On Everett's Formulation of Quantum Mechanics

Abstract

Everett wanted a formulation of quantum mechanics that (i) took the linear dynamics to be a complete and accurate description of the time-evolution of all physical systems and (ii) logically entailed the same subjective appearances predicted by the standard formulation of quantum mechanics. While most everyone would agree with this description of Everett's project, there is little agreement on exactly how his relative-state formulation was supposed to work. In this paper, I consider two very different readings of Everett: the bare reading and the splitting-worlds reading. What distinguishes these is their interpretation of the wave function and how one accounts for the experiences of observers. The difficulty in interpreting Everett, however, is illustrated by the fact that neither reading is entirely compatible with his own description of his project.

I

Everett provided two extended descriptions of his relative-state formulation of quantum mechanics, and in both cases he started with a description of the standard von Neumann formulation.[1] Everett believed that there were physical situations in which von Neumann's formulation failed to make coherent predictions, but he also wanted to present his own formulation as a simplified and more general version of von Neumann's. The standard von Neumann formulation of quantum mechanics then is a natural place to begin a discussion of Everett's.

Von Neumann described "two fundamentally different types of interventions which can occur in a system *S*" (1955, 351). The first type, Process 1, is characterized by "the arbitrary changes by measurements" where the state of the system being measured instantaneously jumps into

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an eigenstate of the measured observable (a state in which the observed property of the system has a determinate value). Process 1 is random, discontinuous, and *nonlinear*. The second type of intervention, Process 2, is characterized by "the automatic changes that occur with the passage of time" where the state of a system smoothly evolves in a way determined by its energy properties. Process 2 is deterministic, continuous, and *linear*. On von Neumann's formulation of quantum mechanics then the time-evolution of every physical system is linear unless it is being measured, and when it is measured the state of the system jumps, instantaneously and nonlinearly, into one of the eigenstates of the observable being measured.[2]

In order to see how this works, consider an x-spin measurement of a spin-1/2 system S (the x-spin of a spin-1/2 system is always measured to be either "up" or "down"). Suppose that M is a perfect x-spin measuring device in that when S is in an eigenstate of x-spin, the position of M's pointer ends up perfectly correlated to the x-spin of S without disturbing it: If M begins in a state where it is ready to make a measurement $|r\rangle_M$ and S begins in an x-spin up eigenstate $|\uparrow\rangle_S$, then the composite system $M + S$ will end up in a state where M reports that its result was x-spin up and S is undisturbed $|\uparrow\rangle_M|\uparrow\rangle_S$; and if M begins in a state where it is ready to make a measurement $|r\rangle_M$ and S begins in an x-spin down eigenstate $|\downarrow\rangle_S$, then the composite system $M + S$ will end up in a state where M reports that its result was x-spin down and S is undisturbed $|\downarrow\rangle_M|\downarrow\rangle_S$. So what happens when M measures the x-spin of a system S that is not initially in an eigenstate of x-spin? Suppose $M + S$ is initially in the state

$$\overline{}_{2}\,|r\rangle_M(\qquad _S + \quad \downarrow\rangle_S$$

If we suppose that interacting with M counts as a measurement of S, then the standard theory tells us that S will instantaneously jump to either $|\uparrow\rangle_S$ or $|\downarrow\rangle_S$, each with probability 1/2. Then, since it is a good measuring device, M's pointer will become correlated with the x-spin of S, so there is an equal chance here of the final state being $|\uparrow\rangle_M|\uparrow\rangle_S$ and being $|\downarrow\rangle_M|\downarrow\rangle_S$, which is (apparently) just what we find whenever we perform this experiment.

It is important to be clear about the role that the collapse dynamics plays in the standard formulation of quantum mechanics. If there were no collapse of the state upon measurement, then it would follow from the linear dynamics and the properties of M that the final state would be

$$\overline{\sqrt{2}} \quad \uparrow\rangle_M \quad \rangle_S + \ \downarrow\rangle_M |\downarrow\rangle_S). \tag{2}$$

But the standard interpretation of states, the eigenstate-eigenvalue link, tells us that a system determinately has some physical property if and only if it is in an eigenstate of having the property. Likewise, the system determinately does not have the property if and only if it is in an eigenstate of not having the property.[3] Since (2) is not an eigenstate of M recording "up" and it is not an eigenstate of M recording "down," on the standard interpretation of states, it is a state where M has no determinate record of its result and thus makes no determinate report. Hence, the argument goes, (2) cannot be the post-measurement state since we do in fact get determinate results of one sort or the other to such measurements; rather, it must be that the post-measurement state is actually an *eigenstate* of recording a determinate result, either $|\uparrow\rangle_M|\uparrow\rangle_S$ or $|\downarrow\rangle_M|\downarrow\rangle_S$), which means that there must have been a nonlinear collapse of the state somewhere over the course of the measurement. So while the linear dynamics (Process 2) accounts for the wave-like behavior of quantum-mechanical systems, it is the nonlinear collapse dynamics (Process 1) that accounts for the fact that we (apparently) never find a system in a superposition of eigenstates of the observable that we measure.

The nonlinear dynamics also accounts for the results of repeat measurements. We know from experience that if S is undisturbed between measurements, then the result of a second x-spin measurement of S will *with certainty* give the same result as the first. The reason, according to von Neumann's formulation, is that the first measurement causes the object system to collapse to one or the other of the two x-spin eigenstates, so if it is undisturbed, then it will still be in that eigenstate when it is measured a second time. If $M + S$ were in the state $|\uparrow\rangle_M|\uparrow\rangle_S$ after the first measurement, then M would get "down" for its second result, and this is simply because M is a good measuring device, so its pointer would become correlated with the x-spin of S, which is *now* either determinately up or determinately down.

Finally, the nonlinear dynamics also accounts for the observed relative frequencies of results. We know from experience that most likely about half of *M*'s results would be "up" if it made a series of *x*-spin measurements on systems prepared like *S* above. The reason, according to von Neumann's formulation, is that in each case there would be a probability of 1/2 that at some point in the measurement process a nonlinear collapse would occur that caused the composite system to end up in an eigenstate of recording the result "up" for that measurement.

