NOTES

Mo Fiorina's Advice to Children and Other Subordinates

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In recent work ([2], [3]) Morris Fiorina, a political scientist at the California Institute of Technology, dealt with the question of constituency influence on representatives' roll call voting by contrasting the predictions of an expected utility maximizing model with those of what he refers to as a "maintaining" model. In Fiorina's maintaining model, Congressmen are assumed to set an "acceptable" level of reelection probability and to vote in such a way as to maintain their (subjectively estimated) reelection probability at this level. Fiorina uses this model to account for a number of anomalies in congressional voting behavior.

In this note we generalize Fiorina's maintainer model so as to make it applicable to any dichotomous choice where an actor is confronted with (potential) demands or pressures from sources which may conflict with his own preferences. These outside sources may make incompatible demands, may differ in their ability to reward and/or punish the actor for disobedience, and may also differ in their likelihood of administering the promised rewards (or threatened punishments). We confine ourself to a three-actor model consisting of an actor (a child) and two outside sources of (potential) demands or pressures (her parents), and give an analysis as seen from the perspective of the actor (the child). Although the discussion is presented in terms of child-parent conflict, the analysis is a perfectly general one, applicable to a wide variety of conflict situations, e.g., a worker torn between demands of union and management; a student torn between demands of teacher and of peer group; a legislature torn between pressures from competing interest groups; or a soldier torn by conflicting commands from his immediate commanding officer vs. general directives of remote superiors.

We consider six cases, based on the extent to which the actor's preferences (e.g., those of the child) and those of the sources of demands (the parents) are in conflict. These cases are summarized in the following chart:

	Sources (Parents) are	
Actor (Child)	In Agreement	Split
Is indifferent	Case I	Case IV
Prefers to do as neither source (parent) wishes	Case II	-------
Prefers to do as stronger source (parent) wishes	Case III	Case V
Prefers to do as weaker source (parent) wishes	Case III	Case VI

For each of these six cases we consider the implications for compliance behavior of the two alternative models used by Fiorina: maintaining and expected utility maximizing. (In a longer version of this paper, available upon request from the author, we consider the implications for compliance behavior of two other strategies called maximin and minimax regretting. (See Luce and Raiffa [4]; Ferejohn and Fiorina [1].))

The expected utility maximizing model is the standard approach to problems of choice under risk (see e.g., Luce and Raiffa [4]). The maintaining model appears most appropriate in a context where outcomes can be readily divided into two classes, satisfactory and unsatisfactory (e.g., winning or losing an election, or getting fired vs. not getting fired). In the context of child-parent conflict this model seems appropriate for children who are not seeking the best of all possible worlds, but merely one which provides no worse level of satisfaction than that they regard as acceptable and to which they have become accustomed.

Dear Professor Fiorina:

Sometimes my parents want me to do what they tell me to. This seems unfair. What should I do?

Virginia

Dear Virginia:

There are various cases of interest. For example, your parents may or may not be in agreement. First, let us consider the times when both your parents are in agreement on what they want you to do. We shall assume that when they reward you for obedience, the reward is worth x, and when they punish you for disobedience, the value of the punishment is $-z$. Since your parents are only human, they don't always watch to see what you're doing. However, let us assume that they are conscientious and always administer the proper reward (or punishment) when they know it is due, but never otherwise. Let the probability of their monitoring your behavior be labeled c. Finally, since you felt the need to write to me, it seems probable that your parents are somewhat strict, and therefore we shall assume $z>x$. On the other hand, I don't really know that much about you, so I will give you advice based on two different kinds of strategies, maximizing and maintaining, thereby allowing you to pick the decision rule which best fits your own personality and attitudes toward risk. If you are a maximizer you will seek the strategy which maximizes your own expected payoff; if you are a maintainer, you will seek the strategy which gives you some constant (and satisfactory level) of expected satisfaction.

Case I. In the absence of rewards or punishments from your parents, you'd be indifferent between doing what your parents want and doing the opposite. We may represent this decision problem in the following game matrix:

Child:	Parents: Monitor ($\text{Pr}=c$)	Don't Monitor ($\text{Pr}=1-c$)
Obey	x	0
Disobey	$-z$	0

Maximizers: The expected payoff of obeying is cx; while the expected payoff of disobeying is $-cz$. Since the former is positive and the latter is negative, as a maximizer you always obey your parents.

