LEARNING ABOUT THE INTERDEPENDENCE BETWEEN THE MACROECONOMY AND THE STOCK MARKET

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Abstract. How strong is the interdependence between the macroeconomy and the stock market?

This paper estimates a New Keynesian general equilibrium model, which is extended to include a wealth effect from asset price fluctuations to consumption, to assess the quantitative importance of interactions among the stock market, macroeconomic variables, and monetary policy.

The paper relaxes the assumption of rational expectations and assumes that economic agents learn over time and form near-rational expectations from their perceived model of the economy. The stock market, therefore, affects the economy through two channels: through a traditional “wealth effect” and through its impact on agents’ expectations. Monetary policy decisions also affect and are potentially affected by the stock market.

The empirical results show that the direct wealth effect is modest, but asset price fluctuations have important effects on future output expectations. Through this expectational channel, shocks in the stock market can account for a large, but varying, portion of output fluctuations.

Keywords: New Keynesian Model, Stock Market, Wealth Channel, Monetary Policy, Constant-Gain Learning, Bayesian Estimation, Non-Fully-Rational Expectations.

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1. Introduction

How strong are the links between the macroeconomy and the stock market?

The New Keynesian models that are widely employed to characterize the joint dynamics of output, inflation, and monetary policy choices, have, for a long time, ignored the stock market altogether.\(^1\)

It is well known, however, that asset prices can influence the economy through a variety of channels. Policy discussions emphasize, in particular, the impact that asset price fluctuations can have on consumption spending decisions: this is the so-called “wealth effect”. Monetary policymakers may consider actively responding to asset prices if the wealth channel is sizable. But the size of the effect is still controversial: although most regressions that have been estimated in the literature show a positive and significant causal effect of wealth on consumption,\(^2\) recent studies, which exploit cointegrating relationships or panel data sets, conclude that the effect is smaller than previously thought (Lettau and Ludvigson, 2004, Case et al., 2005).\(^3\)

Another central area of interdependence involves the link between asset prices and monetary policy decisions. Researchers have been interested in understanding both whether monetary policy responds to asset price fluctuations and how strongly the latter are affected by policy shocks (e.g., Rigobon and Sack, 2003, 2004, Bernanke and Kuttner, 2005, Biørnland and Leitemo, 2009) or other macroeconomic fundamentals (e.g., Chen et al., 1986).

This paper adopts a structural New Keynesian model, which will be estimated on U.S. data, to infer the strength of the interdependence among macroeconomic variables, monetary policy, and the stock market. The model, which is based on a Blanchard-Yaari overlapping generations framework, includes a wealth effect from asset prices to consumption, whose magnitude depends on the length of the households’ planning horizon. Current output is affected by expectations of future output, real interest rates, and by current financial wealth, which is influenced by swings in stock prices. Current stock values depend on their

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\(^1\)Notable exceptions are models with a “financial accelerator”, as in Bernanke et al. (1999). Interest in the stock market and financial variables has expectedly surged after the 2007-2008 Financial Crisis.

\(^2\)Research goes back to Ando and Modigliani (1963); Poterba (2000) and Davis and Palumbo (2001) offer recent surveys.

\(^3\)Additionally, asset prices can affect real activity through other channels, such as through a Tobin’s Q effect on investment and through a balance sheet/credit channel effect. This paper will focus only on the wealth channel, while the Tobin’s Q and balance sheet effects will be, instead, omitted from the analysis.
own expected future values, on expectations about future real activity, and on the ex ante real interest rate.

In modeling the expectations formation, the paper relaxes the traditional informational assumptions imposed by rational expectations and it assumes that agents form subjective – near-rational – expectations and that they attempt to learn the model parameters over time.

Some critics of the conventional wealth channel effect have argued that changes in stock wealth mainly affect consumption through changes in expectations and consumer confidence, but no direct wealth effect exists.\(^4\) This paper includes both effects: a direct wealth effect of asset prices on consumption and output, and an effect of asset prices on future expectations.

The estimation tries to empirically disentangle the two effects.

The model is estimated using Bayesian methods on monthly U.S. data. The constant gain coefficient is jointly estimated with the structural parameters of the economy, so that the learning process can be extracted from the data, rather than arbitrarily imposed.

1.1. Results. The empirical evidence suggests a small direct wealth effect of stock prices on output. Fluctuations in the stock market, however, affect economic agents’ expectations of future real activity. The effect has considerably varied over the sample: in the first half, economic agents believed changes in the stock market to have a strong effect on output, while they revised their beliefs downward in the second part of the sample.

Through such effect on expectations, therefore, the stock market plays an important role for macroeconomic variables. In the 1960s-1970s, a sizable proportion of fluctuations in the output gap were explained by shocks that originated in the stock market; the stock market also acted to amplify the transmission of monetary policy shocks. The importance of stock market shocks has, however, declined over the sample: in the 1990s-2000s, they typically accounted for less than 20% of the variability in output. Fluctuations in the stock price gap were mainly driven by its own innovations until the 1970s, but they have been increasingly affected by demand shocks afterwards. Monetary policy shocks account for at most 10% of fluctuations in the stock market and their effect has also changed over time.

\(^4\)Examples are Hymans (1970), who argues that stock market wealth has small effects on consumption after accounting for changes in consumer confidence, Otoo (1999), who shows that the relation between stock prices and consumer confidence is counterfactually similar between stock owners and non-owners, and Jansen and Nahuis (2003), who find that the short-run impact of changes in the stock market depends on their effect on perceptions about future real activity, rather than personal finances, as would be expected under a traditional wealth channel.
The data show that the estimated response of monetary policy to the stock price gap has been positive if computed over the full sample. But monetary policy has reacted less to the stock market in the post-1984 sample. Moreover, post-1984 policy has responded to the stock market only to the extent that it affected output and inflation forecasts: when those forecasts are included in the policy rule, the estimated reaction to stock prices drops to zero.

1.2. Related Literature. This paper aims to contribute to the literature on the interaction between the stock market and macroeconomic variables. Their linkages have interested researchers for a long time (e.g., Fischer and Merton, 1985, for a general discussion, Blanchard, 1981, for an early theoretical analysis), but empirical analyses in a general equilibrium setting have been rare. The paper's main objective, therefore, is to offer quantitative estimates about the role of such linkages using a theory-based general equilibrium model.

The paper is related to the studies that seek to estimate the wealth effect (e.g., Poterba, 2000, Davis and Palumbo, 2001, Lettau and Ludvigson, 2004), typically using single-equation regressions or error-correction models, and to those that analyze the interaction between asset prices and monetary policy from a positive or normative perspective (e.g., Rigobon and Sack, 2003, 2004, Bjørnland and Leitemo, 2009, Bernanke and Gertler, 1999, 2001, Cecchetti et al., 2000, and Gilchrist and Leahy, 2002). This paper provides estimates of the wealth effect in a structural model, which permits to control for general equilibrium effects, and it reveals a quantitative important channel through which asset prices affect the economy and that operates through expectations. The paper also adds to the evidence on the interrelationship between monetary policy and asset prices, by showing that both monetary policy has reacted to stock prices – but, after Volcker, not beyond their role as leading indicators – and that stock prices are affected by policy shocks, and that both responses have varied over time.

The paper is also related to the countless empirical studies that adopt the New Keynesian model, as it hints that the typical omission of stock market variables may represent an important misspecification of the model, to the empirical studies that replace rational expectations with adaptive learning (e.g., Adam, 2005, Milani 2006, 2007, 2008, Orphanides and Williams, 2003), and to the studies that illustrate how learning can help in explaining asset price dynamics (e.g., Timmermann, 1993, Guidolin and Timmermann, 2007, Carceles-Poveda and Giannitsarou, 2008, Adam et al., 2007, and Branch and Evans, 2010, 2011).
The paper is more closely related to the recent papers by Castelnuovo and Nisticó (2010), Airaudo et al. (2015), and Challe and Giannitsarou (2014). In particular, the work by Castelnuovo and Nisticó (2010) shares many of the same objectives. They estimate a similar general equilibrium model, but maintaining the assumption of rational expectations. They find a somewhat larger wealth effect. Adding learning, however, makes it possible to separate between the traditional wealth effect and the effect due to changes in expectations (the two effects can be quite different under non-fully-rational expectations); moreover, this paper shows how the interaction between financial markets and the macroeconomy has evolved over time, while their rational expectations framework treat it as constant over the 1954-2007 period. Challe and Giannitsarou (2014), instead, exploit the implications of a New Keynesian model for stock-price dynamics to investigate whether the model can match the qualitative and quantitative evidence on the effects of monetary policy shocks on stock prices. Airaudo et al. (2015) consider a similar model to the one used in this paper and focus on deriving determinacy and E-stability conditions.

