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Investment and Monetary Policy: Learning and Determinacy of Equilibrium

We explore determinacy and expectational stability (learnability) of rational expectations equilibrium (REE) in New Keynesian (NK) models that include capital. Using a consistent calibration across three different models—labor-only, firm-specific capital, or an economy-wide rental market for capital, we provide a clear picture of when REE is determinate and learnable and when it is not under a variety of monetary policy rules. Our findings make a case for greater optimism concerning the use of such rules in NK models with capital. While Bullard and Mitra's (2002, 2007) findings for the labor-only NK model do not always extend to models with capital, we show that determinate and learnable REE can be achieved in NK models with capital if there is (i) plausible capital adjustment costs, (ii) some weight given to output in the policy rule, and/or (iii) a policy of interest rate smoothing.

JEL codes: D83, E43, E52
Keywords: determinacy, learning, e-stability, monetary policy, new Keynesian model, investment, firm-specific capital.

TAYLOR'S PRINCIPLE, THAT STABILIZING rule-based monetary policy requires a more-than-proportional rise in the central bank's target interest rate in response to higher inflation, has attracted considerable attention in the monetary policy literature. Woodford (2001, 2003a) shows that Taylor's principle ensures determinacy (local uniqueness) of rational expectations equilibria (REE) in New Keynesian (NK) models, that is, dynamic, stochastic general equilibrium models with imperfect competition and Calvo-style staggered price setting. Bullard and Mitra (2002, 2007) show that Taylor's principle further implies that the REE of NK models are learnable (expectationally stable in the sense of Evans and Honkapohja 2001) by agents who do not initially possess rational expectations. However, these results are obtained in "labor-only" versions of the NK model that lack capital or investment.

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Received August 29, 2008; and accepted in revised form December 27, 2010.

Journal of Money, Credit and Banking, Vol. 43, No. 5 (August 2011)
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It is important to add capital to forward-looking, NK models as such models are more general, allowing for an analysis of investment decisions, an important and volatile component of aggregate demand. As Woodford (2003a, p. 352) observes, "one may doubt the accuracy of the conclusions obtained [using the simple labor-only model], given the obvious importance of variations in investment spending both in business fluctuations generally and in the transmission mechanism for monetary policy in particular." Furthermore, in the labor-only model, both inflation and output are nonpredetermined "jump" variables (in the language of Blanchard and Kahn 1980); in the absence of serially correlated shocks, the forward-looking system would not display any dynamic persistence at all! The addition of capital—a predetermined, nonjump variable—to the NK model thus provides for endogenous dynamics and greater persistence.

Two approaches have been taken to adding capital to NK models. The first, and perhaps most straightforward, approach involves adding an economy-wide rental market for the capital stock. The capital good is demanded by firms for use in combination with labor to produce output and is free to flow to any firm in the economy in response to firms' demands for the capital good. A second approach, advocated by Woodford (2003a, ch. 5, 2005) and developed further by Sveen and Weinke (2005, 2007), imagines that capital is firm-specific; once the capital good has been purchased for use by a specific firm, that capital cannot be reallocated for use by other firms unlike in the economy-wide rental market model. Woodford's purpose in proposing this firm-specific model of capital was to make price adjustment by those firms who are free to adjust prices (under the standard Calvo pricing assumption) less rapid—more sticky—by comparison with the rental market model of capital. Indeed, an advantage of the firm-specific model of capital is that it does not require an unrealistically high degree of price stickiness to match empirical facts, as is the case in the rental market formulation.

Analyses of the impact of Taylor-type monetary policy rules in NK models with either a rental market for capital or with firm-specific capital have yielded more pessimistic findings with regard to the usefulness of Taylor's principle for insuring a *determinate* equilibrium outcome, relative to the labor-only model. For instance, Carlstrom and Fuerst (2005) show that in a NK model with a rental market for capital, Taylor's principle does not suffice to ensure equilibrium determinacy under a "forward-looking" policy rule where the interest rate is adjusted only in response to future expected inflation. Sveen and Weinke (2005) show that in a NK model with firm-specific capital, the Taylor principle does not suffice to ensure equilibrium determinacy even under a "current data" version of Taylor's rule.

The focus of Carlstrom and Fuerst (2005) and Sveen and Weinke (2005) is on the *determinacy* of REE in NK models with capital. Determinacy, or local uniqueness of equilibrium is desirable in that it enables policymakers to correctly anticipate the impact of policy changes (e.g., to the interest rate target) on inflation and output. However, as Bullard and Mitra (2002) emphasize, a "necessary additional criterion for evaluating policy rules" is that they induce REE that are *learnable* by agents who may not initially possess rational expectations knowledge and instead form forecasts using some kind of adaptive real-time updating process such as recursive least squares. The conditions under which a REE is learnable (or expectationally stable in the sense of Evans and Honkapohja 2001) will generally differ from the conditions under which that same REE is determinate, so it is important to check that *both* conditions are satisfied when evaluating policy rules.

The first contribution of this paper is that we study the *learnability* of REE in NK models with capital under a wide variety of policy rules that have appeared in the literature. With the exception of some limited analysis by Kurozumi and Van Zandweghe (2008)—discussed in the next section— the question of the learnability of REE in NK models with capital has not been previously explored. In addressing that question, we consider *all* of the interest rate policy rules that Bullard and Mitra (2002, 2007) explore using a "labor-only" version of the NK model. The main finding from our analysis of learning is that Bullard and Mitra's conclusions regarding the important role of the Taylor principle for the learnability of determinate REE in the labor-only version of the NK model do not necessarily carry over to NK models with both labor and capital—a finding that policymakers will want to bear in mind.

A second contribution of this paper is that we reexamine the (in)determinacy of REE issue within a framework in which three different NK models—the labor-only model, the model with a rental market for capital, and the model with firm-specific capital—are all calibrated using the *same* structural model parameterization. In their labor-only model, Bullard and Mitra (2002) use a different calibration of these parameters than are used, for example, by Sveen and Weinke (2005) in their firm-specific capital model and such differences can impact on the extent of the indeterminacy problem. Our consistent calibration approach provides the reader with a clear picture of the extent of the indeterminacy problem identified by Carlstrom and Fuerst (2005) and Sveen and Weinke (2005) across all three models and enables us to shed greater light on possible remedies. While confirming the indeterminacy problem, our results suggest that in many cases the severity of that problem can be reduced or even eliminated by small changes in a structural parameter or in the specification of a policy rule. Specifically, we find that determinate and learnable REE are more likely with (i) plausible capital adjustment costs, (ii) placing some weight on output in a Taylor-type policy rule, and (iii) pursuing a policy of interest rate smoothing. While all of these remedies have been explored to some extent by the authors mentioned above, our consistent calibration enables us to better assess the relative importance of these potential remedies.

Finally, we believe our findings make a case for greater optimism with regard to the use of Taylor-type monetary policy rules or the Taylor principle as a rough, though imperfect guide for ensuring determinacy and learnability of equilibrium. For instance, one important take-away message from our consistent calibration approach is that there exist empirically plausible specifications and calibrations of Taylor rules, for instance, Taylor's original 1993 policy rule specification and choice of policy weights—for which determinacy and learnability of REE are assured in all three versions of the NK model. Further, as our consistent calibration approach makes clear, in several cases the determinacy/learnability conditions in the labor-only model

TABLE 1

Summary of the Literature Examining the Impact of Policy Rules on the Determinacy and Learnability of REE in NK Models (Topics Addressed in These Papers are Indicated by an "X")

	Labor-only model		Rental capital mkt.		Firm-specific capital mkt.	
Paper	Determinacy	Learnability	Determinacy	Learnability	Determinacy	Learnability
Bullard and Mitra (2002)	X	X				
Dupor (2001)			X			
Carlstrom and Fuerst (2005)			X			
Sveen and Weinke (2005)			X		X	
Bullard and Mitra (2007)	X	X				
Kurozumi and Van Zandweghe (2008)			X	X		
Huang et al. (2009)			X		X	
This paper	X	X	X	X	X	X

are not very different from those for the two models with capital. Many of the policy implications that emerge from our analysis of NK models with capital reinforce some of the more general conclusions that Bullard and Mitra (2002) develop for the labor-only version of the NK model. Again, these findings should be of great interest to policymakers.

1. RELATED LITERATURE

The related literature on determinacy and learnability of REE in NK models is summarized in Table 1. Bullard and Mitra (2002, 2007) study determinacy and learnability of REE under various monetary policy rules in a *labor-only* version of the NK model. Dupor (2001) was the first to observe that the Taylor principle can induce an indeterminate REE in a continuous-time sticky price model with money and an economy-wide rental market for capital. Carlstrom and Fuerst (2005) show that Dupor's finding is sensitive to the continuous time framework he uses; in a discrete-time variant of Dupor's model, Carlstrom and Fuerst show that the Taylor principle can suffice to induce a determinate REE, in contrast to Dupor's finding, provided that the policy rule depends on *current* inflation. On the other hand, as noted earlier, if the policy rule depends on expected future inflation (a "forward-looking" policy), Carlstrom and Fuerst show that, consistent with Dupor's finding, the Taylor principle does not suffice to implement a determinate REE and will almost always implement an indeterminate equilibrium unless capital adjustment costs are sufficiently high.

Kurozumi and Van Zandweghe (2008) consider the robustness of Carlstrom and Fuerst's (2005) findings. They demonstrate that the indeterminacy result under the forward-looking rule is sensitive to the specification of the policy rule; if the rule gives weight to expected future inflation as well as to either *current* (and *not* expected future) output, or to the lagged interest rate, then the Taylor principle will suffice to implement a determinate REE in the model studied by Carlstrom and Fuerst. On the other hand, Huang et al. (2009) show that Kurozumi and Van Zandweghe's (2008) finding is sensitive to whether the elasticity of labor supply is assumed to be infinite (as Kurozumi and Van Zandweghe 2008 assume) or is at a finite and empirically plausible level, in which case indeterminacy remains a problem. However, they go on to show that if the model involves both sticky prices and sticky wages then forward-looking rules that also condition on current output or lagged interest rates generally serve to implement determinate RE equilibria. The latter finding applies in economies with either an economy-wide rental market for capital or a firm specific capital market.

Kurozumi and Van Zandweghe (2008) also briefly investigate whether learning (E-stability) might provide a remedy to the indeterminacy problem under the forward-looking rule studied by Carlstrom and Fuerst (2005) when no weight is given to either current output or to the lagged interest rate and there are no capital adjustment costs. When equilibrium is indeterminate, there will be many rational expectations solutions; the general solution form will allow for nonfundamental "sunspot" variables to matter. In an effort to resolve the indeterminacy problem, Kurozumi and Van Zandweghe suppose that agents are boundedly rational and make use of a fundamental or "minimal state variable" solution that does not condition on any sunspot variables; such solutions are just one (of many) possible solution representations when the REE is indeterminate. They show that, using such a rule, agents will learn the REE but only under conditions that also satisfy the Taylor principle, that is, if the weight attached to future expected inflation is sufficiently greater than 1. Under these assumptions, a sunspot-free REE can be reached, as long as rational agents are replaced with adaptive learners.