Maintainers: As a modern child free of compulsive habits, you might wish to adopt a more hang-loose behavioral rule, e.g., to obey with probability q, so that your expected change in payoff is zero. This maintains your status quo level of satisfaction. To follow such a rule, you seek q so that

$$qcx+q(1-c)0+(1-q)(c)(-z)+(1-q)(1-c)0=0,$$

from which it follows readily that $q=z/(x+z)$. In other words, the greater your reward for obeying, the less you need obey; on the other hand, the more you're punished for disobeying, the more you should obey. Hence, under these assumptions, if parents believe you to be using a maintaining rule, they should always punish and never reward. (You probably should not reveal the results of this analysis to your parents.)

Case II. Suppose that what you want to do and what your parents want you to do are opposite. Assume that doing what you want generates its own reward of r, while not doing what you want to do has value $-y$. Let us also assume that you are more displeased at doing what you don't want to do, than you are pleased doing what you do want to do, i.e., $y>r$. Let us define the strength of your parents as $x+z$ and your strength as $y+r$. Your strength may be thought of as a measure of your ability to generate internal motivations for your actions, and thus it can be taken as a measure of ego-strength. The strength of your parents is a measure of their ability to provide external motivations for your behavior. I know, Virginia, that this may be painful for you to accept, but your parents can do more to affect your life (either positively or negatively) than you can do for yourself; hence, we shall assume $z>y$ and $x>r$. This decision problem may be represented as follows:

Child :	Parents : Monitor ($\mathrm{Pr}=c$)	Don't Monitor ($\mathrm{Pr}=1-c$)
Obey	$x-y$	$-y$
Disobey	$-z+r$	r

Maximizers: Maximizers choose to obey their parents iff $c>(r+y)/(x+z)$. Let us define the effective strength of an actor (child, father, mother, or combination thereof) as the product of strength and vigilance. Parental vigilance is c, the probability of monitoring, while for children vigilance is one, so strength and effective strength are identical. In these terms the rule for maximizing can be rephrased as "obey your parents only if your effective strength is less than their effective strength." (This result can be thought of as a special case of Proposition 11 of Fiorina [**2**], p. 35.) For parents of maximizers, high vigilance is the price of obedience.

Maintainers: Maintainers obey their parents with a probability $q=(cz-r)/(cz-r+cx-y)$. In order that there be a maintaining strategy, we require (see Fiorina [**2**], p. 35) $cx-y$ and $cz-r$ either be both negative, or both positive. If they are negative, then the greater the parents' effective reward, the more the child needs obey them, but the greater the parents' effective punishment, the less he needs obey them. Moreover, under these conditions, the greater the self-generated reward for behavior in compliance with a parental directive, the greater the obedience to that directive (and conversely for self-generated punishments). If, however, these terms are positive, then the impact of effective punishments or rewards on a maintainer is just the opposite. In particular, if these terms are positive, then the greater the (internally generated) value of a given preferred activity (as measured by r), the less often the maintaining child needs to do it. Thus, under these conditions, the things a maintainer likes to do best he needn't do often.

We might also note that, regardless of which condition holds, if their child has maintaining strategy, the more parents monitor their child's behavior, the less likely he or she is to obey them —a rather paradoxical result, indeed. Furthermore, if the terms $cx-y$ and $cz-r$ are both negative, then maintainers obey their parents with a probability less than $1/2$. Thus, Virginia, if you use a maintaining rule, even though you are not as strong as your parents you may still, under some circumstances, disobey them more than half the time!

Case III. Suppose that you prefer to do what your parents want you to do. We may represent this decision problem by the matrix:

Child:	Parents: Monitor (Pr $= c$)	Don't Monitor (Pr $= 1-c$)
Obey	$x+r$	r
Disobey	$-z-y$	$-y$

Maximizer: Since obedience is a dominant strategy (i.e., it is better no matter what monitoring choice is made by the parents), a maximizer clearly obeys his parents (and satisfies his own preferences at the same time).