While the nonlinear dynamics plays an important role in the explanations provided by the standard theory, it also leads to problems. Von Neumann conceded that having mutually incompatible dynamical laws in the same theory was at least counterintuitive:

> one should expect that [the linear dynamics (Process 2)] would suffice to describe the intervention caused by a measurement: Indeed, a physical intervention can be nothing else than the temporary insertion of a certain energy coupling into the observed system . . . [in which case, Process 2 would describe the time-evolution of the composite system] (1955, 352).

So where does the nonlinear collapse dynamics come from? Here we get the first glimmer of the quantum measurement problem. As von Neumann put it, "we have then answered the question as to what happens in the measurement of a quantity. To be sure, the 'how' remains as unexplained as before" (1955, 217). But while von Neumann expressed his sense of mystery at the prospect, he nonetheless ultimately believed that Process 1 and Process 2 could peacefully live together in the same theory.

Everett, however, did not like the standard formulation of quantum mechanics with its two, mutually incompatible, dynamical laws. Everett complained that "Not all conceivable situations fit the framework of [von Neumann's] mathematical formulation" (1957, 315). In particular, he was worried about how one would apply the standard formulation to systems containing measuring devices, systems like the universe itself. When should one use the nonlinear dynamics? Whenever a measurement is made. But when is a measurement made? The standard formulation provides neither necessary nor sufficient conditions. Moreover, as von Neumann himself had already suggested, one would expect that if the linear dynamics correctly described every other sort of interaction, then it would also suffice to describe measurement interactions. One would expect then that every physical system, even a system containing a measuring device, would always obey the linear dynamics (Process 2). And this is just what Everett proposed.

Everett proposed dropping Process 1 from the standard formulation of quantum mechanics and taking the resultant theory as complete and accurate:

> This paper proposes to regard pure wave mechanics (Process 2 only) as a complete theory. It postulates that a wave function that obeys a linear wave equation everywhere and at all times supplies a complete mathematical model for every isolated physical system without exception (1957, 316).

And he intended to deduce the standard formulation of quantum mechanics from this new, more general formulation:

> The aim is not to deny or contradict the conventional formulation of quantum theory . . . but rather to supply a new, more general and complete formulation, from which the conventional interpretation can be *deduced* (1957, 315).

While Everett's proposal might look perfectly straightforward, it turns out that it is not at all clear what he had in mind.

As Richard Healey once described the problem, Everett's interpretation itself stands in need of interpretation (1984, 539). It is not my aim to argue for a particular interpretation of Everett or to systematically evaluate the several mutually incompatible ways of reading Everett that have been proposed. Rather, I will describe two very different readings in order to illustrate at least some of what is at stake in reading Everett and why finding an entirely satisfactory interpretation is difficult: I will refer to these as the *bare* reading and the *splitting-worlds* reading. It will turn out that neither of these is entirely consistent with Everett's own description of his project.

II

The splitting-worlds reading is perhaps the most popular reading of Everett. It postulates the existence of a different world corresponding to each term in the quantum-mechanical state when written in a specified preferred basis. Each world is then taken to have the state expressed by the corresponding term. Here the standard interpretation of states applies to the *local* state of each world, but the *global* state is interpreted differently—given the preferred basis, the global state tells us how many worlds there are and what their local states are.[4] The preferred basis is supposed to be such that every observer in each world always gets a determinate measurement result, always has a determinate record of his experience.[5]

I take the splitting-worlds reading to be DeWitt's reading of Everett. As DeWitt understands it, Everett's interpretation is "one of the most bizarre and at the same time one of the most straightforward interpretations of quantum mechanics that has ever been put forward" (1971, 167). According to DeWitt, Everett's interpretation entails that "our universe must be viewed as constantly splitting into a stupendous number of branches, all resulting from the measurement-like interactions between its myriads of components." He grants that "the idea of 10^{100+} slightly different copies of oneself all constantly splitting into further copies, which ultimately become unrecognizable, is hard to reconcile with the testimony of our senses"; that is, until we realize that "*the laws of quantum mechanics do not allow us to feel ourselves split*" (1971, 178–9). One of the primary advantages of Everett's interpretation, we are told, is that it allows "a return to naïve realism and the old-fashioned idea that there can be a direct correspondence between formalism and reality. . . . The symbols of quantum mechanics represent reality just as much as do those of classical mechanics" (1971, 168). What DeWitt apparently has in mind here is a correspondence between the terms in an expansion of the wave function of the universe with respect to the preferred basis, "the symbols of quantum mechanics," and actual worlds, "reality."

In the Preface to their Everett anthology, DeWitt and Graham claim that Everett's formulation of quantum mechanics

> asserts that it makes sense to talk about a state vector for the whole universe. This state vector never collapses, and hence reality as a whole is rigorously deterministic. This reality, which is described *jointly* by the dynamical variables and the state vector, is not the reality we customarily think of, but is a reality composed of many worlds. By virtue of the temporal development of the dynamical variables the state vector decomposes naturally into orthogonal vectors, reflecting a continual splitting of the universe into a multitude of mutually unobservable but equally real worlds, in each of which every good measurement has yielded a definite result and in most of which the familiar statistical quantum laws hold (DeWitt and Graham 1973, v).

Indeed, Everett apparently took some sort of branching process seriously. On the assumption that his readers would want an explanation of why they do not feel themselves "split," Everett compared his formulation of quantum mechanics to the Copernican theory.

> From the viewpoint of the theory *all* elements of a superposition (all "branches") are "actual," none any more "real" than the rest. It is unnecessary to suppose that all but one are somehow destroyed, since all the separate elements of a superposition individually obey [the linear dynamics] with complete indifference to the presence or absence ("actuality" or not) of any other elements. This total lack of effect of one branch on another also implies that no observer will ever be aware of any "splitting" process.[6]
>
> Arguments that the world picture presented by this theory is contradicted by experience, because we are unaware of any branching processes, are like the criticism of the Copernican theory that the mobility of the earth as a real physical fact is incompatible with the common-sense interpretation of nature because we feel no such motion. In both cases the argument fails when it is shown that the theory itself predicts that our experience will be what it in fact is (1957, 320–321 footnote).