Our findings for the NK model with an economy-wide rental market for capital (the environment studied by Carlstrom and Fuerst 2005, Kurozumi and Van Zandweghe 2008, Huang et al. 2009) differ from these earlier findings in several important respects. First, we consider a wider variety of monetary policy rules than these papers consider; in particular, the policy rules considered by Carlstrom and Fuerst (2005), Kurozumi and Van Zandweghe (2008), and Huang et al. (2009) all involve future expected or current inflation whereas we also consider rules that involve lagged inflation or current expectations of inflation; the latter two rules are *operationalizable* in the sense of McCallum (1999) and may therefore be of greater interest and relevance to policymakers. Second, our learning analysis is restricted to REE that are determinate or locally unique under the given policy rule and we suppose that agents use fundamental, MSV solutions in an effort to learn REE; the minimal state variable (MSV) solutions are more appropriate in the case where REE is determinate as in that case, solutions that condition on nonfundamental sunspot variables cannot comprise RE solutions. Determinacy of equilibrium does not guarantee stability under learning and so it is important to check that determinate equilibria are also learnable as we do in this paper. By contrast as noted earlier, Kurozumi and Van Zandweghe primarily use learning dynamics as a means of resolving the *in*determinacy of equilibrium problem. Summarizing, our analysis focuses attention on REE that are both determinate and learnable as these are the equilibrium properties that should be of greatest interest to policymakers.

The discrete-time models of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) suppose there is *an economy-wide rental market for capital*. As noted in the introduction, Sveen and Weinke (2005) consider a discrete time version of the NK model without money but with firm-specific capital and convex capital adjustment costs. They show that this model is conceptually similar to the rental market for capital NK model—the key difference lies in the parameterization of the NK Phillips curve. However they go on to show that in the firm-specific capital NK model, the Taylor principle does not suffice to ensure determinacy of REE under a policy rule that responds to *current* inflation only. This finding stands in contrast to Carlstrom and Fuerst's findings for the NK model with a rental market for capital. Sveen and Weinke show that interest rate rules that respond to both current inflation and output or that involve some policy smoothing are better able to induce determinate REE than is an interest rate rule that responds only to current inflation. Huang et al. (2009) generalize these findings to settings where there is disutility from labor supply, both price and wage stickiness and habit persistence in preferences for consumption. Neither of these papers consider the learnability (E-stability) of REE in the NK model with firm-specific capital; another novelty of our paper is that we do explore this issue.

A general impression of this literature is that in NK models with capital, the Taylor principle does not suffice to insure determinacy of REE. However, much less is known about E-stability of the REE of these models; with the exception Kurozumi and Van Zandweghe (2008), no authors have explored the learnability of REE in NK models with capital, and the case of firm-specific capital has not been previously considered.[1] More generally, comparisons of determinacy and learnability results between models with and without capital (labor-only) have not been made and there is not much consistency in the choice of interest rate rules and model calibrations used across studies. In this paper, we provide a thorough and consistent analysis of determinacy and learnability of REE in three versions of the NK model—the first-generation labor-only models and the second-generation models with either a rental market for capital or firm specific capital. In addition, we consider the five main interest rate rules that have appeared in the literature: (i) a current data rule, (ii) a forward expectations rule, (iii) a lagged data rule, (iv) a contemporaneous expectations rule, and finally, (v) a policy-smoothing rule. Many of our findings, for example, the E-stability of REE in NK models with firm-specific capital under the five policy rules we consider, *are new*. Some other findings, for example, for the labor-only model, are previously known, but in the latter case the value added of our paper lies in considering a consistent calibration and set of policy rules across all model specifications (labor-only and the two models of capital).[2] Our approach provides the reader with the clearest available picture to date of the conditions under which Taylor-type interest rate rules work to implement determinate and learnable REE in the most commonly studied versions of the NK model of the monetary transmission mechanism (with or without capital).

1. Xiao (2008) also considers learnability of REE in an NK model with capital, but one that involves increasing returns in production.

2. Our analysis thus extends and encompasses that of Bullard and Mitra (2002, 2007).

2. A NEW KEYNESIAN MODEL WITH CAPITAL

2.1 The Environment

We consider two different environments that differ in their treatment of capital. Our benchmark model is one involving an economy-wide rental market for capital. The alternative model has firm-specific capital. The labor-only model is shown to be a special case of these two models.

Rental market for capital. The economy is composed of a large number of infinitely lived consumers. Each consumes a final consumption good C_t, and supplies labor N_t. Savings can be held in the form of bonds B_t, or capital K_t. Consumers seek to maximize expected, discounted life-time utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi} \right],$$

where $\sigma, \chi > 0$ and $0 < \beta < 1$. The budget constraint is given by

$$C_t + \frac{B_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \frac{R_t}{P_t} K_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t, \tag{1}$$

where P_t and i_t denote the time t price level and nominal interest rate and investment,

$$I_t = I\left(\frac{K_{t+1}}{K_t}\right) K_t. \tag{2}$$

The consumer's sources of income are its real labor income $(W_t/P_t)N_t$, its real capital rental income $(R_t/P_t)K_t$, its real return on one-period bonds B_{t-1} purchased in period $t-1$ and earning a gross nominal return of $1 + i_{t-1}$, and its dividends from ownership of firms, D_t. The consumers allocate this income among consumption C_t, new bond purchases B_t/P_t, and new investment I_t. To allow comparisons between this environment and the one with firm specific capital (described below), we follow Woodford (2003) and suppose that each firm faces *capital adjustment costs*. Denote these costs by $I(\frac{K_{t+1}}{K_t})$, where the function $I(\cdot)$ is assumed to satisfy the steady-state conditions: $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \varepsilon_\psi$. Here, $0 < \delta < 1$ denotes the depreciation rate and $\varepsilon_\psi > 0$ characterizes the curvature of the adjustment cost function.[3]

The first-order conditions for the consumer's problem can be written as:

$$N_t^{\chi} = C_t^{-\sigma} \frac{W_t}{P_t}, \tag{3}$$

3. The parameter ε_ψ has been interpreted as the elasticity of the investment/capital ratio with respect to Tobin's q, in the steady state.

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{4}$$

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} (1 + i_t), \tag{5}$$

$$\frac{dI_t}{dK_{t+1}} = \beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{R_{t+1}}{P_{t+1}} - \frac{dI_{t+1}}{dK_{t+1}} \right). \tag{6}$$

There exists a continuum of monopolistically competitive firms producing differentiated intermediate goods. The latter are used as inputs by perfectly competitive firms producing the single final good.

The final good is produced by a representative, perfectly competitive firm with a constant returns to scale technology

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where Y_{jt} is the quantity of intermediate good j used as an input and $\varepsilon > 1$ governs the price elasticity of individual goods. Profit maximization yields the demand schedule

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t, \tag{8}$$

which, when substituted back into (7), yields

$$P_t = \left(\int_0^1 P_{jt}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{9}$$

The intermediate goods market features a large number of monopolistically competitive firms. The production function of a typical intermediate goods firm is:

$$Y_{jt} = K_{jt}^{\alpha} N_{jt}^{1-\alpha}, \tag{10}$$

where K_{jt} and N_{jt} represent the capital and labor services hired by firm j.

These firms' real marginal cost φ_{jt} is derived by minimizing costs:

$$\varphi_{jt} = \frac{1}{(1-\alpha)} \frac{W_t}{P_t} \frac{N_{jt}}{Y_{jt}} = \frac{1}{\alpha} \frac{R_t}{P_t} \frac{K_{jt}}{Y_{jt}}. \tag{11}$$

From this we can derive the expression

$$\frac{K_{jt}}{N_{jt}} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t}, \tag{12}$$

which implies that the capital–labor ratio is equalized across firms, as is marginal cost itself.

Intermediate firms set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1 - \omega$ each period, independent of the time that has elapsed since the last price adjustment and does not reset its price with probability ω. A firm resetting its price in period t seeks to maximize:

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+i}} Y_{jt+i} - \varphi_{jt+i} Y_{jt+i} \right), \tag{13}$$

where P_t^* represents the (common) optimal price chosen by all firms resetting their prices in period t. This maximization problem yields the first-order condition,

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} Y_{jt+i} \left(\frac{P_t^*}{P_{t+i}} - \frac{\varepsilon}{\varepsilon - 1} \varphi_{t+i} \right) = 0. \tag{14}$$

The equation describing the dynamics for the aggregate price level is

$$P_t = \left[\omega P_t^{1-\varepsilon} + (1 - \omega) P_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{15}$$

Finally, market clearing in the factor and goods markets implies that: $N_t = \int_0^1 N_{jt}\, dj$, $K_t = \int_0^1 K_{jt}\, dj$, $Y_t = \int_0^1 Y_{jt}\, dj$, and $C_t + I_t = Y_t$.

Firm-specific capital. Woodford (2003a, 2005) proposes a different version of the NK model in which an economy-wide rental market for capital does not exist. Instead, firms are assumed to accumulate capital for their own use only. This assumption implies that a firm's price-setting decision is no longer separate from its capital accumulation decision (as it is in the rental market case), and this change leads to important changes in the dynamics of the NK model with capital. The main advantage of the firm-specific approach to capital accumulation is that it does not require an unrealistically high degree of price stickiness to match empirical facts relative to the NK model with economy-wide rental markets that was examined in the previous section.

With firm-specific capital, the model needs to be modified as follows. First, the consumer's budget constraint (1) is restated as

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t. \tag{16}$$

That is, consumers no longer make investment decisions given the absence of any economy-wide capital market.

Second, the firm's problem is now defined as

$$\max \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{t+i}}{C_t} \right)^{-\sigma} \left(\frac{P_{jt+i}}{P_{t+i}} Y_{jt+i} - \frac{W_{t+i}}{P_{t+i}} N_{jt+i} - I_{jt+i} \right) \tag{17}$$

subject to constraints (8), (10), where firm-specific investment is given by

$$I_{jt} = I\left(\frac{K_{jt+1}}{K_{jt}}\right)K_{jt}. \tag{18}$$

Notice that investment demand (18) is in the same form as (2) (i.e., it involves the same convex adjustment function $I(\cdot)$), but here it is *firm specific*. Note also that $P_{jt+i+1} = P_{jt+i}$ with probability ω.

