Agreement Beyond Polarization: Spectral Network Analysis of Congressional Roll Call Votes[1]

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Abstract

This paper investigates the structure and dynamics of political agreement in the United States Congress since 1887. We develop new methodologies for the analysis of Congressional behavior revealed by roll call votes. We introduce a new econometric identification strategy of the underlying patterns of political agreement by synthesizing, for the first time, recent advances in random matrix theory, network analysis and boosting regression procedures. We identify networks of agreement that cross partisan and ideological lines, and analyze how they evolve both through history and across the legislative policy space. We uncover major differences between the House of Representatives and the Senate, supporting institutional theories of the Congress that stress the complex interaction of incentives, constraints and preferences. We also discover a unique dynamic of conflict and cooperation that characterizes legislative behavior in the Senate, suggesting a dynamic model of reputation building and turf wars to exercise political power within the Senate and respond to electoral incentives. The new evidence also links periods of high agreement with national crises, and points to defense policy and a conservative agenda as the basis of most common political agreement in recent years. This paper is also the first to provide systematic evidence that credits the rise of Republican Party's power in recent years with high levels of polarization in American politics today.

1 Introduction

Increasing partisan polarization is widely recognized to be one of the key features of American politics in recent decades. Current research on voting behavior and the US Congress has linked the trends in polarization to increasing inequality, the role of race in American politics, and a widening ideological party conflict based on different visions of American religious and secular values (e.g. McCarty, Poole and Rosenthal, 2006). In focusing on polarization, what is often missed is a systematic and persistent tendency of some political leaders to cross partisan battle-lines, strike agreements, and push the nation's agenda forward in the midst of political conflict.

It is often claimed that ideology explains most of the observed voting patterns in Congress. Such claims rely on the statistical application of asymptotic results, but can be very substantially upward biased in finite samples (Harding, 2006a). Inference on the extent to which the first two dimensions explain the observed variation in the data is particularly problematic in the case of large (N, T) panel data such as roll call votes. This implies that although our asymptotic distributional results seem to imply that ideology explains a substantial proportion of the observed voting behavior, this proportion is severely upward biased and in fact ideology has a much smaller explanatory power in the kind of samples we encounter when analyzing roll call votes. This opens the challenging question of identifying additional dimensions that explain Congressional voting behavior and which is the subject of this paper.

In this paper we aim to uncover patterns of agreement and follow their evolution over time in the US Congress. Using complete roll call data on all US Congresses since 1887 (the 50th Congress), we analyze the evolution of agreement in the House and the Senate, identifying pivotal historical periods in which agreement across polarization lines was the strongest. We identify policy areas, both in the House of Representatives and in the Senate, where agreements were most likely to occur, and study how the propensity of legislators to reach across the aisle changed over time. Our results paint a more optimistic picture of American politics over the past century, suggesting that even the most deeply entrenched partisan divides can be overcome as a matter of routine democratic politics to advance the Congressional legislative agenda.

An examination of the dynamics of political agreement in both houses of Congress across history has the potential to help discriminate between increasingly complex theories of the Congress. Following the advances in political economy and social choice, a number of institutional theories of the Congress have been put forward to model political competition between parties and representatives. An analysis of networks of political agreement over time can help us better understand patterns of cooperation and conflict that are structured by the different institutions in the House of Representatives and the Congress. Some of the key open research questions date back to the classics in the study of the Congress, and include the legislators' responsiveness to re-election incentives and the role of incumbent protection devices (Mayhew, 1974), the role of Congressional capacity and power (Dodd, 1977; Sundquist, 1981), the power of the majority party (Rhode, 1991; Cox and McCubbins, 1994; Cox, 2001), the dynamics of coalitional behavior (Schickler, 2001), and interest groups pressures and their influence through the process of campaign funding (Groseclose, Levitt and Snyder, 1999; Shapiro, 2006), among others.

Our empirical analysis reveals that the structure of the networks of agreement is defined by party ideologies and policy platforms. Most importantly, we find that the power of the Republican party in the House of Representatives is directly correlated with the degree of polarization in the Congress; when the Republican party majority is powerful, networks of agreement collapse in the House. While policy dimensions and party ideologies create layers and structures of agreement, individual legislators' interests and re-election incentives determine where they locate themselves in these networks of agreement. While students of American political history are well aware of the pivotal role played by the Southern Democrats in earlier periods of Congressional politics, our data reveals that conservative Southern Democrats in recent years have formed the core of bipartisan agreement in the House of Representatives. In contrast, progressive Democrats find themselves more isolated in the periphery of Congressional networks of agreement.

Our findings also have important implications for understanding party competition in the US Congress and the role of the majority party. Interestingly we find that the strength of the majority party is a significant determinant of polarization and belief heterogeneity only in the case of the Republican party. Moreover, the strategic elements of cartel party behavior induce a dynamic process of high levels of agreement, punctuated by periods of conflict in the US Senate. However, party dominance does not completely determine all dimensions of agreement, as the underlying possibility of political consensus remains dormant and surfaces during periods of national crises and exogenous shocks.

In Section 2, we develop the concept of agreement beyond polarization and construct a statistical representation of revealed Congressional agreements as a network consisting of both coincidental agreements and agreements structured along policy dimensions. This section also analyzes the network topology, discusses clustering of agreements, and identifies the most central legislators to those agreement networks. Section 3 investigates the stochastic properties of the identified networks, and measures the extent of aggregate agreement over time as well as the ease of forming coalitions in a particular Congress. In Section 4, we turn our attention to the identification of the number and nature of structured agreements, employing random matrix theory and a boosting regression procedure. Section 5 concludes by discussing the main empirical results.

2 Voting and Agreement

Since the aim of this paper is to understand agreement we shall describe in this section precisely what is meant by this term and how this process is to be measured. This will help us construct networks of agreement which will be analyzed in more detail in the remaining sections of the paper. Our goal is to understand the patterns of agreements and the underlying issues which give rise to patterns of behavior that can be usefully characterized as agreement.

The basic unit of analysis in most studies of Congress corresponds to the roll call vote. This is particularly helpful to us since it provides a comprehensive quantitative account of the behavior of members of Congress going back to the early years of the Republic. In this study we use roll call votes from the 50th to the 108th Congress (1887-2005) as complied by Keith Poole and Nolan McCarty. While roll call votes encode several categories, we choose to divide the votes into "Y" (Yea), "N" (Nay) and "M" (Missing).

Our statistical analysis will be performed separately for each House and Senate in each Congress. Since the procedure is identical in each case we will refrain from indexing our notation with subscripts for each case. Each sample consists of a set N of individuals observed over T different votes. We denote by $V_{n,t}$ the vote of individual n in roll call t. Table 1 gives the values of N and T for the ten most recent Congresses. In the first step of our analysis we aim to construct a correlation matrix between individuals. The standard definition of a correlation matrix cannot be applied directly due to the non-negligible amount of missing data in the sample. Some individuals vote so rarely that we drop them from the sample altogether. "No Vote" individuals are defined as those individuals who vote less than 5% of the time relative to the average number of times individuals vote in a particular sample. The number of "No Vote" individuals dropped from the sample is also reported in Table 1.

The number of actual votes for each individual in the remaining sample can, nevertheless, vary substantially. We now define the point correlation matrix C between individuals as:

$$(C)_{i,j} = \frac{\sum_{t=1}^{T} 1(V_{i,t} = Y) 1(V_{j,t} = Y)}{T - \sum_{t=1}^{T} 1(V_{i,t} = M) - \sum_{t=1}^{T} 1(V_{j,t} = M) + \sum_{t=1}^{T} 1(V_{i,t} = M) 1(V_{j,t} = M)} \quad (1)$$

where $1(x)$ is the indicator function which is equal to 1 if the logical expression x is true and 0 otherwise. This matrix of correlations is a consistent estimate of the true correlation matrix in the presence of missing data for large T.

Recent studies of Congress have focused extensively on the nature of polarization as estimated by an ideal point analysis (McCarty, Poole and Rosenthal, 2006; Poole and Rosenthal, 1997). By its very nature an ideal point analysis such as NOMINATE is a discriminant procedure aimed at achieving maximum separation between the individuals (Takane, Bozdogan and Shibayama, 1987). The first dimension is usually interpreted as the liberal-conservative dimension, while the second dimension appears to be rather more unstable over time and may be interpreted as North-South or the civil rights dimension.

Since ideal point estimation is related to factor models, we use a computationally convenient approximation to the NOMINATE procedure based on Principal Components Analysis (Brady, 1989; Heckman and Snyder, 1997). A number of authors have shown that the first two dimensions of the NOMINATE analysis can be very accurately estimated by the first two principal components of the point correlation matrix C (Heckman and Snyder, 1997; Jakulin and Buntine, 2004). Since our interest lies in the agreements that exist once we look past the divides captured by the NOMINATE dimensions, we will use the the factor analytic approximation to extract the residual correlations after we remove the effect of the first two NOMINATE dimensions. The resulting correlation matrix $\tilde{C}$ measures the extent to which voting patterns are correlated in excess of the correlations due to the two NOMINATE dimensions. Thus, in effect we will be using the residuals of the NOMINATE fits after we have partialled out the effect of the first two dimensions.

It is important to look beyond the NOMINATE dimensions since reliance on these two dimensions may produce false inference as to the extent to which they can explain observed behavior. Harding (2006a) shows that measures of the explanatory power of the first few dimensions are severely upward biased. In finite samples it may appear that these dimensions have substantial explanatory power, when in fact they explain a much smaller proportion of behavior. Harding (2006a) shows that the bias is a function of $c = N/T$, where N is the number of individuals and T is the number of periods in our sample. The bias only disappears asymptotically if $c \to 0$ as $N \to \infty$ and $T \to \infty$. In the case of roll call votes this asymptotic requirement is not satisfied since both N and T are large and thus the bias is particularly problematic.

We now wish to establish the extent to which the remaining correlations $\tilde{c}_{i,j}$ are statistically significant. We would expect many of the remaining correlations to be close to zero. But if our hypothesis that there are agreements beyond the polarization observed by the NOMINATE procedure is correct, we would expect some correlations to persist. Our eventual aim is to analyze these correlations statistically.

Consider now the following transformation of each observed correlation coefficient $\tilde{c}_{i,j}$ (known as Fisher's transformation of the correlation):

$$z_{i,j} = \frac{1}{2} \log \left(\frac{1 + \tilde{c}_{i,j}}{1 + \tilde{c}_{i,j}} \right). \tag{2}$$

This transformation of the correlation coefficient maps the estimated coefficient from a $[-1, 1]$ range to the real line and has an approximate asymptotically Normal distribution with mean $\frac{1}{2}\log(\frac{1+\rho}{1+\rho})$ and variance $1/(\hat{T} - 3)$, where ρ is the true correlation coefficient between individuals i and j, while $\hat{T}$ is the denominator in the equation 1 above, and which corresponds to the number of votes where both individuals voted.

This suggests a statistical approach for choosing which correlations are deemed to be statistically significant by using an appropriate cut-off parameter. Thus, only correlations such that $z_{i,j} < -\overline{z}$ or $z_{i,j} > \overline{z}$ can be characterized as statistically significant. The most intuitive choices for $\overline{z}$ are given by the appropriate percentiles of the Normal distribution, such that we can reject the estimate $z_{i,j}$ at a common level of statistical significance, such as the 90-th or 95-th percentiles. In this study we use the 95-th percentile for the correlation matrix in the House and the 90-th percentile for that in the Senate. This discrepancy is due to the fact that since the number of individuals in the Senate is much smaller than the number of individuals in the House we need to allow for a large enough number of individuals in order to generate out network. This may introduce some additional noise in the estimating procedure, but as we shall argue later, we expect to be able to filter the noise out at later stages of our statistical procedure.

Define the matrix A such that

$$(A)_{i,j} = \begin{cases} 1 \quad \text{if} \quad z_{i,j} < -\overline{z} \quad \text{or} \quad z_{i,j} > \overline{z} \quad \text{for} \quad i \neq j \\ 0 \quad \text{if} \quad z_{i,j} \geq -\overline{z} \quad \text{and} \quad z_{i,j} \leq \overline{z} \quad \text{for} \quad i \neq j \\ 0 \quad \text{if} \quad i = j \end{cases}, \tag{3}$$

where the matrix A is a sparse binary matrix which records if individuals i and j have correlated voting behavior after removing the effect of the NOMINATE dimensions. Notice the additional restriction that $(A)_{i,j} = 0$ if $i = j$, which corresponds to the removal of the trivial correlations of individuals with themselves.

