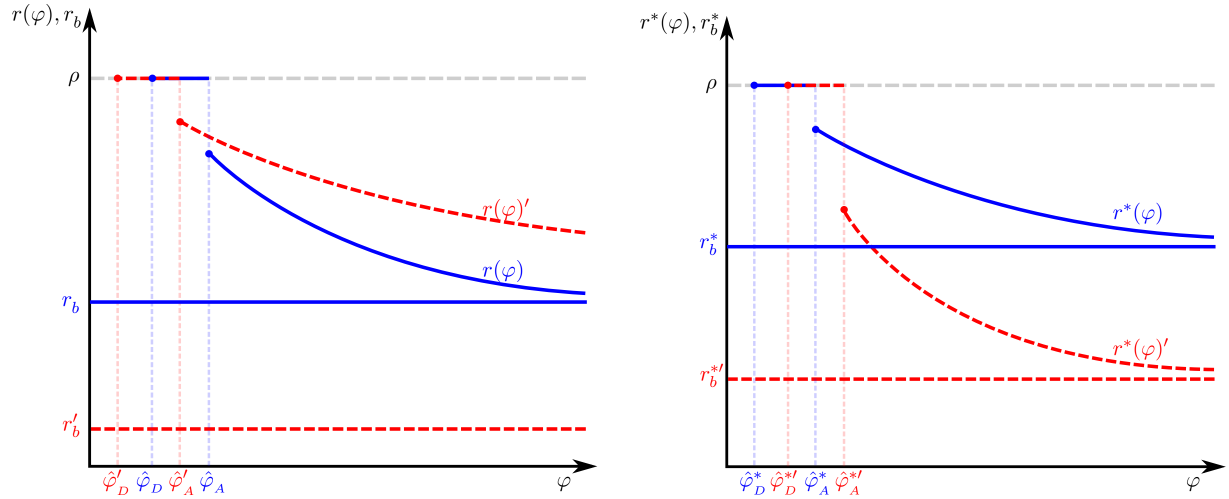
6.2 Liquidity Crises

The financial crisis of 2007-2008 had its origins on bad private assets (subprime mortgage backed securities) that were widely held by financiers. As the financial system realized its exposure to these bad assets, many other private assets were downgraded by rating agencies. For example, according to the IMF (2012), 63 percent of AAA-rated mortgage-backed securities issued from 2005 to 2007 had been downgraded by 2009. The financial crisis was followed by a substantial decline in economic activity, an even larger decline in trade, and an increase in the rate-of-return spread between U.S. government bonds and almost any other type of asset.

This section uses the full-blown model of section 5 to analyze the effects of a liquidity crisis that resembles the one of 2007-2008. In particular, Home will resemble the U.S. by being the source of the liquidity shock on private assets, while still issuing the most liquid asset in the world (U.S. government bonds). In our framework, this type of crisis can be studied with either *(i)* a decline in $\mu_{\mathfrak{p}}$ while keeping $\mu_{\mathfrak{p}}^*$ and $\mu_{\mathfrak{g}}^*$ constant, or *(ii)* a decline in the parameter β of the loan-to-value function, $\lambda(\varphi) = 1 - (\hat{\varphi}_D/\varphi)^{\beta}$. In the first case, the fraction of financial matches in which Home private assets are acceptable, $\mu_{\mathfrak{p}}^* + \mu_{\mathfrak{g}}^* + \mu_{\mathfrak{p}}$, declines, and in the second case, the fraction of a liquid asset's value that can be pledged as collateral is lower. The first case resembles a world financial system's general rejection of Home private assets (whether the Home asset comes, for example, from Apple or Dell), while the second case resembles downgrades of ratings for Home assets.