Most first-order conditions, such as (3), (4), and (5), continue to hold in the NK model with firm-specific capital. However, three differences between this setup and the rental-market setup will eventually lead to differences in the dynamics of the model.

First, the first-order condition associated with capital is different in the firm-specific capital model than in the rental market for capital model (cf. (6)). Maximizing (17) with respect to capital yields:

$$\frac{dI_{jt}}{dK_{jt+1}} = \beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{MS_{jt+1}}{P_{t+1}} - \frac{dI_{t+1}}{dK_{t+1}}\right), \tag{19}$$

where MS_{jt+1} denotes the nominal reduction in firm i's labor cost associated with having an additional unit of capital in period $t+1$, and is derived from the firm's maximization problem as

$$MS_{jt} = W_t\frac{MPK_{jt}}{MPL_{jt}}, \tag{20}$$

where MPK and MPL represent firm j's marginal product of capital and of labor, respectively.

Second, marginal cost is now derived from the firm's maximization problem as

$$\varphi_{jt} = \frac{W_t/P_t}{MPL_{jt}}.$$

The critical feature here is that marginal costs are no longer equalized across firms. They depend on each firm's specific level of capital and labor.

Third, the first-order condition associated with P_{jt+i} looks identical to (14), but after substituting in the expression for marginal cost, pricing decisions become a function of firm-specific capital. Since a firm's marginal cost is affected by its current and future capital levels, its pricing decisions must also depend on its current and future capital levels. Future capital levels, on the other hand, depend in turn on today's price and the future prices set by the firm. This complicated mechanism is absent in the rental-market case. Woodford (2005) shows that a linearized inflation equation can be computed by applying the method of undetermined coefficients; we adopt his method in our later analysis.

Labor only model. For comparison purposes, we also study a version of the model in which labor is the only input in production. Setting $I = K = 0$ in our benchmark case will reduce the model to a generic, labor-only NK model. We assume production has constant returns to scale in labor:

$$Y_{jt} = N_{jt}.$$

The consumer's budget constraint is the same as (16), and the economy-wide resource constraint is simply $Y_t = C_t$. The key first-order conditions are (3), (4), (5), and (14).

2.2 Reduced Linear Systems

In the next three subsections we describe the system of linearized equations we use in our analysis of the determinacy and E-stability of REE in each of the three models that we consider. We use lower case letters to denote percentage deviations of a variable from its steady-state value.

Benchmark model: Rental market for capital. In the benchmark model with a rental market for capital, there are six nondynamic equations and four dynamic equations. The first equation is the linearized version of the labor supply schedule (3):

$$\chi n_t + \sigma c_t = w_t - p_t. \tag{21}$$

The second and third equations are the linearized versions of (11). We are interested in the average level of marginal costs, which are given by

$$\varphi_t = n_t + (w_t - p_t) - y_t, \tag{22}$$

$$= k_t + (r_t - p_t) - y_t. \tag{23}$$

The fourth equation is the linearized production function

$$y_t = \alpha k_t + (1 - \alpha) n_t. \tag{24}$$

The first dynamic equation is the NK Phillips curve, which is derived by solving the firm's dynamic price-setting problem and combining it with (15). This equation is given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \varphi_t, \tag{25}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.

The second dynamic equation is the linearized version of (6), which describes the evolution of capital:

$$\Delta k_{+1} = \beta E_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\{[1-\beta(1-\delta)]E_t(r_{t+1} - p_{t+1}) - (i_t - E_t\pi_{t+1})\}. \tag{26}$$

The third dynamic equation is the Euler equation (5), which can be linearized as

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}). \tag{27}$$

The last dynamic equation is the market clearing condition

$$y_t = \frac{C}{Y}c_t + \frac{K}{Y}[k_{t+1} - (1-\delta)k_t], \tag{28}$$

where C, I, and Y represent steady-state levels of consumption, investment, and output.

Finally, we add the interest rate rule and use the nondynamic equations to substitute out seven variables $k_t^* = \Delta k_{t+1}$, $w_t - p_t$, $r_t - p_t$, x_t, i_t, φ_t, and y_t. The system becomes a four-dimensional linear difference equation system consisting of $s_t = (c_t, n_t, k_t, \pi_t)'$:

$$E_t s_{t+1} = J s_t. \tag{29}$$

Firm-specific capital. With firm-specific capital, the NK Phillips curve becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa^* \varphi_t. \tag{30}$$

This equation is similar to (25), but the parameter κ^* differs from the parameter κ in (25). Woodford (2005) develops an algorithm that utilizes the method of undetermined coefficients to compute κ^*. Sveen and Weinke (2004) show that κ^* can be approximated by $\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\kappa$. Since $0 < \alpha < 1$ is capital's share of output and $\varepsilon > 1$ governs the price elasticity of individual goods, using Sveen and Weinke's approximation we have that $\kappa > \kappa^*$, so that inflation is *less* responsive to changes in marginal costs in the firm specific model of capital as compared with the rental market model of capital. That is, as Sveen and Weinke (2005) point out, for any given value of the Calvo sticky price parameter ω, prices will be *stickier* in the firm-specific model of capital than they will be in the rental market for capital model.

We compared Sveen and Weinke's approximation for κ^* with the results of applying Woodford's algorithm and we found almost no difference, even for our later sensitivity analysis that departs in certain dimensions from Sveen and Weinke's calibration. Nevertheless, in all our analysis we use Woodford's method to directly compute κ^*.

The marginal return to capital can be derived from (20) as

$$ms_t = w_t - p_t + n_t - k_t,$$

and the aggregate capital accumulation equation is a linearized version of (19):

$$\Delta k_{+1} = \beta E_t \Delta k_{t+2} + \frac{1}{\varepsilon_\psi}\{[1 - \beta(1-\delta)]E_t ms_{t+1} - (i_t - E_t\pi_{t+1})\}.$$

As in the rental market for capital case, the model with firm-specific capital can be reduced to a four-dimensional linear system of expectational difference equations with the same variables as in (29).

Labor-only model. The labor-only NK model can be reduced to the New Keynesian Phillips curve (NKPC) and the expectational IS curve,

$$\pi_t = \beta E_t \pi_{t+1} + (\sigma + \chi)\kappa y_t,$$
$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}),$$

together with the first-order condition (4). The model is closed by an interest rate policy rule.

2.3 Monetary Authority

The central bank sets the nominal interest rate i_t every period according to a simple, linear Taylor-type policy rule contingent on information about output and inflation. Following Bullard and Mitra (2002, 2007), we consider five variants of this interest rate rule. The first is the "contemporaneous data" rule:

$$i_t = \tau_\pi \pi_t + \tau_y y_t, \tag{31}$$

where $\tau_\pi \geq 0$ and $\tau_y \geq 0$, and i_t, π_t and y_t denote percentage deviations of the interest rate, the inflation rate, and output from their steady-state values. Rule (31) is a version of Taylor's original (1993) policy rule whereby the nominal interest rate changes with changes in current inflation and output.[4] The "Taylor principle" is that interest rate changes should be more than proportional to changes in inflation; in (31) this is captured by the restriction that $\tau_\pi > 1$. Note, however, that the Taylor

4. Taylor-type interest rate rules typically condition on inflation and output *gaps*, that is, deviations of inflation from a target level and of output from potential output, rather than on the *levels* of these variables by themselves. As the determinacy/learnability conditions of the systems we consider depend only on the magnitudes of the coefficients impacting on inflation and output *levels*, we choose to work with interest rate rules such as (31) (as well as the four other types of rules that follow) that condition on these levels only; of course, all of our findings will continue to apply to rules that condition on inflation and output *gaps*.

principle is not to be confused with a Taylor *rule* (such as (31)) which is an equation relating how the central bank's interest rate target changes in response to realizations of inflation, output, and possibly other variables.

A second Taylor-type policy rule that is commonly considered (e.g., by Clarida et al. 1999), is the "forward expectations rule":

$$i_t = \tau_\pi E_t \pi_{t+1} + \tau_y E_t y_{t+1}, \tag{32}$$

where policymakers use expectations of *future* inflation and output using information available at time t to determine the current interest rate target.

Since current data for output and inflation may not be available at time t, some have suggested restricting attention to the use of time $t-1$ data on output and inflation in the determination of the interest rate target. This consideration gives rise to the next two rules we consider. The third rule is the "lagged data" rule, which may be seen as an alternative to the current data rule (31). It is given by

$$i_t = \tau_\pi \pi_{t-1} + \tau_y y_{t-1}. \tag{33}$$

Similarly, the fourth rule we consider, the "contemporaneous expectations" rule may be seen as an alternative to the forward expectations rule (32) and is given by

$$i_t = \tau_\pi E_{t-1} \pi_t + \tau_y E_{t-1} y_t, \tag{34}$$

where policy depends on forecasts of output and inflation that are formed using data available through time $t-1$.

In addition to the above four rules, we also consider an interest rate smoothing rule, where the policymaker gives some weight ρ to past interest rates and remaining weight $1-\rho$ to the predictions of an interest rate rule such as rules 1-4 given above. Policy-smoothing rules have been considered by Bullard and Mitra (2007) for the labor-only model; results for the two models with capital have not been previously examined.

3. METHODOLOGY AND CALIBRATION

3.1 General Methodology

We now turn to our analysis of the determinacy and E-stability of REE under the three models and five different interest rate rules. When we study E-stability properties, we focus only on REE that are determinate.[5] We use the benchmark model to explain our general methodology. Precise conditions for E-stability of equilibrium under all policy rules considered in this paper are available from the authors on request.

5. For an analysis of the E-stability properties of indeterminate rational expectations equilibria, see, for example, Honkapohja and Mitra (2004) and Evans and McGough (2005).

The determinacy of REE is assessed by computing the eigenvalues of the system (29). Since there is only one predetermined variable k_t and the system is of dimension four, the REE will be determinate in this case if the number of explosive roots is three and the number of stable roots is one (Blanchard and Kahn 1980). If the number of stable roots exceeds one, we have an indeterminate REE. If there is no stable root, the system is explosive.