2. A Model with Wealth Effects

2.1. Households. The model follows Nisticó (2012), who extends Blanchard (1985) and Yaari (1965)’s perpetual-youth setting to include risky equities and adapts it to a New Keynesian framework.\(^5\) While a detailed step-by-step derivation can be found in Nisticó (2012), this section presents a sketch of the main features of the model.

An indefinite number of cohorts populates the economy. Each cohort may survive in any period with probability \((1 - \gamma)\), which may be more generally interpreted as the probability of remaining active in the market;\(^6\) each cohort is assumed to have fixed size \(\gamma\).

Each household of age \(j\) maximizes the lifetime utility at time 0

\[
E_0 \sum_{t=0}^{\infty} \beta^t (1 - \gamma)^t [\zeta_t \log(C_{j,t}) + \log(1 - N_{j,t})]
\]

\[\text{(2.1)}\]

\(^5\)A similar framework has also been used to introduce equity prices in otherwise standard New Keynesian models by Airaudo et al. (2015), Di Giorgio and Nisticó (2007), Milani (2011a), and Castelnuovo and Nisticó (2010). Dai and Spyromitros (2012) use the same framework as this paper to study optimal monetary policy under model uncertainty when asset prices matter. Other recent applications of the perpetual youth model can be found in Benassy (2007), Piergallini (2004), and Smets and Wouters (2002).

\(^6\)Therefore, \(1/\gamma\) can be interpreted as the households’ time horizon when taking consumption and financial decisions. The size of the cohort remains fixed, since by assumption a fraction \(\gamma\) of the total population is born and dies every period.
where $C_{j,t}$ denotes an index of consumption goods, $N_{j,t}$ indicates hours worked, and $\zeta_t$ is an aggregate preference shock. Consumers discount utility at the rate $0 \leq \beta \leq 1$, which denotes the usual intertemporal discount factor, and $1 - \gamma$, where $0 \leq \gamma \leq 1$, to account for their limited lifespan. Consumers can invest in two types of financial assets: bonds and equity shares, which are issued by monopolistically-competitive firms, to which they also supply labor.\footnote{The economy is “cashless” as in Woodford (2003).} Their portfolio, therefore, consists of a set of state-contingent assets with payoff $B_{j,t+1}$ in $t + 1$, which they discount using the stochastic discount factor $F_{t,t+1}$, and equity shares $Z_{j,t+1}(i)$ issued by firm $i$ at the real price $Q_t(i)$ and on which they receive dividends $D_t(i)$. Consumers maximize (2.1), subject to a sequence of budget constraints

$$P_tC_{j,t} + E_t F_{t,t+1}^j B_{j,t+1} + P_t \int_0^1 Q_t(i) Z_{j,t+1}(i) di \leq W_t N_{j,t} - P_t T_{j,t} + \Omega_{j,t},$$

(2.2)

where $P_t$ is the aggregate price level, $(W_t N_{j,t} - P_t T_{j,t})$ is net labor income, financial wealth $\Omega_{j,t}$ is given by

$$\Omega_{j,t} \equiv \frac{1}{1 - \gamma} \left[ B_{j,t} + P_t \int_0^1 (Q_t(i) + D_t(i)) Z_{j,t}(i) di \right],$$

(2.3)

and subject to a No-Ponzi-game condition

$$\lim_{k \to \infty} E_t \{ F_{t,t+k}^j (1 - \gamma)^k \Omega_{j,t+k} \} = 0.$$

(2.4)

Financial wealth $\Omega_{j,t}$ not only includes the portfolio of contingent claims and equity shares, but also, following Blanchard (1985), the return on the insurance contract that redistributes among surviving cohorts the financial wealth of those that have exited the market.\footnote{This is why financial wealth is multiplied by $\frac{1}{1 - \gamma}$.}

The intra and inter-temporal optimality conditions are given by

$$C_{j,t} = \frac{W_t}{P_t} (1 - N_{j,t}) \zeta_t,$$

(2.5)

$$F_{t,t+1} = \beta \frac{U_c(C_{j,t+1}) \zeta_{t+1}}{U_c(C_{j,t}) \zeta_t},$$

(2.6)

$$P_t Q_t(i) = E_t \{ F_{t,t+1} P_{t+1} [Q_{t+1}(i) + D_{t+1}(i)] \}.$$

(2.7)

Equation (2.5) governs the trade-off between consumption and leisure. Using equation (2.6), taking expectations, and using the no-arbitrage condition $E_t F_{t,t+1} = (1 + i_t)^{-1}$, where $i_t$ is the nominal return on a risk-free bond, yields the familiar stochastic Euler equation

$$(1 + i_t) \beta E_t \left[ \frac{P_t C_{j,t} \zeta_{t+1}}{P_{t+1} C_{j,t+1} \zeta_t} \right] = 1.$$

(2.8)
Equation (2.7), instead, defines the nominal price of an equity share of the \( i \)-th firm as the discounted value of the future expected payoff. By iterating forward the budget constraint (2.2) and using (2.4), (2.7), and (2.8), nominal individual consumption can be expressed as a function of total financial (\( \Omega \)) and labor market wealth (\( H \))

\[
P_{t}C_{j,t} = \Sigma_{t}^{-1}(\Omega_{j,t} + H_{j,t}),
\]

where \( H_{j,t} \equiv E_{t}\{\sum_{k=0}^{\infty} F_{t,t+k}(1 - \gamma)^{k}(W_{t+k}N_{j,t+k} - P_{t+k}T_{j,t+k})\} \) and \( \Sigma_{t}^{-1} \) denotes the marginal propensity to consume out of financial and non-financial wealth, which is constant across cohorts, but which can vary over time depending on the preference shock.

**Aggregation.** Individual variables are aggregated by computing the corresponding weighted average in the generation, using the cohort sizes as weights. That is, the aggregate value \( X_{t} \) for each cohort-specific variable \( X_{j,t}, X = C, N, B, T, H, Z(i), F \), is found as

\[
X_{t} \equiv \sum_{j=-\infty}^{t} n_{j,t}X_{j,t} = \sum_{j=-\infty}^{t} \gamma(1 - \gamma)^{t-j}X_{j,t}.
\]

Since the equilibrium conditions are linear with respect to the cohort-specific variables, they maintain the same structural form when expressed in terms of the corresponding aggregate variable.

2.2. **Firms.** There is a continuum of monopolistically-competitive firms in the economy, indexed by \( i \), which produce differentiated goods and set prices à la Calvo: only a fraction \( 0 < 1 - \alpha < 1 \) of firms are allowed to set an optimal price in a given period. Firm \( i \) is a monopolistic supplier of good \( i \), which is produced according to the production technology \( y_{t}(i) = A_{t}N_{t}(i) \), where \( A_{t} \) is an exogenous aggregate technology shock and \( N_{t}(i) \equiv \sum_{j=-\infty}^{t} \gamma(1 - \gamma)^{t-j}N_{j,t}(i) \) is labor input, aggregated across cohorts. Firms face a common demand curve \( y_{t}(i) = Y_{t}\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\theta} \) for their product, where \( Y_{t} \) is the aggregate output, given by \( Y_{t} = \left[\int_{0}^{1} y_{t}(i)\frac{\theta-1}{\theta} di\right]^{\theta-1} \), \( P_{t} = \left[\int_{0}^{1} P_{t}(i)^{1-\theta} di\right]^{1/(1-\theta)} \), and \( \theta > 1 \) is the elasticity of substitution among differentiated goods. Aggregating across firms, we have \( A_{t}N_{t} = Y_{t} \int_{0}^{1} \left(\frac{P_{t}(i)}{P_{t}}\right)^{-\theta} di \), where \( N_{t} \equiv \int_{0}^{1} N_{t}(i)di \).

