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A Cross-Country Empirical Investigation of the Aggregate Production Function Specification

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Many growth models assume that aggregate output is generated by a Cobb-Douglas production function. In this article we question the empirical relevance of this specification. We use a panel of 82 countries over a 28-year period to estimate a general constant-elasticity-of-substitution (CES) production function specification. We find that for the entire sample of countries we can reject the Cobb-Douglas specification. When we divide our sample of countries up into several subsamples, we find that physical capital and human capital adjusted labor are more substitutable in the richest group of countries and are less substitutable in the poorest group of countries than would be implied by a Cobb-Douglas specification.

Keywords: production function, Cobb-Douglas, CES, endogenous growth, multiple equilibria, panel data studies

JEL classification: O40, O47

1. Introduction

Many models of growth and development assume that output is generated by a two-factor, Cobb-Douglas specification for the aggregate production function with physical capital and labor or human capital adjusted labor serving as inputs. The Cobb-Douglas specification is the only linearly homogenous production function with a constant elasticity of substitution in which each factor's share of income is constant over time. Since the latter implication of the Cobb-Douglas specification is thought to be consistent with one of Kaldor's (1961) "stylized facts" of growth—that the shares of income accruing to capital and labor are relatively constant over time—most researchers have not questioned the use of a Cobb-Douglas production function to study questions of growth and development. Of course, the linear homogeneity and constant elasticity of substitution properties of the Cobb-Douglas specification may also explain the popularity of this functional form.

Nevertheless, some researchers have expressed doubts about the Cobb-Douglas orthodoxy. While Solow (1957) was perhaps the first to suggest the use of the Cobb-Douglas specification to characterize aggregate production, he noted that there was little in the way of evidence to support the choice of such a specification. Moreover, Solow (1958) pointed out that Kaldor's stylized fact is *not* that factor shares have been absolutely constant, as the Cobb-Douglas specification literally implies, but rather that these shares have been *relatively* constant over the short period of time for which we have available data. Solow notes that slight departures from a Cobb-Douglas specification, in the form of a constant-elasticity-of-substitution (CES) production technology with an elasticity of substitution that is only slightly different from unity, result in small trends in factor shares of income that are not inconsistent with the observed "relative stability" of these shares over longer periods of time. Indeed, in his seminal 1956 growth paper, Solow presented the CES production function as one of the example technologies for the modeling of long-run growth. The implications of the neoclassical growth model with a CES production technology were further spelled out by Pitchford (1960), who showed that certain parameterizations of this version of the model admitted the possibility of sustained long-run growth, of the variety recently resurrected by Jones and Manuelli (1990) and Rebelo (1991).

Long-run endogenous growth due to the production technology arises whenever the marginal product of capital (more generally, the marginal product of the cumulative, productive input) does not tend to zero in the limit as the capital stock grows large, in violation of the Inada condition. Instead, the marginal product of capital asymptotically achieves some lower bound that is greater than zero, thus eliminating the need for some kind of exogenous technological progress as the long-run steady-state engine of growth. A necessary condition for this type of endogenous growth in the Solow-Pitchford model of neoclassical growth with a CES production function is that the elasticity of substitution between capital and labor is greater than one.

In addition to long-run endogenous growth, a one-sector neoclassical growth model with a CES production technology admits another interesting possibility—the possibility of multiple steady states for per capita output. Azariadis (1993, 1996) for example, shows that in a two-period overlapping-generations model with productive capital, a two-factor CES production technology with an elasticity of substitution between capital and labor that is less than one admits the possibility of multiple, nontrivial steady states for per capita output. Galor (1996a) suggests how this same finding might carry over to the Solow descriptive growth model.

The necessary condition for multiple steady states with a CES production technology—an elasticity of substitution that is less than unity—is *precisely opposite* to the necessary condition for long-run endogenous growth in a Solow-Pitchford model, which requires an elasticity of substitution that is greater than unity. Both of these interesting possibilities are ruled out by a Cobb-Douglas specification, in which the elasticity of substitution is precisely equal to one.

Motivated by these two different and mutually exclusive possibilities, we chose to estimate a general CES production function for a cross section of 82 countries over a period of 28 years. Our goal was to determine whether a Cobb-Douglas specification is an empirically relevant specification for the aggregate production function in cross-country analyses of economic growth. In estimating the CES production function, we consider as inputs the physical capital stock of each country and the supply of labor, and we also consider a measure of labor adjusted for human capital.

While Cobb-Douglas specifications for aggregate output have been estimated for individual countries and even for small groups of countries (see, e.g., Chenery, Robinson, and Syrquin, 1986), we are not aware of any serious efforts to estimate other types of production functions using aggregate data from a large cross section of countries.[1] We suspect that this is mainly due to the absence of aggregate data for a large cross section of countries, especially data on aggregate stocks of physical and human capital. We believe we now have an adequate data source for these aggregate capital stocks for a sufficiently large cross section of countries—the World Bank data that we describe below.

Rather than estimating production functions, the more common approach has been to note that labor's share of income appears to be relatively constant over time and to use this finding to justify a Cobb-Douglas specification (see, e.g., Mankiw, Romer, and Weil, 1992, and Prescott, 1998). This interpretation of the time-series evidence, however, remains subject to Solow's (1958) critique. Furthermore, the cross-country evidence suggests that countries at different stages of development may have vastly different labor shares of income (see, e.g., Gollin, 1998); this finding makes it hard to justify the use of a Cobb-Douglas production function for cross-country analysis of growth. Even after careful adjustments are made to the way labor income is measured, Gollin (1998) continues to find that labor's share of national income across 31 countries has a standard deviation of around 10 percent. One possible explanation for this finding that cannot be rejected a priori is that the elasticity of substitution between capital and labor is not equal to unity, so that factor shares are not constant and instead vary with the accumulation of factor inputs.

To perform our estimation exercise, we make use of a World Bank dataset that includes data on aggregate capital stocks in constant U.S. dollars for a sample of 82 countries over 28 years. Using this data sample, we find that we can reject a Cobb-Douglas specification for the aggregate production function with capital and labor (or human capital adjusted labor) used as inputs. Instead, the data support the use of a more general CES specification with an elasticity of substitution between capital and labor that is significantly greater than one. We also estimate a CES specification for aggregate production for four different subsamples of countries, grouped according to the initial level of capital per worker. For these subsamples we find evidence that the elasticity of substitution may vary with the stage of development. In particular, for the richest group of countries we find that when human capital adjusted labor is used as an input, the elasticity of substitution between capital and labor is significantly greater than unity. However, for the poorest group of countries, the estimated elasticity of substitution is found to be significantly less than unity.

In the next section we motivate our estimation exercise by discussing in further detail how the two interesting possibilities discussed above can arise from a CES specification for the aggregate production function within a simple neoclassical growth framework. In Section 3 we present and discuss the results of our estimation of a CES specification for aggregate production for the entire sample of 82 countries and for various subsamples of countries. Section 4 concludes.

2. A One-Sector Neoclassical Growth Model with a CES Production Function

Consider first a one-sector, Diamond (1965) overlapping generations economy without national debt (there is no government). Agents are identical and live for two periods. At every date t a new generation is born. These agents are endowed with a single unit of leisure in the first period of their lives, which they inelastically supply in exchange for the competitive wage at time t, w_t. The population is assumed to grow at the constant rate $n > 0$.

Output of the economy's single, perishable consumption good is produced according to a CES production function

$$Y_t = F(K_t, L_t) = A\left[\delta K_t^{-\rho} + (1-\delta)L_t^{-\rho}\right]^{-\frac{\nu}{\rho}},$$

where Y_t is the real aggregate level of output (GDP), K_t is the aggregate capital stock, L_t is the aggregate labor supply, and A, δ, ρ, and ν are parameters satisfying $A > 0$, $\delta \in (0, 1)$, $\rho \geq -1$, and $\nu > 0$. We follow most of the existing growth literature in assuming that capital and labor are separate and distinct inputs into production. We further assume constant returns to scale in production by imposing the restriction that $\nu = 1$. Later, in the empirical analysis we will test this restriction. Given our assumption, we can rewrite the CES production function in the intensive form

$$y_t = f(k_t) = A[\delta k_t^{-\rho} + (1-\delta)]^{-\frac{1}{\rho}},$$

where $y_t = Y_t/L_t$ and $k_t = K_t/L_t$. We abstract from the possibility of exogenous, labor-augmenting technological progress that would provide us with sustained long-run growth, as we will later want to focus attention on the possibility of long-run endogenous growth.[2]

Agents have preferences over consumption in the two periods of their lives given by $U(c_t^1, c_{t+1}^2)$, where c_{t+j}^i denotes period i consumption by the representative agent in period $t + j$, $j = 0, 1$, and U: $\Re_+^2 \to \Re$ is a homothetic, increasing, strictly quasi-concave utility function with partial derivatives that satisfy the conditions $\lim_{c^1 \to 0} U_1(c^1, c^2) = \lim_{c^2 \to 0} U_2(c^1, c^2) = +\infty$.

The representative agent maximizes $U(c_t^1, c_{t+1}^2)$ subject to the constraint

$$c_t^1 + \frac{c_{t+1}^2}{R_{t+1}} \leq w_t,$$

where w_t and R_{t+1} represent the factor returns to labor and capital, respectively. Since we assume constant returns to scale in production, we have

$$\begin{aligned} R_{t+1} &= f'(k_{t+1}) + 1 - \mu = \delta A^{-\rho}\left[\frac{f(k_{t+1})}{k_{t+1}}\right]^{1+\rho} + 1 - \mu, \\ w_t &= f(k_t) - k_t f'(k_t) = (1-\delta)A^{-\rho}[f(k_t)]^{1+\rho}, \end{aligned}$$

where $\mu \in (0, 1)$ denotes the constant rate of depreciation of the capital stock. For ease of exposition we shall set $\mu = 1$.

The representative agent's optimal decision can be characterized by a savings function

$$s^t(w_t, R_{t+1}) = \gamma w_t \in [0, w_t], \quad \text{with } \gamma = \gamma(R) \in (0, 1)\ \forall R > 0.$$

Market clearing requires that all savings are invested for purposes of producing next period's output so that

$$k_{t+1} = \frac{\gamma}{1+n} w(k_t) = \frac{\gamma}{1+n}(1-\delta)A^{-\rho}[f(k_t)]^{1+\rho} \equiv h(k_t).$$

Steady states for k are solutions to the polynomial equation

$$k - h(k) = 0. \tag{1}$$

2.1. The Possibility of Multiple Steady States

It is well known that equation (1) may yield to zero, one or a maximum of two nontrivial—that is, *positive* steady state values for k, in addition to the trivial, $k = 0$, steady state (see, e.g., Azariadis, 1993, pp. 203–204). The number of positive steady states for k depends on the value of the elasticity of substitution between capital and labor, σ, defined by

$$\sigma = \frac{1}{1+\rho},$$

and may also depend on the value of the scale factor A. If $\sigma \geq 1$ ($\rho \leq 0$), then there *always* exists one unique positive steady state for k, since in this case, $\lim_{k\to 0} h'(k) > 1$, and $\lim_{k\to +\infty} h'(k) = 0$. Note that the familiar Cobb-Douglas specification where $\sigma = 1$ is included in this case. On the other hand, if $\sigma < 1$ ($\rho > 0$), then there are either *zero* or *two* positive and distinct steady-state values for k depending on the value of the scale factor A (see Azariadis, 1993). This case where $\sigma < 1$ is interesting because it leads to a dynamical system that is qualitatively different from the system with a Cobb-Douglas specification for the aggregate production function.