To study adaptive learning, we rewrite the system as

$$b_z z_t + b_k k_t = d_k E_t k_{t+1} + d_z E_t z_{t+1}, \tag{35}$$

$$k_{t+1} = e_z z_t + e_k k_t, \tag{36}$$

where the second equation is derived from the capital accumulation equation, which does not involve any expectations and so does not need to be learned. We assume that agents use the perceived law of motion (PLM)

$$\begin{aligned} z_t &= a_1 + \psi k_t, \\ k_t &= a_2 + m k_t, \end{aligned}$$

which is in the same form as the MSV RE solution. By contrast with RE, learning agents do not initially know the parameter vectors a_1, a_2, ψ, and m and must learn these over time. Given the PLM, we calculate the forward expectations as

$$\begin{aligned} E_t k_{t+1} &= a_2 + m k_t, \\ E_t z_{t+1} &= a_1 + \psi E_t k_{t+1} = a_1 + \psi a_2 + \psi m k_t. \end{aligned}$$

Substituting these expressions into (35), we obtain a T-mapping from $(a_1, a_2, \psi, m)'$ to the actual law of motion of the model. Following Evans and Honkapohja (2001), we say the REE is E-stable (learnable by adaptive agents) if the differential equation, $\frac{d}{d\tau}(a_1, a_2, \psi, m) = T(a_1, a_2, \psi, m) - (a_1, a_2, \psi, m)$, evaluated at the REE solution, is stable. This condition requires that all eigenvalues of $D[T(a_1, a_2, \psi, m) - (a_1, a_2, \psi, m)]$ evaluated at the REE have real parts that are less than zero. Evans and Honkapohja (2001) provide conditions under which this differential equation approximates the limiting behavior of the recursive algorithms that characterize adaptive agent learning.

It is worth pointing out that assumptions about the agents' information set can be crucial in assessing E-stability results. In the baseline case outlined earlier, we implicitly assume that both the private sector and the central bank can observe current values of the variable k_t. They use this information to obtain forecasts $E_t z_{t+1}$ and $E_t k_{t+1}$, which in turn determine the current values of z_t. This assumption applies in models using the current data rule or the forward expectation rule. However, this assumption is sometimes criticized as being unrealistic, since current data are usually not available to economic agents.[6] An alternative assumption is to assume

6. The case with the current data rule is especially controversial. As pointed out by Bullard and Mitra (2002), it implies that the central bank has "superior information" in that it reacts to current values of y_t and π_t while the private sector does not possess such information.

TABLE 2

Calibrations Used in Our Numerical Analyses, Quarterly Frequency

Parameter	Description	Value
α	Capital's share of output	0.36
β	Period discount factor	0.99
σ^{-1}	Intertemporal elasticity of substitution	0.5
χ^{-1}	Labor supply elasticity	1
ε	Elasticity of substitution between varieties of consumption goods	11
δ	Depreciation rate	0.025
ε_{ψ}	Curvature of the adjustment cost function	3
$1-\omega$	Fraction of firms free to adjust prices each period	0.25
κ	Parameter relating to the degree of price stickiness	0.086[a]

[a]As implied by the relationship $\kappa = (1-\omega)(1-\beta\omega)/\omega$ – see Walsh (2010) for a derivation. We estimate the value of κ^* following the procedure in Woodford (2005).

that the agents can observe current exogenous variables but only lagged values of the endogenous and state variables at time t. We apply this assumption in models using the lagged data rule or the contemporaneous expectations rule. Both the central bank and the private sector are assumed to have symmetric knowledge of the lagged data. With these assumptions, we derive the specific E-stability conditions for each interest rate rule.

3.2 Calibration

Table 2 provides the calibration of model parameters that we use in our analysis of all three models, labor-only, rental market for capital, and firm-specific capital (also used by Sveen and Weinke 2005). Note that Bullard and Mitra's calibration (2002, 2007) differs in some respects from the calibration we adopt, for example, in the intertemporal elasticity of substitution and the parameter relating to the degree of price stickiness among other parameters.[7]

The reader may have noticed that we have excluded exogenous disturbance processes from all three versions of the NK model we have considered. This was in the interest of simplicity, as our determinacy and learnability findings do not depend in any way on the calibration of these shock processes.

3.3 Determinacy and Learnability of REE under Various Interest Rate Rules

Ideally, we would like to provide analytic results concerning the determinacy and learnability of REE under various interest rate rules. Unfortunately, except in a few special cases, such as those studied by Bullard and Mitra (2002) and Carlstrom and Fuerst (2005), analytic results are not possible. The reason for this is simple: with the addition of capital, the dimension of the systems we are considering is either four or

7. Results using Bullard and Mitra's calibration are available from the authors on request.

five and too complicated to reduce to a system that would allow for analytic findings. This situation necessitates that we adopt a numerical approach. Still, to the extent possible, we will try to provide some intuition for our numerical findings.[8]

Our approach is as follows. In all simulation exercises, we vary the weights τ_π and τ_y, in the various interest rate rules. The ranges allowed for these weights cover all empirically relevant cases; in particular, we search over a fine grid of values for τ_π between 0 and 5 and for τ_y between 0 and 4. We use an increment stepsize of 0.02. For each possible pair of weights $[\tau_\pi, \tau_y]$ in this grid, we check whether the eigenvalues satisfy the conditions for (i) determinacy and (ii) E-stability. If the REE is indeterminate, we do not consider whether it is E-stable; such regions are simply labeled "indeterminate" and are not shaded—the white or blank regions in the figures below. If both conditions are satisfied, we indicate this in the figures below with some shading and the label "determinate and E-stable." If the REE is determinate but not E-stable, we use a different shading and the label "determinate and E-unstable." Finally, we use a different shading to indicate weight pairs for which all roots are explosive (i.e., greater than one), and we label such regions "explosive."

Current data rule. Determinacy and E-stability findings using the current data rule (31) are shown in the three panels of Figure 1 . The top panel shows the labor-only model while the two bottom panels show the rental- and firm-specific models of capital. (Subsequent figures have a similar layout.) Values of the monetary policy rule weight, τ_π, are indicated on the horizontal axis and values of the monetary policy rule weight, τ_y, are indicated on the vertical axis in these (and all subsequent) figures. Notice that under the current data rule, all three figures show that the REE is either determinate and E-stable (shaded regions) or indeterminate (unshaded, white regions). The reason for this finding is that, in the case of the current data rule (31), if a REE is determinate, it is also E-stable in all three models.

The coincidence of the determinacy and learnability conditions under the current data rule in the labor-only model is known from the work of Bullard and Mitra (2002). They show that REE will be both determinate and learnable provided that (in our notation) $\tau_\pi + \frac{1-\beta}{(\sigma+\chi)\kappa}\tau_y > 1$, which they refer to as the "long-run" Taylor principle following Woodford (2003a, chap. 4). Notice that if $\tau_y = 0$, as in a pure inflation targeting rule, this condition reduces to what we shall term the "simple" Taylor principle: $\tau_\pi > 1$. Using our model calibration, it can be shown that Bullard and Mitra's long-run Taylor principle inequality precisely characterizes the border between determinacy/E-stability and indeterminacy/E-instability seen in Figure 1 for the labor-only model.[9]

For some intuition, suppose this inequality is not satisfied, so that the nominal interest rate adjusts less than proportionately to an increase in inflation. Then, higher inflation is associated with lower real interest rates. Under rational expectations, an

8. The Matlab code we used in our numerical analysis is available on request.

9. For models with capital, we are unable to derive a similar condition due to the higher dimensionality of the model. However, from Figure 1 it seems clear that a similar, linear restriction between τ_π and τ_y exists in the two NK models with capital.

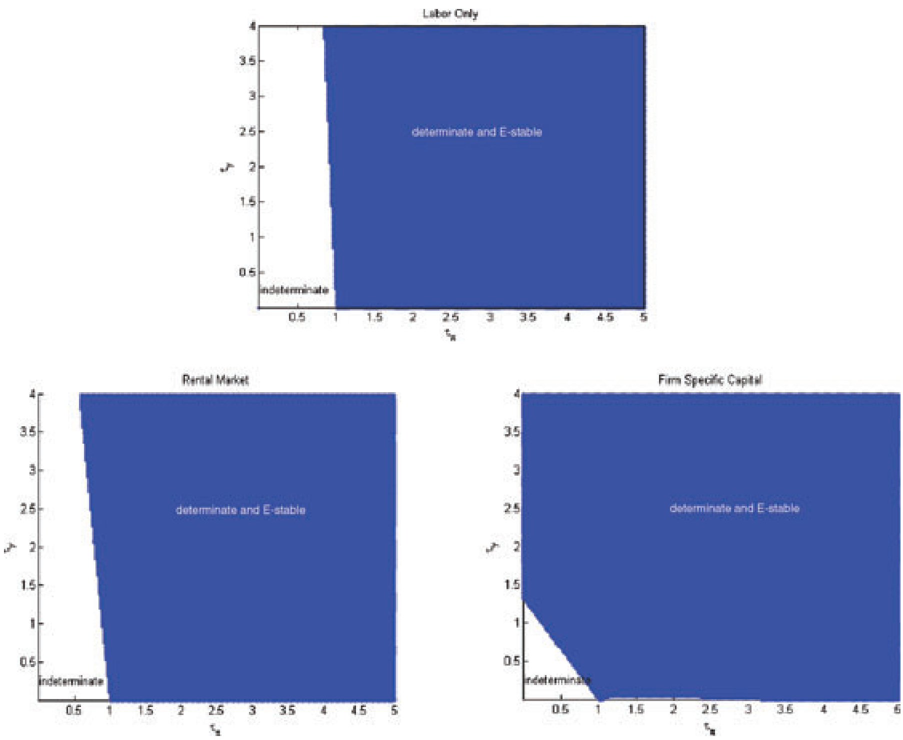

FIG. 1. Determinacy and E-Stability Results under the Current Data Rule.

increase in $E_t\pi_{t+1}$ or under learning, an upward departure of inflation forecasts from rational expectations values is associated with a lower real interest rate and an increase in the output gap, y_t, via the expectational IS equation, which serves in turn to ratify the increase in π_t via the NKPC. Under rational expectations the increase in $E_t\pi_{t+1}$ is self-fulfilling, while under learning there is no mechanism to reverse a departure of expectations from rational expectations: the higher realization of inflation will lead agents to adjust their next forecast of inflation still higher.