Each firm faces the same decision problem and, if allowed to re-optimize, sets the same price \( P_{t}^{*}(i) \) to maximize the expected present discounted value of future profits

\[
E_{t}\left\{\sum_{t=0}^{\infty} \alpha^{k} F_{t,t+k} \left[\Pi_{t+k}(P_{t}^{*}(i))\right]\right\},
\]

(2.11)
subject to the demand curve for its product, where \( \Pi_{t+k}(\cdot) \equiv Y_{t+k}(i) (P_t(i) - P_{t+k}MC_{t+k}) \) denotes firm’s nominal profits in period \( t+k \), and \( MC_t = \frac{w_t}{A_tP_t} \) are real marginal costs.

The first-order condition (here in loglinearized form) implies that firms choose the common price

\[
p_t^*(i) = \left(1 - \alpha \tilde{\beta}\right) \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\alpha \tilde{\beta})^k (mc_{t+k} + p_{t+k}) \right\}, \tag{2.12}
\]

where \( \tilde{\beta} \equiv \frac{\beta}{1+\psi} \).

2.3. Equilibrium. It is assumed that the government simply aims to maintain a balanced budget, by using lump-sum taxation \( T_t \) to finance government expenditures \( G_t \), which are assumed to be a constant fraction of aggregate income \( G_t = \omega Y_t \). Fiscal policy is, therefore, Ricardian and does not affect the main relationships of the economy. The net supply of bonds is equal to zero in equilibrium, i.e. \( B_t = 0 \).

The total amount of issued shares by firms is normalized to equal 1, i.e. \( \int Z_t(i) = 1 \).

Total real dividends and the aggregate real stock price index are defined by integrating over the continuum of firms as

\[
D_t \equiv \int_0^1 D_t(i) di, \quad Q_t \equiv \int_0^1 Q_t(i) di. \tag{2.13}
\]

The aggregate demand side of the economy in equilibrium is, therefore, characterized by the following relations

\[
Y_t = C_t + \omega Y_t, \tag{2.14}
\]

\[
P_tY_t = N_t W_t + P_t D_t, \tag{2.15}
\]

\[
C_t = \frac{W_t}{P_t}(1 - N_t) \zeta_t, \tag{2.16}
\]

\[
(\Sigma_t - 1)C_t = \gamma Q_t + (1 - \gamma) \mathbb{E}_t \left\{ F_{t,t+1} \Pi_{t+1} \Sigma_{t+1} C_{t+1} \right\}, \tag{2.17}
\]

\[
Q_t = \mathbb{E}_t \left\{ F_{t,t+1} \Pi_{t+1} [Q_{t+1} + D_{t+1}] \right\}. \tag{2.18}
\]

In particular, the aggregate Euler equation (2.17) makes clear how aggregate consumption is affected by fluctuations in the stock price \( Q_t \), which itself evolves according to (2.18). The effect of stock wealth on consumption is a positive function of \( \gamma \), the probability of exiting the market parameter.

2.4. Aggregate Dynamics. After some additional algebra (see Nisticó, 2012), log-linearization of the model’s first-order conditions around a zero-inflation steady state gives the following
equations, which summarize the aggregate dynamics of the economy:

\[
x_t = \frac{1}{1+\psi} \tilde{E}_t x_{t+1} + \frac{\psi}{1+\psi} s_t - \frac{1}{1+\psi} (i_t - \tilde{E}_t \pi_{t+1} - r^n_t) \tag{2.19}
\]

\[
s_t = \tilde{\beta} \hat{E}_t s_{t+1} + \lambda \hat{E}_t x_{t+1} - (i_t - \tilde{E}_t \pi_{t+1} - r^n_t) + e_t \tag{2.20}
\]

\[
\pi_t = \tilde{\beta} \hat{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{2.21}
\]

\[
i_t = \rho i_{t-1} + (1-\rho) [r^n_t + (1+\chi) \pi_{t-1} + \chi x_{t-1} + \chi s_{t-1}] + \varepsilon_t \tag{2.22}
\]

where \(x_t\) denotes the output gap, \(s_t\) denotes the real stock price gap, \(\pi_t\) denotes inflation, \(i_t\) denotes the short-term nominal interest rate, and \(\tilde{E}_t\) stands for subjective near-rational expectations.\(^9\) Four disturbances affect the economy: \(r^n_t\) denotes the natural rate of interest, \(e_t\) is a shock that originates in the stock market and that can be rationalized as an equity premium shock (as done in Nisticó, 2012) or can account for fluctuations in asset prices that are not linked to fundamentals (e.g. bubbles, “irrational exuberance”, fads, etc.),\(^10\) \(u_t\) is a cost-push shock, and \(\varepsilon_t\) is a monetary policy shock. The disturbances \(r^n_t, e_t,\) and \(u_t\) follow AR(1) processes, while \(\varepsilon_t\) is assumed to be i.i.d.

Equation (2.19) represents the log-linearized intertemporal Euler equation that derives from the households’ optimal choice of consumption. As in the standard optimizing IS equation in the New Keynesian model, the output gap depends on the expected one-period-ahead output gap and on the ex-ante real interest rate. The novelty in the model is the inclusion of a wealth channel, i.e. a direct effect of stock price fluctuations on the output gap, which depends on the size of the reduced-form coefficient \(\psi\). The coefficient \(\psi\) is a combination of structural parameters, \(\psi \equiv \gamma \frac{1-\beta (1-\gamma)}{(1-\gamma)} \frac{\Omega}{PC}\), where \(\frac{\Omega}{PC}\) denotes the steady-state real financial wealth to consumption ratio. The magnitude of \(\psi\) and hence the magnitude of the wealth effect positively depends on the structural parameter \(\gamma\), which as seen in (2.1), denotes the span of the agents’ planning horizon. A high survival probability – or equivalently

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\(^9\)The output gap is given by the deviation of total output \(Y_t\) from \(Y^n_t\), the natural level of output, i.e. the equilibrium level of output under flexible prices. Similarly the real stock price gap is defined as \(s_t \equiv q_t - q^n_t\), where \(q_t\) is the real stock price and \(q^n_t\) is the corresponding flexible-price equilibrium level.

\(^10\)As customary in the adaptive learning literature, near-rational expectations are assumed starting from the same log-linearized conditions that would be obtained under rational expectations. For the conditions under which this is justifiable, see Honkapohja, Mitra, and Evans (2003). Preston (2005) presents an alternative approach in which learning enters from the primitive assumptions of the model and he shows that long-horizon expectations also turn out to matter. Analyzing the model under Preston’s approach, however, is beyond the scope of this paper.

\(^11\)Learning may potentially generate endogenous bubbles in the model (e.g., Branch and Evans, 2011); the disturbance term \(e_t\), however, captures exogenous bubbles that are not rationalized by such learning dynamics. Unmodeled changes in the stock market risk premium will also end up in \(e_t\).
a long planning horizon (i.e. a low $\gamma$) – implies a weaker wealth effect. Also, a shorter planning horizon reduces the degree of consumption smoothing and the responsiveness of consumption to the real interest rate.

The stock-price dynamics is characterized by equation (2.20). Stock prices are forward-looking: the stock price gap depends on its own one-period ahead expectations, on expectations about future output gap, on the ex-ante real interest rate, and on the stock market shock. In Nisticó (2012) and Airaudo et al. (2015), the composite coefficient $\lambda \equiv \left(1 + \frac{1 + \phi}{\mu - 1} - 1\right)$ depends on the steady-state markup $\mu = \frac{\theta}{\theta - 1}$ and on the inverse of the Frisch elasticity of labor supply $\phi$, and it enters the stock price equation with a negative sign: expectations of future expansions imply lower stock prices. This might be seen as contrary to what commonly thought and hinges on the assumption of a flexible labor market (which in the model would generate countercyclical profits and dividends, which is at odds with the evidence). In this paper, I assume marginal costs that can deviate from the value implied by the flexible labor market assumption, by allowing for labor rigidities following Blanchard and Galí (2007). The coefficients $\lambda$ now becomes equal to $\left(1 - \delta\right)\left(1 + \frac{1 + \phi}{\mu - 1} - 1\right)$, which can be positive or negative, and where $\left(1 - \delta\right)$ accounts for the rigidity. The relationship between marginal cost and the output gap is potentially attenuated. Although admittedly ad hoc, for the purposes of the paper, this assumption permits to avoid biases in the results that are due to imprecisions in modeling the labor market. Castelnuovo and Nistico’ (2010) also introduce rigidity in the labor market to induce procyclical dividends. They assume nominal wage stickiness and wage indexation, while here I keep the scale of the model as small as possible to allow for learning.