Figure 1 provides an illustration of the case where $\sigma < 1$ and the scale parameter A is sufficiently large that there are two positive and distinct steady states for k. Using the parameter values indicated in Figure 1 and equation (1) one can verify that there are two positive steady states, $k^1 = 0.38$ and $k^2 = 2.62$.

While the value of the scale factor A may matter for the existence of multiple positive steady states, a *necessary* condition for the existence of multiple steady states is that $\sigma < 1$ or $\rho > 0$.[3] When there are two positive steady states, $0 < k^1 < k^2$, as in Figure 1, the larger of these two positive steady-state values, k^2, is locally asymptotically stable. The trivial, $k = 0$ steady state is also locally asymptotically stable in this case. The domains of attraction of these two stable steady states are distinct and clearly depend on whether the initial capital stock, k_0, lies above or below k^1. Of course, it is always possible to add a constant to the production function so as to make the $k = 0$ steady state a more plausible, *low-income* "poverty trap" where income per worker is small but positive. With this modification, and the assumption that $\rho > 0$, two different development paths become possible: countries may either converge to a steady state with high per capita income and capital or to a steady state with low per capita income and capital. The existence of such multiple, steady-state equilibria is consistent with recent empirical work by Quah (1996a, 1996b) and Durlauf and Johnson (1995), who use methodologies that are not based on aggregate production function specifications.

The overlapping generations model differs from the more commonly studied Solow and optimal growth models in one important respect: in the overlapping generations model, individual savings must come out of wage income—that is, $s_t \leq w_t$. The Solow and the optimal-growth models impose no such restriction. In the Solow model, savings is

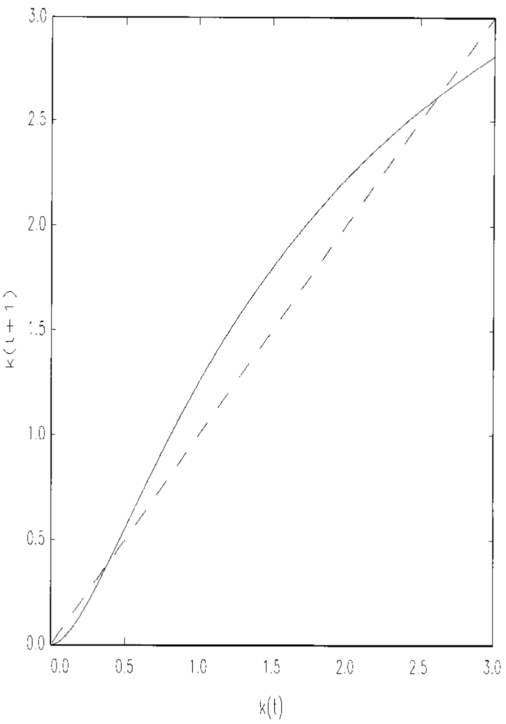

Figure 1. Illustration of the dynamical system ($\rho = 1$, $n = 0$, $\delta = \gamma = 0.5$, $A = 5$).

some constant fraction, $s \in (0, 1)$, of per capita *output*, $f(k_t) \geq w_t$. As Galor (1996a) has noted, the differences between the one-sector, overlapping-generations-growth model and the Solow growth model might be reconciled under the assumption of a neoclassical, linearly homogeneous production function with constant returns. Following Galor (1996a), suppose the fraction saved out of wage income s^w may generally differ from the fraction saved out of rental income s^r. The possibility of differential saving rates could be due to any number of factors, such as agent preferences or heterogeneous endowments. With this distinction, we may use Euler's theorem to write the law of motion for capital in the Solow model as

$$k_{t+1} = \frac{s^w}{1+n}[f(k_t) - f'(k_t)k_t] + \frac{s^r}{1+n} f'(k_t)k_t.$$

If $s^w = \gamma$ and $s^r = 0$, then we see immediately that all of our results for the overlapping generations economy readily extend to the Solow growth model as well. More generally, Galor (1996b) provides conditions under which values for $s^w \in [0, 1]$, $s^r \in [0, 1]$, and $\rho > 0$ give rise to multiple (that is, two) locally stable steady states in the Solow growth model. This result follows via a simple continuity argument. A necessary condition for this result, however, is that $\rho > 0$.

2.2. The Possibility of Endogenous Growth

The other interesting case with a CES specification for aggregate production occurs when $-1 \le \rho < 0$ ($\sigma > 1$). In this case there is the potential for long-run *endogenous growth*, due to the production technology á la Jones and Manuelli (1990) and Rebelo (1991).[4] This type of endogenous growth is not possible in Diamond's neoclassical growth model, where all savings must come from wage income (see, e.g., Jones and Manuelli, 1992, and Boldrin, 1992). However, as is well known, long-run endogenous growth due to the specification of the aggregate production function is possible in the descriptive and optimal growth frameworks of Solow (1956) and Cass (1965) and Koopmans (1965). The growth rate implied by the Solow-Cass-Koopmans model for the per capita capital stock is given by

$$\frac{k_{t+1}}{k_t} - 1 = \frac{s\frac{f(k_t)}{k_t} - (n+\mu)}{1+n}.$$

Positive growth occurs if $f(k)/k > (n+\mu)/s$. Of course, we are interested in positive growth *in the long run* as $k \to +\infty$. For this we require that

$$\lim_{k\to+\infty}\left[\frac{f(k)}{k}\right] = \lim_{k\to+\infty} f'(k) > \frac{n+\mu}{s} > 0.$$

Long-run endogenous growth arises from the possibility that $\lim_{k\to+\infty} f'(k) = b > 0$–that is, in the limit, the marginal product of capital does not diminish to zero but instead attains some lower bound $b > 0$. If $b > (n+\mu)/s$, then there is the potential for long-run endogenous growth.[5]

With a more general CES specification for the aggregate production function we have that

$$f'(k) = \delta A[\delta + (1-\delta)k^{\rho}]^{-\frac{1}{\rho}-1}.$$

For $\rho \ge 0$, it is clear that $\lim_{k\to+\infty} f'(k) = 0$. However, for $-1 \le \rho < 0$, we have that $\lim_{k\to+\infty} f'(k) = A\delta^{-1/\rho} > 0$. It follows that in the latter case, positive endogenous growth occurs in the long run of the Solow/optimal-growth model provided that $A\delta^{-1/\rho} > (n+\mu)/s$.[6]

Whether or not the aggregate production technology will admit either the possibility of multiple steady states or endogenous growth is clearly an empirical question that can be resolved only by estimating a CES specification for the aggregate production. We now turn our attention toward this estimation exercise.

3. Estimation of a CES Production Function

Our estimation of a CES specification for aggregate production involved data on 82 countries for 28 years from 1960 to 1987. We considered both nonlinear and linear least-squares regressions in combination with panel data techniques and instrumental variable approaches to obtain our parameter estimates. We begin by briefly describing the data used in our estimation.

3.1. The Data

All of the raw data that we used were obtained from the World Bank's STARS database. From this database we obtained measures of GDP and the aggregate physical capital *stock*, both of which were denominated in constant, end of period 1987 local currency units (converted into constant, end of period 1987 U.S. dollars) for all 82 countries over the period 1960 to 1987. The database also provided us with data on the number of individuals in the workforce between the ages of 15 to 64, as well as data on the mean years of schooling of members of the workforce. Further details concerning the construction of this data are provided in the appendix. We note however, that aside from our manipulations of this raw data, we did not construct any of the raw data used in this study.

Of particular note is our use of a new dataset on physical capital stocks. The relevant reference concerning the construction of these capital stock estimates is Nehru and Dhareshwar (1993), and the dataset is available as part of the World Bank's STARS databank. Several previous empirical studies involving physical capital have used proxies for the physical capital stock constructed from the Summers-Heston (1991) cross-country data on the ratio of investment to GDP at international prices—for example, Benhabib and Spiegel (1994) and Jones (1997). By contrast, the Nehru and Dhareshwar (1993) data on physical capital stocks makes use of World Bank data on gross domestic fixed investment at constant local prices and draws on additional data sources. Nehru and Dhareshwar use the "perpetual inventory method" to calculate capital stocks as briefly discussed in the data appendix. They show that their capital stock estimates are positively correlated with other, more limited, datasets on physical capital stocks. While capital stock estimates necessarily involve some guesswork, we believe the Nehru and Dhareshwar dataset is the best that is currently available. It has the further advantage of being widely accessible to other researchers. For these reasons, we chose to work with this dataset for physical capital stocks.

Prior to estimation, we made some simple transformations to the data in our sample. In particular, we converted all of the GDP and physical capital stock data into units of constant 1987 U.S. dollars using the 1987 exchange rate (also obtained from the STARS database) between the local currency and the U.S. dollar. We did this so as to avoid scale effects that might arise from differences in currency units across countries.

Let Y_{it} denote real (constant 1987 U.S. dollar) GDP, and let K_{it} denote the real capital stock (in constant 1987 U.S. dollars), where $i = 1, 2, \ldots, 82$ indexes each country and $t = 0, 1, 2, \ldots, 27$ indexes the 28 years of our sample period, 1960 to 1987. Similarly, let L_{it} denote the number of people in the workforce in country i in year t. In estimating the CES production function we will make use of the data in this *level* form, and we will also make use of our data in *per worker* terms: $y_{it} = Y_{it}/L_{it}$, $k_{it} = K_{it}/L_{it}$.

In addition to considering raw (unadjusted) labor, L, as an input in our CES specification, we also examined whether labor input, adjusted in some way for human capital accumulation, might alter our results. The motivation for including human capital adjusted labor supply in the production function comes from Romer (1986), Lucas (1988) and others who have stressed the importance of human capital in accounting for economic growth. Several previous empirical studies of economic growth across countries—such as Mankiw, Romer, and Weil (1992), Tallman and Wang (1994), Islam (1995), and Caselli, Esquivel, and Lefort

(1996)—have revealed that production function parameter estimates can change significantly when measures of human capital or labor adjusted for human capital are included as inputs. Here we follow Tallman and Wang (1994) and adopt a simple proxy for human capital adjusted labor input. First, we define the stock of human capital in country i at time t, H_{it}, as

$$H_{it} = E_{it}^{\phi},$$

where E_{it} denotes the mean years of schooling of the labor force (workers between the ages of 15 and 64 as in the measure of L) in country i at time t, and $\phi > 0$ is a parameter. The mean school years of education, E, is defined as the sum of the average number of years of primary, secondary, and postsecondary education.[7] We note that the data we use on mean years of schooling is also somewhat novel in that it is available *annually* for a large number of countries (85) and has been adjusted for differential drop-out and mortality rates and corrected for grade repetition.[8] Details on the construction of this data are provided in Nehru, Swanson, and Dubey (1995).