Maintainer: A maintainer obeys his parents with probability $q=(cz+y)/(cz+y+cx+r)$, which is greater than $1/2$ for all c. The stronger punishments are (relative to rewards), the greater the obedience; the stronger rewards are (relative to punishments), the greater the disobedience. So, even though his interests and his parents are perfectly coincident, if a child is a maintainer he will still sometimes disobey.

It is easy to see that, to maximize the child's obedience, parents should set $c=1$ if $yx<zr$, and they should set $c=0$ if $yx>zr$. Thus, if $yx>zr$, for a child who is a maintainer, the best way to insure his obedience is never to check up on him.

Now, Virginia, let us consider the cases where your father and mother are split in their preferences. We shall assume that one of your parents (whose strength we shall denote x_1+z_1) is stronger than the other (whose strength we shall denote x_2+z_2), so $x_1+z_1>x_2+z_2$. For simplicity, we shall assume that your father is the stronger parent (albeit not necessarily the most vigilant). Your mother wants you to do one thing; your father would have you do the opposite. Finally, assume that your father monitors with probability c_1; your mother with probability c_2.

Case IV. Suppose you would be indifferent as to what to do were it not for the possibility of rewards or punishments offered by your parents. This decision problem may be represented by

		Father Monitors		Father Doesn't Monitor	
		Mother Monitors	Mother Doesn't	Mother Monitors	Mother Doesn't
	Probability	c_1c_2	$c_1(1-c_2)$	$(1-c_1)c_2$	$(1-c_1)(1-c_2)$
Child:	Obey Father	x_1-z_2	x_1	$-z_2$	0
	Obey Mother	$-z_1+x_2$	$-z_1$	x_2	0

This case is equivalent to the heterogeneous constituency case in Fiorina ([**2**], [**3**]), although our notation is somewhat different.

Maximizers: If $c_1=c_2$, maximizers vote exclusively with the stronger parent. For $c_1 \neq c_2$, maximizers vote with the parent with greater effective strength: with the weaker parent if $c_1(z_1+x_1)<c_2(z_2+x_2)$, and with the stronger parent otherwise.

Maintainers: Where $c_1=c_2$, Fiorina ([**2**], pp. 32–39) shows that for a maintaining strategy to exist requires that $x_1 \geqslant z_2$; in this case the maintainer obeys the stronger parent with probability

$$q=\frac{z_1-x_2}{(x_1-z_2)+(z_1-x_2)}>\frac{1}{2}.$$

If $c_1=c_2$ and $x_1=z_2$, then the maintaining child always obeys the stronger parent. Further, if we assume (not unreasonably) that, when parents are grossly unequal in strength, $x_1 \gg z_2$ (x_1 considerably greater than z_2) and $z_1 \gg x_2$, then we obtain the paradoxical result that maintaining children exhibit greater variance in their choice of which parent to obey when one parent is considerably stronger than the other than in the case where both parents are of near equal strength.

Fiorina ([2], pp. 35–37) shows that if $c_1 \neq c_2$, the probability of a maintainer obeying the stronger parent is

$$q = \frac{c_1 z_1 - c_2 x_2}{c_1(x_1 + z_1) - c_2(x_2 + z_2)},$$

while maintaining strategy exists if and only if either $c_1/c_2 \leqslant x_2/z_1$, or $c_1/c_2 \geqslant z_2/x_1$. (These conditions are mutually exclusive.)

In the former case, maintainers obey the weaker parent with probability greater than $1/2$ while in the latter case they obey the stronger parent with probability greater than $1/2$. Hence, maintainers (like maximizers) may vote with the weaker parent provided that parent is the more vigilant. However, if $c_1 > c_2$ and $c_1/c_2 \geqslant z_2/x_1$, then the higher c_1 the lower the probability that a maintainer obeys the stronger parent. In other words, the more the stronger parent monitors the child's behavior, the less likely he is to obey. This result contrasts with what is true for maximizers, for whom it is always true that the greater a parent's vigilance, the more likely he or she is to be obeyed.