Equation (2.21) is the forward-looking New-Keynesian Phillips curve. Inflation depends on expected inflation in $t+1$ and on current output gap. The parameter $\kappa$ denotes the slope of the Phillips curve and negatively depends on $\alpha$, the Calvo price stickiness parameter.

Equation (2.22) describes monetary policy. The central bank follows a Taylor rule by adjusting its policy instrument, a short-term nominal interest rate, in response to changes in inflation, output gap, and stock price gap. The policy feedback coefficients are denoted by $\chi^\pi_t$, $\chi^x_t$, and $\chi^s_t$, while $\rho$ accounts for interest-rate smoothing.

12Monetary policy is assumed to react to the stock price gap, not to the level. This is similar to Nisticó (2012) and Gilchrist and Saito (2007). The estimation results remained comparable when policy responds to $s_t$ rather than to $s_{t-1}$. 
An advantage of this framework is that it permits to deal with interactions between macroeconomic variables and the stock market by maintaining a parsimonious structure, which potentially allows large wealth effects and nests the standard New Keynesian model as special case.\textsuperscript{13} The evidence from this paper can be seen as complementary to that coming from ‘financial accelerator’ models as in Bernanke et al. (1999) or Gilchrist and Saito (2007), which emphasize a different channel through which financial variables can affect the economy.

2.5. \textbf{Expectations}. The paper relaxes the assumption of rational expectations, by assuming that economic agents form near-rational expectations and learn about economic relationships over time (e.g., Evans and Honkapohja, 2001).

Agents are assumed to use a linear model as their Perceived Law of Motion

\[
Z_t = a_t + b_t Z_{t-1} + \epsilon_t, \tag{2.23}
\]

where \(Z_t \equiv [x_t, s_t, \pi_t, i_t]'\), \(a_t\) is a 4 \times 1 vector, and \(b_t\) is a 4 \times 4 matrix of coefficients, to form expectations about \(Z_{t+1}\). Agents are assumed not to know the relevant model parameters and they use historical data to learn them over time. Each period, they update their estimates of \(a_t\) and \(b_t\) according to the constant-gain learning formula

\[
\widehat{\phi}_t = \widehat{\phi}_{t-1} + \overline{g} R_t^{-1} X_t (Z_t - \widehat{\phi}_{t-1} X_t)'
\]

\(R_t = R_{t-1} + \overline{g} (X_t X_t' - R_{t-1}) \tag{2.25}
\]

where (2.24) describes the updating of the learning rule coefficients collected in \(\widehat{\phi}_t = (a_t, b_t)\), and (2.25) characterizes the updating of the precision matrix \(R_t\) of the stacked regressors

\(X_t \equiv [1, x_{t-1}, s_{t-1}, \pi_{t-1}, i_{t-1}]'\). \(\overline{g}\) denotes the constant gain coefficient. Economic agents are assumed to use only observables in their perceived model: they do not know, instead, the realizations of the unobservable shocks.\textsuperscript{14}

\textsuperscript{13}The model is mostly aimed at studying the influence of stock prices on the main macroeconomic variables that matter for monetary policy; the model, instead, doesn’t aim to provide the best possible characterization of stock price dynamics, since that would likely involve higher-order terms, which are here lost in the linearization, and which would significantly complicate the estimation of the general equilibrium model.

\textsuperscript{14}Adding learning in the model results in a number of advantages. The relationship between stock prices and macroeconomic variables may have not been stable over time: learning allows the model to incorporate the time variation in a parsimonious way. As in Milani (2007), learning introduces lags in the model, without the need to change the microfoundations – by assuming habit formation in consumption or inflation indexation, for example – thereby helping in capturing the persistence in the data. Allowing for deviations from fully-rational expectations is also useful in fitting the persistence and volatility of the stock price variable.
3. Estimation

The vector $\Theta$ collects the coefficients that need to be estimated:

$$\Theta = \{ \gamma, \lambda, \kappa, \rho, \chi_\pi, \chi_x, \chi_s, \chi_g, \rho_r, \rho_e, \rho_u, \sigma_r, \sigma_e, \sigma_u, \sigma_\varepsilon \}$$ (3.1)

I use monthly data on industrial production, the S&P 500 stock price index, the CPI, and the federal funds rate. The output gap $x_t$ is computed by detrending the log of the industrial production series using the Hodrick-Prescott filter. The real stock price gap $s_t$ is calculated as the S&P 500 index deflated using the CPI and then detrended using the Hodrick-Prescott filter.\(^{15}\) Inflation $\pi_t$ is constructed as the monthly change in the CPI, and the Federal Funds rate $i_t$ is taken in levels and converted to monthly units.\(^{16}\) Figure 1 displays the output and stock price gap series. The stock price gap is about four times more volatile than the output gap. Booms and busts in the stock market anticipate economic expansions and recessions: essentially all recessions in the sample have been preceded by a fall in the stock price gap (as known, however, not all stock market busts develop into a recession). The relation between the two series appears attenuated starting from the early 1980s (for instance, the cross-correlation between the output gap and the eight-months-lagged stock price gap goes from 0.66 in the pre-1979 sample to 0.13 in the post-1984 sample).

In the estimation, I consider a sample from 1960:M1 to 2007:M8. To initialize the learning algorithm in (2.24) and (2.25), I use pre-sample data from 1951:M1 to 1959:M1 (estimating (2.23) by OLS over this period).

The model is estimated by likelihood-based Bayesian methods to fit the output gap, real stock price gap, inflation, and Federal Funds rate series. The estimation technique follows

\(^{15}\)Obviously, the empirical measures for the output and stock price gap obtained by detrending the data with the Hodrick-Prescott filter may not correspond to the theoretical definitions of deviations from their corresponding flexible price level. The flexible price potential stock price level in the model would be strictly connected to the flexible price potential output, as they both depend on the technology shock. I have preferred not to impose this restriction on the data, at least in the baseline estimation, and to focus on a more data-driven decomposition. This choice, therefore, differs from the one in Castelnuovo and Nisticó (2010), who assume, instead, that output, consumption, real wages, and stock prices all share a common trend, although they include a measurement error in the stock price equation. The robustness of the results to using different detrending methods and also to the use of the theoretical gap measures and of growth series as observables is, however, analyzed later in the paper.

\(^{16}\)The series on industrial production, the CPI, and the Federal Funds rate were downloaded from FRED, the Federal Reserve Economic Database, hosted by the Federal Reserve Bank of St. Louis. Industrial production is the Industrial Production Index, Seasonally Adjusted (INDPRO), CPI is the Consumer Price Index for all Urban Consumers, All Items, Seasonally Adjusted (CPIAUCSL), the Federal Funds Rate is the Effective Federal Funds Rate, in percent, average of daily figures (FEDFUNDS). The S&P 500 was downloaded from IHS-Global Insight.
Milani (2007), who extends the approach described in An and Schorfheide (2007) to permit the estimation of DSGE models with near-rational expectations and learning by economic agents. The results may depend on the assumed learning dynamics, if this is imposed \textit{a priori}. Therefore, here, I instead estimate also the learning process (which depends on the constant gain coefficient) jointly with the rest of structural parameters of the economy. In this way, the best-fitting learning process is extrapolated from the data along with the best-fitting preference and policy parameters.\footnote{We do not impose the assumption that the coefficients satisfy any determinacy or E-stability condition in the estimation. However, the choice of priors forces the monetary policy response coefficient to inflation to remain above one.}

I use the Metropolis-Hastings algorithm to generate draws from the posterior distribution. At each iteration, the likelihood is evaluated using the Kalman filter. I consider 300,000 draws, discarding the first 25\% as initial burn-in.

