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Rein Taagepera's Approach to the Study of Electoral Systems

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This essay will consist of two main parts. In the first section we provide a brief overview of the historical evolution of the study of electoral rules, and a 2 x 2 typology of perspectives on electoral systems research; in the next section, which makes up the bulk of the paper, we focus on Rein Taagepera's unique contributions to electoral system research. In that section, (a) we identify key elements of Professor Taagepera's methodological approach, (b) highlight some of the innovative analytic tools he developed, such as the Laakso-Taagepera index (Laakso and Taagepera 1979), and (c) consider Professor Taagepera's views on how electoral system issues fit into the broader set of constitutional design questions.

An Overview of the Historical Evolution of the Study of Electoral Rules

While there is a long history of the study of voting rules, going back at least as far as Aristotle, the modern era is often thought to begin with the work of Condorcet (1785) and Borda (1781), which serve as precursors for important work in social choice theory. The 18th Century is also notable for the introduction of methods of apportionment for the U.S. House of Representatives that are identical in their form to the most common methods of proportional representation (list PR) now used for elections in Europe and elsewhere (Balinski and Young 1982).[1] The 19th century is notable for the invention of methods such as the limited vote (now used in Spain) and the single transferable vote (now used in Ireland).

When we move to the twentieth century, and look at work by political scientists and interested non-academics, with a few notable exceptions (e.g., work by Maurice Duverger (1948, 1955), by Grumm (1958) and by Eckstein (1963)),[2] and the tradition of work in social choice theory exemplified by Arrow (1963) and Black (1958), until the late 1960s, at least in the English-speaking world, the debate about choice (and consequences) of electoral systems was cast in polemic terms, with works such as Lakeman and Lambert (1955; updated as Lakeman 1974) and Hermens (1940, 1941) largely talking past one another, and staking out positions with something rather like messianic zeal. Hermens looked at post-WWI European history (particularly the rise of the Nazis in Germany) and attributed a variety of undesirable consequences (e.g., the political inclusion of those with extremist views, and what he saw as the inevitable concomitant political instability) to the use of proportional representation. Enid Lakeman, until the end of her long life associated with the Electoral Reform Society of Great Britain, in contrast, followed John Stuart Mill in arguing for the merits of the single transferable vote, a system that allows minorities to elect candidates of choice.

Not only was the early 20th century political science literature on electoral systems effects generally methodologically unsophisticated from a statistical perspective, it was also limited in its focus on only a few key dependent variables (e.g., proportionality, on the one hand, cabinet duration as a measure of stability on the other). For example, for Lakeman (and, for many others, to this day), the litmus test of an electoral system is the degree of proportionality between seats and votes. The publication in 1969 of Douglas Rae's Ph.D. dissertation, "The Political Consequence of Electoral Laws" (2nd edition, 1971) marked a sea change.[3] With this book, the study of electoral systems moved clearly into the mainstream of comparative politics research, making first steps in drawing on analytic tools from game theory, and making use of quantitative multivariate analyses.[4]

In the 1980s the growing maturity of the electoral systems field was signaled by the publication of a number of important books offering summaries and syntheses of data-oriented research findings as well as new analytic perspectives: e.g., Katz (1980), Bogdanor and Butler (1983), Lijphart (1984), Lijphart and Grofman (1984), Grofman and Lijphart (1986), Taagepera and Shugart (1989).[5] Moreover, there was important work on seats-vote issues and redistricting done by political geographers such as Johnston, Taylor, Gudgen and Shelley (Johnston, Shelley and Taylor 1990, Taylor and Johnston 1979, Gudgin and Taylor 1979, also see review in Grofman 1982). Also, and perhaps even more importantly, during this period, the international journal *Electoral Studies* was founded. In addition to depolemicizing the debate, this new research cast doubt on the simplistic notion that the only relevant electoral choice is based on a dichotomy between plurality and proportional representation (PR).[6] It also cast strong doubt on the empirical claims made about the inherent political instability of PR systems as compared to plurality systems (see esp. Lijphart 1984).