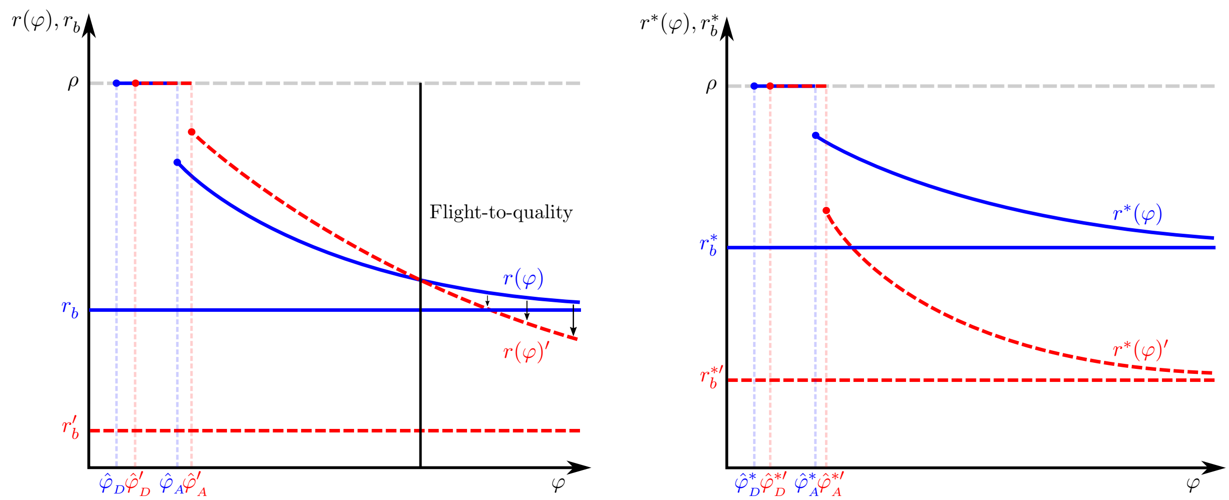
Figure 5 shows the effects of a decline in $\mu_{\mathfrak{p}}$ on the full structure of Home and Foreign interest rates. As in the recent financial crisis, the return on Home government bonds declines despite Home being the origin of the crisis. On the other hand, interest rates increase for most private assets, with the exception of some low productivity assets that become liquid after the decline in $\hat{\varphi}_A$ caused by the crisis. Intuitively, the liquidity crisis has such a strong negative effect on the aggregate amount of Home private liquidity, that financiers start to use some lower-quality assets to compensate. This type of shock has negative real effects in Home that are very similar to those observed in section 6.1 for Foreign after an increase in $\mu_{\mathfrak{p}}$. In contrast, most Foreign assets experience higher liquidity premiums. The exceptions are some low productivity firms that become illiquid after the increase in $\hat{\varphi}_A^*$.

Lastly, Figure 6 presents the structure of interest rates before and after a decline in β. For Foreign the effects are similar to those described in Figure 5. For Home, however, there are important



(a) Home interest rates (b) Foreign interest rates

Figure 5: Liquidity crisis in Home private assets: A decline in $\mu_{\mathfrak{p}}$ (crisis—dashed lines)



(a) Home interest rates (b) Foreign interest rates

Figure 6: Liquidity crisis in Home private assets: A decline in β (crisis—dashed lines)

differences. In contrast to Figure 5a, $\hat{\varphi}_D$ and $\hat{\varphi}_A$ increase in Figure 6a, which imply increases in Home aggregate productivity and in the average quality of Home collateral used in financial transactions. Moreover, Figure 6a shows an interesting flight-to-quality phenomenon that not only involves Home government bonds (whose rate of return declines), but also high-productivity Home firms—note that the interest rate increases for low-productivity firms but declines for high-productivity firms. Remarkably, these findings are consistent with the results of Dick-Nielsen, Feldhütter, and Lando (2012), who find evidence of flight-to-quality toward AAA-rate corporate bonds during the 2007-2008 financial crisis. In the end, these effects cushion the negative impact of the liquidity crisis on the real economy: although the amount of Home private liquidity, A, declines, the total capitalization of Home firms may even increase.