We are now ready to formally introduce the notion of agreement which forms the subject of study in this paper. Two individuals i and j **agree** if and only if $(A)_{i,j} = 1$. Notice that by definition the property of agreement is symmetric $(A)_{i,j} = (A)_{j,i}$. In order to simplify the rest of the analysis we ignore self-referential agreement, and let $(A)_{i,i} = 0$ by definition. This has no implications on the conclusions of the analysis but reduces the complexity of the analysis by removing some of the combinatorial problems resulting from reflexivity. It is important however to note that our notion of agreement is not transitive, that is $(A)_{i,j} = (A)_{j,k} = 1$ does not imply that $(A)_{i,k} = 1$! This is very natural to understand in a multi-dimensional issue space which is commonly assumed to underly voting behavior. Consider for example a situation where three issues $\{\alpha, \beta, \gamma\}$ are under considerations. Individual i votes for issues $\{\alpha, \gamma\}$, individual j votes for issues $\{\alpha, \beta\}$ and individual k votes for issues $\{\gamma, \beta\}$. Thus, following the logic of our definition individual i agrees with individual j on issue α, individual j agrees with individual k on issue β, but individuals i and k fail to agree on any issues. If this pattern of behavior is evident in the voting behavior, it will be captured by a corresponding pattern of agreement in the matrix A.

Furthermore, the matrix A captures **agreement beyond polarization** since by construction the underlying correlations are constructed so as to capture the residual correlations after removing the influence of the first two NOMINATE dimensions.

The agreement relationships between member of Congress are particularly well suited to be modeled by a graphical model $\Gamma = (V, E)$, where $V = \{1, 2, ..., N\}$ is the set of individuals in either the House or the Senate corresponding to the vertices of the graph and E is the set of edges defined on the Cartesian product $V \times V$. An edge exists between vertices i and j if and only if $(A)_{i,j} = 1$, that is the two individuals corresponding to i and j agree, where agreement follows the definition above. Notice that our model of agreement corresponds to a simple undirected graph represented by the adjacency matrix A.

By construction, it is possible to find vertices that are unconnected to other vertices. These correspond to individuals whose behavior is completely characterized by the first two NOMINATE dimensions. These individuals reveal no further information beyond the fact that they vote mostly along the liberal-conservative dimension and only very rarely deviate from it. We present the number of such individuals in Table 1 for the last ten Congresses. While they will be dropped from the rest of the study, it is interesting to note that the number of Republicans in this category was extremely high for the 108th and 107th Congress in the House and the 108th Congress in the Senate relative to historical trends. This may provide further support to the much discussed claim that American politics has become increasingly polarized in recent years. The number of Republicans in this category seems to have increased tenfold, while no discernible corresponding trend seems to be evident for the Democrats.

The data on agreement represented by the matrix A can be visually represented as a network (Nooy, Mrvar, Batagelj, 2005). We use the Fruchterman and Reingold (1991) algorithm to visualize the network by relating the distance between individuals to the extent to which they agree with each other. The exact equilibrium procedure is described in more detail in Appendix A. Figure 1 gives the resulting network of agreement in the House for the 108th Congress and Figure 2 gives the corresponding figure for Senate. At first glance it seems that the House is split between four clusters, two for each party. Since our visualization procedure clusters individuals with similar agreement patterns closer together we can investigate the nature of these clusters by looking at their membership.

The configuration for the House can be described as being composed of one Democratic and one Republican cluster that are closely merged and two other periphery clusters occupying a more distant location. The central two clusters are composed of the moderate Representatives in both parties. The periphery cluster consisting of Democratic Representatives is composed of the more liberal Democratic Representatives. Some of the Representatives on the outer boundary of this cluster are Hilda Solis, the pro-choice Representative of the 32nd District of California, closely associated with labor unions and

Tammy Baldwin, the Representative from Wisconsin and first openly gay candidate to be elected to the House. The Republican periphery cluster is composed of a more conservative subset of the Republican representatives. Some of the Representatives on the outer boundary of this cluster are Nathan Deal, representing the 10th District of Georgia, who most recently fought against extending the Voting Rights Act for minorities and Mike Pence from Indiana's 6th District, a strong opponent of minimum wage increases and supporter of the elimination of the estate tax. By contrast the Senate does not exhibit the same clustering pattern. Senators are clustered most strongly along party lines.

The discussion above tells us that agreement is most likely to be found between close ideological positions. But that is only one side of the story. We can now ask which individuals are most likely to reach across the aisle and agree with others on the opposite side. We call these individuals "agreers". The main agreers are those which agree with the largest number of other individuals. Formally we count the number of individuals that agree with any one individuals by the following score:

$$d_i = \sum_{j=1}^{N} (A)_{i,j}. \tag{4}$$

The individuals with the largest such five such scores are reported in Table 2 for the last ten Congresses. For the 108th Congress in the House of Representatives the main agreer was Gene Taylor from the 4th District of Mississippi, one of the most conservative Democrats in Congress on a variety of issues from his pro-life stance to gun control, immigration and the death penalty. For the 108th Congress we find that the main agreers in the House are Democrats from Southern States with a very conservative record.

The main agreer in the Senate is Blanche Lincoln, Democrat of Arkansas, followed by Ben Nelson (Democrat of Nebraska), John McCain (Republican of Arizona), John Breaux (Democrat of Louisiana) and Mark Pryor (Democrat or Arkansas). Similar to the results for the House the Democrats reaching across the aisle tend to be very conservative on a variety of issues, but particularly social issues such as abortion or gay marriage.

Thus, it seems that the main agreers are socially conservative individuals in both the House and the Senate. This result is consistent with the recent theoretical evidence provided by Alexander and Harding (2006) who show that deliberation aimed at reaching consensus converges towards the position of the most conservative member of the group.

3 Spectral Properties of Agreement

The methods employed so far have relied on the visual inspection of the resulting network of agreements and on simple counting measures of the edges. In this Section we will develop additional methods for the statistical analysis of the network of agreements based on the spectral decomposition of the matrix of agreements A. Since the matrix A is symmetric, we can find a matrix U with columns that are orthogonal to each other such that:

$$A = U'DU = U' \begin{pmatrix} \lambda_1 & 0 & ... & 0 \\ 0 & \lambda_2 & ... & 0 \\ 0 & 0 & ... & 0 \\ 0 & 0 & ... & \lambda_N \end{pmatrix} U. \quad (5)$$

The matrix $D = \text{diag}\{\lambda_1, \lambda_2, ..., \lambda_N\}$ contains the set of eigenvalues of the matrix A, while the columns of A are the eigenvectors of A. These quantities will play an important role in our analysis. The set of eigenvalues of A is also called the spectrum of the graph Γ which encodes our network of agreement in Congress. The empirical distribution of eigenvalues of A is given by:

$$F^A(\lambda) = \frac{1}{N}\{\text{Number of Eigenvalues of } A \leq \lambda\} = \frac{1}{N}\sum_{\lambda_i \leq \lambda} 1, \tag{6}$$

where λ_i are the eigenvalues of A.

We can use the spectral properties of the networks of agreement described above to extract a variety of insightful characteristics of the patterns of agreement as revealed by the voting behavior of members of Congress. These will form the basis of the discussion in this paper.

In the previous section we characterized agreers as those individuals with the highest number of connections to other members of Congress. Newman (2004) argues that while this measure correctly captures the extent to which individuals are connected to each other it may not necessarily be the right measure of influence in a network. In particular it seems likely that if someone is connected to other well-connected individuals she is more likely to be able to exert greater influence than another person with the same number of connections but who is connected to more isolated individuals. Thus, we may wish to weigh the extent to which an individual is actually connected to other well-connected individuals. Newman (2004) shows that if we let the influence scores be ξ_i, then for some constant λ, we can weigh the influence of i by the influence scores for the individuals to which i is connected through the following equation:

$$\lambda \xi_i = \sum_{j=1}^{N} (A)_{i,j} \xi_j. \tag{7}$$

If we now stack the equations for ξ_i and solve the resulting system simultaneously, we obtain the matrix equation $\lambda\xi = A\xi$, which defines ξ to be an eigenvector of A associated with the eigenvalue λ. Moreover, the optimal choice of λ corresponds to the largest eigenvalue of the spectrum of A.

In Table 3 we list the most influential members of Congress for both the House and the Senate from the 99th to the 108th Congress. If we compare this list to the the one in Table 2 we will find some of same names but in a slightly different ordering. There is no perfect one-to-one mapping between the main agreers and the most influential members of Congress. But we nevertheless find that agreers tend to agree mostly with other agreers. Thus agreement and influence are closely related. In particular we notice that influential members tend to be listed as permutations of the list of agreers.

This provides further evidence as to the importance of a key network of agreement formed between influential agreers in order to reach out across the aisle. Agreement seems to often require reciprocity and agreement is mostly formed between central members rather than by reaching across to the fringe of either party. Both the main agreers and the most influential members of Congress, as described by our agreement metric, tend to agree on largely conservative issues.

We will now use the spectral characteristics of the network of agreement in each Congress to characterize the patterns of agreement rather than the players involved in these agreements. We will employ a series of statistics which are summarized in Table 4 and discussed below.

In our model we wish to distinguish between two sources of agreement beyond polarization, coincidental agreement and structured agreement. Coincidental agreement corresponds to agreement that occurs because heterogeneous lawmakers that happen to find themselves in agreement as a result of their background interests. Structured agreement on the other hand occurs because lawmakers agree on a number of salient political dimensions. Coincidental agreement is best described as a micro-phenomenon that captures the attention of different lawmakers without necessarily implying agreement on broad political lines which are characterized by the structured dimensions of agreement in our model. Coincidental agreement is similar to the stochastic error term in more conventional statistical models. It captures all that is left outside a model in order to allow us to focus on the few structural elements that are the main focus of discovery.

In order to avoid confusion, we refer to the full extent of agreement estimated from the data as aggregate agreement and understand that it originates both in coincidental agreement and in structured agreement. It measures the extent to which lawmakers are able to engage in agreements once they look past both ideological polarization. Aggregate agreement captures the extent to which lawmakers are able to relate to each other because of shared beliefs and values. These shared beliefs may be coincidental or structured along clearly defined policy dimensions. It is the existence of these shared beliefs which allows lawmakers to reach across the aisle.

Aggregate agreement is thus also measures a certain type of belief heterogeneity. More heterogeneous individuals will have a broader spectrum of beliefs than those individuals who stick closely to some ideological dimensions. A variety of held beliefs will enable them to find commonalities across party divides or big political issues of national interest.

From a modeling point of view, the probability of aggregate agreement is $P\{(A)_{i,j} = 1\}$, that is the probability of agreement between any two random individuals in Congress. We will relate this property to the spectrum of the adjacency matrix A (Juhasz, 1982). We assume that this probability if fixed for a given Congress but may vary between Congresses as membership changes. In order to measure the probability of aggregate agreement we can use the following result:

Proposition 1: *Let* $(A)_{i,j}$ *be an* $N \times N$ *matrix encoding the incidence of edges in a random network* with $(A)_{i,i} = 0$. Let $(A)_{i,j}$ *for* $i > j$ *be independent random variables. Suppose* $P\{(A)_{i,j} = 1\} = p$ *and* $P\{(A)_{i,j} = 0\} = 1 - p$. *Denote by* $\lambda_1 = \lambda_1(N)$ *be the* *largest eigenvalue of* A, *then* $\text{plim}_{N\to\infty}\frac{\lambda_1}{N} = p$.