The renaissance of the electoral systems field continues. Recently we have seen the publication of Lijphart (1994), Cox (1997) and numerous detailed studies of particular electoral systems and their effects.[7] Likewise, various essays in Colomer (2004) force us to rethink claims often made about the causal impact of changes in electoral systems. Electoral system research has moved into the political science mainstream and established itself as an important subfield. Moreover, strong links have been forged between electoral systems research and the study of party systems, on the one hand, and constitutional design, on the other (see e.g., Carey 1996). The most important aspect of this body of work is that electoral systems and their consequences are seen as embedded in a larger political and institutional framework (see Grofman 1999).[8]

While it is important to emphasize the extent to which the field of electoral studies has matured and grown, including both new substantive foci (such as the role of electoral systems in mediating ethnic conflict)[9] and new analytic tools (including an explicit use of game-theoretic models),[10] there is a continuity with the past in that the field continues to reflect three central concerns about the consequences of electoral systems, namely about (1) degree of proportionality, (2) party proliferation, and (3) tendencies to facilitate or suppress political instability.[11] In each of these areas Professor Taagepera has made seminal contributions.[12]

One useful way to think about the history of the study of electoral systems is in terms of a simple 2 x 2 typology in which one dimension involves the degree to which the focus is on normative as opposed to empirical concerns, and the other involves the degree of formalization (see Table 1).[13]

Table 1. A Simple Typology of Electoral System Research

	primarily simple statistics or purely verbal presentations	**primarily formal models/ axiomatization**
primarily normative/ prescriptive	Mill (1861); Hoag and Hallett (1926); Lijphart (1997)	Condorcet (1785); Black (1949, 1958); Arrow (1951, 1962); May (1953); Balinski and Young (1980); Sugden (1984); Saari (1994, 1995)
primarily empirical	Rae (1969, 1971); Lijphart (1994)	Downs (1957)[14]; Taagepera and Shugart (1989); Cox (1997)

In Table 1 we have placed Rein Taagepera's work in the box that signals both empirical interests and the use of formal modeling techniques. We view this simple 2 x 2 table as realistically indicating the place of Taagepera's work in the centuries-long electoral and voting systems research tradition -- combining modeling skills with a concern for making sense of empirical patterns in the real world. Yet, what our table signally fails to indicate is the uniqueness and originality of Taagepera's approach

-- one which renders it almost *sui generis* with respect to the vast literature that preceded and follows it.[15]

At the cost of typological neatness, we propose to extract Taagepera from the lower right hand box and elevate his work to a category of its own. This gives rise to a five-fold division among the research traditions in the electoral systems area. The first one is strongly rooted in normative policy concerns (see e.g., the work of Ferdinand Hermens or of Enid Lakeman). A second one, which is also very normative in tone but rooted in mathematics, not political science, is axiomatic social choice theory, especially that branch of it deriving from Kenneth Arrow's *Social Choice and Individual Values* or Robin Farqhuarson's *Theory of Voting*. Another tradition is strongly empirical (see e.g., the work of Douglas Rae or Arend Lijphart), while a fourth, arguably the most recent mode of research, is game-theoretic in orientation (Gary Cox's *Making Votes Count* is an excellent synthesis of, and an original contribution to, this approach).

The fifth approach is best represented by the work of Rein Taagepera. Unlike game theory and rational choice approaches, which draw inspiration from economics, Taagepera's modeling work is inspired by ideas from physics. Taagepera seeks to specify a set of *fundamental* variables giving rise to "law-like" relationships with one another, and in terms of which other variables of interest can be defined. Simultaneously, however, he draws heavily on formal statistical models of large-scale aggregates, in the tradition of statistical thermodynamics.

While each of the five perspectives outlined above has made and will continue to make important contributions to the understanding of electoral systems, in the remainder of this essay we will focus more particularly on the ideas of Rein Taagepera.

The Approach to Electoral Systems of Rein Taagepera

As a scholar influenced by notions of physics as well as statistical methods, Rein Taagepera's approach to electoral systems is embedded first and foremost in a number of methodological principles or ideas about how knowledge can be achieved. These principles are often general in their application, and thus frequently go beyond Taagepera's formal contributions to electoral studies. At the same time, to the extent that Professor Taagepera has taught these principles to dozens of students of electoral systems over the years, their spread within the field can clearly be seen as an important aspect of his professional impact. In this section, we will summarize some of these methodological commandments.

(P1) Try to eliminate proper names if you can.