7 Credit Frictions

This paper relates a country's financial development to its ability to generate liquid assets. The traditional literature on financial development and trade pioneered by Kletzer and Bardhan (1987), however, relates a country's financial development to the country's extent of credit-market imperfections. In line with this tradition, this section expands our model to include credit-market frictions.

7.1 The Model with Credit-Market Frictions

Up to now we have assumed that entrepreneurs get immediate access to funding from financiers to cover the sunk entry cost in the differentiated-good sector. In contrast, here we assume that entrepreneurs face searching costs for funds, and that the rate at which they get access to credit depends on the measure of financiers, which will now be endogenous. To simplify notation, in this section we abstract from certification costs, $f_A = f_A^* = 0$, and assume that all private assets from the same country are equally acceptable as collateral in the OTC market, $\beta = \beta^* \to \infty$; as mentioned in section 6.1, these assumptions imply that $r(\varphi) = r$ for every $\varphi \geq \hat{\varphi}_D$, and $r^*(\varphi) = r^*$ for every $\varphi \geq \hat{\varphi}_D^*$.

Let ψ denote the measure of Home financiers, and let it also denote the Poisson rate at which Home entrepreneurs get access to credit—more financiers imply more credit opportunities for entrepreneurs. Home entrepreneurs face a flow searching cost for funds, κ. Given the arrival rate of a funding opportunity, ψ, the total expected cost of accessing the credit market for a Home entrepreneur is $\frac{\kappa}{\psi}$. This cost represents Home's credit market frictions. The free-entry condition in the Home differentiated-good sector then becomes

$$\int_{\hat{\varphi}_D}^{\infty} \left[\frac{\pi_D(\varphi) - f_D}{r + \delta} \right] g(\varphi) d\varphi + \int_{\hat{\varphi}_X}^{\infty} \left[\frac{\pi_X(\varphi) - f_X}{r + \delta} \right] g(\varphi) d\varphi = f_E + \frac{\kappa}{\psi}, \tag{56}$$

where the left-hand side is the expected value of entry, and the right-hand side accounts for the entry costs, which are the sum of the initial investment to create the firm (f_E—provided by the financier), and the expected credit-market access cost. Analogously, the free-entry condition at Foreign is

$$\int_{\hat{\varphi}_D^*}^{\infty} \left[\frac{\pi_D^*(\varphi) - f_D^*}{r^* + \delta} \right] g(\varphi) d\varphi + \int_{\hat{\varphi}_X^*}^{\infty} \left[\frac{\pi_X^*(\varphi) - f_X^*}{r^* + \delta} \right] g(\varphi) d\varphi = f_E^* + \frac{\kappa^*}{\psi^*}, \tag{57}$$

where κ^* is the flow searching cost for Foreign entrepreneurs, and ψ^* is the measure of Foreign financiers.

Financiers are born as *investors*. These are entities that have the outside option of investing in a technology to produce some amount of the homogeneous good. Investors are heterogeneous in their homogeneous-good production capacity. In particular, a Home investor can produce a flow γ of the homogeneous good, where γ is drawn from a probability distribution with cumulative function $\Psi(\gamma)$. Whether an investor becomes a financier or not, it must pay a lump-sum tax of Υ. The investor's lifetime surplus from homogeneous-good production is then $\frac{\gamma-\Upsilon}{\rho}$, which is compared against its lifetime expected surplus from being a financier—which involves providing credit to entrepreneurs in the differentiated-good sector and participating in the OTC financial market.

