

Econometric Issues in DSGE Models

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March 16, 2006

The paper by An and Schorfheide reviews an important body of literature that takes the empirical implications of DSGE models seriously. Often, in the past, macroeconomic models have abstracted from estimation: the assessment of the models relied on calibration exercises and on the comparison between simulated and actual moments. But papers adopting the Bayesian approach are rapidly changing this habit.

And the results are promising. Recent studies, starting from Smets and Wouters (2003), build from microfoundations and estimate models that compete in fit with unrestricted Bayesian VARs. The early calibrated DSGE models can be thought of being built on dogmatic priors for structural parameters chosen by simplistic readings of past empirical studies. Here we will offer some suggestions for obtaining nondogmatic priors.

The empirical success of new-generation DSGE models has also encouraged cross-fertilization between academics and central bankers. The models embed more and more realistic features to serve the needs of policy makers, and policy is more and more informed by models rooted in economic theory. The prototypical model presented by An and Schorfheide is an example of a relatively small-scale New Keynesian model, on which most of this literature builds. The authors correctly emphasize the risks of misspecifications and identification that Bayesian methods have to address in DSGE models. Here, we want to point out three additional issues that receive less attention and need to be clarified in future research.

First, some of the major potential misspecifications are typically taken as given in the estimation. For example, how do we measure potential output? Theory clearly specifies that the measure of potential output that enters the model is the level of output that would prevail under flexible prices. But good empirical proxies are missing. An and Schorfheide derive output under flexible prices analytically in their simple model. But other researchers typically follow a shortcut: they estimate models using deterministic time trends, Hodrick-Prescott filters, or the CBO estimate, as empirical measures of potential output. None of them is correct as a measure of the theoretical variable in the model.

A better approach would be to estimate potential output or some common trend among real variables jointly within the system. Del Negro et al. (2005) provide an example, but this is rarely

done. Unfortunately, the common trend among variables in their paper is rejected by the data. In any case, the results are likely to be strongly model-dependent. Nonetheless, more research should shed light on the correct variable that should appear in the estimation.

Second, even more important improvements for future Bayesian DSGE models should stem from theory. There is now a trade-off between the empirical fit of the models and the rigor of their microfoundations. An and Schorfheide work with a parsimonious model. As it is their model is known to have problems in fitting the data. The well-fitting models we encounter in the literature, instead, need a variety of mechanisms to match the sluggishness of macroeconomic variables: the typical model would include habit formation in consumption, inflation and wage stickiness, inflation and wage indexation, capital adjustment costs, and persistent exogenous disturbances. These are the micro-foundations that we need to improve the fit. But are they correct microfoundations? The answer is crucial, for the welfare implications of different policies strongly depend on them. Faust (2005) and Sims (2005) make a similar point.

For example, Milani (2004) shows that replacing rational expectations with learning in a simple monetary DSGE model makes some of the mechanical sources of persistence superfluous. The microfoundations are different, and therefore, the welfare-maximizing policies also differ.

Besides learning, other microfoundations that can empirically improve the models consist of adding frictions in the labor market (through search and matching), or financial frictions. Now that we have found a reduced-form that fits the data, we ought to go back and refine our understanding of the frictions that are really important. Thanks to work by Schorfheide and others, now, we have the techniques to test the success of different DSGE models. And possibly microeconometric studies may provide even more compelling evidence than macro studies.

Third, another perennial issue that interests DSGE modelers is identification. The paper suggests that potential improvements may arise from the use of nonlinear approximation and a particle filter to estimate the model. Linearization is the most common strategy to compute approximate solutions of DSGE models. But this strategy has important limitations. Fernandez-Villaverde, Rubio-Ramirez, and Santos (2006) show that with linearization the approximated and exact likelihoods diverge as the sample size goes to infinity. Also, they show, the errors induced by linearization might lead VARs to outperform the true DSGE model that has generated the data. An important advantage of the computationally-intensive nonlinear approach would arise from a better identification of structural parameters. The results are still unclear. From what we learned in Fernandez-Villaverde and Rubio-Ramirez (2004c), nonlinear and linear estimation lead to posterior estimates that are very close to each other. The nonlinear estimation does, instead, reduce the uncertainty in the estimates. In general, the nonlinear estimation seems to put more curvature on parameters that are already identified. Whether it helps estimating parameters that are not identified

in the linear case is less clear.

