# Constructing surfaces and contours in displays of color from motion: The role of nearest neighbors and maximal disks 

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#### Abstract

Color-from-motion displays consist of a sparse array of dots which never move but change color according to various algorithms. Yet such displays can trigger human vision to construct apparent motion of a subjective surface which is uniformly colored and bounded by a subjective contour. We show that the perceptual strength of this construction depends on the density and regularity of dot placement. We studied three objective measures of density and regularity: nearest-neighbor distance, mean of maximal disks, and variance of maximal disks. We found that nearest-neighbor mechanisms alone are inadequate to account for the perceptual strength of the subjective surfaces and contours. Mechanisms sensitive to areal gaps provide a more adequate account.


## 1 Introduction

Human vision can construct surfaces and contours from sparse visual data in dynamic displays (eg Kaplan 1969; Cortese and Andersen 1991; Kellman and Shipley 1991). Here we study how the density and regularity of the sparse data affect this construction process.

As an example of our constructive ability, consider our perception of the flounder Bothus ocellatus, a species that is expert at camouflage. When placed in a new environment, this flounder quickly scans the surface below and, within seconds, reorganizes its pigment granules so that its skin coloring resembles the texture of this surface (Ramachandran et al 1996). For example, if the surface is gravel, the flounder changes its pigments so that its skin resembles the texture of the gravel. As a result, if the flounder lies motionless, as it often does when it detects a predator, it is nearly invisible to us. But if the flounder moves, we can then construct both its motion and shape.

You can mimic this 'kinetic occlusion' effect as follows. Take a white sheet of paper and place on it, at random, many small black dots. Photocopy the sheet, and cut a 2 inch disk from the Xerox copy. Toss this disk onto the first sheet of paper. Like the flounder, the disk blends into the background and disappears. Now grasp the sheet of paper and shake it back and forth. The disk suddenly appears. Its bounding contour is betrayed by the appearance and disappearance of dots in the background as the disk slides around, and its surface is betrayed by the common motion of the dots on the disk. Both the disk and flounder illustrate that kinetic occlusion is adequate for the detection and construction of surfaces by human vision.

Kaplan (1969) found that we can compute shape and motion from kinetic occlusion cues alone. His stimuli can best be understood as follows. Imagine a white sheet of paper covered with black dots. Cut it from top to bottom, dividing it into two sheets. Slide these sheets back and forth, one partly behind the other. The dots on one sheet disappear and reappear as this sheet slides back and forth behind the other, creating kinetic occlusion.

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Kaplan simulated this situation using animations recorded on 16 mm film, minimizing all cues other than kinetic occlusion. He found that subjects could construct two surfaces in motion, together with a bounding contour between them.

Andersen and Cortese (1989) studied how subjects use kinetic occlusion to discriminate between different shapes. They found that accuracy of discrimination increased with speed of the shapes and with density of the texture elements within the display, with density being the stronger factor. They also found that static cues, such as presence or absence of texture elements, did little to aid discrimination.

Cortese and Andersen (1991) studied how subjects use kinetic occlusion to construct 3-D shapes and motion. Their stimuli can best be understood as follows. Imagine many randomly placed white dots on a black computer screen. Further, imagine a rotating black ellipsoid just in front of the screen so that, as the ellipsoid rotates, certain dots on the screen appear and disappear. Cortese and Andersen simulated this situation with a dynamic 2-D dot display by appropriately adding and deleting dots from frame to frame. The pattern of accretion and deletion of these dots tracked the projection of the bounding contour of the ellipsoid onto the screen as it deformed over time. They found that from this 2-D display, subjects perceived an ellipsoid rotating in three dimensions. These results illustrate the power of kinetic occlusion to trigger the visual construction of 3-D shapes and motions.

Cunningham et al (1998a) created displays in which texture information was contained (i) only inside, (ii) only outside, or (iii) on both sides of a figure defined through kinetic occlusion. They found that shape identification was most accurate when texture information was contained only outside a figure. They concluded that texture information inside the figure interfered with the visual construction of its shape and boundaries.