The priors for the model parameters are described in Table 1. The main parameter of interest is $\gamma$, the probability of exiting the market, which influences the size of the wealth effect. Since there isn’t much existing evidence on its value, and in order to rule out a large influence from the prior, I assume a non-informative Uniform $[0,1]$ prior for $\gamma$. I choose prior Gamma distributions for the slope of the Phillips curve $\kappa$ and for the monetary policy feedback coefficients to inflation and output gap; the Gamma priors assure that only values within the positive region are assigned positive probability. I select, instead, a Normal prior with mean 0 and standard deviation 0.15 for the policy feedback to the stock price gap and with mean 0 and standard deviation 0.25 for $\lambda$. I also assume a Gamma prior distribution for the constant gain coefficient (I have re-estimated the model under a Uniform$[0,0.3]$ prior for the gain coefficient, however, and the results are unaffected, as the data seem highly informative about its values). Finally, Beta distributions are used for the autoregressive coefficients (to constrain them to assume values between 0 and 1) and Inverse Gamma distributions for the standard deviations of the shocks.\footnote{I need to fix some of the steady-state parameters that appear in the reduced-form coefficients: $\beta$ equals $(1 + \frac{0.04}{12})^{-1} = 0.9967$, while the real financial wealth to consumption ratio in steady-state $\Omega$ is fixed to 4 (this value is consistent with the information in the Households Balance Sheet in the Federal Reserve System’s Flow of Funds accounts).}
4. Empirical Results

Table 2 presents the posterior estimates for the baseline model, summarized by equations (2.19) to (2.22), with expectations formed using (2.23), (2.24), and (2.25). Table 3 displays the estimates for a selection of alternative cases, including estimations based on different detrending procedures or different observables. Table 4 and Figures 6 to 11 present the outcome of selected impulse response functions and variance decompositions, which are time-varying in the model as a consequence of learning dynamics.

4.1. How Large is the Wealth Effect? The data indicate a low value for the probability of exiting the market parameter $\gamma$. The mean posterior estimate equals 0.0084, which implies a decision planning horizon of $1/\gamma = 119$ months, or 10 years. The implied wealth effect from changes in asset prices on output, measured by the composite reduced-form parameter $\frac{\psi}{1+\psi}$ is extremely small: the 95% highest posterior density interval does not contain values higher than 0.0025. The estimate for $\gamma$ implies that the degree of consumption smoothing and the sensitivity of output to the real interest rate remain high and close to the level they would assume in the nested case of a New Keynesian model with no wealth effect.

The estimates point to a longer planning horizon and a weaker wealth effect than those found by Castelnuovo and Nisticó (2010) in a model that retains the assumption of fully-rational expectations.$^{19}$

Turning to the other parameters, the posterior mean for the constant gain coefficient equals 0.014, which is lower, but not far from the value estimated in Milani (2007) on quarterly data. The sensitivity of the stock price gap to output expectations $\lambda$ has posterior mean 0.09, but the estimate is characterized by large uncertainty (the 95% HPD contains values between -0.11 and 0.29). The monthly Phillips curve is relatively flat ($\kappa = 0.008$).

The estimated autoregressive parameters for the shocks are moderate: this shows that learning can account for most of the persistence in the model, so that strongly serially-correlated exogenous shocks are not necessary.

4.1.1. Post-1984 Sample. The rate of equity ownership (direct or indirect through mutual funds) has doubled from below a quarter in the 1970s to more than half in the 1990s (see

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$^{19}$Direct comparisons, however, are difficult as the data series used, the sample, the frequency of the data (monthly here, quarterly in their case), the detrending methods, and some model features are different between the two papers.
Duca, 2006). It is, therefore, possible that the size of the wealth effect has increased in the second part of the sample, since a larger fraction of the population can now be affected by swings in asset values. The estimates for the post-1984 sample (Table 3), however, indicate a posterior mean for $\gamma$ equal to 0.009, which is only marginally higher than the full-sample result.\footnote{Estimates of the wealth effect appear stable over sub-samples. This finding differs from the evidence of sub-sample instability detected by Ludvigson and Steindel (1999), who estimate a much larger effect before 1985 than afterwards. Such instability may easily reflect changes in the impact of stock prices on expectations, more than a decline of the direct wealth effect over time.}

When the model is estimated assuming that stock prices do not affect the formation of expectations (by setting to zero the coefficients on stock prices in expression (2.23), the agents’ PLM), the posterior mean for $\gamma$, in the full sample case, becomes larger ($\gamma = 0.033$, Table 3). The fit of the model, however, worsens (the marginal likelihood falls from -1303 to -1351).

4.2. **Evolving Economic Agents’ Beliefs.** Figure 2 illustrates the evolving beliefs by economic agents about the coefficient $b_{12,t}$, which refers to the perceived effect of stock prices on the output gap. Agents appear to use information in the stock market when forming expectations about future output: the effect on their expectations, however, declines over the sample.

Figure 3 provides some supportive evidence that such behavior is consistent with what a rational forecaster would do. Economic agents that use information in stock prices to forecast future output gaps obtain much lower root mean squared errors in the early part of the sample and for most of the 1970s compared with forecasters that exclude asset prices from their perceived model. Their forecasting performances become very similar at the end of the sample (when, in fact, learning agents start to believe that stock prices have only a small effect on output). If agents had kept their initial belief of a large influence of the stock market on the economy, retaining their 1965 estimate of $b_{12}$ over the whole sample (that is $b_{12,t} = 0.056$ for all $t$’s), they would have done well until the 1970s, but poorly starting from 1985. This evidence is consistent with Stock and Watson (2003)’s finding that asset prices are useful in forecasting for some periods, but not others.

Turning to the other beliefs, I find that the intercept and the autoregressive parameter in the inflation equation are revised upward in the middle of the sample and decline again later
on. The perceived degree of monetary policy inertia jumps after 1979. In the stock price equation, the perceived stock price persistence declines over time, while its sensitivity to the real interest rate is stronger in the late 1960s and 1970s.

The agents’ forecasting performance appears satisfactory: Figure 4 shows the actual and forecast values for the output gap and the real stock price gap, together with the implied absolute forecast errors. The forecasts fall usually close to the realized values, although some episodes, as the stock market crash in October 1987, clearly took agents by surprise. Overall, through the evolution of agents’ beliefs, the model can explain a substantial part of stock market fluctuations.

4.3. Do Stock Market Shocks Matter? Effect Through Expectations. As evidenced by the estimation, the direct wealth effect of short-run stock price changes on output is close to zero. Is the stock market hence irrelevant for output fluctuations?

Under rational expectations, a low direct wealth effect would imply a trivial effect of stock price fluctuations in the model. Agents would form expectations from the rational expectations solution:

\[ Z_t = F Z_{t-1} + \Psi w_t, \]  

(4.1)

where \( w_t \) collects the structural innovation terms. Agents would use (4.1) in the formation of their expectations; in the expression, the reduced-form effect of stock prices on output is captured by the appropriate element of \( F \) that we can denote as \( f_{x,s} \) (the coefficient in the MSV solution of output on stock prices). The value of \( f_{x,s} \) is constant over the sample and mostly influenced by the deep parameter \( \gamma \). With \( \gamma \) small, as estimated, agents would form expectations about future output at each point in the sample largely ignoring the stock market (\( f_{x,s} \approx 0 \)).

In the model with learning, however, asset prices affect the economy through a second channel, by leading economic agents to revise their beliefs over time. Under learning, agents do not have perfect knowledge about the model and its coefficients. Among other things, they do not know the value of \( \gamma \), and they cannot infer the magnitudes of the reduced-form relationships among variables, such as \( f_{x,s} \). Hence, they attempt to learn about those relationships using past data by estimating the PLM (2.23). At each point in the sample \( t \), they form a belief \( b_{xs,t} \) (the element of the belief matrix \( b_t \) corresponding to the perceived sensitivity of output to stock prices). Their perception about the effect of stock prices on real
activity is driven by the past dynamic relations among those variables, it is time-varying, and it may be far from $f_{x,s}$, the value that would be justified by the size of the wealth channel only. The state of beliefs about the impact of stock price fluctuations affects the formation of expectations and, through such expectation channel, affects macroeconomic realizations. Stock price changes, therefore, may lead to overoptimism or overpessimism in the formation of output forecasts, and hence additional business cycle fluctuations.

