Stochastic Asymmetric Blotto Games: An Experimental Study

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Abstract. We consider a model where two players compete for n items having different common values in a Blotto game. Players must decide how to allocate their common budgets across all n items. The winner of each item is determined stochastically using a lottery mechanism which yields a unique equilibrium in pure strategies. We analyze behavior under two competing payoff objectives found in the Blotto games literature that have not been previously compared: (i) players aim to maximize their total expected payoff and (ii) players maximize the probability of winning a majority value of all n items. We report results from an experiment where subjects face both payoff objectives and we find support for the differing theoretical predictions.

Keywords: Colonel Blotto game, Contests, Resource Allocation, Lotteries, Electoral College, Game Theory, Political Theory, Experimental Economics.

JEL Classification Nos. C72, C73, C92, D72, D74.

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1. Introduction

Resource allocation plays a central role in both economics and politics. A well-studied game-theoretic representation of the resource allocation problem is the Colonel Blotto game (Borel, 1921) which models the problem as a two-player non-cooperative game. In the canonical "Blotto" game, there are $(2k+1)$ battlefields of equal value. The two players have fixed endowments of resources, e.g., troops or money, which they must simultaneously allocate to each of the $(2k+1)$ battlefields. The winner of each battlefield is determined according to which player allocated the greater amount of resources to that battlefield. The standard objective function for each player is to win $k+1$ or more (a majority) of the battlefields although an alternative objective function is to win as many battlefields as possible.

In this paper we experimentally study behavior in two-player, n-item (or battlefield) *stochastic asymmetric* Blotto games where the n items have commonly known but asymmetric values, and the winner of each item is determined stochastically using a simple lottery mechanism. The lottery mechanism for awarding each item makes the payoff function continuous, by contrast with the canonical deterministic (or auction) version of the Blotto game, resulting in a unique pure strategy equilibrium allocation of players' endowments across the n items. Thus, the stochastic Blotto game environment provides crisp predictions for an experimental evaluation by contrast with the deterministic version of the Blotto game which yields a multiplicity of possible equilibria typically in mixed strategies. Within the stochastic Blotto game environment, we compare and contrast behavior under an objective function where players seek to win a *majority* of the values of the n battlefields with an alternative objective function where players seek to maximize their *total* expected earnings from allocating their budget across all n battlefields. To our knowledge there is no prior experimental work comparing these two different, but commonly used objective functions for Blotto games.

A motivating example for the majority rule version of the stochastic asymmetric Blotto game is the U.S. electoral college mechanism for determining the President of the United States. The electoral college system is a two-player, stochastic asymmetric majority-rule Blotto game in the sense that: 1) there are typically just two presidential candidates; 2) the 51 states (including the District of Columbia) have different numbers of electoral votes (values) ranging from a minimum of 3 to a maximum (as of 2012) of 55 votes (for the state of California); 3) the presidential candidate who spends the most resources (campaign expenditures) on any given state does not necessarily win that state's electoral votes, i.e., the winner is stochastically determined;[1] 4) the overall winner of the Presidency is the candidate who earns a majority (currently 270 or more) of the total electoral votes (currently 538), that is, the objective is to achieve a majority of the asymmetric and stochastically awarded prize values (electoral votes).

A motivating example for the total rule version of the stochastic asymmetric Blotto game (due to Friedman (1958)) comes from advertising decisions by two firms in a duopoly setting. The two firms produce and sell an identical good and must decide how to allocate their advertising budgets over n markets having various different sales potentials that are

[1] For example in the 2012 U.S. Presidential election, candidate Barak Obama and affiliated political action committees spent \$69.3 million USD in the state of Florida while candidate Mitt Romney and affiliated political action committees spent \$81.3 million USD in Florida (Bell and Wilson (2012)). Despite being out spent in Florida, candidate Obama nevertheless won Florida's 29 electoral votes in the 2012 presidential election.

common to both firms. As advertising is not perfectly effective in stimulating sales of the good, increased expenditures in any given market may have only a stochastically larger impact on each firms' sales of the good in that market. However, both duopolists' goal is to maximize the *total market share* for their product and not just a majority share as in the electoral college example.

We note that while these two objective functions (majority and total) might seem to be quite similar, equilibrium resource allocation by the two players under the majority rule objective is dramatically different from equilibrium resource allocation under the total rule objective and this observation is what motivates our experimental study evaluating allocation decisions under the two different rules. We present results from a within-subjects experimental design involving two different versions of a 4-item stochastic asymmetric Blotto game where subjects make resource allocation decisions under both the total and majority rules. Our experiment yields support for the different equilibrium allocation predictions under the two different payoff objective functions.

The rest of this paper is organized as follows. The next section reviews the related literature. Section 3 provides a theoretical framework for comparing equilibria under the two different objective functions for the stochastic asymmetric Blotto game. Section 4 describes our experimental design and Section 5 reports our main experimental findings. Section 6 concludes with a summary and some suggestions for future research.

2. Related Literature

The Blotto game is one of the oldest games in game theory. It was originally proposed by Borel (1921), who considered the $n = 3$ battlefield case. Many researchers have subsequently analyzed and extended the n-battlefield Blotto game in a number of important directions. See, for example, Tukey (1949), Gross and Wagner (1950), Blackett (1954, 1958), Bellman (1969), Young (1978), Shubik and Weber (1981), as well as more recent publications: Laslier (2002), Laslier and Picard (2002), Roberson (2006), Sahuguet and Persico (2006), Kvasov (2007), Powell (2007, 2009), Hart (2008), Adamo and Matros (2009), Golman and Page (2009), Hortala-Vallve and Llorente-Saguer (2010, 2012), Weinstein (2012), Roberson and Kvasov (2012), Kovenock and Roberson (2012) and Macdonell and Mastronardi (2012). There is also a long tradition in the literature of considering various types of majority rule Blotto games to analyze elections, for example Brams and Davis (1973, 1974), Colantoni, Levesque, and Ordeshook (1975).

Friedman (1958) was the first to provide analytic results for the stochastic asymmetric "total rule" Blotto game where the winner of each item i is determined by a lottery in which the chance of winning is proportional to each player's allocation of resources to item i.[2] Friedman describes a unique pure-strategy Nash equilibrium which we test in our experiment. Lake (1978) was the first to study a stochastic asymmetric "majority rule" Blotto game. Lake studied the case of equal budget constraints, which we also employ in our experiment. The equilibrium prediction in the majority rule case with equal budget constraints is that allocation of resources to item i should equal the Banzhaf Power Index for item i. In a companion paper, Duffy and Matros (2015), we generalize Lake's (1978) theoretical findings to asymmetric stochastic Blotto games involving n players and asymmetric budget

[2] Monahan (1987) and Robson (2005) generalize Friedman's model. Monahan allows costly effort and Robson considers more general versions of Tullock's (1980) contest success function.

constraints, i.e., $X \neq Y$ and we theoretically compare and contrast the predictions for the stochastic asymmetric Blotto game under the total and majority rule objectives.

Snyder (1989) analyzes an election model where players have cost functions instead of given budgets (or endowments). He was the first to compare equilibrium behavior under two different assumptions about the candidates' objectives 1) the total rule, where candidates maximize the expected total number of votes, and 2) the majority rule, where candidates maximize their probabilities to win a majority of the total votes. Klumpp and Polborn (2006) use Snyder's framework to develop a costly advertising model of political competition (both simultaneous and sequential) in which candidates have to win the majority of a number of electoral districts in order to obtain a certain prize. In both of these papers, the setting differs from the environment we study due to the presence of cost functions as opposed endowments and budget constraints. More importantly in both Snyder (1989) and Klumpp and Polborn (2006), the value of each item (or of each electoral district) is the same. By contrast we study the more general case where the value of the individual items may be different from one another, as in the number of electoral votes per state in the U.S. electoral college, or the sales potential of different markets for a good.

More relevant to this paper are several different experimental studies of Blotto games. Avrahami and Kareev (2009) consider a lottery version of an 8-item (box) Blotto game in which players have identical valuations for the items but different budgets. In their setting, only one box is open for each player and the winner is determined by comparing these two boxes. Modzelewski, Stein, and Yu (2009) analyze the classic (auction) Blotto game with 6 identically valued battlefields and equal resources. Arad and Rubinstein (2012) consider a large web-based experiment of a tournament version of the Blotto game with 6 battles and equal resources. Chowdhury, Kovenock, and Sheremeta (2013) investigate two types of Blotto games: stochastic (lottery) and deterministic (auction) with 8 battlefields (boxes) all having identical values but where the two players may have asymmetric resources. Several recent experimental studies consider Blotto games with asymmetric item values as in this paper, but these studies all use deterministic (auction) rules for determining the winner of each item. Hortala-Vallve and Llorente-Saguer (2010) examine an incomplete information Blotto game with both asymmetric and heterogeneous battlefield valuations so that the game is no longer zero-sum. Hortala-Vallve, Llorente-Saguer and Nagel (2013) and Hortala-Vallve, Llorente-Saguer (2015) report on further experiments in similar settings but where the players can either communicate with one another or there is complete information about the player's heterogeneous valuations. Montero, Possajennikov, Sefton and Turocy (2016) conduct an experimental test of Young's (1978) model where two lobbyists allocate resources to politicians having different voting powers and seek to obtain a majority of votes cast for their opposed positions.

By contrast, the contribution of this paper is to experimentally compare and contrast Blotto games where: 1) the items (battlefields) have asymmetric but commonly known values 2) the items (battlefields) are awarded according to a (stochastic) lottery mechanism so that the unique equilibrium prediction is always in pure strategies and 3) the objective function is either to maximize total expected payoff or to win a majority of the value of all items. Our main focus is on whether subjects appreciate the subtleties of the two different objective functions in allocating resources across the differently-valued battlefields. We are not aware of any prior experimental companions of these two different, but commonly studied objective functions for Blotto games. As noted in the introduction, these two different versions of the asymmetric, stochastic Blotto game are of real-world interest in understanding advertising decisions by competing duopolists and resource allocation by U.S. presidential candidates competing to win the Electoral College.

3. Stochastic Asymmetric Blotto Games

The game we study involves two players x and y, and n items. Player x has a given budget of size X and player y has a given budget of size Y. Let $N=\{1,...,n\}$ denote the set of the n items (or battlefields). Each item i has a known value, $W_i > 0$, that is the same for both players. The two players compete for these items by simultaneously allocating their budgets across all n items. A pure strategy for player x is a nonnegative $n-$dimensional vector $(x_1, ..., x_n)$, such that $\sum_{j=1}^{n} x_j = X$ and x_i is player x's spending on item i. A pure strategy for player y, $(y_1, ..., y_n)$, is determined analogously with $\sum_{j=1}^{n} y_j = Y$. Each item is allocated by means of a lottery in which player x obtains item i with probability $x_i/\left(x_i + y_i\right)$ and player y obtains item i with probability $y_i/\left(x_i + y_i\right)$.[3,4] Without loss of generality, for the rest of the paper we shall assume that

$$W_1 \geq ... \geq W_n > 0. \tag{1}$$

Denote the total value of all n items by:

$$W = \sum_{i=1}^{n} W_i.$$

3.1. Plurality: Maximizing the expected value. Friedman (1958) describes a Nash equilibrium where both players seek to maximize their expected item values:

$$\max_{x_1,...,x_n} \sum_{j=1}^{n} \frac{x_j}{x_j + y_j} W_j,$$

$$s.t. \sum_{j=1}^{n} x_j = X \text{ and } x_j \geq 0 \ \forall j,$$

for player x and analogously for player y.

Theorem 1. *(Friedman, 1958) The stochastic Blotto game where players seek to maximize their expected item values has a unique Nash equilibrium. In this Nash equilibrium,*

$$(x_1, ..., x_n) = \left(\frac{W_1}{W}, ..., \frac{W_n}{W}\right) X, \tag{2}$$

$$(y_1, ..., y_n) = \left(\frac{W_1}{W}, ..., \frac{W_n}{W}\right) Y. \tag{3}$$

The expected equilibrium payoffs are $\frac{X}{X+Y}W$ *for player* x *and* $\frac{Y}{X+Y}W$ *for player* y.

[3] We assume that if $x_i = y_i = 0$, then each player has .5 probability to win item i.

[4] We assume that all lotteries are statistically independent.

There are several corollaries to this Theorem. First, note that the Nash equilibrium described is unique. Second, both players compete for all items in the Nash equilibrium of this version of the Blotto game. The intuition for the latter result is straightforward: if one player were to allocate zero resources to any single item, his opponent would win it with certainty with only a very small allocation to that item in which case the first player could very cheaply allocate enough resources to that same item to yield a significant probability of winning it. It follows that zero allocation to any item is not an equilibrium best response in the Blotto game where both players seek to maximize their expected item values. Third, the unique Nash equilibrium has a monotonic property: the player with the *greater budget* has a greater chance to win *each item*.