Cunningham et al (1998b) performed two experiments which show that human vision can construct a dark, extended, opaque surface based on dynamic information alone. They created displays in which all static cues for the detection of a surface were removed and only dynamic cues remained. Subjects nevertheless reported the perception of an opaque surface. They also demonstrated that these dynamic cues greatly improve performance at shape identification.

In this paper we focus on 'color-from-motion' or 'dynamic color spreading' displays (Cicerone and Hoffman 1992; Shipley and Kellman 1993, 1994; Cicerone et al 1995), as illustrated in figure 1 (for the real display see http://aris.ss.uci.edu/cogsci/personnel/hoffman/ colordiskexp.html; also available on http://www.perceptionweb.com/perc0500/fidopiastis.html and scheduled to be archived on the annual CD-ROM supplied with issue 12 of Perception). Begin with a white computer screen and a collection of 900 randomly placed black dots. A color-from-motion display is created from this set of dots as follows. Imagine a virtual disk on the screen with center at p and radius $r$. If a dot falls within this disk, color it green; if not, color it red. Save this frame of colored dots. Now translate the disk by a small fixed amount in a fixed direction and again color the dots as before. Thus, different subcollections of dots in the display will be colored green and red. Save this second frame. Continue this process of translating the disk, and coloring dots and save, say, ten of the resulting frames. Four such frames are given in figures $1 \mathrm{a}-1 \mathrm{~d}$, where the green dots are depicted as smaller dots and the red dots as larger dots.

When subjects view these frames in rapid sequence, they see a subjective green disk with a bounding contour translating over red dots. Even though no dots move, our visual system constructs the green disk, its bounding contour, and its motion from the changes in color of these unmoving dots. If, instead of static dots and a moving virtual disk, we show moving dots and a static virtual disk, almost no color spreading is seen.


Figure 1. Four frames from a color-from-motion display. Green dots are shown as the smaller dots, and red dots are shown as the larger dots.

In color-from-motion displays a static view of a single frame gives little or no color spreading. It is only when the display is put in motion that the color spreading appears. This is a key difference between standard neon color spreading and color from motion.

The strength of these constructions in color-from-motion displays is affected by many factors. Miyahara and Cicerone (1997) found that, if the red and green dots are isoluminous with each other, then subjects still see a green disk but no longer see a distinct bounding contour. Cicerone et al (1995) found that, within a certain range, the perceived strength of the disk and its bounding contour increases with increasing visual angle and increasing velocity of the disk. These results led Prophet et al (2000) to present two algorithms for constructing bounding contours in color-from-motion displays.

In this paper we investigate two more factors that affect the perceived strength of the subjective disk and its contour: the density and regularity in placement of the dots. We show that linear nearest-neighbor mechanisms alone are inadequate to account for the perceptual strength of the subjective surfaces and contours. Mechanisms sensitive to areal gaps provide a more adequate account.

In the first experiment we test two hypotheses. The first is that increasing the density of dots increases the perceived strength of the disk and its contour. The reasoning here is that a higher density of dots leads, on average, to smaller 'gaps' between dots over which the visual system must interpolate to construct the disk and its bounding contour. This makes the construction process easier and the resulting percept stronger. The second hypothesis is that a more regular placement of dots also increases the perceived strength of the disk and its contour. Our reasoning here is that a more regular placement of dots leads to smaller variance in distance between dots, so that there are fewer large gaps over which the visual system must interpolate to construct the disk and its bounding contour. This again makes the construction process easier and the resulting percept stronger.

## 2 Experiment 1: Modulating dot density and dot regularity

### 2.1 Method

The five subjects were students at the University of California, Irvine. Two subjects were naïve to the purposes of the experiment. All subjects had normal or corrected-to-normal vision (20/40, Snellen eye chart) in at least one eye.