Given our definition for human capital, $H_{it} = E_{it}^{\phi}$, we define the *human capital adjusted labor supply*, HL_{it}, as

$$HL_{it} = H_{it} \times L_{it} = E_{it}^{\phi} L_{it}.$$

In estimating the CES specification for aggregate production, we will use both L and HL as measures of labor input.[9]

The parameter $\phi > 0$ in the definition of HL captures the returns to education. Given the large cross-section of 82 countries we are considering and their disparate educational systems, the appropriate choice for ϕ is not clear. We tried estimating ϕ in our nonlinear production function regressions, but the estimates were either implausibly negative or the iteration procedure failed to converge. We therefore chose to consider a grid of values for ϕ, ranging from 0 to 2 (by tenths) in both our nonlinear and linear regressions. We found that, for the entire sample of 82 countries, the log likelihood from both our nonlinear and linear production function regressions was always maximized in the case where $\phi = 0$, though this was *not* the case when we considered subsamples of countries as discussed in Section 3.6. Since $\phi = 0$ corresponds to the use of raw labor input only, L, we chose to follow Lucas (1988), Rebelo (1991), and many others and also consider the case where ϕ is simply assumed to be equal to 1. Thus for our regressions involving the *entire* sample of 82 countries, $HL_{it} = E_{it} L_{it}$. Later, when we consider production function estimates for several different subsamples of countries grouped according to capital per worker ratios (Section 3.6), we will relax this restriction on ϕ.

Finally, we note that in using human capital adjusted labor, HL, in place of raw labor, L, in our regression model specifications we are implicitly assuming that HL (like raw labor) is separate from capital as an input into production. Beginning with the work of Griliches (1969), some researchers have noted that there appears to be a strong complementarity between the level of *skilled* labor (our HL) and the level of capital, while unskilled labor (our raw labor input, L) and the capital stock are more likely to be highly substitutable. Consequently, the aggregate input-output production relationship might be better approximated by a function of *three* inputs: capital, skilled labor, and unskilled labor.[10] While we recognize the possibility of capital-skill complementarities, the conventional theoretical framework, which we seek to test here, imposes a Cobb-Douglas specification for the aggregate production function with just *two* inputs into production, capital, and either raw labor or human capital adjusted labor. We leave the testing of even more general production function specifications to further research.

3.2. A Look at the Data

Before turning to our production function regression results, we provide an illustration of our data on real GDP, capital, and labor for a few of the countries in our sample. Figure 2 plots output data—the log of real GDP, $\log Y_{it}$, against input data—the log of real capital, $\log K_{it}$, and the log of labor supply, $\log L_{it}$, for four of the 82 countries we consider over the sample period, 1960 to 1987: the United States, Chile, Ghana, and Ethiopia. To save space, we chose to present data for one country from each of the four subsamples of countries that we will consider later in Section 3.6. The input-output relationships depicted for these four countries are representative of the input-output relationships observed across the other 78 countries in our sample. Figure 3 is similar to Figure 2, except that the log of labor supply has been replaced with the log of human capital adjusted labor supply, $\log HL_{it}$ for the same four representative countries.

If the input-output relationship is characterized by a Cobb-Douglas specification for the aggregate production function, then $\log Y$ should be a strictly linear function of $\log K$ and $\log L$ (or $\log HL$). We see that for the one developed country in our illustration, the United States, the logarithmic input-output relationship is approximately, though not perfectly, linear. For the other countries, this relationship is clearly nonlinear; indeed for many of the countries in our sample, the function mapping the log of inputs into the log of output appears to be better approximated by a concave rather than by a linear function. Figures 2 and 3 thus provide a data-based justification for our consideration of the more general, nonlinear, CES specification for the aggregate production function.

3.3. Nonlinear Estimation

We began our empirical analysis by specifying the aggregate input-output production relationship as the nonlinear equation:

$$Y_{it} = A_0 \left[\delta K_{it}^{-\rho} + (1-\delta) L_{it}^{-\rho}\right]^{-\frac{\nu}{\rho}} e^{\lambda t + \epsilon_{it}}.$$

Here, A_0 denotes the initial (1960) value of the scale factor A, and we allowed Hicks-neutral exogenous technological growth at rate λ–$A_t = A_0 e^{\lambda t}$. We assume for now that A_0 and λ are common across countries.

Taking logarithms of both sides gives us

$$\log Y_{it} = \log A_0 + \lambda t - \frac{\nu}{\rho} \log\left[\delta K_{it}^{-\rho} + (1-\delta) L_{it}^{-\rho}\right] + \epsilon_{it}. \tag{2}$$

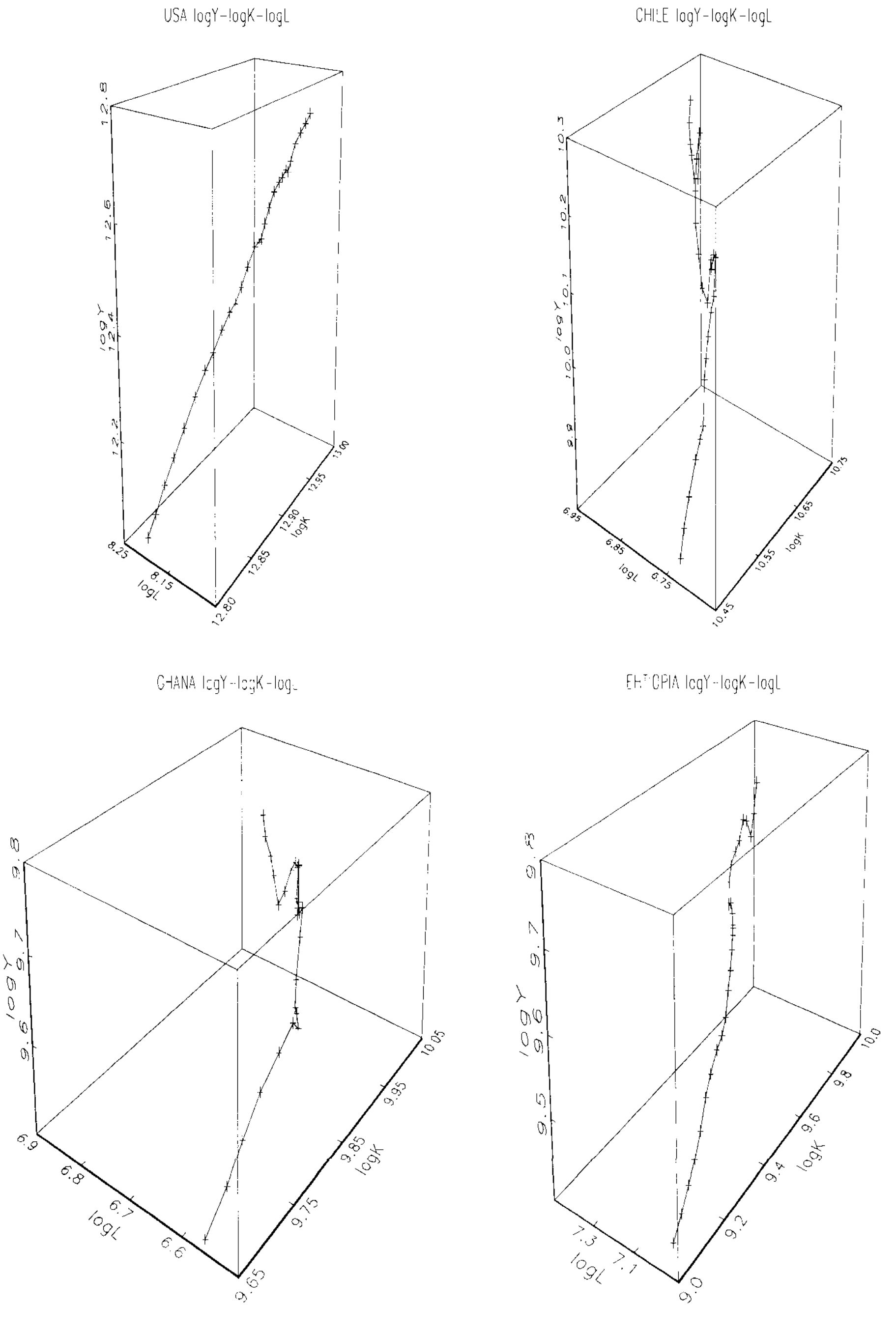

Figure 2. Log of input-output data from four representative countries (unadjusted labor input).

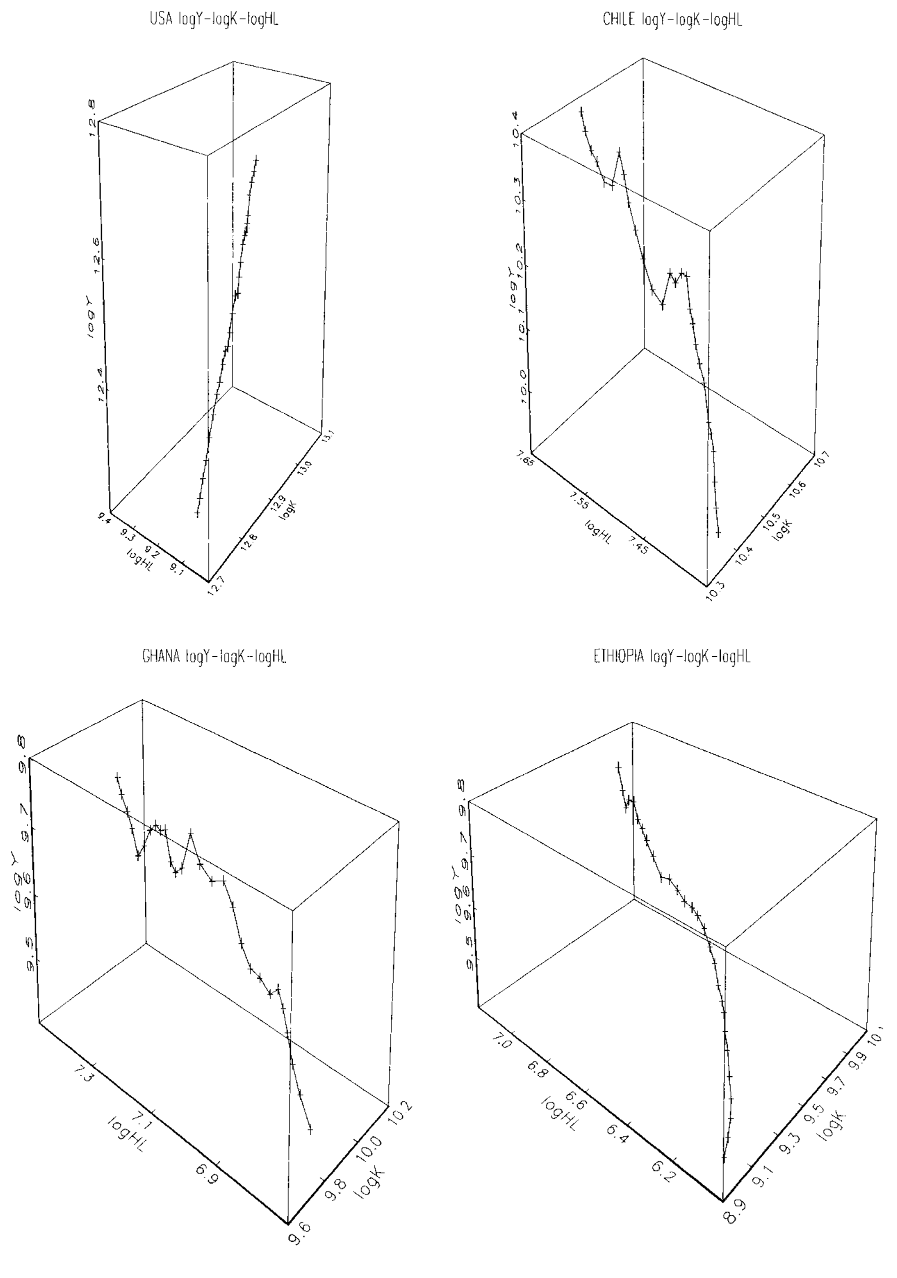

Figure 3. Log of input-output data from four representative countries (adjusted labor input).