One might wonder why Everett would have thought that such an explanation was necessary if he were not personally committed to some genuine, ontological splitting of the world over time. On the other hand, one might wonder why he would use scare quotes throughout if he were so committed. In any case, there is evidence that Wheeler, with whom Everett worked closely in developing his formulation of quantum mechanics, accepted something akin to the splitting-worlds reading.[7]

III

The splitting-worlds reading is not universally accepted as the right reading of Everett. Bell, for example, believed that DeWitt and Everett may have had something very different in mind (Bell 1987, 137–138). Along these lines, Lockwood argues that "one widespread misconception—encouraged by sloppy popular exposition—is that, on the Everett view, the observer somehow splits the entire universe simply by measuring a quantum system" (1989, 225). According to Lockwood, the essential aspect of Everett's formulation is that there is no collapse or reduction of the wave function, and he takes this to be logically incompatible with the claim that the universe, or anything else, actually splits when an observation occurs (this last claim is something that we will return to later). Any talk of "'dividing' or 'splitting', in the relative-state view, is essentially a metaphor for going into a superposition; or more specifically, for going into a *macroscopic* superposition" (1989, 227).

This denial that anything at all splits when an observation occurs suggests a *one-world* reading of Everett. The one-world reading we will consider here is what Albert has called the "bare" theory (1992, 116–125).[8] According to the bare theory, the time-evolution of the physical world is always correctly described by the linear dynamics and the standard interpretation of states is taken to be true. In other words, the bare theory is simply the standard formulation of quantum mechanics dropping the collapse postulate but keeping the standard interpretation of states. There is only one world, and because the linear dynamics is assumed to be true, it is usually in a complicated superposition of eigenstates of the observables that interest us.

The bare reading is suggested by Everett's initial description of his project. He claimed that his goal was to regard "pure wave mechanics" (the standard theory without the collapse dynamics) as a complete physical theory and then to deduce the "conventional interpretation." But since the *linear* dynamics will never by itself generate a *nonlinear* evolution no matter how complicated a system is, it is clearly impossible to *deduce* the standard formulation of quantum mechanics from pure wave mechanics, so this cannot be what he meant. Indeed, Everett explicitly argued that since his theory contained only the linear dynamics "nothing resembling [a formal collapse (process 1)] can take place" (1973, 60). In order to clarify what he meant when he claimed that he would deduce the conventional interpretation from his formulation, he said that he would

> deduce the probabilistic assertions of Process 1 as *subjective* appearances to . . . observers, thus placing the theory in correspondence with experience. We are then led to the novel situation in which the formal theory is objectively continuous and causal, while subjectively discontinuous and probabilistic (1973, 9).

Given this formulation of the project, one might expect Everett to say something about the relationship between an observer's physical state and his experience, and he does. Everett claimed that "as models for observers we can, if we wish, consider automatically functioning machines, possessing sensory apparatus and coupled to recording devices capable of registering past sensory data and machine configurations" (1957, 318) and he claimed that "in order to make deductions about the subjective experience of an observer it is sufficient to examine the contents of the [observer's physical] memory" (1973, 64). He even gave a very quick dispositional argument for believing that mental states could be understood in terms of physical states:

> For such machines we are justified in using such phrases as "the machine has perceived A" or "the machine is aware of A" if the occurrence of A is represented in the memory, since the future behavior of the machine will be based upon the occurrence of A (1973, 64).

On Everett's model, then, a good observer is a machine: a measuring device with a physical memory to record the results of its measurements. And just like any other physical system, the time-evolution of an observer's physical state is taken as completely and accurately described by the linear dynamics.

His discussion of the relationship between physical and mental states allowed Everett a final refinement of his project:

> Our problem is, then, to treat the interactions of such observer-systems with other physical systems (observations), within the framework of wave mechanics, and to deduce the resulting memory configurations, which we can then interpret as the subjective experiences of the observers (1973, 65).

In other words, his project was to take the linear dynamics to be a complete and accurate description of the time-evolution of every physical system without exception, then deduce, in terms of the physical memory configurations of an observer, the same subjective appearances predicted by the standard formulation of quantum mechanics with the collapse postulate.

Had Everett succeeded, he would have accomplished something quite remarkable. If one could show that the standard theory of quantum mechanics *without* the collapse postulate makes exactly the same empirical predictions as the standard theory of quantum mechanics *with* the collapse postulate, then one would have shown that the collapse postulate was redundant, that the addition of a nonlinear, probabilistic dynamics to Schrödinger's pure wave mechanics was entirely unnecessary. The measurement problem would be nothing more than an unfortunate misunderstanding resulting from the addition of something to quantum mechanics that it never needed. Everett said that "it remains a matter of intellectual interest that the statistical assertions of the usual interpretation do not have the status of independent hypotheses, but are deducible (in the present sense) from the pure wave mechanics, which results from their omission" (1973, 119). This would indeed be a matter of intellectual interest.

Having said how remarkable this would be, however, one probably ought to be a little skeptical. Without the collapse postulate, quantum mechanics entails that even macroscopic systems would generally be in complicated superpositions of eigenstates of most observables, and whenever something like this happens, the standard interpretation of states tells us that the physical quantities corresponding to the observables have no determinate values. Even household objects would routinely fail to have determinate properties like position. My refrigerator would most likely be in a superposition of being not only in my kitchen, but in your kitchen, one mile directly above the north pole, in the center of the sun, orbiting α-Centauri, having never been constructed, and so forth, and thus would not have anything even close to a determinate position. By most accounts, this violates common sense. After all, I keep food in my refrigerator—I can usually just walk into the kitchen, open it, and find something to eat. What is worse, at least from a philosophical point of view, is that if refrigerators failed to have definite positions, then so would the pointers on measuring devices, and more generally, there would typically be no determinate physical records (on paper, in RAM, on CD ROM, in human brains, etc.) of an observer's measurement results, which on Everett's account of experience would presumably mean that there would typically be no determinate experiences of the sort that we believe there are. I would not have the determinate experience of seeing my refrigerator sitting peacefully in the kitchen; rather, everything, my refrigerator and my brain and the ink on this page and your brain, would typically be in a complicated, entangled superposition of mutually incompatible states.