The stimuli (created with Mathematica) were color-from-motion displays, as described above, except that the central region was shaped not like a disk, but, instead, like a square or a square with rounded corners. Each display consisted of twelve frames, which were shown repeatedly in a loop mode. The central green square translated vertically 0.125 deg per frame, starting from 0.75 deg below the center of the display.

The independent variables were central shape, dot density, and dot regularity. The two levels of central shape were square or square with rounded corners. The three levels of dot density were 100,400 , and 900 dots per frame resulting in dot densities of 0.16 , 0.64 , and 1.44 dots $\mathrm{deg}^{-2}$, respectively. The three levels of dot regularity (or dot placement) were random, pseudo-random, and aligned. For the random displays, as shown in figure 2 a , dots were placed within the display according to a uniform distribution. For the pseudo-random displays, as shown in figure 2 b , the display was partitioned into a square array of 10 by 10,20 by 20 , or 30 by 30 cells and then a single dot was placed within each cell according to a uniform distribution. For the aligned displays, as shown in figure 2c, dots were placed in regular rows and columns.


Figure 2. Three types of regularity of dot placement used in experiments 1 and 2: (a) random, (b) pseudo-random, and (c) aligned.

Subjects rated, on a scale of $0-4$, (i) the strength of the square's boundary contour and (ii) the strength of the square's surface. Subjects gave a rating of 0 for weak or nonexistent attribute strength, 1 for fairly weak strength, 2 if they were fairly confident of attribute strength, 3 if they were very confident of attribute strength, and 4 if they were absolutely certain of its presence. We asked subjects to use the full range of ratings.

Experiments were run on an Apple Quadra 840 AV computer and displayed on a 17 inch Sony Trinitron Color Monitor. Subjects viewed the displays in a darkened room at a distance of about 110 cm . Each dot in the displays subtended a visual angle of 3 min of arc and the virtual square subtended $1.2 \mathrm{deg} \times 1.2$ deg. Background dots were red (CIE $x=0.6213, y=0.3444$; luminance $=20.1 \mathrm{~cd} \mathrm{~m}^{-2}$ ) and those within the virtual square were green (CIE $x=0.2797, y=0.6102$; luminance $=62.4 \mathrm{~cd} \mathrm{~m}^{-2}$ ). The screen area between dots was white (luminance $=76.9 \mathrm{~cd} \mathrm{~m}^{-2}$ ). Each display subtended 5 deg of visual angle and frames were shown at a rate of 5.1 frames $\mathrm{s}^{-1}$, so that the virtual square translated at a velocity of $0.6375 \mathrm{deg} \mathrm{s}^{-1}$. This velocity was chosen on the basis of the results of Cicerone et al (1995) which showed that it is effective for producing color from motion. Subjects had unlimited viewing time on each trial and gave oral responses, recorded by the experimenter. There were a total of eighteen displays ( 2 central shape$\mathrm{s} \times 3$ dot densities $\times 3$ dot regularities). Each subject participated in three sessions, and
viewed each display three times in each session, for a total of 162 responses per subject. In each session displays were presented in pseudo-random order.

### 2.2 Results and discussion

There was no effect of central shape on surface strength ( $F_{1,4}=0.48, p=0.526$ ) or on contour strength ( $F_{1,4}=0.97, p=0.381$ ). This suggests that aliasing (Galvin and Williams 1992) between the dots and the straight edges of the central square does not affect the perceived strengths of boundary and surfaces. As expected, ratings for contour strength and surface strength increased as dot density increased (contour strength, $F_{2,8}=375, p=0.0001$; surface strength, $F_{2,8}=371, p=0.0001$ ). Also as expected, ratings for contour strength and surface strength increased as dot regularity increased (contour strength, $F_{2,8}=24.7, p=0.0004$; surface strength, $F_{2,8}=15.7, p=0.0017$ ). This is shown in figure 3, which combines results from both central shapes.