Proof: By the Perron-Frobenius Theorem we have

$$\min_{1\le i\le N}\sum_{j=1}^{N}(A)_{i,j} \le \lambda_1 \le \max_{1\le i\le N}\sum_{j=1}^{N}(A)_{i,j} \tag{8}$$

Since for any given i we have that $P\left(|\frac{1}{N}\sum_{j=1}^{N}(A)_{i,j} - p| > \delta\right)$ is exponentially small by the Central Limit Theorem, it implies that

$$\lim_{N\to\infty}P\left(\max_{1\le i\le N}|\frac{1}{N}\sum_{j=1}^{N}(A)_{i,j} - p| > \delta\right) = 0 \tag{9}$$

$$\lim_{N\to\infty}P\left(\min_{1\le i\le N}|\frac{1}{N}\sum_{j=1}^{N}(A)_{i,j} - p| > \delta\right) = 0. \tag{10}$$

This proposition implies that we can use the largest eigenvalue to measure the probability of aggregate agreement, since a consistent estimator can be constructing by scaling the largest eigenvalue by the number of individuals that compose the network. In Table 4 we list the largest eigenvalue for the ten most recent Congresses and in Figure 3 we plot the time series of the probability of aggregate agreement over the period 1887-2005 for both the House and the Senate.

The time series plots reveal that over this period the probability of aggregate agreement was higher in the Senate than in the House for almost all Congresses. Both series increase during most of the 20th century but show a downward trend starting in the late 1970s. Both series appear to follow a trend similar to that of the measure of majority party heterogeneity plotted by Schickler (2001). Schickler relates majority party heterogeneity to a number of institutional changes over the history of the Congress. Note that our measure of heterogeneity is much broader in scope since it is based on all the individuals in Congress and the issues on which they agree as revealed by their voting behavior.

In Figure 6 (a,c) we relate the probability of aggregate agreement to the percentage of Republican members of the total members over the period 1887-2005 for both the House and the Senate. We find a negative and statistically significant relationship, which indicates that the probability of aggregate agreement tends to be lower during Republican controlled Congresses. This implies that in Congresses with a higher Republican majority the likelihood of agreement is reduced, as members focus more on ideological divisions and less on the shared values and beliefs.

In the next section we will investigate the extent and source of structured agreements driven by a series of important political dimensions, but first we wish to enquire into the ease with which bipartisan agreements can happen in Congress. As we have seen above, the probability of aggregate agreement varies with the composition of Congress over time, so we would expect the ease with which legislators can reach across the aisle to also vary over time.

In order to answer this question we will introduce a new mathematical construct termed the Laplacian which is based on the matrix A that captures agreement relations in Congress. We define the Laplacian by:

$$L = \begin{pmatrix} \sum_{j=1}^{N} (A)_{1,j} & -(A)_{1,2} & \dots & -(A)_{1,N} \\ -(A)_{2,1} & \sum_{j=1}^{N} (A)_{2,j} & \dots & -(A)_{2,N} \\ \dots & \dots & \dots & \dots \\ -(A)_{N,1} & -(A)_{N,2} & \dots & \sum_{j=1}^{N} (A)_{N,j} \end{pmatrix}. \tag{11}$$

The Laplacian is the negative of A with added terms on the diagonal corresponding to the number of individuals that agree with a particular individual. Similar to the procedure outlined above, we can compute the spectrum of the Laplacian and it can be shown that the resulting eigenvalues carry important information on the design of the underlying network of agreement (Cvetkovic, Doob and Sachs, 1979; Cvetkovic, Rowlinson and Simic, 1997, 2004).

First let us answer the question whether there was ever a time when Congress was so fragmented that no reaching across the aisle was possible. It can be shown that the smallest eigenvalue of the Laplacian is always zero and that the multiplicity of this zero eigenvalue gives the number of connected components of the graph that characterizes agreement. A connected component is a subgraph of the original graph such that there are no connections between this subgraph and the rest of the graph. We find that with the exception of the 108th Congress the underlying network for both the House and the Senate over the period 1887-2005 consisted of a single connected component, thus bipartisan agreements have always existed. For the 108th Congress we find that for the House there is a small group of Representatives which are separated from the main network of agreements. However, we have found this feature not to be robust to variations in the cut-off parameter used to construct the network and thus we do not have sufficient evidence to conclude that this is a permanent outcome of the recent polarization of Congress.

Given that bipartisan agreements have always happened we can ask whether there are periods when these agreements were easier to implement. As we have seen in the previous section it appears that the underlying networks can be divided in clusters of individuals between which the density of edges is higher. This corresponds to clusters for which the density of agreements is higher due to shared beliefs or vote clustering along policy dimensions. Bipartisan agreements, however, require the ability to bridge differences between individual clusters of agreement. The ease with which a bipartisan agreement is reached can be modeled as the ease with which a random walk along the edges of a graph steps from one cluster to the other. Since deal makers need to work along the lines of existing propensities for agreement, this provides a useful metaphor to think about the ability to induce bipartisan agreements. If the network of agreement is strongly clustered, a random walk will step between clusters only with a low probability and will have the tendency to stay within the cluster, but, if the network of agreement has more uniformly distributed edges, a random walk will frequently step between different parts of the network. The second smallest eigenvalue of the Laplacian defined above provides a measure of the connectivity of the network of agreement. A more connected network corresponds to a network where bipartisan agreements are easier to enforce. We list the values of the second smallest eigenvalue of the Laplacian in Table 4 (labeled "Laplacian") for the last ten Congresses and plot the time series for both the House and the Senate over the period 1887-2005 in Figure 4.

We notice that bipartisan agreements are easier to implement in the Senate than in the House during most Congresses in our sample. Starting in the mid 1980's this however becomes increasingly difficult with the ease of such agreements reaching historical lows over the past few Congresses. The time series for the Senate shows increased ease of bipartisan agreements during the Great War of 1914-1918, the Great Depression, at the end of World War II, after the assassination of President Kennedy and during the 1970s. The time series for the House shows a less pronounced profile which peaks at the end of the Great War, the end of World War II and during the 1970s.

4 Dimensions of Agreement

In the previous sections we focused on different aspects of agreement beyond polarization and described aggregate agreement as the joint measure of agreement originating both in coincidental shared beliefs and in structured interests along policy dimensions. In this section we will turn our attention towards structured agreement and explore the concept further. We will identify the extent of structured agreement as distinct from coincidental agreement. Additionally we will uncover the main dimensions which produce structured agreement. While coincidental agreement happens as a result of the underlying belief heterogeneity that allows individuals to agree with some but not others, structured agreement addresses the convergence of beliefs on a small set of issues deemed important enough to bridge ideological polarization.

Thus the crucial element of this section will be an identification strategy that allows us to distinguish between coincidental and structured agreement. In effect we wish to decompose the network of agreement into those agreements which are coincidental and those which are structured. Moreover, we will introduce a method based on recent advanced in boosting regression analysis which allows us to identify what the dimensions of structured agreement correspond to in terms of real political issues. For this latter procedure we will employ a careful textual analysis of the content of roll call votes.

Consider the matrix A which encodes the set of agreements in a given Congress. Since A is symmetric it has a spectral decomposition in terms of its eigenvalues and eigenvectors (Cvetkovic, Rowlinson and Simic, 1997):

$$A = \lambda_1 P_1 + \lambda_2 P_2 + ... + \lambda_N P_N, \tag{12}$$

where $\{\lambda_1, \lambda_2, ..., \lambda_N\}$ corresponds to the spectrum of A ordered from the smallest eigenvalue λ_1 to the largest eigenvalue λ_N and possibly contains multiplicities. Let $E_i = \text{diag}\{0, 0, ..., 1, 0, ..., 0\}$ with 1 in position i. Moreover, $P_i = UE'U'$, for U the eigenvectors of A defined in the previous section.

By analogy to the factor analysis procedure used to the study of covariance matrices we wish to use the spectral decomposition above to separate the noise part of A corresponding to coincidental agreements from the structural part due to structured agreements along a set of well-defined political dimensions (Harding, 2006b). In contrast to the usual factor analysis method however, the spectrum of A is not bounded by zero from below and the eigenvalues of A can be both positive and negative (Juhasz, 1981).

Consider the following result:

Proposition 2: *Let $(A)_{i,j}$ be the matrix defined in Proposition 1. Then the second largest eigenvalue of A, $\lambda_2(N)$ behaves in probability like $\sqrt{N}$*, i.e. $\lambda_2 = O(N^{1/2+\epsilon})$.

Together with Proposition 1, this result implies that if there were no structured agreements and all observed agreements in Congress are only due to coincidental agreements then we should observe a substantial gap in the spectrum of the matrix A as we move from the largest to the second largest eigenvalue. This is due to the fact that the largest eigenvalue scales as N while the second largest eigenvalue scales as $\sqrt{N}$. This is easy to check by plotting the histogram of the empirical eigenvalue distribution. However, we do not observe such a spectral gap, thus suggesting that the additional observed eigenvalues correspond to structured agreements.

This reasoning is similar to that of standard principal components analysis where we look for large eigenvalues to uncover the structured agreements in excess of the randomly occurring coincidental agreements. The additional complication in our case comes from the fact that the notion of large eigenvalues must be considered in absolute terms, ie. $|\lambda_k|$ since the spectrum of A can take both positive and negative values. Moreover, we need to disregard the largest eigenvalue which is captures both coincidental and structured agreement.

Thus, our identification strategy for structured agreements requires us to choose eigenvalues which are large in the sense that they are larger in absolute terms than the second largest eigenvalue of a random network due to coincidental agreements, but smaller than the largest eigenvalue. Notice that in finite samples this does not necessarily guarantee that all dimensions of structured agreement can actually be identified. It is known, that in finite samples, the effect of a structural component can be too weak compared to that of the random component and it cannot necessarily be identified (Harding, 2006a). In our model this implies a situation where an eigenvalue due to a dimension of structured agreement, may actually be smaller than the second largest eigenvalue of a random network of coincidental agreements, thereby not being identified by our strategy. Harding (2006a) shows that the extent to which this occurs depends on various model parameters but most importantly on the number of observations T available for each individual. In this particular application, the large sample sizes of roll call votes makes this problem less likely to occur.

Given that we have identified where to look for the dimensions of structured agreements we now have to decide on a statistically consistent procedure to separate the eigenvalues due to structured agreement from those due to coincidental agreement. We will be performing a procedure similar to that of separating the signal from the noise in more standard factor analysis methods. Harding (2006b) develops a classical minimum distance procedure by matching the theoretical and empirical moments of the spectral density for covariance models. Here we show how such a method can be adapted to solve the problem of separating structured agreements from coincidental agreements in a random network of agreements such the one underlying this paper.

Define the following linear spectral statistics on the spectrum of A:

$$m_A^k = \int \mu^k dF^A(\mu), \tag{13}$$

defined over the monomials μ, μ^2, ..., μ^k. The sample equivalents of these quantities are given by:

$$\hat{m}_A^k = \frac{1}{N}\,\mathrm{tr}(A^k). \tag{14}$$

In finite samples we have found the results to be more accurate if we exclude the first eigenvalue from the calculations and compute the above quantity using the rest of the spectrum of A. Results from random matrix theory tell us that the spectral distribution of a random network converges to Wigner's semi-circle law (Juhasz, 1981; Bauer and Golinelli, 2001). However, recent numerical results by Farkas et. al. (2001) show substantial deviations from this asymptotic law in finite samples. Therefore, we will not employ a method that relies on all the moments of the asymptotic spectral distribution in order to account for finite sample issues. Instead, we will rely on the result below which tells us that the spectral distribution is symmetric around zero in the absence of structured agreements:

Proposition 3: *Let* $(A)_{i,j}$ *be the matrix defined in Proposition 1. Then* $m_A^k = 0$ *if* $k = 2y + 1$ *for* $y = 0, 1, 2, 3, \ldots$.

In order to choose the number of dimensions of structured agreement we can use the following minimum distance objective function similar to that in Harding (2006b):

$$J = N \begin{pmatrix} \hat{m}_A^1 & \hat{m}_A^3 & \hat{m}_A^5 & \ldots \end{pmatrix} \hat{W}' \begin{pmatrix} \hat{m}_A^1 \\ \hat{m}_A^3 \\ \hat{m}_A^5 \\ \ldots \end{pmatrix}, \tag{15}$$

for some weighting function W corresponding to the covariances between the spectral moments. Since in this case we do not estimate additional unknown parameters and do no have to worry about the efficiency of those estimators, we can take W to be the identity. Alternatively we can use the bootstrap to estimate W.