Table 1. Nonlinear regression estimates.

	Unrestricted NLLS	Restricted ($\nu = 1$) NLLS	Restricted with Fixed Effects GMM
Labor (L)			
ρ	-0.56966^{***}	-0.56584^{***}	-0.19074^{***}
	(0.06176)	(0.05542)	(0.06805)
δ	0.05627^{**}	0.05755^{**}	0.08629^{**}
	(0.02507)	(0.02354)	(0.03885)
λ	-0.01160^{***}	-0.01164^{***}	0.00707^{***}
	(0.00087)	(0.00086)	(0.00169)
A_0	63.364^{***}	61.617^{***}	—
	(19.527)	(15.787)	—
ν	0.99899^{***}	—	—
	(0.00483)	—	—
$-\ln L$	836.61	836.63	—
Adjusted Labor (HL)			
ρ	-0.44833^{***}	-0.36200^{***}	-0.69172^{*}
	(0.10730)	(0.08297)	(0.39429)
δ	0.22290^{*}	0.30016^{**}	0.00586
	(0.12493)	(0.12076)	(0.01653)
λ	-0.01392^{***}	-0.01536^{***}	-0.01308^{***}
	(0.00092)	(0.00090)	0.00123
A_0	20.346^{*}	9.8487^{**}	—
	(11.211)	(4.4941)	—
ν	0.97271^{***}	—	—
	(0.00462)	—	—
$-\ln L$	961.35	979.02	—
Obs.	2,296	2,296	2,132

Notes: The GMM coefficients appearing in the third column are estimated using the Newey and West (1987) estimator. Standard errors are given in parentheses.
*** Significantly different from 0 at the 1 percent level.
** Significantly different from 0 at the 5 percent level.
* Significantly different from 0 at the 10 percent level.

We estimated equation (2) by nonlinear least squares (NLLS) for the entire panel of 2,296 observations using our data on real GDP, physical capital, and either raw labor supply L or human capital adjusted labor supply HL (with $\phi = 1$) in place of L. The coefficient estimates from a NLLS regression using the unrestricted model are provided in the first column of Table 1.[11]

We see in this first column that all of the estimated coefficients are significantly different from zero and economically plausible, regardless of whether L or HL is used for labor input. The most important finding is that the sign of ρ is found to be negative for both types of labor input, implying that the elasticity of substitution between capital and labor, σ, is greater than one, in contrast to the Cobb-Douglas specification. Our NLLS estimate for ρ

suggests that for our 28-year, 82-country sample, we may rule out the possibility of multiple steady states arising from the specification of the general CES production technology, and more important, we may rule out the Cobb-Douglas specification as being rejected by the data.

A second interesting finding from our NLLS estimates of the unrestricted model is that ν, the returns-to-scale parameter, is essentially equal to 1 when raw labor L is used as input implying that there are constant returns to scale in this case. Thus the constant-returns-to-scale restriction seems reasonable for the case where raw labor is used as input. When we replace raw labor input with human capital adjusted labor, HL, the estimated value of ν is found to be 0.97271, which is significantly different from unity, suggesting that there are slightly decreasing returns to scale in this case. However, since the theory supposes that there are constant returns to scale in production, we will focus our attention on this restricted version of the model, as discussed further below.

A third interesting finding from our NLLS estimation of the unrestricted model concerns the estimates for δ. Arrow, Chenery, Minhas, and Solow (1961) refer to δ as the *distribution* parameter. In the special Cobb-Douglas case, δ is readily interpreted as capital's share of output. However, the interpretation of δ is more complicated in the more general CES specification, where capital's share of output is given by $s_K = \frac{\delta K^{-\rho}}{\delta K^{-\rho}+(1-\delta)L^{-\rho}}$ and therefore depends on values of K, L, and ρ in addition to δ. The restriction that $s_K \in [0, 1]$ implies that $\delta \in [0, 1]$, a restriction that is satisfied by our NLLS estimates for δ. Moreover, $\partial s_K/\partial\delta > 0$, so that for a given ρ, K, and L, a higher value for δ is associated with a higher s_K. Note however, that as ρ becomes more negative, the value of δ that is needed to keep s_K constant becomes smaller; this relationship is born out in our NLLS estimates of δ and ρ.

Finally, we note that our NLLS estimates for λ, the coefficient on the time trend in the unrestricted model, are significantly different from zero and have negative signs, indicating that for the 82 countries of our sample, the log of real GDP has, on average, declined over the period 1960 to 1987. We note that our sample period, 1960 to 1987, was first marked by high productivity growth, especially among the more developed nations, and was later followed by a productivity slowdown in growth beginning after 1973 and coincident with a worldwide oil price shock (see, e.g., Perron, 1989; Greenwood and Yorukoglu, 1997). We have examined the robustness of our NLLS findings (both the unrestricted version and the restricted version discussed below) to the addition of an exogenously imposed *broken time trend*. Following Perron (1989) we estimated one time trend coefficient for the period 1960 to 1973 and a different time trend coefficient for the period 1974 to 1987; these two trends effectively cover the first and second halves, respectively, of our sample period. We found that the addition of this type of "broken" time trend did not alter our NLLS findings. The values, signs, and statistical significance of the estimated parameters ρ, δ, A_0, and ν remain largely unchanged, and the coefficients on the two time trends (pre- and post-1974) are both found to be significantly different from zero and slightly negative.[12] We conclude that, on average, there was a very slight decline in the log of real GDP across the 82 countries of our sample over the period 1960 to 1987.

In the second column of Table 1 we report NLLS estimates from the "restricted" version of the model, where $\nu = 1$. This restricted version, where the returns to scale are constant, corresponds to the theoretical case we considered in Section 2, and we will focus exclusively on this case in the remainder of our empirical analysis, as it is the most commonly studied case, and in the case of raw labor input L the restricted version of the model is not rejected by our unrestricted NLLS estimation results. We see that while the magnitude of the NLLS estimates for all parameters in the restricted model differ slightly from those obtained using the unrestricted model, the signs and statistical significance of the coefficient estimates are largely unchanged by comparison.

Thus far, the aggregate input-output production relationship we have estimated using NLLS does not allow for the presence of *fixed effects* across countries. A fixed-effects specification would allow us to capture country-specific characteristics—such as geography, political factors, or culture—that might affect aggregate output. Islam (1995) has emphasized the importance of allowing for such country-specific fixed effects in cross-country, linear growth regression analyses, and his same arguments apply to the aggregate input-output production relationship that we consider here. Forcing all countries to have the same, initial-period scale factor may lead to biased coefficient estimates due to an omitted variables problem.

Admitting the possibility of fixed effects implies that the error term in (2) can be written as $\epsilon_{it} = \eta_i + \upsilon_{it}$, where η_i captures time-invariant fixed factors in country i. Given this specification, first differencing (2) gets rid of the fixed-effect component in the error term, yielding the nonlinear equation

$$\log \frac{Y_{it}}{Y_{i,t-1}} = \lambda - \frac{1}{\rho} \log \left[\frac{\delta K_{it}^{-\rho} + (1-\delta) L_{it}^{-\rho}}{\delta K_{i,t-1}^{-\rho} + (1-\delta) L_{i,t-1}^{-\rho}} \right] + \upsilon_{it} - \upsilon_{i,t-1}. \tag{3}$$

Note that in (3) we have imposed the restriction that $\nu = 1$.

While it is straightforward to estimate (3) using NLLS, the first-difference specification leads to another difficulty in that the lagged error term $\upsilon_{i,t-1}$ is likely to be correlated with time t values of the explanatory variables, K_{it} and L_{it}. More generally, the capital-accumulation equation used to construct the capital stock values (see the data appendix for details) implies that K_{it} will *always* depend on such lagged error terms.[13] Consequently, some kind of instrumental variables approach such as two-stage nonlinear least squares estimation would appear to be required. We chose to use a generalized method of moments (GMM) approach to estimate the parameters in (3), which is a more general estimation method than nonlinear two-stage estimation in that the GMM approach allows for the possibility of both autocorrelation and heteroskedasticity in the disturbance term, $\upsilon_{it} - \upsilon_{i,t-1}$, which seems appropriate in this case. In our GMM estimation of (3) we used $\log K_{i,t-1}, \log K_{i,t-2}$ and $\log L_{i,t-1}, \log L_{i,t-2}$ (or $\log HL_{i,t-1}, \log HL_{i,t-2}$) as instruments.[14] The coefficient estimates we obtained using this estimation approach are given in the third column of Table 1.

Looking at these GMM estimates, we see that while the estimated value of ρ changes in magnitude relative to the NLLS estimates, it remains both negative and significantly different from zero regardless of whether raw labor L or human capital adjusted labor input HL is used as input. The GMM estimates of δ also change relative to the NLLS estimates but are positive and less than unity, and, in the case where raw labor is used, δ is significantly different from zero. Finally, note that in this first-difference version of the model the interpretation of the λ coefficient is different than in the log-linear model specification. In particular, λ is now an estimate of the exogenous average annual *growth rate* of real GDP for our sample period. We find that when raw labor is used as input, the estimated annual growth rate is 0.7 percent and significantly different from zero, but when human capital adjusted labor is used as input, the estimated annual growth rate is -1.3 percent and also significantly different from zero.

We note that for the entire 82-country sample, we are somewhat less confident in the nonlinear model estimates obtained using human capital adjusted labor HL as input as compared with the estimates using raw labor L as input. Recall that when HL was used as input, our unrestricted NLLS estimates did not support the constant returns to scale restriction, $\nu = 1$. More important, for the entire sample, our choice of $\phi = 1$ to construct human capital adjusted labor from raw labor input was also based on theoretical rather than empirical grounds. Nevertheless, these two theoretical restrictions ($\nu = 1, \phi = 1$) are frequently encountered in the literature on growth and human capital accumulation so that it seems reasonable to empirically test such versions of the model.

Of course, the main finding to take away from our nonlinear estimation exercises is that for the entire sample of countries, the estimates of ρ are always found to be negative and significantly different from zero, implying an elasticity of substitution between capital and labor that is greater than unity, in contrast to the Cobb-Douglas specification.