Generalizations about electoral systems are usually stated in terms of the proper names of those electoral systems, e.g. about what we expect in single member district (SMD) plurality systems as opposed to what we expect in list PR systems, or what differences in proportionality should obtain in comparing list PR elections under the d'Hondt method of vote aggregation to elections under the Sainte Laguë method, or elections under single-transferable-vote systems vs. elections under an alternative system. But we don't really want those proper names if there is any way to avoid them. To avoid them we take advantage of the fact that many of the key properties of any electoral system may be expressed in terms of T_E (the threshold of exclusion) or T_I (the threshold of inclusion) or T_{ave} (the average of the two). In turn, these thresholds may be expressed as functions solely of M (the average district magnitude or the number of seats in a constituency that are to be filled) or, perhaps, as functions of M and n (the number of candidates) and S (assembly size) when we look at national thresholds -- with the particular functions, of course, varying from electoral system to electoral system.[16]

Thus, in principle, we may put results of *all* district elections in *all* electoral systems into a *single* graph, where the independent variable is expressed in terms, say, of some function of the theoretical value, T_{ave}, i.e., in terms of some function of M. And in any case, we still can go from a theory with lots of proper names of electoral system variants in it to a theory that only contains a handful of key variables such as M, n and S.

Physical scientists distinguish between fundamental and derived quantities. For example, in mechanics, distance (s) and time (t) are fundamental, velocity (v) and acceleration (a) are derived in that $v = ds/dt$ and $a = d^2s/dt^2$. In the electoral systems area, in like manner, Professor Taagepera has been seeking to identify a very small set of fundamental electoral system variables such that the link between virtually any two variables of interest can be restated simply in terms of some function of the handful of variables in this set.

Thus, Professor Taagepera finds that, say, the number of seat winning parties, p, in the legislature can be approximated as:

$$p \approx (MS)^{.25}.$$

where M is mean district magnitude (the number of seats being filled in each constituency) and S is the size of the legislature, and also that the seat share of the largest party is given by:

$$s_l \approx p^{(-0.5)}.$$

Now, s_l can also be expressed as a function of M by substituting in $p \approx (MS)^{.25}$, so as to get

$$s_l \approx (MS)^{-.125}.$$

Here p as well as s_l have each been shown to be replaceable by a function of M and S. Now we may treat the first two of these variables (like velocity or acceleration in physics) as derived variables, when we treat the latter two, M and S, as fundamental values, directly observable from the data.

In like manner we would ideally want to restate other variables used in the electoral systems literature as functions of M (or perhaps, as functions of M and S). Indeed a good portion of Professor Taagepera's recent work is about doing exactly that, and then checking to see whether the results match up with theoretically derived expectations.

In the above example, we may restate the last results as

$$s_l\,(MS)^{.125} \approx 1\ .$$

Taagepera (2001) reports values of this expression for the Netherlands, Finland, the UK and New Zealand in the post WWII period ranging from .86 to 1.24, with a mean of 1.06.

Taagepera (2002) restates $s_l \approx p^{(-0.5)}$ in the form

$$s_l^2 p \approx 1.$$

He then standardizes this expression by looking at the values of $log\ (s_l^2 p)\ /\ logp$ rather than those of $s_l^2 p$ directly. Here we would expect that

$$\log (s_l^2 p)\ /\ \log p \approx 0.$$

Using the Mackie and Rose (1991, 1997) collection of electoral data, Taagepera looks at the distribution of values of this expression for a total of 952 elections. He finds a mean of -.006 with a standard deviation of 0.3. (Taagepera 2002, 25)

This "architectonic" aspect of Taagepera's work is crucial in understanding his ambitions -- to reduce the study of electoral systems to interrelationships among a small set of variables. If one does not understand that this activity of identifying fundamental variables for theory building is, in effect, what Professor Taagepera's research on electoral systems is all about, then one has not understood his research, or the key reasoning underlying the methodological choices that he makes in model development and model testing.

(P2) Before we try to make sense of the particular features of a set of data it is often useful to begin with some statistical insights about the general distribution of the data or about parameters of interest such as the mean.

In an article published in the *Journal of Theoretical Politics*, Professor Taagepera outlined an important statistical insight that could be called the *principle of ignorance* (Taagepera 1999a). For example, if we don't know anything about a (univariate) distribution of data, we can often reasonably assume that the data will be (approximately) normally distributed. A second principle employed extensively by Taagepera can be called the *principle of bounds*, namely, when we know that a variable is bounded in its values from above or below, we should estimate that variable using nonlinear models which cannot give out-of-bounds estimates. Lastly, if we combine these two principles, we find that they often work together.