In order to estimate the number of dimensions of structured agreement, we can employ the following procedure. First order all the eigenvalues (except for the largest one) in descending order of absolute magnitude. An example of such a sequence would be $\{3, -2.5, -2.1, 1.9, -0.5, 0.1, ...\}$. Now recursively drop the eigenvalue with the largest absolute value from this sequence and evaluate the J statistic from above at each stop. This procedure will sequentially reduce the value of the J statistic until the estimate of the number of dimensions of structured agreement has been reached. Dropping too many eigenvalues increases the value of the statistic again. We can think of the eigenvalues corresponding to structured agreement as outliers in the spectral distribution which are eliminated from the spectrum until the remaining spectrum is symmetric. This identifies the dimensions of structured agreement since we know that the remaining spectrum, which is almost symmetric, corresponds to those eigenvalues due to random coincidental agreements.

In Table 4 we list the number of dimensions of agreement estimated for the most recent ten Congresses. In Figure 5 (a,b) we plot the time series of estimated dimensions of agreement over the period 1887-2005 for both the House and the Senate. The first thing to note is the very different behavior of the estimated time series of the number of dimensions for the House and the Senate. The series for the House seems to oscillate around a mean of approximately 7 dimensions of agreement over the past 50 Congresses. The highest number of dimensions of agreement is recorded during the Great Depression while the fewest dimensions are observed during the Populist rebellion at the end of the 19th Century, during the extremely divisive 61st Congress which introduced the modern income tax legislation and during the entire Civil Rights era. By contrast, the time series of the dimensions of political agreement in the Congress shows a very different profile which oscillates between 15 and 1 dimensions, with no obvious historical pattern.

While both the pattern in the House and in the Senate are related to underlying political issues of national importance, it seems that the pattern in the House is easily related to the extent to which the political issues of the day were divisive or not. By contrast the pattern in the Senate requires a more complicated explanation. In Figure 6 (b,d) we relate the estimated number of dimensions of agreement to the percentage of Republicans in the House and Senate respectively. We find no statistical connection between the two variables. This provides further evidence to the claim that coincidental agreements (and the two NOMINATE dimensions) capture many of the agreements based on belief heterogeneity and ideological position.

The pattern of substantial structured agreement punctuated by disagreement in the Senate is rather unusual compared to the one observed for the House and it cannot be explained by reference to important historical events. By contrast we hypothesize that it is due to very different competitive pressures within the Senate dynamics. A pattern of regime switches between cooperation and intense breakdowns in cooperation is familiar to economists who often rely on such mechanisms to explain the pricing behavior of firms (Porter, 1981). From a game theoretic point of view, periods of noncooperative behavior are used to provide punishment incentives that guarantee cooperative behavior in other periods. Such models are known to produce cooperation punctuated by occasional breakdowns in cooperation and may provide an interesting insight into the competitive behavior of members of the Senate.

Given the complex behavior of structured agreement over time, we now aim to explain the main dimensions of agreement for the 108th Congress. The aim is to relate the observed structure of agreements to the underlying political issues debated in Congress. As we noted before, the network of agreements is completely characterized by the spectral decomposition into eigenvalues and eigenvectors. In this section we have uncovered those eigenvalues associated with structured agreements as opposed to coincidental ones. Explaining these dimensions of structured agreements is thus equivalent to explaining the eigenvectors associated with these eigenvalues.

If we consider each dimension of structured agreement individually, the corresponding eigenvector records the weight per individual in Congress that this dimension carried in determining the network of agreements. Lawmakers may place different weights on different issues as expressed by their roll call votes. Thus a procedure for uncovering the meaning of the structured dimensions of agreement involves relating the weights per individual recorded by the eigenvectors to the roll call votes. By examining textual details of the roll call votes mostly associated with a particular dimension of agreement we can uncover the underlying political issue that corresponds to our dimension of interest.

This, however, presents a major statistical problem since the number of potential explanatory variables T (the roll call votes in a Congress) is much larger than the sample size N (the number of individuals in Congress). In order to address this issue we need a consistent variable selection procedure that would allow us to relate the elements of the eigenvectors for each dimension of agreement to particular roll call votes in Congress. We employ the L_2-Boosting with componentwise linear least squares procedure of Buhlmann (2006). The following analysis will be conducted separately for the House and the Senate for the 108th Congress.

Let Y be an $N \times 1$ eigenvector of A corresponding to one of the eigenvalues of the spectrum of A attributed to structured agreements and let X be the $N \times T$ set of roll call votes for a given Congress. We wish to estimate the linear model:

$$Y_i = \beta' X_i + \epsilon_i, \tag{16}$$

consistently given that $T \gg N$. Define the following base procedure for an arbitrary set of response variables W, $g(X, W)$. Let $g(X, W) = \hat{\beta}_s \mathrm{X}_s$ for some column vector X_s of X.

Moreover for each $r = 1..T$ we have

$$\hat{\beta}_r = \left(X_r' X_r\right)^{-1}\left(X_r' W\right). \tag{17}$$

Then, let

$$s = \text{argmin}_{1 \leq r \leq T}\left\{\sum_{i=1}^{N} W_i - \hat{\beta}_r (X_r)^i\right\}. \tag{18}$$

The idea behind the boosting algorithm is to repeatedly apply the base procedure to residuals from previous fits. This produces a sequential estimate of the function of interest $F = \beta' X$, thus consistently estimating the parameter vector β corresponding to each vote. The recursive procedure is as follows. First, let $F^0 = g(X, Y)$. Let $Z = Y - F^0$ and $F^1 = F^0 + pg(X, Z)$, for some scaling factor $0 < p < 1$. The algorithm is then iterated as $F^{m+1} = F^m + pg^m(X, Z)$, where at each step the base procedure g is applied to X and the current residuals $Z = Y - F^{m-1}$. Buhlmann (2006) defines an information criterion that allows us to determine the optimal number of iterations.

We apply this procedure to each of the 5 eigenvectors identified as corresponding to structured agreements in the House and 9 eigenvectors identified as corresponding to structured agreements in the Senate in the 108th Congress For each estimated vector of coefficients β we then investigate the roll call votes corresponding to the elements of β which are largest in magnitude. This allows us to identify the roll call votes which are most significant in explaining that particular dimension of structured agreement. Once we identify a list of roll call votes for each dimension we use detailed textual analysis to understand the political issues corresponding to that particular dimension. We find that for each dimension most of the votes correspond to a well-defined policy area such a defense or health care, thereby allowing us to label the identified dimensions of structured agreement.

In Table 5 we list the main policy issues as revealed from an analysis of roll call votes.

For both the House and the Senate the main dimension of agreement was defense. The issues identified cover a range of current political dimensions from security and foreign trade to economic regulation and tax policy. It is interesting to note that one of the dimensions of agreement corresponds to Congressional procedure, voting on the Journal. Since this is a non-controversial procedure expected to draw bipartisan support, we are encouraged to find it among the issues identified by our statistical procedure. It provides a pleasing reminder that our procedure correctly estimates the policy dimensions underlying structured agreements.

5 Discussion and Conclusion

Polarization in American politics today can be attributed, in part, to a dramatic change in Republican leadership's Congressional behavior. Over the 107th and 108th Congress, the number of Congressional Republicans who refused to engage in agreement across ideological lines increased tenfold, while Congressional Democrats' willingness to cross the partisan divide remained the same.

We uncover evidence that networks of agreement between the US Senate and the House differ significantly, providing systematic quantitative evidence for institutional theories of Congress that stress the importance of committees and rules in structuring cooperation. But contrary to established wisdom, we find that the Senate is a more partisan body than the House of Representatives. We find that ideology explains voting behavior in the Senate to a larger extent than in the House. This may be due to the fact that the number of dimensions of agreement is larger on average in the Senate than in the House. At the same time the Senate is characterized by frequent breakdowns in bipartisan agreements.

The network of agreements in the House of Representatives shows clear divisions into four clusters, two in each party; therefore, pointing to important intra-party differences.

Inter-party differences in the House are both systematic and important for understanding representation in the Congress. Moderate wings from the two parties form the core of bipartisan agreements in the House, while each party also has a separate cluster of representatives with more extreme voting behavior. In the extreme wings of both parties, we uncover evidence that identity, ideology and positions on redistribution play an important role in the legislators' behavior. On the Democratic side, strong advocates of labor unions and the first openly gay representative assume voting positions most distant from the center. On the Republican side, the opponents of the Voting Rights Act, the minimum wage and the estate tax show the strongest disagreement with the center. These findings suggest an important role of re-election incentives and legislators' policy preferences in their strategic behavior.

Looking across Congressional history, we find that agreement along major policy dimensions in the US House was most likely to occur during periods of crises, such as the First World War and the assassination of President Kennedy.

Remarkably, we find that the likelihood of agreement tends to decrease with an increase in the power of the Republican Party in the House of Representatives. But in the Senate, the dynamics of agreement networks exhibit major differences from the House. First, there is no discernable trend over time, and the influence of Republican Party power on polarization is not evident in the Senate. Instead, we find a cycle of high and low possibility of agreement, suggesting a dynamic of on-going propensity to agree, punctuated by periods of total breakdowns as a result of stronger strategic cartel behavior.

Finally, looking at the present-day Congress, we identify defense and national security issues to be the most common points of agreement across the main lines of polarization. However, once we move past defense and national security, we see that other issues, especially those related to budget appropriations, can also form very frequent bases for bi-partisan agreements.

A Appendix

Consider the graph $\Gamma = (V, E)$, defined as a set of vertices V and edges E, given by the observed patterns of agreement beyond polarization in Congress. The set of vertices is given by the set of individuals in our sample. We wish to display the network of agreements, while clustering by the number of agreements within a subset of the network such as ideologically close individuals. To achieve this aim, we use a version of the Fruchterman and Reingold (1991) algorithm, which chooses the positions of the vertices through simulation of an $n-$body mechanical system. On each vertex we impose a system of attraction and repulsion forces. The system of forces is artificial in the sense that it does not reflect any actual moving bodies physical properties but rather a simplified abstraction from physical laws.

Each vertex is given an initial location vector in the two dimension space ${v_i}^0 = ({x_i}^0, {y_i}^0)'$. The aim is to create correlation based clusters, while keeping the overall layout sufficiently spread out in order to easily distinguish the visual properties of the graph. We follow the numerical implementation of (Nooy, Mrvar, Batagelj, 2005).

The final location vector for each vertex ${v_i}^*$ is chosen so as to balance the attraction and repulsion forces between vertices. Since the acting forces will be functions of the Euclidean distance between vertices, the optimal location of each vertex will be determined by the optimal distance between vertices only. Thus, our clustering algorithm will be invariant to a number of transformations such as rotations and reflections.

We can write this clustering problem as a non-linear optimization problem over $n(n-1)/2$ distances between vertices. Let $\Delta_{\text{is}} = v_i - v_s$ be a difference vector, and $||\Delta_{\text{is}}||$ denotes the Euclidean distance between two location vectors, i.e. $||\Delta_{\text{is}}|| = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2}$.

Consider a re-arrangement of the locations of the vertices on the place such that the distance between two vertices i and j is given by k_{ij}. The location of vertex i can be written as

$$v_i = {v_i}^0 + \sum_{s \neq i} \frac{{\Delta^0}_{\text{is}}}{||{\Delta^0}_{\text{is}}||} k_{\text{is},} \tag{19}$$

where ${\Delta^0}_{\text{is}}$, and $||{\Delta^0}_{\text{is}}||$ are computed with respect to initial location vectors.