3.4. Linear Estimation Results

While the use of a nonlinear estimation technique would seem to be the most appropriate method for estimation of a CES specification for the aggregate production technology, we have also estimated a *linearized* version of the CES specification. We consider a linearized version of the CES specification for several reasons. First, much of the cross-country empirical-growth literature has made use of ordinary least squares (linear) regressions, under the assumption of a Cobb-Douglas specification for the aggregate production technology. We want to show how the Cobb-Douglas specification might be replaced by a (linearized) CES specification within the context of this very large body of empirical work. Second, we want to consider our production function estimation exercise for several nonoverlapping *subsamples* of our full sample of 82 countries. These subsamples of countries were grouped according to initial-period (1960) levels of capital per worker (which serve as a proxy for the state of development). In considering these smaller subsamples of countries, we found that nonlinear estimation methods generally failed to converge or led to empirically implausible estimates. The reason for this failure is that the panel of countries in these subsamples becomes unbalanced; there is too little heterogeneity within a subsample to properly identify the parameter estimates of a CES production function using nonlinear methods. A linear approximation works to eliminate these difficulties albeit at the expense of imposing some further restrictions on the model. A final justification for a linearized version of the CES production function is that this version provides us with a useful robustness check: our linearization of the CES production function is based on a simple first-order Taylor series expansion of the model where $\rho = 0$, the Cobb-Douglas version of CES. Hence our linear approximation of the CES production technology provides the Cobb-Douglas specification with its *best opportunity* to characterize the aggregate input-output production relationship.

For all of these reasons, we think it is sensible to consider estimates from a linearized version of the CES production function. We will, of course, compare our linearized estimates with those we obtained using nonlinear estimation methods.

The linearization begins with the nonlinear specification for the input-output production relationship as given by equation (2). A first-order linearization of this equation around $\rho = 0$ yields (see, e.g., Kmenta, 1967)

$$\begin{aligned}\log Y_{it} &= \log A_0 + \lambda t + \nu\delta \log K_{it} + \nu(1-\delta)\log L_{it} \\ &\quad - 1/2\nu\rho\delta(1-\delta)[\log K_{it} - \log L_{it}]^2 + \epsilon_{it}.\end{aligned}$$

Since the theory supposes that there are constant returns to scale in production, we impose this assumption on our linear specification by setting $\nu = 1$.[15] We can then rewrite the linear specification in *per worker* terms dividing through by L_{it} to obtain

$$\log y_{it} = \log A_0 + \lambda t + \delta \log k_{it} - 1/2\rho\delta(1-\delta)[\log k_{it}]^2 + \epsilon_{it}.$$

This change allows us to estimate the following specification:

$$\log y_{it} = \alpha + \lambda t + \beta_1 \log k_{it} + \beta_2 [\log k_{it}]^2 + \epsilon_{it}. \tag{4}$$

After estimating this specification, we can recover the CES parameters according to

$$\begin{aligned}\rho &= -2\beta_2/(\beta_1(1-\beta_1)),\\ \delta &= \beta_1,\\ A_0 &= e^{\alpha}.\end{aligned}$$

It is also possible to recover the associated standard errors using standard approximation techniques.

Notice that this linear specification essentially involves the addition of a quadratic term, $[\log k_{it}]^2$, to the standard, Cobb-Douglas log-linear specification. If the estimated coefficient β_2 is not significantly different from zero, then neither will be the implied estimate of ρ, and we will be unable to reject the Cobb-Douglas specification as characterizing the input-output production relationship. Recall, however, from the representative illustrations presented in Figures 2 and 3 that for many of the countries in our sample, the log-linear Cobb-Douglas specification appears to be readily violated.

The results from estimating equation (4) using OLS are provided in the first column of Table 2. We see that regardless of whether L or HL is used as input, all estimated coefficients are significantly different from zero and empirically plausible. A comparison of the linear model, OLS estimation results with the restricted model ($\nu = 1$) NLLS estimation results (as reported in the second column of Table 1) reveals differences in the magnitudes of the estimated coefficients only; the signs and statistical significance of the estimated coefficients are unchanged. In particular, the OLS estimates of ρ remain significantly negative, while the estimates of δ are significant, positive, and less than unity. As in the nonlinear estimation results, the estimates of λ are negative but close to zero.

The second column of Table 2 presents OLS results from a modified version of equation (4) in which we have allowed for a broken time trend. As in case of the nonlinear model, we

Table 2. Estimates for various different specifications of the linear model.

	OLS: Common Intercept & Trend	OLS: Common Intercept and Broken Trend	Two-Way Model: No Instruments	Two-Way Model: With Instruments
L				
ρ	-0.29204^{***}	-0.29360^{***}	-0.32952^{***}	-0.32300^{***}
	(0.05150)	(0.05191)	(0.12256)	(0.13350)
δ	0.32089^{***}	0.31934^{***}	0.15788^{***}	0.14761
	(0.05203)	(0.05196)	(0.04854)	(0.04976)
A_0	15.941^{***}	15.579^{***}	—	—
	(3.6001)	(3.5241)	—	—
λ	-0.01194^{***}	—	—	—
	(0.00092)	—	—	—
λ^{60-73}	—	-0.00708^{***}	—	—
	—	(0.00265)	—	—
λ^{74-87}	—	-0.01088^{***}	—	—
	—	(0.00112)	—	—
R^2	0.940	0.940	0.993	0.992
HL				
ρ	-0.22721^{***}	-0.22838^{***}	-1.2685	-0.90970
	(0.05087)	(0.05344)	(1.2224)	(0.69151)
δ	0.44411^{***}	0.44233^{***}	0.08159	0.10782
	(0.08280)	(0.08279)	(0.07669)	(0.07865)
A_0	6.9923^{***}	6.8436^{***}	—	—
	(2.1308)	(2.0822)	—	—
λ	-0.01546^{***}	—	—	—
	(0.00099)	—	—	—
λ^{60-73}	—	-0.01089^{***}	—	—
	—	(0.00284)	—	—
λ^{74-87}	—	-0.01447^{***}	—	—
	—	(0.00121)	—	—
R^2	0.873	0.872	0.982	0.980
Observations	2,296	2,296	2,214	2,214

Notes: Standard errors are given in parentheses and were recovered using standard approximation methods for testing nonlinear functions of parameters. White's heteroskedasticity correction was used.
*** Significantly different from 0 at the 1 percent level.
** Significantly different from 0 at the 5 percent level.
* Significantly different from 0 at the 10 percent level.

added two time trends to the linear model—one for the first half of our sample period (1960 to 1973) with coefficient λ^{60-73} and one for the second half (1974 to 1987) with coefficient λ^{74-87} in accordance with the structural break that has been identified by many researchers around 1973. We find again, that the coefficients on both of these time trends remained negative and significant, though the coefficient on the second trend, λ^{74-87}, is found to be more negative than the coefficient on the first trend when either L or HL is used as input.[16] As in our NLLS estimation results, the addition of such a broken time trend does not lead to any substantive change in the magnitude, sign, or statistical significance of any of the other estimated coefficients of the model.

We have also conducted OLS regressions for a related version of the basic linear model in which each country i in the 82-country sample is allowed to have its own time trend, with coefficient λ_i. These results, which are not reported in Table 2 due to the large number of time trend coefficients, suggest that allowing for individual country time trends does not lead to any significant changes in either the magnitude or the statistical significance of the estimated production function parameters by comparison with the values reported in the first two columns of Table 2. However, this exercise does reveal that there is considerable heterogeneity in the estimated time trend coefficients for each country. When raw labor L is used as input, we find that 78 percent (64/82) of our estimated λ_i coefficients are significantly different from zero and that there is a mixture of positive and negative values for these λ_i estimates. Among the estimates of λ_i that are significantly different from zero, a large majority of the estimates, 86 percent (55/64) have negative signs. The countries with significantly positive estimates for λ_i tend to be among the more developed countries. We obtain very similar results when HL is used as input. This finding helps to account for the negative coefficient estimates we found on time trends in the model specifications with a single common time trend or a broken time trend.

To account for the possibility of country-specific fixed effects as well as for time effects, we have also estimated a "two-way" or "covariance" version of the basic linear model, equation (4). With a sample size of $N = 82$ countries and $T = 28$ periods, the two-way fixed effects specification involves the addition of 81 ($N - 1$) country-specific dummy variables and 27 ($T - 1$) time dummy variables to the basic linear model.[17] In practice, the estimation of this two-way covariance model involves taking deviations of all variables from the time and individual mean values for each country but adding in the overall mean value for each country.[18] Note that this specification allows for country-specific growth factors and for the rate of exogenous technological progress to differ over time. After estimating this linear equation we can continue to recover the CES parameters as before. However, as there are more coefficients to estimate, we lose degrees of freedom; our sample size is reduced from 2,296 to 2,214 observations.

The estimation results for the model with country-specific fixed effects and time effects are reported in the third column of Table 2.[19] We see that when raw labor L is used as input, the only difference between our two-way model estimates and previous regression estimates lies in the magnitude of the coefficient estimates; the estimates of ρ and δ continue to have the same signs and remain significantly different from zero. One notable feature of the fixed-effects estimation results is that the estimated value of the distribution parameter δ is much lower as compared with the estimates reported in the first two columns or in Table 2. Changes in the estimate of this parameter may be due to the correction for omitted variable bias that the fixed-effects model makes possible.[20]

When we use HL as input in our two-way model specification, the estimate of ρ remains negative but is no longer significantly different from zero and seems implausibly high. This finding can be attributed to the estimate we obtain for the distribution parameter δ, which, when HL is used as input, is quite low as compared with estimates from the other linear model specifications using HL as input and is not significantly different from zero.

(Recall from our discussion above that the implied estimate of ρ depends on the estimate of δ.) A likely explanation for these findings is that we have not correctly adjusted labor for human capital accumulation with our assumption that $\phi = 1$ for the entire 82-country sample, so that our human capital adjusted labor variable HL is poorly measured (recall that $\phi = 0$ is the preferred choice for the entire sample). As Griliches and Hausman (1986) note, in regressions using panel data with fixed-effects specifications, measurement error in the explanatory variables can lead to coefficient estimates that are "too low" and therefore insignificant; in controlling for the various individual effects, the relative importance of measurement errors in the explanatory variables becomes greatly exacerbated and works to bias coefficient estimates downward. Since we did not find such radical changes in our coefficient estimates when we used raw labor input L and switched from a "no-effects" to a two-way model specification, it seems reasonable to conclude that measurement error is responsible for the magnitude changes and lack of significance of the estimates we obtain for the two-way model when HL is used as input.

The OLS estimates reported in the first three columns of Table 2 may suffer from another bias in that the model specification does not allow for the possible contemporaneous correlation of the regressors, in particular k, with the error term. Such correlation is possible, as noted earlier, because of the way in which the capital stocks are estimated, via a simple first-order capital-accumulation equation. The lack of attention to such econometric issues in the earlier empirical-growth literature was first pointed out by Caselli, Esquivel, and Lefort (1996), and consistent with the more recent empirical literature we now turn to regression results that address the problem of contemporaneously correlated disturbances by using an instrumental variable, two-stage least-squares (2SLS) approach in combination with our two-way model.