In particular, if a variable is bounded to take on only positive values, an appeal to the principle of ignorance no longer suggests an (approximately) normal distribution. Rather, for a variable bounded from below with only positive values we would, absent other information, expect a (truncated) (approximately) lognormal distribution. For example, since there are physical limits on the maximum size of animals given earth's gravity (albeit with water-breathing creatures able to grow larger than the land-bound), and no animal can have a negative weight, we would expect that the distribution of animal weights would be approximately lognormal, with very few animals that are very large and lots of animals that are rather small.[17]

The combination of these two principles can give unexpected insights in the analysis of electoral or other data. In particular, for variables bounded from both above and below, Taagepera makes use of the principle of indifference to model the expected mean value of that variable as a function of its bounds so as to give rise to a square root principle that he has applied in a number of different contexts. Taagepera's most famous application of this square root law is his claim that the expected (effective) number of seats-winning parties will be given by $\sqrt{M}$, where M is again district magnitude. By appeal to the statistical principles of indifference and the method of bounds, in the form shown above, Taagepera was able to significantly extend the work of Duverger (1955) -- by eliminating proper names of voting methods (e.g., plurality, proportional representation) and replacing them with a continuum, district magnitude, in a fashion which allowed him to generate quantitative, and not merely qualitative, predictions.

(P3) If you can't answer why, then you don't have a law, you only have an empirical generalization. If you can't trace further (testable) implications and linkages to other variables, then you don't really have a useful theory.[18]

(P4) Modeling and data analysis go hand in hand.

In particular, the process of model development involves both deduction from first principles and inductive aspects in which insights are derived from eyeballing data. Moreover, this is not a one-stage process, but a repeated interactive one.

(P5) The fact that a given model fits well does not prove it's right; the fact that the model fits badly for a particular data set does not necessarily mean that it is a bad model.

The best fitting line we have for some particular data set may not really give us a law-like relationship no matter how high an r^2 we get for the fit of that line. The fact that a linear model fits well does not mean that a linear model is the correct model. On the other hand, lawful relationships may well be masked by a limited range of variation in the data, but would be revealed were there more data points over a wider range. Professor Taagepera has made this point with respect to the link between population and the size of representative assemblies. (Taagepera 1972, 1973, 1986) For the world, this relationship is well fitted by the simple function:

$$\text{assembly size of (national) parliaments} = \text{population}^{(1/3)}.$$

If we include state and regional parliaments in this model, the model still does well. Yet, if we were to look at the relationship between population and assembly size just for the lower chambers of the 50 U. S. states, the fit of the model would be atrocious. Why? Basically because for this limited data set, there is a lot of scatter in the data (relative to the range) and the population data range (0.5 to 30 million or so) among the 50 states is very restricted compared to the range of population variation among nations of the world (0.1 to 1000 million or so). The pattern of a positive increase in assembly size as population increases (according to a cube root law) would be invisible over such a limited population range.

(P6) Theoretical justification is the only real reason to include variables in a model; parsimony is usually a good enough reason to exclude variables.

Ideally, what we would like to do is to develop law-like generalizations (from a combination of deductive and inductive reasoning) and then see whether the generalizations hold, *ceteris paribus,* i.e., subject only to what appears to be a random error. Physical scientists take the idea of *ceteris paribus* very seriously. They are looking for a handful of key variables that can be used to describe (lawful) underlying relationships, recognizing that the fit of any theoretical model will never be perfect because of the failure of the *ceteris paribus* assumption. Rather than trying to control for every possible contaminating effect, physicists tend to look for powerful factors whose impact should be clearly visible.

Social scientists frequently say that in the physical sciences it is reasonable to develop *ceteris paribus* models, but that in the social sciences this is unreasonable because in the social sciences the *ceteris* is rarely if ever *paribus.* Professor Taagepera's view is that the *ceteris* may, indeed, be rarely if ever *paribus* in the social sciences but the *same is true in the natural sciences*. Especially when it's windy, feathers do not actually fall as fast as cannon balls (or may even, temporarily, rise); at the same time, that failure of *ceteris paribus* doesn't mean that there isn't a force of gravity that may be characterized in terms of a planetary gravitational constant translating into a particular rate of acceleration in the planet's gravitational field. Moreover, since the planet isn't perfectly round or uniformly dense, even the gravitational force isn't really constant -- but we can still go a very long way by pretending that it is.

(P7) It helps to have competing hypotheses/models.