Let's explore this further in the context of $M_1(L)$ versus $M_1(Q)$. We use the phrase *structural parameters* to describe parameters for which we can contemplate a change in any one while holding all other parameters unchanged. To the extent that DSGE models involve structural parameters, this may suggest priors which assume the parameters are mutually independent - in the spirit of such structural autonomy. An and Schorfheide pick independent priors in Tables 2 and 3. As a standard sensitivity analysis, we suggest a minor tweaking of An and Schorfheide's priors.

Let $\theta_{-(\kappa)}$ denote all the elements in θ (defined in Section 2.5) except κ , and for notational convenience, reorder the elements so that $\theta = [\theta_{-(\kappa)}', \kappa]'$. The slope coefficient of Phillips-curve relationship (29) is

$$\kappa = \left[\frac{\tau \left(\frac{1-v}{v} \right) \phi^{-1}}{(1 + \pi^{(A)})^2} \right]$$

where τ^{-1} is the intertemporal elasticity of substitution, v^{-1} represents the elasticity of demand for each intermediate good, $\pi^{(A)}$ is related to the steady state inflation rate associated with the final good, and ϕ governs the price stickiness in the economy. In the linear approximation scheme, $\theta_{-(\kappa)}$ (which includes τ and $\pi^{(A)}$) and κ are identified, but the three parameters g^{-1} ($1-g^{-1}$ is the steady state government spending output ratio), v , and ϕ are *not* identified. In the quadratic approximation scheme, all three are identified.

Since κ is the function of v and ϕ that is identified in the case of the linear approximation, we understand the authors' decision to parameterize in terms of κ . But κ is *not* a structural parameter, and so we find the assumption that κ is independent of v , τ , and $\pi^{(A)}$ *not* compelling. An implication of prior dependence between v and κ , *even in the linear approximation case*, is that prior beliefs about the unidentified v are updated to the marginal posterior $v|Y, M_1(L)$ [see Poirier (1998)]. The prior dependence between v and κ allows learning marginally about v as "spillover" from the learning about the identified parameter κ . Because g is a priori independent of κ , no learning about g occurs under $M_1(L)$.

We are interested in comparing the marginal learning for v in the linear and quadratic approximation cases, and in particular the value added from the quadratic approximation over the linear approximation. Note that for some elements in $\theta_{-(\kappa)}$ (namely, κ , $r^{(A)}$, $\pi^{(A)}$, σ_R , σ_G , and σ_Z), An and Schorfheide's priors differ across $M_1(L)$ (Table 2) and $M_1(Q)$ (Table 3). We instead propose using the prior in Table 3 for both the linear and the quadratic approximations with one important caveat. We wish to replace the marginal independent gamma priors for κ with a prior that exhibits *dependence* between κ and $[v, \tau, \pi^{(A)}]$. We do this by assuming the marginal prior density for ϕ^{-1} is the gamma density $p_\gamma(\phi^{-1}|\mathbf{a}, \mathbf{b})$ with mean \mathbf{a} and standard deviation \mathbf{b} . Then the conditional prior

distribution of $\kappa = [\tau(1-\nu)/\nu(1+.0025\pi^{(A)2})]\phi^{-1}$, given ν , τ , and $\pi^{(A)}$, is

$$p(\kappa|\nu, \tau, \pi^{(A)}) = \left| \frac{\nu(1+.0025\pi^{(A)2})}{\tau(1-\nu)} \right| p_{\gamma} \left(\frac{\kappa\nu(1+.0025\pi^{(A)2})}{\tau[1-\nu]} \mid \mathbf{a}, \mathbf{b} \right).$$

For the prior hyperparameters we suggest $\mathbf{a} = .057$ and $\mathbf{b} = .037$. Using this prior, we would like the authors to show us the marginal posteriors for ν given $M_1(L)$ and given $M_1(Q)$, The difference between these two posteriors will then measure the additional learning about ν from the quadratic approximation.

Finally, there are other possible benefits from the use of nonlinear estimation. For example, it makes possible to estimate non-normal economies. Therefore, it can allow researchers to include time variation in the volatility of the disturbances, an element that Sims and Zha (2004) found to be crucial in VARs, in DSGE models [the paper by Justiniano and Primiceri (2005) is a first example].

Additional References

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