The coincidence of the determinacy and learnability results under the current data rule in the NK models with a rental or firm-specific approach to capital is a new finding of this paper. For both models, the intuition is similar to that given for the labor-only model, though deriving a meaningful analytic condition such as the long-run Taylor principle is not possible in NK models with capital given the greater dimensions of those models.[10] For the firm-specific capital model, while determinacy implies learnability, we also observe that the "simple" Taylor principle, $\tau_\pi > 1$, no longer

10. Kurozumi and Van Zandweghe (2008) provide a similar Taylor-principle type analytic condition for determinacy (but not learnability) of REE in the rental-market model of capital, though their conditions are derived under an interest rate rule that gives weight to *future expected inflation*, $E_t\pi_{t+1}$, and to current

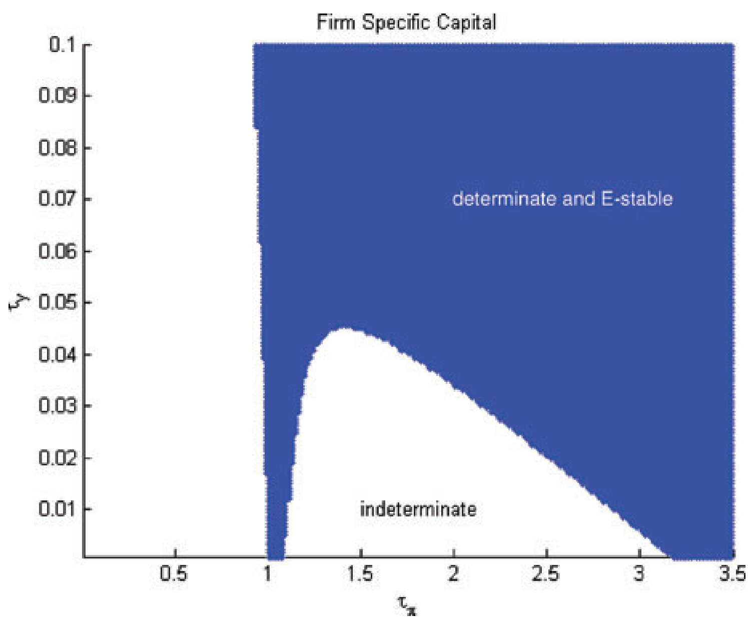

FIG. 2. Blown-Up View of the Indeterminate and E-unstable Region of the Firm-Specific Model of Capital under the Current Data Rule.

suffices to insure that the REE is determinate. Notice that in the firm-specific capital model (bottom right panel of Figure 1), there is a very small "sliver" where $\tau_y \approx 0$ and $\tau_\pi > 1$, where the REE is both indeterminate and unstable under learning. While this sliver may seem inconsequential in our figure, in the case of policy rules that condition only on inflation ($\tau_y = 0$) this region is a very plausible range for inflation responses and is therefore a very important region of the parameter space to consider.

Figure 2 provides a blown-up view of this region. The presence of this region of indeterminacy is consistent with Sveen and Weinke's (2005) findings under a current data rule that gives zero weight to output [$\tau_y = 0$]. Indeed, they report that for various values of ω, measuring the stickiness of prices (which include our calibrated value $\omega = 0.75$), the Taylor principle may not suffice to guarantee determinacy of the REE unless sufficient weight is given either to real activity or to lagged interest rates (as in a policy-smoothing rule).

The intuition for the difference between the rental market and firm specific capital cases must lie with the different parameterizations of the NKPC (25) and (30) since, as Sveen and Weinke (2005) note, this is the only difference between the two linearized versions of the models with capital. Recall that the difference between these two NKPC equations lies in the coefficient on marginal costs φ_t, that is, κ in the rental

output (or its components consumption, investment)—a hybrid rule that differs from the current data rule (31) that we consider here.

market case and κ^*, in the firm-specific case, with $\kappa > \kappa^*$. Why is this the case? In the case of firm-specific capital, an increase in the demand for a firm's output will raise the firm's marginal costs, but as these are specific to the firm it will not affect the marginal costs of other firms. Thus, in the firm-specific model, a firm that is experiencing increased demand (and is free to adjust prices) will take into account the impact of price changes on its future relative demand and adjust prices *less* than it would in the rental market model where changes in marginal costs are homogeneous across all firms. The resulting decline in the firm's future relative demand leads to a fall in its future relative marginal cost as well, which reinforces the incentive to avoid a large price increase today. Consequently, price setting is more forward looking (and will appear to be much more sluggish) in the firm-specific case relative to the economy-wide rental market case, where capital is perfectly mobile across firms making the marginal costs firms face independent of the demand for their output.

To see why stickier price adjustment might lead to indeterminacy under the current data rule, consider whether an exogenous (sunspot driven) investment boom could be self-fulfilling. The answer depends on how it affects current and future marginal costs and inflation and on how capital is modeled. Under a rental market for capital, the increase in investment demand will immediately drive up the marginal costs that all firms face and via the NKPC, will increase current inflation. An activist monetary policy focused on current inflation only ($\tau_\pi > 1$, $\tau_y = 0$) responds to the increase in current inflation by raising interest rates, thereby, killing off the speculative investment boom. By contrast, under a firm-specific model of capital because investment is firm specific, price setting is more strategic (forward looking) with the result that price adjustment (by those firms free to adjust prices) is more sluggish. An increase in investment will raise marginal costs, but the impact on inflation will be reduced relative to the rental market case for the reasons given earlier. Furthermore, in the firm-specific model, the increase in firm-specific investment will lead to lower, future firm-specific marginal costs, lower future inflation, and hence lower future real interest rates, and with the more forward-looking view of firms making firm-specific investments, this can serve to make the investment boom self-fulfilling. Note that this indeterminacy possibility would be reduced, if not eliminated, if monetary policy also put some weight on current output, as the investment boom would increase y_t and lead to an even higher increase in current interest rates.

Alternatively put, *for the baseline calibration we use*, $\omega = 0.75$, prices will be sufficiently flexible in the rental market model of capital to avoid the indeterminacy outcome when the Taylor principle holds, but the same will not be true in the firm-specific model of capital.[11] As we shall see, this same "sliver of a region" of indeterminacy/E-instability in the firm-specific capital model can also arise under all four of the interest rate rules we consider that do not involve policy smoothing.

11. For calibrations other than the one we consider, for example, higher, but empirically implausible values for ω, the small sliver of indeterminacy we observe for the firm-specific model of capital under the current data rule when $\tau_y \approx 0$ will also appear in the rental market model of capital under the current data rule so that the Taylor principle will not suffice to insure determinacy and learnability of REE *for such calibrations*.

Adding some policy inertia may work to eliminate this region of indeterminacy as will be shown later in the paper.

Of course, a judicious (and empirically plausible) choice of policy rule weights will also ensure that the REE is determinate and learnable in all three models under a current data rule. For instance, Taylor's (1993) original calibration of the (current data) Taylor rule, adapted for the quarterly frequency of our calibrations, has $\tau_\pi = 1.5$ and $\tau_y = 0.125$.[12] This calibration succeeds in implementing a determinate and learnable REE in all three models as Figures 1 and 2 confirm. The clear recommendation that follows from our findings using the current data rule is that the Taylor principle, in tandem with some positive weight being given to real activity will reliably implement both a determinate and learnable REE in models with capital.

Aside from policy rule changes, we can also eliminate the sliver of indeterminacy in the firm-specific capital model under the current data rule by assuming more flexible prices, for example, values of ω that are closer to zero, which raises κ and hence κ^*.[13] Alternatively, holding ω fixed, we can reduce α or ϵ or both, which will also increase κ^*. The impact of such changes (higher values for κ^*) on the area of indeterminacy in the firm-specific model under the current data rule are shown in Figure 3, where κ^* is varied from 0.005 to 0.01 to 0.025 to 0.035. We see that for sufficiently high levels of κ^*—our baseline calibration value is 0.012—the indeterminacy problem is eliminated.

The forward expectations rule. Determinacy and E-stability results for the forward expectations rule (32) in the three models are shown in Figure 4. Under this rule, the Taylor principle *does not* suffice to insure determinacy and learnability of REE in *any* of the three NK models and there are large differences in the regions giving rise to determinate and learnable REE across the three models. Specifically, the addition of capital either via an economy-wide rental market or via firm-specific demand leads to a big reduction in the parameter region for which REE is determinate and learnable relative to the labor-only case. An important observation from Figure 4 is that in models with capital, the weight assigned to output under a forward expectations rule that obeys the Taylor principle should neither be too aggressive nor too modest. Notice, however, that the weight regions giving rise to determinate, E-stable REE appear to be empirically plausible ones. For instance, Taylor's (1993) calibration, adapted for the quarterly time frequency of our model will again work to insure determinacy/learnability of the REE in all three NK models.

The finding that the Taylor principle does not suffice for *determinacy* of REE in the *labor-only model* under the forward expectations rule was previously shown by Bullard and Mitra (2002). They showed that learnability (of MSV REE solutions) continues to be guaranteed by the long-run version of the Taylor principle,

12. As noted by Woodford (2003, p. 245) and Gali (2008, p. 83), Taylor's original calibration of the weight on the output gap is 0.5, but Taylor used annualized rates for interest and inflation. Thus, Taylor's calibration of the weight on the output gap under our quarterly model frequency is, appropriately, $0.5/4 = 0.125$.

13. However, Sveen and Weinke (2007) show that if wages are also modeled as being sticky, much lower values for ω (greater price flexibility) does not eliminate the indeterminacy problem.

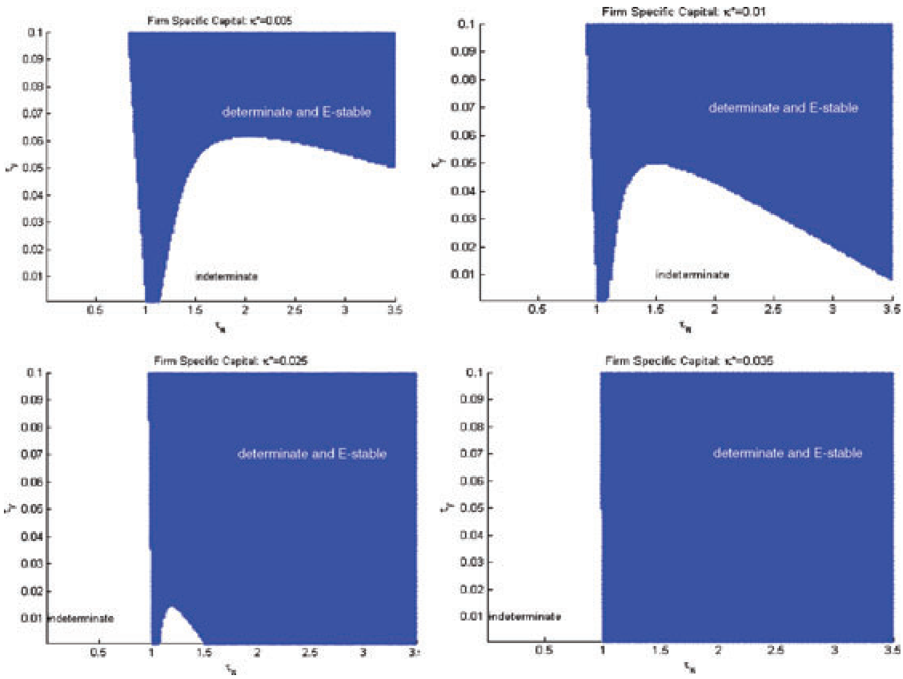

FIG. 3. Determinacy and E-Stability Results in the Firm-Specific Model under the Current Data Rule and Various Values for κ^*.