Suppose that my refrigerator is actually in a superposition of being here and on α-Centauri, that its physical state is something like

$$\alpha \,|\, \text{on } \alpha\text{-Centauri}\rangle_R + \beta \,|\, \text{in kitchen}\rangle_R, \tag{3}$$

And suppose that I am initially in a ready-to-look-for-the-refrigerator state $|r\rangle_J$ such that (i) if the refrigerator were in the kitchen, then I would see it in the kitchen, I would record this fact, and I would report what I had recorded and (ii) if the refrigerator were on α-Centauri, then I would not see it in the kitchen, I would record this fact, and I would report what I had recorded. Given all this, when I look for the refrigerator, I would end up in the entangled state

$$\alpha |\text{don't see it}\rangle_I \, |\text{on } \alpha\text{-Centauri}\rangle_R + \beta |\text{see it}\rangle_I \, |\text{in kitchen}\rangle_R. \quad (4$$

which on the standard interpretation of states, assuming Everett's physical model of an observer, presumably means that there would be no determinate matter of fact concerning what I saw, what I remember seeing, or what I would report as the result. Indeed, since my state would be entangled with the refrigerator's state, I would even fail to have a pure quantum-mechanical state of my own.

Everett himself emphasized the difficulty of his task. He recognized that taking the linear dynamics (Process 2) as a complete and accurate description of the time-evolution of every physical system without exception meant that an object would typically be in a complicated superposition of most interesting observables.

> Suppose, for example, that we [coupled a spin] measuring device to a cannonball, so that if the spin is up the cannonball will be shifted one foot to the left, while if the spin is down it will be shifted an equal distance to the right. If we now perform a measurement with this arrangement upon a particle whose spin is a superposition of up and down, then the resulting total state will also be a superposition of two states, one in which the cannonball is to the left, and one in which it is to the right. There is no definite position for our macroscopic cannonball! (1973, 61).[9]

Everett conceded that "this indefinite behavior seems to be quite at variance with our observations, since macroscopic objects always appear to us to have definite positions." He then posed his central problem: "Can we reconcile this prediction [the indefiniteness of properties like position] of the purely wave mechanical theory with experience, or must we abandon it as untenable?" (1973, 61–62). Here Everett did not deny that physical properties like position are typically indeterminate; rather, he sought to *reconcile* this indeterminateness with our experience. One might argue then that the bare reading is suggested not only by Everett's initial description of his theory but also by the way he described the problem he faced.

Everett explained that taking the linear dynamics to be a complete and accurate description of the time-evolution of every physical system

> has the far reaching implication that for any possible measurement, for which the initial system state is not an eigenstate, the resulting state of the composite system leads to *no* definite system state nor any definite apparatus state (1973, 60).

Further

> There does not, in general, exist anything like a single state for one subsystem of a composite system. Subsystems do not possess states that are independent of the states of the remainder of the system, so that the subsystem states are generally *correlated* with one another. One can arbitrarily choose a state for one subsystem, and be led to the relative state for the remainder. Thus we are faced with a fundamental *relativity of states*, which is implied by the formalism of composite systems. It is meaningless to ask the absolute state of a subsystem—one can only ask the state relative to a given state of the remainder of the subsystem (1957, 317).[10]

He then asked us to consider any measuring apparatus interacting with any object system

> As a result of the interaction the state of the measuring apparatus is no longer capable of independent definition. It can be defined only *relative* to the state of the object system. In other words, there exists only a correlation between the states of the two systems. It seems as if nothing can ever be settled by such a measurement (1957, 318).

The puzzle in reading Everett is in trying to figure out why he thought something *could* be settled by such a measurement.

IV

In order to show that pure wave mechanics makes the same predictions for the subjective experiences of observers as the standard theory of quantum mechanics, Everett first considered an experiment that is essentially equivalent to looking for a refrigerator that is in a superposition of being at different positions. Concerning the post-measurement state, one like that described by (4) above, Everett said

> There is no longer any independent system state or observer state, although the two have become correlated in a one-one manner. However, in each *element* of the superposition . . . the object-system state is a particular eigenstate of the observation, and *furthermore the observer-system state describes the observer as definitely perceiving that particular system state*. This corre-

> lation is what allows one to maintain the interpretation that a measurement has been performed (1957, 320).

Here we are supposed to see how the relativity of states, the correlation between the observer's state and the object system's state, explains the experience of the observer; but it is not at all clear from what is said here how the explanation goes. What does the observer experience? More specifically, in a state like (4), do I see the refrigerator or not? Do I both see the refrigerator and not see it? Or do I somehow split and one of me sees the refrigerator and the other does not?

In addition to recovering the standard theory's predictions for single measurements, Everett also wanted to recover the predictions for repeat measurements. In the kitchen story, if I blinked and looked again, then I would end up in a state like

$$\alpha|\text{didn't see it, don't see it}\rangle_J|\text{on }\alpha\text{-Centauri}\rangle_R + \beta|\text{saw it, see it}\rangle_J|\text{in kitchen}\rangle_R. \quad (5)$$

"Thus," Everett concluded, "for every separate state of the observer in the final superposition the result of the observation was repeatable, even though different for different states" (1957, 320). Right, but it is unclear precisely how this fact is supposed to account for my experience.

Finally, Everett wanted to recover the standard statistical predictions of quantum mechanics. After considering repeating a measurement on a system, he described what the linear dynamics would predict for an infinite series of identical measurements on identically prepared systems. In short, one gets an increasingly complicated superposition of mutually incompatible sequences of results, where, as Everett described it, "A typical element . . . of the final superposition describes a state of affairs wherein the observer has perceived an apparently random sequence of definite results for the observations" (1957, 320). A typical element of the superposition, if one uses the right notion of "typical" here (a notion where each element is weighted by the norm-squared of its coefficient in the superposition), also describes an observer who got the same relative frequencies of results that the standard formulation of quantum mechanics predicts.