3.2. Weighted Majority: Maximizing the probability of winning a majority. We now assume that each player wants to maximize her probability to win a majority of all items' values as in the U.S. electoral college example. Note that each possible coalition of items $\{W_1, ..., W_n\}$ can be represented by a binary, n-dimensional characteristic vector $(t_1, ..., t_n)$, where $t_i \in \{0, 1\}$ for any $i = 1, ..., n$. If $t_i = 1$, then item i belongs to the coalition, and if $t_i = 0$, then item i does not belong to the coalition. We will use the corresponding characteristic vector to represent a coalition in the rest of the paper. There are 2^n such coalitions.

Denote by $\mathbf{V}$ the set of winning coalitions under the win-a-majority-of-item-values objective. Then coalition $(t_1, ..., t_n) \in \mathbf{V}$, if

$$\sum_{j=1}^{n} t_j W_j > \frac{W}{2}.$$

We next make a technical assumption that guarantees a unique majority winner in all realizations of individual lotteries.

Assumption 1.

$$\sum_{j=1}^{n} t_j W_j \neq \frac{W}{2} \text{ for any coalition } (t_1, ..., t_n)\,. \tag{4}$$

A stronger version of Assumption 1 is typical in the literature. Usually in this literature, all values are the same, $W_1 = ... = W_n$, in which case Assumption 1 becomes $n = 2k + 1$, $k = 1, 2, ...$

A player wins the stochastic majority Blotto game if she gets a winning coalition. Without loss of generality, a player receives a payoff of 1 from winning the game and a payoff of 0 from losing the game. Player x maximizes her chance to get a winning coalition by solving the following maximization problem:

$$\max_{x_1,...,x_n} \sum_{(t_1,...,t_n)\in\mathbf{V}} \prod_{j:t_j=1} \frac{x_j}{x_j + y_j} \prod_{k:t_k=0} \frac{y_k}{x_k + y_k}, \tag{5}$$

$$s.t. \sum_{j=1}^{n} x_j = X, \tag{6}$$

where $\prod_{j:t_j=1} \frac{x_j}{x_j+y_j}$ is the probability to win all the items that belong to the coalition $(t_1, ..., t_n)$ and $\prod_{k:t_k=0} \frac{y_k}{x_k+y_k}$ is the probability to lose all the items that do not belong to the coalition $(t_1, ..., t_n)$.

Similarly, player y solves the following maximization problem:

$$\max_{y_1,\dots,y_n} \sum_{(t_1,\dots,t_n)\in\mathbf{V}} \prod_{j:t_j=1} \frac{y_j}{x_j+y_j} \prod_{k:t_k=0} \frac{x_k}{x_k+y_k}, \tag{7}$$

$$s.t. \sum_{j=1}^{n} y_j = Y. \tag{8}$$

We will need the following definition.

Definition 1. *An item i is pivotal in coalitions $(t_1, ..., t_{i-1}, 1, t_{i+1}, ..., t_n)$ and $(t_1, ..., t_{i-1}, 0, t_{i+1}, ..., t_n)$ if $(t_1, ..., t_{i-1}, 1, t_{i+1}, ..., t_n)$ is a winning coalition and $(t_1, ..., t_{i-1}, 0, t_{i+1}, ..., t_n)$ is a losing coalition.*

Denote by $\mathbf{V}_i$ a set of winning coalitions where item i is pivotal. Denote by $Piv\,(i)$ the number of winning coalitions in which item i is pivotal, or

$$Piv\,(i) = \|\mathbf{V}_i\| = \sum_{(t_1,\dots,t_n)\in\mathbf{V}_i} 1. \tag{9}$$

We now introduce the Banzhaf Power Index[5] for item i in the following way:

$$BPI(i) = \frac{Piv(i)}{Piv(1)+...+Piv(N)} = \frac{\sum_{(t_1,\dots,t_n)\in\mathbf{V}_i} 1}{\sum_{(t_1,\dots,t_n)\in\mathbf{V}_1} 1 + ... + \sum_{(t_1,\dots,t_n)\in\mathbf{V}_n} 1}. \tag{10}$$

Intuitively, the Banzhaf Power Index for item i measures the probability that item i will be pivotal as part of a winning coalition. Using this definition, and following Lake (1978), we can now characterize the Nash equilibrium for the asymmetric stochastic Blotto game where both players seek to maximize their probability of winning a *majority* of all items' values.

Theorem 2. *(Lake, 1978) Suppose that conditions* (4) *and $X = Y$ hold. Then, the stochastic Blotto game where players seek to maximize their chance of winning a majority of the item values has a unique pure-strategy Nash equilibrium in which $x_i = BPI\,(i)\,X$ and $y_i = BPI\,(i)\,Y$, for $i = 1, 2, ..., n$.*

A comparison of equilibrium allocations under the two different, but seemingly similar objective functions – the total and the majority rules – as given in Theorems 1 and 2 reveals that the unique equilibrium allocations of resources to the n items can be quite different from one another. A natural question is whether these differences are simply a theoretical curiosity or whether they are indeed empirically relevant to subjects incentivized to play according to the two different objective functions. To address this important question we designed an experiment to test equilibrium predictions in stochastic asymmetric Blotto games under the two different objective functions.

[5]See Banzhaf (1965) for discussion.

4. Experimental Design

The main objective of our experiment is to test the predictions found in Theorems 1 and 2 regarding the equilibrium allocation of players' endowments toward winning the n prizes under the two different payoff rules. Toward that goal we chose to consider a two player, $n = 4$ item asymmetric value, stochastic Blotto game. We focus on the $n = 4$ case because it represents the smallest value of n for which the differences in allocations under the two payoff rules are sufficiently distinct from one another. The equilibrium allocation under the total rule is unique for any n (Theorem 1). For simplicity, we also study the case where the two players have equal budgets, leaving the case of asymmetric budgets to future research.

Our experiment involves a 2×2 experimental design where the four main treatment variables are 1) the payoff rule: players either seek to maximize the total expected value of prize items – "the total rule" or to maximize their chance of winning a majority of the value of all items – "the majority rule." and 2) the vector of values of the four prize items (labeled Version 1 and Version 2). We chose to consider two different prize value vectors to test some of the comparative statics implications of the theory as detailed below.

Under the total rule, a player receives in points the total value of any and all items won. Under both prize Versions 1 and 2, the total value of all four items sums to 100 points. Thus under the total rule, a player can get a positive number of points if she wins one or more of the four items. The game is constant sum: one player's earnings are 100 minus the earnings of the other player.

By contrast, under the majority rule, a player wins the game if the value of all her won items is greater than the value of the items won by her opponent. The winner of the majority rule game receives 100 points and the loser receives *zero* points. The prize valuations respect our Assumption 1 (condition (4)) so the majority winner is always unambiguous in our setup. Further, since $n = 4$ and the two players have equal budget constraints, the equilibrium allocations follow the Banzhaf power index predictions given in Proposition 3 and are uniquely determined.

In all four treatments, the play of a round of the asymmetric stochastic Blotto game proceeds as follows. First, subjects are randomly and anonymously paired with one another with all possible pairings being equally likely in their matching group of size six. Then, both players in each pair are given a budget of 120 tokens and instructed that each must allocate their 120 tokens toward winning the four items. Both players submit their token allocations simultaneously and without communication using a computer interface developed for this study. The program checks that the four allocations made by each player to the four items sum up to 120; if not, then a player is prompted to resubmit his or her allocation until the budget of 120 tokens has been fully exhausted. Thus, the pure strategy of a player j is a 4-dimensional vector $\left(x_1^j, ..., x_4^j\right)$, such that $\sum_{i=1}^4 x_i^j = 120$.

The four items have different but commonly known valuations, W_i, $i = 1, ..., 4$. These valuations remained constant across all rounds of a session. Item i is allocated by means of a lottery in which player j wins item i with probability $x_i^j / \left(x_i^j + x_i^k\right)$ where x_i^k is the bid made for the same item i by player k, who is player j's opponent. We assume that if $x_i^j = x_i^k$ (including the case where $x_i^j = x_i^k = 0$) then each of the two players has a 0.5 probability of winning item i. The lottery mechanism for awarding prizes was carefully explained to subjects in the written instructions. Example scenarios were presented and subjects had to successfully complete a quiz demonstrating their knowledge of the lottery mechanism used to determine the winners of each of the four items prior to the start of the experiment.

As mentioned, we considered two versions for the distribution of values across the four items.

4.1. Version 1. In this treatment, the four items had the known values:

$$W_1 = 35,\ W_2 = 30,\ W_3 = 25,\ W_4 = 10.$$

The value of these four items sum to 100 points. Given that both players have a fixed budget of 120 tokens, Table 1 gives the unique equilibrium predictions regarding the allocation of tokens across the four items under the total and majority rules for prize vector Version 1. These predictions follow from Theorems 1 and 2 and Proposition 3.

Prize Vector	Eq. Token Allocation Under	
Version 1	Majority Rule	Total Rule
$W_1 = 35$	40	$120 \times \frac{35}{100} = 42$
$W_2 = 30$	40	$120 \times \frac{30}{100} = 36$
$W_3 = 25$	40	$120 \times \frac{25}{100} = 30$
$W_4 = 10$	0	$120 \times \frac{10}{100} = 12$

Table 1: Equilibrium Predictions for Prize Vector Version 1

Notice that our parameterization of the experiment induces distinct differences in equilibrium bids between the two rules. For example, if a player wins only items 1 and 2 under the Majority rule, then she has a majority of the 100 possible prize points ($35 + 30 = 65 > 35 = 25 + 10$) and so her payoff is 100 while the payoff of her opponent is 0. By contrast, if a player wins only items 1 and 2 under the Total rule, then her payoff is $35 + 30 = 65$ and the payoff of her opponent is $25 + 10 = 35$. A complete characterization of the points and dollar earnings possible in this version of the experiment is given in the payoff table of the experimental instructions found in Appendix A.

4.2. Version 2. In this treatment, the four items had the known values:

$$W_1 = 45,\ W_2 = 25,\ W_3 = 20,\ W_4 = 10.$$

The values of the four items again sum to 100 points. Notice that Version 2 differs from Version 1 only in the values assigned to the first three items; the value of the fourth prize is 10 in both treatments, (prize versions), a feature we will later exploit in our analysis of the experimental data. Under the maintained assumption that both players have a fixed budget of 120 tokens, Table 2 gives the equilibrium predictions regarding the allocation of tokens across the four items under the majority and total rules for prize vector Version 2.

Notice that the two different versions for the vector of prize values (Versions 1 and 2) yield strikingly contrasting predictions as to how subjects should allocate their tokens, and indeed, that is why we chose these two different sets of prize values. In particular, observe

Prize Vector	Eq. Token Allocation Under	
Version 2	Majority Rule	Total Rule
$W_1 = 45$	60	$120 \times \frac{45}{100} = 54$
$W_2 = 25$	20	$120 \times \frac{25}{100} = 30$
$W_3 = 20$	20	$120 \times \frac{20}{100} = 24$
$W_4 = 10$	20	$120 \times \frac{10}{100} = 12$

Table 2: Equilibrium Predictions for Prize Vector Version 2

that for Version 1, under the Majority rule, there should be zero allocation to an item that can never be pivotal (item 4). Further, when payoffs are determined by the Majority rule, allocations should be the same across the first three items under Version 1 while under Version 2, allocations should be the same across the last three items. By contrast, for both prize vector versions, allocations under the Total rule should be proportional to the prize values.

4.3. Interface. On the first decision screen, subjects chose how to allocate their tokens across the four items. Subjects were reminded on this first decision screen of the values of the four items which were presented to subjects either in descending order from highest to lowest value or in ascending order from lowest to highest value (a further treatment condition). Next to each prize value, subjects entered their bids and the program checked that bids for all items were non-negative and summed to 120, the common endowment of tokens for each subject. After all pairs had submitted their bids the winners of each of the four items in each pair were determined by the computer program and the results were shown to subjects using a novel graphical display that utilized pie charts to convey the chances that both subjects in a pair had of winning the four different items. An illustration of this second "results" screen for Version 1 under the total rule is shown in Figure 1. This results screen also revealed the number of tokens that each player in the pair had allocated toward winning the four items and reported on whether a player had won or lost each prize, their total points earned (according to either the total or majority rule in place) and their dollar earnings for that round. It was public knowledge that the point earnings of the other player in the match were 100 minus the player's point earnings. Subjects only saw results for their own pair.

4.4. Session Characteristics. Each session involved 18 subjects divided up into 3 matching groups of size 6. Subjects were randomly matched only with the five other members of their matching group in all rounds of each session. Thus, each session yields three independent observations or what we label 'groups'. We conducted two sessions of each of our four treatments. Thus we have data from 8 sessions involving $8 \times 18 = 144$ subjects divided up into 24 matching groups.