In particular, it is helpful to have multiple hypotheses that are sufficiently distinct in their predictions that at least one of them can clearly be rejected once we look at data. For example, in looking at the link between the (effective) number of parties (N_s) and the number of issue dimensions (I), Professor Taagepera distinguished between the hypothesis that $N_s = I + 1$ (Taagepera and Grofman 1985; Taagepera and Shugart 1989) and the at least as plausible hypothesis that $N_s = 2^I$. Examining data, he showed that the latter hypothesis could clearly be rejected, while the former hypothesis fits the data rather well.

(P8) In looking for explanations, don't confuse correlation with causation; but do try to get straight the causality, recognizing that causal arrows may point both ways.

For example, in looking at the link between the *(effective) number of parties* (N_s) and the *number of issue dimensions* (I), while Professor Taagepera stated the hypothesis that $N_s = I + 1$ (Taagepera and Grofman 1985; Taagepera and Shugart 1989), he was also interested in considering the possibility that $I = N_s - 1$, i.e., that the nature of the party system (and the number of parties, in particular) structures the nature of cleavages, for example, by limiting the number of cleavages that can be expressed. Professor Taagepera was also quite prepared to entertain the hypothesis that N and I acted interactively, with each having an effect on the other.

(P9) In looking for explanations, be prepared to have fun.

Having had the pleasure of sitting in on a number of Professor Taagepera's lectures I can attest that he tries to motivate his students to explore ideas, and to be playful in doing so. As Professor Taagepera is fond of reminding his students, (paraphrasing Johan Huizinga) man is at least as much *homo ludens* as *homo sapiens*.

Some Helpful Indices

In very practical terms, one of the most famous dimensions of Rein Taagepera's contribution to electoral studies has been the creation and popularization of a number of empirical indices to analyze party systems and electoral outcomes. In this section, we profile two of these indices.

Number of parties

To the average observer, it might seem rather obvious how many political parties there may be running for office in, or be elected from, a given constituency or a given parliament during any given election. Yet, in cases where we have independent or non-affiliated candidates, how do we then count the real number of parties? For example, do we simply count all such independents as "parties", thus giving us what seems like an implausibly large party system? Or do we omit independents from consideration? If we follow the latter path, then where do we draw the line with minor parties that field multiple candidates but receive only a trivial share of the vote? Clearly not all parties are equal in their vote or seat shares or even their influence.

The puzzle that Professor Taagepera concerned himself with was therefore how best to characterize party systems. For example, was a four party constellation where the parties (ranked from the highest seat vote share down) had vote shares of 41, 41, 19, 19 more like a four party constellation where each party had one quarter of the vote, or a three party constellation where each party had one third of the vote, or perhaps even most like a two-party constellation where each party had approximately half the vote? The answer he gave, based on what has come to be known as the Laakso-Taagepera index, is that this constellation is actually closer to two-party politics than to either three-party or four-party politics (Laakso and Taagepera 1979).[19] Let us see why, based on the formal equation

$$N = 1 / \sum_{i=1}^{n} (s_i^2).$$

Using the inverse of another measure (known as the Herfindahl-Hirschman index of concentration), we can generate a numerical indication of dispersion, which suits our need to assess the meaning of different parties' vote shares. (Taagepera and Grofman 1981). When we plug into this equation the values of 41/120, 41/120, 19/120 and 19/120, we get 2.45 as the "effective" number of parties. In other words, given this distribution of votes, we can say there are actually two meaningful parties and less than half of a third.

While other measures of the "effective" number of parties have been proposed, none has come even close to winning the acceptance of the Laakso-Taagepera index. It would now be a rare month that passes by without at least one political science application of that index being published. But, just as Taagepera's work changed how specialists thought about the "effective" number of parties, it also had an impact on how scholars thought about disproportionality in elections. Until the work of Rein Taagepera, the standard way in political science to represent the disproportionality between a party's vote share and its seat share in parliament was in terms of a difference approach, e.g., $|v_i - s_i|$ or $(v_i - s_i)^2$. Professors Taagepera and Laakso, however, realized that a ratio approach had a more powerful appeal in terms of simplicity, thus giving rise to the Taagepera-Laakso Advantage Ratio and closely related measures (Taagepera and Laakso 1980; see also Taagepera and Shugart 1989):

$\mathbf{A_i}$ = Taagepera-Laakso Advantage Ratio
= the ratio of seat shares to vote shares for party i

and

B = *break-even point of the Taagepera-Laakso Advantage Ratio*
= vote share value at which the Advantage Ratio rises to 1