With no further argument or explanation, Everett concluded

> It will thus *appear* to the observer, as described by a typical element of the superposition, that each initial observation on a system caused the system to

> "jump" into an eigenstate in a random fashion and thereafter remain there for subsequent measurements on the same system. Therefore . . . the probabilistic assertions of Process 1 *appear* to be valid to the observer described by a typical element of the final superposition (1957, 320).

> Therefore all predictions of the usual theory will [as the number of all types of observations goes to infinity] appear to be valid to the observer in almost all observer states, since these predictions hold for almost all memory sequences (1973, 77).[11]

So

> We have now completed the abstract treatment of measurement and observation, with the deduction that the statistical predictions of the usual form of quantum theory (Process 1) will appear to be valid to all observers. We have therefore succeeded in placing our theory in correspondence with experience, at least insofar as the ordinary theory correctly represents experience (1973, 85).

> We see, therefore, how the classical appearance of the macroscopic world to us can be explained in the wave theory (1973, 90).

And finally

> We have shown that our theory . . . can be put in satisfactory correspondence with experience. We saw that the probabilistic assertions of the usual interpretation of quantum mechanics can be *deduced* from this theory . . . as subjective appearances to observers [where the observers are regarded as ordinary physical systems] (1973, 109).

That's it. Filling in the details of the deduction was left as an exercise for the reader.

How might this work on the bare reading? Suppose a kitchen observer were in a state like (4). On the bare reading, there would be no determinate physical record that says that he "sees it" and no determinate physical record that says that he "doesn't see it." When asked which particular result he got, he would give a superposition of two mutually incompatible answers—his lips would be in a superposition of moving the way they would if he were saying "I see it" and the way they would if he were saying "I don't see it." It is a property of such a state, however, that he would report that he got a determinate result of either one or the other of the two classically possible results—that either he sees the refrigerator or does not. If he were in the state $|\text{don't see it}\rangle_J|\text{on } \alpha\text{-Centauri}\rangle_R$, then he would report that he either sees the refrigerator or does not; and if he were in the state $|\text{see it}\rangle_J|\text{in kitchen}\rangle_R$, then he would report that he either sees the refrigerator or does not; so by the linearity of the dynamics, even though he did not in fact determinately get either result on the standard interpretation of states, his lips would be in an eigenstate of moving in such a way that he says "Yes, I either see it or I don't" if he were in the state described by (4). But again, on the standard interpretation of states, this report would be *false*. On Everett's physical account of experience, it might be thought of as the result of an illusion: the observer might believe (record) that he got a determinate result to his observation, that he either saw the refrigerator or failed to see it, but he would neither determinately believe (record) that he saw the refrigerator nor would he determinately believe (record) that he failed to see the refrigerator; rather, he would be in an entangled superposition of the two states.

On hearing this story someone familiar with quantum mechanics might complain that the Hermitian operator that is supposed to represent the question "Do you either see or not see the refrigerator?" here is just the identity operator and thus cannot possibly represent an interesting question.[12] First, this is not quite right: $|\text{haven't looked yet}\rangle_J|\text{in kitchen}\rangle_R$, for example, is not an eigenvector of the operator. Second, and more important, to worry about whether this ought to be thought of as an identity operator is to miss the point. That the observer would report that he either sees the refrigerator or does not see it when in the superposition described by (4) *is simply a matter of the physical dispositions that he would have as a good observer given the linear dynamics*. Again, if he had the disposition to report "*P* or *Q*" in the state $|P\rangle_J$ and to report "P or Q" in the state $|Q\rangle_J$ (a property of a good observer), then by the linearity of the dynamics he would have the disposition to report "*P* or *Q*" in the state $\alpha|P\rangle_J + \beta|Q\rangle_J$.

Not only is it in a property of states like (4) that the observer would report that he determinately got exactly one of the classically possible results even though he did not in fact determinately get either, but if the observer very carefully repeated the measurement and ended up in a state like (5), then since each term describes a situation where he would report that he got the same result for both observations, it follows from the linearity of the dynamics that he would determinately report that he got the same result for both observations. Given this he might incorrectly conclude that there had been a nonlinear collapse of the state. On Everett's account of experience, one might argue that it would *appear* to the observer that the state of the object system had in fact collapsed to one of the two eigenstates of the observable being measured, when all that had in fact happened was that his measurement records had become perfectly correlated with the physical property being measured, which was not determinate.

As for the statistical predictions of quantum mechanics, one can show that if a series of identical measurements were performed on identically prepared systems, then the observer and the object systems would approach an eigenstate of the observer reporting that his results were randomly distributed with the relative frequencies predicted by the standard theory.[13] Consequently, on Everett's account of experience, one might argue that if an observer were to begin in an eigenstate of being ready to perform an infinite number of identical measurements on identically prepared systems, then (i) it would appear to him that he got a determinate, repeatable result for each measurement and (ii) in the limit he would approach a state where it would appear to him that his results were randomly distributed just as the standard theory predicts that they would be.

Does this mean that the bare theory entails the same empirical predictions as the standard theory of quantum mechanics? Consider a case where one determinately records that P or Q occurred but does not determinately record that P occurred and does not determinately record that Q occurred. One might call this a *disjunctive record* (or on Everett's account of experience, a *disjunctive experience*). A proponent of the bare reading of Everett counts on a disjunctive record and an ordinary record corresponding to one of the disjuncts being subjectively indistinguishable so that one can claim that one has deduced the subjective experiences predicted by the standard theory. Such records are indeed subjectively indistinguishable in the sense that in both cases an observer would report that he got a determinate result, but a proponent of the bare reading needs to argue that subjective indistinguishability in this sense is enough for the records (or experiences) to count as the same in the sense that we are, or ought to be, interested in when we consider the empirical predictions of a theory.

So does the bare theory entail the same empirical predictions as the standard theory of quantum mechanics? The most plausible answer, I believe, is *No*. If one assumes that the state of an observer's physical record of an experience fully determines the observer's subjective experience, then the standard theory and the bare theory typically predict different subjective experiences since they typically predict different post-measurement states for an observer's records. The standard theory tells us that a good observer would record an ordinary determinate result for the measurement, but the bare theory tells us that while he might report that he had recorded an ordinary determinate result, there would in fact be no such record. Indeed, according to the bare theory, an observer's records would typically not have pure states of their own nor would the observer typically even be in an eigenstate of existing as a sentient being, which on the standard interpretation of states presumably means that there would fail to be *any* determinate experience (see Albert 1992, 124–25).