To confirm that aliasing has no effect, we replicated this experiment with five subjects using disks instead of squares as the central shape, and obtained similar results. The design of this replication was 1 central shape $\times 3$ dot densities $\times 3$ dot regularities $\times 3$ repetitions, for a total of 81 trials per subject.

Experiment 1 shows that increasing dot density and dot regularity increases ratings of contour strength and surface strength in displays of color from motion. This accords with the hypotheses that increasing dot density decreases the size of gaps over which human vision must interpolate to construct contours and surfaces, while increasing dot regularity increases the consistency of the size of the gaps over which human vision must interpolate, and therefore facilitates the constructive process.


Figure 3. Results for the five subjects from experiment 1 showing the effects of dot density and regularity of dot placement on contour strength and surface strength ratings. Error bars represent the standard deviation.

## 3 Experiment 2: Tracking is not responsible for contour or surface ratings

One could argue that the ratings for surface strength in experiment 1 depend on subjects tracking the virtual figure with their eyes. This tracking might smear the green color within the virtual figure, creating the appearance of color spreading. In experiment 2 , we check that color spreading is not due to such smearing.

### 3.1 Method

Five subjects from the University of California, Irvine, different from those used in experiment 1, were used in experiment 2 . In this experiment we used the displays from experiment 1 with rounded corners, and added one new factor: fixation versus tracking. In fixation trials, a fixation cross appeared in the display, and subjects were asked to fixate the cross while making their ratings. Fixation trials were blocked separately from tracking trials.

### 3.2 Results and discussion

Results are shown in figure 4. Figure 4 a shows how judgments of contour strength vary with regularity of dot placement and with dot density, and figure 4 b shows how judgments of surface strength vary with regularity of dot placement and with dot density. There was no effect of fixation on ratings (contour strength, $F_{1,4}=0.805$, $p=0.42$; surface strength, $F_{1,4}=0.163, p=0.71$ ).

We also see the same effect of density and regularity of dot placement on ratings as in experiment 1.


Figure 4. Results for the five subjects from experiment 2 showing the effects of fixation on (a) contour strength ratings, and (b) surface strength ratings. Figures 4 a and 4 b show how judgments of contour strength and surface strength vary with regularity of dot placement and with dot density.

## 4 Experiment 3: Nearest-neighbor measure of regularity

Experiments 1 and 2 suggest that, as gaps between dots decrease, human vision more easily interpolates contours and surfaces in displays of color from motion. However, these experiments leave open what precisely constitutes a 'gap' for human vision. In many computer-vision algorithms a standard measure for gaps is the 'nearest-neighbor' distance, ie the linear distance between pairs of points that are nearest each other (Levy et al 1990; Prubert 1992). Experiment 3 was designed to test whether this measure can account for the performance of subjects on displays of color from motion.

### 4.1 Method

We created displays, each with a constant nearest-neighbor distance, $r$, using the following algorithm. Place one dot on the screen at random (uniform measure). Randomly place a second dot on a (virtual) circle of radius $r$ centered on the first dot. Place a third dot on the screen at random, but at least $r$ units from each of the first two dots. Using the same $r$, randomly place a fourth dot on a (virtual) circle of radius $r$ centered on the third dot, but at least $r$ units from each of the first two dots. Now place a fifth dot on the screen
at random, but at least $r$ units from all previous dots. Continue this process until the correct number of dots is reached. The nearest-neighbor measure evaluated on this display gives the constant value $r$. This assumes, of course, that $r$ is smaller than the display size and that the number of dots is small enough to allow the algorithm to work.

The independent variables were dot density (420, 220, and 110 dots or dot densities of $0.672,0.352$, and 0.176 dots $\mathrm{deg}^{-2}$ ) and nearest-neighbor distance ( 12,6 , and 3 min of arc). Although in general dot density and nearest-neighbor distance are not independent, since as dot density gets very high the maximal nearest-neighbor distance must diminish, nevertheless with all dot densities used in our displays it was possible to independently manipulate the nearest-neighbor distance. In all other respects, the displays were as in experiment 1 . Each subject rated contour and surface strength of virtual squares using the same rating scale as in experiment 1.