The break-even value marks the boundary where we observe a seats bonus above proportionality for the parties with more than that vote share and a less than proportional result for parties with below that vote share (at least for situations where the ratio of party seat shares to party vote shares is monotonically rising with party strength). The scattergram of A_i values versus v_i values is called by Professor Taagepera a *proportionality profile*.[20]

Electoral System Issues and Broader Issues of Constitutional Design

Taagepera's work has also dealt with the relative importance of electoral institutions as compared to other factors (such as socio-political cleavages) in shaping the structure of party competition (Taagepera and Grofman 1985) and, even more broadly, he has looked at the importance of electoral system design for democracy (Taagepera 1997, 1998c; see also Taagepera and Shugart 1989, ch. 18). Here, as in other ways, Professor Taagepera is temperate in his opinions. He is neither a dogmatist nor a zealot. While he believes that electoral systems matter, and that we do have a range of knowledge that helps us judge what will work and what will not, and which allows us to anticipate some of the most likely consequences of electoral system choice, perhaps Taagepera's key piece of advice to institutional designers is "keep it simple!" The more complex the political apparatus, the harder it is to predict its effects, and the more likely that there will be effects that are completely unanticipated.[21]

While clearly identified in political science as an "institutionalist", any fair reading of Taagepera's work reveals his deep appreciation of so-called "cultural" factors. For example, in discussing democracy he has emphasized the importance of attitudes and norms of behavior among political elites such as civility, tolerance for opposing points of view, willingness to compromise to reach political agreement, and patience (Taagepera 1997). Also, Taagepera has been far more willing to allow for the strong possibility of human error than is the case for the often hyper-rational models of game-theory oriented scholars in political science (Grofman, Mikkel and Taagepera 1999). And his work takes a long-term historical perspective that is lacking in much of political science (e.g. his very recent work on the prospects for Islamic democracy, see Taagepera 2003).

Some Final Observations

Although now technically an emeritus research professor, the reduction in Professor Taagepera's teaching load has only made it easier for him to pursue research. Professor Taagepera continues to make major contribution to the fields of electoral and party studies, publishing an average of over three articles per year just in this area over the past five years -- *in addition to* the major work he continues to do in other areas of scholarship such as Baltic studies. Professor Taagepera's most recent work on electoral system effects has dealt with some outstanding important theoretical issues. These include:

(1) How can we best move from district level analysis to national level analysis, and vice versa? (Taagepera 1998a, b)[22]

(2) Is it possible to predict the nature of the overall distribution of party vote shares from key electoral system features, or are the feasible shapes of such distributions too much affected by factors that are exogenous to the electoral system? (Taagepera 1999a, 1999b, 2001)

(3) What are the factors that determine the size of legislatures? (Taagepera 1972, Taagepera and Recchia 2002)

In my view, many of Rein Taagepera's most important contributions to the electoral and party systems literatures have been made in the past decade. Moreover, given that Professor Taagepera is presently in the rising part of his productivity curve (comparing the pre-retirement and post-retirement period), we can anticipate many more such contributions still to come.[23]