One might try weakening the relationship between an observer's subjective experience and the state of the observer's physical record of the experience so that the record state $1/\sqrt{2}(|P\rangle_J + |Q\rangle_J)$—call this ★—and the record state $|P\rangle_J$ correspond to the same experience, but this does not work. If ★ corresponds to the same subjective experience as $|P\rangle_J$, then by symmetry ★ also corresponds to the same experience as $|Q\rangle_J$, which presumably means that the record $|P\rangle_J$ corresponds to the same experience as the record $|Q\rangle_J$, which is a serious problem—this would mean that an observer would have the same subjective experience if he were in an eigenstate of recording that he sees the refrigerator as he would have if he were in an eigenstate of recording that he does not see the refrigerator!

One might avoid this conclusion by stipulating that ★ only *sometimes* corresponds to the same subjective experience as $|P\rangle_J$. But if ★ only sometimes corresponds to the same subjective experience as $|P\rangle_J$, then an observer's experience would not be determined by the quantum-mechanical state alone since knowing that ★ was the state would not tell us what the observer's subjective experience was. One might add a new parameter to the theory that would determine the observer's subjective experience (something like the particle positions in Bohm's theory, for example), but this would require one to sacrifice Everett's claim that pure wave mechanics was complete.

There are other problems with the bare reading, but the most serious, at least in the context of searching for a satisfactory interpretation of Everett, is the difficulty in deducing the same empirical predictions from the bare theory as made by the standard theory.

V

On the splitting-worlds reading there is a determinate matter of fact concerning which result a particular observer recorded for a measurement. This is because an observer's world typically splits on measurement into many worlds each inhabited by a different observer and each described by a different term in the post-measurement state written in the preferred basis, which is stipulated to be such that each observer in each world has a determinate record of his measurement result.

It is the ontology that does the real work on this reading. Once one buys into the existence of many post-measurement worlds each containing a slightly different copy of the pre-measurement world, it is easy to explain why every observer gets a determinate measurement result—there are many post-measurement observers, and each has a determinate record. In the refrigerator story, with an appropriate choice of a preferred basis, there would be two of me after my first measurement. One would see the refrigerator *because it is right there in front of his eyes* and the other would not see it *because it is on α-Centauri and he cannot see that far*. It is also easy to explain why each observer would get the same result when he repeats my initial measurement. When the first me blinks and looks again, he still sees the refrigerator. Why? Because *in his world the refrigerator is still in his kitchen*. When the second me blinks and looks again, he still sees no refrigerator. Why? Because *in his world the refrigerator is still on α-Centauri*. There is nothing subtle here—both observers get determinate results to their measurements just as the standard theory predicts, and the explanation is perfectly straightforward—after the first measurement, there would be two observers living in ontologically distinct worlds, each with a complete (though mutually incompatible) set of determinate records of past measurement results.

Explaining why our experimental results exhibit the usual quantum statistics on the splitting-worlds reading is more difficult. Since it is generally the case that *most* elements in the superposition after n measurements would *not* exhibit the relative frequencies predicted by the standard theory, we need some other way of interpreting Everett when he said that an observer in a *typical* element would get the same statistical results predicted by the standard theory with the collapse dynamics.[14] If one randomly chose an element in the superposition using the measure determined by the squares of the coefficients on each term, then one *would* typically choose an element that describes an observer's measurement results as being randomly distributed with the relative frequencies predicted by the standard theory. But exactly how is this fact supposed to help us explain why our experimental results exhibit the usual quantum statistics?

The problem here has to do with the interpretation of probability in the theory. One might suppose that it would be necessary to distinguish between possible measurement results and actual measurement results in order to sensibly discuss the probability of some result, but according to Everett, all possible measurement results are actual.

> The whole issue of the transition from "possible" to "actual" is taken care of in the theory in a very simple way—there is no such transition, nor is such a transition necessary for the theory to be in accord with our experience. From the viewpoint of the theory *all* elements of a superposition (all "branches") are "actual," none any more "real" than the rest (1957, 320 footnote).

For its part, the splitting-worlds reading is thoroughly deterministic, and it tells us that every possible measurement result is actual in some world. But if we know that every possible measurement outcome is in fact realized, then how do we interpret the probabilities associated with the various outcomes? One might want to say that only one of these equally real worlds would be *the world where I would find myself* and then interpret the norm-squared of the coefficient on a term as the probability that the term correctly describes *that world*. If there really were a world described by every term in the superposition, however, then there would typically be many post-measurement copies of me inhabiting different worlds, and it is consequently difficult to see how one of these would acquire the special status of being *the world where I would find myself.*[15]

The fact that the splitting-world reading requires a preferred basis also leads to a well-known problem. One would want the physically preferred basis to be such that every observer in every world would end up with a determinate record of his past experience. Which basis would do this, however, depends on how our most immediate records of past experiences are actually recorded, which depends on the actual physiology and practice of observers. But presumably there would be a physical matter of fact about which basis was the preferred one. So why should one suppose that this basis would be one that would give every sentient being determinate records of his past experience? It has been argued that the preferred basis problem can be solved, or at least made less worrisome, by considering decoherence effects, though it is, I believe, still unclear exactly how this would work.[16]

Another problem with the splitting-worlds reading is that it is difficult to see how one could claim that the linear dynamics correctly describes the time-evolution of every system and that there is more than one post-measurement world. Consider an observable A of the composite system $J + R$ that has (4) as an eigenstate with eigenvalue $+1$ and any state orthogonal to (4) as an eigenstate with eigenvalue -1. How would an A-measurement of $J + R$ turn out if the composite system were in fact in the state described by (4)? If the linear dynamics were right, then one would *with certainty* get the result $+1$. But we are supposed to explain why each observer got a determinate result on the splitting-worlds reading by appealing to the fact that they live in different worlds, each with determinate records; and if the worlds were in fact different, if (4) represented two ontologically distinct worlds described by its two terms, then the probability of getting the result $+1$ would presumably be $1/2$ in either world (since the state of a world must determine, insofar as they are determined, all events in that world—this is presumably just part of what it means to be a world). So here's the dilemma: if the post-measurement worlds were really *separate worlds* then one could explain why an observer would get determinate measurement results; but if the linear dynamics is taken to be a complete and accurate description of the time-evolution of states, as Everett wanted, then it is difficult to see how they could be.[17]