A total of five subjects were run, of which two were naïve to the experimental hypothesis. Otherwise the methods were the same as in experiment 1.

### 4.2 Results and discussion

Results are summarised in figure 5. As expected, ratings increased with dot density (contour strength, $F_{2,8}=450.1, p=0.0001$; surface strength, $F_{2,8}=278.97, p=0.0001$ ). Contrary to what would be expected on the nearest-neighbor hypothesis, ratings for both contour and surface strength increased as nearest-neighbor distance increased (contour strength, $F_{2,8}=23.9, p=0.0004$; surface strength, $F_{2,8}=19.94, p=0.0008$ ).


Figure 5. Results for the five subjects from experiment 3 showing the effects of dot density and nearest-neighbor distance on contour strength and surface strength ratings. Error bars represent the standard deviation.

This demonstrates that nearest-neighbor distance, by itself, is an inadequate measure of regularity in dot placement. One possible reason for this is that nearest-neighbor distance is insensitive to gaps of area between dots, since it only measures linear gaps between pairs of dots. Moreover, in displays generated with the nearest-neighbor algorithm, it is the displays with the largest nearest-neighbor distances, not the smallest, that give the more regular dot placement. This idea is illustrated in figure 6. Figure 6a shows a dot pattern in which the nearest-neighbor distance between dots is large, and figure 6 b shows a dot display in which the nearest-neighbor distance between dots is small. Notice that the dots of figure 6 b cover the area much more uniformly than those in 6 a .

Nearest-neighbor distance, although by itself an inadequate measure of dot regularity, might still be an important aspect of dot regularity. One shortcoming with the nearestneighbor measure is that, for each dot, it encodes information only about a single direction, and in this sense is just a linear measure. What might help is a measure of regularity that is two-dimensional, ie one which encodes information about many


Figure 6. Dot displays with two different values of the nearest-neighbor measure. The nearestneighbor distance is greater in (b) than in (a). Notice how this leads to the dots in (b) having more regular placement.
(if not all) directions from a given dot. This measure would be sensitive to gaps in area, not just to linear gaps. In figure 6a, although the nearest-neighbor gaps are small, there are much bigger gaps of area than in figure 6 b , and it is these bigger gaps of area that lead to weaker visual constructions of contours and surfaces.

One measure that seems to capture gaps of area is the mean of 'maximal disks'.
Maximal disks can be defined as follows. For a given dot display, define the 'closestdot' function, $c(x, y)$, whose value at each point $(x, y)$ in the display is the distance from that point to the closest dot. This function is zero at all points which happen to be dots, and positive everywhere else. At each local maximum of this function, draw a disk whose radius is the value of the function at that point and which is centered at that point. This is a 'maximal disk', ie any other disk is smaller if it (i) is centered anywhere in an infinitesimal neighborhood of that point and (ii) contains no dots. For a given display, there are numerous maximal disks (a different one at each local maximum of the function).

Let us re-examine the notion of regularity using these maximal disks. Intuitively, a display is 'regular' if the size of the area gaps is consistent, ie roughly the same throughout the display. This consistency can be measured by the variance of the radii of the maximal disks: higher consistency entails lower variance. Also, the mean of these radii is a measure of dot density, or, equivalently, of average gap area.

For example, in experiments 1 and 2 the variance of maximal disks is highest in the random displays, less in the pseudo-random displays, and least (in fact, zero) in the aligned displays.