Similarly we can define Δ_{ij} and $||\Delta_{\text{ij}}||$ in terms of any location vectors of each vertex. That is,

$$\Delta_{\text{ij}} = v_i - v_j = \left({v_i}^0 - {v_j}^0\right) + \sum_{s \neq i} \frac{{\Delta^0}_{\text{is}}}{||{\Delta^0}_{\text{is}}||} k_{\text{is}} - \sum_{s \neq j} \frac{{\Delta^0}_{\text{js}}}{||{\Delta^0}_{\text{js}}||} k_{\text{js}}, \tag{20}$$

with $||\Delta_{\text{ij}}||$ the corresponding Euclidean distance. Each pair of vertices exert a repulsion force on each other given by:

$$f_r(v_i, v_j) = -\frac{(k_{\text{ij}})^2}{||\Delta_{\text{ij}}||}. \tag{21}$$

The clustering procedure thus involves the choice of optimal distances between vertices such as to stabilize the network by balancing the forces acting on each vertex:

$$(k_{\text{ij}})_{i,j=1..n} = \text{argmin} \sum_{i=1}^{n} \sum_{j \neq i} \left\{ \frac{(||\Delta_{\text{ij}}||)^2}{k_{\text{ij}}} (A)_{i,j} - \frac{(k_{\text{ij}})^2}{||\Delta_{\text{ij}}||} \right\}, s.t. \; k_{\text{ij}} = k_{\text{ji}} \text{ and } k_{\text{ii}} = 0. \tag{22}$$

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Table 1: Network characteristics

House of Representatives

Congress	Sample N	Sample T	No Vote			Unconnected		
			Democrats	Republicans	Other	Democrats	Republicans	Other
108	441	1218	6	7	0	8	33	0
107	444	990	5	11	0	0	38	0
106	440	1207	6	7	1	0	0	0
105	445	1166	12	7	0	0	2	0
104	446	1321	16	6	0	0	6	0
103	443	1094	8	5	0	8	5	0
102	442	901	6	6	0	0	0	0
101	443	879	9	5	0	0	0	0
100	442	939	13	5	0	3	5	0
99	440	890	10	5	0	0	0	0

Senate

Congress	Sample N	Sample T	No Vote			Unconnected		
			Democrats	Republicans	Other	Democrats	Republicans	Other
108	100	675	4	0	0	4	20	0
107	102	633	0	2	2	0	1	0
106	106	672	1	3	0	4	3	0
105	101	612	1	0	0	2	0	0
104	104	919	3	2	0	0	1	0
103	103	724	2	1	0	0	0	0
102	103	550	1	2	0	0	0	0
101	102	638	1	2	0	0	0	0
100	102	799	4	1	0	0	0	0
99	102	740	0	3	0	0	0	0

Table 2: Main agreers

House of Representatives

Congress	Main Agreers (Name, Party, State, Score) 1	2	3	4	5
108	Taylor (D-MS, 161)	Stenholm (D-TX, 133)	Davis (D-TN, 133)	John (D-LA, 132)	Cramer (D-AL, 117)
107	Taylor (D-MS, 176)	Costello (D-IL, 151)	Lobiondo (R-NJ, 140)	Peterson C (D-MN, 140)	Ramstad (R-MN, 132)
106	Ramstad (R-MN, 203)	Taylor (D-MS, 197)	Lobiondo (R-NJ, 195)	Shays (R-CT, 194)	Peterson C (D-MN, 186)
105	Shays (R-CT, 192)	Morella (R-MD, 165)	Ramstad (R-MN, 163)	Taylor (D-MS, 161)	Boehlert (D-NY, 152)
104	Shays (R-CT, 242)	Zimmer (R-NJ, 238)	Martini (R-NJ, 224)	Franks (R-NJ, 212)	Ramstad (R-MN, 201)
103	Shays (R-CT, 231)	Stenholm (D-TX, 229)	Geren (D-TX, 222)	Sarpalius (D-TX, 221)	Orton (D-UT, 215)
102	Shays (R-CT, 209)	Montgomery (D-MS, 207)	Clay (D-MO, 206)	Geren (D-TX, 201)	Green (R-NY, 194)
101	Schroeder (D-CO, 269)	Clay (D-MO, 249)	Sikorski (D-MN, 236)	Shays (R-CT, 228)	Jacobs (D-IN, 224)
100	Sikorski (D-MN, 244)	Schroeder (D-CO, 236)	Clay (D-MO, 230)	Montgomery (D-MS, 211)	Nichols (D-AL, 210)
99	Schroeder (D-CO, 245)	Leach (R-IA, 239)	Clay (D-MO, 236)	Jacobs (D-IN, 235)	Montgomery (D-MS, 209)

Senate

Congress	Main Agreers (Name, Party, State, Score) 1	2	3	4	5
108	Lincoln (D-AR, 29)	Nelson (D-NE, 27)	McCain (R-AZ, 25)	Breaux (D-LA, 24)	Pryor (D-AR, 23)
107	Bayh (D-IN, 30)	Feingold (D-WI, 28)	Breaux (D-LA, 28)	Chafee (R-RI, 27)	Specter (R-PA, 27)
106	Collins (R-ME, 38)	Snowe (R-ME, 37)	Smith (R-NH, 29)	Feingold (D-WI, 25)	Jeffords (R-VT, 25)
105	Stevens (R-AK, 37)	Ashcroft (R-MO, 34)	Collins (R-ME, 34)	Jeffords (R-VT, 33)	Chafee (R-RI, 33)
104	Snowe (R-ME, 46)	Cohen (R-ME, 45)	Chafee (R-RI, 41)	Jeffords (R-VT, 38)	Specter (R-PA, 37)
103	Chafee (R-RI, 51)	Kohl (D-WI, 44)	Breaux (D-LA, 44)	Bradley (D-NJ, 42)	Cohen (R-ME, 42)
102	Chafee (R-RI, 51)	Conrad (D-ND, 46)	Ford (D-KY, 46)	Cohen (R-ME, 43)	Nunn (D-GA, 43)
101	Exon (D-NE, 60)	Ford (D-KY, 55)	Pressler (R-SD, 52)	Shelby (D-AL, 52)	Conrad (D-ND, 51)
100	Heflin (D-AL, 65)	Evans (R-WA, 64)	Packwood (R-OR, 58)	Chafee (R-RI, 57)	Stafford (R-VT, 56)
99	Chafee (R-RI, 64)	Exon (D-NE, 60)	Heflin (D-AL, 60)	Stafford (R-VT, 59)	Boren (D-OK, 56)

Table 3: Most Influential Members

House of Representatives

Congress	Most Influential (Name, Party, State, Score)				
	1	2	3	4	5
108	John (D-LA, 1.1704)	Davis (D-TN, 1.1384)	Stenholm (D-TX, 1.1382)	Lucas (D-KY, 1.0949)	Turner (D-TX, 1.0570)
107	Costello (R-IL, 1.0701)	Taylor (D-MS, 1.0679)	Peterson C (R-MN, 0.9987)	Lobiondo (R-NJ, 0.9226)	Waters (D-CA, 0.8984)
106	Boehlert (R-NY, 0.7069)	Shays (R-CT, 0.7037)	Taylor (D-MS, 0.6942)	Porter (R-IL, 0.6901)	Ramstad (R-MN, 0.6799)
105	Shays (R-CT, 0.7940)	Morella (R-MD, 0.7610)	Boehlert (R-NY, 0.7257)	Taylor (D-MS, 0.7213)	Gilman (R-NY, 07131)
104	Shays (R-CT, 0.7451)	Skelton (D-MS, 0.7195)	Browder (D-AL, 0.7172)	Montgomery (D-MS, 0.7135)	Stenholm (D-TX, 0.7042)
103	Sarpalius (D-TX, 0.6737)	Stenholm (D-TX, 0.6737)	Skelton (D-MS, 0.6659)	Geren (D-TX, 0.6592)	Montgomery (D-MS, 0.6561)
102	Montgomery (D-MS, 0.6799)	Sarpalius (D-TX, 0.6611)	Skelton (D-MS, 0.6581)	Shays (R-CT, 0.6544)	Clay (D-MO, 0.6443)
101	Schroeder (D-CO, 0.6844)	Clay (D-MO, 0.6463)	Shays (R-CT, 0.6352)	Montgomery (D-MS, 0.6214)	Skelton (D-MS, 0.6107)
100	Sikorski (D-MN, 0.77168)	Clay (D-MO, 0.7678)	Schroeder (D-CO, 0.7620)	Montgomery (D-MS, 0.7341)	Nichols (D-AL, 0.7331
99	Leach (R-IA, 0.7164)	Schroeder (D-CO, 0.6999)	Jacobs (D-IN, 0.6975)	Clay (D-MO, 0.6854)	Montgomery (D-MS, 0.6853)

Senate

Congress	Most Influential (Name, Party, State, Score)				
	1	2	3	4	5
108	Lincoln (D-AR, 5.3455)	Nelson (D-NE, 4.9743)	Breaux (D-LA, 4.5358)	Pyor (D-AR, 4.4963)	McCain (R-AZ, 4.4073)
107	Cleland (D-GA, 3.5158)	Lincoln (D-AR, 3.4420)	Nelson (D-NE, 3.4347)	Breaux (D-LA, 2.3777)	Bayh (D-IN, 3.3644)
106	Collins (R-ME, 3.9313)	Snowe (R-ME, 3.8463)	Smith (R-NH, 3.2622)	Specter (R-PA, 2.9013)	Jeffords (R-VT, 2.8505)
105	Stevens (R-AK, 3.2677)	Ashcroft (R-MO, 3.1181)	Smith (R-NH, 2.9492)	Domenici (R-NM, 2.9351)	Chafee (R-RI, 2.9096)
104	Snowe (R-ME, 2.6680)	Chafee (R-RI, 2.4613)	Cohen (R-ME, 2.4495)	Breaux (D-LA, 2.3603)	Heflin (D-AL, 2.2881)
103	Chafee (R-RI, 2.3849)	Breaux (D-LA, 2.3382)	Jeffords (R-VT, 2.1089)	Bradley (D-NJ, 2.0334)	Byrd (D-WV, 2.0157)
102	Chafee (R-RI, 2.5144)	Ford (D-KY, 2.4212)	Heflin (D-AL, 2.2246)	Cohen (R-ME, 2.1723)	Jeffords (R-VT, 2.1227)
101	Exon (D-NE, 2.1614)	Ford (D-KY, 2.0253)	Conrad (D-ND, 1.9351)	Breaux (D-LA, 1.9265)	Shelby (D-AL, 1.8868)
100	Heflin (D-AL, 2.1123)	Evans (R-WA, 2.0330)	Packwood (R-OR, 1.9835)	Chafee (R-RI, 1.9517)	Stafford (R-VT, 1.9205)
99	Chafee (R-RI, 2.0955)	Heflin (D-AL, 1.9663)	Stafford (R-VT, 1.9636)	Exon (D-NE, 1.8932)	Zorinsky (D-NE, 1.7940)

Table 4: Spectral characteristics

House of Representatives

Congress	Largest Eig	Smallest Eig	Laplacian	Components	Dimensions
108	61.9951	-16.7206	0.0000	2	5
107	63.9586	-14.8647	0.3821	1	6
106	91.0087	-20.1034	0.5845	1	7
105	81.0255	-19.5185	0.9755	1	5
104	106.1047	-22.4682	0.9382	1	8
103	116.8246	-22.2490	0.5803	1	10
102	106.4908	-18.3581	0.9758	1	8
101	116.6840	-20.8638	1.9729	1	6
100	101.6936	-21.3536	0.9420	1	7
99	110.0984	-17.7410	0.8223	1	8

Senate

Congress	Largest Eig	Smallest Eig	Laplacian	Components	Dimensions
108	13.7531	-5.5543	0.2002	1	9
107	17.7520	-5.7742	0.6429	1	11
106	14.7243	-6.5910	0.7712	1	13
105	21.4933	-6.3567	0.8081	1	1
104	22.4434	-8.3075	0.9859	1	3
103	26.5328	-7.0288	1.9564	1	14
102	27.3796	-7.4972	0.9364	1	1
101	32.5600	-8.0035	4.8510	1	2
100	36.8880	-8.7914	5.7599	1	14
99	37.8586	-8.0111	6.6434	1	14

Table 5: Votes corresponding to main dimensions of agreement for the 108th Congress

Dimension	House	Senate
1	Defense Funding	Defense, Homeland Security
2	Foreign Operations, Security	Energy, Disabilities
3	Labor, Health and Safety	Tax policy, Foreign Operations
4	Military Constructions, Marriage Defense	Market regulation
5	Journal	Health care, Pensions
6	-	Confirmations
7	-	Agriculture
8	-	Foreign Relations, Intelligence
9	-	Free trade