In each session, the vector of prize values $\{W_1, W_2, W_3, W_4\}$ was set either according to Version 1 or Version 2 and the prize vector remained constant for the duration of the session. We used a within-subjects design where subjects in each matching group/session played the first 20 rounds under either the majority or the total rule treatment. Following the 20th round, the experiment was paused and subjects were given new, continuation

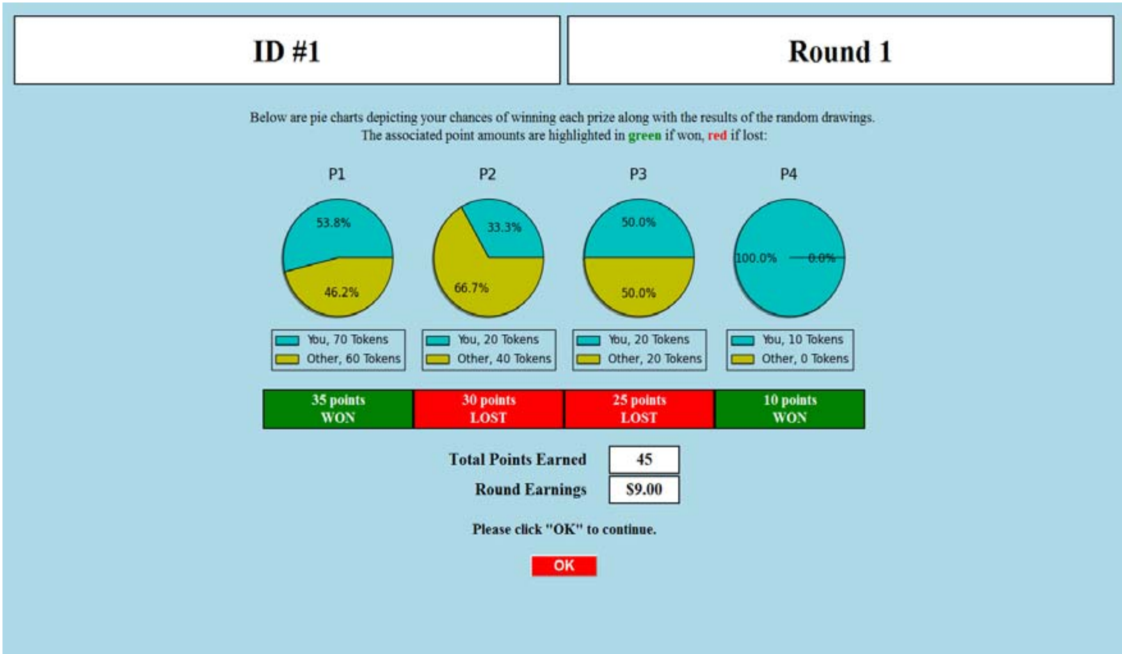

Figure 1: Illustration of the depiction of the results from play of treatment version 1 under the total rule.

instructions informing them that in the remaining 20 rounds they would be playing under the opposite rule, either the total or the majority rule treatment. This treatment change was *not* announced in advance. Thus each session involved play of 20 rounds of both payoff rules (40 rounds total) all under the *same* vector of item values. Sessions 1-4 used Version 1 for the prize values, $\{35, 30, 25, 10\}$, while sessions 5-8 used Version 2 for the prize values, $\{45, 25, 20, 10\}$. We chose a within-subjects design as it is statistically more powerful than a between-subject design since in a within-subject design, each subject serves as their own control (e.g., being in both objective function treatments) thereby minimizing the effects of individual differences.[6] At the same time, within-subject designs mean that treatment order can matter and to minimize this possibility, in one-half of the sessions of each payoff vector treatment (Version 1 or Version 2) subjects played under the majority rule for the first 20 rounds followed by play under the total rule for the last 20 rounds, while in the other half of the sessions of each payoff vector treatment this order was reversed. As noted earlier, we also varied the order in which the list of prize values was presented to subjects and their resource allocations were elicited. In half of the sessions of each payoff vector treatment, prize values were presented and resource allocations were elicited in descending order (e.g., $\{35, 30, 25, 10\}$) while in the other half they were presented and elicited in ascending order (e.g., $\{10, 25, 30, 35\}$). The precise details of our eight experimental sessions are summarized in Table 3.

A copy of the written instructions used in session 1 where the majority rule was played

[6] See, e.g., Camerer (2003), pp. 41-42.

Session Number	Item Value List Order	Payoff Rule Treatment Order	No. of Subjects	No. of Indep. Obs.	Group Nos.	Avg. Total Earnings
1	$\{35, 30, 25, 10\}$	Majority then Total	18	3	1–3	\$21.33
2	$\{35, 30, 25, 10\}$	Total then Majority	18	3	4–6	\$22.33
3	$\{10, 25, 30, 35\}$	Majority then Total	18	3	7–9	\$27.22
4	$\{10, 25, 30, 35\}$	Total then Majority	18	3	10–12	\$25.39
5	$\{45, 25, 20, 10\}$	Majority then Total	18	3	1–3	\$25.78
6	$\{45, 25, 20, 10\}$	Total then Majority	18	3	4–6	\$23.89
7	$\{10, 20, 25, 45\}$	Majority then Total	18	3	7–9	\$29.67
8	$\{10, 20, 25, 45\}$	Total then Majority	18	3	10–12	\$26.44

Table 3: Characteristics of Experimental Sessions

in the first 20 rounds and the total rule was played in the last 20 rounds all under prize Version 1 with descending prize values is given in Appendix A. Instructions for other sessions/treatments are similar.

The experiment was programmed using Willow, a Python-based toolkit for conducting economics experiments.[7] Subjects were recruited from the undergraduate population of the University of Pittsburgh and the experiments were all conducted in the Pittsburgh Experimental Economics Laboratory. No subject participated in more than one session of this study.

4.5. Payments. Subjects were paid their point earnings from two randomly chosen rounds, one from the first 20 rounds played and one from the second 20 rounds played. Points from these two randomly chosen rounds were converted into dollars at the fixed and known rate of 1 point = 20 cents (so 100 points = \$20). In addition, subjects were given a \$5 show-up payment. Thus the maximum total payoff that subjects could earn (including the \$5 show-up payment) was \$45, the minimum payment was \$5 and the average payoff was \$25.26 for participation in a 90-minute experimental session. Note that the two rounds chosen for payment were randomly drawn for each subject in each session, which accounts for the different average total earnings reported in Table 3.

5. Experimental Results

The main comparative statics implications of our treatments concern the allocation of subjects' endowment of 120 tokens across the four items having the prize values of Version 1 or Version 2 and under the different payoff rules, the majority rule or the total rule. Our experimental results consist of a number of different findings.

Finding 1. *Mean bids are qualitatively similar to Nash equilibrium predictions, however quantitatively mean bids differ from Nash equilibrium predictions in three of the four main treatments.*

[7]Willow was developed by Jaap Weel at George Mason University and is available at: http://econwillow.sourceforge.net/

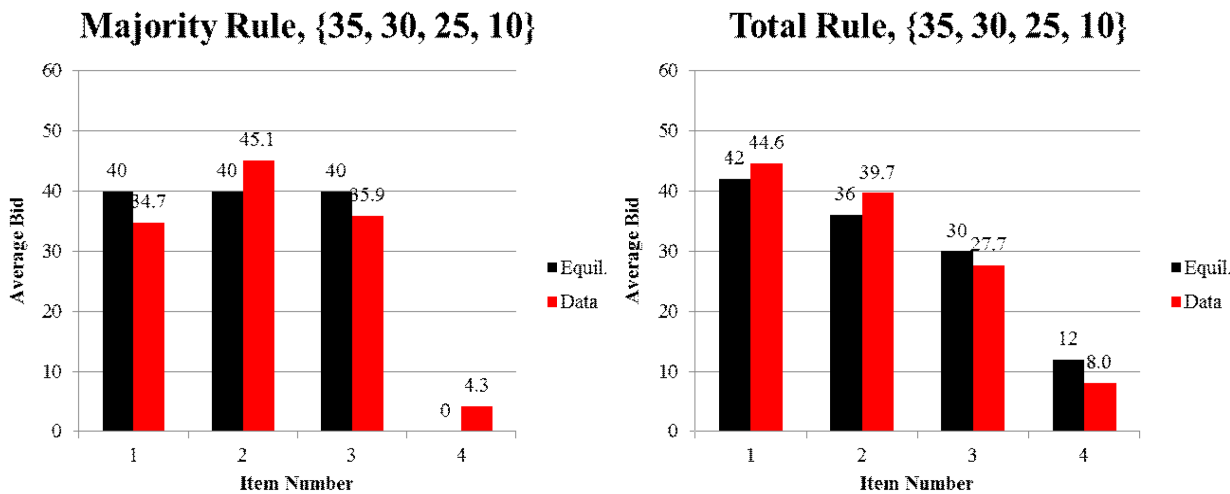

Figure 2: Average Bids under the Majority and Total Rules, Version 1 {35, 30, 25, 10}. Pooled Data from All Rounds Played by All 12 Groups

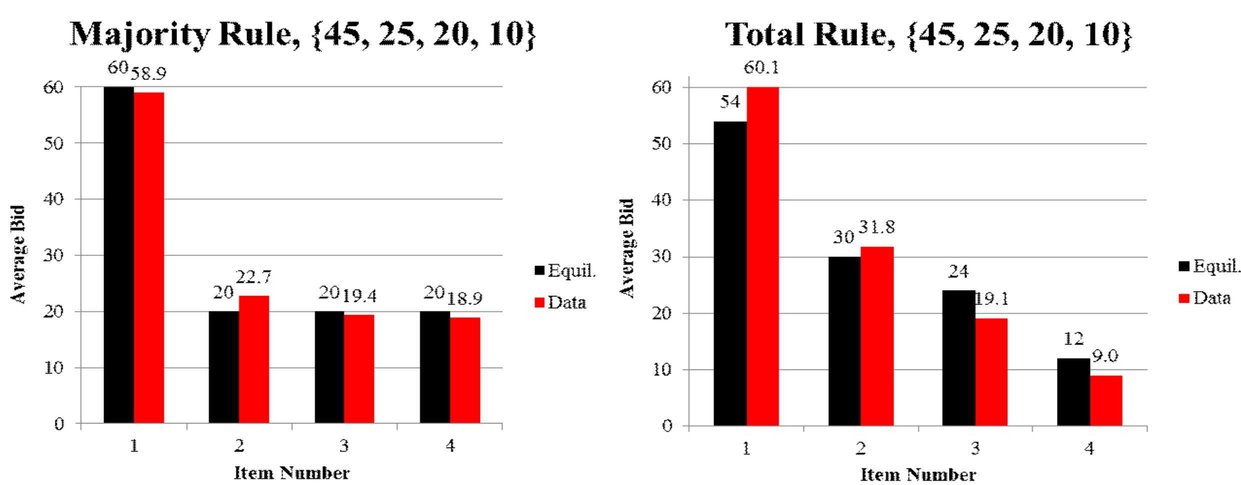

Figure 3: Average Bids under the Majority and Total Rules, Version 2 {45, 25, 20, 10}. Pooled Data from All Rounds Played by All 12 Groups

Treatment	Majority Rule Version 1				Total Rule Version 1			
Prize Value	35.00	30.00	25.00	10.00	35.00	30.00	25.00	10.00
Nash Bid	40.00	40.00	40.00	0.00	42.00	36.00	30.00	12.00
Mean Bid	34.73	45.08	35.92	4.26	44.58	39.69	27.70	8.04
(StDev)	(8.29)	(5.66)	(6.39)	(1.95)	(4.48)	(3.78)	(2.76)	(2.79)
WSRT p-value	0.06	0.01	0.12	0.00	0.07	0.01	0.02	0.00
Mean Bid L5	33.33	45.87	36.20	2.92	44.71	38.95	28.18	8.16
(StDev) L5	(10.41)	(6.93)	(8.94)	(1.94)	(4.39)	(4.56)	(2.81)	(3.15)
WSRT L5 p-value	0.07	0.02	0.21	0.00	0.05	0.03	0.05	0.01
Treatment	Majority Rule Version 2				Total Rule Version 2			
Prize Value	45.00	25.00	20.00	10.00	45.00	25.00	20.00	10.00
Nash Bid	60.00	20.00	20.00	20.00	54.00	30.00	24.00	12.00
Mean Bid	58.94	22.72	19.42	18.92	60.06	31.84	19.07	9.03
(StDev)	(6.90)	(3.48)	(4.60)	(4.66)	(5.73)	(1.64)	(4.40)	(3.10)
WSRT p-value	0.35	0.01	0.72	0.58	0.00	0.00	0.00	0.01
Mean Bid L5	56.89	22.78	20.56	19.77	61.43	31.21	18.67	8.70
(StDev) L5	(9.32)	(5.17)	(6.27)	(6.58)	(9.46)	(2.23)	(5.16)	(4.45)
WSRT L5 p-value	0.39	0.12	0.81	0.88	0.03	0.10	0.01	0.03

Table 4: Mean bids compared with Nash bids

Support for finding 1 can be found in Figures 2-3 which show the mean bids for each of the four items by all subjects participating in each of the four treatments using pooled data from all 12 groups of the four main treatments: Version 1, Majority and Total Rule and Version 2, Majority and Total Rule.[8] The Nash equilibrium bid for each item is given by the left-most bar (the one colored black), while the second (red (or lighter)–colored) bars represent the mean amounts bid on each item by subjects. Figures 2-3 give the impression as conveyed in Finding 1 that Nash equilibrium provides a good but imperfect way of characterizing bidding behavior in our experiment. Table 4 reports the same mean bids shown in Figures 2-3 along with their associated standard deviations for each treatment/prize value over all 20 rounds and over the last 5 rounds only (L5).[9] In addition, Table 4 reports p-values from a two-sided, one-sample Wilcoxon signed rank test (WSRT) of the null hypothesis that the median of the 12 independent group-level mean bids for each prize/treatment condition, reported in Tables 9-12 of Appendix B, equals the Nash equilibrium predicted bids for that same prize/treatment condition. The p-values from applying this test are reported both for mean bids over all rounds and for mean bids over the last 5 rounds (L5) only. Table 4 reveals that mean bids over all rounds or over the last 5 rounds are close to but often statistically significantly different from Nash equilibrium predictions at the 5 percent level. The sole

[8]For now we ignore other treatment variables such as the order of the payoff rules or whether prizes were presented in ascending or descending order. We will address the impact of these treatment variables below.