The mean and variance of maximal disks accurately accounts for the results of experiments 1 and 2: ratings of contour and surface strength increase as a result of decreases in both mean maximal-disk size and maximal-disk variance. So we take the results of experiments 1 and 2 as supporting the following hypotheses: (i) As the mean maximal-disk size decreases, human vision more easily or confidently constructs contours and surfaces in displays of color from motion. (ii) As the variance of maximal disks decreases (and thus the regularity of a dot display increases), human vision more easily or confidently constructs contours and surfaces in displays of color from motion.

Given that the mean and variance of maximal disks accounts for our results in experiments 1 and 2, the obvious next question is whether this is the whole story: Can we construct displays for which the mean and variance is the same for each display, and still have differences in these displays that lead to differences in perceived contour and surface strengths? This is the topic of experiment 4.

## 5 Experiment 4: Maximal-disk measure of regularity

One way to create displays which all have the same mean and variance of maximal disks, but might nonetheless vary in regularity, is to place the dots in differing symmetric patterns. In such displays the mean of the maximal disks can be made constant, and the variance of maximal disks is in fact zero. Yet the different symmetries might still reflect, for human vision, different degrees of regularity. Thus in experiment 4 we compare ratings of contour and surface strength across several different symmetric patterns of dots. If these differences in symmetry lead to differences in ratings, then this shows that there is more to perceptual regularity than just mean and variance of maximal disks.

### 5.1 Method

The subjects were six graduate students from the University of California, Irvine, naïve to the purposes of the experiment. Each subject sat 70 cm from the monitor. The independent variables were dot symmetry (rectangular, shifted rectangular, square, and hexagonal) and radius of mean maximal disk ( 0.88 deg and 0.47 deg ), for a total of eight displays. The four displays with the larger maximal-disk radius are shown in figure 7. Each display subtended 7.76 deg. The virtual disk subtended 1.72 deg, started its motion 0.7 deg below the center of the display, translated along a $45^{\circ}$ diagonal at 0.24 deg per frame (resulting in a velocity of $0.6375 \mathrm{deg} \mathrm{s}^{-1}$ ), and made a total excursion of 2.9 deg. The eight displays were presented to subjects in pseudo-random order. Figure 8 illustrates the maximal disks for the different types of symmetry. In all other respects the stimuli were as in experiment 1.

(a)

(c)

(b)

(d)

Figure 7. The four different symmetries of dot placement used in experiment 4: (a) shifted rectangular, (b) rectangular, (c) square, and (d) hexagonal. The maximal disk size is constant in all displays.

(a)

(c)

(b)

(d)

Figure 8. Locations of maximal disks in displays with the four levels of symmetry of dot placement: (a) shifted-rectangular array, (b) rectangular array, (c) square array, and (d) hexagonal array.

There were 24 practice trials (eight displays seen three times each) followed by 56 experimental trials (eight displays seen seven times each). Subjects orally rated contour and surface strength using the same rating scale as in experiment 1 .

### 5.2 Results and discussion

The results are shown in figure 9. As expected, there was a main effect of mean radius of maximal disk (contour strength, $F_{1,5}=487.818, p=0.0001$; surface strength, $F_{1,5}=67.726, p=0.0004$ ). There was also a main effect of dot symmetry (contour strength, $F_{3,15}=22.296, p=0.0001$; surface strength, $F_{3,15}=23.013, p=0.0001$ ). There was also a significant interaction between disk radius and symmetry (contour strength, $F_{3,15}=13.085, p=0.0002$; surface strength, $F_{3,15}=14.992, p=0.0001$ ). A posteriori analyses showed that for the larger disk radius, but not for the smaller radius, the rectangle and shifted-rectangle displays were rated significantly higher than the square and hexagon displays (see figure 9), and that for both disk radii there were no significant differences between rectangle and shifted-rectangle, or between square and


Figure 9. Results for the six subjects from experiment 4 showing the effects of maximal-disk radius and symmetry of dot placement on contour strength and surface strength ratings. Error bars represent the standard deviation. For a fixed level of dot density, dot numerosity varied between displays. For a fixed degree of symmetry, there was a substantial, although not pre-determined difference in dot numerosity between low and high dot density displays.
hexagon displays. The lack of any significant differences at the smaller disk radius is probably a ceiling effect. Displays with the smaller disk radius had higher densities of dots, and these displays were uniformly rated quite high. It was only when the disk radius was large, and thus dot densities were lower, that the differential effects of symmetry appeared. These results indicate that the mean and variance of maximal disks are not solely responsible for the perceived strengths of surface and contour of the subjective disk.