Figure 1: Agreement in the House of Representatives
108th Congress

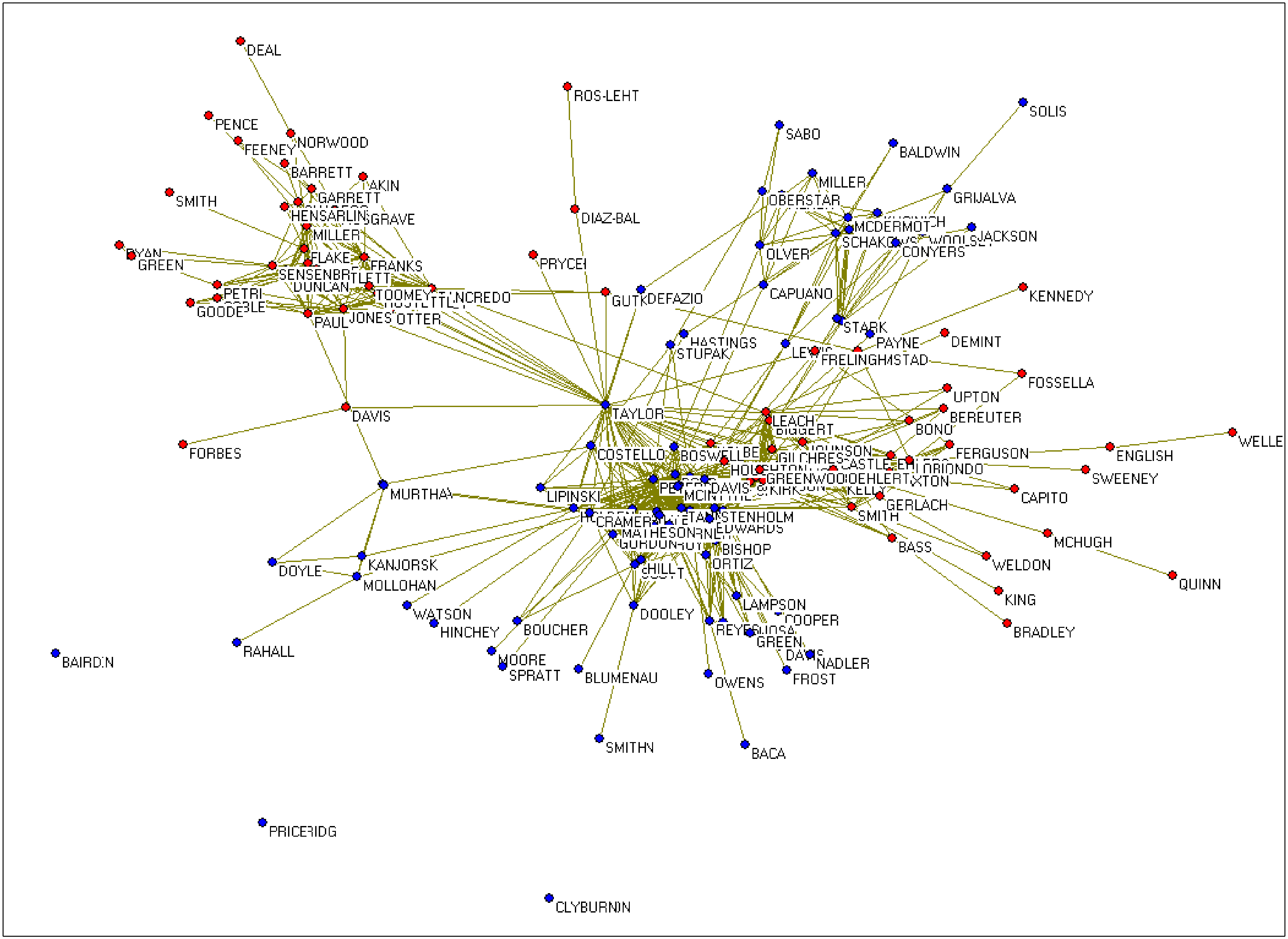


Figure 2: Agreement in the Senate
108th Congress

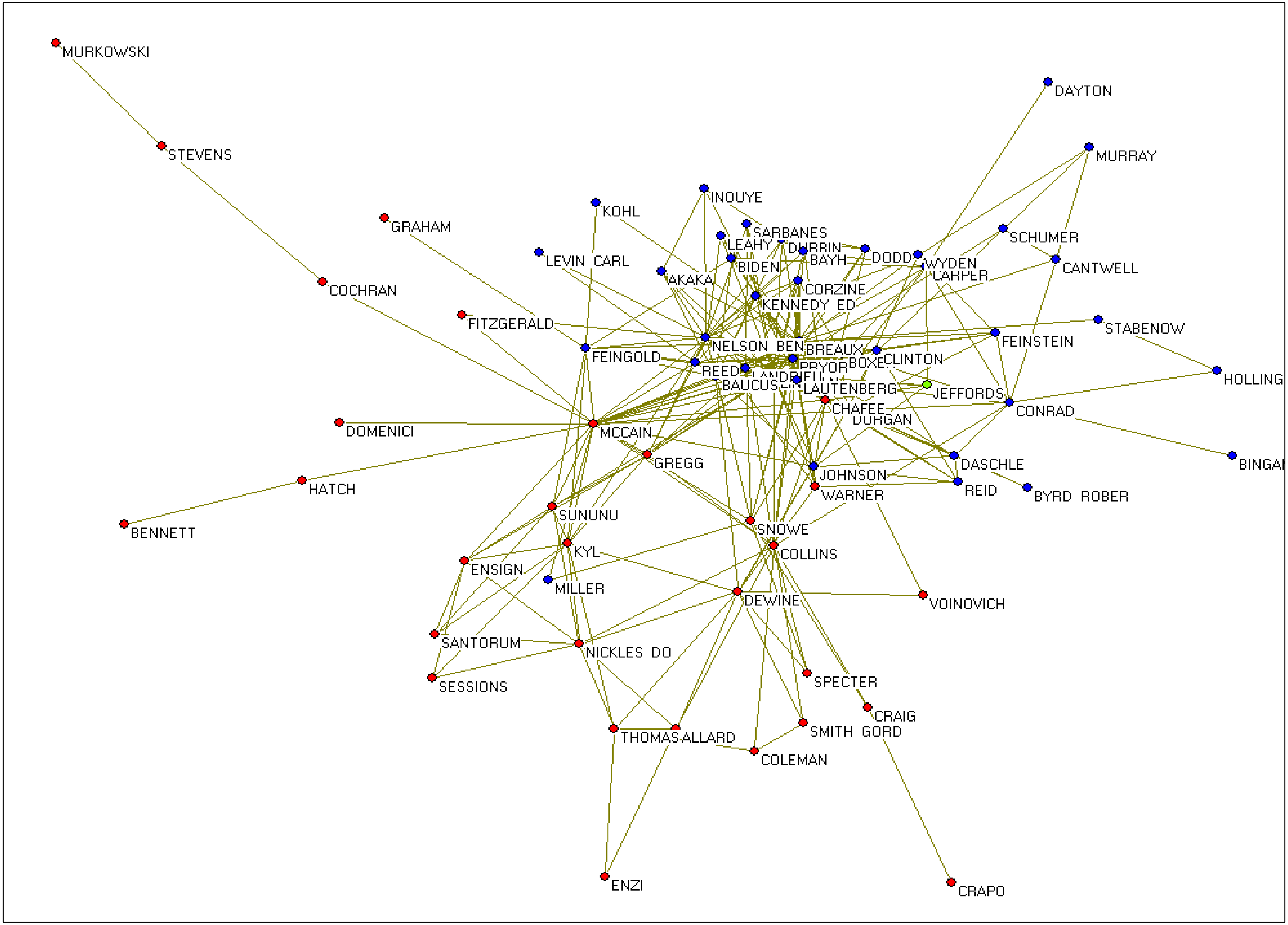


Figure 3: Probability of aggregate agreement

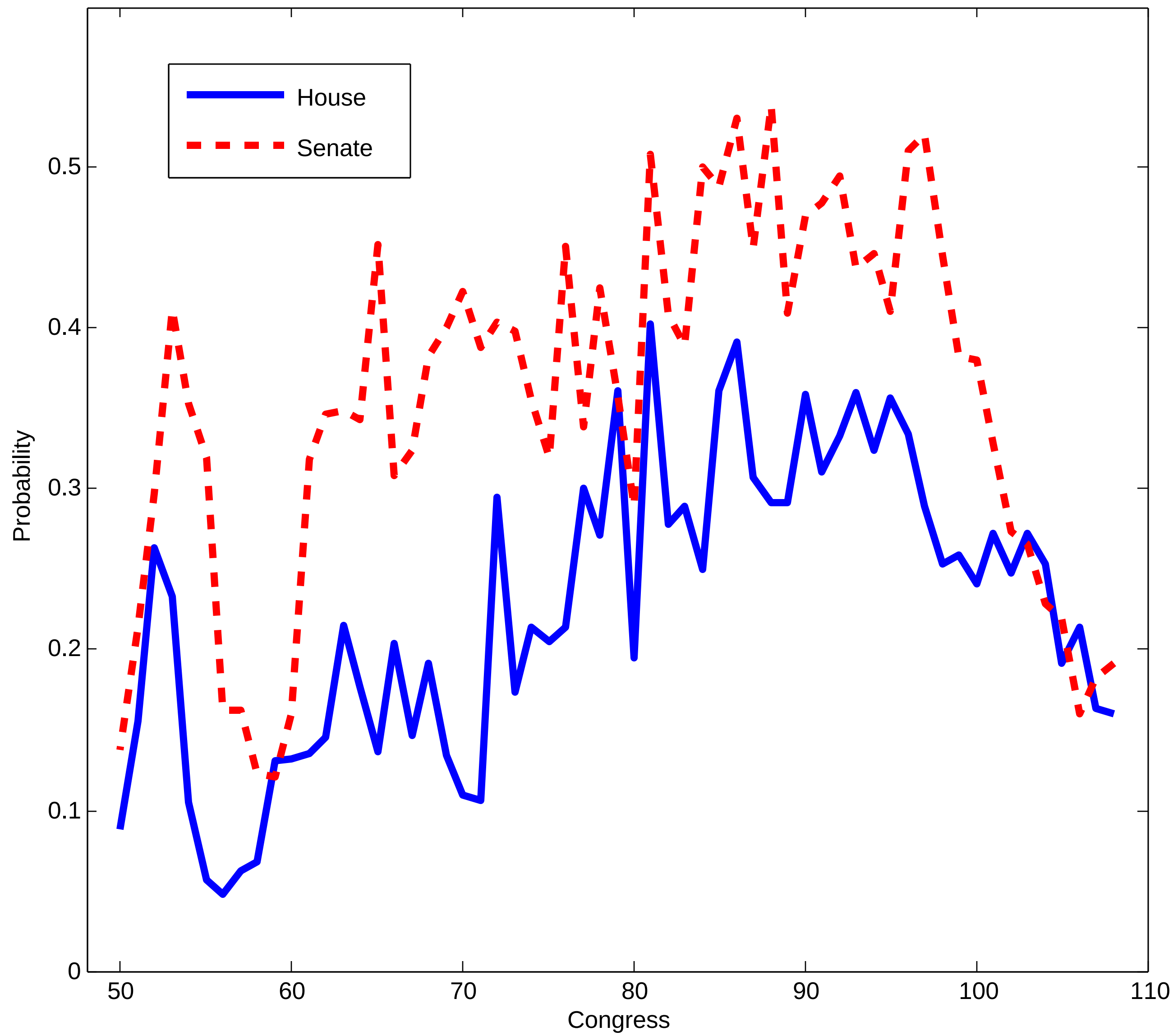


Figure 4: Ease of Implementing Bipartisan Agreements
(Laplacian Eigenvalue)