The final column of Table 2 reports regression results for the two-way linear model with country-specific fixed effects and time effects as well as instrumental variables (the result of a 2SLS estimation procedure). The coefficient estimates in this case are similar to those for the two-way model without instruments. In particular, the estimates of ρ remain negative, and the estimates of δ remain positive and less than unity. However, these coefficient estimates are found to be significantly different from zero only in the case where raw labor L is used as input.

We conclude from these exercises that our rejection of the null hypothesis that the aggregate production function is Cobb-Douglas, in particular that $\rho = 0$, appears robust to several different linear-regression model specifications, including broken time trends, individual country time trends, and country-specific fixed effects and time effects with or without instrumental variables, especially when raw labor L is used as input.

3.5. Implications of Our Estimates for the Entire Sample

Recall that when $\rho < 0$, as we have found, the neoclassical growth model has a unique, nontrivial steady state and there is the potential for endogenous growth, as the $\lim_{k\to+\infty} f'(k) = A\delta^{-1/\rho}$ under the assumption of constant returns to scale—$\nu = 1$. Whether long-run endogenous growth is possible depends, in this case, on whether the limiting value of the marginal product of capital, $A\delta^{-1/\rho}$, is greater than the sum of the average, cross-country population growth and depreciation rates divided by the average, cross-country savings rate, $\frac{\mu+n}{s}$. Using the values for A_0, δ, and ρ that we have estimated using either nonlinear or linear regression methods, we can take a first step toward addressing the question of whether endogenous growth due to the production technology is a real possibility. While ideally it would have been useful to have estimates of parameters such as ρ and δ for *individual countries*, we found that there were too few observations (28) for any single country to properly identify or obtain meaningful and significant estimates of these CES model parameters for individual countries. We must therefore be content to ask whether endogenous growth due to the production technology is possible, *on average*, for the 82 countries of our sample.

To make such as assessment, we will need to have average values of n and s and μ. The labor data we use for our empirical analysis can be used to construct estimates of the average labor-force growth rate across the 82 countries of our sample over the period 1960 to 1987. We found that using the raw labor data, the average growth rate of the labor force is $n_L = .0223$, while if the human capital adjusted labor data is used (with $\phi = 1$), we have $n_{HL} = .0507$. To estimate an average savings rate s, we used data from the Penn World Tables (Summers and Heston, 1991) on the investment (I) to output (Y) ratio (I/Y) for each country to proxy for a country's savings rate. Data on I/Y were available for the 82 countries of our sample, but only over the shorter period 1965 to 1987. We calculated the average value of I/Y for all 82 countries over the 22 years for which this variable was available and found an average savings rate of $s = 0.1728$. Finally, we chose to set the average depreciation rate $\mu = 0.06$, which is the same common value that has been used in other empirical analyses of cross-country growth (see, e.g., Bils and Klenow, 1996, and Hall and Jones, 1999). Let us define $\xi = \frac{\mu+n}{s}$. Then using our choices for μ, n, and s we have $\xi_L = 0.4762$ and $\xi_{HL} = 0.6406$.

Consider first the case of raw labor input. For endogenous growth due to the production technology, we need $A\delta^{-1/\rho} > \xi_L = 0.4672$. Using any of the estimated values for A (we use A_0), δ, and ρ from those regressions involving the restricted $\nu = 1$ version of the model, where all three parameters were identified and raw labor L was used as input (as reported in the second column of Table 1 or the first two columns of Table 2), we find that $A\delta^{-1/\rho}$ takes on values ranging from a high of 0.3966 to a low of 0.3192.

For the case of human capital adjusted labor input we come to a similar finding. For long-run endogenous growth due to the production technology, we need $A\delta^{-1/\rho} > \xi_{HL} = 0.6406$. Using any of the estimated values for A, δ, and ρ from those regressions involving the restricted $\nu = 1$ version of the model, where all three parameters were identified and human capital adjusted labor HL was used as input (as reported in the second column of Table 1 or the first two columns of Table 2), we find that $A\delta^{-1/\rho}$ takes on values ranging from a high of 0.3759 to a low of 0.1924.

It thus appears that our estimated CES production function specifications using either raw labor L or human capital adjusted labor HL as input do not admit the possibility of long-run endogenous growth due to the production technology. In this respect, the estimated CES function is similar to the commonly used Cobb-Douglas specification.

Nevertheless, our estimation results for the entire panel of 82 countries over 28 years suggest that for empirical cross-country analyses of economic growth, a Cobb-Douglas specification for the aggregate production function may be an inappropriate choice. Consequently, many earlier empirical analyses of economic growth across countries that simply presumed a Cobb-Douglas specification for aggregate production and that provided support for the conditional convergence hypothesis may well have been based on a misspecified model of the aggregate input-output production relationship, thus calling the results of these studies into question. On the other hand, it is a simple matter to correct the specification of the aggregate production function in these cross-country growth analyses and examine whether the results obtained under a Cobb-Douglas specification continue to hold.

3.6. Alternative Samples

In addition to testing the robustness of our empirical findings using different linear regression model specifications, we have also reconsidered our findings for certain subsamples of countries. In particular, we have divided our sample of 82 countries up into four subsamples of roughly equal size (approximately 20 countries per subsample), and we have reestimated the linearized production function for each of these subsamples. The subsamples were constructed by first ranking all 82 countries according to their initial-period, 1960 level of capital per worker, k_{1960}, in constant 1987 U.S. dollars, and then dividing this ranking of countries up into four groups: *high–k*, *middle–k*, *low–middle–k*, and *low–k*. We sorted countries by initial levels of capital per worker rather than by initial income per worker so as to avoid sample selection biases that might arise from sorting by the dependent variable in our regressions.[21] Of course, the correlation between capital per worker and income per worker is quite high; the high-k group generally contains the world's richest countries, while the low-k group generally contains the world's poorest countries. A list of the countries in each subsample is provided in the appendix. The capital per worker cut-off levels that were used to construct the four subsamples are indicated in Table 3. The subsample groups of countries we consider are similar to other groupings of countries that have appeared in the empirical growth literature (e.g., Durlaug and Johnson, 1995).

In estimating the basic linear model (equation (4) for the four subsamples we used a time-series cross-section (TSCS) estimation method that allows for the possibility of both groupwise autocorrelation and groupwise heteroskedasticity. We adopted this estimation method because our OLS estimates for the various subsamples indicated the presence of both groupwise autocorrelation and groupwise heteroskedasticity.[22] In obtaining these subsample estimates, we also allowed the value of the ϕ parameter, which represents the returns to education, to vary across the different subsamples. Recall that the choice of ϕ matters only in those regressions where we use human capital adjusted labor HL in place of raw labor input L. Our maintained assumption up to now has been that $\phi = 1$ (as a grid search indicated that the value of ϕ that maximized the log-likelihood function for the full sample of countries was 0) in keeping with a frequently encountered theoretical assumption. In our subsample regressions, we again considered a grid of different possible values for ϕ, including the $\phi = 1$ case, and we applied our TSCS estimation procedure for each of these different values of ϕ. As in the case of the full sample, for each subsample we conducted regressions for a grid of different ϕ values in [0, 2] with step size .01. In contrast with the grid search we conducted for the full 82-country sample, we found that for the high k subsample, the log-likelihood function from our TSCS procedure was maximized when $\phi = 0.68$.

Table 3. TSCS estimates for four subsamples using the linear model.

a. High-*k* subsample: 21 Countries, initial capital per worker greater than \$10,000, $\phi = 0.68$

Labor	ρ	δ	A	λ	Log-Likelihood Function
L	−0.15210	0.48843*	10.859	−0.00550***	1545.3
	(0.11915)	(0.29934)	(16.963)	(0.00077)	
HL	−0.08209*	0.68247***	3.4951*	0.00620***	1555.5
	(0.04772)	(0.12358)	(1.8985)	(0.00039)	

b. Middle-*k* subsample: 20 Countries, initial capital per worker \$3,000 to \$10,000, $\phi = 0$

Labor	ρ	δ	A	λ	Log-Likelihood Function
L, HL	−0.08992***	1.5989***	0.05810	−0.00183	1116.8
	(0.01669)	(0.17728)	(0.04763)	(0.00158)	

c. Low- middle-*k* subsample: 18 Countries, initial capital per worker \$1,000 to \$3,000, $\phi = 0.20$

Labor	ρ	δ	A	λ	Log-Likelihood Function
L	−0.19285	0.37928*	22.573	−0.01309***	984.9
	(0.15865)	(0.23010)	(22.180)	(0.00151)	
HL	−0.20294	0.36716	22.666	−0.01483***	987.3
	(0.17768)	(0.23689)	(2.135)	(0.00135)	

d. Low-*k* subsample: 23 Countries, initial capital per worker less than \$1,000, $\phi = 0.18$

Labor	ρ	δ	A	λ	Log-Likelihood Function
L	−0.08884	0.32380***	32.528***	0.00118	1235.5
	(0.09097)	(0.10958)	(11.761)	(0.00079)	
HL	0.21204*	0.72550***	8.7699***	−0.00223***	1244.5
	(0.12484)	(0.09979)	(2.8278)	(0.00051)	

Notes: Standard errors are given in parentheses and were recovered using standard approximation methods for testing nonlinear functions of parameters. White's heteroskedasticity correction was used.

*** Significantly different from 0 at the 1 percent level.

** Significantly different from 0 at the 5 percent level.

* Significantly different from 0 at the 10 percent level.

Similarly, the log-likelihood function from our TSCS procedure was maximized by choices of $\phi = 0.20$ and $\phi = 0.18$ for the low-middle-*k* and low-*k* subsamples, respectively. For the remaining middle-*k* subsample, we found that over the allowable range [0, 2], the log-likelihood function from our TSCS regressions was maximized when $\phi = .00$, just as we found for the entire 82-country sample. Since $\phi = 0$ corresponds to the case where raw labor L is used as input, and we found $\phi > 0$ for the other three subsamples, we decided not to adjust labor for human capital for this middle-*k* subsample of countries. Accordingly, for the middle-*k* subsample, there is no difference in the estimates we report for model that uses HL in place of L. Our estimation results for the four subsamples are reported in Table 3.

The most striking results in Table 3 are obtained for the two extreme groups—the high-*k* and the low-*k* subsamples when human capital adjusted labor HL is used as input (see Tables 3a and 3d). In these two cases, all of the estimated production function coefficients are empirically plausible and significantly different from zero. Note in particular that when

HL is used, the estimate of λ is positive and significant for the high-k subsample and negative and significant for the low-k subsample. More important, in the high-k subsample, when HL is used as input, the coefficient estimate for ρ is significantly *negative*, while in the low-k subsample when HL is used as input, the coefficient estimate for ρ is significantly *positive*. By contrast, when raw labor L is used as input in the high-k subsample, the coefficient estimate for ρ, while negative, is not significantly different from zero. Similarly, when L is used as input in the low-k subsample, the coefficient estimate for ρ is not significantly different from zero, in contrast to the case where HL is used as input. Note, however, that in these two extreme subsamples, the log-likelihood criterion favors the model with $\phi > 0$—that is, the model in which HL is used as input, over the model in which $\phi = 0$ and raw labor, L is used as input. We therefore focus attention on our model estimates using HL as input for these two cases.