We can define the 'degree of symmetry' for each of the four figures (namely hexagon, square, rectangle, and shifted rectangle) that are used to generate our symmetric displays. This is done by counting the number of rigid motions that keep the figure unchanged as a geometric object (more precisely, we count bijective isometries from the figure to itself; see, eg, Jacobson 1985). For example, one can rotate the hexagon about its center through an angle of $0^{\circ}, 60^{\circ}, 120^{\circ}, 180^{\circ}, 240^{\circ}$, or $300^{\circ}$ without changing its appearance as a geometric object. One can also reflect the hexagon across horizontal, vertical, and diagonal lines going through its center without changing its appearance. Since the number of shape-preserving rigid transformations is less for the rectangle and shifted rectangle than for the square and hexagon, the rectangle and shifted rectangle have lower degrees of symmetry.

What is striking is that, for displays with the large disk radius, these 'less symmetric' displays produce higher perceptual ratings. These higher ratings can be explained through an analysis of dot density. For displays with the large disk radius, geometrical considerations dictate that dot numerosities vary, ${ }^{(1)}$ from $120\left(0.192\right.$ dots $\left.\mathrm{deg}^{-2}\right)$ in the hexagon display $(12 \times 10)$ to $169\left(0.27\right.$ dots deg $\left.^{-2}\right)$ in the square display $(13 \times 13)$ to 297 in both of the rectangular displays $(9 \times 33)$. Having 33 dots place horizontally across the screen in the rectangle and shifted-rectangle displays led to a horizontal separation of about 0.189 deg between dots; having 10 and 13 dots placed horizontally across the screen in the square and hexagon displays led to a separation of about 0.78 deg between dots. Thus the nearest-neighbor distance between points in the rectangle and shifted-rectangle displays was smaller than in the square and hexagon displays. Experiment 3 showed that nearest-neighbor distance cannot, by itself, account for ratings of surface and contour strength in displays of color from motion. However, the results of the present experiment suggest that nearest-neighbor distance, in coordination with mean and variance of maximal disks, does play a role. Smaller nearest-neighbor distances make it easier for subjects to construct the surface and bounding contour of the subjective disk.

## 6 General discussion

Displays of color from motion trigger a remarkable construction by human vision. We construct a uniformly colored surface bounded by a clear subjective contour, and we construct a motion of the surface and contour, when in fact no surface, contour, or motion is explicitly displayed on the computer screen. Casual inspection confirms what visual intuition might expect: the larger the gaps over which these constructions must take place, the more difficult and less compelling the constructions. We found that nearest-neighbor mechanisms alone are inadequate to account for the perceptual strength of the subjective surfaces and contours. Mechanisms sensitive to areal gaps provide a more adequate account.
${ }^{(1)}$ For the square array, distance between dots was chosen arbitrarily to be 0.65 deg. This determined the parameters of the hexagonal array. For the rectangular display, 0.65 deg was a lower bound for the length of the longer side (oriented vertically). By our choice of maximal-disk diameter, this side was further constrained to be less than 0.92 deg . Choosing the length of the longer side to be midway between these two numbers would force the shorter-length side of the rectangle to be about 0.1 deg . This would require about 66 dots in a single row of the display compared with 10 for the hexagonal pattern. To avoid a discrepancy this large, we decided to use shorter sides of length 0.19 deg , which required 33 dots in a single row of the display and longer sides to be of length 0.9 deg. The parameters for the shifted-rectangle displays were chosen similarly.