VI

Everett sometimes insisted that there was only one physical observer after a measurement. In one of his most detailed explanations of his metaphysical commitments he said

> At this point we encounter a language difficulty. Whereas before the observation we had a single observer state afterwards there were a number of

> different states for the observer, all occurring in a superposition. Each of these separate states is a state for an observer, so that we can speak of the different observers described by the different states. On the other hand, the same physical system is involved, and from this viewpoint it is the *same* observer, which is in different states for different elements of the superposition (i.e., has had different experiences in the separate elements of the superposition). In this situation we shall use the singular when we wish to emphasize that a single physical system is involved, and the plural when we wish to emphasize the different experiences for the separate elements of the superposition (1973, 68).

So while Everett claimed that there was only one physical observer at the end of the measurement, he also claimed that this observer had different experiences in different elements of the superposition. Does this mean that there is one physical observer with many mutually incompatible experiences? One physical observer who both sees and does not see the refrigerator? How is this possible?

Everett said that "the discontinuous 'jump' into an eigenstate is . . . only a relative proposition, dependent upon our decomposition of the total wave function into the superposition, and relative to a particular chosen apparatus value" (1973, 59–60). But how could an explanation of an *actual event*, the experience of a real observer, depend on *a choice* of how to write the total wave function and *a choice* of a particular apparatus value from among the terms in the expression of the state when both of these seem to be arbitrary matters of convention?

Saunders has proposed understanding physical facts as relations in Everett.[18] This reading captures some of what Everett said concerning relative states that is difficult to make sense of on other readings. In order to distinguish his relative-fact reading from the splitting-worlds reading, Saunders argues that "what is involved in the Everett procedure [for determining relative states] is poorly made out in terms of the notion of a set-theoretic collection of worlds" (1995, 236). Rather, the point of Everett's talk of relative states is that facts *are* relations—a fact concerning a physical system is typically only a fact relative to a specification of the state of every system with which it has interacted. Where is the Eiffel Tower? It does not have a determinate position itself; rather, it has a determinate position only relative to a specification of the state of all other systems that are somehow correlated with its position. Relative to Newt Gingrich being Speaker of the U.S. House of Representatives, etc. the

Eiffel Tower is in Paris. But relative to Gingrich being the President of the United States, etc. the Eiffel Tower may be next to the Washington Monument. So is Gingrich President or not? Just as the location of the Eiffel Tower is a relative fact, who is the President of the United States is a relative fact: relative to the Eiffel Tower being in Paris, Gingrich may not be President; but relative to it being in Washington D.C., he may be.

One might object that the Eiffel Tower is in fact in Paris and that Gingrich is in fact not the President of the United States. But again, on this reading of Everett, facts are relative. One cannot say that the Eiffel Tower is in Paris. One cannot even say that one remembers seeing it in Paris. Given the current quantum-mechanical state, it might be true that the Eiffel Tower is in Paris relative to my remembering seeing it there, Gingrich being Speaker of the House, etc. But it might also be true that the Eiffel Tower is in Washington D.C. relative to my remembering not seeing it in Paris, Gingrich being President, etc.

There are many things that might be said concerning the relative-fact reading of Everett, but there are two particularly straightforward things to say. First, if one believes that there are absolute matters of fact concerning the location of the Eiffel Tower, who is the President of the United States, whether one has a refrigerator in the kitchen, etc. then the relative-fact reading is unacceptable as a formulation of quantum mechanics. Second, concerning its adequacy as a reading of Everett, it apparently violates his goal of deducing the empirical predictions of the standard theory. The standard formulation of quantum mechanics tells us that there are simple facts concerning what an observer recorded (or experienced) and that these facts are fully determined by the current quantum-mechanical state. The relative-fact reading, however, does not account for an observer's determinate records (or experience); rather, it tells us that there are no simple facts concerning what the observer recorded (or experienced), only relative facts. On Everett's account of experience then the relative-fact reading fails to make the same predictions for an observer's subjective experiences as the standard formulation of quantum mechanics makes. This is not to say that a relative-fact or some other strongly perspectival formulation of quantum mechanics is necessarily unacceptable; one might ultimately be willing to make the sacrifices required to embrace such a formulation, but such a formulation would be, I believe, inconsistent with Everett's straightforward claim that pure wave mechanics makes the same predictions for the subjective experiences of observers as the standard theory.

Another way to make sense of Everett's claim that there is just one observer with different experiences for each separate element in the superposition is described by Albert and Loewer (1988). They adopt the plausible-sounding maxim that observers typically have simple, determinate experiences, and they are led to the conclusion that Everett requires there to be many conscious minds associated with each physical observer. On their many-minds formulation of quantum mechanics, there is a continuous infinity of minds associated with every physical observer, and each of these minds has a mental state determined by a term in the observer's physical state when written in what might be called his determinate-belief basis. This formulation makes sense of Everett's claim that there is just one physical observer with many mutually incompatible experiences, and like the relative-fact reading, it also fits well with other claims that he makes concerning his formulation of quantum mechanics. While it avoids many of the problems encountered by other readings of Everett, it is also easy to see why one might find the many-minds theory to be an unpleasant alternative.[19]

VIII

The failure of the bare and splitting-worlds readings and some of the problems encountered more recently illustrate the difficulty in finding an entirely satisfactory reconstruction of Everett's project. Everett was right to worry about von Neumann's formulation of quantum mechanics—we do not want a theory that has mathematically incompatible dynamical laws and no clear prescription for when each obtains. And Everett's proposal to take the linear dynamics as always correctly describing the time-evolution of the usual quantum-mechanical state seems particularly natural. The problem is that it is difficult to recapture the standard empirical predictions of quantum mechanics. One response would be to give up our basic intuitions concerning the nature of physical facts or what it means for a theory to be empirically adequate. Another response, perhaps more conservative, would be to give up the close relationship between an observer's experience and the quantum-mechanical state of the observer's records, then to add a parameter to the usual-state description that typically makes exactly one measurement result an absolute matter of fact and tells us what it was. Such an addition, however, would be incompatible with Everett's claim that pure wave mechanics was complete.[20]

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NOTES

1. The first was his 1957 paper, and the second was the longer work published in the DeWitt and Graham anthology in 1973. Many of the most important passages are word for word the same in the two works, but Everett (1973) provides more details. The references to Everett (1957) are to page numbers in the Wheeler and Zurek anthology since this is perhaps the most popular source for this paper now. The references to Everett (1973) are to the DeWitt and Graham anthology.