The coefficient estimates for ρ in the two extreme subsamples when HL is used as input provide the first evidence ever for the notion that the substitutability of physical capital and human capital adjusted labor may depend on the stage of economic development (more precisely the extent of capital accumulation per worker) and that at the two extreme stages of development, a Cobb-Douglas specification may not adequately capture the aggregate input-output relationship. The negative estimate for ρ in the high-k subsample, which includes the world's richest countries, suggests that in this group, the elasticity of substitution is greater than unity, that there is a unique, nontrivial steady state, and that there is the possibility of long-run endogenous growth as discussed earlier. The positive estimate for ρ in the low-k subsample, which includes the world's poorest countries, suggests that the elasticity of substitution is less than unity, so that in these countries physical capital and human capital adjusted labor can be regarded as more complementary to production than would be implied by a Cobb-Douglas specification. Furthermore, as discussed earlier, the positive estimate for ρ is consistent with multiple, nontrivial steady states for this low-k subsample; in particular, the possibility that some of these countries are stuck in poverty traps cannot be ruled out. The negative estimate for ρ for the most developed nations and the positive estimate for ρ for the least developed nations would seem to accord well with the results of earlier empirical studies, cited above, which also find support for the notion that the most developed countries are achieving convergence in per capita incomes but find less clear evidence of convergence for other groups of countries, particularly for the least developed countries.

For the middle-k subsample of countries, the regression results are more difficult to interpret since the estimate for the distribution parameter δ is implausibly greater than unity and significantly different from zero.[23] Therefore, some caution seems warranted in assessing our estimates for this subsample. Note, however, that the estimate of ρ for this middle-k subsample is significantly negative. As was the case for the high-k subsample, this finding would imply rejection of a Cobb-Douglas specification for the aggregate production function for this middle-k group of countries.

For the low-middle-k subsample of countries, all of the estimated coefficients are empirically plausible. The estimates of ρ are negative but not significantly different from zero when either L or HL is used as input. These findings would appear to suggest that a Cobb-Douglas specification for the aggregate production function is an appropriate choice for this group of countries. However, some caution is again warranted in arriving at this conclusion. Notice that for the model where HL is used as input, the estimates of δ and A for the low-middle-k subsamples are not significantly different from zero. For this subsample, the model with HL as input is preferred by the log-likelihood criterion. Thus, our production parameter estimates for this low-middle-k subsample do not allow us to reach firm conclusions concerning the appropriate specification of the aggregate production function for this group of countries.

We note that we have experimented with several different ways of dividing up the entire sample of countries into various subsamples but the results never varied much from those reported in Table 3. We always found empirically implausible or insignificant values for δ or A for both the middle-k and low-middle-k group of countries, regardless of the various cut-off values that we used to define these subsamples. Accordingly, we do not take our estimates for these middle-k countries too seriously. Our results would seem to indicate that for the middle and low-middle-k countries a CES specification, *including the Cobb-Douglas special case*, may not be the correct specification for the aggregate input-output relationship.

3.7. Discussion of the Subsample Estimates

Our subsample estimates provide the first evidence of an evolving elasticity of substitution between physical capital and human capital augmented labor along the development path. What are the theoretical implications that might be drawn from this finding? Theoretically, our estimates suggest a growth model with a CES specification for the aggregate production function in which the elasticity of substitution between physical capital and human capital adjusted labor increases with increases in the level of output per capital. Since we observe statistically significant changes in the value of σ only when we include H, and since the mean years of education of the labor force ($H = E$) is strongly positively correlated with output per capita y (correlation coefficient using our data is 0.71922), we might suppose that $\Delta\sigma = g(H_t)$, where g is a function that is increasing in H.

The possibility that a production technology parameter evolves over the development process has been suggested before. Indeed, following Romer (1990), many endogenous-growth models have postulated that the productivity-scale factor A evolves over time as a function of endogenously determined input stocks. Here we are suggesting a model in which endogenous changes are possible in a different production technology parameter, the elasticity of substitution between physical capital and human capital adjusted labor σ. Low substitutability at the initial levels of development would allow for the possibility of multiple steady states and poverty traps. At later stages of development, higher substitutability might allow for the possibility of sustained long-run endogenous growth. Furthermore, changes in the value of σ itself may be an important source of economic growth (see Duffy and Papageorgiou, 1999).

We have also attempted to assess the possibility of long-run endogenous growth using our subsample estimates for A_0, δ, and ρ. We do this only for the high-k subsample when adjusted labor (HL) is used as input, as this was the only subsample in which the estimated value of ρ was found to be significantly negative and all other model parameter estimates were empirically plausible. Following the same procedures used in Section 3.5 we calculated the savings and population growth rates for this high-k subsample of countries, which we found were given by $s^{hi} = 0.2522$ and $n_{HL}^{hi} = 0.0281$, and we assumed, as before that $\mu = 0.06$. Using our parameter estimates, we find that $A\delta^{-1/\rho} = 0.0033 < \xi_{HL}^{hi} = 0.3493$, which implies that these estimates do not admit the possibility of long-run endogenous growth due to the production technology specification for the high-k subsample.

4. Conclusion

We began this article by arguing that the conventional use of the two-factor, Cobb-Douglas specification for aggregate production needed to be empirically justified. We showed how the more general CES specification could lead to two mutually exclusive results—the possibility of multiple steady states and the possibility of long-run endogenous growth. Both of these possibilities are excluded by a Cobb-Douglas specification for the aggregate production function.

We have presented empirical evidence suggesting that the Cobb-Douglas specification for the aggregate production function may not be empirically valid. Instead, we have found, using aggregate data on a panel of 82 countries over 28 years and a variety of different regression-model specifications, empirical support in favor of a more general CES specification of the aggregate input-output production relationship where the elasticity of substitution between capital and labor or effective labor is significantly greater than unity. In this case, there is the possibility of endogenous growth. We caution, however, that endogenous growth is only a *possibility*; the finding that the elasticity of substitution is greater than unity is necessary but not sufficient for long-term perpetual growth to occur. Indeed, our estimated coefficient values suggest that the conditions for such long-run growth may not be in place, on average, for the 82 countries of our sample.

We also estimated the CES specification for aggregate production for four different subsamples of countries, grouped according to initial capital per worker. For these subsamples we find evidence suggesting that the elasticity of substitution may vary with the stage of development. Our subsample estimates provide the first evidence of an evolving elasticity of substitution between physical capital and human capital augmented labor along the development path.

We believe that our findings call into question a number of earlier cross-country growth accounting exercises that simply presumed a Cobb-Douglas specification for the aggregate input-output relationship. In fairness, it should be noted that these earlier growth-accounting exercises used different data sets from the one considered in this article, including various different proxies for physical and human capital. Nevertheless, a simple test for model specification, such as the one we have conducted, would seem to be a natural prerequisite for these kinds of cross-country growth analyses.

It is our hope that the findings reported in this article will encourage other researchers, both theoretical and applied, to consider the more general CES specification for the aggregate input-output production relationship (or perhaps some other specification), when constructing models of economic growth across countries. Indeed, some researchers, such as Stokey (1996) and Bencivenga and Smith (1997), have already begun to venture along this path.

Appendix

The Data

The data used in this article were obtained from the STARS database of the World Bank. This data set was recently revised to include comparable annual data on physical capital stocks and the average numbers of years of education of the workforce (used as a proxy for the stock of human capital) for a large number of countries. Both gross domestic product and the stock of physical capital were denominated in constant 1987 local currency units. For cross-country comparability purposes and the common econometric issues that might arise without it, we made transformations to these data sets and converted the series on GDP and physical capital into constant 1987 U.S. dollars.

Data Obtained from STARS

Income (GDP) Gross domestic product at the end of each period in constant 1987 local currency units. For comparison across countries, GDP measured in local constant 1987 currency was converted into constant 1987 U.S. dollars amounts ($US) using official exchange rates for 1987 (Ex87).

Exchange Rate (Ex87) Official end-of-period (annual) bilateral dollar exchange rate (foreign exchange per U.S. dollar) in 1987. In some cases an alternative rate was used when the official rate had been revalued (such as for Brazil or Argentina).

Education (E) Total mean years of education is the sum of the average number of years of primary, secondary, and tertiary education in labor force. These series were constructed from enrollment data using the perpetual-inventory method, and they were adjusted for mortality, drop-out rates, and grade repetition. For a detailed discussion on the sources and methodology used to build this data, set see Nehru, Swanson, and Dubey (1995).

Physical Capital (K) The data on physical capital stocks are taken from the dataset compiled by Nehru and Dhareshwar (1993), who use a modified version of the Harberger-Armington technique to estimate initial capital stocks for each country. Given an initial capital stock K_0 and a sequence of annual investment amounts $\{I_{t-i}\}_{i=0}^{t-1}$, the perpetual-inventory method was used to calculate the capital stock in period t, K_t, according to

$$K_t = \sum_{i=0}^{t-1}(1-\lambda)^i I_{t-i} + (1-\lambda)^t K_0,$$

where the rate of decay of the capital stock λ was set at .04. Since the initial capital stocks and investment amounts are all denominated in constant, 1987 local currency units and are regarded as end of period values, the same holds true for the physical capital stock estimates.

As was the case for GDP, we converted the physical capital stock estimates into constant 1987 U.S. dollar amounts using the official, end-of-period, bilateral dollar exchange rate for 1987 (Ex87) for each country.

Labor Force (L) The population between the ages of 15 and 65 was used a a rough proxy for the labor force.

Countries in the Comprehensive Sample

Our comprehensive sample included 82 countries for which annual data on *both* physical and human capital were available for every year of the sample period, 1960 to 1987. Table 4 provides a list of these countries along with the mean value of all of the variables used in our empirical analysis.

Countries in the Subsamples

The comprehensive sample of 82 countries was divided into four roughly equal-sized subsamples based on initial-period, 1960 levels of capital per worker in each country in constant 1987 U.S. dollars. Below is a list of the countries in each of the four subsamples as well as the capital per worker cut-off levels that we used to determine membership in each group. The ordering of countries within each subsample reflects their rank in terms of 1960 capital per worker.

High-k Subsample (initial capital per worker greater than $10,000, 21 countries) Switzerland, United States, Denmark, Sweden, Iceland, Germany, Netherlands, Norway, New Zealand, Canada, France, Finland, Australia, United Kingdom, Austria, Belgium, Italy, Israel, Ireland, Venezuela, Algeria.

Middle-k Subsample (initial capital per worker $3,000–$10,000, 20 countries) Japan, Uruguay, Jamaica, Cyprus, Spain, Mauritius, Argentina, Iraq, Chile, Peru, Portugal, Greece, Zimbabwe, Bolivia, Brazil, Colombia, Ecuador, Costa Rica, Mexico, Jordan.

Low- middle-k Subsample (initial capital per worker $1,000–$3,000, 18 countries) Panama, El Salvador, Uganda, Malaysia, Iran, Honduras, Guatemala, Senegal, Cameroon, Côte d’Ivoire, Paraguay, Turkey, Tunisia, Korea Rep., Ghana, Philippines, Morocco, Singapore.