$\tau_\pi + \frac{1-\beta}{(\sigma+\chi)\kappa}\tau_y > 1$, but this same condition was shown to be necessary but not sufficient for determinacy of equilibrium. To ensure a determinate and learnable REE requires a further upper bound on τ_y, a condition we also find in the two NK models with capital. As we show below, this upper bound on τ_y in the NK models with capital varies with capital adjustment costs.

The fact that the forward expectations rule (32) makes it more likely (relative to the labor-only case) that REE is indeterminate in the rental market for capital model finds support in the work of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008), while our finding for the firm-specific model of capital is new. Carlstrom and Fuerst and Kurozumi and Van Zandweghe show that the use of a forward expectations rule *nearly always* results in an indeterminate REE in the rental market for capital model, whereas we find that there exists a large and plausible set of policy weights for the forward expectations rule under which the REE is both determinate and learnable in the rental market case, as well as in the firm-specific capital market case.[14] What accounts for our different finding?

14. Indeed, Kurozumi and Van Zandweghe (2008, p. 1498) state that, under the forward expectations rule, the addition of "[a] policy response to expected future output cannot ameliorate the indeterminacy problem."

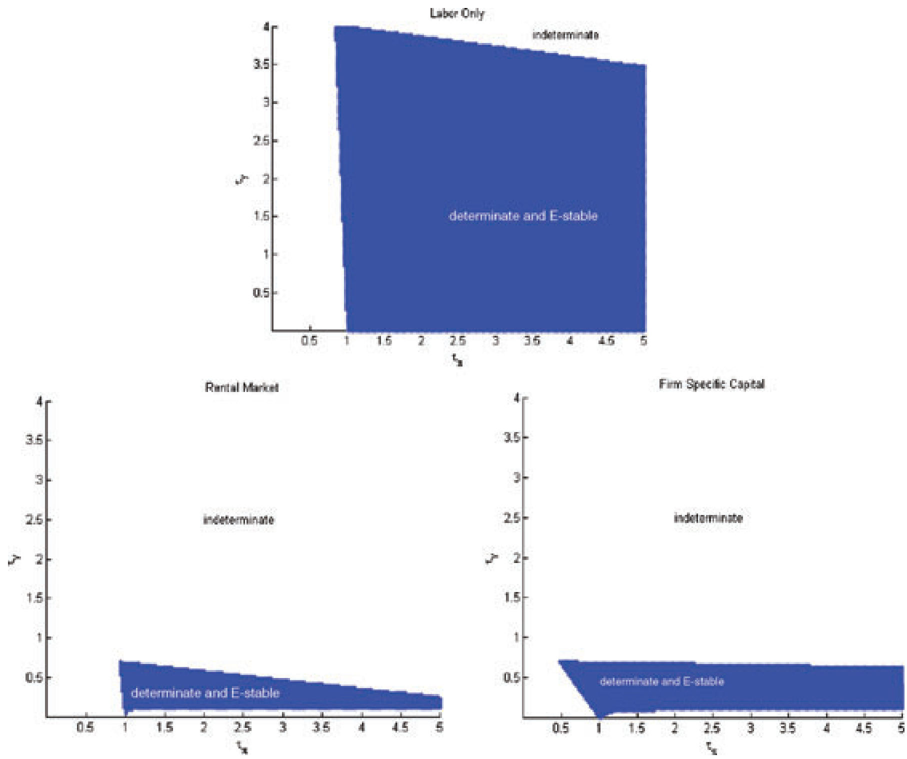

FIG. 4. Determinacy and E-Stability Results under the Forward Expectations Rule.

We begin by noting that the addition of investment to the NK model imposes an arbitrage relationship between the return on bonds and capital. As Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) argue, under a purely forward-looking policy rule, this arbitrage relationship will hinge entirely on future expected variables, producing a zero eigenvalue in the system that is *not* present in the system where the interest rate rule conditions on *current* variables. This zero eigenvalue forces the only state variable, the capital stock, to become a jump variable, and this insures that the equilibrium will be indeterminate.

More intuitively, suppose that under the forward expectations rule we have an activist policy regime with $\tau_\pi > 1$ and $\tau_y = 0$ and a sunspot driven increase in expected future inflation $E_t\pi_{t+1}$. Given the active policy response, this leads to an increase in the real interest rate that, via the arbitrage relationship between bonds and capital, leads to an increase in the future real rental price of capital, $\frac{R_{t+1}}{P_{t+1}}$, and, via (23) to an increase in future real marginal costs, φ_{t+1}. The rise in future real marginal costs leads, via the NK Phillips curve to a self-fulfilling increase in π_{t+1}.

The critical feature in the model we consider, and the reason our results differ from Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) is that we include *capital adjustment costs* in both versions of the NK model with capital.[15] Capital adjustment costs make capital accumulation dependent on *current* and not just future capital; this makes the arbitrage relationship not entirely forward looking and works to eliminates the zero eigenvalue, which, in combination with a purely forward-looking interest rate rule, will implement a determinate REE in certain cases, that is, with sufficiently high costs of adjustment.[16]

To see this more clearly, consider how the first-order condition, (6), changes if we *do not* assume capital adjustment costs. In place of (2) we instead follow Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) and suppose that $I_t = K_{t+1} - (1-\delta)K_t$. In that case, the first-order condition (6) is replaced by:

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}(1+i_t). \tag{37}$$

Combining (37) with (5) we have the arbitrage relationship:

$$\frac{(1+i_t)}{1+\pi_{t+1}} = \frac{R_{t+1}}{P_{t+1}} + 1 - \delta, \tag{38}$$

where for simplicity we have assumed perfect foresight. By contrast, under the model *with* capital adjustment cost, the combination of first order conditions (6) and (5) yields a different arbitrage relationship (again under perfect foresight):

$$\frac{(1+i_t)}{1+\pi_{t+1}} = \frac{\dfrac{R_{t+1}}{P_{t+1}} - \dfrac{dI_{t+1}}{dK_{t+1}}}{\dfrac{dI_t}{dK_{t+1}}}. \tag{39}$$

Linearized versions of the two arbitrage conditions (38) and (39), which do not assume perfect foresight (respect expectations), are given by:

$$(i_t - E_t\pi_{t+1}) = [1-\beta(1-\delta)]\, E_t(r_{t+1} - p_{t+1}), \tag{40}$$

$$(i_t - E_t\pi_{t+1}) = [1-\beta(1-\delta)]\, E_t(r_{t+1} - p_{t+1}) - \epsilon_\psi\left(\beta E_t \Delta k_{t+2} - \Delta k_{t+1}\right), \tag{41}$$

where $\Delta k_{t+i} = k_{t+i} - k_t$. Notice that the only difference between (40) and (41) is the additional right-hand-side term $-\epsilon_\psi(\beta E_t \Delta k_{t+2} - \Delta k_{t+1})$ in the latter. This additional term in (41) breaks the direct link between real interest rates and marginal costs that

15. The inclusion of capital adjustment costs is a standard practice in neoclassical investment theory. Carlstrom and Fuerst (2005) briefly discuss the addition of capital adjustment costs to the rental market for capital model they examine, and note that such adjustment costs may overturn their conclusions for forward-looking policy rules. However they consider a simpler, exponential form of capital adjustment costs. Kurozumi and Van Zandweghe (2008) do not consider capital adjustment costs.

16. An alternative mechanism for achieving the same end, as pursued by Kurozumi and Van Zandweghe (2008), is to have a *hybrid* policy rule that conditions on future expected inflation but on *current* output or its components (consumption, investment).

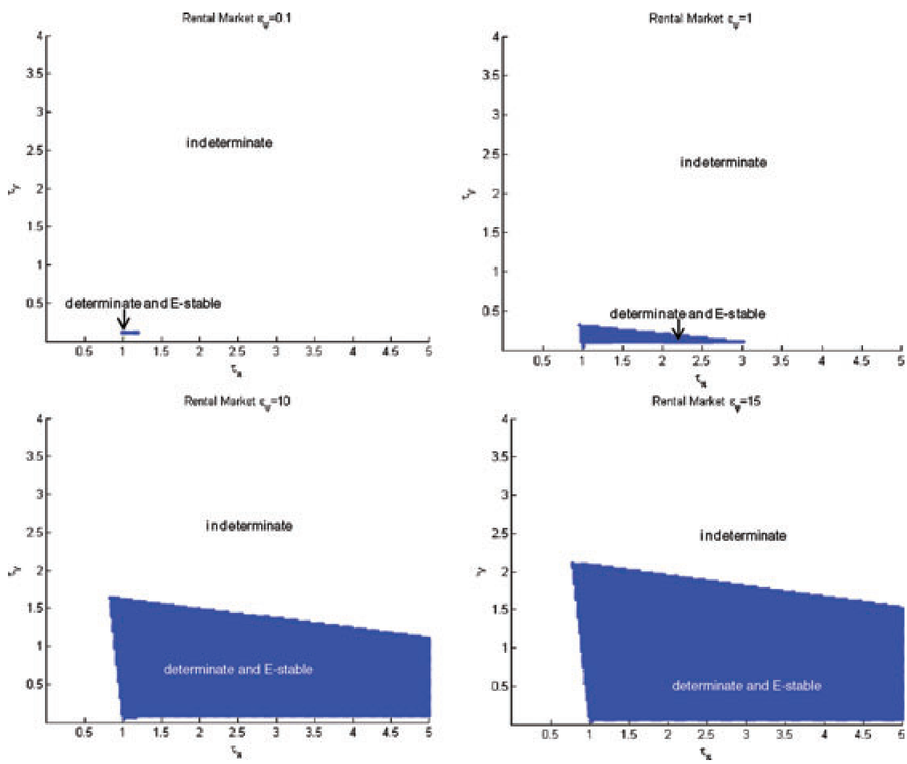

FIG. 5. Determinacy and E-Stability Results under the Forward Expectations Rule in the Rental Market for Capital for Various Values of the Parameter ε_{ψ} (0.1, 1, 10, and 15) Characterizing the Degree of Adjustment Costs.

was critical to the self-fulfilling inflation scenario described earlier. Operationally, we see that in (41) the current capital stock plays a role in determining the real interest rate; that is, the system is not entirely forward looking, a feature that serves to eliminate the zero eigenvalue as discussed earlier.