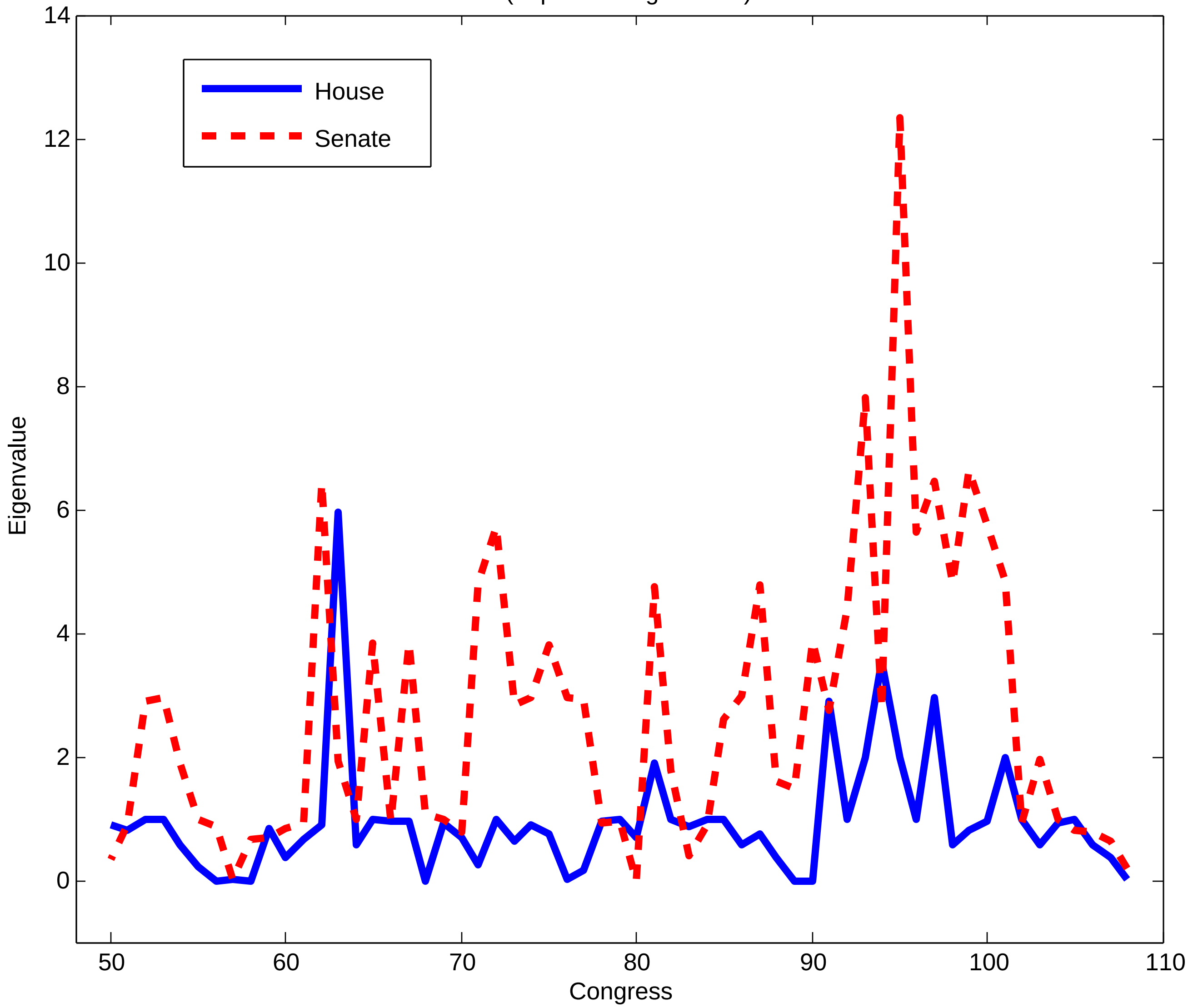


Figure 5(a): Structured Agreement (House)

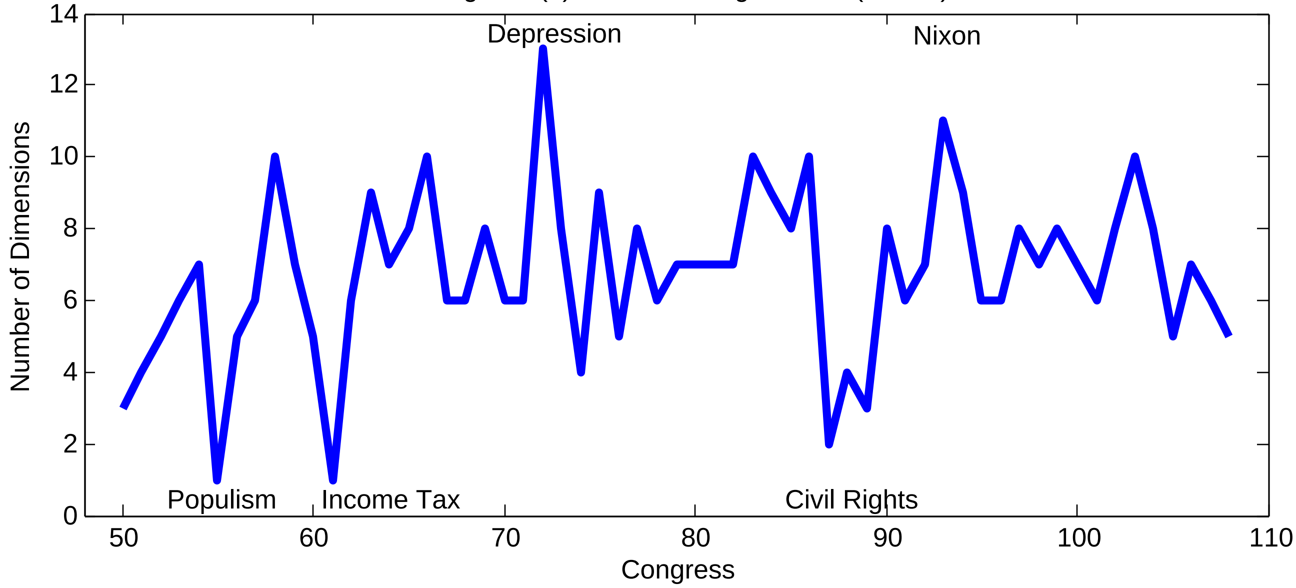
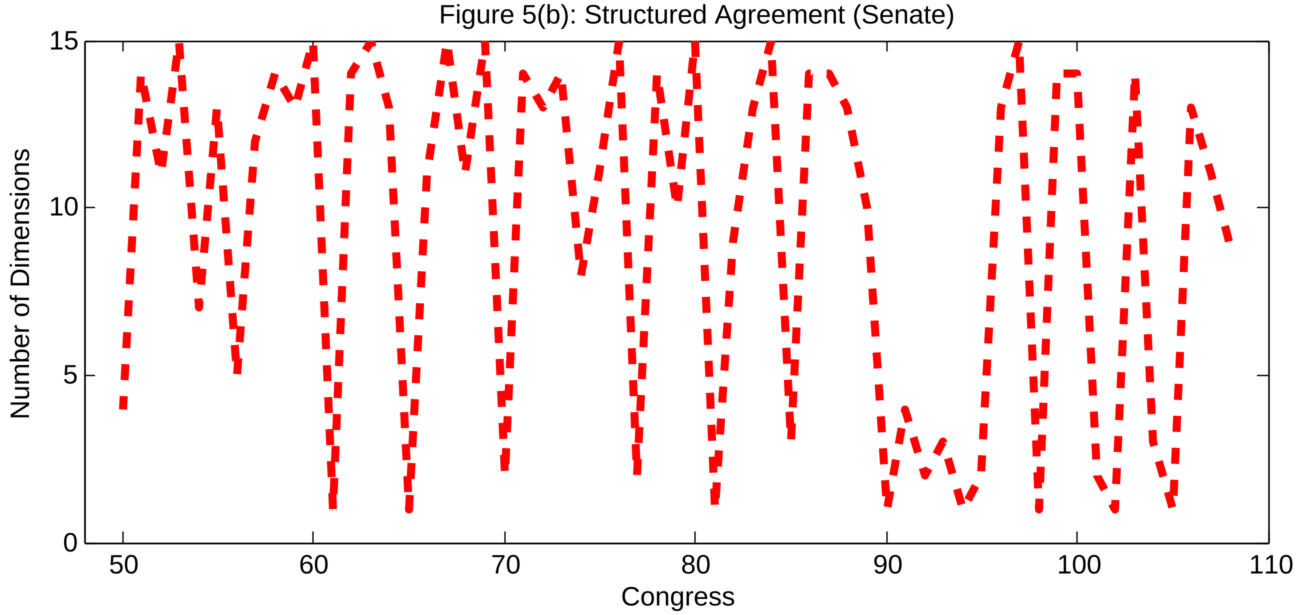


Figure 5(b): Structured Agreement (Senate)



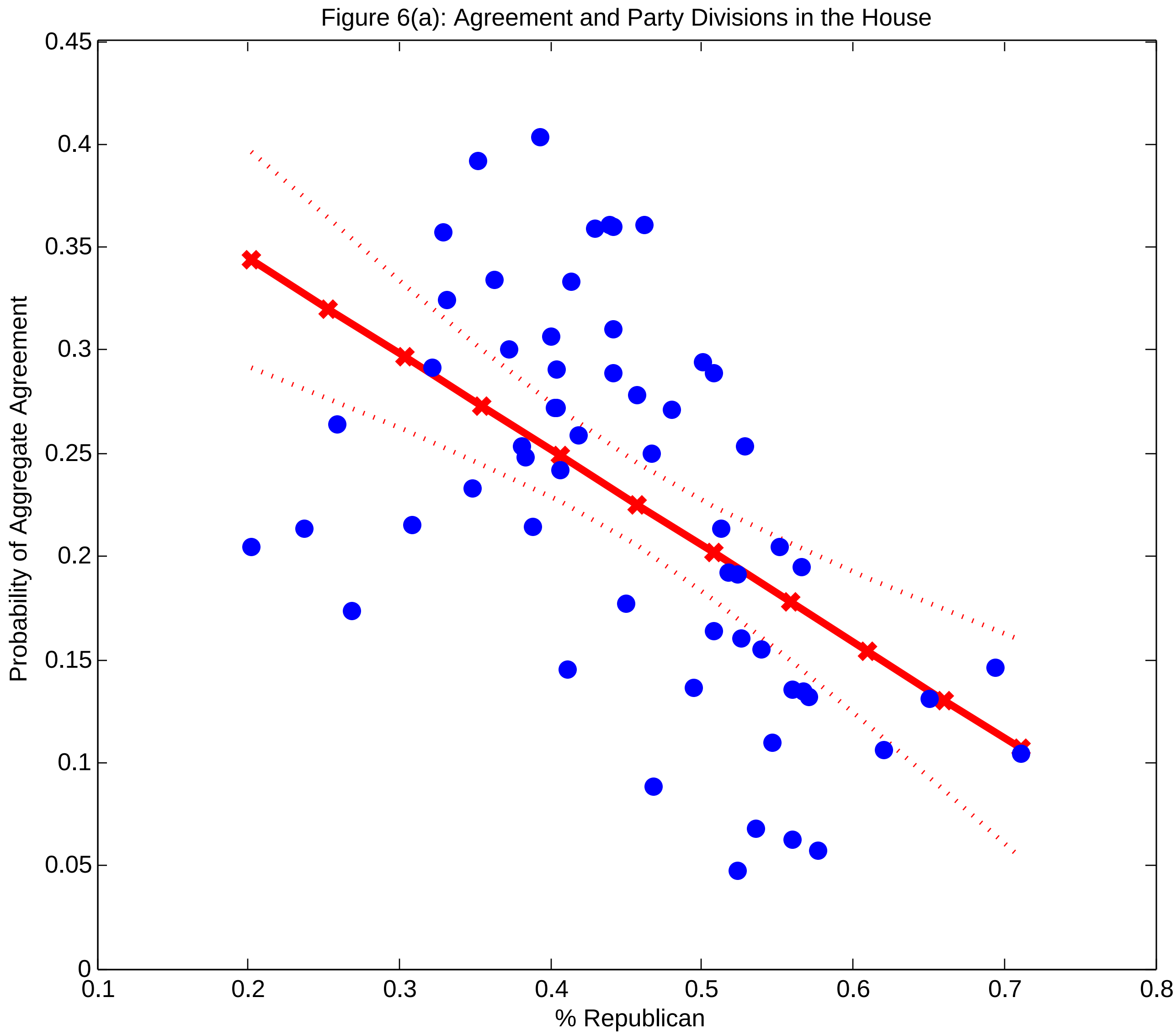


Figure 6(a): Agreement and Party Divisions in the House
Probability of Aggregate Agreement
% Republican
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