Figure 5 shows the responses of output gap expectations in the case in which the wealth effect is present, but expectations are formed according to the rational expectations hypothesis, and in the alternative in which expectations are formed from the learning model with PLM (2.23). It is apparent that expectations in the learning model respond much more to stock market developments than justified by the wealth channel alone. Through this additional expectational effect, the impact on actual output can be magnified.

From an empirical point of view, the size of the wealth and expectation effects can be disentangled through the use of the learning model and the existence of time variation in the effects of stock prices. While under rational expectations and a wealth channel, the response of output to stock prices would be constant, the model with learning allows us to capture time-varying responses over the sample in a parsimonious way (through the addition of a single extra-parameter, the gain coefficient).

From the estimation, it appears that stock market shocks do matter and that the expectation channel plays the more sizable role, although the effect through beliefs has varied over time.

I investigate the importance of stock prices on the economy by looking at the variance decomposition over the sample. Figure 6 reports the percentage of variance in the output gap that is explained by shocks in the stock price gap variable, shown across forecast horizons (from one month to ten years) and over time.

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21 In both cases, the parameter are fixed at their posterior means obtained from the estimation and shown in Table 2. For the learning model, I show the average response over the sample.

22 It could also be argued that the wealth channel itself may have varied over time. We tested this in section 4.1.1, but did not find large differences. Moreover, most would expect the size of the wealth effect to increase over time, as a consequence of more widespread stock market participation, rather than decrease, as the data seem to indicate for the overall stock market effect.

23 The effect of stock prices through expectations, rather than through a direct wealth channel, is consistent with the microeconometric evidence that uncovers a similar consumption response between households that own or do not own equities to stock price changes (e.g., Otoo, 1999).
Shocks in the stock market play a significant role in explaining output fluctuations. In the 1960s and 1970s, on average around 40% of output fluctuations are due to stock price shocks. The stock market appears to play a more limited role in the second half of the sample, the 1990s and 2000s: output fluctuations are due in large part to shocks in the natural interest rate and usually to less than 20% to stock market shocks.\footnote{Doan, Litterman, and Sims (1983), in a paper with unrelated focus, find similar evidence that stock price shocks are important in a structural VAR on data up to 1983. The percentage of variance they explain amounts to 30-40% after 1960, which was already declining from more than 60% in their 1948:M7-1960:M1 sample.}

Figure 7 exhibits the impulse response of the output gap to a stock price shock as it varies over the sample. In the early part of the sample and until the early 1980s, stock market shocks induce a sharp increase in output that lasts about a year, before falling below its initial level and reverting to zero in less than three years. The effect becomes much smaller in the second half of the sample. Inflation is also affected by stock market shocks: the effect is larger in the 1970s (figure 8). These shocks explain, on average, around 10% of the variance in inflation (Table 4).

But why has the role of stock market shocks faded over time?

One possible interpretation is that economic agents have slowly learned over the sample and are converging to the true REE estimate of a wealth effect close to zero.

The decline in the stock market effects on the real economy may also be related to the “Great Moderation”. The standard deviation of the output gap measure has fallen from 2.68 before 1984 to 1.24 afterwards, while the standard deviation of the stock price gap did not experience a similar decline (it went from 8.61 to 6.60). The stock market has remained volatile, but the volatility of asset price fluctuations has not translated into macroeconomic volatility. The improved monetary policy, which is one of the major candidates as driver of the Great Moderation, may have induced agents to expect small deviations of output from potential and, therefore, it may have reduced the usefulness of asset prices in forecasting the output gap.

4.3.1. Stock Market Shocks and Consumer Confidence. The estimation provides evidence that the effect of stock market shocks operates mostly through expectations than through a conventional wealth effect on consumption decisions.
Here, we provide some additional evidence that stock market innovations affect economic confidence. We construct a rather typical indicator of consumer confidence by using the University of Michigan monthly Consumer Sentiment Index (obtained from FRED) and adding it to a VAR that also includes growth in industrial production, CPI inflation, and the Federal Funds rate (the same observables as in the theoretical model). We then extract the consumer sentiment innovation from the VAR, which can now be interpreted as consumer sentiment, but ‘purified’ from its endogenous reaction to macroeconomic conditions (a similar VAR has been used in Matsusaka and Sbordone, 1995, to identify and study the effects of consumer confidence shocks on economic fluctuations).

We test whether there is a relation between these empirical sentiment shocks and the stock market shocks identified in the paper from the DSGE model. We regress the ‘purified’ sentiment innovation on stock market shocks (denoting $\tilde{e}_t$ the innovation component of $e_t$) and obtain the following results (numbers in parenthesis denote standard errors):

$$Sentiment_t = -0.048 + 0.136\tilde{e}_{t-1} + 0.198\tilde{e}_{t-2} + 0.134\tilde{e}_{t-3} + \tilde{e}_t$$ (4.2)

$$R^2 = 0.11$$

$$S.E. = 3.35.$$ 

Therefore, the regression indicates that stock market shocks in the paper lead unexplained consumer sentiment one to three months into the future.\(^{25}\) While more work is definitely needed, the results suggest an interesting interpretation for stock market shocks in the paper, as potential drivers of overall ‘sentiment’ in the economy. In this respect, they may have points of contact with the sentiment shocks estimated in Milani (2011b, 2013) in models that abstracted from the stock market.

4.3.2. Stock Market and the Propagation of Shocks. The stock market, mainly through its effect on expectations, plays also a significant role in propagating non-financial shocks. Figure 9 displays the mean impulse responses, across sub-samples, of the output gap to a monetary policy shock for the baseline model and for an alternative model in which the effect of stock prices on expectations is shut down. In periods when economic agents assign a relatively large weight to stock prices in their forecasting model, the stock market considerably amplifies

\(^{25}\)The optimal number of lags was selected based on Schwarz’s Bayesian Information Criterion; lags at $t - 4$ or further were insignificant. There is also no significant contemporaneous correlation or any statistically significant coefficient when the opposite direction of causality is tested.
the propagation of monetary policy shocks (this is evident in the 1960s and 1970s). A monetary contraction, in fact, depresses both output and stock prices, which in turn, through their effect on expectations, cause an even larger reduction in output. The initial effect is, therefore, magnified. The role of the stock market, however, varies over the sample. In the 1984-1999 sub-sample, the model that allows for stock price effects on expectations displays an attenuated and more transient response (this is mostly due to a perceived negative effect of past output on current stock prices in this period, which weakens the original output effect). Finally, when agents’ beliefs assign a small weight to asset prices in their perceived law of motion (as in the 2000-2007 period), the impulse responses with or without stock price effects are virtually indistinguishable.

If the stock market channel is entirely shut down, demand shocks would pick up most of the effect of stock price shocks in explaining output fluctuations (the percentage of variance they explain rises to 60% in the first half and 75% in the second half). Monetary policy shocks would also matter more and they would account for a larger part of the variation in inflation.

4.4. Does Monetary Policy React to Stock Market Fluctuations? The full-sample estimates indicate that monetary policy has responded to the stock price gap. The posterior mean estimate for $\chi_s$ in table 2 equals 0.135.