To establish that capital adjustment costs are responsible for our different determinacy/E-stability findings under the forward expectations rule, Figure 5 shows the consequences of varying these adjustment costs in the *rental* market model of capital (similar results obtain if we vary adjustment costs in the firm-specific model of capital). More precisely, we vary the parameter governing the curvature of the capital adjustment cost function, ε_{ψ}, in the four panels of Figure 5 from a value very close to 0–0.1 (i.e., no adjustment costs), to 1, and then to 10, and finally to 15. Recall that our baseline calibration had $\varepsilon_{\psi} = 3$ (compare Figure 5 with Figure 4, rental market case). When there are no/very small capital adjustment costs, as in the upper left panel of Figure 5, REE is always indeterminate under the forward expectations rule. As capital adjustment costs increase, firms increasingly avoid adjustments to their capital stock and the model increasingly resembles the labor-only model. In

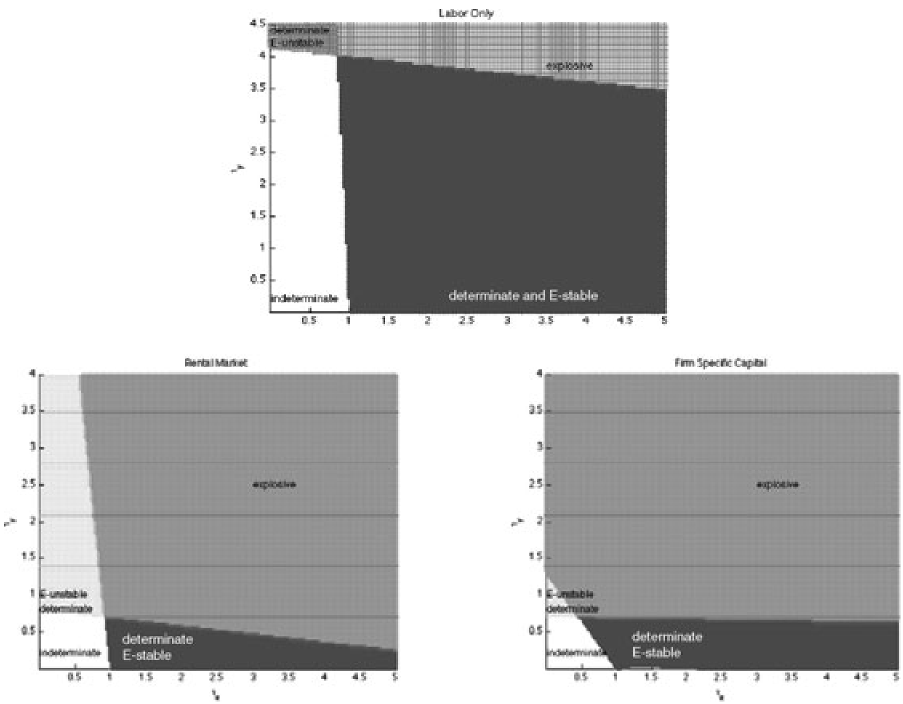

Fig. 6. Determinacy and E-Stability Results under the Lagged Data Rule.

policy terms, the increase in capital adjustment costs also increases the upper bound on τ_y that, together with the long-run Taylor principle condition, suffice to insure determinacy and learnability of REE under the forward expectations rule.

Figure 5 shows clearly that when ε_ψ is close to zero, there are essentially no weight pairs for which the REE is both determinate and E-stable, consistent with the findings of Carlstrom and Fuerst (2005) and Kurozumi and Van Zandweghe (2008) (e.g., compare the upper left panel of our Figure 5 with Figure 1 in Kurozumi and Van Zandweghe 2008). As ε_ψ is steadily increased above zero, the determinacy/E-stability region increases as well, which is consistent with the intuition we have provided: the increasing convexity of adjustment costs means investment becomes both more costly and more tied to the current level of the capital stock; as the latter variable is predetermined, it makes the indeterminacy (and E-instability) outcome less likely.

The lagged data rule. Results for the lagged data policy rule (33) in the three models are shown in Figure 6. In these figures, four different regions with different labels are now visible. We observe that there exist weight pairs (τ_π, τ_y) for which the REE is both determinate and learnable, or determinate but not learnable (E-unstable), or where all roots are explosive and finally the case where the REE is indeterminate (unshaded regions). The top panel of Figure 6 depicting the labor-only model, has an expanded range of $[\tau_\pi, \tau_y]$ pairs in order to reveal all four possibilities.

Under the lagged data rule we observe that *in all three models*, the simple Taylor principle $[\tau_\pi > 1]$ does not suffice to insure both a determinate and learnable REE. This finding, for the labor-only model only, was earlier reported by Bullard and Mitra (2002); indeed, the labor-only case, (top panel of Figure 6) is as in Bullard and Mitra (2002, Figure 2). The novel finding we report under the lagged data rule is for the two models *with* capital: the determinate and learnable parameter regions in the models with capital are considerably smaller relative to the labor-only model (using our own baseline calibration) suggesting that a much more modest response to output is needed for determinacy and learnability of REE. This finding is similar to what we found under the forward expectations rule; indeed, we have verified that for both models with capital, the upper bound on τ_y increases with increases in capital adjustment costs (the graph is similar to Figure 5 and for this reason we do not show it here). In the firm-specific model of capital there is again a small sliver of indeterminate REE for values of τ_y that are close to 0 and values for τ_π between 1 and 3.

Notice that the upper bound on τ_y needed to ensure determinacy and learnability of REE under the lagged data rule is to prevent REE from becoming *locally explosive*, that is, diverging away from the steady state and *not* from becoming *indeterminate*. However, the same transition from a determinate/learnable REE to an explosive system occurs under the labor-only model as shown by Bullard and Mitra (2002). Thus it seems that the addition of capital to the NK model is not the cause of this transition in the dynamics of the system.

Notice further in Figure 6 that under the lagged data rule, determinate but E-unstable equilibria exist in all three models, but only for values of $\tau_\pi < 1$ and for sufficiently large values of τ_y, while for $\tau_\pi < 1$ and low values of τ_y, REE is indeterminate. Similar findings using the lagged-data rule in the labor-only NK model were previously documented by Bullard and Mitra (2002), so again, our new determinacy and learnability findings for the two NK models with capital do not appear to result from adding capital to the system. Policy-based intuition for these findings is difficult to provide as determinacy of REE under the lagged policy rule can obtain in all three models for *all* values of τ_π (even $\tau_\pi < 1$) provided that τ_y lies in some narrow bands; for this reason, the lagged data rule may be undesirable for practical use. On the other hand, we can claim that a necessary condition for *E-stability* of *determinate* REE under the lagged data policy rule is that some version of the Taylor principle, for example, the simple version, $\tau_\pi > 1$, is satisfied.

Comparing the two different approaches to modeling capital, the firm-specific case leads to a slightly larger region of determinate and learnable REE, though the firm-specific case continues to have a sliver of a region where equilibrium is both indeterminate and E-unstable. Nevertheless, for reasonable parameterizations of the lagged data version of the Taylor rule, for instance, Taylor's original (1993) calibration (adapted to quarterly data) $\tau_\pi = 1.5$ and $\tau_y = 0.125$, determinacy and learnability of the REE are assured in all three models.

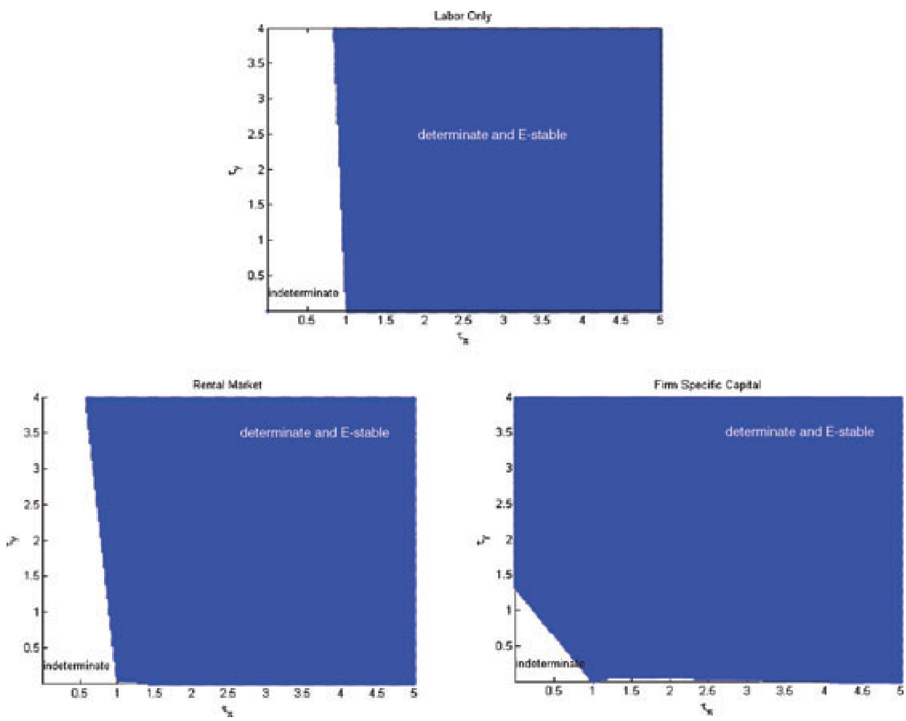

FIG. 7. Determinacy and E-Stability Results under the Contemporaneous Expectations Rule.

The contemporaneous expectations rule. Results for the contemporaneous expectations rule (34) in the three models are shown in Figure 7. This case yields results that appear similar to the current data rule (compare Figure 7 with Figure 1). Indeed, under the contemporaneous expectations rule the Taylor principle again suffices to implement a determinate and learnable REE in the labor-only model as shown by Bullard and Mitra (2002). However, by contrast with the current data rule, under the contemporaneous expectations rule, the Taylor principle no longer suffices to insure both determinacy and learnability of REE in either NK model with capital, a new finding of this paper. In both the rental and firm-specific models of capital, the determinacy conditions under the contemporaneous expectations rule are exactly the same as under the current data rule. The Taylor principle suffices to insure determinacy of REE in the rental market case, but in the firm-specific case, there is the same small (but empirically important) sliver of a region where $\tau_y \approx 0$ and τ_π is between 1 and 3 for which the REE is indeterminate. Thus, regarding determinacy, the findings are the same as for the case of the current data rule, and the intuition provided for that case extends to the case of the contemporaneous expectations rule. However, under the contemporaneous expectations rule there is a difference with regard to learnability of the determinate REE: in both the rental and firm-specific models of capital there is now a small sliver of a region where τ_y is close to 0 and τ_π is between 1 and 1.5 (rental market) or between 1 and 3 (firm-specific) for which the REE *is* determinate but is *not* E-stable (labeled "determinate and E-unstable"). Thus, in the rental market model under contemporaneous expectations, the Taylor principle may suffice for determinacy of REE but it no longer suffices for E-stability of REE. In the firm-specific model under the contemporaneous expectations rule, the region of determinate but E unstable REE is a very small sliver (which is admittedly difficult to see) but which lies along the border between the indeterminate (unshaded) and determinate and E-stable regions in Figure (7). We conjecture that the nonoverlap between determinacy and E-stability conditions must arise from the different timing of the information set used to form expectations under the contemporaneous expectations policy rule, as we do not observe this kind of divergence under either the current data or forward expectations rules.