The lifetime expected utility of a Home financier, $W(a, a^*, b, b^*)$, is given by

$$\begin{aligned}\rho W(a, a^*, b, b^*) =& \mu_{\mathfrak{p}}^* \theta \left[F(y_{\mathfrak{p}}^*) - y_{\mathfrak{p}}^*\right] + \mu_{\mathfrak{g}}^* \theta \left[F(y_{\mathfrak{g}}^*) - y_{\mathfrak{g}}^*\right] + \mu_{\mathfrak{p}} \theta \left[F(y_{\mathfrak{p}}) - y_{\mathfrak{p}}\right] \\ &+ \mu_{\mathfrak{g}} \theta \left[F(y_{\mathfrak{g}}) - y_{\mathfrak{g}}\right] + ra + r^*a^* + r_b b + r_b^* b^* - \Upsilon, \end{aligned} \tag{58}$$

which accounts for the financier's surpluses in each type of match and the returns on asset holdings net of taxes. The financier chooses the portfolio of assets that maximizes $W(a, a^*, b, b^*) - (a + a^* + b + b^*)$, and therefore, an investor with draw γ becomes a financier if and only if

$$\frac{\gamma - \Upsilon}{\rho} \leq \max_{a,a^*,b,b^*} \left\{W(a, a^*, b, b^*) - (a + a^* + b + b^*)\right\}. \tag{59}$$

It follows that there exists a cutoff level $\hat{\gamma}$ that determines whether or not an investor becomes a financier, which by (58) and (59) is given by

$$\begin{aligned}\hat{\gamma} \equiv \max_{a,a^*,b,b^*} &\{\mu_{\mathfrak{p}}^* \theta \left[F(y_{\mathfrak{p}}^*) - y_{\mathfrak{p}}^*\right] + \mu_{\mathfrak{g}}^* \theta \left[F(y_{\mathfrak{g}}^*) - y_{\mathfrak{g}}^*\right] + \mu_{\mathfrak{p}} \theta \left[F(y_{\mathfrak{p}}) - y_{\mathfrak{p}}\right] + \mu_{\mathfrak{g}} \theta \left[F(y_{\mathfrak{g}}) - y_{\mathfrak{g}}\right] \\ &- (\rho - r)a - (\rho - r^*)a^* - (\rho - r_b)b - (\rho - r_b^*)b^*\}. \end{aligned} \tag{60}$$

The measure of Home financiers is then given by $\psi = \Psi(\hat{\gamma})$ and contains all those investors whose opportunity costs of participating in the credit market are no more than $\hat{\gamma}$.

Analogous expressions hold for Foreign investors, who will face the same opportunity cost cutoff, $\hat{\gamma}$. We allow, however, for a different distribution of opportunity costs for Foreign investors, $\Psi^*(\gamma)$, so that the measure of Foreign financiers, $\psi^* = \Psi^*(\hat{\gamma})$, is not necessarily equal to the measure of Home financiers, $\psi = \Psi(\hat{\gamma})$. Of course, this can be a source of asymmetric credit-market imperfections. For example, all else equal, if $\Psi(\gamma) > \Psi^*(\gamma)$ for every γ (so that Ψ^* stochastically dominates Ψ) then there are more Home financiers than Foreign financiers, $\psi > \psi^*$, and thus there are lower credit-market frictions at Home.

As before, the supplies of Home and Foreign private liquidity (A and A^*) are given by (38) and (39), and the optimal portfolio of a financier solves (40)-(43). However, recall that under assumptions $f_A = f_A^* = 0$ and $\beta = \beta^* \to \infty$, the model yields not only the same return for all private assets from the same country, but also $\hat{\varphi}_A = \hat{\varphi}_D$, $\hat{\varphi}_A^* = \hat{\varphi}_D^*$, $\lambda(\varphi) = \lambda^*(\varphi) = 1$ for every producing firm, $N_A = N_D$, and $N_A^* = N_D^*$. As Home and Foreign financiers hold identical portfolios, and given that there is a mass $\psi + \psi^*$ of financiers in the world, market clearing requires that $(\psi + \psi^*)a = A$, $(\psi + \psi^*)a^* = A^*$, $(\psi + \psi^*)b = B$, and $(\psi + \psi^*)b^* = B^*$. Thus, the quantities of financial services traded in each type of OTC match are