[9]Tables 9-12 in Appendix C provide mean bids for each of the 12 groups per treatment over various subintervals of time.

exception occurs for mean bids under the Majority Rule, Version 2 treatment where by the last 5 rounds, we cannot reject the null hypothesis that the median of the mean bids differs from the Nash equilibrium bid for all four items ($p > .10$ for all four tests). Despite these differences between mean bids and Nash equilibrium bids, there is considerable support for the comparative statics predictions of the theory with regard to the change in the payoff rule on individual behavior. Specifically, we have:

Finding 2. *Individual bids respond to the change in the payoff rule in a manner that is consistent with equilibrium predictions.*

Support for finding 2 comes from Table 5 which reports results from a GLS random effects estimation of the linear model:

$$b_{W,i,t} = \alpha + \beta_1 \delta^T + \beta_2 \delta^{RO} + \beta_3 \delta^A + \epsilon_{i,t}.$$

Here $b_{W,i,t}$ denotes the amount bid on the item with value W by player i in round t, δ^T is a dummy variable equal to 1 when the total rule was in effect (so the baseline is the majority rule), δ^{RO} is a dummy variable equal to 1 when the Rule Order was 20 rounds of the total rule followed by 20 rounds of the majority rule (so the baseline is majority then total) and δ^A is a dummy variable equal to 1 when the four items were presented to subjects for bidding in ascending order of valuation, e.g., $\{10, 25, 30, 35\}$ (so the baseline is descending order, e.g., $\{35, 30, 25, 10\}$). The regression (5) was run separately for each prize value under either Version 1 or Version 2 (8 regressions). The regressions make use of bid data from all four sessions of either Version 1 or Version 2, consisting of 2,880 individual-level observations on amounts bid over all 40 rounds for each prize value, W. The regression estimates and robust standard errors obtained from clustering on matching groups are reported in Table 5.

Table 5 reveals that for nearly every prize, the coefficient on the dummy variable, δ^T, is significantly different from zero, and in all cases where this coefficient is significantly different from zero, the coefficient has the theoretically predicted sign (negative or positive). For example, consider bids for the prize with value 35, b_{35}, under Version 1 – the first column of Table 5. In the baseline, majority rule case, the coefficient on the constant term indicates an average bid of 31.46 for this prize which is somewhat less than the predicted bid of 40 for this treatment condition. However, under the total rule there is a statistically significant increase in the average bid by the amount 9.85 (the coefficient on δ^T) taking the average bid up to 41.31 (ignoring other explanatory factors); the latter is close to the Nash equilibrium bid prediction of 42 for the total rule treatment. Bids for the other three prizes under Version 1 and bids b_{25} and b_{10} under Version 2 display similarly theoretically consistent reactions when the payoff rule is made the total rule (from the baseline majority rule). The two exceptions are for b_{45} and b_{20} under Version 2; in those two cases the coefficient on δ^T is not significantly different from zero, so that the payoff rule change does not affect bidding for those two prizes. Notice that the estimated mean bids under the baseline majority rule treatment (the coefficients on the constant terms as reported in Table 5) are already close to the Nash equilibrium bids under the Total Rule for Version 2 for these two prize amounts i.e., $b_{45}^{ne} = 54$ and $b_{20}^{ne} = 24$, and consequently the change in the payoff rule does not have much effect on bidding for these two prize items.

	Version 1: $\{35, 30, 25, 10\}$				**Version 2:** $\{45, 25, 20, 10\}$			
	b_{35}	b_{30}	b_{25}	b_{10}	b_{45}	b_{25}	b_{20}	b_{10}
cons	31.46***	45.25***	39.69***	3.59***	55.26***	22.54***	21.06***	21.15***
	(2.18)	(1.73)	(1.45)	(0.93)	(2.28)	(1.12)	(1.70)	(1.14)
δ^T	9.85***	-5.39***	-8.24***	3.78***	1.12	9.13***	-0.35	-9.89***
	(3.22)	(2.17)	(2.02)	(0.86)	(1.57)	(1.19)	(1.34)	(1.52)
δ^{RO}	1.99	2.11	-4.62***	0.54	3.36	-1.24	-0.56	-1.55
	(1.45)	(1.42)	(1.13)	(1.05)	(2.91)	(0.82)	(2.07)	(1.39)
δ^A	4.63***	-2.50*	-2.97***	0.83	4.00	1.60**	-2.70	-2.90**
	(1.45)	(1.42)	(1.13)	(1.05)	(2.91)	(0.82)	(2.07)	(1.39)
R^2	0.08	0.05	0.11	0.07	0.01	0.11	0.01	0.17
Nobs	2880	2880	2880	2880	2880	2880	2880	2880

***,**,*, indicate significance, respectively, at the .01, .05, and .10 levels.

Table 5: Regression Analysis of Bids for the Four Prizes under Version 1 (Left Columns) or Version 2 (Right Columns) All Data From All Sessions

Another important observation from Table 5 is that the payoff rule treatment ordering does not appear to matter for bidding behavior; with a single exception (b_{25} for Version 1), the coefficient on the dummy variable δ^{RO} is never significantly different from zero indicating that whether the payoff rule treatment order was 20 rounds of majority rule followed by 20 rounds of the total rule (the baseline rule order) or the reverse rule order was not much of a factor in subjects' bidding behavior. On the other hand, Table 5 also reveals that the coefficient on the δ^A dummy variable is frequently significantly different from zero (in 5 out of the 8 bid regressions), indicating that the presentation of the four values in ascending order as opposed to descending order (the baseline) has some effect on bidding behavior. While the latter finding is surprising as it is not predicted by the theory, two mitigating factors are that: 1) the impact of prize order on bids is economically speaking rather small, amounting to a change of less than 5 tokens (4.2% of a player's budget) in all cases and 2) the signs of the significant coefficients on δ^A are a mix of both positive and negative, indicating no systematic effects. If anything, the latter finding suggests that we were correct to consider some variation in the prize order so that overall mean bids as presented in Figures 2-3 and Table 4 reflect the changes in bids that can be induced by variations in the presentations of the prize values.

The next findings address in further detail some of the comparative statics predictions of the theory, according to whether the majority or total rule was in place.

Finding 3. *Under the majority rule, consistent with the theoretical predictions there is near zero spending on an item that can never be pivotal. Spending on such an item is significantly less than spending on other items that can be pivotal.*

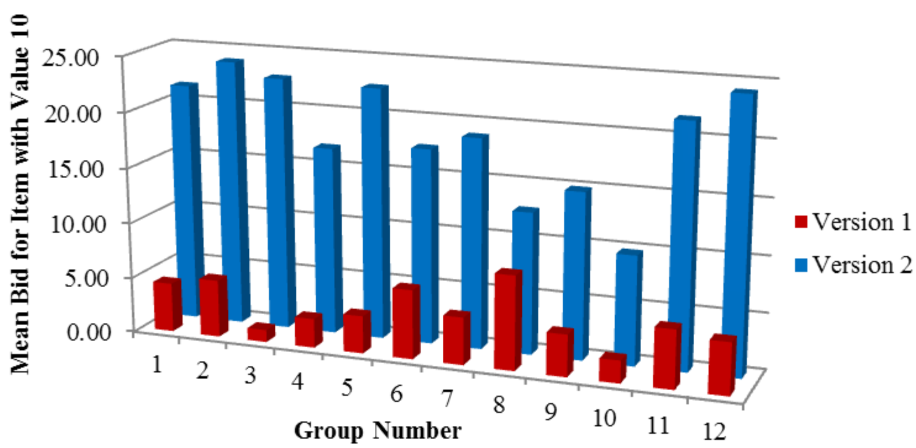

Figure 4: Mean Bids Over all 20 Rounds for the Item with Value 10: Each of the 12 Groups for the Majority Version 1 Treatment Compared with Each of the 12 Groups for the Majority Version 2 Treatment.

Under the majority rule treatment, the item with the lowest prize value of 10 can never be pivotal when the prize values are as given in Version 1 and thus bids for this item should be zero. Table 4 reveals that bids for the item with prize value 10 under Majority Rule Version 1 are indeed low, averaging 4.26 over all rounds and just 2.92 over the last 5 rounds of this treatment. While these bid amounts are different from zero, it is instructive to compare bids for the item with value 10 under both versions of the Majority rule treatment. In particular, under the Majority Rule Version 2, the lowest valued item also has a prize value of 10 and can be pivotal. Thus bids for this prize should be strictly positive (the Nash prediction is a bid of 20). Consider the 12 independent group-level mean bids over all 20 rounds for the prize with value 10 under Majority Rule, Version 1 as reported in Table 9 and under the Majority Rule, Version 2 as reported in Table 11 (Appendix B). For convenience these bids are also presented in Figure 4.

This figure makes it clear that bids for the item with value 10 under the Majority Rule Version 1 are stochastically dominated by bids for the item with value 10 under the Majority Rule Version 2. Indeed a Wilcoxon Mann-Whitney test using the 12 independent (group) observations for each treatment condition confirms that we can reject the null hypothesis of no difference in bids for the item with value 10 in favor of the alternative that bids for this item are higher under Version 2 than under Version 1 ($p < .01$, one-sided test). Note that this same finding holds for *any* of the 7 mean bid subsamples reported for the item with a prize value of 10 in Tables 9 and 11.[10] This is strong evidence that pivotality concerns play

[10]That is, this finding holds not only for all rounds played but also for the first round, for rounds 1-5, rounds 6-10,.. etc.

	Majority Rule Version 1			Majority Rule Version 2		
Null Hypothesis	$\bar{b}_{35} = \bar{b}_{30}$	$\bar{b}_{30} = \bar{b}_{25}$	$\bar{b}_{35} = \bar{b}_{20}$	$\bar{b}_{25} = \bar{b}_{20}$	$\bar{b}_{20} = \bar{b}_{10}$	$\bar{b}_{25} = \bar{b}_{10}$
WSRT p-value	0.0386	0.0063	1.0000	0.3877	0.7744	0.1460
WSRT L5 p-value	0.3877	0.0386	0.7744	0.3877	0.3877	0.7744

Table 6: Pairwise Bidding Companions, Majority Treatments

an important role as identified in the theory.

Finding 4. *Under the majority rule, consistent with the theoretical predictions, we observe significantly higher spending on items that can be pivotal more often, e.g., item 1 in Version 2.*

Using a Wilcoxon Signed Ranks test for matched pairs on mean bids from the Majority Rule Version 2 treatment (see Table 11) we test whether the 12 independent (group) mean bids (over all 20 rounds) for the item with value 45 are significantly greater than mean bids for each of other three items having values 25, 20 and 10, respectively. We find that we can easily reject the null hypothesis of no difference in mean bids on the item with value 45 versus each of the items having values 25, 20 or 10, respectively, in favor of the alternative that bids are higher for the item with value 45. ($p < .01$ for all three pair-wise comparisons, one-sided test).

Finding 5. *Under the majority rule, we observe roughly similar levels of spending on all items that can be pivotal the same number of times, e.g., items 1-3 under Version 1 and items 2-4 under Version 2, albeit with some variance, especially in the case of Version 1.*

Support for this finding comes from Table 6 which reports the results of several Wilcoxon Signed Ranks test for matched pairs on bid data from the two Majority Rule treatments. The test is performed on matched pairs of the group level bid averages for two items ($\bar{b}_i$) over all rounds or over the last 5 rounds only (L5). The table reports the p-value from various pairwise null hypotheses of no difference in bidding behavior (two-sided tests in all cases).