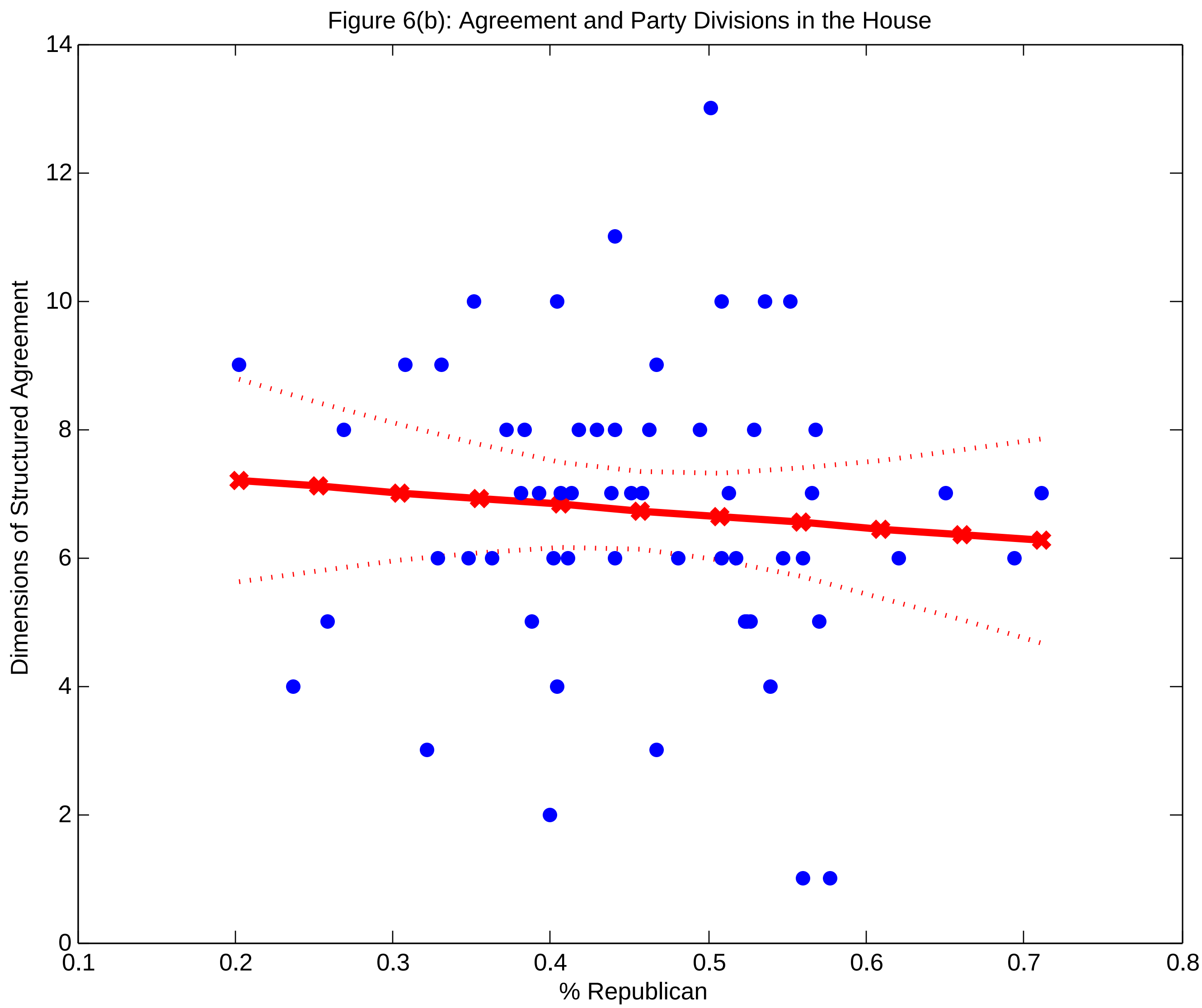
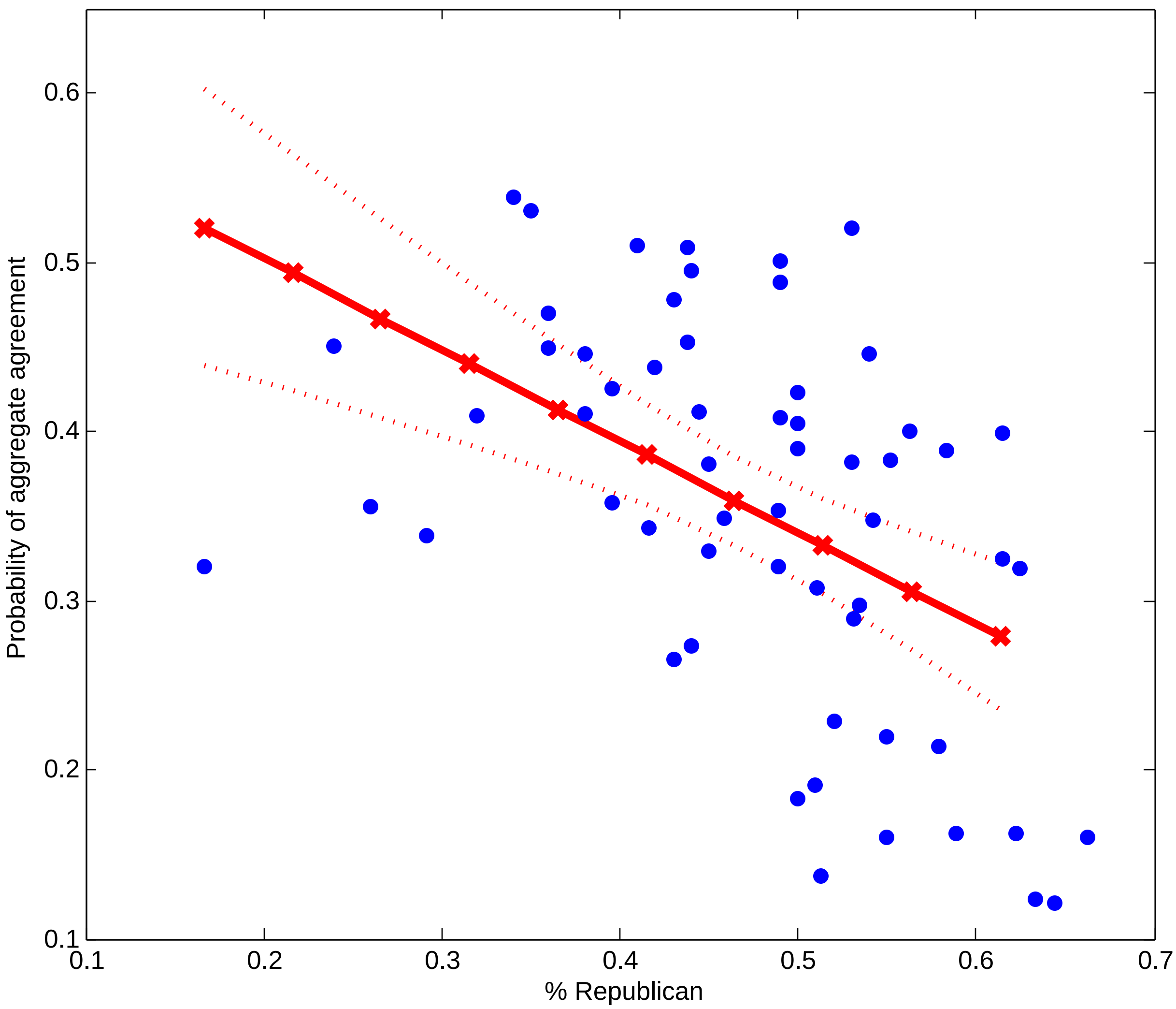


Figure 6(b): Agreement and Party Divisions in the House
Dimensions of Structured Agreement
% Republican
0
2
4
6
8
10
12
14
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Figure 6(c): Agreement and Party Divisions in the Senate



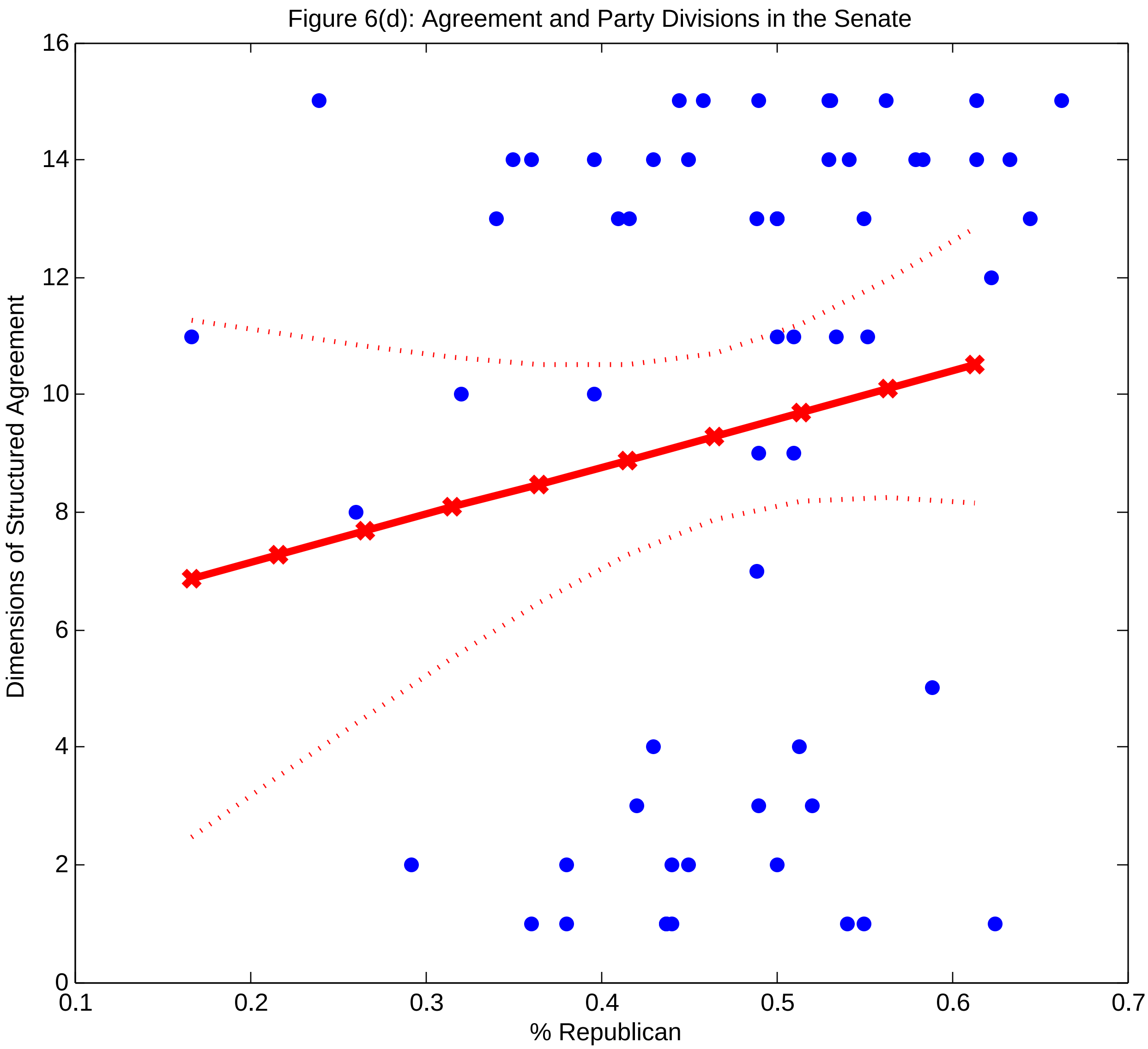


Figure 6(d): Agreement and Party Divisions in the Senate
Dimensions of Structured Agreement
% Republican
0
2
4
6
8
10
12
14
16
0.1
0.2
0.3
0.4
0.5
0.6
0.7