$$y_{\mathfrak{p}}^* = \min\left\{\frac{A + A^* + B + B^*}{\Psi(\hat{\gamma}) + \Psi^*(\hat{\gamma})}, \hat{y}\right\}, \tag{61}$$

$$y_{\mathfrak{g}}^* = \min\left\{\frac{A + B + B^*}{\Psi(\hat{\gamma}) + \Psi^*(\hat{\gamma})}, \hat{y}\right\}, \tag{62}$$

$$y_{\mathfrak{p}} = \min\left\{\frac{A + B}{\Psi(\hat{\gamma}) + \Psi^*(\hat{\gamma})}, \hat{y}\right\}, \tag{63}$$

$$y_{\mathfrak{g}} = \min\left\{\frac{B}{\Psi(\hat{\gamma}) + \Psi^*(\hat{\gamma})}, \hat{y}\right\}. \tag{64}$$

The definition of equilibrium follows.

Definition 3. *A steady-state equilibrium in the two-country model with credit frictions is a list*

$$\left(\hat{\varphi}_D, \hat{\varphi}_X, \hat{\varphi}_D^*, \hat{\varphi}_X^*, \hat{\gamma}, A, A^*, y_{\mathfrak{p}}^*, y_{\mathfrak{g}}^*, y_{\mathfrak{p}}, y_{\mathfrak{g}}, r^*, r_b^*, r, r_b\right)$$

that solves (25)*,* (26)*,* (38)*,* (39)*,* (40)*-*(43)*,* (56)*,* (57)*, and* (60)*-*(64)*.*

The steady-state equilibrium solves for the cutoff productivity levels that indicate the tradability of Home and Foreign goods in each market $(\hat{\varphi}_D, \hat{\varphi}_X, \hat{\varphi}_D^*, \hat{\varphi}_X^*)$, the investors' cutoff opportunity cost for becoming a financier, $\hat{\gamma}$, the amounts of Home and Foreign private liquidity (A, A^*), the amount of financial services traded in each type of match $(y_{\mathfrak{p}}^*, y_{\mathfrak{g}}^*, y_{\mathfrak{p}}, y_{\mathfrak{g}})$, and the interest rates (r^*, r_b^*, r, r_b).

7.2 Credit Frictions, Economic Activity, and Trade

We now study the allocation of economic activity and the impact of trade liberalization when countries differ in their degree of credit frictions. Similar to the analysis in section 6.1, to highlight the role of credit-market frictions we study a tractable case with a Pareto distribution of productivity and identical production structures at Home and Foreign. We further assume that all Home and Foreign assets are equally liquid in all OTC matches ($\mu_{\mathfrak{p}} = \mu_{\mathfrak{g}}^* = \mu_{\mathfrak{g}} = 0$ and $\mu_{\mathfrak{p}}^* = 1$, which is equivalent to $\mu = 0$ in section 6.1), and thus $r = r^*$.

Credit frictions are captured by $\frac{\kappa}{\Psi(\hat{\gamma})}$ at Home, and by $\frac{\kappa^*}{\Psi^*(\hat{\gamma})}$ at Foreign. For simplicity, we let $\Psi(\gamma) = \Psi^*(\gamma)$ for every γ, and focus on a situation where the only difference between Home and Foreign is that $\kappa^* > \kappa$, so that credit-market frictions are larger at Foreign. Under these assumptions, the model with credit frictions and scarce liquidity—*i.e.*, $\frac{A+A^*+B+B^*}{2\Psi(\hat{\gamma})} < \hat{y}$ so that $r < \rho$—can be reduced to a system of four equations and four unknowns,

$$A = \frac{(\sigma-1)\eta}{\sigma k(r+\delta)}\left[\frac{\tau^k}{\tau^k - \Lambda(\kappa,\kappa^*,\hat{\gamma})} - \frac{1}{\tau^k\Lambda(\kappa,\kappa^*,\hat{\gamma}) - 1}\right], \tag{65}$$