We observe that for Majority rule Version 2, the null hypothesis of no difference in average bids between items with values 25, 20, and 10 is never rejected ($p > .10$ in all pairwise comparisons). However, for Version 1, we observe that there is excess bidding on the item with prize value 30 relative to the items with prize values 35 and 25, respectively. The difference in mean bids for items with prize values 35 and 30 disappears by the final 5 rounds but the difference remains significant between mean bids for the two prizes with values 30 and 25 even in the final 5 rounds. The excessive bids for the item with value 30 relative to the items with values 35 and 25 which are equally pivotal is hard to reconcile with risk aversion, since the token budget must be fully allocated. It may instead reflect a strategic calculation that one's opponent is more likely to bid for the item with the highest (35) or lowest (25) prize value among the three items that are equally pivotal toward winning a majority of item values in this treatment.

Finding 6. *Under the total rule, consistent with theoretical predictions, we generally find that 1) mean bids $\bar{b}_i$, are positive for all items and 2) (over all 20 rounds) under Version 1 we generally have that $\bar{b}_{35} > \bar{b}_{30} > \bar{b}_{25} > \bar{b}_{10}$ while under Version 2 we have that $\bar{b}_{45} > \bar{b}_{25} > \bar{b}_{20} > \bar{b}_{10}$, that is, token allocations are, on average, increasing with item values.*

Specifically, using the data for all rounds as reported in Tables 10 and 12, this ordering is observed to hold for 10 of the 12 groups under Version 1 and for all 12 groups under Version 2. Where it breaks down under the total rule for Version 1 is for groups 3 and 6 (see Table 10) where overall average bids for the prize with value 35 are less than overall average bids on the prize with value 30, though average bids on the highest prize of 35 exceeded average bids on the two lowest prizes with values 25 and 10 in these two sessions. This finding is again suggestive of some type of strategic avoidance of bidding on the highest valued item in this treatment. However, we note that under Version 1, there is not as large a difference between the valuations of the first two prizes (a difference of just 5) and this small difference may have also played a role in bidding behavior. When the difference in valuations between the highest and second highest items is more substantial as in Version 2 (a difference of 20) evidence of strategic avoidance of bidding on the highest valued item under the total rule disappears completely.

The lowest prize, with a valuation of 10 is the same across all four of our treatments and is therefore a natural focus for comparisons across all treatments. Under the total rule, bids on this item are predicted to equal 12 under both treatments (Versions 1 and 2). Under the majority rule Version 1, as previously discussed in Finding 3 bids on this item should be 0 while under the majority rule Version 2, bids on this item should be 20.

Finding 7. *Consistent with theoretical predictions, bids for the lowest prize having a common value of 10 in all four treatments are: a) not significantly different from one another under both Versions 1 and 2 of the total rule, b) significantly lower under majority rule Version 1 than for the other three treatments and c) significantly higher under the majority rule Version 2 than for the other three treatments.*

Support for this finding comes from conducting pairwise Wilcoxon Mann-Whitney tests using the 12 independent (group) observations on average bids for the item with value 10 for each of the four main treatments over all 20 rounds as reported in Tables 9-12 (Appendix B). The p-values from the pairwise tests are summarized in Table 7. In all but one case, we can reject the null hypothesis of no difference in favor of the alternative directional prediction of the theory. The one case where we cannot reject the null hypothesis is in the comparison between the Total Version 1 and Total Version 2 treatments, where consistent with the theory, bids on the item with prize value 10 are predicted to be exactly the same; the fact that we cannot reject the null hypothesis in this case is thus also consistent with the theoretical prediction.

Having examined the behavior of *mean* bids across treatments we next consider the distribution of *individual* bids across our four treatments so as to assess whether these distributions also conform to predictions of the theory.

Finding 8. *The distribution of individual bids is not degenerate at equilibrium predictions. However, these bid distributions are ordered in such a way as to be consistent with the comparative statics predictions of the theory.*

Treatment & Prediction	Total V1, $\bar{b}_{10} = 12$	Total V2, $\bar{b}_{10} = 12$	Majority V2, $\bar{b}_{10} = 20$
Majority V1, $\bar{b}_{10} = 0$	.0012	.0005	.0000
Total V1, $\bar{b}_{10} = 12$		.4356†	.0001
Total V2, $\bar{b}_{10} = 12$			.0001

†= two-sided test; all other tests are one-sided.

Table 7: p-values from pairwise tests of the null hypothesis of no difference in bid amounts between treatments for the prize with the lowest value of 10. Tests are performed on session-level bid averages over all 20 rounds.

Support for Finding 8 can be found in Figures 5–6 which show the cumulative distribution functions (CDFs) of bid amounts between 0 to 120 tokens for each of the four prizes in each of the four treatments.

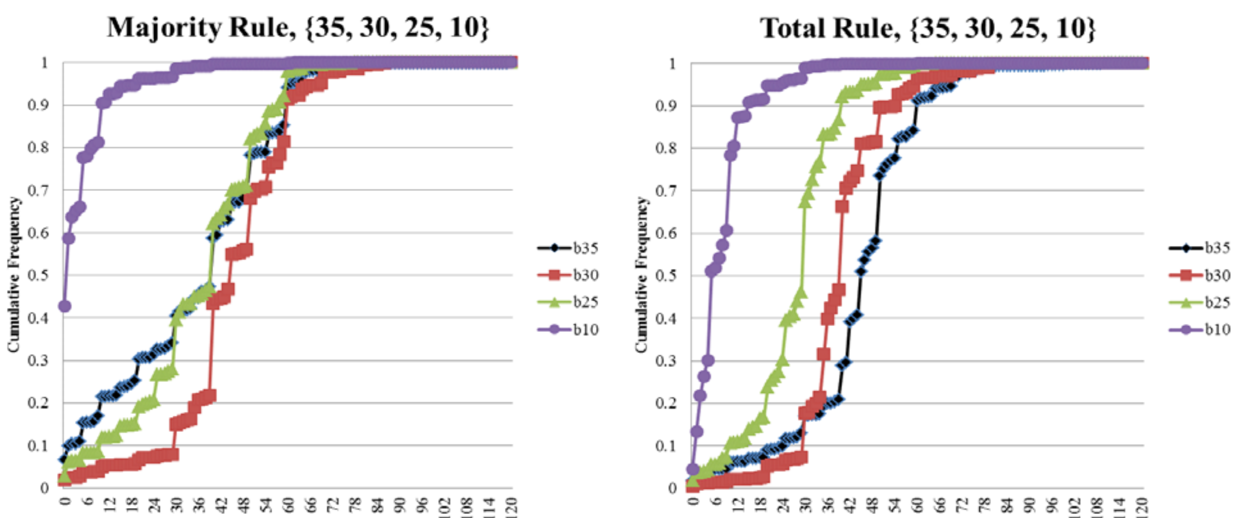

Figure 5: Cumulative Distribution of Bid Amounts on the Four Items under the Majority and Total Rules, Version 1 {35, 30, 25, 10}

The CDFs presented in Figures 5–6 reveal that the distribution of bid amounts for the four prizes do not correspond precisely with theoretical predictions. However, the observed *differences* between the bid distributions for the four prizes are strikingly consistent with the comparative statics predictions of the theory. More precisely, consider the CDFs for bid amounts under prize vector 1 as shown in Figure 5. Under the majority rule (left panel of Figure 5) equilibrium bids for the 3 highest prize values should all be 100 percent at a bid of 40. While these three bid distributions are clearly not degenerate at 40, the bid distributions for the prizes with values 35, 30, and 25 are all centered around 40 and are similar to one another. By contrast, under the majority rule the equilibrium bid for the lowest prize with a value of 10 should be 0 and indeed there is a large mass of bids (42.7%) at a

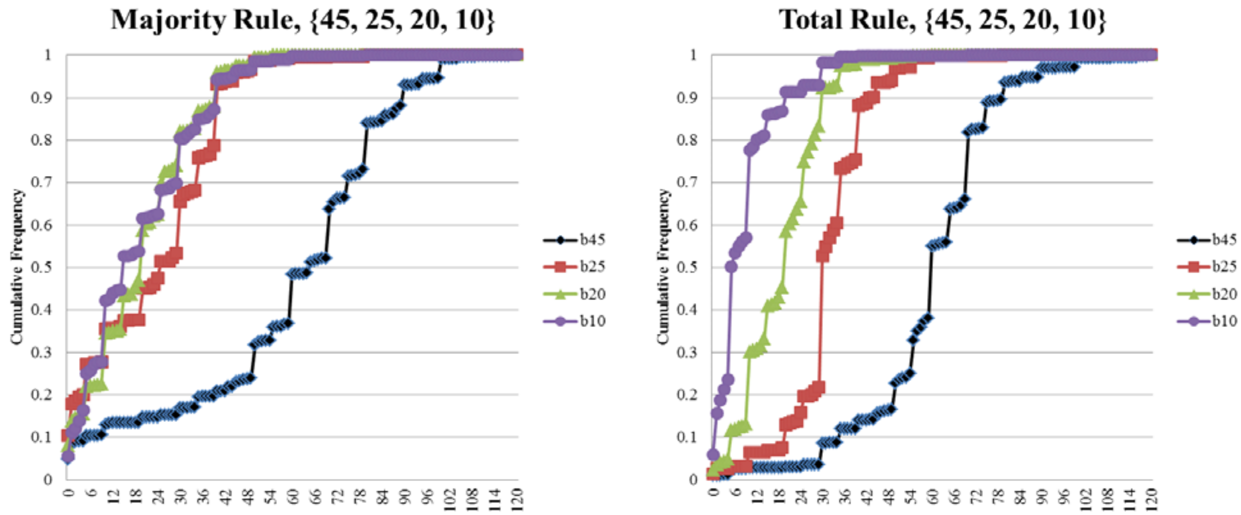

Figure 6: Cumulative Distribution of Bid Amounts on the Four Items under the Majority and Total Rules, Version 2 {45, 25, 20, 10}

bid of 0. Importantly, the bid distribution for this lowest value prize is clearly distinct from the bid distributions for the 3 highest value prizes. Consider next the distribution of bids under the total rule for prize vector 1 (right panel of Figure 5). These distributions are all rather distinct from one another and reflect the monotonic prediction between bids and prize amounts for this treatment. Similar results are found in the CDFs of bids under prize vector 2 as shown in Figure 6. Under the majority rule (left panel of Figure 6) equilibrium bids for the 3 lowest value prizes should all be 100 percent at a bid of 20. While the distributions are clearly not degenerate at 20, they are closely clustered together and centered around 20. By contrast equilibrium bids for the highest prize with a value of 45 should all be at 60. While the distribution of bids for this highest value prize is not degenerate at 60, the distribution of bids for this prize is clearly distinct from the distribution of bids for the other three lower valued items. Under the total rule for Version 2 (right panel of Figure 6) bid distributions are again distinct from one another and correspond precisely to the monotonic prediction that higher prize values are associated with higher bids.

Finally, we discuss the adjustment of bids over time. At the aggregate level, there is some evidence of learning over time in comparisons of the mean bids made by groups over the first rounds 1-5 and the mean bids made by these same groups over the final rounds, 16-20, using the data of Tables 9-12. For example, consider mean group bids for the prize with value 35 under the Majority Rule, Version 1 as reported in top panel of Table 9. The predicted Nash equilibrium bid for this item is 40. Notice that over the first five rounds (Rnds 1-5) 10 of the 12 groups have mean bids below 40 while only two groups have mean bids above 40. Of the 10 groups with mean initial bids below 40, 7 of these 10 groups had increased mean bids for this same item over the last 5 rounds (Rnds 16-20). Of the 2 groups with mean bids initially above 40, 1 had decreased its mean bid by the final 5 rounds. Thus, 8 of the 12 groups who bid for this item in this treatment – a majority – exhibit some evidence of aggregate equilibration toward Nash equilibrium bids over time. Carrying out a similar analysis for all other prize value/treatment conditions we can report that for at least 6 of the 12 groups, if the average group bid for an item over the first rounds 1-5 was below (above) the predicted Nash equilibrium bid for that item then the average bid by that same group for that same item over the last rounds 16-20 was higher (lower) in 14 of the 16 prize/treatment conditions reported on in Tables 9-12.[11] This finding provides some evidence of aggregate equilibration toward equilibrium bids.

We look for further evidence of learning behavior by exploring how *individual* bids differed from Nash equilibrium bids over time. To do this, let us define the mean squared deviation of individual i's 4-element bid vector in period t from the vector of Nash equilibrium bids by $MSD_i(t) = \frac{1}{4}\sum_{j=1}^{4}(b_{i,j}(t) - b_j^{ne})^2$, where b_j^{ne} refers to the Nash equilibrium bid for item j, which depends on the prize vector, Version 1 or 2, and the rule, majority or total that was in place in period t. Table 8 reports results from GLS random effects regressions of the MSD variable, as defined above, on the same dummy variables δ^T, δ^{RO} and δ^A that were defined and used earlier in connection with Table 5, again with robust standard errors clustered on an individual's group membership. In addition to these variables, we also include the round number, "Round(t)" so as to assess the impact of experience on the adjustment of bids toward equilibrium predictions. We also explore separate regressions of the MSD on the same explanatory variables under either the "Majority" or the "Total" rule for each prize vector (thus excluding the δ^T variable); the columns labeled "All Data" includes MSDs under both rules.