Low-k Subsample (initial capital per worker less than $1,000, 23 countries) Zambia, Haiti, Madagascar, Rwanda, Thailand, Nigeria, Zaire, Egypt, Kenya, Sri Lanka, India, Mali, Myanmar, Indonesia, Sudan, Pakistan, Mozambique, Bangladesh, Tanzania, Sierra Leone, Malawi, Ethiopia, China.

Table 4. Mean values of data from the 82 country sample.

Country	Code	GDP (billions of U.S. dollars)	Capital (billions of U.S. dollars)	Labor (millions age 15–64)	Average Years of Education
Algeria	DZA	38.7	142	7.87	2.51
Argentina	ARG	90	250	16	6.38
Australia	AUS	136	426	8.51	6.55
Austria	AUT	83.7	240	4.74	8.7
Bangladesh	BGD	11.3	22.4	38.3	2.56
Brazil	BRA	162	420	59	3.13
Belgium	BEL	104	274	6.24	7.87
Bolivia	BOL	3.62	13.3	2.59	4.29
Cameroon	CMR	6.4	9.75	4.03	1.68
Canada	CAN	260	600	14	8.98
Chile	CHL	13.8	35.5	5.9	6.06
China	CHN	103	309	513	3.36
Colombia	COL	21.5	48.2	13	3.54
Côte d'Ivoire	CIV	6.65	14	3.35	0.93
Costa Rica	CRI	2.87	10.9	1.04	6.14
Cyprus	CYP	1.91	6.15	0.38	6.91
Denmark	DEN	74.5	199	3.21	8.36
Ecuador	ECU	6.52	20.1	3.60	4.22
Egypt	EGY	17.2	25.5	19.9	3.59
El Salvador	SLV	3.71	6.19	1.96	3.54
Ethiopia	ETH	3.93	3.86	17.2	0.24
Finland	FIN	58.6	199	3.11	8.2
France	FRA	629	1620	33	8.01
Germany	DEU	831	2420	42	8.43
Ghana	GHA	4.27	8.77	4.97	2.98
Greece	GRC	31.5	82.1	5.90	7.76
Guatemala	GTM	5.08	10.1	3.04	2.72
Haiti	HTI	1.78	2.18	2.66	1.9
Honduras	HND	2.58	5.03	1.55	3.23
Iceland	ICE	3.08	7.96	0.129	7.58
Indonesia	IND	39	59.3	72.3	2.91
India	IND	155	365	343	2.37
Iran	IRN	109	183	17	2.02
Iraq	IRQ	49	71.6	5.62	2.33
Ireland	IRL	19.5	47.8	1.84	14.55
Israel	ISR	21.9	59.8	1.95	4.69
Italy	ITA	511	1480	36	6.96
Jamaica	JAM	2.71	13.3	1.04	6.89
Japan	JPN	1400	3600	74	10.67
Jordan	JOR	3.06	5.44	1.21	3.11
Kenya	KEN	4.36	19.2	6.53	2.48
Korea, Republic of	KOR	51.6	87.7	19.1	5.12
Madagascar	MDG	2.33	3.83	4.05	2.4
Malawi	MWI	0.77	2.03	2.68	3.34
Malaysia	MYS	16.3	34.5	6.54	4.32
Mali	MLI	1.36	3.34	3.04	0.49
Mauritius	MUS	1.02	3.63	0.5	5.41
Mexico	MEX	89.2	206	31	4.36
Morocco	MAR	11.1	25.1	8.7	1.33
Mozambique	MOZ	1.59	5.91	5.67	1.65

Table 4. Continued.

Country	Code	GDP (billions of U.S. dollars)	Capital (billions of U.S. dollars)	Labor (millions age 15–64)	Average Years of Education
Myanmar (Burma)	MMR	6.95	12	16.7	1.68
Netherlands	NLD	159	483	8.59	8.1
New Zealand	NZL	26.8	77.5	1.8	7.06
Nigeria	NGA	22.4	68.8	37.6	1.34
Norway	NOR	51.9	204	2.48	8.87
Pakistan	PAK	16.7	31.8	36.3	1.49
Panama	PAN	3.14	7.04	0.92	5.66
Paraguay	PRY	2.12	4.12	1.41	5.42
Peru	PER	18.6	52.8	8.0	4.79
Philippines	PHI	23.5	49.5	22.5	6.14
Portugal	PRT	23.9	75.6	5.98	4.44
Rwanda	RWA	1.33	1.09	2.16	2.09
Senegal	SEN	3.32	6.55	2.61	0.98
Sierra Leone	SLE	0.44	0.83	1.59	1.21
Singapore	SGP	9.26	24.5	1.39	4.68
Spain	ESP	201	494	22	6.01
Sri Lanka	LKA	3.95	7.5	7.59	5.15
Sudan	SDN	12.4	13.8	8.44	0.88
Sweden	SWE	120	320	5.25	9.12
Switzerland	CHE	134	374	4.09	6.62
Tanzania	TZA	2.39	7.44	7.84	1.23
Thailand	THA	23.3	48.6	21.7	4.61
Tunisia	TUN	5.36	16.6	3.01	3.0
Turkey	TUR	37.1	93.2	21.9	3.11
Uganda	UGA	5.33	9.31	5.31	2.1
United Kingdom	GRB	510	1220	36	9.66
United States	USA	3100	8300	135	10.91
Uruguay	URY	5.96	18.4	1.77	6.07
Venezuela	VEN	37.2	116	6.71	4.28
Zaire	ZAR	6.2	8.1	11.9	2.57
Zambia	ZMB	1.76	11.9	2.42	2.55
Zimbabwe	ZWE	3.62	12.6	2.94	3.54

Note: The source for this data is the World Bank database STARS. Country-specific mean values presented above have been rounded. The data set used in this study is available in its entirety from the authors on request.

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Notes

1. See, however, Boskin and Lau (1992), who estimate a transcendental logarithmic production function for the group of five countries. Benhabib and Spiegel (1994) note in a footnote (note 7) that they have run growth-accounting-type regressions with a CES specification for the aggregate production function but do not elaborate on their findings.

2. In our empirical analysis we will allow for exogenous technological progress by including time trends in our regressions.
3. If the estimated value of σ is found to be less than 1, then it remains to be shown whether the estimated value of A is sufficiently large to admit the possibility of multiple steady states.
4. See also Barro and Sali-i-Martin (1995, Chapter 1) for a good discussion of this possibility.
5. More generally, this condition may depend on other factors, such as preference parameters. See, e.g., Jones and Manuelli (1997).
6. While endogenous growth due to the specification of the aggregate production function is a possibility with a CES specification, the implication of this type of endogenous growth is that, in the limit, capital's share of output will be 100 percent. Since there is no evidence that capital's share of output in any country of the world is 100 percent or is even approaching this level (indeed capital's share of output appears to be bounded at around 30 to 40 percent), we might readily dismiss this type of endogenous growth as being empirically implausible. However, some caution seems warranted; while we may not see any evidence of Jones-Manuelli type endogenous growth, it may simply be that we do not yet have enough observations to conclusively determine whether the economies of the world are on a trajectory toward this type of long-run endogenous-growth outcome.
7. In addition to considering the simple unweighted sum of the average years of primary, secondary, and post-secondary education, we also considered using the following weighted sum:

$$0.1(\text{avg. yrs. in primary}) + 0.2(\text{avg. years. in secondary}) + 0.3(\text{avg. yrs. in postsecondary}).$$

   Persson and Tabellini (1994) and Tallman and Wang (1994) use similar indexing techniques to construct human-capital proxies. Using this weighted sum in place of the unweighted sum, however, led to no significant difference in our results, and therefore, we chose to report results using only the unweighted sum. The majority of countries in our sample are developing countries that possess very little postsecondary education. Therefore, weighting schemes, such as the one above, which give the most weight to postsecondary education will have very little impact on the human-capital index for these countries.
8. Indeed, the availability of this data on years of schooling played the greatest role in limiting the panel data set we used to just 82 countries.
9. Bils and Klenow (1996) have suggested that human capital augmented labor input is better represented by the expression $HL_{it} = e^{\Phi(E_{it})}L_{it}$, where $\Phi(E)$ is the return to one unit of labor with E years of education. Assuming as in Hall and Jones (1999) that $\Phi(E)$ is piecewise linear and using Psacharopoulos's (1994) evidence on returns to schooling, we constructed this alternative proxy for human capital adjusted labor supply. We found that this alternative proxy did not lead to any qualitative change in our results.
10. See Stokey (1996), who has developed such a specification.
11. The initial parameter choices for all of the NLLS estimation results reported in Table 1 were based on estimates we obtained from a preliminary OLS regression of $\log Y_{it}$ on a constant, $\log K_{it}$, and $\log L_{it}$ or $\log HL_{it}$. We also considered other initial parameter choices and obtained similar NLLS estimates.
12. We chose not to report these regression results in Table 1 as they are very similar to the results found in the first and second columns of this table.
13. The first paper that examined cross-country growth regressions adjusting for both the fixed-effects problem as well as for the endogeneity problem is Caselli, Esquivel, and Lefort (1996). For further discussion on these issues, the reader is referred to their paper.
14. We have used alternative sets of instruments, including one set with $\log K_{i,t-1}$ and $\log L_{i,t-1}$ (or $\log HL_{i,t-1}$) and another set with $\log K_{i,t-2}$, $\log L_{i,t-2}$ (or $\log HL_{i,2-1}$). We do not report these results as they are very similar to those reported in the third column of Table 1.
15. In addition to estimating the linear model with the restriction that $\nu = 1$, we have also estimated an unrestricted version of the linear model where ν was free to vary. We found that the estimates from this unrestricted model were not qualitatively different from those reported below for the restricted linear model.
16. We find similar results when we use either 1972 or 1974 as the break-point for the first half of our sample.
17. Ideally, we would like to allow for endogenous technical progress, by accounting for country-specific investments in R&D (see, e.g., Coe and Helpman, 1997, and Lichtenberg and Pottelsberghe, 1998). Unfortunately, the requisite data for our cross section of countries are not unavailable.

18. See, e.g., Green (1990) for a discussion of this two-way effects model.
19. In addition to a two-way fixed-effects specification, we also considered a random-effects model. We found that in all cases a Hausman test indicated that the fixed-effects model was preferred to the random-effects model. We have therefore chosen not to report the estimation results from the random-effects model specification. We note, however, that the results for the random-effects model were always quite similar to the results obtained with the fixed-effects specification; in particular, the signs and significance of the coefficient estimates were unchanged.
20. See Islam (1995, pp. 1147–1150) for a more extensive discussion of this finding.
21. In an earlier draft, we sorted and divided countries by initial income per worker and obtained similar groupings. Our regression results for the subsamples based on initial income per worker are similar to those we report below for the groupings based on initial capital per worker.
22. For a discussion of econometric issues regarding the TSCS method used here, see, e.g., Green (1990, Chapter 16).
23. Recall that for the middle-k subsample, HL is the same as L, since the model specification with $\phi = 0$ yielded the highest value of the log-likelihood function for this subsample.

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