Notes

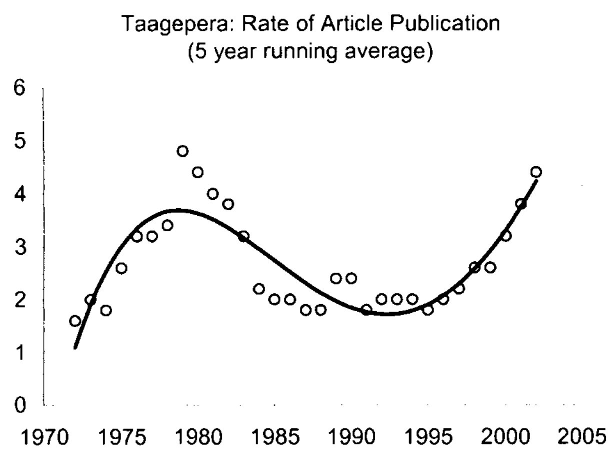
1. For example, the *d'Hondt* list PR rules corresponds to the (Thomas) *Jefferson method* for apportionment; the *greatest remainder rule* corresponds to the (Alexander) *Hamilton method*, and *Ste. Laguë* corresponds to the (Daniel) *Webster method.*
2. Of course, there was also an off again, on again U.S debate about choice of congressional apportionment formula that is very sophisticated in mathematical terms, but quite limited in the issues that it considers (Balinski and Young 1982).
3. However, Giovanni Sartori's work (see esp. Sartori 1968) in some very important ways anticipates that of Rae, as does Rokkan (1968).
4. See further discussion in Grofman (1975).
5. My review of the post-WWII history of the study of electoral systems draws heavily on Taagepera and Shugart (1989, ch. 5).
6. See e.g., Taagepera 1984, Rose 1984, Blais and Carty 1990, cf. Sartori, 1968.
7. For example, Davidson and Grofman 1994, Reynolds 1999, Grofman, Lee, Winckler and Woodall 1999, Bowler and Grofman 2000, Shugart and Wattenberg 2001, Grofman and Lijphart 2002.
8. Thus, for example, Shugart and Carey (1992), and others have begun to look at how the presence of a presidential system, the rules for electing the president, and the timing of parliamentary and presidential elections interact with choice of electoral system for parliamentary elections to produce political consequences. Taagepera and Grofman (work in progress) have suggested a tri-fold classification scheme for electoral system effects, i.e. effects on (1) representation, (2) governance, and (3) party systems.
9. See review in Grofman and Stockwell (2003).
10. See esp. Cox (1997).
11. For example, Lijphart (1992) deals primarily with the votes into seats proportionality aspects of electoral systems research (including how electoral systems can create majority parties in terms of seat share from parties that lack a majority share of the vote).
12. For example, Taagepera and Shugart (1989) have their primary focus on proportionality issues, but also deal with the issue of party proliferation which then becomes linked to political stability via the observed negative link between the (effective) number of political parties and cabinet duration. From a normative perspective, the first and third of these aspects of electoral systems research can be thought of as being in a kind of dynamic tension with one another in that giving priority to one or the other desiderata can determine one's perspectives about what electoral choices should be made.
13. While we have provided a handful of illustrative citations for each of the four cells of Table 1, many works would have to be placed in more than one cell. For example, Katz (2000: 11) makes the point that much of the work on issues related to representation blurs lines between prescription and description (see e.g., Lakeman and Lambert 1959).
14. We have placed Downs (1957) in the formal modeling category even though, by contemporary standards, his modeling efforts are quite "informal." Although his work has been viewed as having important normative implications, we place him in the empirical category because his main interest is in models that predict voter and party behavior; indeed, there is an Appendix that attempts (in an casual "armchair empiricism" sort of way) to test various of the more specific predictions of his model (cf. Grofman 1987).

15. The work to which it is closest in spirit is Theil (1970).
16. See Taagepera and Shugart 1993, Rokkan 1968, Rae 1971, Grofman 1975, Lijphart and Gibberd 1977, Lijphart 1986, Lijphart 1994.
17. For reasons neither Professor Taagepera nor I understand, this insight into the likely lognormal distribution of bounded variables seems rare in the literatures we are familiar with. Of course, in many situations, a lognormal distribution is approximately normal, and using the normal approximation to it may make perfect sense. But not always! In particular, the lognormal will be rather different than the normal if only one tail of the distribution is being eliminated, e.g., if the bounds are 0 and $+\infty$ (plus infinity). For binomial sampling distributions we will get distributions that are approximately lognormal if p is considerably lower than .5, i.e., a negatively skewed distribution.
18. In the jargon of philosophy of science: "Good theory is nomothetically embedded."
19. See, however, Taagepera (1999c).
20. Customarily Professor Taagepera plots the A_i values on the y axis and the v_i values on the x axis and draws in the line A = 1 so that we can readily see at what point on the x axis (i.e., at how large a seat share) the Advantage Ratio comes to exceed 1, indicating a bonus for the larger parties.
21. Estonia can be used to illustrate what happens when that advice is not taken. See Grofman, Mikkel and Taagepera (1999), an article on which my name appears as first author largely through the accident of typography, in that G comes before M and T.
22. This is a problem analogous to developing macroeconomics from purely microeconomic foundations.
23. The fitted loglinear curve is given by:

Taagepera publications = 66.637Ln(year) - 503.28
(R^2 = 0.03)

However, when we use five year running averages, a polynomial function (x = year) is a much better fit:

Taagepera publications = $-4E\text{-}05x^4 + 0.2952x^3 - 883.79x^2 + 1E{+}06x - 6E{+}08$
(R^2 = 0.76)



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