The feedback to the stock price gap is much lower ($\chi_s = 0.034$) in the post-1984 sample: this is consistent with the reduced influence of stock prices on output expectations and with the common perception that Fed’s policy under Greenspan did not react to the bubble in the 1990s. Moreover, if the estimation is repeated using a Taylor rule that responds to forecasts of inflation and the output gap (i.e., to $\hat{E}_{t+1}\pi_t$ and $\hat{E}_{t+1}x_t$, and assuming that the Fed uses the same forecasting model (2.23) as the private sector), rather than to their lagged values, the response to the stock price gap is quite precisely estimated around zero (and the model fit improves). This signals that policy reacts to stock prices only to the extent that they act as leading indicators of future inflation and real activity, but no separate response exists.\footnote{Fuhrer and Tootell (2008) similarly find little evidence of an independent response to stock values when Greenbook forecasts are included in Taylor rules. In the full-sample estimation (table 2), instead, allowing the Taylor rule to respond to expectations would reduce the estimate for $\chi_s$ from 0.135 to 0.07.}

Our full sample estimate of the monetary policy reaction to the stock market is similar to the one in Castelnuovo and Nisticó (2010): they estimate a value of 0.12 for a sample
starting in 1954. Other papers have estimated a more modest response to stock market fluctuations after 1980. Chadha et al. (2004) estimate augmented Taylor rules using GMM and obtain values between 0.01 and 0.036. Furlanetto (2011) uses a VAR identified through heteroskedasticity and estimates that the response of monetary policy to stock prices has been close to zero in the post-1985 period. He similarly discusses how the response of monetary policy to stock prices seems to have declined over time. Ravn (2012) provides evidence that the Federal Reserve’s response to the stock market has been asymmetric, but small. The implied value for the parameter corresponding to $\chi_s$ would be equal to 0.0246 (in response to stock price drops only), based on a 1998-2008 sample. The magnitude of the response is significantly smaller than the one estimated, using the same techniques, by Rigobon and Sack (2003) on the preceding (pre-1999) sample.

Recent papers find that if central banks react to asset prices, they may increase the chances of indeterminacy in the economy. Carlstrom and Fuerst (2007) find determinacy only if the response to asset prices remains below a certain threshold. Aireantu et al. (2015) study determinacy and learnability conditions in a similar model as the one used in this paper, but without the assumption of wage rigidity. In that context, the expectation of an economic expansion leads to a falling stock market (through a negative value of $\lambda$ in equation 2.15). As a result of such a counterfactual implication, a positive reaction to the stock price gap enlarges the indeterminacy region, particularly if the wealth effect in (2.14) is small. In this paper, the coefficient $\lambda$ is positive. Therefore, a positive response of the central bank to stock prices may help stabilize the economy. Figure 10 shows the indeterminacy and determinacy regions obtained by varying the monetary policy coefficients $\chi_\pi$ and $\chi_s$ and leaving the remaining coefficients fixed at their posterior mean estimates. A larger $\chi_s$ now unequivocally enlarges the determinacy region.

4.5. **The Effect of Monetary Policy and Macro Shocks on Stock Prices.** Figure 11 presents the impulse responses of the stock price gap to one-standard-deviation monetary policy, demand, and supply shocks. Stock prices seem more responsive to monetary policy surprises in the 1960-1970s. The decline is even more pronounced if examined on 1970s data alone. The response in the latest part of the sample, instead, is much smaller (the plotted response, however, conceals some variation that exists in the post-1984 period).²⁷

²⁷The small response may be consistent with Davig and Gerlach (2007)’s estimate of a distinct regime in the late 1990s-early 2002, in which stock prices’ response to policy shocks is insignificant and volatile.
This may suggest a recent more limited effect of monetary policy, but it might also reflect the difficulty in identifying monetary policy shocks in the second part of the sample on monthly data (stock price responses that may be found on high-frequency data may have become extremely short-lived and may be lost in the monthly averaging).

Shocks in the natural interest rate lead to an immediate jump in the stock price gap, which turns negative after six-eight months, before reverting to zero. Inflationary shocks lead to a decline in the stock price gap, with a less sluggish adjustment in the post-1984 sample.

Table 4 reports the outcome of the forecast error variance decomposition at alternative horizons. Regarding the stock price gap, in the 1960-1970s, monetary policy shocks account for up to 7.5% of the variance, demand shocks for 6.2%. Fluctuations in the stock market are mostly driven by shocks that originate in the stock market. In the second half of the sample, monetary policy shocks account for up to 9.7% of fluctuations, and demand shocks for more than 20%; the stock market hence appears not as isolated from the rest of the economy as it was in the past.

4.6. Robustness to alternative detrending procedures and choice of observables. The baseline estimation in the paper used the Hodrick-Prescott filter to separate between trend and business cycle fluctuations in industrial production and stock prices. Here, I check whether the estimates are robust, first, to a different detrending assumption, by removing quadratic trends from both series and, later, by assuming, instead, a stochastic trend driven by technology shocks and estimating the model on growth rates rather than on detrended variables.

Table 3 reports the posterior estimates under these alternatives. The finding of a modest wealth channel does not depend on the detrending method used to compute the gap variables: the probability $\gamma$ has a posterior mean equal to 0.0067, slightly lower than before, when a quadratic trend is assumed (eighth and ninth columns in the table). The other estimates are also largely unaffected and the main conclusions of the paper remain valid (the same holds if linear or cubic time trends are assumed instead).

The gaps obtained in the paper for output and stock prices using a statistical detrending procedure, however, may differ from their respective theoretical definitions (i.e., the deviation of output and real stock prices from the values that they would assume in the same economy, but under flexible prices). I also evaluate the robustness of the results to this
choice, by re-estimating the model now exploiting the theoretical model-consistent definitions of output and stock price gap, rather than adopting a statistical filter. Therefore, I extend the model to include a non-stationary neutral technology shock $A_t$, as in Castelnuovo and Nisticó (2010), and use $\Delta Y_t \equiv Y_t - Y_{t-1}$ and $\Delta q_t \equiv q_t - q_{t-1}$, the log monthly differences in industrial production and in stock prices, as the new observable variables that need to be matched in the estimation (rather than using directly gap variables as observables). The measurement equations corresponding to the growth rates of industrial production and stock prices, therefore, now assume the form $\Delta Y_t = x_t - x_{t-1} + z_t$ and $\Delta q_t = s_t - s_{t-1} + z_t$, where $z_t = \log (A_t/A_{t-1})$.

Even with the model-consistent gap measures, the estimation indicates a small posterior mean for the probability $\gamma$ ($\gamma = 0.011$), which still implies a modest wealth effect of asset price fluctuations on economic activity ($\frac{\phi}{1+\psi} = 0.007$). The evolution of agents’ beliefs regarding the perceived effect of asset prices on output still closely resembles the pattern shown in Figure 2. As a result, the variance decomposition and impulse responses over the sample remain similar to those presented in Figure 5 and 6.\textsuperscript{28}

We also assess the sensitivity of the results to the use of different observables. First, we repeat the estimation using data on consumption rather than industrial production (in the model, the market clearing condition $y_t = c_t$ applies); it can be argued that the use of a consumption series is more appropriate to identify the wealth effect from asset price fluctuations. The estimation results are shown in Table 3. The results, however, are similar: the posterior mean for $\gamma$ is 0.009 and the direct wealth effect remains as small as before.

As a final robustness check, we maintain the consumption data, but now change the frequency of all observable series from monthly to quarterly. The estimate for $\gamma$ rises to 0.026 (now indicating the quarterly, rather than monthly, rate of market exit), which still implies a planning horizon around ten years, as before. The reduced-form wealth effect is still small, with a mean equal to 0.0044.

5. Conclusions

The paper has provided evidence from a structural model on the empirical relevance of interactions between macroeconomic variables and the stock market. One of the main channels that are usually emphasized in policy discussions, the wealth channel, appears

\textsuperscript{28}The additional graphs are not shown here for brevity. They are available upon request.
modest. But the stock market plays a significant role through its impact on expectations about future real activity.

Monetary policy seems to have reacted to stock price fluctuations, but, in the post-1984 sample, only to the extent that they influence output and inflation forecasts. A monetary policy response may be justified if non-rational movements in the stock market affect expectations, as found in the data, and if non-fundamental stock market shocks are an important source of fluctuations. But both these effects are now less important. Yet, the welfare implications of different monetary policy rules in a model in which asset prices affect private sector’s expectations and learning remains an important topic that deserves future study.