The regression results reported in Table 8 indicate that individual means squared deviations of bid amounts from NE predictions are significantly lower under the total rule treatment as compared with the majority rule treatment for both prize vectors, Versions 1 and 2. Indeed, the rule change seems to be the most significant factor in explaining the MSDs, judging from the large impact that the rule change has on the MSD. Evidence for this can be found in the significantly negative coefficient attached to the δ^T dummy variable in Table 8. This finding is likely owing to the fact that equilibrium bids under the total rule are proportional to prize values and this type of bidding behavior comes more naturally to subjects than thinking about pivotality considerations as is required for equilibrium bidding under the majority rule. Other explanatory variables that sometimes play a significant role in these regressions are the dummy variable for whether the prize values were presented in ascending order and, in one instance only, the round number. An ascending prize order has a marginally negative impact on MSD, particularly under Version 2 where the ascending prize order is 10, 20, 25, 45. The coefficient on the round number is almost always negative suggesting that there is a slight reduction in MSD over time, but this coefficient is only significantly negative under the total rule for prize vector 1 and the coefficient is small in magnitude. Taken together, these results, along with those reported earlier in Table 5 suggest that much of the adjustment in subjects' bids is a consequence of the rule change with very little modification to bids in response to experience or other factors under a given rule. We summarize this last finding as follows.

[11]The exceptions are for a prize value of 10 under the Majority rule, Version 2 (Table 11) and for a prize value of 45 under the Total rule, Version 2 (Table 12). In these two cases only 5 of the 12 groups exhibit evidence for equilibration in terms of the difference in their mean bids over the first and last 5 rounds.

	Version 1: $\{35, 30, 25, 10\}$ MSD of bids from NE Pred.			**Version 2:** $\{45, 25, 20, 10\}$ MSD of bids from NE Pred.		
	All Data	Majority	Total	All Data	Majority	Total
cons	302.07***	274.38***	208.28***	404.97***	396.07***	223.03***
	28.16	40.87	27.95	53.28	52.37	53.09
δ^T	-121.08***			-190.84***		
	23.32			29.17		
δ^{RO}	40.54	70.07	11.27	20.76	43.37	45.70
	34.56	45.21	34.29	57.97	77.75	-0.04
δ^A	-63.77	-41.61	-86.19*	-107.54*	-142.45*	-72.63
	34.51	45.21	34.32	57.96	77.62	45.70
Round(t)	-1.82	-1.64	-1.97*	-0.10	1.33	-1.53
	1.13	1.98	1.08	1.67	2.74	1.78
R^2	0.09	0.03	0.05	0.13	0.05	0.03
Nobs	2880	1440	1440	2880	1440	1440

***,**,*, indicate significance, respectively, at the .01, .05, and .10 levels.

Table 8: Regression Analysis of MSDs of Bids from NE Predictions in Version 1 (Left Columns) or Version 2 (Right Columns) All Data From All Sessions

Finding 9. *Individual bids are significantly closer to equilibrium bids under the total rule than under the majority rule. Individual bids under both rules adjust toward equilibrium predictions only slightly, or not at all with experience.*

6. Summary and Conclusions

The stochastic, asymmetric value Blotto game has many applications, e.g., to warfare, advertising and political campaigns. In this paper we present results from an experimental study of this version of the Blotto game under two commonly used objective functions: a majority rule objective and a total expected payoff objective. The majority rule objective is particularly relevant to understanding electoral competitions in two party systems, e.g., the electoral college system for electing the U.S. president, while the total expected payoff version is relevant to understanding competition between duopoly firms for market share. Despite the seeming similarity between the two objective functions, equilibrium bid allocations under the majority rule objective are quite different than under the total expected payoff objective. In particular, for the equal budget constraint case that we study, bids for each item under the majority rule objective are proportional to the Banzhaf index of an item's power. By contrast, bids under the total rule are proportional to the relative value of each item.

To test these theoretical predictions, we report the results of a laboratory experiment comparing bidding behavior in stochastic, asymmetric 4-item Blotto games under the majority rule objective with bidding for the same items under the total rule objective using a within-subjects design. We consider two different prize vectors so as to further test some of the comparative statics implications of the theory. Our experimental results are shown to be qualitatively (if not perfectly quantitatively) consistent with the theoretical predictions for how players should allocate their bids across the four items, confirming that the differing payoff function objectives matter for allocations.

Future research on this topic might proceed in several dimensions. First, one could attempt to incorporate some other potentially important features of the U.S. electoral college system (that we have left out) for instance, the fact that certain states (items) are ex-ante more likely to be won by one player or the other, or relaxing the assumption that the winner of an item gets all of that items' value.[12] Another possible extension would be to consider super-majority rules and examine how allocations are affected relative to the majority rule case. We leave these extensions to future research.

Appendix A: Experimental Instructions

Here we provide the instructions from the Version 1 treatment with the majority rule used for the first 20 rounds followed by 20 rounds of the total rule. The prize order is descending. Other experimental instructions (for the reverse treatment orders or for Version 2) are similar.

Overview Welcome to this experiment in the economics of decision-making. Funding for this experiment has been provided by the University of Pittsburgh. Please read these instructions carefully as they explain how you earn money from the decisions you make in today's session. There is no talking for the duration of the session. If you have a question please raise your hand and your question will be answered in private.

This experiment will consist of two parts. You will receive instructions for the second part after the first part has been completed.

At the start of the first part of the experiment, all participants will be randomly assigned to a group of 6 participants. In this first part of the experiment, you will participate in 20 rounds of decision-making. In each of these 20 rounds you will be randomly and anonymously paired with one of the other 5 members of your 6-member group. All possible pairings with the other 5 participants in your group are equally likely in each round. You will not know the identity of any participant you are paired with in any round nor will they be informed of your identity even after the session is over.

Specific Details Each round proceeds as follows. At the start of the round, both you and the participant with whom you are randomly paired for that round – your current "match"– are given 120 tokens each. The two of you must then decide simultaneously and without any communication how many of your 120 tokens you will allocate toward winning each of four different prizes which are labeled P1, P2, P3 and P4. Each prize is associated with a certain number of points as given in the table below.

[12]Of the 51 states in the electoral college, all but two assign all of their electoral votes to the winner of the state. The two exceptions, Maine and Nebraska, assign electoral votes in a more proportional manner: 1 electoral vote is awarded to the winner of each Congressional district within the state and the remaining 2 electoral votes are awarded to the state-wide winner.

Prize	Points Earned by Winner
P1	35
P2	30
P3	25
P4	10

This table has also been written on the white board for all to see.

You enter the number of tokens you wish to commit toward winning each prize in the four input boxes next to each prize number P1, P2, P3 and P4 that appear on the decision screen for each round. You must enter between 0 and 120 tokens inclusive in each input box, and the sum of the tokens you have entered in all four boxes must exactly equal your allocation of 120 tokens. As you enter token amounts, the counter at the top of the decision screen will indicate how many tokens you have left from your initial allocation of 120 tokens. If the sum across all four boxes does not exactly equal 120 you will be prompted to re-enter your choices for each prize. Once you are satisfied with your choices, click the red OK button. You may change your choices anytime prior to clicking the red OK button.

After both participants have submitted their token allocations for the four prizes, the computer program computes the sum of the tokens that you and the other participant contributed to each prize. Specifically, if C_i^{You} denotes the number of tokens that you contributed toward winning prize Pi, $i = 1, 2, 3$ or 4, and C_i^{Other} denotes the number of tokens the other participant (your match for the round) contributed toward winning prize Pi, then the sum of the contributions toward winning prize i is $C_i^{\text{You}} + C_i^{\text{Other}}$.

Your chance of winning prize i is given by the formula:

$$\text{Your chance of winning prize } i = \frac{C_i^{\text{You}}}{C_i^{\text{You}} + C_i^{\text{Other}}}.$$

Your match's chance of winning prize i is similar and is given by:

$$\text{Your match's chance of winning prize } i = \frac{C_i^{\text{Other}}}{C_i^{\text{You}} + C_i^{\text{Other}}}.$$

For example, if you allocate 25 tokens toward winning one of the four prizes and your match allocates 15 tokens toward winning that same prize, then your chance of winning the prize is $\frac{25}{25+15} = \frac{25}{40} = 5/8$ while your match's chance of winning that same prize is $\frac{15}{25+15} = \frac{15}{40} = 3/8$.

Notice that the two chances for you and your match always add up to 100 percent and that your chance of winning prize i will be higher, the higher is the number of tokens that you contributed toward winning prize i, and your chance of winning prize i is lower the higher is the number of tokens that your match contributed toward winning prize i. If you both contribute the exact same number of tokens toward winning prize i, that is, if $C_i^{\text{You}} = C_i^{\text{Other}}$, even if you both contributed zero tokens toward winning prize i, then you each have a 50 percent chance of winning that prize.

Using these chances for each contest, the computer program determines the winner of each prize for each pair of participants. Specifically, for each pair and for each prize P1, P2, P3 and P4, the computer program randomly draws a number from 1 to 100 and depending on the two participants' chances of winning each prize, this random draw determines which participant wins that prize. For example, suppose you have a 60 percent chance of winning a prize and your match has a 40 percent chance of winning that same prize. If the random number drawn by the computer program is in the set of numbers assigned to you based on your 60 percent chance of winning, e.g., the numbers between 1-60, then you win the prize but if the random number drawn was instead between 61-100 then your match wins the prize. Thus, even if you have a higher chance of winning the prize than your match, it is still possible, though less likely, that your match will win the prize. Note that the computer program will draw a different random number for each prize for each pair and use the token-determined chances for each participant for each prize to determine the winner of each prize. After the round is over you will be informed of your chances of winning each prize and which participant in your pair, you or your match were awarded each prize.

Specifically, at the end of each round, for each prize, P1, P2, P3 and P4, you and your match will see a pie chart (4 pie charts total). Each chart will show graphically your chances of winning prize i, $\frac{C_i^{\text{You}}}{C_i^{\text{You}}+C_i^{\text{Other}}}$, as one portion of the pie and your match's chance of winning the prize, $\frac{C_i^{\text{Other}}}{C_i^{\text{You}}+C_i^{\text{Other}}}$, as the remaining portion of the pie. You will also learn the number of tokens that you and your match contributed toward winning each of the four prizes. Below each pie chart you will see whether you won the prize (as indicated by "WON" in a green box) or lost the prize (as indicated by "LOST" in a red box), along with the points for each prize. Finally, you will learn your total points earned for the round which is the sum of the points you earned from the prizes that you won, if any. For some examples, if you won prizes P1 and P3, then your total points earned for the round would be $35+25=60$ points; if you won all four prizes then your total points earned for the round would be $35+30+25+10=100$; if you won prizes P2 and P4 then your total points earned for the round would be $30+10=40$; if you did not win any of the four prizes, then your total points earned for the round would be 0.

Your objective Your objective is to win a *majority* of the prize points that are available in each round. Since there are exactly 100 prize points possible in each round, to win a majority means that your total points earned for the round are greater than 50 points. If your total points earned in a round are greater 50, then YOU win that round and your earnings for that round are \$20.00. In that case your match loses the round and his/her earnings for that round are \$0.00. However, if your total points earned in a round are less than 50, then you LOSE that round and your earnings for that round are \$0.00. In that case your match wins the round and his/her earnings for that round are \$20.00. A complete list of the 16 possible prize outcomes, the total points that you earn from each outcome and your dollar payoff for that outcome for the round are given in the table below.

Prize(s) P1–P4 *you* win [Points won]				Your Total Points	Dollar Payoff
P1 [35]	P2 [30]	P3 [25]	P4 [10]	100	$20.00
P1 [35]	P2 [30]	P3 [25]		90	$20.00
P1 [35]	P2 [30]		P4 [10]	75	$20.00
P1 [35]		P3 [25]	P4 [10]	70	$20.00
P1 [35]	P2 [30]			65	$20.00
	P2 [30]	P3 [25]	P4 [10]	65	$20.00
P1 [35]		P3 [25]		60	$20.00
	P2 [30]	P3 [25]		55	$20.00
P1 [35]			P4 [10]	45	$0.00
	P2 [30]		P4 [10]	40	$0.00
P1 [35]				35	$0.00
		P3 [25]	P4 [10]	35	$0.00
	P2 [30]			30	$0.00
		P3 [25]		25	$0.00
			P4 [10]	10	$0.00
				0	$0.00

Notice several things in this table. First, the total point earnings you could possibly win have been sorted from highest to lowest. Second, the dollar amount earned by your match for the round is $20.00 minus the dollar amount that you earn. That is, there is a total of $20.00 at stake in each round. Third, because of the way the prize points were chosen, no point total can ever add up to exactly 50, so there is no possibility of ties in determining who won the majority of points in any round.

At the end of each round, please record on your record sheet the number of tokens that you allocated to each of the four prizes. Then circle the prizes that you won for the round. Finally record your total points earned for the round and your round earnings.