2. More specifically, the probability P_n of a system in the state represented by the vector $|\psi\rangle$ jumping into the eigenstate represented by the vector $|\phi_n\rangle$ is given by the norm-squared of the inner product of the two vectors $|\langle\psi|\phi_n\rangle|^2$. Taking $|\langle\psi|\phi_n\rangle|^2$ as the probability of getting the measurement result corresponding to the state $|\phi_n\rangle$ when the state of one's object system is initially $|\psi\rangle$ is sometimes called "Born's rule."

3. For the sake of simplicity, I will consider only pure states in this paper.

4. These are not possible worlds in one of the standard philosophical senses; rather, each of these worlds is supposed to be equally actual and produced by events in the world that splits.

5. There are other many-worlds interpretations. Tipler, for an example of the range of many-worlds interpretations out there, claims that most skeptics "have a mistaken idea of what the [many-worlds interpretation] really means." The problem, he says, is that "many presentations of the [many-worlds interpretation] have made it appear more counter-intuitive than it really is." One example of this is when it is claimed that the entire universe is split by a measurement. According to Tipler, "this is not true. Only the observed/observer system splits; only that restricted portion of the universe acted on by the measurement operator M splits." Apparently, Tipler takes this splitting very seriously; he argues that it cannot go on forever "since the information stored in human beings is finite, the set of all possible measurements can split a human being into only a finite number of pieces." Tipler goes on to estimate that a human being can only be split into about "2 raised to the 10^{26} power" pieces (Tipler 1984, 204–206). I must admit, however, that I don't really understand this.

6. We will see later that if Everett took the linear dynamics to be universally true and if he meant to say here that the existence of other branches can make no empirical difference, then he was simply wrong.

7. Wheeler seems to have supported some version of the splitting-worlds reading of Everett until his 1977 paper titled "Include the Observer in the Wave Function?" Tipler reports that "in a private conversation, John Wheeler told me that the main reason for his

current rejection of what was once called the 'Everett-Wheeler theory', was his distaste for the large ontology it definitely implies" (Tipler 1984, 207).

8. As was the case with the splitting-worlds reading, there are several mutually incompatible readings of Everett that are described as one-world interpretations. For example, Healey refers to his interactive interpretation as a one-world reading of Everett (1989, 211) and Gell-Mann and Hartle take their many-histories formulation to be a one-world reading of Everett (1990, 455).

9. Given that he could have used almost any macroscopic object, that Everett chose a cannonball as his example of an object that would not have a determinate position may go some way in explaining how he ended up working at the Pentagon's Institute for Defense Analysis. I take the correct interpretation of my example of a macroscopic object to be obvious.

10. I removed the italics from this quotation to make it easier to read. Given that Everett referred to his version of quantum mechanics as the relative-state formulation, one might expect that one would understand his formulation when one understands what he means by the relativity of states. But this is precisely where one runs into the most serious problems in interpreting Everett. While it is perfectly clear from Everett's exposition what relative states are, it is unclear what role they are meant to play in his formulation of quantum mechanics. Indeed, when Everett introduces his notion of relative state here (1957, 317–318), the first thing he uses it for is to explain why one might believe that a typical measurement would in fact *fail* to yield a physical state compatible with our experience.

11. Taken literally, what Everett said here is false, so for "almost all" read "typical" in the sense described above.

12. For such an objection see Weinstein (1996).

13. There is a long tradition of such arguments. For an early example see Hartle (1968), and for a more recent one see Barrett (1994). See Farhi, Goldstone, and Gutmann (1989) for a particularly good account of the relative-frequency result.

14. This is the problem mentioned earlier. Graham (1972, 232–236) thought that Everett *ought* to have meant most. Graham's idea was to make worlds split so that the *numerical proportion* of worlds with some property would always be equal to the sum of the norm-squared of the coefficients on those terms describing worlds with that property. All this would do, however, is replace the norm-squared measure on worlds by a counting measure, which does little to help us explain why our experimental results exhibit the usual quantum statistics.

15. This puzzle has a long tradition. It is recently discussed in Albert and Loewer (1988). See Saunders (1995) for a recent attempt to make sense of probability in Everett.

16. See Butterfield (1995) and Saunders (1995) for recent discussions of decoherence and the preferred basis problem.

17. See Albert and Barrett (1995) for a more detailed discussion of this problem. Also see Clifton (1996) for a nice examination of possible many-worlds responses.

18. This proposal is similar to others, but it differs in the type of relativity of facts proposed. Putnam, for example, held that facts are relative to the specification of an observer (or more specifically, relative to the specification of a von Neumann cut). He held that "when we choose to measure the 'mortality condition' of [Schrödinger's] cat (*alive* or *dead*), we choose to institute a frame *relative to which* the cat *has* a determinate property of being alive or a determinate property of being dead *and the measurement finds out which*, we are, so to speak, 'realists' *about the property we measure*; but we are not committed to realism about properties *incompatible* with the ones we measure. Relative to

this observer *these* properties are 'real' (i.e., there to be discovered); but relative to a different observer different properties would be 'real'. There is no 'absolute' point of view" (1981, 209). Putnam explained that there were no physical jumps; but rather, such talk is only an "expression of the relativity of truth to the observer" (209).

19. See Barrett (1995) for a discussion of some of its virtues and problems.

20. I would like to thank D. Albert, J. Butterfield, M. Hemmo, and B. Loewer for comments on an earlier version of this paper.

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