$$A^* = \frac{(\sigma-1)\eta\Lambda(\kappa,\kappa^*,\hat{\gamma})}{\sigma k(r+\delta)}\left[\frac{\tau^k}{\tau^k\Lambda(\kappa,\kappa^*,\hat{\gamma}) - 1} - \frac{1}{\tau^k - \Lambda(\kappa,\kappa^*,\hat{\gamma})}\right], \tag{66}$$

$$r = \rho - \theta\left[F'\left(\frac{A+A^*+B+B^*}{2\Psi(\hat{\gamma})}\right) - 1\right], \tag{67}$$

$$\hat{\gamma} = \theta\left[F\left(\frac{A+A^*+B+B^*}{2\Psi(\hat{\gamma})}\right) - \frac{A+A^*+B+B^*}{2\Psi(\hat{\gamma})}\right] - (\rho - r)\left[\frac{A+A^*+B+B^*}{2\Psi(\hat{\gamma})}\right], \tag{68}$$

where $\Lambda(\kappa,\kappa^*,\hat{\gamma}) = 1 + \frac{\kappa^*-\kappa}{f_E\Psi(\hat{\gamma})+\kappa} > 1$ is a measure of Foreign credit frictions relative to Home credit frictions. An interior solution requires $\frac{2\tau^k}{\tau^{2k}+1} < \Lambda(\kappa,\kappa^*,\hat{\gamma}) < \frac{\tau^{2k}+1}{2\tau^k}$, which we assume always holds. Taking the ratio of (65) and (66) it follows that

$$\frac{A}{A^*} = 1 + \frac{2\tau^k[\Lambda^2(\kappa,\kappa^*,\hat{\gamma}) - 1]}{\Lambda(\kappa,\kappa^*,\hat{\gamma})\left[\tau^{2k} - 2\tau^k\Lambda(\kappa,\kappa^*,\hat{\gamma}) + 1\right]} > 1.$$

Hence, similar to the impact of differences in liquidity properties across countries' assets, asymmetries in credit market frictions produce an allocation of economic activity in favor of the country with less frictions. Also as before, Home enjoys of higher average productivity and a lower aggregate price.

Based on the parsimonious system in (65)-(68), Table 2 shows comparative statics for A, A^*, r and $\hat{\gamma}$, with respect to the flow searching cost for funds for Foreign entrepreneurs, κ^*, the iceberg trade cost, τ, and Home and Foreign public liquidity, B and B^*. Note from the first and second columns in Table 2 that r and $\hat{\gamma}$ are independent of κ^* and τ. This can be seen by noticing from (65) and (66) that $A + A^* = \frac{2(\sigma-1)\eta}{\sigma k(r+\delta)}$, which then implies that (67) and (68) do not depend on κ, κ^*, or τ. Thus, total world capitalization of firms, $A+A^*$, is not affected by changes in κ^* or τ and therefore, $\frac{dA}{d\kappa^*} = -\frac{dA^*}{d\kappa^*}$ and $\frac{dA}{d\tau} = -\frac{dA^*}{d\tau}$. Similar to an increase in μ in section 6.1, an increase in Foreign credit frictions (κ^*) cause an expansion in the differentiated-good sector at Home and a contraction in Foreign. Also as before, trade liberalization (a decline in τ) widens the total-capitalization gap between the countries.

In this version of the model—with no liquidity differences across countries' assets—changes in B and B^* have the same effects on all variables. The third column in Table 2 shows that a rise in either B or B^* increases both the interest rate (liquidity is more abundant and hence its value declines, *i.e.*, $\rho - r$ declines) and the cutoff opportunity cost for investors, $\hat{\gamma}$, which then causes an increase in the measure of financiers in each country, $\Psi(\hat{\gamma})$. The rise in r partially crowds out total world private liquidity ($A + A^* = \frac{2(\sigma-1)\eta}{\sigma k(r+\delta)}$ declines), but Foreign total capitalization may go up. The

exogenous → endogenous ↓	κ^*	τ	B or B^*
A	$+$	$-$	$-$
A^*	$-$	$+$	$+/-$
r	0	0	$+$
$\hat{\gamma}$	0	0	$+$