The stock market dynamics is affected by macroeconomic fundamentals, but a large part of fluctuations is due to exogenous stock price shocks. A better modeling of the stock market, which retains second-order terms, will be needed to shed more light on the nature of financial shocks (Challe and Giannitsarou, 2014, offer a general equilibrium framework in this direction). Future extensions should also move away from the linear/Gaussian framework: including stochastic volatility in the structural innovations, for example, would allow researchers to study the relation between output and stock price volatility, as well as between expectations of future booms and busts and volatility. Finally, it is necessary to check whether the evidence is robust to the use of a larger model and the inclusion of different financial sector channels: in this respect, Christiano et al. (2008)’s findings, in a different framework, similarly identify an important role for financial shocks.

References


### Table 1 - Prior Distributions.

(U = Uniform, N = Normal, Γ = Gamma, B = Beta, Γ⁻¹ = Inverse Gamma)

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Distr.</th>
<th>Support</th>
<th>Prior Mean</th>
<th>95% Prior Prob. Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob. of Leaving the Mkt.</td>
<td>γ</td>
<td>U</td>
<td>[0,1]</td>
<td>0.5</td>
<td>[0.025, 0.975]</td>
</tr>
<tr>
<td>Sensit. Stock Prices to Output</td>
<td>λ</td>
<td>N</td>
<td>R</td>
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<td>[-0.98, 0.98]</td>
</tr>
<tr>
<td>Slope PC</td>
<td>κ</td>
<td>Γ</td>
<td>R⁺</td>
<td>0.25</td>
<td>[0.03, 0.70]</td>
</tr>
<tr>
<td>MP Inertia</td>
<td>ρ</td>
<td>B</td>
<td>[0,1]</td>
<td>0.8</td>
<td>[0.459, 0.985]</td>
</tr>
<tr>
<td>MP Inflation feedback</td>
<td>χᵣ</td>
<td>Γ</td>
<td>R⁺</td>
<td>0.5</td>
<td>[0.06, 1.40]</td>
</tr>
<tr>
<td>MP Output Gap feedback</td>
<td>χₓ</td>
<td>Γ</td>
<td>R⁺</td>
<td>0.25</td>
<td>[0.03, 0.70]</td>
</tr>
<tr>
<td>MP Stock Price Gap feedback</td>
<td>χₛ</td>
<td>N</td>
<td>R</td>
<td>0</td>
<td>[-0.29, 0.29]</td>
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<tr>
<td>Std. Demand Shock</td>
<td>σᵣ</td>
<td>Γ⁻¹</td>
<td>R⁺</td>
<td>0.11</td>
<td>[0.038, 0.31]</td>
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<td>Std. Stock Price Shock</td>
<td>σₑ</td>
<td>Γ⁻¹</td>
<td>R⁺</td>
<td>0.33</td>
<td>[0.11, 0.92]</td>
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<td>σᵤ</td>
<td>Γ⁻¹</td>
<td>R⁺</td>
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<td>[0.038, 0.31]</td>
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<td>Std. MP Shock</td>
<td>σₑ</td>
<td>Γ⁻¹</td>
<td>R⁺</td>
<td>0.11</td>
<td>[0.038, 0.31]</td>
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<tr>
<td>Autoregr. coeff. $rₙ^N$</td>
<td>ρᵣ</td>
<td>B</td>
<td>[0,1]</td>
<td>0.8</td>
<td>[0.459, 0.985]</td>
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<td>ρₑ</td>
<td>B</td>
<td>[0,1]</td>
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<td>[0.459, 0.985]</td>
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<td>ρᵤ</td>
<td>B</td>
<td>[0,1]</td>
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<td>[0.459, 0.985]</td>
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<td>Γ</td>
<td>R⁺</td>
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<td>Parameter</td>
<td>Posterior Mean</td>
<td>95% HPD</td>
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<td>------------------------------------------------</td>
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<td>Prob. of Leaving the Mkt.</td>
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<td>[0.0004,0.023]</td>
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<td>Slope PC</td>
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<td>[0.001,0.017]</td>
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<td>[0.06,0.265]</td>
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<td>[0.00005,0.0025]</td>
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Table 2 - Posterior Estimates.

Notes: Full Sample 1960:M1-2007:M7, Baseline Case. The table shows the posterior mean (standard deviation in brackets) and the 95% Highest Posterior Density Interval.
Table 3 - Posterior Estimates: Alternative Models.

Notes: The table shows the posterior mean, standard deviations, and the 95% Posterior Probability Intervals. The second and third columns refer to the estimate for the 1984:M1-2007:M7 sample, the fourth and fifth columns to the 1984:M1-2007:M7 sample using a model with a Taylor rule that responds to expected inflation and output gap $i_t = \rho_i e_{t-1} + (1 - \rho)(1 + \chi_x)\tilde{E}_t \tilde{\eta}_{t+1} + \chi_x \tilde{E}_t \tilde{E}_{t+1} + \chi_s s_{t-1} + \varepsilon_t$, the sixth and seventh to the full-sample estimation of a model in which the stock price gap $s_t$ is assumed not to affect economic agents’ expectations in (2.23), the eighth and ninth columns to the full-sample estimation under a different detrending method (using quadratic trends for output and stock prices rather than the HP filter), the tenth and eleventh columns to the full-sample estimation assuming a stochastic trend driven by a neutral technology shock and using growth rates of industrial production and stock prices as observable variables, the twelfth and thirteenth columns to estimation with consumption, rather than industrial production, as observable, and the fourteenth and fifteenth columns to the estimation with consumption and quarterly, rather than monthly, data for all series.
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<th>Inflation Shock</th>
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<td>0.072</td>
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Table 4 - Forecast Error Variance Decomposition.
Figure 1. Output Gap and Real Stock Price Gap series. Note: the series are expressed in percentage deviations from potential; the left scale refers to the output gap, right scale to the stock price gap. The light-yellow shaded areas denote NBER recession dates.
Figure 2. Agents' Beliefs: Perceived Sensitivity of the Output Gap to Stock Price Gap Movements. Note: The solid line denotes the posterior mean of beliefs across Metropolis-Hastings draws. The dotted lines denote the 2.5% and 97.5% error bands.
Figure 3. Rolling Root Mean Squared Error. Note: The graphs shows the rolling RMSE calculated using a window of 60 observations (for the first five years, the RMSE is recursively calculated). The baseline case refers to the agents’ PLM in (2.23), the second assuming a zero effect of the real stock price gap in the agents’ PLM, the third assuming a constant (large) effect of the stock price gap on output expectations, which is fixed at the agents’ belief in 1965 (i.e., $b_{12} = 0.056$).
Figure 4. Actual Output and Real Stock Price Gap versus Estimated Agents’ Forecasts. The figure shows (panel 1 and 3) the actual values for the output gap and the real stock price gap together with the mean agents’ forecasts (mean across MH posterior draws). Panel 2 and 4 report the absolute value of the corresponding forecast errors for each variable.
Figure 5. Impulse response of Output Gap expectation $E_t x_{t+1}$ to stock market shocks. The upper panel refers to case with a wealth channel, but expectations formed as under RE; the bottom panel to the case with a wealth channel and expectations formed from the learning model, according to the PLM (2.23).
Figure 6. Variance Decomposition: Variance of the output gap, $x_t$, due to stock price gap shocks, shown across forecast horizons and over the sample.
Figure 7. Impulse Response Function of the Output Gap to a one-standard-deviation Stock Price Gap Shock, shown across horizons and over the sample.
Figure 8. Impulse Response Functions of the Output Gap and Inflation to one-standard-deviation Stock Price Gap Shocks.
Figure 9. Impulse Response Functions of the Output Gap to a one-standard-deviation Monetary Policy Shock. Note: The solid line denotes the impulse responses in the baseline estimated model, which includes a direct wealth effect and allows for an effect of stock prices on expectations. The dashed line refers to an alternative model, in which the effect of stock prices on expectations is shut down.
Figure 10. Determinacy and Indeterminacy Regions as a function of the Taylor rule reaction coefficients to inflation ($\chi_\pi$) and to the stock price gap ($\chi_s$). Note: the white region denotes Indeterminacy, the grey region denotes Determinacy.
Figure 11. Impulse Response Functions of the Real Stock Price Gap to one-standard-deviation Monetary Policy, Natural Rate, Stock Market, and Cost-Push Shocks.