Of course, as Figure 7 shows, these regions of indeterminacy or E-instability can be easily avoided by setting τ_y sufficiently high. Indeed we observe that there is again a very wide range of plausible calibrations (e.g., Taylor's 1993 calibration, $\tau_\pi = 1.5$ and $\tau_y = 0.125$), which result in determinate and learnable REE in all three models under the contemporaneous expectations rule.

Interest rate smoothing. Finally, we consider a policy rule involving interest rate smoothing, that is, giving some weight to lagged values of the interest rate so that policy does not adjust too quickly to changes in inflation or output. We focus on a policy-smoothing version of the lagged interest rate rule (33) as given by:

$$i_t = \rho i_{t-1} + (1 - \rho)[\tau_\pi \pi_{t-1} + \tau_y y_{t-1}], \tag{42}$$

where $\rho \in (0, 1)$ is the weight given to the past interest rate target. Results for policy-smoothing versions of the other three rules we consider are broadly similar. We know from Bullard and Mitra (2007) that the addition of policy inertia in the labor-only model can work to enlarge the region of policy weights for which a policy rule satisfying the Taylor principle yields determinate and learnable REE.

Here, in contrast to Bullard and Mitra (2007), we follow the convention in much of the literature on monetary policy rules (e.g., Rudebusch 2002) and imagine that the weight assigned to the lagged interest rate, i_{t-1}, and to the prescription of the policy rule [in square brackets] add up to unity; in this case the interest rate rule without smoothing can be regarded as the special limiting case where $\rho \to 0$. Woodford (2003b) has shown how such a "partial adjustment" model of monetary policy inertia may result from optimizing behavior on the part of the central bank.[17] Thus, we add

17. Some authors, for example, Rotemberg and Woodford (1998) and Giannoni and Woodford (2003), have derived optimal policy rules where the coefficient on the lagged interest rate is greater than 1. However, such a *superinertial* policy rule appears to be at odds with estimated interest rate rules. For instance, using U.S. data, Amato and Labauch (1999) estimate the current data rule (31) with the addition of a lagged interest rate (dependent) variable and report that the unrestricted coefficient estimate on the lagged interest rate is always less than one. While we think it would be of interest to consider superinertial interest rate rules, a virtue of the partial adjustment model we examine is that it requires just one additional

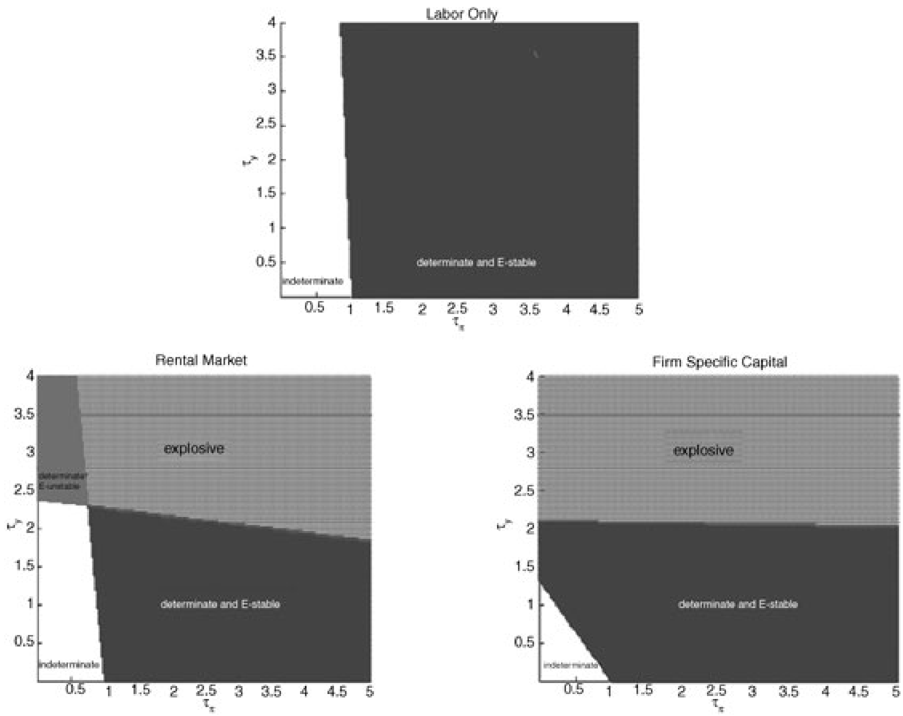

FIG. 8. Determinacy and E-Stability Results under the Lagged Data Policy-Smoothing Rule.

the choice of $\rho = 0.5$ to our baseline calibration (Table 2) for the policy-smoothing rule (42) but we later explore the impact of changes in ρ.

Determinacy and learnability results for the three models under the policy-smoothing rule (42) and our baseline calibration are shown in Figure 8. We see that in this case, the Taylor principle suffices to guarantee both determinacy and learnability of REE in the labor-only model but not in the two models that include capital. Comparing Figure 8 with Figure 6, which showed results for the lagged data rule without inertia [$\rho = 0$], we observe that the addition of policy inertia [specifically, $\rho = 0.5$] greatly enlarges the range of policy weights for which REE are determinate and E-stable in both models with capital. Policy inertia acts like a positive weight attached to output and thus helps policymakers avoid indeterminacy and E-instability. As in the case of the other rules, one can find a large range of empirically plausible values for the policy weights (τ_π, τ_y), for which the REE is both determinate and learnable, for example, Taylor's original calibration.

We also explore the sensitivity of our findings using the policy smoothing rule (42) to changes in the persistence parameter ρ. We focus on the rental market for capital

parameter, ρ, making it easier to see whether our findings without inertia generalize to the addition of some inertia.

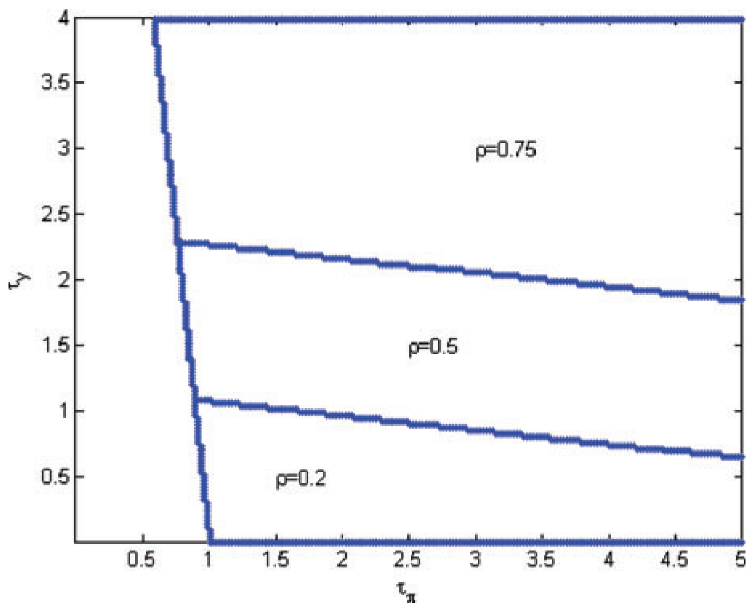

FIG. 9. Sensitivity Analysis for the Rental Market Model under the Policy-Smoothing Rule (42) Showing How the Determinate and E-stable Region Varies with Changes in the Value of ρ.

model as the results are similar for the firm-specific model of capital. Figure 9 reveals that in the rental market for capital model, the region of weight pairs for which REE is both determinate and E-stable increases as ρ increases. For instance, the determinate and E-stable polygon for the baseline $\rho = 0.5$ case in Figure 9 corresponds to the determinate and E-stable region of Figure 8. As ρ is lowered to 0.2, the upper bound to this determinate and E-stable region falls relative to the baseline case and as ρ is raised to 0.75, the upper bound to the determinate and E-stable region rises relative to the baseline case as Figure 9 illustrates. The main finding from this analysis is that increasing persistence in policy [the value of ρ] in models with capital leads to an expansion in the range of policy rule weights for which equilibrium is both determinate and learnable.

4. CONCLUSIONS

We have studied determinacy and learnability of REE in three different NK models, one with labor only and two that add productive capital via an economy-wide rental market for capital or via firm-specific demand for capital. The addition of capital to the NK model allows for the study of investment decisions, an important component of aggregate demand.

Determinacy and learnability are two highly desirable properties for REE and it should be the aim of central banks to adopt interest rate policies that implement equilibria possessing both of these properties. While Bullard and Mitra (2002, 2007) find that the Taylor principle nearly always suffices for both determinacy and learnability of REE in the labor-only model, the addition of capital to the NK model requires some further qualifications to this conclusion. In particular, we find that (i) in the model with a rental market for capital, the Taylor principle continues to suffice to insure both determinacy and learnability of REE if the interest rate rule responds to current data on inflation and output. However, we also find (ii) the Taylor principle need not suffice for both determinacy and learnability of equilibrium if the interest rate rule responds to future or contemporaneous expectations of inflation and output or to lagged values of these variables or if the central bank uses a policy-smoothing rule. We further find (iii) that in the model with firm-specific capital the Taylor principle *never* suffices to insure both determinacy and learnability of REE for the calibration we consider. Finally, (iv) an important policy finding is that the Taylor principle appears to be necessary, but not sufficient, for E-stability (learnability) of determinate REE in all of the models that we consider.

While the Taylor principle does not suffice to guarantee determinacy and learnability of equilibrium, we can still reach several practical conclusions that should be of interest to central bankers. First, while the specific findings for the labor-only model do not generalize to models with capital and investment decisions, some of the policy recommendations for the labor-only model appear to carry over to models that include capital. Specifically, two of the rules we consider, the current data rule and the contemporaneous expectations rule, both of which are operational in the sense of McCallum (1999), fare the best in all three models in terms of admitting the largest possible regions of determinate and learnable REE. Second, the results for the firm-specific and rental models of capital suggest that there is high value to policy rules that obey both the Taylor principle *and* give some weight to output and/or to policy smoothing so as to avoid indeterminacy and instability under learning. Under a forward-looking policy rule, the response to output should not be too modest nor too aggressive, and under a policy-smoothing rule, the weight attached to past interest rates should not be too small. Finally, we note that some (τ_π, τ_y) pairs succeed in implementing determinate and learnable REE in *all* models and for *all* interest rate rules that we have considered. In particular, the parameterization proposed by Taylor (1993) adjusted for the quarterly frequency of our model,—$\tau_\pi = 1.5$ and $\tau_y = 0.125$—belongs to that class. Perhaps the empirical success of Taylor's (1993) policy rule rests as much with the policy weights he chose for that rule as with the principle that also bears his name.

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