If the 20th round has not yet been played, then we will play a new round of this same decision-making task. Click the OK button after you have finished your record keeping to start this new round. In the new round you will again be randomly and anonymously matched with one of the other 5 participants in your 6-member group. Following completion of the 20th round, please wait for further instruction.

Payments For showing up today and completing this experiment you are guaranteed $5. At the end of today's session we will pick one round at random from all 20 rounds played in the first part of today's session and we will pay you your dollar earnings from that one randomly chosen round. Since you do not know in advance which round will be chosen, you will want to do your best in every round. You will have the opportunity to earn an additional money amount in the second part of today's session, but that will be discussed later in the instructions that follow the completion of this first part of today' session.

Questions? Now is the time for questions. If you have any questions please raise your hand and the experimenter will come to you and answer your question in private.

Quiz Before we begin the first part of today's session, we ask that you answer the following questions that are designed to check your comprehension of the written instructions. Feel free to consult the instructions in answering these questions. Write your answers to the questions in the spaces provided or circle the correct answer. When you are done answering these questions we will come around to check your answers. If there are mistakes, we will go over the relevant part of the instructions again.

1. True or false: The member of each pair who contributes the most tokens toward winning a prize always wins that prize. Circle one: True False.
2. Suppose you have decided to allocate a total of 82 of your tokens to the three prizes P1, P3 and P4. How many tokens must you allocate to prize P2? _______.
3. If you contribute 10 tokens toward winning one of the four prizes and your match contributes 40 tokens toward winning that same prize, what is your chance of winning the prize? _______. What is your match's chance of winning that prize? _______.
4. If you win prizes P1 and P2, what are your total point earnings for the round? _______. What are your dollar earnings for the round? _______.
5. If you win prizes P3 and P4, what are your total point earnings for the round? _______. What are your dollar earnings for the round? _______.
6. True or false: I am matched with the same other participant in all 20 rounds of this first part of today's session. Circle one: True False.
7. True or false: At the end of today's session 1 round will be randomly chosen from the first 20 rounds played in today's session and I will be paid my dollar earnings from that randomly chosen round. Circle one: True False.

The following instructions were read following the completion of the first 20 rounds.

Continuation Instructions We now begin the second part of today's session. In this second part you will again be assigned to a 6-member group and you will participate in 20 rounds of decision-making. As in the first part, in each round you will be randomly and anonymously paired with one of the other 5 members of your group. You will not know the identify of the participant matched to you in each round nor will they know your identity even after the session is over. In each round you and your match for the round will face a similar choice to the one you faced in the first 20 rounds. Specifically, you will again each be given 120 tokens and will have to choose how to allocate those tokens toward winning 4 different prizes. The four prizes P1, P2, P3, and P4 yield the same number of points as in the first part. These points are reported once again in the table below.

Prize	Points Earned by Winner
P1	35
P2	30
P3	25
P4	10

You and your match's chances of winning each prize and the manner in which those prizes are awarded is the same as in the first part of the session. The *only* difference from the first part is that in this second part, your earnings each round do not depend on whether you obtained a majority of the 100 possible points. Instead, in this second part, the total points that you earn in each round will be converted into dollars at the fixed rate of 1 point = $0.20. Thus your objective is to win as many points as you can in each round. A complete list of the 16 possible prize outcomes, the total points that you earn from each outcome and your dollar payoff for that outcome for the round are given in the new payoff table below which replaces the table used in the first part of the session.

Prize(s) P1–P4 *you* win [Points won]				Your Total Points	Dollar Payoff
P1 [35]	P2 [30]	P3 [25]	P4 [10]	100	$20.00
P1 [35]	P2 [30]	P3 [25]		90	$18.00
P1 [35]	P2 [30]		P4 [10]	75	$15.00
P1 [35]		P3 [25]	P4 [10]	70	$14.00
P1 [35]	P2 [30]			65	$13.00
	P2 [30]	P3 [25]	P4 [10]	65	$13.00
P1 [35]		P3 [25]		60	$12.00
	P2 [30]	P3 [25]		55	$11.00
P1 [35]			P4 [10]	45	$9.00
	P2 [30]		P4 [10]	40	$8.00
P1 [35]				35	$7.00
		P3 [25]	P4 [10]	35	$7.00
	P2 [30]			30	$6.00
		P3 [25]		25	$5.00
			P4 [10]	10	$2.00
				0	$0.00

Notice several things in this new table. First, the total point earnings you could possibly win have again been sorted from highest to lowest. Second, the dollar amount earned by your match for the round is again $20.00 minus the dollar amount that you earn. That is, there is a total of $20.00 at stake in each round. Finally, and differently from the first part, you can now earn a positive dollar payoff for the round if you win as few as one of the four prizes, and the more prizes you win in a round, the greater are your dollar earnings for that round.

At the end of each round, please record on your record sheet the number of tokens that you allocated to each of the four prizes. Then circle the prizes that you won for the round. Finally record your total points earned for the round and your round earnings.

If the 20th round has not yet been played, then we will play a new round of this same decision-making task. Click the OK button after you have finished your record keeping to start this new round. In the new round you will again be randomly and anonymously matched with one of the other 5 participants in your 6-member group. Following completion of the 20th round, the session is over.

Payments Following completion of this second part of today's session we will pick one round at random from all 20 rounds played in this second part and we will pay you your dollar earnings from that one randomly chosen round. Since you do not know in advance which round will be chosen, you will want to do your best in every round. In addition, as discussed in the first set of instructions, one round will be randomly chosen from the first 20 rounds played and you will also earn your dollar payoff from that randomly chosen round. Finally, you also receive your $5 show-up payment. All payments will be made in cash and in private following the completion of this second part of today's session.

Questions? Now is the time for questions. If you have any questions please raise your hand and the experimenter will come to you and answer your question in private.

Appendix B: Tables

Prize= 35

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	25.67	23.03	32.93	35.07	32.00	34.67	30.76	40
2	D	M,T	10.00	25.50	25.67	32.50	30.50	30.50	28.54	40
3	D	M,T	30.00	30.07	38.90	33.47	37.83	36.67	35.07	40
4	D	T,M	38.17	34.40	37.50	38.17	35.27	41.17	36.33	40
5	D	T,M	37.50	17.33	10.77	13.27	19.73	18.00	15.28	40
6	D	T,M	48.33	38.67	45.50	53.17	46.17	46.67	45.88	40
7	A	M,T	45.00	47.30	40.83	37.10	25.07	22.17	37.58	40
8	A	M,T	36.83	35.43	42.77	39.80	37.07	36.17	38.77	40
9	A	M,T	23.33	35.00	30.13	26.60	16.37	15.00	27.03	40
10	A	T,M	35.06	39.42	37.91	34.50	26.17	24.44	37.65	40
11	A	T,M	37.67	39.90	26.27	49.50	50.13	38.33	41.45	40
12	A	T,M	44.00	44.40	42.37	39.40	43.63	41.50	42.45	40
All 12			34.30	34.20	34.30	36.04	33.33	32.11	34.73	40

Prize= 30

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	51.33	49.30	41.63	41.57	38.57	39.83	42.77	40
2	D	M,T	55.00	46.67	44.10	46.33	46.83	48.17	45.98	40
3	D	M,T	45.00	46.90	38.37	38.90	37.80	38.50	40.49	40
4	D	T,M	43.33	40.47	44.90	43.70	43.30	39.83	43.09	40
5	D	T,M	46.67	58.17	59.47	62.67	56.67	56.17	59.24	40
6	D	T,M	35.00	42.00	37.67	30.67	37.80	40.00	37.03	40
7	A	M,T	36.33	41.97	47.33	46.17	57.33	57.83	48.20	40
8	A	M,T	38.50	41.00	40.93	39.17	41.73	39.50	40.71	40
9	A	M,T	47.67	39.93	43.87	45.00	52.97	51.33	45.44	40
10	A	T,M	51.83	48.10	46.90	48.90	46.20	46.33	47.53	40
11	A	T,M	41.17	45.63	50.10	49.40	48.93	54.00	48.52	40
12	A	T,M	33.00	39.40	44.63	41.60	42.30	45.00	41.98	40
All 12			43.74	44.96	44.99	44.51	45.87	46.38	45.08	40

Prize= 25

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	37.17	41.43	42.43	40.70	43.67	40.17	42.06	40
2	D	M,T	50.00	42.13	43.17	36.30	39.67	38.33	40.32	40
3	D	M,T	39.83	39.80	41.97	47.40	44.20	44.67	43.34	40
4	D	T,M	34.83	42.37	32.73	35.43	41.33	38.83	37.97	40
5	D	T,M	30.83	42.33	46.43	39.53	40.17	42.50	42.12	40
6	D	T,M	27.50	31.70	30.30	30.67	30.93	28.33	30.90	40
7	A	M,T	28.67	24.27	27.70	32.63	35.33	38.17	29.98	40
8	A	M,T	31.83	32.00	27.30	33.10	36.00	38.67	32.10	40
9	A	M,T	40.67	39.43	40.67	45.23	49.77	53.33	43.78	40
10	A	T,M	42.33	39.27	37.93	28.37	25.67	27.00	32.81	40
11	A	T,M	31.50	27.00	36.27	18.23	17.73	24.50	24.81	40
12	A	T,M	39.17	31.53	28.33	33.90	29.90	28.50	30.92	40
All 12			36.19	36.11	36.27	35.13	36.20	36.92	35.92	40

Prize= 10

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	5.83	6.23	3.00	2.67	5.77	5.33	4.42	0
2	D	M,T	5.00	5.70	7.07	4.87	3.00	3.00	5.16	0
3	D	M,T	5.17	3.23	0.77	0.23	0.17	0.17	1.10	0
4	D	T,M	3.67	2.77	4.87	2.70	0.10	0.17	2.61	0
5	D	T,M	5.00	2.17	3.33	4.53	3.43	3.33	3.37	0
6	D	T,M	9.17	7.63	6.53	5.50	5.10	5.00	6.19	0
7	A	M,T	10.00	6.47	4.13	4.10	2.27	1.83	4.24	0
8	A	M,T	12.83	11.57	9.00	7.93	5.20	5.67	8.43	0
9	A	M,T	8.33	5.63	5.33	3.17	0.90	0.33	3.76	0
10	A	T,M	1.67	3.10	2.23	1.00	1.73	1.67	2.02	0
11	A	T,M	9.67	7.47	7.37	2.87	3.20	3.17	5.23	0
12	A	T,M	3.83	4.67	4.67	5.10	4.17	5.00	4.65	0
All 12			6.68	5.55	4.86	3.72	2.92	2.89	4.26	0

Values: D=Descending, A=Ascending; Order: M,T=Majority then Total, T,M=Total then Majority.

Table 9: Average Bids Over Time, Version 1 {35, 30, 25, 10}, Majority Rule

Prize= 35

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	38.00	40.57	46.97	49.97	43.83	40.67	45.33	42
2	D	M,T	46.00	44.73	44.50	46.17	47.53	50.33	45.73	42
3	D	M,T	36.17	37.97	39.40	38.87	37.90	31.50	38.53	42
4	D	T,M	29.83	36.40	45.53	44.53	45.23	34.50	42.93	42
5	D	T,M	47.50	57.13	46.03	47.27	41.43	38.33	47.97	42
6	D	T,M	27.50	34.33	36.07	35.73	38.83	42.50	36.24	42
7	A	M,T	35.00	35.67	41.27	43.37	39.63	38.33	39.98	42
8	A	M,T	37.67	42.03	44.80	45.73	46.30	49.17	44.72	42
9	A	M,T	41.00	47.50	53.33	53.57	51.20	58.33	51.40	42
10	A	T,M	55.00	48.00	48.17	48.93	49.60	49.17	48.68	42
11	A	T,M	34.67	40.03	45.80	48.00	46.70	47.33	45.13	42
12	A	T,M	45.50	47.20	46.27	51.43	48.30	43.50	48.30	42
All 12			39.49	42.63	44.84	46.13	44.71	43.64	44.58	42

Prize= 30

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	35.83	43.87	38.53	36.80	35.83	35.67	38.76	36
2	D	M,T	41.67	41.07	38.83	37.67	37.23	36.00	38.70	36
3	D	M,T	37.50	40.57	46.77	45.67	44.83	47.00	44.46	36
4	D	T,M	48.67	47.80	43.90	39.80	38.57	44.17	42.52	36
5	D	T,M	39.83	39.10	41.90	43.33	47.23	48.83	42.89	36
6	D	T,M	47.50	46.83	48.43	48.93	45.50	45.00	47.43	36
7	A	M,T	43.33	41.50	33.67	33.47	36.63	36.00	36.32	36
8	A	M,T	39.50	39.23	37.90	37.77	37.80	37.50	38.18	36
9	A	M,T	39.67	40.27	35.07	37.70	31.93	22.50	36.24	36
10	A	T,M	29.50	36.70	39.50	40.03	38.93	39.00	38.79	36
11	A	T,M	39.50	34.60	34.57	36.63	35.70	35.83	35.38	36
12	A	T,M	28.50	34.70	36.90	37.63	37.23	39.50	36.62	36
All 12			39.25	40.52	39.66	39.62	38.95	38.92	39.69	36