This work expands on the results of Andersen and Cortese (1989), who found that dot density and translational velocity both affect the perception of kinetic occlusion, but that velocity has the greater effect. In the study reported here, we held figure velocity constant and explored the factor of density in more detail, finding that the pattern of placement of the dots has a strong effect on the perceived strength of the subjective curves and surfaces.

In all experiments described here, we asked subjects to rate the perceptual strength of the subjective contours and surfaces. In experiment 1, we varied the dot density, the regularity of dot placement, and the shape of the illusory figure between displays, and found that ratings increased with increasing dot density and regularity of dot placement. In experiment 2 subjects were allowed to track the figure with their eyes or not, and we found that ratings were statistically indistinguishable in both conditions, indicating that these ratings are not simply due to optical smearing. In experiment 3 , we varied dot density and level of linear nearest-neighbor measure, and found that ratings increased with increasing dot density, but decreased with decreasing nearestneighbor distance, suggesting that nearest-neighbor distance, by itself, is an inadequate measure of regularity in dot placement. In experiment 4, we varied the symmetry of dot placement and the radius of the maximal disk, and found that ratings increased with decreasing mean and variance of the maximal disks, suggesting that mean and variance of maximal disks provide a useful measure of regularity in dot placement.

In these experiments we considered several precise measures of gap size and regularity, and tested how well they track human performance in the construction of visual curves and surfaces. We found that three measures together (but none of them alone) accurately track human performance: mean maximal-disk radius, maximal-disk variance, and linear nearest-neighbor distance.

Several researchers have studied algorithms for computing contours and surfaces given a collection of dots (eg Grimson 1981; Weiss 1990; Zhao et al 1998; Prophet et al 2000). Algorithms such as these, together with measures of regularity and gap size as studied here, should lead to predictions of perceived shapes and perceptual strengths in displays of color from motion.

There are several questions for future research. First, does the degree to which maximal disks overlap affect the perceptual strength of the curves and surface that we construct in displays of color from motion?

Second, how do dot numerosity and mean radius of maximal disks interact in determining perceptual strength? One test of this interaction is to fix the dot numerosity while varying the mean radius of maximal disks. For instance, the square array display of experiment 4 had dimensions $13 \times 13$, containing 169 dots. We created a rectangular display with dimensions $7 \times 24$ ( 168 dots), dramatically increasing the height of the individual rectangles. As a result, the mean maximal-disk size was smaller for the square and hexagon displays than for the rectangle and shifted-rectangle displays. Pilot studies with these displays suggest, as predicted by the mean-maximal-disk hypothesis, that the perceptual effect is opposite of what we found in experiment 4: the subjective disk is now seen as more compelling in the square and hexagon displays than in the rectangle and shifted-rectangle displays. This suggests that changes in mean maximal disk size, even with fixed dot numerosities, can affect the construction of contours and surfaces in displays of color from motion.

Third, we have created nonrigid color-from-motion displays in 2-D and rigid color-from-motion displays in 3-D. For the nonrigid 2-D displays, rather than use a constant-sized virtual disk from frame to frame, we used a varying-sized disk or some other sequence of changing virtual regions. As long as the virtual regions do not change too much in shape from frame to frame, human vision readily constructs a smoothly deforming 2-D shape in color. For the rigid 3-D displays, in each frame we chose the
virtual area in which dots are colored green to be the inside of the silhouette of a rotating 3-D ellipsoid. Again, as long as the virtual regions do not change too much in shape from frame to frame, human vision readily constructs a rigid 3-D ellipsoid in motion, and fills it with subjective color. One question to be answered in both cases is this: Do the same measures of gap size and variability studied in this paper for rigid 2-D shapes also track human visual performance in nonrigid 2-D and rigid 3-D displays?

These are all interesting topics for further research. However, the measures tested in this paper provide strong constraints on computational theories of the construction of surfaces and contours in displays of color from motion.
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