Case V. Suppose you prefer to do what the stronger parent wants you to do:

	Father Monitors		Father Doesn't Monitor	
	Mother Monitors	Mother Doesn't	Mother Monitors	Mother Doesn't
Probability	c_1c_2	$c_1(1-c_2)$	$(1-c_1)c_2$	$(1-c_1)(1-c_2)$
Child:				
Obey Father	$x_1 - z_2 + r$	$x_1 + r$	$-z_2 + r$	r
Obey Mother	$-z_1 + x_2 - y$	$-z_1 - y$	$x_2 - y$	$-y$

Maximizers: Maximizers obey the stronger parent when $c_1(x_1 + z_1) > c_2(x_2 + z_2) - (r + y)$. The child will disobey the stronger parent only when the expected strength of this coalition (child + stronger parent) fails to exceed the effective strength of the weaker parent. If $r + y > x_2 + z_2$, the above condition will always be true, under our assumptions. Thus, if the child is stronger than his weaker parent, he will always do what he (and his stronger parent) want him to do. For maximizing children, high vigilance can compensate (within limits) for low strength to insure obedience to the will of the weaker parent.

Maintainers: Maintainers must obey the stronger parent with a probability

$$q = \frac{c_1 z_1 - c_2 x_2 + y}{(c_1 z_1 - c_2 x_2 + y) + (c_1 x_1 - c_2 z_2 + r)}.$$

A maintaining strategy exists if and only if either $c_1 z_1 < c_2 x_2 - y$ or $c_1 x_1 > c_2 z_2 - r$. (These conditions are mutually exclusive.) If the former holds, then maintainers obey the weaker parent with probability greater than $1/2$, while if the latter holds, then they obey the stronger parent with probability greater than $1/2$. Hence, in the first case, we again obtain the paradoxical result that the higher c_1, the lower the probability that a maintainer obeys the stronger parent; in other words, the more the stronger parent monitors the maintaining child's behavior, the less likely is the child to obey him.

Case VI. Suppose your preferences coincide with those of the weaker parent. Since the analysis is straightforward, we leave this case for you to work out. As a helpful hint, if you need one, here is the decision matrix:

	Father Monitors		Father Doesn't Monitor	
	Mother Monitors	Mother Doesn't	Mother Monitors	Mother Doesn't
Probability	c_1c_2	$c_1(1-c_2)$	$(1-c_1)c_2$	$(1-c_1)(1-c_2)$
Child:				
Obey Father	$x_1 - z_2 - y$	$x_1 - y$	$-z_2 - y$	$-y$
Obey Mother	$-z_1 + x_2 + r$	$-z_1 + r$	$x_2 + r$	r

Our analysis has been written from the child's perspective. We showed how choice can be affected by the structure of the rewards (or punishments) administered externally by authority figures such as parents and also by the child's internally generated reward structure. Choices were determined as a function of these rewards. Our analysis was determined by just two decision rules which we assumed the child could adopt to govern his behavior: maximizing and maintaining.

We could extend our result in one of two ways. One natural extension would be to reexamine our decision problems from the parental perspective. We would make reasonable assumptions about the costs involved in monitoring the child's behavior and the payoff to parents of achieving obedience and determine optimal parental strategies in a 2-person or 3-person game. We might (a la Ferejohn and Fiorina **[1]**) construct a decision problem in which the alternatives open to parents are "to command x," "to command not x," or "to leave the child to do as he pleases." Another direction would be to look at other decision rules, e.g., maximin or minimax regret.

Our analysis led to some paradoxical conclusions (e.g., under some circumstances the less surveillance, the more compliance) and often led to non-obvious results about the differing impacts of rewards and punishments under the two different decision rules. Although presented in the context of child-parent conflict, our analysis is intended to be applicable to a wide variety of situations in which some command and others choose whether or not to obey. We leave to the reader the task of detailing such alternative scenarios.

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References

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Sequentially So

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Which elements in the sequence $N=\{1,2,3,4,\ldots,n,\ldots\}$ of natural numbers can be written as a sum of two or more consecutive elements of N? We exemplify by this simple number theoretic problem the paradigm: "Discovery consists of seeing what all have seen and thinking what nobody else has thought." Such is the process of mathematics as done by a mathematician.