Prize= 25

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	37.67	29.83	25.20	23.70	30.60	34.33	27.33	30
2	D	M,T	22.00	24.77	30.10	29.33	28.40	26.83	28.15	30
3	D	M,T	36.33	35.87	29.30	29.80	31.57	35.67	31.63	30
4	D	T,M	32.50	26.30	22.63	26.13	28.07	33.33	25.78	30
5	D	T,M	25.00	17.57	23.67	22.57	23.73	24.83	21.88	30
6	D	T,M	32.50	30.20	27.00	26.27	27.00	26.67	27.62	30
7	A	M,T	31.67	33.37	34.97	29.00	27.67	27.33	31.25	30
8	A	M,T	35.17	31.47	30.33	30.03	29.90	27.83	30.43	30
9	A	M,T	35.67	29.50	29.20	26.10	33.40	36.83	29.55	30
10	A	T,M	26.33	27.70	27.27	25.23	25.23	25.33	26.36	30
11	A	T,M	29.83	27.67	25.90	26.30	27.30	26.17	26.79	30
12	A	T,M	31.00	27.80	26.87	22.33	25.30	28.33	25.58	30
All 12			31.31	28.50	27.70	26.40	28.18	29.46	27.70	30

Prize= 10

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	8.50	5.73	9.30	9.53	9.73	9.33	8.58	12
2	D	M,T	10.33	9.43	6.57	6.83	6.83	6.83	7.42	12
3	D	M,T	10.00	5.60	4.53	5.67	5.70	5.83	5.38	12
4	D	T,M	9.00	9.50	7.93	9.53	8.13	8.00	8.78	12
5	D	T,M	7.67	6.20	8.40	6.83	7.60	8.00	7.26	12
6	D	T,M	12.50	8.63	8.50	9.07	8.67	5.83	8.72	12
7	A	M,T	10.00	9.47	10.10	14.17	16.07	18.33	12.45	12
8	A	M,T	7.67	7.27	6.97	6.47	6.00	5.50	6.68	12
9	A	M,T	3.67	2.73	2.40	2.63	3.47	2.33	2.81	12
10	A	T,M	9.17	7.60	5.07	5.80	6.23	6.50	6.18	12
11	A	T,M	16.00	17.70	13.73	9.07	10.30	10.67	12.70	12
12	A	T,M	15.00	10.30	9.97	8.60	9.17	8.67	9.51	12
All 12			9.96	8.35	7.79	7.85	8.16	7.99	8.04	12

Values: D=Descending, A=Ascending; Order: M,T=Majority then Total, T,M=Total then Majority.

Table 10: Average Bids Over Time, Version 1 {35, 30, 25, 10}, Total Rule

Prize= 45

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	59.17	64.00	55.57	55.13	39.97	41.67	53.67	60
2	D	M,T	55.00	43.83	49.33	47.17	53.50	52.50	48.46	60
3	D	M,T	57.50	62.33	58.77	64.27	57.63	56.00	60.75	60
4	D	T,M	67.17	63.03	56.67	50.47	40.17	40.50	52.58	60
5	D	T,M	87.17	69.50	52.63	49.87	52.77	39.83	56.19	60
6	D	T,M	67.83	66.13	64.83	66.70	65.50	65.17	65.79	60
7	A	M,T	68.67	56.53	54.93	53.57	62.67	55.33	56.93	60
8	A	M,T	54.67	55.07	57.83	63.83	63.83	65.83	60.14	60
9	A	M,T	43.33	57.80	56.33	61.07	59.80	57.33	58.75	60
10	A	T,M	59.17	70.50	80.30	82.67	68.50	68.33	75.49	60
11	A	T,M	59.50	59.10	56.30	59.30	54.10	54.33	57.20	60
12	A	T,M	60.33	61.90	60.40	58.73	64.27	64.83	61.33	60
All 12			61.63	60.81	58.66	59.40	56.89	55.14	58.94	60

Prize= 25

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	21.00	18.00	21.47	20.70	30.80	29.50	22.74	20
2	D	M,T	24.67	30.27	27.50	25.17	18.50	18.33	25.36	20
3	D	M,T	20.83	18.13	19.80	18.60	16.30	15.67	18.21	20
4	D	T,M	17.83	19.00	22.47	21.33	26.33	25.83	22.28	20
5	D	T,M	5.17	15.23	18.77	19.03	18.23	24.83	17.82	20
6	D	T,M	24.17	23.30	20.43	22.47	22.97	23.17	22.29	20
7	A	M,T	13.67	21.00	20.40	26.43	19.50	27.83	21.83	20
8	A	M,T	27.50	29.20	30.10	29.50	29.50	28.33	29.58	20
9	A	M,T	32.50	29.20	27.97	26.77	25.93	30.00	27.47	20
10	A	T,M	30.83	28.93	19.07	18.00	26.83	26.00	23.21	20
11	A	T,M	24.33	21.40	23.63	21.43	22.97	20.83	22.36	20
12	A	T,M	36.33	26.17	18.57	17.63	15.47	15.50	19.46	20
All 12			23.24	23.32	22.51	22.26	22.78	23.82	22.72	20

Prize= 20

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	28.00	17.63	19.27	24.37	27.17	24.17	22.11	20
2	D	M,T	18.67	23.90	24.33	22.83	18.00	20.83	22.27	20
3	D	M,T	19.17	17.97	20.87	16.90	17.53	15.83	18.32	20
4	D	T,M	23.33	24.83	25.37	28.87	34.17	34.17	28.31	20
5	D	T,M	16.67	18.10	21.37	25.77	28.77	35.50	23.50	20
6	D	T,M	12.17	12.10	14.23	14.87	16.50	16.83	14.43	20
7	A	M,T	11.17	19.63	25.70	23.93	20.40	22.67	22.42	20
8	A	M,T	19.67	19.10	18.23	16.17	16.83	15.83	17.58	20
9	A	M,T	24.83	15.40	20.77	19.27	20.10	18.50	18.88	20
10	A	T,M	15.67	11.93	10.17	10.17	13.70	13.33	11.49	20
11	A	T,M	17.33	18.10	18.30	19.20	19.47	22.00	18.77	20
12	A	T,M	19.50	17.20	12.83	15.87	14.13	14.17	15.01	20
All 12			18.85	17.99	19.29	19.85	20.56	21.15	19.42	20

Prize= 10

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	11.83	20.37	23.70	19.80	22.07	24.67	21.48	20
2	D	M,T	21.67	22.00	18.83	24.83	30.00	28.33	23.92	20
3	D	M,T	22.50	21.57	20.57	20.23	28.53	32.50	22.73	20
4	D	T,M	11.67	13.13	15.50	19.33	19.33	19.50	16.83	20
5	D	T,M	11.00	17.17	27.23	25.33	20.23	19.83	22.49	20
6	D	T,M	15.83	18.47	20.50	15.97	15.03	14.83	17.49	20
7	A	M,T	26.50	22.83	18.97	16.07	17.43	14.17	18.83	20
8	A	M,T	18.17	16.63	13.83	10.50	9.83	10.00	12.70	20
9	A	M,T	19.33	17.60	14.93	12.90	14.17	14.17	14.90	20
10	A	T,M	14.33	8.63	10.47	9.17	10.97	12.33	9.81	20
11	A	T,M	18.83	21.40	21.77	20.07	23.47	22.83	21.68	20
12	A	T,M	3.83	14.73	28.20	27.77	26.13	25.50	24.21	20
All 12			16.29	17.88	19.54	18.50	19.77	19.89	18.92	20

Values: D=Descending, A=Ascending; Order: M,T=Majority then Total, T,M=Total then Majority.

Table 11: Average Bids Over Time, Version 2 {45, 25, 20, 10}, Majority Rule

Prize= 45

Group	Values	Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	47.50	55.77	64.90	56.50	69.87	67.50	61.76	54
2	D	M,T	53.33	57.50	52.33	54.17	54.33	53.33	54.58	54
3	D	M,T	55.33	62.57	61.50	52.67	53.50	61.67	57.56	54
4	D	T,M	58.33	52.63	56.90	54.80	47.83	45.00	53.04	54
5	D	T,M	50.00	58.33	66.27	65.13	76.40	74.83	66.53	54
6	D	T,M	58.83	54.30	61.60	61.43	58.97	61.33	59.08	54
7	A	M,T	63.67	63.90	57.17	63.40	66.40	65.33	62.72	54
8	A	M,T	64.17	62.50	61.67	62.00	63.33	64.17	62.38	54
9	A	M,T	54.67	54.40	55.53	56.63	57.93	57.17	56.13	54
10	A	T,M	65.00	72.77	67.87	75.73	77.47	76.67	73.46	54
11	A	T,M	59.17	60.53	58.97	57.17	52.57	51.50	57.31	54
12	A	T,M	66.83	56.27	53.97	55.80	58.50	59.67	56.13	54
All 12			58.07	59.29	59.89	59.62	61.43	61.51	60.06	54

Prize= 25

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	32.17	31.00	27.07	32.73	27.20	29.50	29.50	30
2	D	M,T	31.67	31.33	31.33	29.67	30.00	31.67	30.58	30
3	D	M,T	23.67	28.90	31.67	32.50	33.17	32.50	31.56	30
4	D	T,M	28.17	30.63	27.93	31.57	31.93	30.67	30.52	30
5	D	T,M	31.67	34.50	34.47	35.13	30.17	31.83	33.57	30
6	D	T,M	34.17	37.03	31.77	31.40	33.03	32.00	33.31	30
7	A	M,T	39.83	34.87	37.40	30.30	29.00	27.67	32.89	30
8	A	M,T	28.33	29.33	31.67	30.50	30.00	30.00	30.38	30
9	A	M,T	35.17	35.03	35.73	35.07	33.07	33.83	34.73	30
10	A	T,M	32.50	31.87	33.37	31.13	29.83	30.83	31.55	30
11	A	T,M	26.50	29.33	30.13	30.20	31.83	32.50	30.38	30
12	A	T,M	30.17	31.37	33.20	32.83	35.27	35.83	33.17	30
All 12			31.17	32.10	32.14	31.92	31.21	31.57	31.84	30

Prize= 20

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	21.50	20.60	22.20	23.57	17.73	17.67	21.03	24
2	D	M,T	20.83	19.83	23.00	23.50	22.83	22.50	22.29	24
3	D	M,T	20.50	14.60	14.17	19.00	18.00	14.17	16.44	24
4	D	T,M	21.33	24.80	25.40	25.03	25.60	27.33	25.21	24
5	D	T,M	27.17	19.47	14.40	16.10	10.73	11.00	15.18	24
6	D	T,M	14.67	16.23	18.30	18.53	19.37	17.50	18.11	24
7	A	M,T	11.67	15.33	17.37	20.07	18.20	19.00	17.74	24
8	A	M,T	16.67	17.50	15.83	16.67	15.83	15.00	16.46	24
9	A	M,T	20.17	19.90	18.63	18.30	18.40	18.50	18.81	24
10	A	T,M	14.83	10.93	11.27	8.77	9.07	9.17	10.01	24
11	A	T,M	22.00	20.80	21.53	22.70	25.13	25.67	22.54	24
12	A	T,M	18.00	24.73	25.60	26.67	23.13	22.33	25.03	24
All 12			19.11	18.73	18.98	19.91	18.67	18.32	19.07	24

Prize= 10

Group		Order	Rnd 1	Rnds 1-5	Rnds 6-10	Rnds 11-15	Rnds 16-20	Rnd 20	All Rnds	NE
1	D	M,T	18.83	12.63	5.83	7.20	5.20	5.33	7.72	12
2	D	M,T	14.17	11.33	13.33	12.67	12.83	12.50	12.54	12
3	D	M,T	20.50	13.93	12.67	15.83	15.33	11.67	14.44	12
4	D	T,M	12.17	11.93	9.77	8.60	14.63	17.00	11.23	12
5	D	T,M	11.17	7.70	4.87	3.63	2.70	2.33	4.73	12
6	D	T,M	12.33	12.43	8.33	8.63	8.63	9.17	9.51	12
7	A	M,T	4.83	5.90	8.07	6.23	6.40	8.00	6.65	12
8	A	M,T	10.83	10.67	10.83	10.83	10.83	10.83	10.79	12
9	A	M,T	10.00	10.67	10.10	10.00	10.60	10.50	10.34	12
10	A	T,M	7.67	4.43	7.50	4.37	3.63	3.33	4.98	12
11	A	T,M	12.33	9.33	9.37	9.93	10.47	10.33	9.78	12
12	A	T,M	5.00	7.63	7.23	4.70	3.10	2.17	5.67	12
All 12			11.65	9.88	8.99	8.55	8.70	8.60	9.03	12

Values: D=Descending, A=Ascending; Order: M,T=Majority then Total, T,M=Total then Majority.

Table 12: Average Bids Over Time, Version 2 {45, 25, 20, 10}, Total Rule

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