Table 2: Comparative statics when $\mu = 0$ and $\kappa^* > \kappa$ — Each cell indicates the sign of the derivative of the endogenous variable in the row with respect to the exogenous variable in the column

possibility of A^* increasing arises because the increase in the measure of financiers causes a decline in Foreign credit frictions relative to Home credit frictions ($\Lambda(\kappa, \kappa^*, \hat{\gamma})$, which is defined after (68), declines toward 1). This result is a consequence of our assumption that $\Psi(\gamma) = \Psi^*(\gamma)$.[11] More generally, this example highlights another mechanism through which public liquidity can affect real economic activity (besides the crowding-out mechanism): more public liquidity encourages entry of more financiers, improving credit opportunities in both countries (both $\frac{\kappa}{\Psi(\hat{\gamma})}$ and $\frac{\kappa^*}{\Psi^*(\hat{\gamma})}$ decline), and altering the relative measure of credit-market frictions. The new mechanism offsets the crowding-out effect for the country whose credit frictions decline relatively more, and reinforces the crowding-out effect for the other country.

To sum up, both types of financial development—the extent of credit-market imperfections and the capacity to generate liquid assets—are sources of comparative advantage, causing an allocation of economic activity in favor of the most financially developed country, and generating unequal effects of trade liberalization.

8 Conclusion

The United States is by far the most important provider of public and private liquidity for the world's financial system. But how does this benefit U.S. firms and consumers? This paper showed that a country's capacity to generate liquid assets is a source of comparative advantage in international trade. It increases the size and average productivity of the real-economy sectors that generate liquid assets, and lowers aggregate prices. This happens at the expense of competing foreign sectors that are not as good at providing liquidity.

The proposed framework can be used not only to understand the allocation of economic activity and the effects of trade liberalization when there are differences in liquidity properties or credit frictions across countries, but also to understand interest rate responses and flight-to-liquidity

[11] If $\Psi(\gamma)$ and $\Psi^*(\gamma)$ are not the same, the formula for Foreign credit frictions relative to Home credit frictions becomes $\Lambda(\kappa, \kappa^*, \hat{\gamma}) = \frac{f_E + \kappa^*/\Psi^*(\hat{\gamma})}{f_E + \kappa/\Psi(\hat{\gamma})}$.

phenomena during a liquidity crisis and the effects of changes in the supply of government bonds. The full-fledged model is very rich—two countries, a market for liquidity with several types of assets with different acceptabilities, heterogeneous firms, and international trade—and can be solved numerically, but we also showed that we can work with analytically tractable versions of the model to provide transparent answers to fundamental questions.

The framework is also amenable to applications and extensions. A potential application is in a model of exchange-rate determination. A country's ability to generate liquid assets should be reflected in the exchange rate. In particular, liquidity premiums of a country's assets should carry over to the value of its currency. This idea is closely related to the seminal contribution of Kouri (1983), who argue that equilibrium in the foreign exchange market stems "from demands for and supplies of all kinds of assets and goods and services." The framework here can help explain why during periods of flight-to-liquidity, the currency of the country that generates the most liquid assets appreciates. Engel and West (2010) document, for example, that the U.S. dollar appreciated during the most turbulent period of the financial crisis (March 2008-March 2009) and suggest a liquidity premium story—driven by a large increase in the demand for U.S. Treasuries—as one of the potential explanations.

The model can also be extended to accommodate multinationals or offshoring possibilities. The current setting yields that differences in financial development translate to more economic activity in the most developed country, and that trade liberalization exacerbates the economic activity gap. Accounting for production process fragmentation or horizontal FDI will still yield the same implications regarding total firm capitalization based on country's ownership, but may allow for increases in economic activity in the least developed country through births and expansions of multinational subsidiaries or arm's-length providers.

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